Chapter 3

Equivalence
A Factor Approach

3-1
If you had $1,000 now and invested it at 6%, how much would it be worth 12 years from now?

Solution

F = 1,000(F/P, 6%, 12) = $2,012.00

3-2
Mr. Ray deposited $200,000 in the Old and Third National Bank. If the bank pays 8% interest, how much will he have in the account at the end of 10 years?

Solution

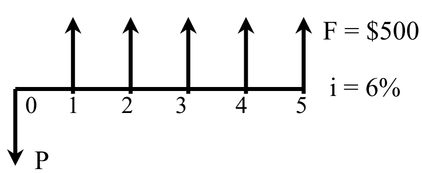
F = 200,000(F/P, 8%, 10) = $431,800

3-3
If you can earn 6% interest on your money, how much is $1,000 paid to you 12 years in the future worth to you now?

Solution

P = 1,000(P/F, 6%, 12) = $497.00

3-4
Determine the value of P using the appropriate factor.



Solution

P = F(P/F, 6%, 5) = $500(0.7473) = $373.65

3-5
Downtown is experiencing an explosive population growth of 10% per year. At the end of 2005

the population was 16,000. If the growth rate continues unabated, at the end of how many years will it take for the population to triple?

Solution

Use i = 10% to represent the growth rate.

$$48{,}000 = 16{,}000(F/P, 10\%, n)$$
$$(F/P, 10\%, n) = 48{,}000/16{,}000 = 3.000$$

From the 10% table n is 12
Note that population would not have tripled after 11 years.

3-6

If the interest rate is 6% compounded quarterly, how long (number of quarters) does it take to earn $100 interest on an initial deposit of $300?

Solution

$$i = 6\%/4 = 1\tfrac{1}{2}\%$$

$$400 = 300(F/P, 1\tfrac{1}{2}\%, n)$$
$$(F/P, 1\tfrac{1}{2}\%, n) = 400/300 = 1.333$$

From the 1½% table n = 20 quarters

3-7

The amount of money accumulated in five years with an initial deposit of $10,000, if the account earned 12% compounded monthly the first three years and 15% compounded semi-annually the last two years is closest to

a. $18,580
b. $19,110
c. $19,230
d. $1,034,285

Solution

$$F = [10{,}000(F/P, 1\%, 36)](F/P, 7.5\%, 4)$$
$$= 10{,}000(1.431)(1.075)^4$$
$$= \$19{,}110.56$$

3-8

One thousand dollars is deposited into an account that pays interest monthly and allowed to remain in the account for three years. If the annual interest rate is 6%, the balance at the end of the three years is closest to

a. $1,180

b. $1,191
c. $1,197
d. $2,898

Solution

$i = 6/12 = ½\%$ $n = (12)(3) = 36$

$F = P(1 + i)^n = 1{,}000(1.005)^{36} = \$1{,}196.68$

or using interest tables

$F = 1{,}000(F/P, ½\%, 36) = 1{,}000(1.197) = \$1{,}197$

The answer is c.

3-9
On July 1 and September 1, Abby placed $2,000 into an account paying 3% compounded monthly. How much was in the account on October 1?

Solution

$i = 3/12 = ¼\%$

$F = 2{,}000(1 + .0025)^3 + 2{,}000(1 + .0025)^1 = \$4{,}020.04$

or

$F = 2{,}000(F/P, ¼\%, 3) + 2{,}000(F/P, ¼\%, 1) = \$4{,}022.00$

3-10
The Block Concrete Company borrowed $20,000 at 8% interest, compounded semi-annually, to be paid off in one payment at the end of four years. At the end of the four years, Block made a payment of $8,000 and refinanced the remaining balance at 6% interest, compounded monthly, to be paid at the end of two years. The amount Block owes at the end of the two years is nearest to

a. $21,580
b. $21,841
c. $22,020
d. $34,184

Solution

$i_1 = 8/2 = 4\%$ $n_1 = (4)(2) = 8$ $i_2 = 6/2 = ½\%$ $n_2 = (12)(2)$

$F = [20{,}000(F/P, 4\%, 8) - 8{,}000](F/P, ½\%, 24)$

$= \$21{,}841.26$

The answer is b.

3-11

How much should Abigail invest in a fund that will pay 9%, compounded continuously, if she wishes to have \$60,000 in the fund at the end of 10 years?

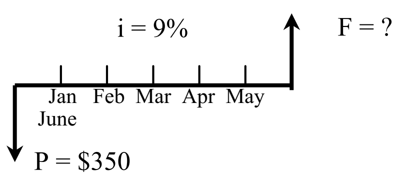
Solution

$r = 0.09$
$n = 10$

$P = Fe^{-rn} = 60{,}000e^{-.09(10)} = \$24{,}394.18$

3-12

Given :



Find: a) F
b) i_{eff}

Solution

a) $F = 350(F/P, .75\%, 6) = \$366.10$
b) $i_{eff} = (1 + i)^m - 1 = (1.0075)^{12} - 1 = 9.38\%$

3-13

Five hundred dollars is deposited into an account that pays 5% interest compounded continuously. If the money remains in the account for three years the account balance is nearest to

a. \$525
b. \$578
c. \$580
d. \$598

Solution

$F = e^{rn} = 500e^{.05(3)}$
$= \$580.91$

The answer is c.

3-14

The multistate Powerball Lottery, worth \$182 million, was won by a single individual who had purchased five tickets at \$1 each. The winner was given two choices: Receive 26 payments of \$7 million each, with the first payment to be made now and the rest to be made at the end of each of the next twenty five years; or receive a single lump-sum payment now that would be equivalent to the 26 payments of \$7 million each. If the state uses an interest rate of 4% per year, the amount of the lump sum payment is closest to

a. \$109,355,000
b. \$111,881,000
c. \$116,354,000
d. \$182,000,000

Solution

P = 7,000,000 + 7,000,000(P/F, 4%, 25)
= \$116,354,000

The answer is c.

3-15

The future worth (in year 8) of \$10,000 deposited at the end of year 3, \$10,000 deposited at the end of year 5, and \$10,000 deposited at the end of year 8 at an interest rate of 12% per year is closest to

a. \$32,100
b. \$39,300
c. \$41,670
d. \$46,200

Solution

F = 10,000(F/P, 12%, 5) + 10,000(F/P, 12%, 3) + 10,000
= \$41,670

The answer is c.

3-16

A woman deposited \$10,000 into an account at her credit union. The money was left on deposit for 10 years. During the first five years the woman earned 9% interest, compounded monthly. The credit union then changed its interest policy so that the second five years the woman earned 6% interest, compounded quarterly.

a. How much money was in the account at the end of the 10 years?
b. Calculate the rate of return that the woman received.

Solution

a) at the end of 5 years:
F = 10,000 (F/P, ¾%, 60)* = \$15,660.00   * i = 9/12 = ¾%   n = (12)(5) = 60
at the end of 10 years:
F = 15,660(F/P, 1½%, 20)** = \$21,094.02   ** i = 6/4 = 1½%   n = (4)(5) = 20

b) 10,000(F/P, i, 10) = 21,094.02
(F/P, i, 10) = 2.1094

try i = 7% (F/P, 7%, 10) = 1.967
try i = 8% (F/P, 8%, 10) = 2.159

7% < i < 8% ∴ interpolate

i = 7.75%

3-17
A young engineer wishes to buy a house but only can afford monthly payments of $500. Thirty-year loans are available at 6% interest compounded monthly. If she can make a $5,000 down payment, what is the price of the most expensive house that she can afford to purchase?

Solution

i = 6/12 = ½% n = (30)(12) = 360

P* = 500(P/A, ½%, 360) = 83,396.00
P = 83,396.00 + 5,000
P = $88,396

3-18
A person borrows $15,000 at an interest rate of 6%, compounded monthly to be paid off with payments of $456.33.

a. What is the length of the loan in years?
b. What is the <u>total</u> amount that would be required at the end of the twelfth month to payoff the entire loan balance?

Solution

a) P = A(P/A, i%, n)
15,000 = 456.33(P/A, ½%, n)
(P/A, ½%, n) = 15,000/456.33
= 32.871

From the ½% interest table n = 36 months = 6 years.

b) 456.33 + 456.33(P/A, ½%, 24) = $10,752.50

3-19
A $50,000 30-year loan with a nominal interest rate of 6% is to be repaid with payments of $299.77. The borrower wants to know how many payments, N*, he will have to make until he owes only half of the amount she borrowed initially.

Solution

The outstanding principal is equal to the present worth of the remaining payments when the payments are discounted at the loan's effective interest rate.

Therefore, let N' be the remaining payments.

$\frac{1}{2}(50{,}000) = 299.77(P/A, \frac{1}{2}\%, N')$
$(P/A, \frac{1}{2}\%, N') = 83.397$
$N' = 108.30 \approx 108$ From i = ½% table
So, $N^* = 360 - N'$
$= 252$ payments

3-20

J.D. Homeowner has just bought a house with a 20-year, 9%, $70,000 mortgage on which he is paying $629.81 per month.

a) If J.D. sells the house after ten years, how much must he pay the bank to completely pay off the mortgage at the time of the 120th payment?
b) How much of the first $629.81 payment on the loan is interest?

Solution

a) $P = 120^{th}$ payment + PW of remaining 120 payments
$= 629.81 + 629.81(P/A, \frac{3}{4}\%, 120)$
$= \$49{,}718.46$

b) $\$70{,}000 \times 0.0075 = \$525$

3-21

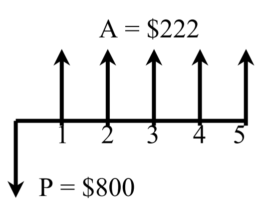
While in college Ellen received $40,000 in student loans at 8% interest. She will graduate in June and is expected to begin repaying the loans in either 5 or 10 equal annual payments. Compute her yearly payments for both repayment plans.

Solution

5 YEARS	10 YEARS
$A = P(A/P, i, n)$	$A = P(A/P, i, n)$
$= 40{,}000(A/P, 8\%, 5)$	$= 40{,}000(A/P, 8\%, 10)$
$= \$10{,}020.00$	$= \$5{,}960.00$

3-22

Given:



Find: i%

Solution

$$P = A(P/A, i\ \%, 5)$$
$$800 = 222(P/A, i\%, 5)$$
$$(P/A, i\ \%, 5) = 800/222 = 3.6$$

From the interest tables i = 12%

3-23

How much will accumulate in an Individual Retirement Account (IRA) in 15 years if $5,000 is deposited in the account at the end of each quarter during that time? The account earns 8% interest, compounded quarterly. What is the effective interest rate?

Solution

$i = 8/4 = 2\%$ $n = (4)(15) = 60$

F = 5,000 (F/A, 2%, 60) = $570,255.00

Effective interest rate = $(1 + .02)^4 - 1 = 8.24\%$

3-24

Suppose you wanted to buy a $180,000 house. You have $20,000 cash to use as the down payment. The bank offers to loan you the remainder at 6% nominal interest. The term of the loan is 20 years. Compute your monthly loan payment.

Solution

Amount of loan: $180,000 - $20,000 = $160,000

i = 6/12 = ½% per month n = (12)(20) = 240

A = 160,000(A/P, ½%, 240) = $1,145.60 per month

3-25

To offset the cost of buying a $120,000 house, James and Lexie borrowed $25,000 from their parents at 6% nominal interest, compounded monthly. The loan from their parents is to be paid off in five years in equal monthly payments. The couple has saved $12,500. Their total down payment is therefore $25,000 + 12,500 = $37,500. The balance will be mortgaged at 9% nominal interest, compounded monthly for 30 years. Find the combined monthly payment that the couple will be making for the first five years.

Solution

Payment to parents:

25,000(A/P, ½%, 60) = $482.50

Borrowed from bank: 120,000 – 37,500 = $82,500

Payment to bank

82,500(A/P, ¾%, 360) = $664.13

Therefore, monthly payments are 482.50 + 664.13 = $1,146.63

3-26

If $15,000 is deposited into a savings account that pays 4% interest compounded quarterly, how much can be withdrawn each quarter for five years?

Solution

A = 15,000(A/P, 1%, 20)
= $831.00 per quarter

3-27

How much will Thomas accumulate in a bank account that pays 5% annual interest compounded quarterly if he deposits $800 at the end of each quarter for 7 years?

Solution

F = 800(F/A, 1.25%, 28)
= $25,824.00

3-28

A consumer purchased new furniture by borrowing $1,500 using the store's credit plan which charges 18% compounded monthly.

(a) What are the monthly payments if the loan is to be repaid in 3 years?
(b) How much of the first payment is interest?
(c) How much does the consumer still owe just after making the 20th payment?

Solution

(a) A = 1,500(A/P, 1½%, 36)
= $54.30 per month

(b) Interest payment = principal × interest rate
Interest payment = 1,500 × 0.015
= $22.50

(c) P = 54.30(P/A, 1½%, 16)
= $767.31

3-29

A company borrowed $20,000 at 8% interest. The loan was repaid according to the following schedule. Find X, the amount that will pay off the loan at the end of year 5.

Year	Amount
1	$4,000
2	4,000
3	4,000
4	4,000
5	X

Solution

20,000 = 4,000(P/A, 8%, 4) + X(P/F, 8%, 5)
6,752 = X(.6806)
X = 6,752/.6806
= $9,920.66

3-30
The local loan shark has loaned you $1,000. The interest rate you must pay is 20%, compounded monthly. The loan will be repaid by making 24 equal monthly payments. What is the amount of each monthly payment?

Solution

i = 20/12 = 1-2/3%

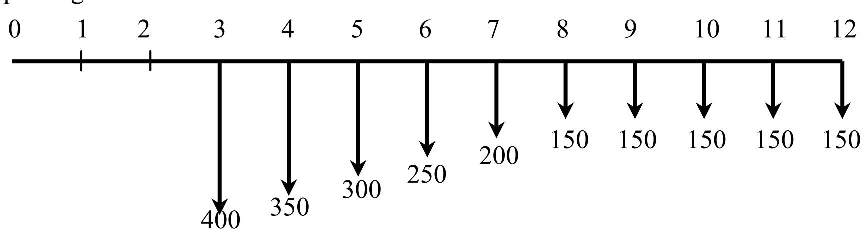
A = 1,000(A/P, 1-2/3%, 24)

There is no 1-2/3% compound interest table readily available. Therefore the capital recovery factor must be calculated.

(A/P, 1.666%, 24) = $[0.01666(1.01666)^{24}]/[(1.01666)^{24} - 1]$ = 0.050892

A = 1,000(0.050892) = $50.90

3-31
Find the uniform annual equivalent for the following cash flow diagram if i = 10%. Use the appropriate gradient and uniform series factors.



Solution

P^1 = [400(P/A, 10%, 6) - 50(P/G, 10%, 6)](P/F, 10%, 2) = $1,039.45

P^2 = [150(P/A, 10%, 4)](P/F, 10%, 8) = $221.82

P = 1,039.45 + 221.82 = $1,261.27

A = 1,261.27(A/P, 10%, 12) = $185.15

3-32

You need to borrow $10,000 and the following two alternatives are available at different banks: a) pay $2,571 at the end of each year for 5 years, starting at the end of the first year (5 payments in total.) or b) pay $207.58 at the end of each month, for 5 years, starting at the end of the first month. (60 payments in total). On the basis of the interest rate being charged in each case, which alternative should you choose?

Solution

Alternative a:

10,000 = 2,571(P/A, i, 5)
(P/A, i, 5) = 10,000/2,571
= 3.890

From the interest tables, $i \approx 9\%$ The nominal annual rate = effective rate.

Alternative b:

10,000 = 207.58(P/A, i, 60)
(P/A, i, 60) = 10,000/ 207.58
= 48.174

From the interest tables i = .75% The nominal annual interest rate is: 12 × .75 = 9% but the effective interest rate is $(1 + 0.0075)^{12} - 1 = 9.38\%$

Therefore, choose the first alternative.

3-33

Using a credit card, Ben Spendthrift has just purchased a new stereo system for $975 and will be making payments of $45 per month. If the interest rate is 18% compounded monthly, how long will it take to completely pay off the stereo?

Solution

i = 18/12 = 1½%

975 = 45(P/A, 1½%, n)
(P/A, 1½%, n) = 975/45
= 21.667

From the 1½% table n is between 26 and 27 months. The loan will not be completely paid off after 26 months. Therefore the payment in the 27th month will be smaller.

3-34

Explain in one or two sentences why (A/P, i%, infinity) = i.

Solution

In order to have an infinitely long annuity (A) series, the principal (present sum P) must never be reduced. For this to occur, only the interest earned each period may be removed. If more than the interest earned is removed it would decrease the original P so that less interest is available the next period.

3-35

An engineer on the verge of retirement has accumulated savings of $100,000 that are in an account paying 6% compounded quarterly. The engineer wishes to withdraw $6,000 each quarter. For how long can she withdraw the full amount?

Solution

i = 6/4 = 1½%

6,000 = 100,000(A/P, 1½%, n)
(A/P, 1½%, n) = 0.0600

From the 1½% table n = 19 quarters or 4¾ years

Note: This leaves some money in the account but not enough for a full $6,000 withdrawal.

3-36

If $3,000 is deposited into an account paying 13.5% interest how much can be withdrawn each year indefinitely?

Solution

$(A/P, i, \infty) = i$

$A = (P)(i)$

$A = 3{,}000 \times .135 = \$405$

3-37

A grandfather gave his grandchild $100 for his 10th birthday. The child's parents talked him into putting this gift into a bank account so that when he had grandchildren of his own he could give them similar gifts. The child lets this account grow for 50 years, and it has $100,000. What was the interest rate of the account?

a. 14.0%
b. 14.8%
c. 15.8%
d. 15.0%

Solution

$\$100{,}000 = \$100(1 + i)^{50}$

$i = 14.8\%$

The answer is c.

3-38

The annual cost to maintain a cemetery plot is \$75. If interest is 6% how much must be set aside to pay for perpetual maintenance?

a. \$1,150
b. \$1,200
c. \$1,250
d. \$1,300

Solution

$$\begin{aligned} P &= 75(P/A, 6\%, \infty) \\ &= 75(1/.06) \\ &= \$1,250 \end{aligned}$$

The answer is c.

3-39

Henry Fuller purchases a used automobile for \$6,500. He wishes to limit his monthly payment to \$200 for a period of two years. What down payment must he make to complete the purchase if the interest rate is 9% on the loan?

Solution

$$\begin{aligned} P &= P' + A(P/A, \tfrac{3}{4}\%, 24) \\ 6,500 &= P' + 200(21.889) \\ P' &= 6,500 - 4,377.80 \\ &= \$2,122.20 \leftarrow \text{down payment} \end{aligned}$$

3-40

To start business, ECON ENGINEERING has just borrowed \$500,000 at 6%, compounded quarterly, which will be repaid by quarterly payments of \$50,000 each, with the first payment due in one year. How many quarters after the money is borrowed is the loan fully paid off?

Solution

$i = 6/4 = 1\tfrac{1}{2}\%$

$$\begin{aligned} 500,000 &= 50,000(P/A, 1\tfrac{1}{2}\%, n)(P/F, 1\tfrac{1}{2}\%, 3) \\ (P/A, 1\tfrac{1}{2}\%, n) &= 500,000/[50,000(.9563)] \\ &= 10.46 \end{aligned}$$

From the 1½% table n = 12 payments plus 3 quarters without payments equal 15 quarters before loan is fully paid off.

3-41
A bank is offering a loan of $20,000 with an interest rate of 12%, payable with monthly payments over a four year period.

a. Calculate the monthly payment required to repay the loan.
b. This bank also charges a loan fee of 4% of the amount of the loan, payable at the time of the closing of the loan (that is, at the time they give the money to the borrower). What is the effective interest rate they are charging?

Solution

a. The monthly payments:

$i = 12/12 = 1\%$, $n = (12)(4) = 48$

20,000(A/P, 1%, 48) = $526.00

b. Actual money received = P = 20,000 - 0.04(20,000) = $19,200

A = $526.00 based on $20,000

Recalling that A = P(A/P, i, n)
526 = 19,200(A/P, i, 48)
(A/P, i, 48) = 526/19,200
= 0.02739

for i = 1¼% the A/P factor @ n = 48 = 0.0278

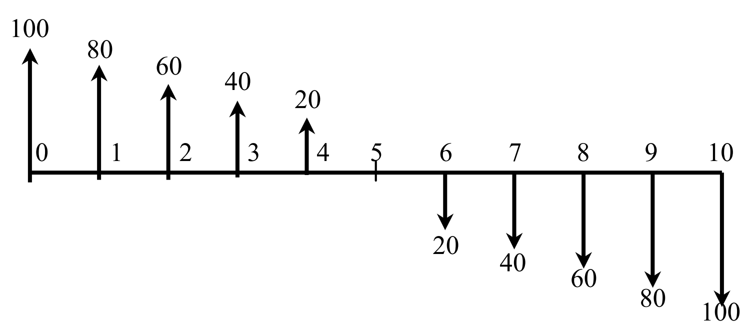
for i = 1% the A/P factor @ n = 48 = 0.0263

by interpolation $i \approx 1 + ¼[(.0263 - .02739)/(.0263 - .0278)]$

$i \approx 1.1817\%$

Therefore $i_{eff} = (1 + 0.011817)^{12} - 1 = 0.1514 = 15.14\%$

3-42
Find the present equivalent of the following cash flow diagram if i = 18%.



Solution

P = 100 + 80(P/A, 18%, 10) - 20(P/G, 18%, 10) = $172.48

3-43

The annual worth of a quarterly lease payment of $500 at 8% interest is nearest to

a. $2,061
b. $2,102
c. $2,123
d. $2,253

Solution

Lease payments are beginning-of-period cash flows.

First find the present worth of the quarterly payments at 8/4 = 2%.

P = 500 + 500(P/A, 2%, 3) = $1,941.95

$A = 1{,}941.95(1 + .02)^4$
$= \$2{,}102$

The answer is b.

3-44

A 30-year mortgage of $100,000 at a 6% interest rate had the first payment made on September 1, 1999. What amount of interest was paid for the 12 monthly payments of 2002?

Solution

Monthly payment A = 100,000(A/P, ½%, 360) = $599.55

Interest periods remaining Jan 1, 2002 = 331
Jan 1, 2003 = 319

P' = 599.55(P/A, ½%, 331) = 599.55(161.624) = 96,901.67

P" = 599.55(P/A, ½%, 319) = 599.55(159.257) = 95,482.53

Interest = 599.55(12) - (96,901.67 – 95,482.53) = $5,775.46

3-45

Holloman Hops has budgeted $300,000 per year to pay for labor over the next five years. If the company expects the cost of labor to increase by $10,000 each year, what is the expected cost of the labor in the first year, if the interest rate is 10%?

Solution

A′ = $300,000
A′ = A + 10,000(A/G, 10%, 5)

300,000 = A + 10,000(1.81)
A = $281,900 first year labor cost

3-46

For the cash flow shown below, determine the value of *G* that will make the future worth at the end of year 6 equal to $8,000 at an interest rate of 12% per year.

Year	0	1	2	3	4	5	6
Cash Flow	0	600	600 + *G*	600 + 2*G*	600 + 3*G*	600 + 4*G*	600 + 5*G*

Solution

P = 8,000(P/F, 12%, 6)
= 8,000(.5066)
= $4,052.80

4,052.80 = 600(P/A, 12%, 6) + *G*(P/G, 12%, 6)
4,052.80 = 600(4.111) + *G*(8.930)
G = $177.63

3-47

Big John Sipes, owner of Sipe's Sipping Shine, has decided to replace the distillation machine his company now uses. After some research, he finds an acceptable distiller that cost $62,500. The current machine has approximately 1200 lbs of copper tubing that can be salvaged and sold for $4.75/lb to use as a down payment on the new machine. The remaining components of the distillation machine can be sold as scrap for $3,000. This will also be used to pay for the replacement equipment. The remaining money will be obtained through a 10-year mortgage with quarterly payments at an interest rate of 8%. Determine the quarterly payment required to pay off the mortgage. Also determine the effective interest rate on the loan.

Solution

i = 8/4 = 2% n = (4)(10) = 40

P = 62,500 - (1,200 x 4.75) - 3,000 = 53,800

A = 53,800(A/P, 2%, 40)

= 53,800(.0366)

= $1,969

$i_{eff} = (1 + .02)^4 - 1 = 8.24\%$

3-48

Ray Witmer, an engineering professor at UTM, is preparing to retire to his farm and care for his cats and dogs. During his many years at UTM he invested well and has a balance of $1,098,000 in his retirement fund. How long will he be able to withdraw $100,000 per year beginning today if his account earns interest at a rate of 4% per year?

Solution

A = $100,000 P = 1,098,000 - 100,000* = $998,000 *First withdrawal is today

100,000 = 998,000(A/P, 4%, n)
(A/P, 4%, n) = 100,000/998,000
(A/P, 4%, n) = .1002

Searching i = 4% table, n = 13 additional years of withdrawals, 14 total years of withdrawals

3-49
Abby W. deposits $75 per month into an account paying 9% interest for two years to be used to purchase a car. The car she selects costs more than the amount in the account. She agrees to pay $125 per month for two more years at 12% interest, and also uses a gift from her uncle of $375 as part of the down payment. What is the cost of the car to the nearest dollar?

Solution

i = 9/12 = ¾% n = (12)(2) = 24

F = 75(F/A, ¾%, 24)
= 75(26.189)
= $1,964.18 ← Amount in account

i = 12/12 = 1% n = (12)(2) = 24

P = 125(P/A, 1%, 24)
= 125(21.243)
= $2,655.38 ← Amount repaid by loan

Total = 1,964.18 + 2,655.38 + 375
= $4,994.56 = $4,995 ← Cost of automobile

3-50
The amount required to establish an endowment to provide an annual scholarship of $20,000 requires a deposit into an account paying 8% is nearest to

a. $1,600
b. $25,000
c. $250,000
d. $500,000

Solution

P = 20,000(P/A, 8%, ∞)
P = 20,000(1/.08)
P = $250,000

The answer is c.

3-51

Abby Motors offers to sell customers used automobiles with \$400 down and payments for 3 years of \$215 per month. If the interest rate charged to its customers is 12%, the cost of the automobile is nearest to

a. \$1,760
b. \$2,160
c. \$6,475
d. \$6,875

Solution

i = 12/12 = 1%   n = (12)(3) = 36

P = 400 + 215(P/A, 1%, 36)
= \$6,873.01

The answer is d.

3-52

A tractor is bought for \$125,000. What is the required payment per year to completely pay off the tractor in 20 years, assuming an interest rate of 6%?

a. \$ 1,150
b. \$ 5,550
c. \$10,900
d. \$12,750

Solution

A = 125,000(A/P, 6%, 20)
= \$10,900

The answer is c.

3-53

Jason W. bought a Mercedes when he came to UTM as an engineering student. The Mercedes was purchased by taking a loan that was to be paid off in 20 equal, quarterly payments. The interest rate on the loan was 12%. Four years later, after Jason made his 16th payment, he got married (no more dating!) and sold the Mercedes to his buddy Houston S. Houston made arrangements with Jason's bank to refinance the loan and to pay Jason's unpaid balance by making 16 equal, quarterly payments at the same interest rate that Jason was paying. Three and ¼ years later, after Houston made his 13th payment, he flunked out of UTM (too many dates!) and sold the car to Jeff M. Jeff paid the bank \$2,000 cash (he had a good summer job!) to pay the loan balance. What was the amount of Jason's loan to purchase the Mercedes when it was new?

Solution

i = 12/4 = 3%

Jason W.

A = P(A/P, 3%, 20)
A = P(.0672) Quarterly payment for Jason

Jason owes

P = .0672P(P/A, 3%, 4) Present worth of four remaining payments
= .0672P(3.717)
= .2498P

Houston S.

A = .2498P(A/P, 3%, 16) Quarterly payment for Houston
= .2498P(.0796)
= .0199P

Jeff M.

P = .0199P(P/A, 3%, 3) Present worth of three remaining payments
= .0199P(2.829)
= .0563P

Set final payment equal to present worth of remaining payments

2000 = .0562P

P = $35,556.75

3-54

A mortgage of $50,000 for 30 years, with monthly payments at 6% interest is contemplated. At the last moment you receive news of a $25,000 gift from you parents to be applied to the principal. Leaving the monthly payments the same, what amount of time will now be required to pay off the mortgage and what is the amount of the last payment (assume any residual partial payment amount is added to the last payment)?

Solution

i = 6/12 = ½% n = (12)(30) = 360 periods

A = 50,000(A/P, ½%, 360)
= $299.77 monthly payment (Note: For more accurate answer $\frac{1}{P/A}$ factor was used)

After reduction of P to 25,000

25,000 = 299.77(P/A, ½%, n)
(P/A, ½%, n) = 83.40

Try n = 104 periods: P/A = 80.942
Try n = 120 periods: P/A = 90.074

Interpolate, n = 108.31 periods = 9.03 years

At 9 years (108 periods): P = 299.77(P/A, ½%, 108)
= 299.77(83.2934)

= $24,968.87

Residual = 25,000 - 24,968.87 = $31.13

Last Payment = Value of residual at time of last payment + last payment
= 31.13(F/P, ½%, 108) + 299.77
= $353.12

3-55

A person would like to retire 10 years from now. He currently has $32,000 in savings, and he plans to deposit $300 per month, starting next month, in a special retirement plan. The $32,000 is earning 8% interest, while the monthly deposits will pay him 6% nominal annual interest. Once he retires, he will deposit the total of the two sums of money into an account that he expects will earn a 4% annual interest rate. Assuming he will only spend the interest he earns, how much will he collect in annual interest, starting in year 11?

Solution

Savings:

F = 32,000(F/P, 8%, 10) = $69,086

Monthly deposits:

i = 6/12 = ½%   n = (12)(10) = 120

F = 300(F/A, ½%, 120) = $49,164

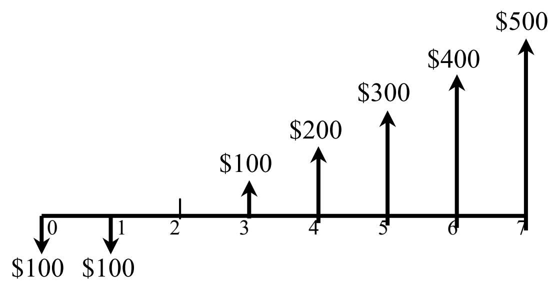
The total amount on deposit at the end of year 10 is

F_T = 69,086 + 49,164 = $118,250

The interest to collect per year = 118,250 × 0.04 = $4,730

3-56

Using the tables for uniform gradients, solve for the future value at the end of year 7 if i = 10%.



Solution

PV = 100(P/G, 10%, 7) - 100(P/A, 10%, 7) - 100
= $689.50

FV = 689.5(F/P, 10%, 7)
= $1,343.84