

Chapter 3

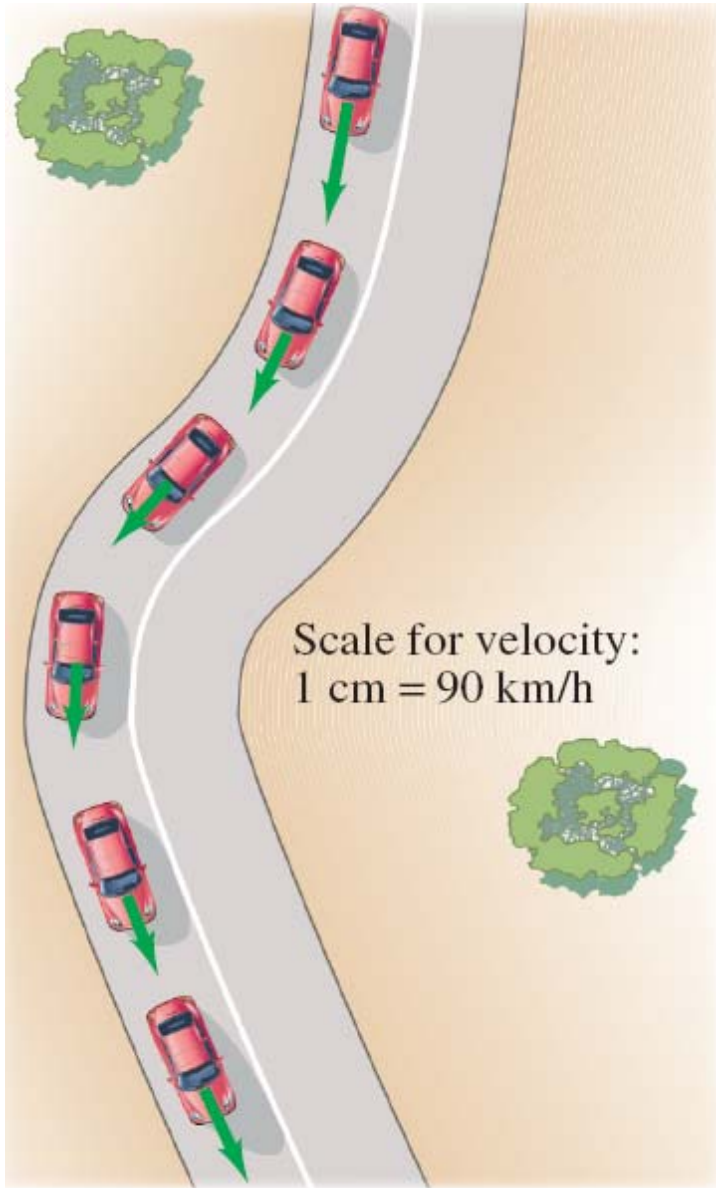
Kinematics in Two or Three Dimensions; Vectors



Units of Chapter 3

- **Vectors and Scalars**
- **Addition of Vectors—Graphical Methods**
- **Subtraction of Vectors, and Multiplication of a Vector by a Scalar**
- **Adding Vectors by Components**
- **Unit Vectors**
- **Vector Kinematics**
- **Projectile Motion**
- **Solving Problems Involving Projectile Motion**
- **Relative Velocity**

3-1 Vectors and Scalars



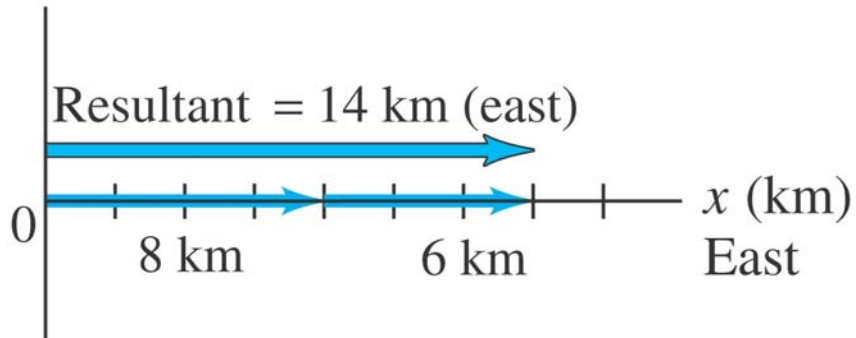
A vector has magnitude as well as direction.

Some vector quantities: displacement, velocity, force, momentum

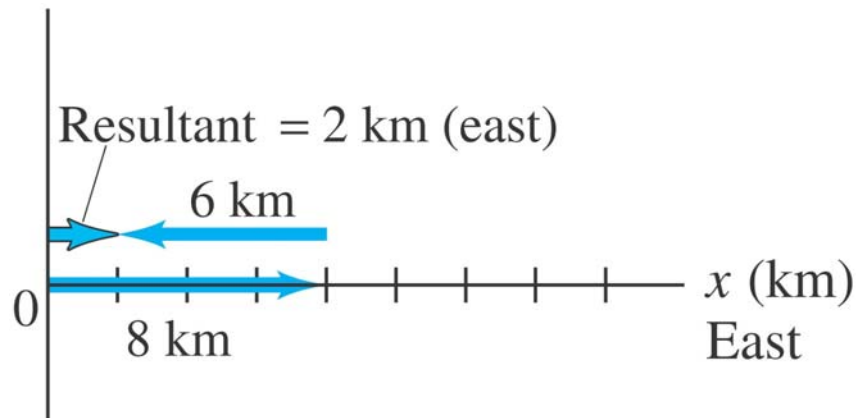
A scalar has only a magnitude.

Some scalar quantities: mass, time, temperature

3-2 Addition of Vectors—Graphical Methods



For vectors in one dimension, simple addition and subtraction are all that is needed.

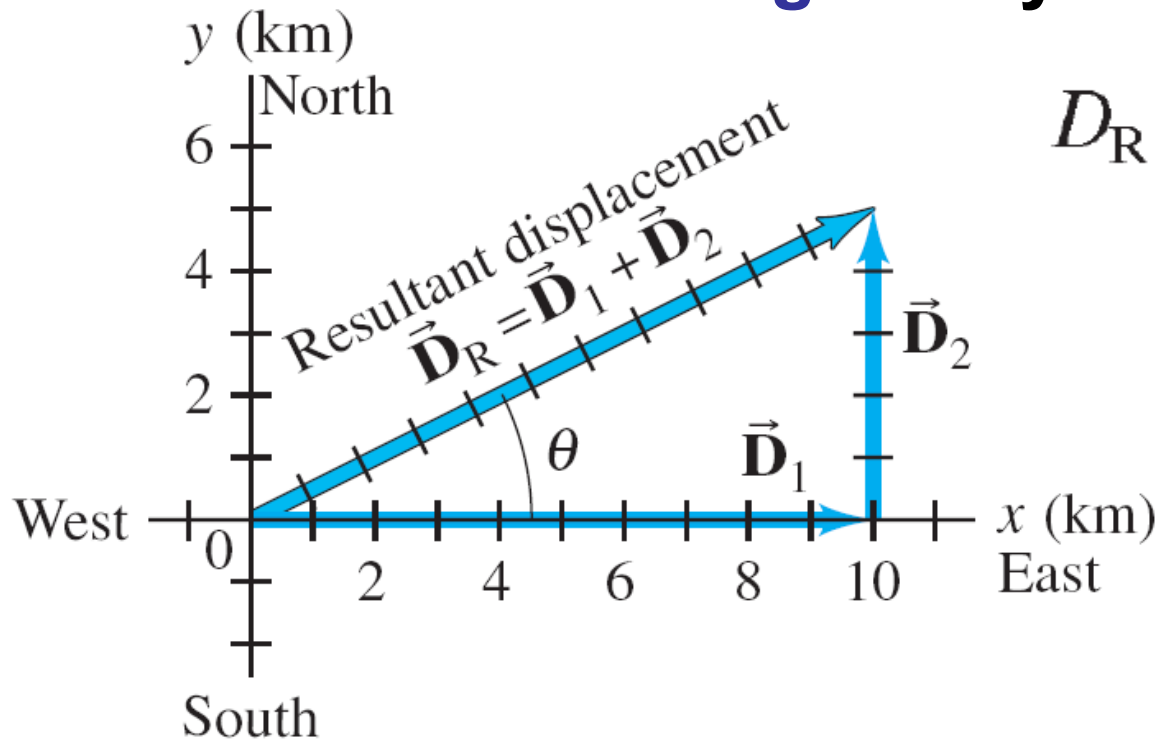


You do need to be careful about the signs, as the figure indicates.

3-2 Addition of Vectors—Graphical Methods

If the motion is in two dimensions, the situation is somewhat more complicated.

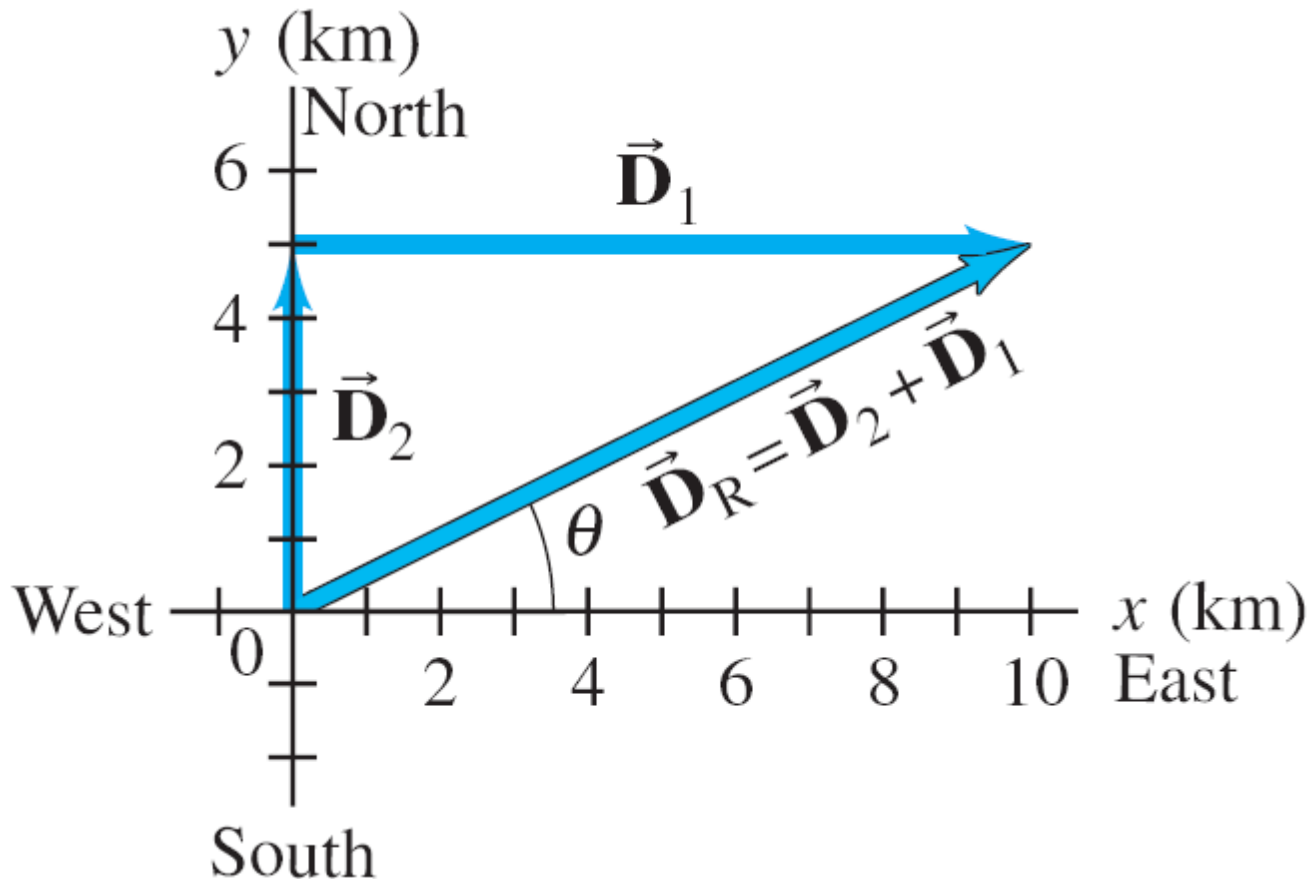
Here, the actual travel paths are at right angles to one another; we can find the displacement by using the Pythagorean Theorem.



$$D_R = \sqrt{D_1^2 + D_2^2}$$

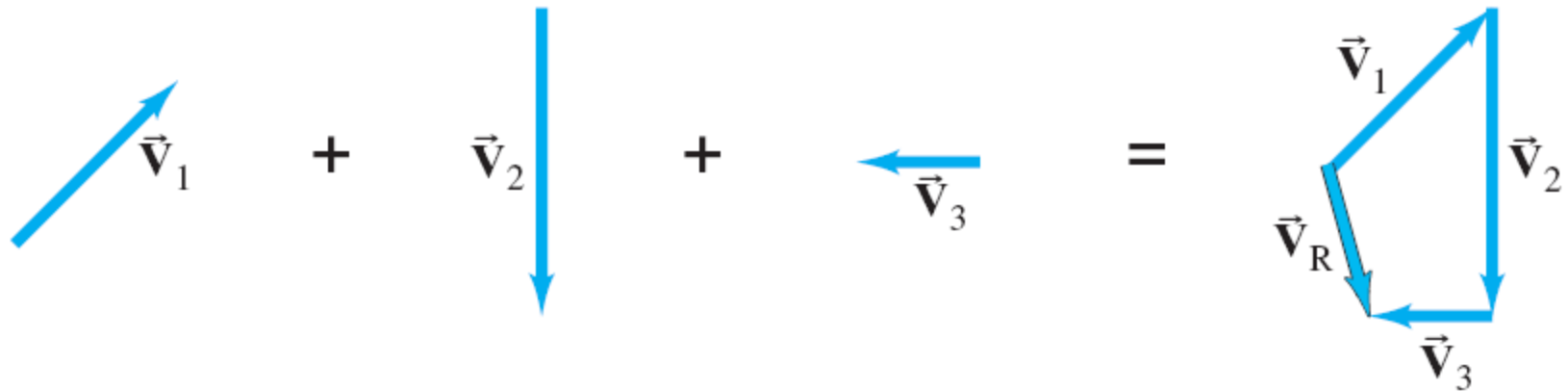
3-2 Addition of Vectors—Graphical Methods

Adding the vectors in the **opposite order** gives the same result: $\vec{V}_2 - \vec{V}_1 = \vec{V}_2 + (-\vec{V}_1)$.



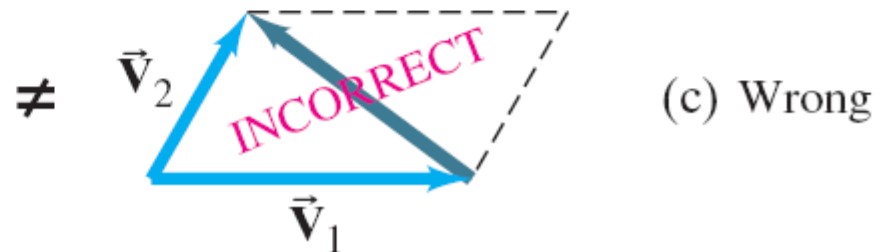
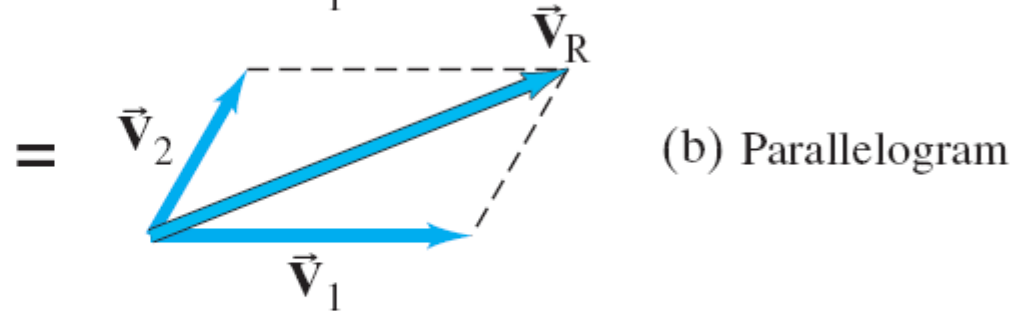
3-2 Addition of Vectors—Graphical Methods

Even if the vectors are not at right angles, they can be added **graphically** by using the **tail-to-tip** method.

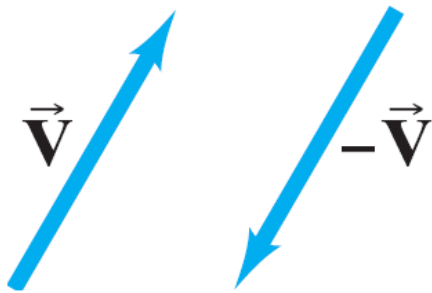


3-2 Addition of Vectors—Graphical Methods

The **parallelogram method** may also be used; here again the vectors must be **tail-to-tip**.



3-3 Subtraction of Vectors, and Multiplication of a Vector by a Scalar



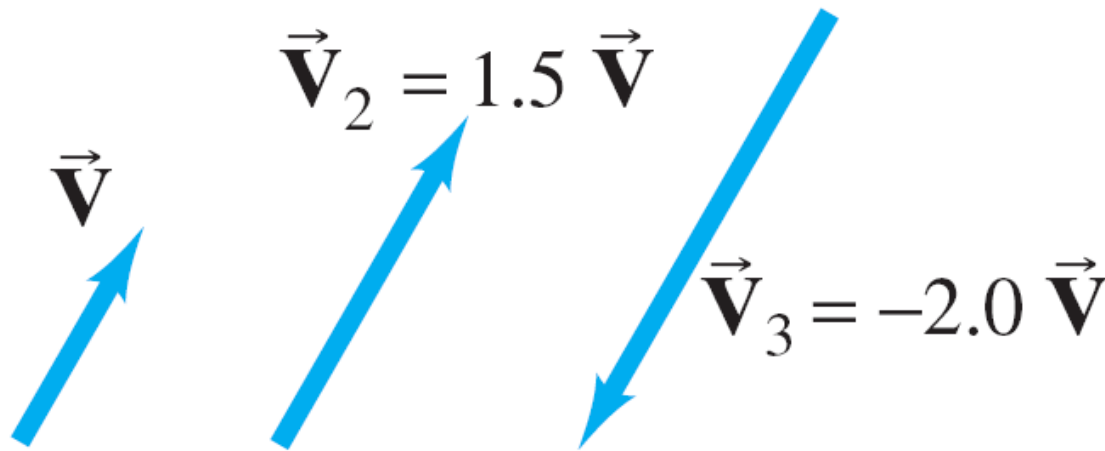
In order to **subtract** vectors, we define the **negative** of a vector, which has the same magnitude but points in the **opposite** direction.

Then we add the negative vector.



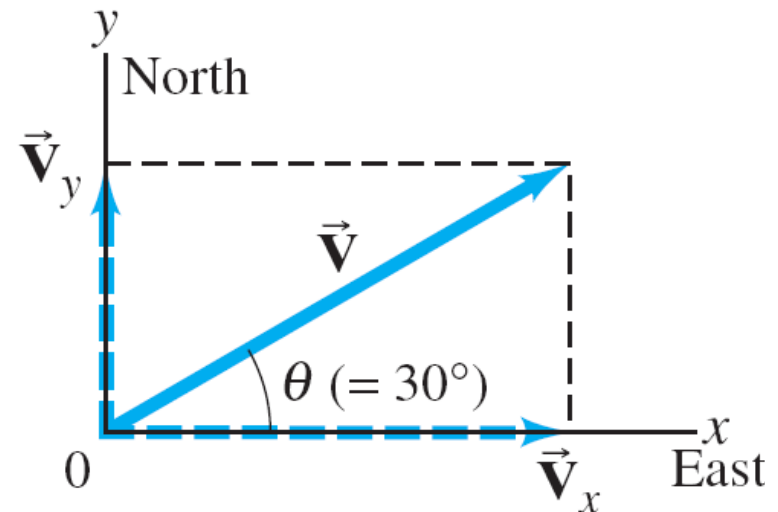
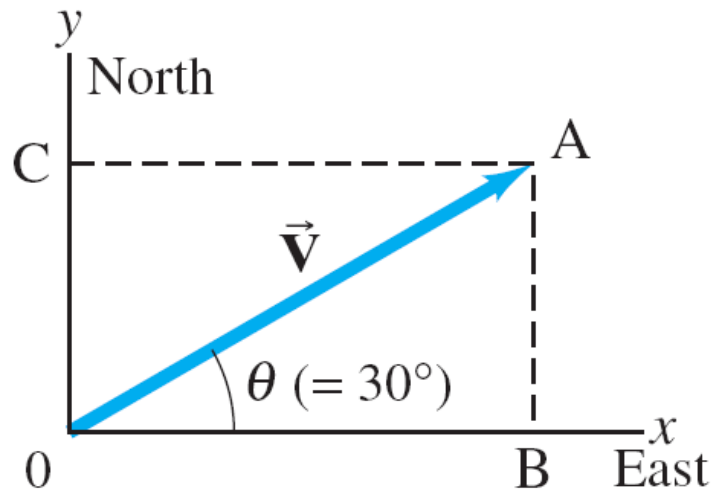
3-3 Subtraction of Vectors, and Multiplication of a Vector by a Scalar

A vector \vec{V} can be multiplied by a scalar c ; the result is a vector $c\vec{V}$ that has the same direction but a magnitude cV . If c is negative, the resultant vector points in the opposite direction.

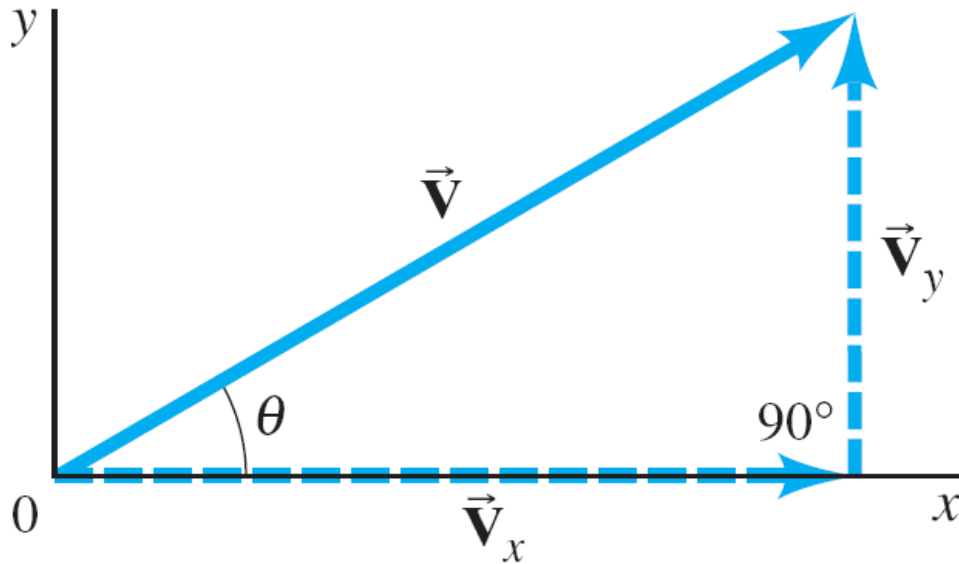


3-4 Adding Vectors by Components

Any vector can be expressed as the **sum** of two other vectors, which are called its **components**. Usually the other vectors are chosen so that they are **perpendicular** to each other.



3-4 Adding Vectors by Components



$$\sin \theta = \frac{V_y}{V}$$

$$\cos \theta = \frac{V_x}{V}$$

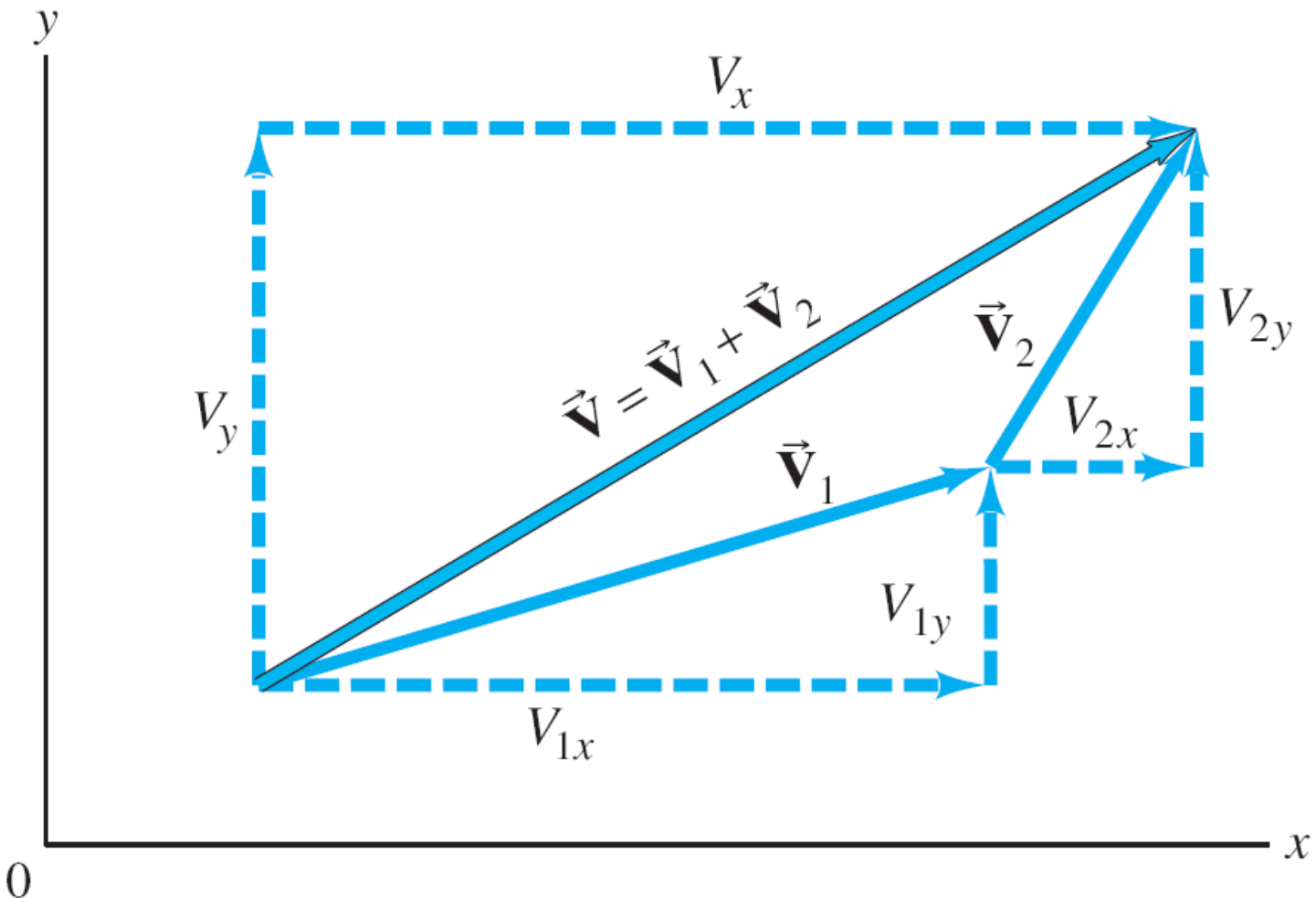
$$\tan \theta = \frac{V_y}{V_x}$$

$$V^2 = V_x^2 + V_y^2$$

If the components are perpendicular, they can be found using trigonometric functions.

3-4 Adding Vectors by Components

The **components** are effectively one-dimensional, so they can be added **arithmetically**.



3-4 Adding Vectors by Components

Adding vectors:

1. Draw a diagram; add the vectors graphically.
2. Choose x and y axes.
3. Resolve each vector into x and y components.
4. Calculate each component using sines and cosines.
5. Add the components in each direction.
6. To find the length and direction of the vector, use:

$$V = \sqrt{V_x^2 + V_y^2}$$

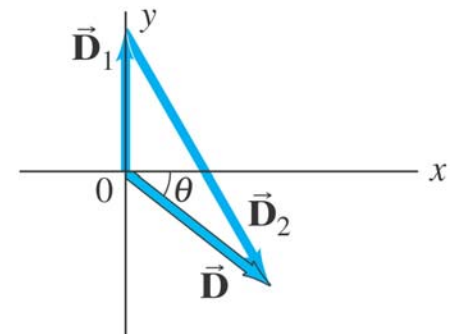
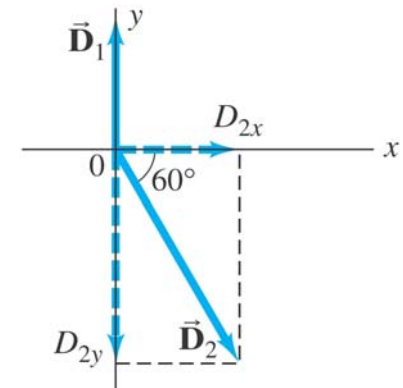
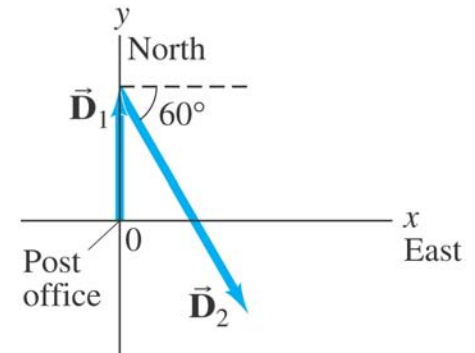
and

$$\tan \theta = \frac{V_y}{V_x} .$$

3-4 Adding Vectors by Components

Example 3-2: Mail carrier's displacement.

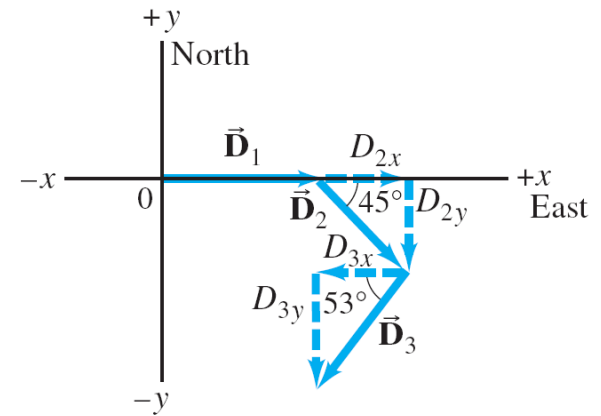
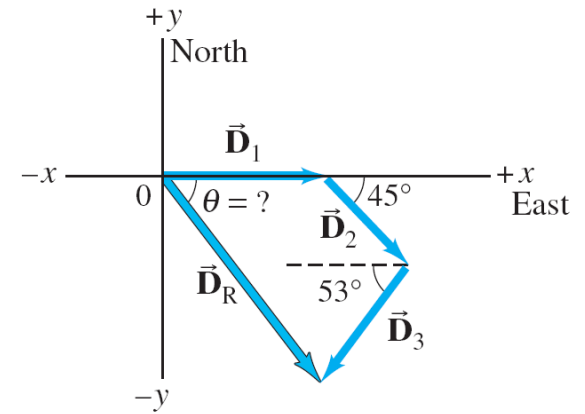
A rural mail carrier leaves the post office and drives 22.0 km in a northerly direction. She then drives in a direction 60.0° south of east for 47.0 km. What is her displacement from the post office?



3-4 Adding Vectors by Components

Example 3-3: Three short trips.

An airplane trip involves three legs, with two stopovers. The first leg is due east for 620 km; the second leg is southeast (45°) for 440 km; and the third leg is at 53° south of west, for 550 km, as shown. What is the plane's total displacement?

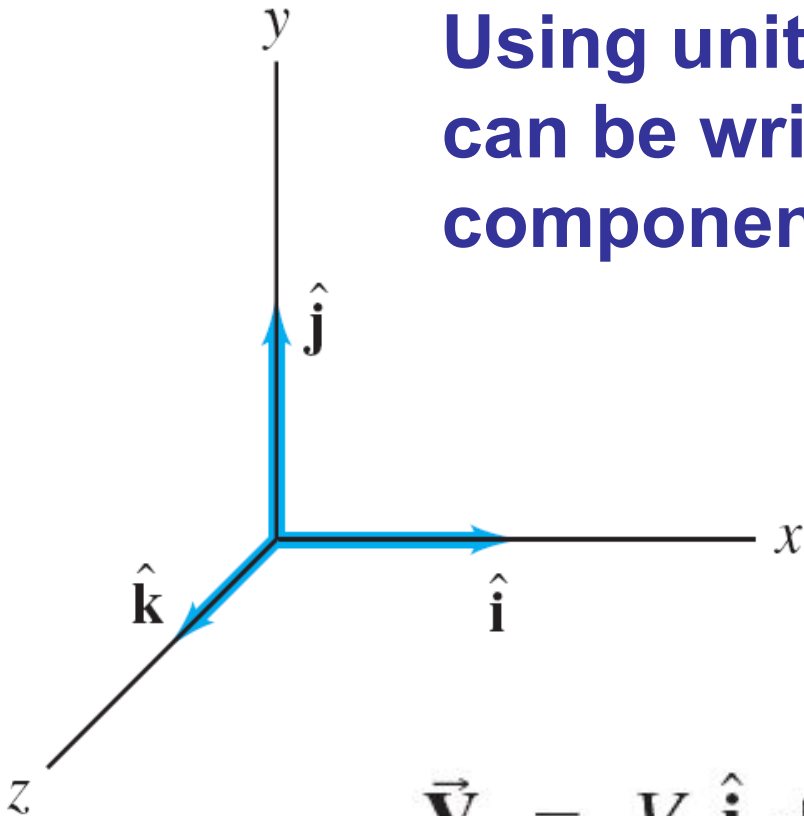


Vector	Components	
	x (km)	y (km)
\vec{D}_1	620	0
\vec{D}_2	311	-311
\vec{D}_3	-331	-439
\vec{D}_R	600	-750

3-5 Unit Vectors

Unit vectors have magnitude 1.

Using unit vectors, any vector \vec{V} can be written in terms of its components:

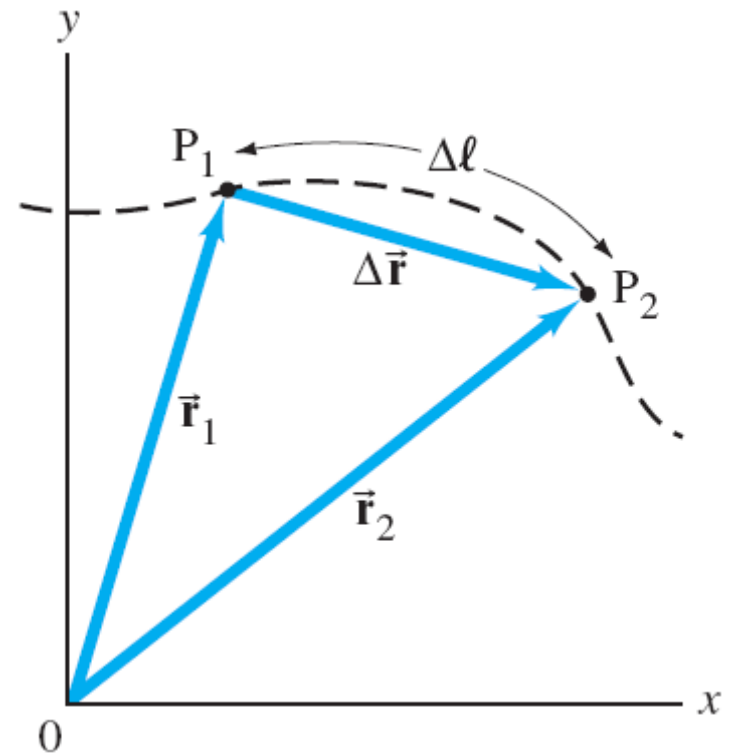


$$\vec{V} = V_x \hat{i} + V_y \hat{j} + V_z \hat{k}.$$

3-6 Vector Kinematics

In two or three dimensions, the displacement is a vector:

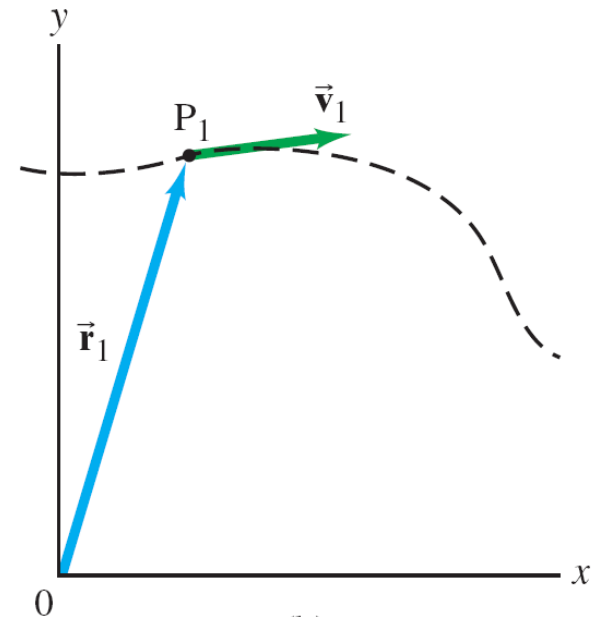
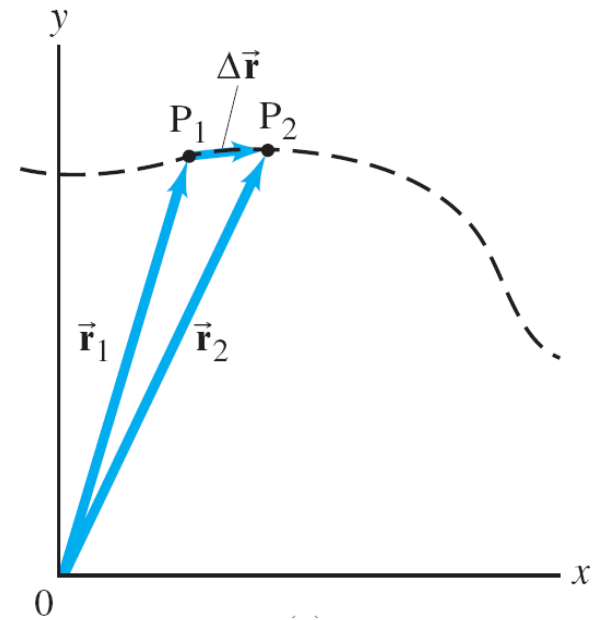
$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1.$$



3-6 Vector Kinematics

As Δt and Δr become smaller and smaller, the average velocity approaches the instantaneous velocity.

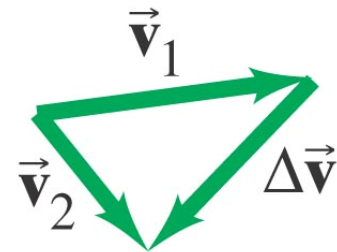
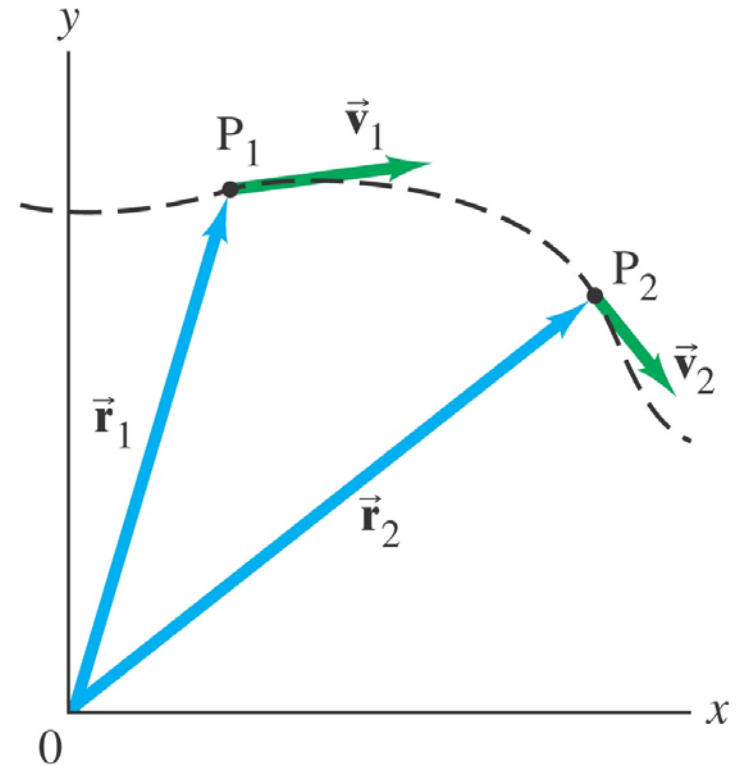
$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$



3-6 Vector Kinematics

The instantaneous acceleration is in the direction of $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$, and is given by:

$$\vec{a} = \lim_{\Delta \rightarrow 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$



3-6 Vector Kinematics

Using unit vectors,

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} + \frac{dz}{dt} \hat{k};$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} + \frac{dv_z}{dt} \hat{k}.$$

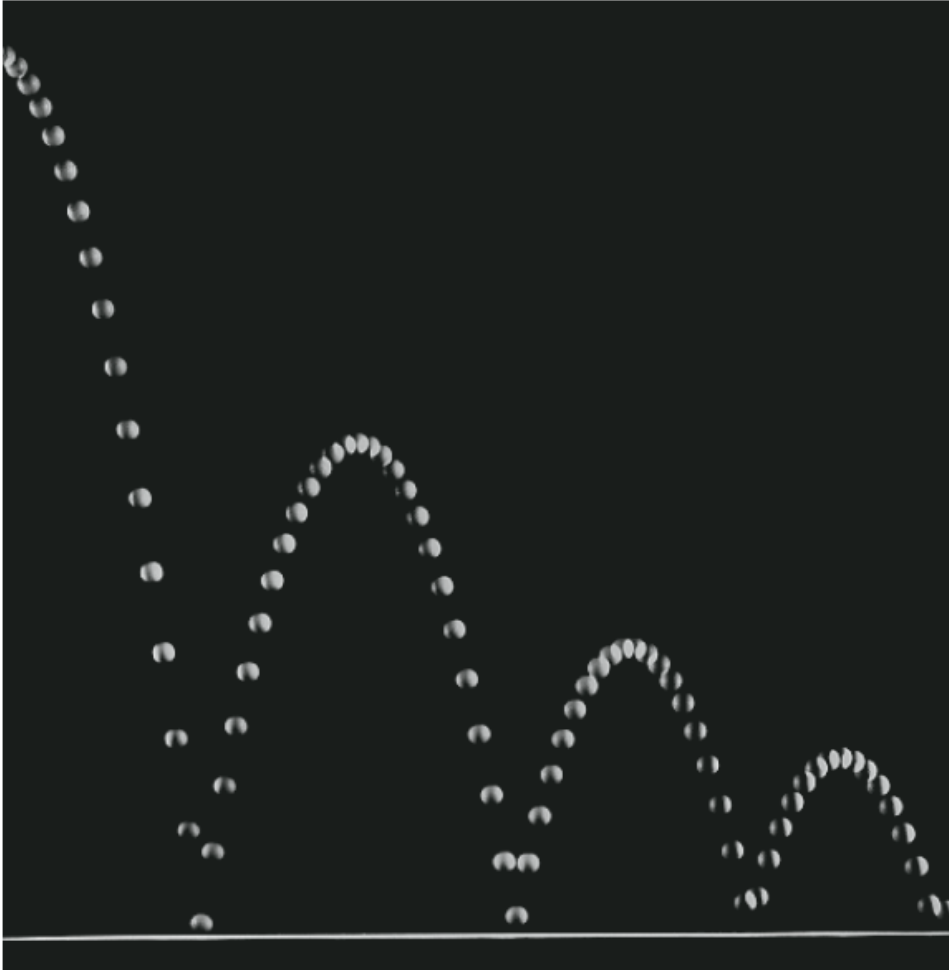
3-6 Vector Kinematics

Generalizing the one-dimensional equations for constant acceleration:

TABLE 3–1 Kinematic Equations for Constant Acceleration in 2 Dimensions

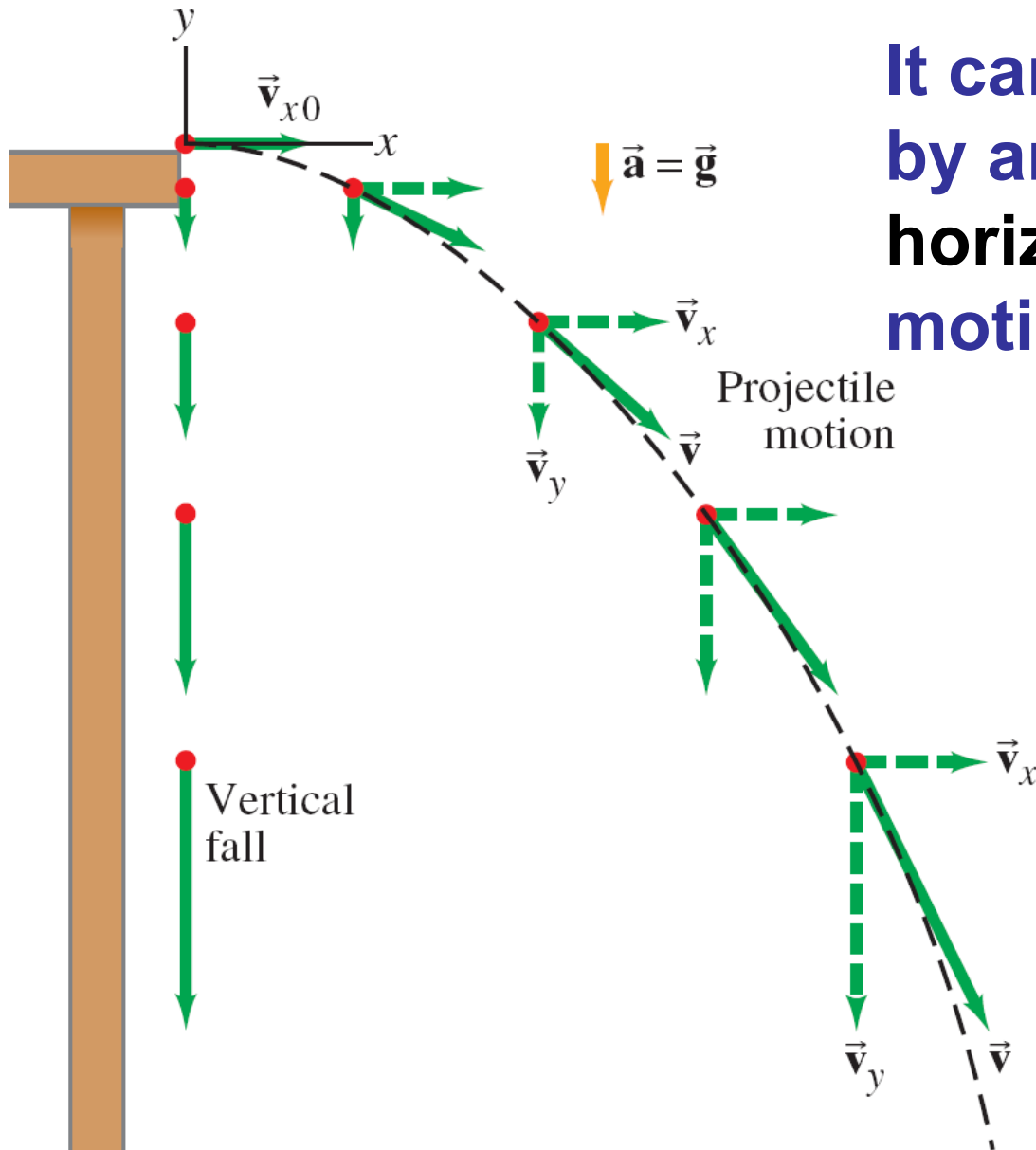
<i>x</i> Component (horizontal)		<i>y</i> Component (vertical)
$v_x = v_{x0} + a_x t$	(Eq. 2–12a)	$v_y = v_{y0} + a_y t$
$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$	(Eq. 2–12b)	$y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$
$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$	(Eq. 2–12c)	$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$

3-7 Projectile Motion



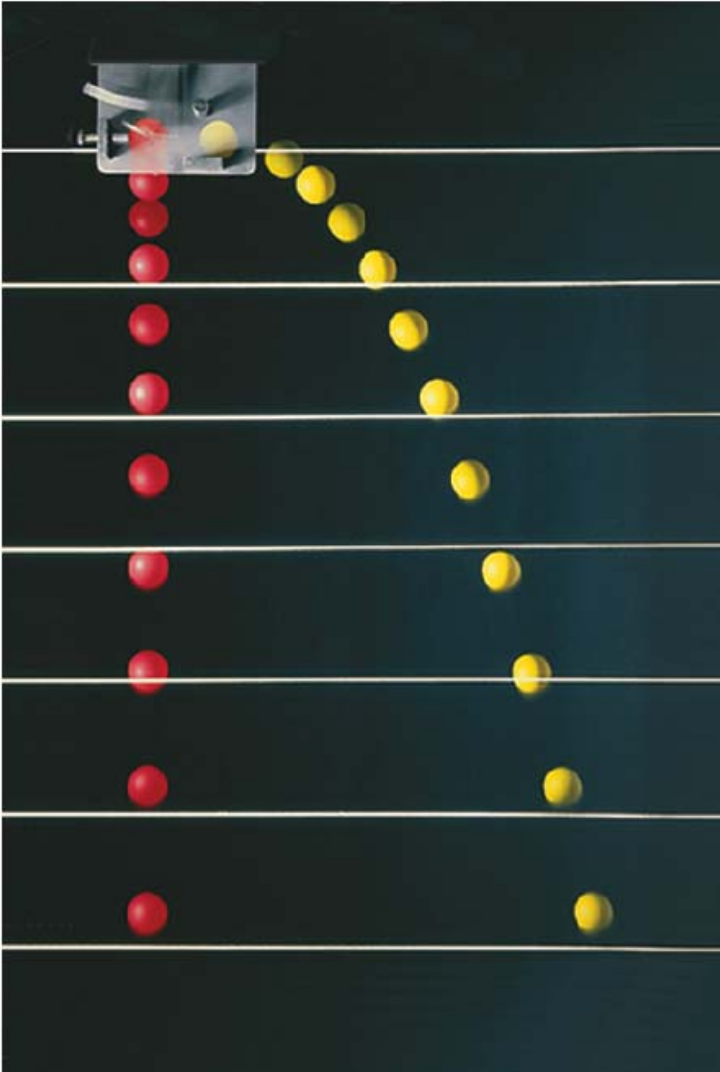
A projectile is an object moving in two dimensions under the influence of Earth's gravity; its path is a parabola.

3-7 Projectile Motion



It can be understood by analyzing the horizontal and vertical motions separately.

3-7 Projectile Motion

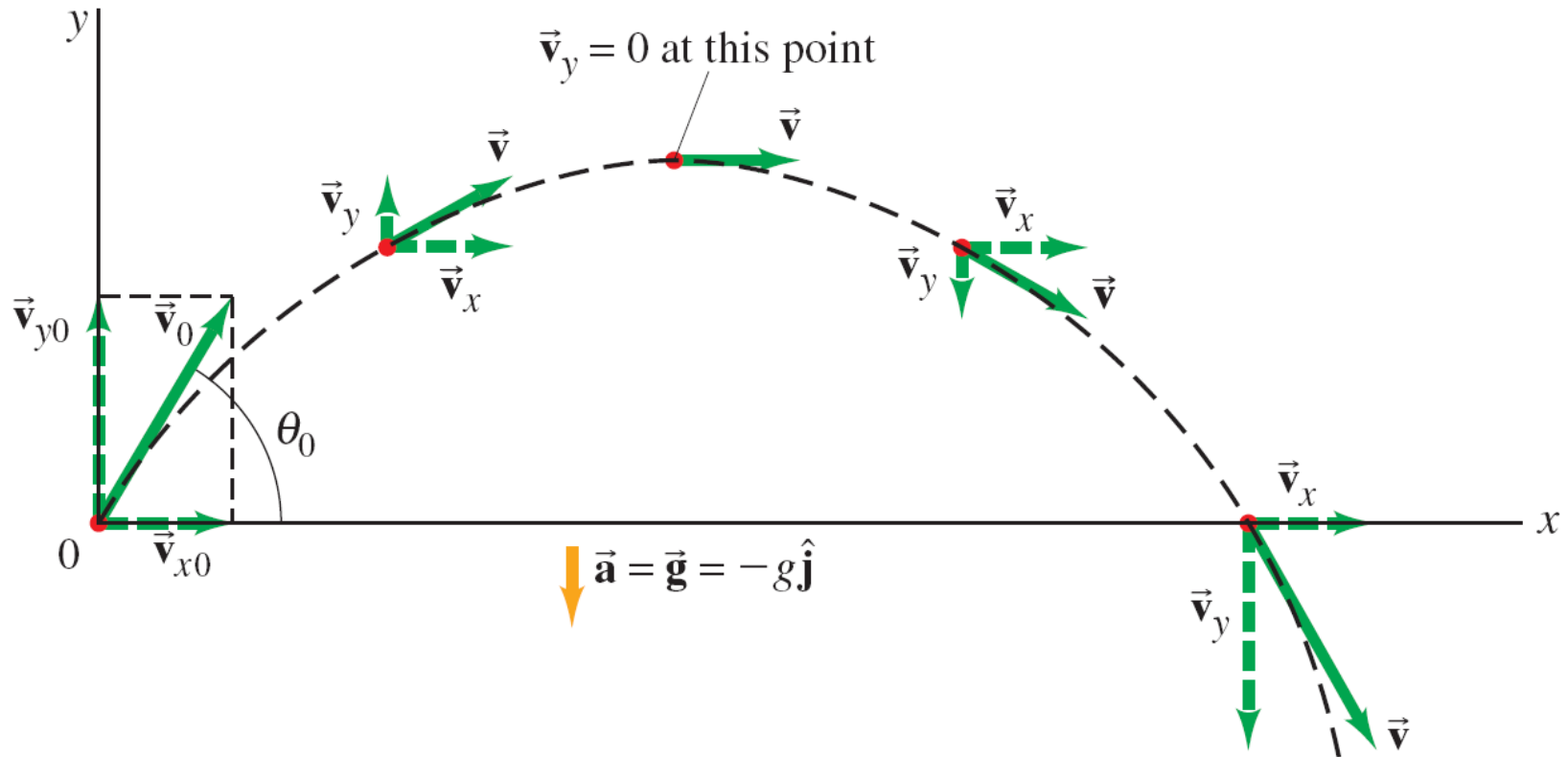


The **speed** in the x -direction is constant; in the y -direction the object moves with constant **acceleration** g .

This photograph shows two balls that start to fall at the same time. The one on the right has an initial speed in the x -direction. It can be seen that vertical positions of the two balls are identical at identical times, while the horizontal position of the yellow ball increases linearly.

3-7 Projectile Motion

If an object is launched at an initial angle of θ_0 with the horizontal, the analysis is similar except that the initial velocity has a vertical component.



3-8 Solving Problems Involving Projectile Motion

Projectile motion is motion with constant acceleration in two dimensions, where the acceleration is g and is down.

TABLE 3–2 Kinematic Equations for Projectile Motion

(y positive upward; $a_x = 0$, $a_y = -g = -9.80 \text{ m/s}^2$)

Horizontal Motion
($a_x = 0$, $v_x = \text{constant}$)

$$v_x = v_{x0} \quad (\text{Eq. 2-12a})$$

$$x = x_0 + v_{x0}t \quad (\text{Eq. 2-12b})$$

$$(\text{Eq. 2-12c})$$

Vertical Motion[†]
($a_y = -g = \text{constant}$)

$$v_y = v_{y0} - gt$$

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

$$v_y^2 = v_{y0}^2 - 2g(y - y_0)$$

[†] If y is taken positive downward, the minus (–) signs in front of g become plus (+) signs.

3-8 Solving Problems Involving Projectile Motion

1. **Read the problem carefully, and choose the object(s) you are going to analyze.**
2. **Draw a diagram.**
3. **Choose an origin and a coordinate system.**
4. **Decide on the time interval; this is the same in both directions, and includes only the time the object is moving with constant acceleration g .**
5. **Examine the x and y motions separately.**

3-8 Solving Problems Involving Projectile Motion

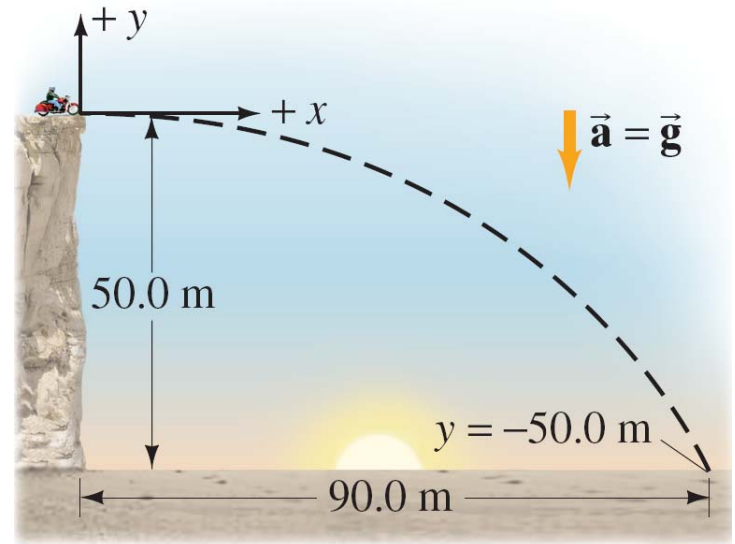
6. List known and unknown quantities. Remember that v_x never changes, and that $v_y = 0$ at the highest point.

7. Plan how you will proceed. Use the appropriate equations; you may have to combine some of them.

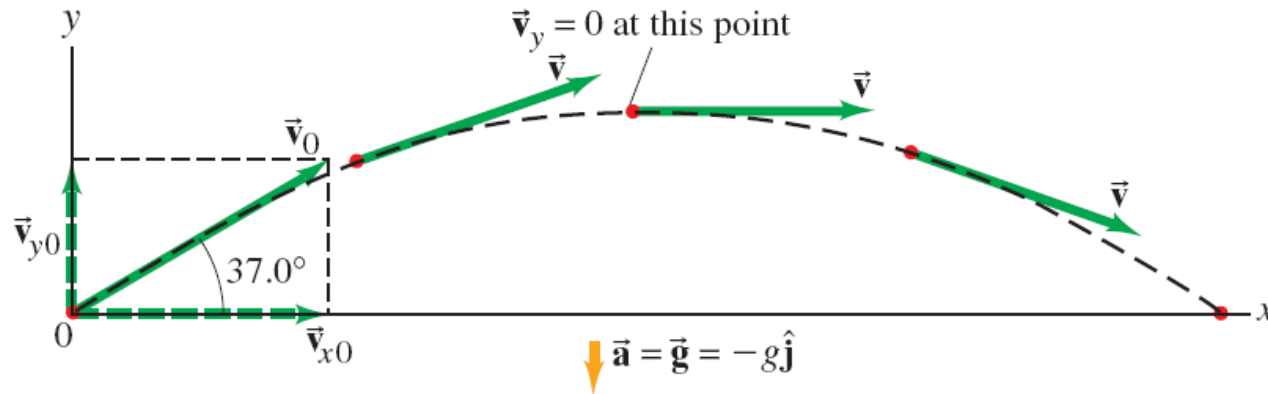
3-8 Solving Problems Involving Projectile Motion

Example 3-6: Driving off a cliff.

A movie stunt driver on a motorcycle speeds horizontally off a 50.0-m-high cliff. How fast must the motorcycle leave the cliff top to land on level ground below, 90.0 m from the base of the cliff where the cameras are? Ignore air resistance.



3-8 Solving Problems Involving Projectile Motion



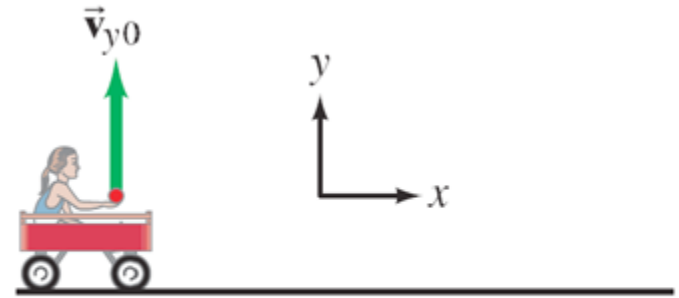
Example 3-7: A kicked football.

A football is kicked at an angle $\theta_0 = 37.0^\circ$ with a velocity of 20.0 m/s, as shown. Calculate (a) the maximum height, (b) the time of travel before the football hits the ground, (c) how far away it hits the ground, (d) the velocity vector at the maximum height, and (e) the acceleration vector at maximum height. Assume the ball leaves the foot at ground level, and ignore air resistance and rotation of the ball.

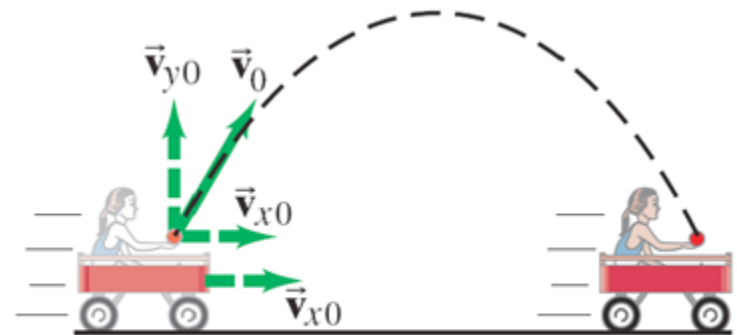
3-8 Solving Problems Involving Projectile Motion

Conceptual Example 3-8: Where does the apple land?

A child sits upright in a wagon which is moving to the right at constant speed as shown. The child extends her hand and throws an apple straight upward (from her own point of view), while the wagon continues to travel forward at constant speed. If air resistance is neglected, will the apple land (a) behind the wagon, (b) in the wagon, or (c) in front of the wagon?



Wagon reference frame

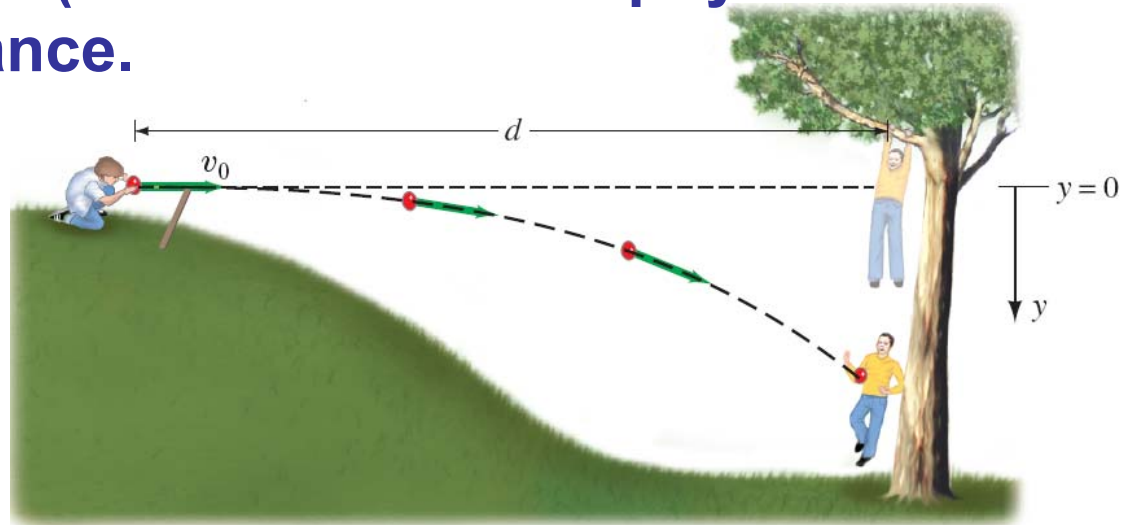


Ground reference frame

3-8 Solving Problems Involving Projectile Motion

Conceptual Example 3-9: The wrong strategy.

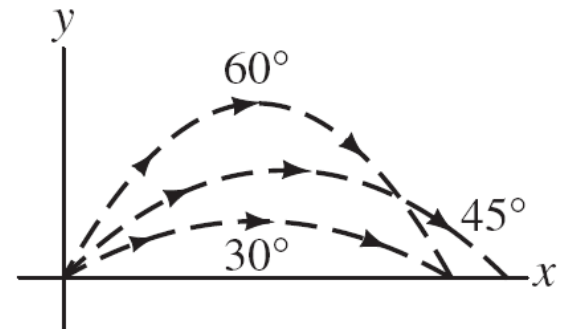
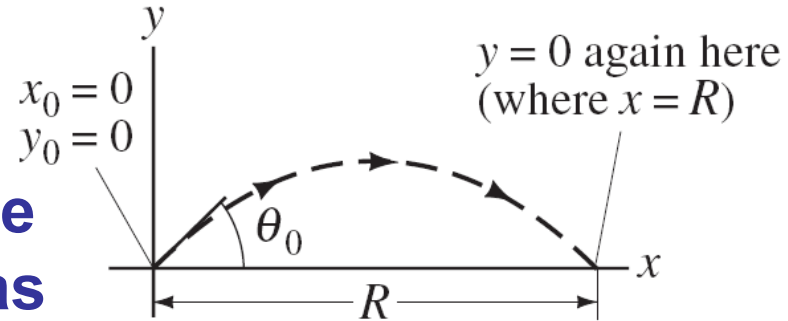
A boy on a small hill aims his water-balloon slingshot horizontally, straight at a second boy hanging from a tree branch a distance d away. At the instant the water balloon is released, the second boy lets go and falls from the tree, hoping to avoid being hit. Show that he made the wrong move. (He hadn't studied physics yet.) Ignore air resistance.



3-8 Solving Problems Involving Projectile Motion

Example 3-10: Level horizontal range.

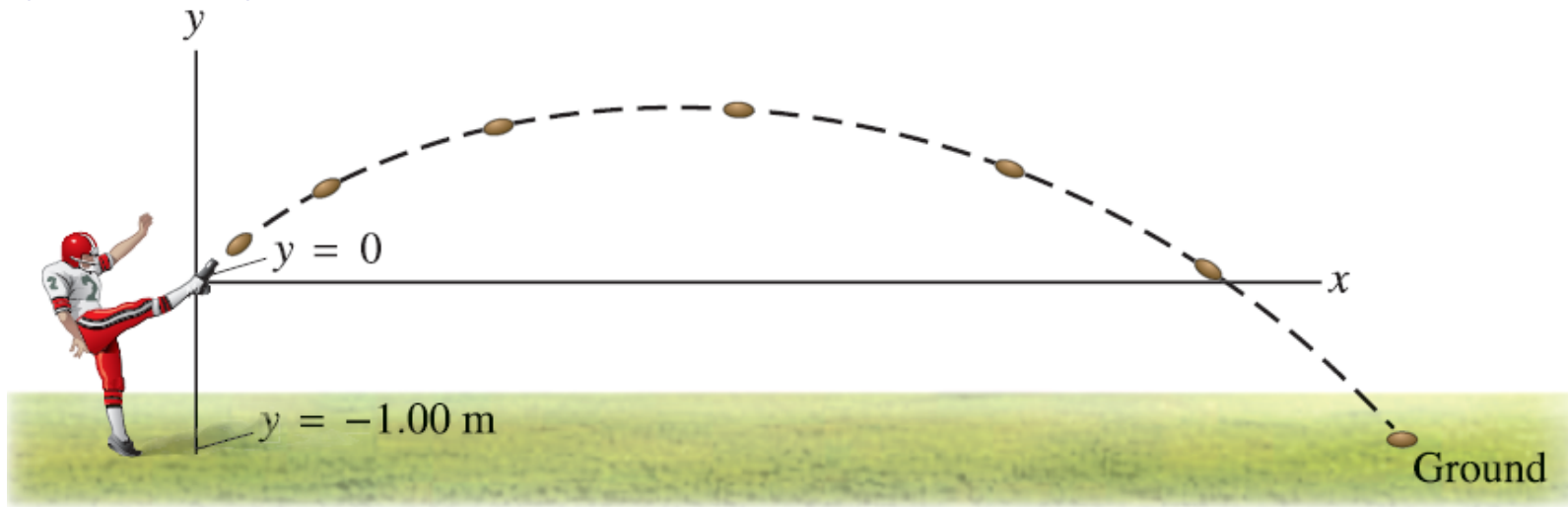
(a) Derive a formula for the horizontal range R of a projectile in terms of its initial speed v_0 and angle θ_0 . The horizontal range is defined as the horizontal distance the projectile travels before returning to its original height (which is typically the ground); that is, $y(\text{final}) = y_0$. (b) Suppose one of Napoleon's cannons had a muzzle speed, v_0 , of 60.0 m/s. At what angle should it have been aimed (ignore air resistance) to strike a target 320 m away?



3-8 Solving Problems Involving Projectile Motion

Example 3-11: A punt.

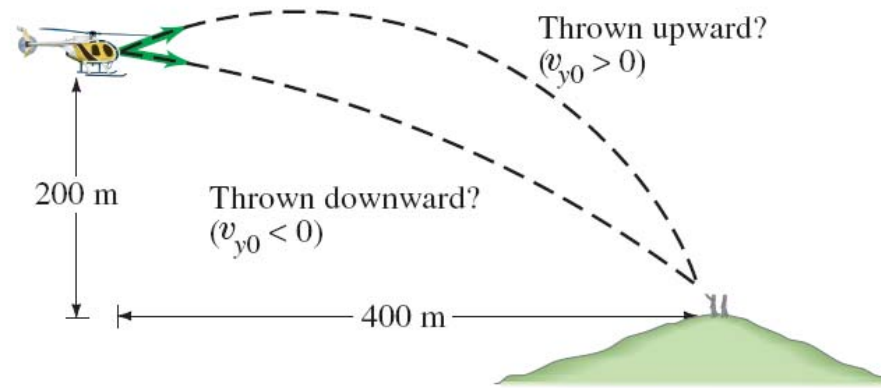
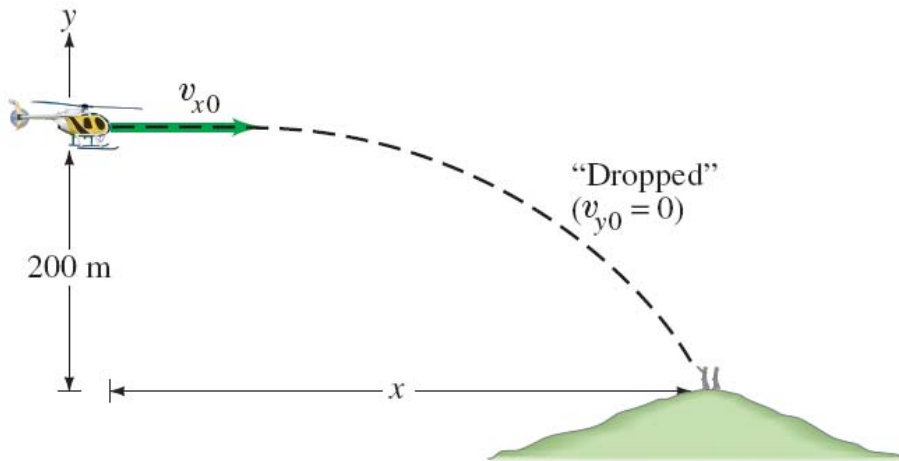
Suppose the football in Example 3–7 was punted and left the punter's foot at a height of 1.00 m above the ground. How far did the football travel before hitting the ground? Set $x_0 = 0, y_0 = 0$.



3-8 Solving Problems Involving Projectile Motion

Example 3-12: Rescue helicopter drops supplies.

A rescue helicopter wants to drop a package of supplies to isolated mountain climbers on a rocky ridge 200 m below. If the helicopter is traveling horizontally with a speed of 70 m/s (250 km/h), (a) how far in advance of the recipients (horizontal distance) must the package be dropped? (b) Suppose, instead, that the helicopter releases the package a horizontal distance of 400 m in advance of the mountain climbers. What vertical velocity should the package be given (up or down) so that it arrives precisely at the climbers' position? (c) With what speed does the package land in the latter case?



3-8 Solving Problems Involving Projectile Motion

Projectile motion is parabolic:

Taking the equations for x and y as a function of time, and combining them to eliminate t , we find y as a function of x :

$$y = \left(\frac{v_{y0}}{v_{x0}} \right) x - \left(\frac{g}{2v_{x0}^2} \right) x^2.$$

This is the equation for a parabola.

3-8 Solving Problems Involving Projectile Motion

Examples of projectile motion. Notice the effects of air resistance.

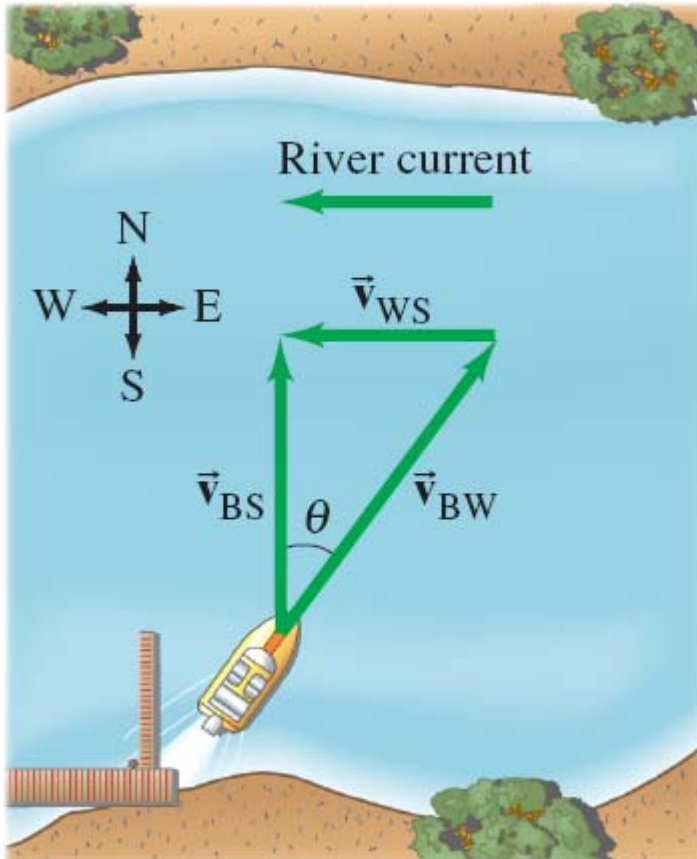


3-9 Relative Velocity

We have already considered relative speed in one dimension; it is similar in two dimensions except that we must add and subtract velocities as vectors.

Each velocity is labeled first with the object, and second with the reference frame in which it has this velocity.

3-9 Relative Velocity



Here, v_{WS} is the velocity of the water in the shore frame, v_{BS} is the velocity of the boat in the shore frame, and v_{BW} is the velocity of the boat in the water frame.

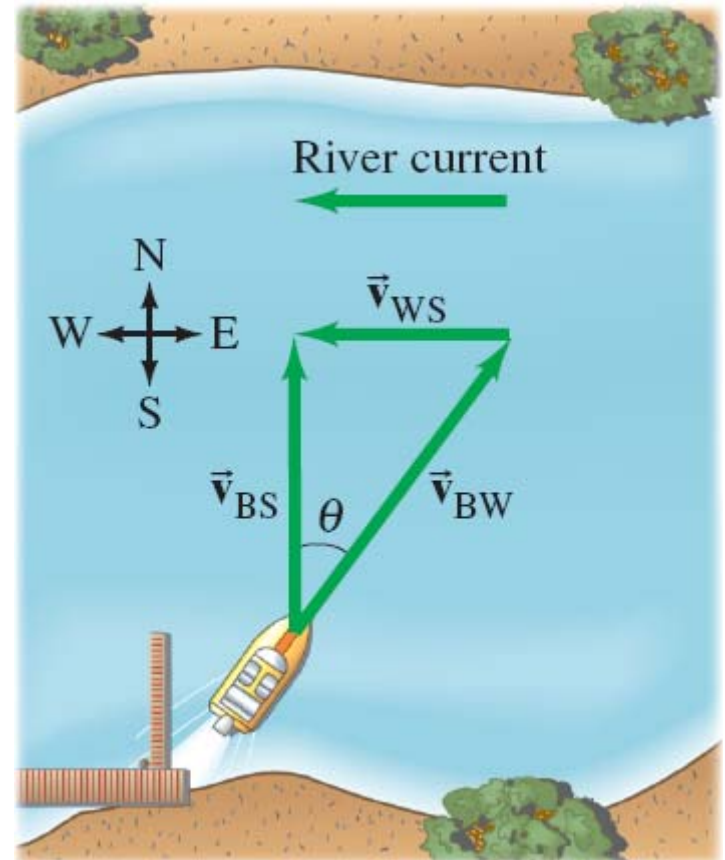
The relationship between the three velocities is:

$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS}.$$

3-9 Relative Velocity

Example 3-14: Heading upstream.

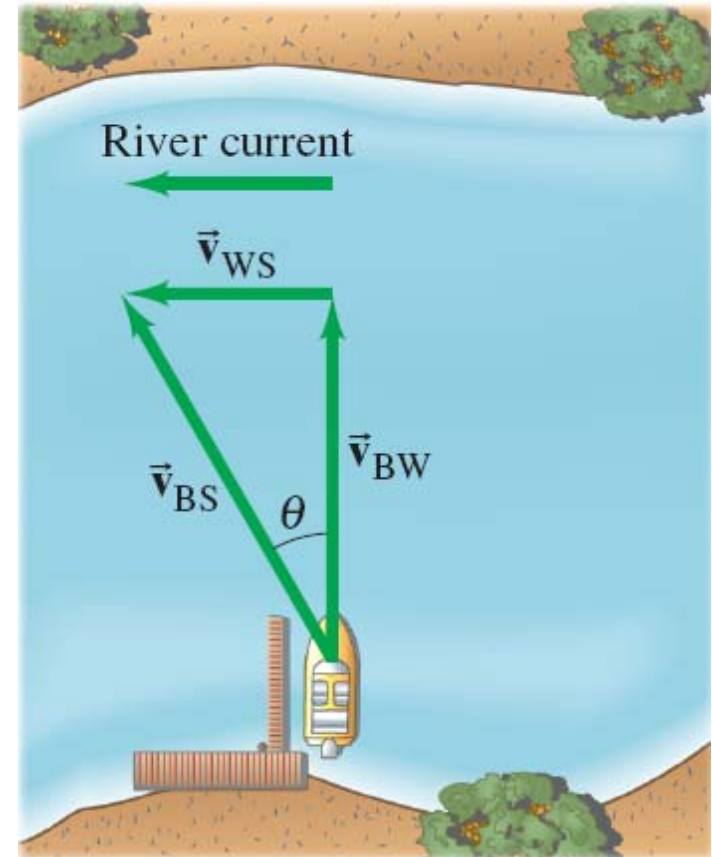
A boat's speed in still water is $v_{BW} = 1.85 \text{ m/s}$. If the boat is to travel directly across a river whose current has speed $v_{WS} = 1.20 \text{ m/s}$, at what upstream angle must the boat head?



3-9 Relative Velocity

Example 3-15: Heading across the river.

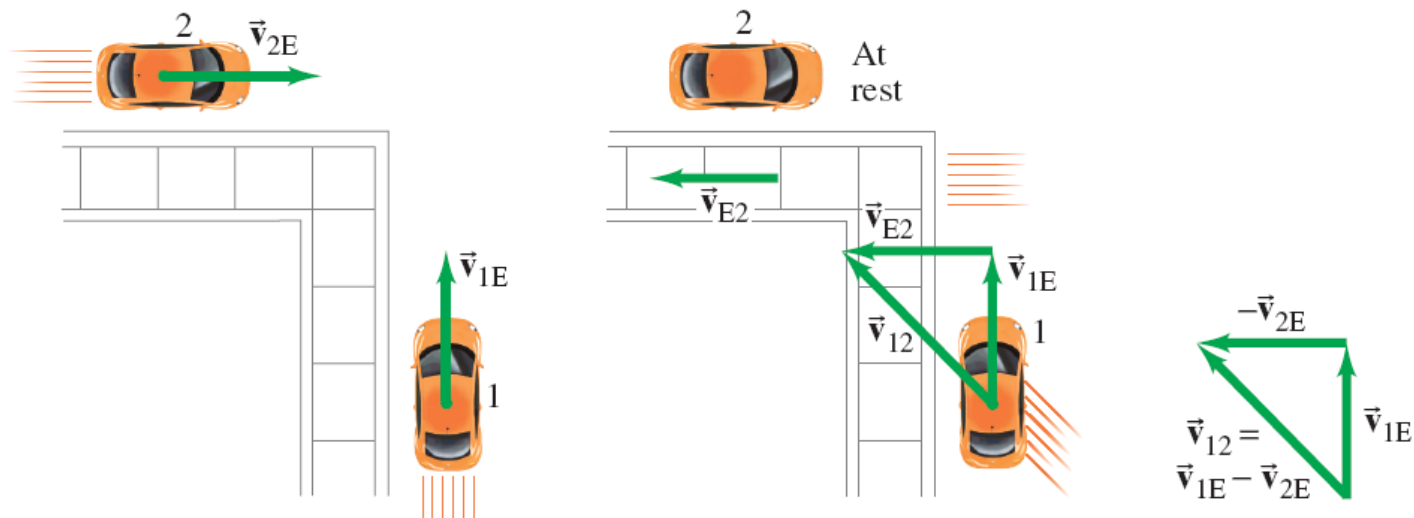
The same boat ($v_{BW} = 1.85 \text{ m/s}$) now heads directly across the river whose current is still 1.20 m/s . (a) What is the velocity (magnitude and direction) of the boat relative to the shore? (b) If the river is 110 m wide, how long will it take to cross and how far downstream will the boat be then?



3-9 Relative Velocity

Example 3-16: Car velocities at 90° .

Two automobiles approach a street corner at right angles to each other with the same speed of 40.0 km/h ($= 11.1 \text{ m/s}$), as shown. What is the relative velocity of one car with respect to the other? That is, determine the velocity of car 1 as seen by car 2.



Summary of Chapter 3

- **A quantity with magnitude and direction is a vector.**
- **A quantity with magnitude but no direction is a scalar.**
- **Vector addition can be done either graphically or by using components.**
- **The sum is called the resultant vector.**
- **Projectile motion is the motion of an object near the Earth's surface under the influence of gravity.**