Chapter 3: Factors and Products:

Skill Builder: Some Divisibility Rules

We can use rules to find out if a number is a factor of another.

- ❖ To find out if 2, 5, or 10 is a factor look at the last digits in the number
  - 2 is a factor when the last digit is even
  - 5 is a factor of any number that ends in 0 or 5
  - 10 is a factor when the last digit is 0
- ❖ To find out if 3 is a factor add the digits in the number to see if it is a factor of the sum

Section 3.1 Factors and Multiples of Whole Numbers:

We can factor 24 in different ways:

A **prime number** has only two factors: itself and 1

The **Prime factorization** of 24 is:

The **prime factorization** of a number is the product of its prime factors

Example 1: Write the prime factorization for each of the following

(A) 50 (B) 72 (C) 324

The **Greatest Common Factor (GCF)** of two numbers is the greatest factor the numbers have in common:

Example 2: Find the GCF between

(A) 9 and 15

(B) 12 and 18

Note:	Prime factorization can be used to find the GCF Use the **common primes** and the **lowest power**

(C) 24 and 36

(D) 224 and 240

Example 3: Two ribbons are 24cm and 42cm long. All pieces from both ribbons are to be cut into equal pieces. What is the greatest possible length of each piece?

The **multiples** of a number are found by multiplying it by a whole number.

For example the multiples of 12 are:

The **Least Common Multiple (LCM)**, is the lowest number that is divisible by both given numbers.

Example 4: Find the LCM between

(A) 4 and 6

(B) 12 and 30

> Note: Prime factorization can be used to find the LCM
> Use **all the primes** and the **highest power**

(C) 30 and 72

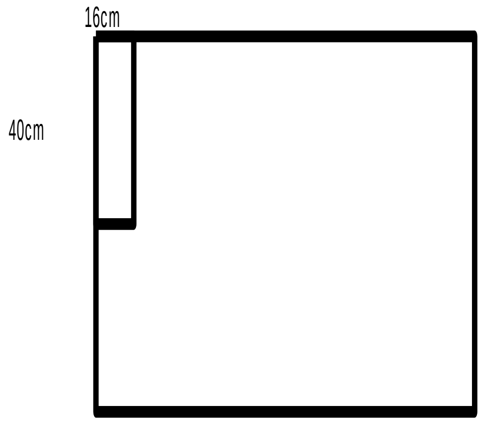
Example 5: Hamburger patties come in packages of 8. Buns come in packages of 6. What is the least number of hamburgers that can be made with no patties or buns left over?

> Questions page 140
> #3a,4ae,5bce,6af,
> 8abe, 10abd

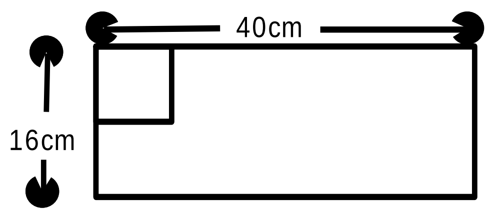
Solving Problems involving GCF and LCM

Example 6:

(A) What is the side length of the smallest square that could be tiled with rectangles that measure 16cm by 40cm? Assume the rectangles cannot be cut.



(B) What is the side length of the largest square that could be used to tile a rectangle that measures 16cm by 40cm? Assume the rectangles cannot be cut.



Questions page 140
#9a,11a,13,16ace,
17,19a

3.2 Perfect Squares, Perfect Cubes & their Roots:

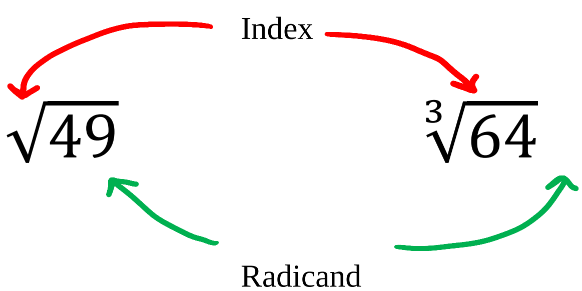
A **Perfect Square** is the square of a whole number.

- For example 16 and 100 are perfect squares. Since $4^2 = 16 \; and \; 10^2 = 100$
- We say, 4 is the square root of 16
- We write, $\sqrt{16} = 4$

A **Perfect Cube** is the cube of a whole number.

- For example 8 and 125 are perfect cubes. Since $2^3 = 8 \; and \; 5^3 = 125$
- We say, 2 is the cube root of 8
- We write, $\sqrt[3]{8} = 2$

Terminology:
A **radical** is an expression that has a square root, cube root, etc.



Note: For square root the index is 2,
it is something that is known and not written in.

Example 1: Determine the square root

(A) $\sqrt{81}$ (B) $\sqrt{225}$ (C) $\sqrt{576}$

Example 2: Determine the cube root

(A) $\sqrt[3]{27}$ (B) $\sqrt[3]{216}$ (C) $\sqrt[3]{729}$

Using roots to solve a problem

Example 3:

(A) The volume of a cube is 512 cubic feet. What is the length of each side of the cube?

(B) The volume of a rectangular prism is 2535 m^3. The prism has a square cross section and a height of 15m. What is the length and width of the prism?

Questions page 146-147
#4, 5, 7, 8, 14, 17

Chapter 4: Roots and Powers:

Section 4.2 Irrational Numbers:

Set of Real Numbers

These are **rational** numbers	These are **irrational** numbers
$\sqrt{100}$ $\sqrt{0.25}$ $\sqrt[3]{8}$ 0.5	$\sqrt{0.24}$ $\sqrt[3]{9}$ $\sqrt{2}$
$\sqrt{\frac{9}{64}}$ $\frac{5}{6}$ 0.8^2 $\sqrt[5]{-32}$	$\sqrt{\frac{1}{3}}$ $\sqrt[4]{12}$ π

- **Rational Numbers** have exact answers. They can be written as a whole number, integer, fraction, repeating decimal or a terminating decimal.
- **Irrational Numbers,** the decimal representation does not terminate or repeat

There are 6 number sets:

Symbol

N Natural = {1,2,3,4,...} the counting numbers greater than zero

W Whole = {0,1,2,3,...} the counting numbers including zero

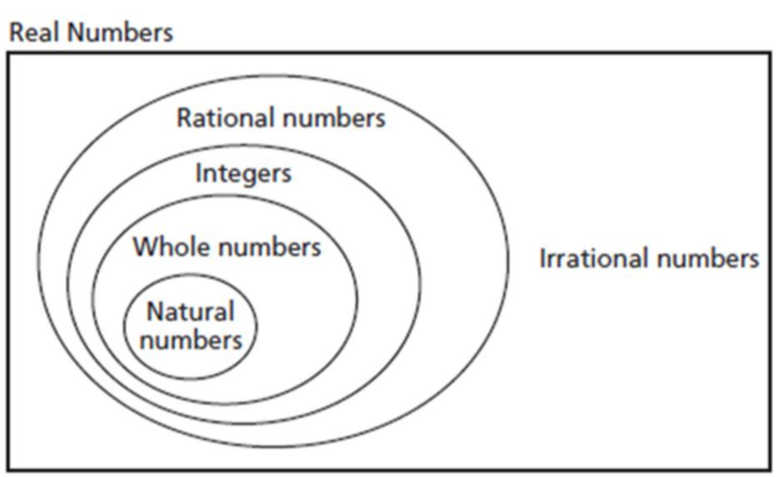
I Integers = { . . .,-3,-2,-1,0,1,2,3,...} all positive and negative numbers (no decimal or fractions)

Q **Rationals** can written as a fraction or a decimal that ends (terminates) or repeats.

Ex. $5, \frac{1}{3}, 0.75, 0.\bar{3}, \sqrt{16}, \sqrt[3]{8}$

$\bar{Q}$ **Irrationals** are numbers that in decimal form do not repeat or do not end.

Ex. $\pi, \sqrt{2}, 1.76435 \ldots$



Example 1: Classify (identify) which set(s) each number belongs to.

(A) -2

(B) $\frac{2}{3}$

(C) $\sqrt{9}$

(D) $\sqrt{5}$

Ordering Numbers from Least to Greatest.

Example 2: Rewrite from least to greatest $\frac{2}{3}, \sqrt[3]{13}, \sqrt[4]{27}, 2$

Questions page 211
#3-6, 12, 14

<u>Section 4.3 Mixed and Entire Radicals:</u>

Examples of **Radicals**: $\sqrt{9}$, $\sqrt[3]{8}$, $\sqrt[4]{16}$, $\sqrt{12}$

Radicals have a Root symbol:

Index

$$\sqrt[n]{x}$$

Radicand

Note: For **square root** the index is **2**,
it is something that is known and not written in.

Entire Radical: All radicals under the root sign ex. $\sqrt{16}$, $\sqrt{200}$, $\sqrt[3]{80}$, $\sqrt[4]{32}$,

Mixed Radical: has number and root part, ex. $3\sqrt{2}$, $10\sqrt{7}$, $2\sqrt[3]{10}$

Multiplication Property: $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Simplifying Square Roots:

Example 1: $\sqrt{36}$

This can also be broken down using the multiplication property.

$\sqrt{36}$ 36 has two perfect square factors

Note:
⇨Not all radicals are perfect square numbers.
⇨To simplify non-perfect square radicands, find the highest perfect square factor of the radicand. {4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169,…}

Example 2: Simplify

(A) $\sqrt{12}$ (B) $\sqrt{90}$ (C) $\sqrt{32}$

Example 3: Write as an entire radical

(A) $4\sqrt{3}$ (B) $7\sqrt{2}$

Questions page 218
#4, 5bdfg, 10

Simplifying Cube Roots:

Perfect Cubes:
{8, 27, 64, 125,…}

Example 1: Simplify

(A) $\sqrt[3]{8}$

(B) $\sqrt[3]{-27}$

(C) $\sqrt[3]{24}$

(D) $\sqrt[3]{-54}$

Example 2: Write as an entire radical

(A) $5\sqrt[3]{2}$

(B) $-2\sqrt[3]{5}$

Questions page 218
#11, 12acegi

Simplifying Radicals:

Square Root: $\sqrt{32}$

Cube Root: (A) $\sqrt[3]{40}$ (B) $\sqrt[3]{-128}$

4^{th} Root: (A) $\sqrt[4]{16}$ (B) $\sqrt[4]{32}$

5^{th} Root: (A) $\sqrt[5]{32}$ (B) $\sqrt[5]{64}$

Questions page 218-219
#14,15,17ab, 18ac,#20

Review:
page 149 #1a, 2a, 3a, 6a, 7a, 10
page 221 #1, 4, 7b, 9,11

Section 4.4 Fractional Exponents and Radicals:

Using your calculator, find

(A) $25^{\frac{1}{2}}$ (B) $8^{\frac{1}{3}}$

- Raising a number to the exponent $\frac{1}{2}$ is the same as finding **square** root.
- Raising a number to the exponent $\frac{1}{3}$ is the same as finding **cube** root.

Example 1, rewrite as a radical

(A) $3^{\frac{1}{2}} =$ (B) $4^{\frac{1}{3}} =$

Example 2, rewrite as a fractional power

(A) $\sqrt{10} =$ (B) $\sqrt[3]{21} =$

Powers with Rational Exponents:

$\sqrt[I]{b^E} = b^{\frac{E}{I}}$ I ⇨ index
or b ⇨ radicand
$\left(\sqrt[I]{b}\right)^E = b^{\frac{E}{I}}$ E ⇨ exponent

Example 3, rewrite as a radical

(A) $8^{\frac{3}{4}} =$ (B) $7^{\frac{3}{2}} =$

Example 4, rewrite as a fractional power

(A) $\sqrt{13^5} =$ (B) $\left(\sqrt[3]{5}\right)^2 =$

Note: handling fractions ➲ Apply power to the numerator and denominator

Example: Evaluate

(A) $\left(\frac{3}{5}\right)^2$

(B) $\left(\frac{16}{25}\right)^{\frac{1}{2}}$

(C) $\left(\frac{8}{27}\right)^{\frac{4}{3}}$

Questions page 227
#3bce,5,6ad,7,8,10ae,11ac

NOTE:

- Power of zero: $b^0 = 1$
- Converting a decimal to a fraction:

  Example: (A) 0.4 (B) 1.5

- Be able to write out the steps when evaluating:

  Example: (A) $27^{\frac{4}{3}}$ (B) $25^{1.5}$

Questions page 227-228
#4ab,10cf,11d,12,16a,17,
18,19

Section 4.5 Negative Exponents and Reciprocals:

- Making a connection to negative powers.

Complete
A) $2^1 =$
B) $2^2 =$
C) $2^3 =$

Complete	Write as a fraction	Write as fraction with base 2
$2^{-1} =$		
$2^{-2} =$		
$2^{-3} =$		

Complete	rewrite as with base 2
A) $\frac{1}{2^{-1}} =$	
B) $\frac{1}{2^{-2}} =$	
C) $\frac{1}{2^{-3}} =$	

NOTE:

> Switching the term from top to bottom, and vice versa, changes the sign on the power

Ex. (A) $\frac{1}{2^{-4}} =$

(B) 2^{-5}

Powers with Negative Exponents:

$$x^{-n} = \frac{1}{x^n} \quad \text{or} \quad \frac{1}{x^{-n}} = x^n$$

NOTE: 1) $x^n y^{-m} = \dfrac{x^n}{y^m}$

Only the term with the negative power moves

2) a negative power is not a negative number

$(-2)^3$ verses $(2)^{-3}$

Example: Rewrite each as a positive exponent and then evaluate.

(A) 3^{-2} (B) 4^{-3} (C) $\frac{1}{5^{-2}}$

(D) $\left(\frac{2}{3}\right)^{-3}$

To make outside power positive,
Take reciprocal of inside fraction

(E) $\left(\frac{4}{5}\right)^{-2}$

(F) $(0.3)^{-4}$

Example: Given that $3^8 = 6561$, what is 3^{-8}?

Questions page 233 #5,6,7,8

Negative Fractional Exponents:

Examples:

(A) $16^{\frac{-1}{2}}$

(B) $8^{\frac{-2}{3}}$

(C) $(-27)^{\frac{5}{3}}$

(D) $\left(\frac{9}{16}\right)^{\frac{-3}{2}}$

(E) $(0.04)^{\frac{-1}{2}}$

(F) $(0.25)^{-1.5}$

Questions page 233-234
#9,12,13,15

Section 4.6 Applying the Exponent Laws :

(I) Multiplication Property

Evaluate: $2^3 \cdot 2^2$

Multiplication Property:

$a^m \bullet a^n =$

(II)Division Property

Evaluate: $\frac{2^6}{2^2}$

Division Property:

$\frac{a^m}{a^n} =$

(III)Power of a power Property

Evaluate: $(2^3)^2$

Power Property:

$(a^m)^n =$

(IV)Power of Product

Evaluate: $(2 \cdot 3)^2$

Power of Product:

$(ab)^m =$

(V)Power of Quotient

Evaluate: $\left(\frac{2}{3}\right)^2 =$

Power of Quotient:

$\left(\frac{a}{b}\right)^m =$

(VI) Zero Exponent

Evaluate: $\frac{2^4}{2^4} =$

Zero Exponent:

$x^0 =$

NOTE: Simplest form:

- Has **NO** negative power
- Has a decimal or a fraction, not both

Example 1: Simplify by expressing as a single power.

(A) $0.3^{-3} \cdot 0.3^{5}$

(B) $\left[\left(-\frac{3}{2}\right)^{-4}\right]^{2} \cdot \left[\left(-\frac{3}{2}\right)^{2}\right]^{3}$

(C) $\frac{(1.4^{3})(1.4^{4})}{1.4^{-2}}$

(D) $\left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}}\right)^{6}$

Questions page 241-242
#3ab,4bcd,5b,6bc,7bc

Example 2: Simplify

(a) $(x^3y^2)(x^2y^{-4})$

(b) $\frac{10a^5b^3}{2a^2b^{-2}}$

Example 3: Simplifying Algebraic expressions with Rational Exponents:

(A) $(8a^3b^6)^{\frac{1}{3}}$

(B) $\left(x^{\frac{3}{2}}y^2\right)\left(x^{\frac{1}{2}}y^{-1}\right)$

(c) $\dfrac{4a^{-2}b^{\frac{2}{3}}}{2a^{2}b^{\frac{1}{3}}}$

(d) $\left(\dfrac{100a}{25a^{5}b^{\frac{-1}{2}}}\right)^{\frac{1}{2}}$

Questions page 241-242
#8cdegh,9cefgh,10cdeg,
11,14,16

Unit 3 Review: Powers and Roots

Sections 3.1-3.2, 4.2-4.6

Section 3.1

1: Write the prime factorization of 144 and 600

2: Find the GCF of each pair of numbers:

A) 44 and 70 B) 36 and 48

3: Find the LCM of each pair of numbers:

A) 12 and 30 B) 16 and 18

4: Hamburger patties come in packages of 8. Buns come in packages of 6. What is the least number of hamburgers that can be made with no patties or buns left over?

Section 3.2

5: Use prime factorization to find each square root

A) $\sqrt{225}$ B) $\sqrt{196}$ C) $\sqrt{1225}$

6: Use prime factorization to find each cube root.

A) $\sqrt[3]{729}$ B) $\sqrt[3]{3375}$ C) $\sqrt[3]{9261}$

Section 4.2

7: Tell whether each number is rational or irrational.

A) $\sqrt{36}$ B) $\sqrt[3]{12}$ C) $\sqrt[3]{-8}$ D) $\sqrt{\frac{81}{4}}$

8: Find each square root. Identify any that are irrational.

A) $\sqrt{4}$ B) $\sqrt{5}$ C) $\sqrt{9}$ D) $\sqrt{100}$

9: Find each cube root. Identify any that are irrational.

A) $\sqrt[3]{-216}$ B) $\sqrt[3]{64}$ C) $\sqrt[3]{1}$ D) $\sqrt[3]{100}$

Section 4.3

10: Simplify each radical.

A) $\sqrt{320}$ B) $\sqrt{735}$ C) $\sqrt{24}$ D) $\sqrt{108}$ E) $\sqrt[3]{40}$

F) $\sqrt[3]{162}$ G) $\sqrt[3]{189}$ H) $\sqrt[3]{576}$

11: Write each mixed radical as an entire radical.

A) $3\sqrt{11}$ B) $2\sqrt{13}$ C) $3\sqrt[3]{4}$ D) $2\sqrt[3]{15}$

Section 4.4

12: Write each power as a radical.

A) $8^{\frac{1}{2}}$ B) $32^{\frac{1}{3}}$ C) $12^{\frac{1}{4}}$

13: Write each radical as a power.

A) $\sqrt{35}$ B) $\sqrt[3]{11}$ C) $\sqrt[4]{6}$

14: Evaluate each power.

A) $100^{\frac{1}{2}}$ B) $125^{\frac{1}{3}}$ C) $81^{\frac{1}{4}}$

15: Find the value of each power and radical to 2 decimal places.

A) $22^{\frac{1}{2}}$ B) $30^{\frac{1}{3}}$ C) $\sqrt[4]{250}$

16: Write each power as a radical in two ways.

A) $52^{\frac{3}{4}}$ B) $114^{\frac{3}{2}}$ C) $92^{\frac{2}{3}}$

17: Write each radical as a power with a fractional exponent.

A) $\sqrt{537^3}$ B) $\left(\sqrt[3]{15}\right)^4$ C) $\left(\sqrt[4]{63}\right)^5$

18: Evaluate each power.

A) $25^{\frac{3}{2}}$ B) $64^{\frac{2}{3}}$ C) $16^{\frac{3}{4}}$

Section 4.5

19: Evaluate each power.

A) 6^{-2} B) 5^{-1} C) 2^{-5} D) $\frac{1}{2^{-4}}$ E) $\frac{1}{3^{-3}}$ F) $\frac{1}{10^{-1}}$

20: Write each power as a fraction with a radical denominator.

A) $3^{-\frac{1}{2}}$ B) $2^{-\frac{1}{3}}$ C) $8^{-\frac{1}{4}}$

21: Write each power with a positive exponent.

A) $3^{-\frac{2}{3}}$ B) $2^{-\frac{3}{2}}$ C) $\left(\frac{5}{6}\right)^{-4}$

22: Evaluate each power.

A) $100^{-\frac{1}{2}}$ B) $(-125)^{-\frac{1}{3}}$ C) $16^{-\frac{1}{4}}$ D) $100^{-\frac{3}{2}}$ E) $(-125)^{-\frac{4}{3}}$ F) $16^{-\frac{5}{4}}$

G) $\left(\frac{5}{6}\right)^{-2}$ H) $\left(\frac{2}{7}\right)^{-3}$ I) 2.9^{-4} (use a calculator)

Section 4.6

23: Write as a power with a positive exponent.

A) $5^7 \cdot 5^4 \cdot 5^{-7}$ B) $\frac{11^2}{11^{-3}}$ C) $\left(3^2 \cdot 3^2\right)^{-2}$ D) $\left(\frac{8^2}{8^3}\right)^{-4}$

24: Evaluate.

A) $\left(7^{-2}\right)^{-1}$ B) $5^3 \cdot \left(5^{-2}\right)^2$ C) $\left(\frac{4^{-3} \cdot 4^{-1}}{4^{-2}}\right)^2$

25: Write as a power with a positive exponent.

A) $5^{\frac{3}{4}} \cdot 5^{-\frac{1}{4}}$ B) $\left(7^{-0.5} \cdot 7^{2.5}\right)^{-2}$ C) $\frac{3^{-\frac{2}{3}}}{3^{\frac{4}{3}}}$ D) $\left(\frac{2^{-1.75}}{2^{-0.25}}\right)^3$

26: Simplify. Write an expression with positive exponents where necessary.

A) $3y \cdot y^{-2} \cdot y^4$ B) $\left(4x^3 \cdot 3x^{-4}\right)^2$ C) $\frac{25b^3}{10b^{-2}}$ D) $\frac{\left(7a^{-3}\right)^2}{a^{-4}}$

E) $x^{-\frac{1}{2}} \cdot x^{-3}$ F) $\frac{16a^{\frac{1}{3}}}{24a^{-1}}$

ANSWERS

Section 3.1

1. $144 = 2^4 \times 3^2$; $600 = 2^3 \times 3 \times 5^2$
2. a. $44 = 2^2 \times 11$; $70 = 2 \times 5 \times 7$; GCF = 2 b. $36 = 2^2 \times 3^2$; $48 = 2^4 \times 3$; GCF = $2^2 \times 3 = 12$
3. a. $12 = 2^2 \times 3$; $30 = 2 \times 3 \times 5$; LCM = $2^2 \times 3 \times 5 = 60$

   b. $16 = 2^4$; $18 = 2 \times 3^2$; LCM = $2^4 \times 3^2 = 16 \times 9 = 144$

4. $8 = 2^3$; $6 = 2 \times 3$; LCM = $2^3 \times 3 = 24$;
   24 hamburgers need to be made so that no patties or buns are left over.

Section 3.2:

5. a. 15 b. 14 c. 35 6. a. 9 b. 15 c. 21

Section 4.2:

7. a. Rational b. Irrational c. Rational d. Rational

8. a. 2 b. 2.23606 . . . irrational c. 3 d. 10

9. a. -6 b. 4 c. 1 d. 4.641588834. . . Irrational

Section 4.3:

10. a. $8\sqrt{5}$ b. $7\sqrt{15}$ c. $2\sqrt{6}$ d. $6\sqrt{3}$ e. $2\sqrt[3]{5}$ f. $3\sqrt[3]{6}$ g. $3\sqrt[3]{7}$ h. $4\sqrt[3]{9}$
11. a. $\sqrt{99}$ b. $\sqrt{52}$ c. $\sqrt[3]{108}$ d. $\sqrt[3]{120}$ 12. a. $\sqrt{8}$ b. $\sqrt[3]{32}$ c. $\sqrt[4]{12}$

13. a. $35^{\frac{1}{2}}$ b. $11^{\frac{1}{3}}$ c. $6^{\frac{1}{4}}$ 14. a. 10 b. 5 c. 3 15. a. 4.69 b. 3.11 c. 3.98

16. a. $52^{\frac{3}{4}} = \sqrt[4]{52}^3$ b. $114^{\frac{3}{2}} = \sqrt{114}^3$ c. $92^{\frac{3}{2}} = \sqrt[3]{92}^2$

17. a. $\sqrt{537}^3 = 537^{\frac{3}{2}}$ b. $\left(\sqrt[3]{15}\right)^4 = 15^{\frac{4}{3}}$ c. $\left(\sqrt[4]{63}\right)^5 = 63^{\frac{5}{4}}$ 18. a. 125 b. 16 c. 8

19. a) $\frac{1}{36}$ b) $\frac{1}{5}$ c) $\frac{1}{32}$ d) 16 e) 27f) 10 20. a) $\frac{1}{\sqrt{3}}$ b) $\frac{1}{\sqrt[3]{2}}$ c) $\frac{1}{\sqrt[4]{8}}$

21.a) $\frac{1}{\sqrt[3]{3}^2}$ b) $\frac{1}{\sqrt{2}^3}$ c) $\left(\frac{6}{5}\right)^4$ 22.a) $\frac{1}{10}$ b) $-\frac{1}{5}$ c) $\frac{1}{2}$ d) $\frac{1}{1000}$ e) $\frac{1}{625}$ f) $\frac{1}{32}$ g) $\frac{36}{25}$ h) $\frac{343}{8}$

23. a) 5^4 b) 11^5 c) $\frac{1}{3^8}$ d) 8^4 24. a) 49 b) $\frac{1}{5}$ c) $\frac{1}{256}$

25.a) $5^{\frac{1}{2}}$ b) $\frac{1}{7^4}$ c) $\frac{1}{3^2}$ d) $\frac{1}{2^{\frac{9}{2}}}$ 26. a) $3y^5$ b) $\frac{48}{x^2}$ c) $\frac{5b^5}{2}$ d) $\frac{49}{a^2}$ e) $\frac{1}{x^{\frac{7}{2}}}$ f) $\frac{2a^{\frac{4}{3}}}{3}$