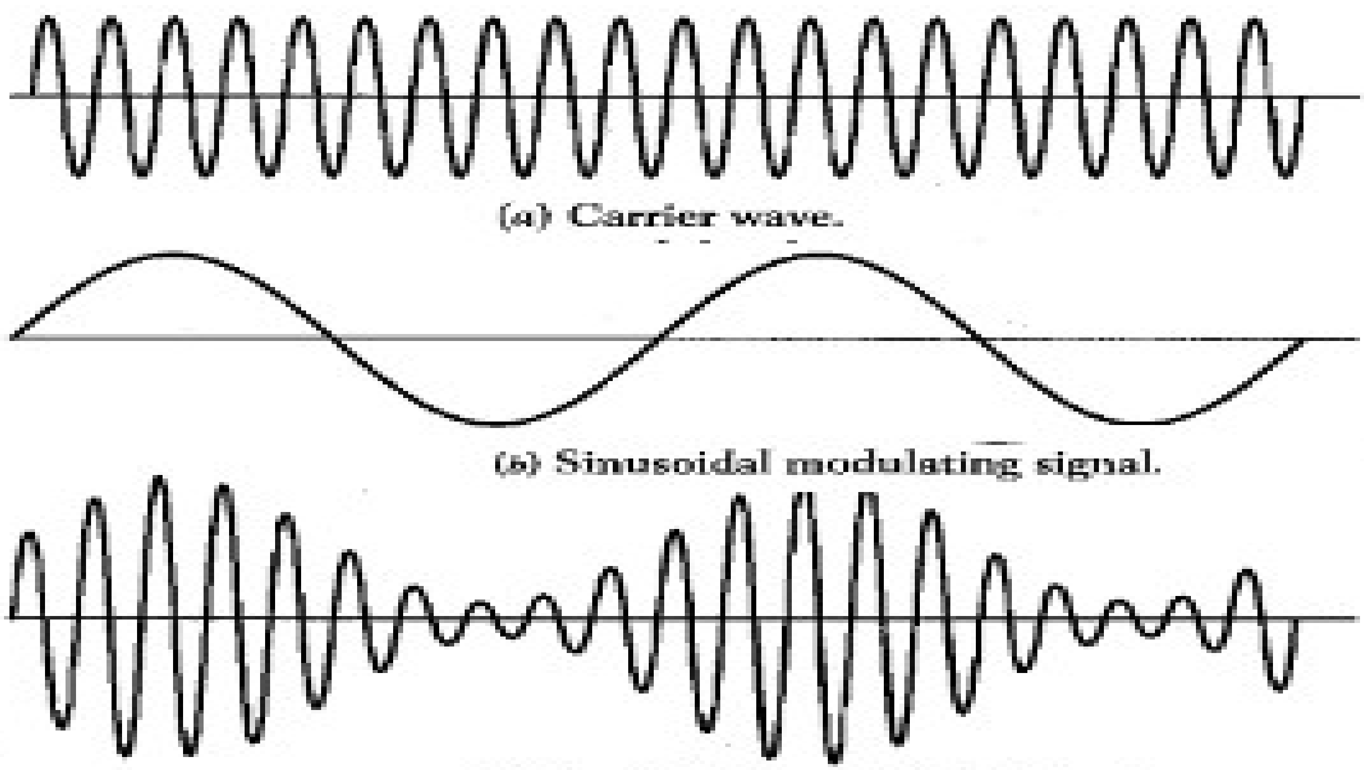
CHAPTER 3
Noise in Amplitude Modulation Systems

NOISE

Review:

- **Types of Noise**
  - **External (Atmospheric(sky),Solar(Cosmic),Hotspot)**
  - **Internal(Shot, Thermal)**
  - **Parameters of Noise**
- Signal to Noise ratio
- Noise Figure or Noise Factor
- Effective Noise temperature
- Noise Bandwidth
- **Narrow Band noise and its Components**

Representation of AM Modulated signal

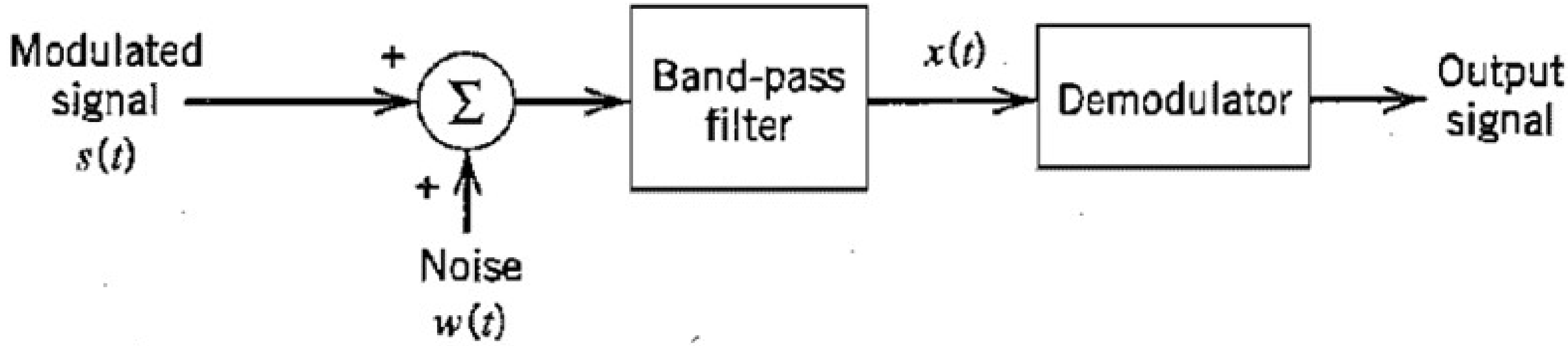


(a) Carrier wave.

(b) Sinusoidal modulating signal.

(c) Amplitude-modulated signal.

Noisy Receiver Model



- where the receiver noise is included in N_0 *given by:*

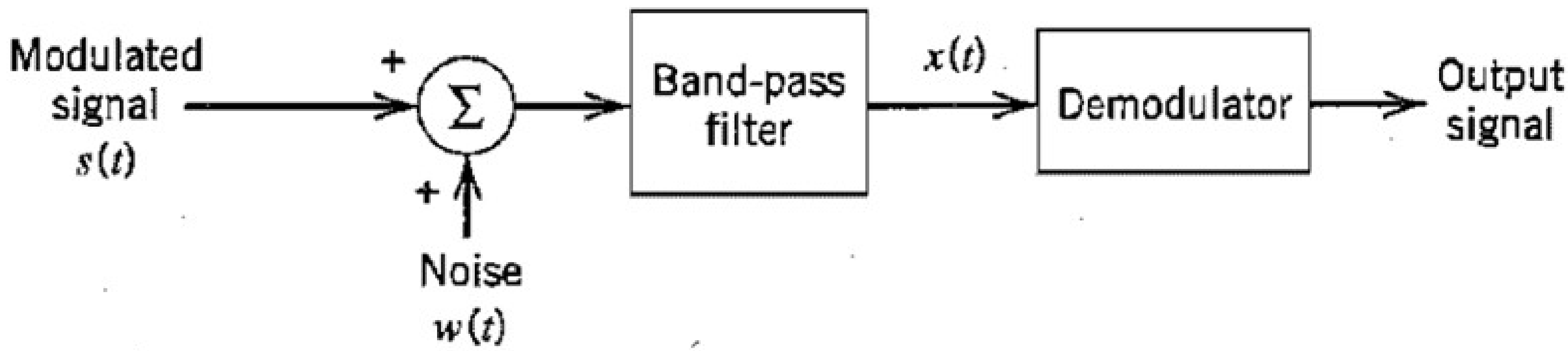
$$N_0 = kT_e$$

the bandwidth and center frequency of ideal band-pass channel filter are identical to the transmission bandwidth B_T *and the center frequency of modulated* waveform, respectively.

- The filtered noisy received signal *x(t) available for demodulation is defined by:*

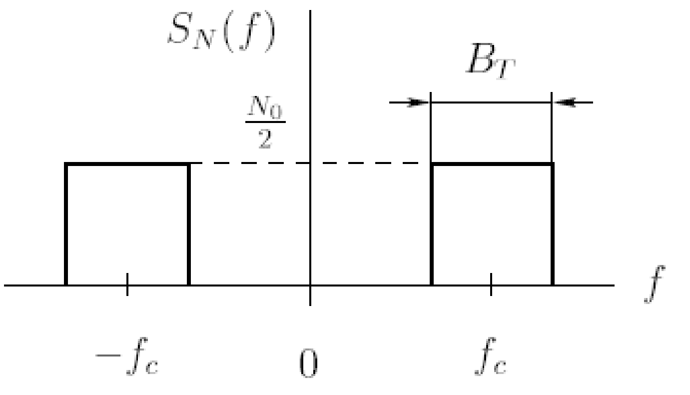
$$x(t) = s(t) + n(t)$$

- Note: Noise *n(t) is the band-pass filtered version of w(t)*



Power spectral density (PSD) of band-pass filtered noise

$$S_N(f) = |H(f)|^2 S_W(f) = |H(f)|^2 \frac{N_0}{2}$$



where $H(f)$ is the frequency response of channel filter and $S_W(f)$ denotes the psd of white noise

- The *average noise power may be calculated from the power spectral density.*
- The average power *N of filtered Gaussian white noise is:*

$$N = \sigma^2 = 2\,\frac{N_0}{2}\,B_T = N_0 B_T$$

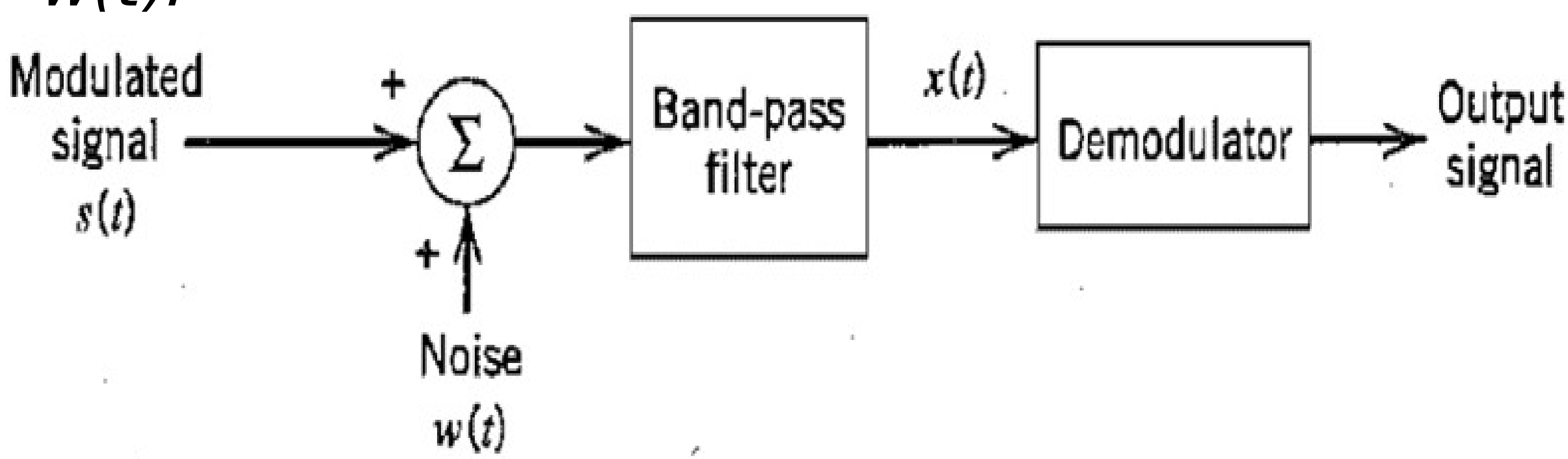
Signal to Noise Ratio (SNR)

- A measure of the degree to which a signal is contaminated with *additive noise* is the ***signal-to-noise ratio (SNR)***

$$\text{SNR} = \frac{\text{average power of a signal } s(t)}{\text{average power of noise } n(t)} = \frac{S}{N}$$

Figure of Merit Of CW Modulation Schemes

- Signal-to-noise ratio (SNR) is a measure of the degree to which a signal is contaminated by noise.
- Assume that the only source of degradation in message signal quality is the additive noise *w(t).*



- **The signal-to-noise ratio at the demodulator input**:

$$(\mathrm{SNR})_I = \frac{\text{average power of the modulated signal } s(t)}{\text{average power of noise } n(t)}$$

- **The signal-to-noise ratio at the demodulator output**:

$$(\mathrm{SNR})_O = \frac{\text{average power of the demodulated message signal } \widehat{m}(t)}{\text{average power of noise } n(t)}$$

- *$(\mathrm{SNR})_O$ is well defined only if the recovered message signal and noise appear additively at demodulator output. This condition is:*
  - *Always valid for coherent demodulators*
  - *But is valid for non-coherent demodulators only if the input signal to-* noise ratio $(\mathrm{SNR})_I$ *is high enough*
- Output signal-to-noise ratio $(\mathrm{SNR})_O$ *depends on:*
  - *Modulation scheme*
  - *Type of demodulator*

Conditions of comparison

- To get a fair comparison of CW modulation schemes and receiver configurations, it must be made on an *equal basis.*
  - *Modulated signal s(t) transmitted by each modulation scheme has* the *same average power*
  - *Channel and receiver noise w(t) has the same average power measured in the message bandwidth W*
- According to the equal basis, the **channel signal-to-noise ratio** is defined as:

$$(\mathrm{SNR})_C = \frac{\text{average power of the modulated signal } s(t)}{\text{average power of noise in the message bandwidth}}$$

$$= \frac{\text{average power of the modulated signal } s(t)}{N_0 W}$$

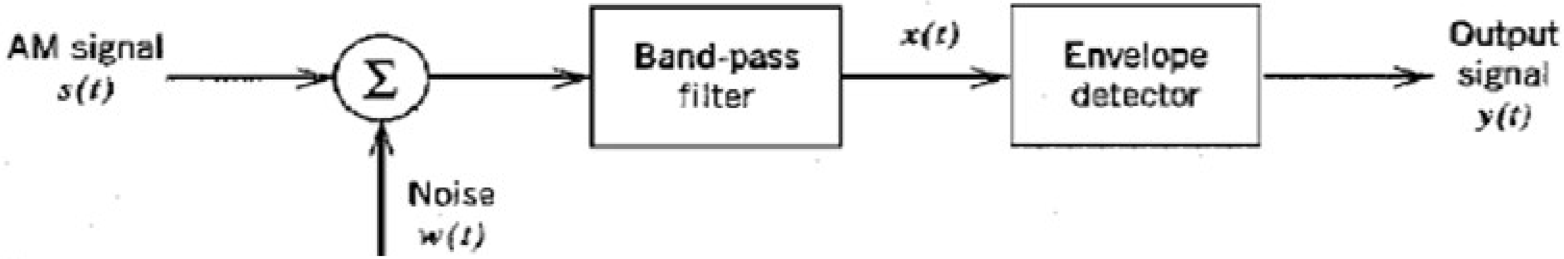
- Noise performance of a given CW modulation scheme and a given type of demodulator is characterized by the ***figure of merit.***

- By definition, the figure of merit is:

$$\text{Figure of merit} = \frac{(\text{SNR})_O}{(\text{SNR})_C}$$

- The higher the value of the figure of merit, the better the noise performance

Noise in AM DSB-FC Receivers



- *AM signal*

$$s(t) = A_C[1 + k_a m(t)]\cos(2\pi f_C t)$$

- Average signal power $= A_C^2(1 + k_a^2 P)/2$
- Average noise power $= WN_0 \quad \leftarrow \quad (2W \times \frac{N_0}{2})$

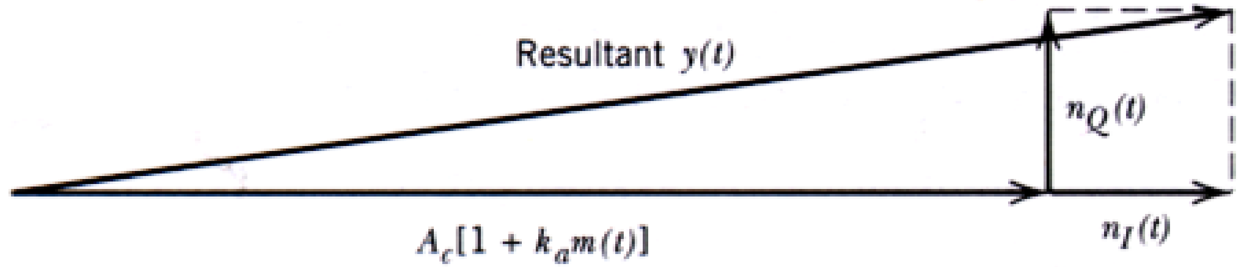
$$(SNR)_{C,AM} = \frac{A_C^2(1 + k_a^2 P)}{2WN_0}$$

- *Filtered* signal

$$x(t) = s(t) + n(t)$$

$$= [A_C + A_C k_a m(t) + n_I(t)]\cos(2\pi f_C t) - n_Q(t)\sin(2\pi f_C t)$$

$y(t) = \text{envelop of } x(t)$



$$= \left\{[A_C + A_C k_a m(t) + n_I(t)]^2 + n_Q^2(t)\right\}^{\frac{1}{2}}$$

$$\textit{Assume} \quad A_C[1 + k_a m(t)] \gg n_I(t), n_Q(t)$$

$$y(t) \cong A_C + A_C k_a m(t) + n_I(t)$$

- $$(SNR)_{O,AM} = \frac{A_C^2 k_a^2 P}{2WN_0}$$

$$\begin{cases} \text{Avg carrier power} > \text{Avg noise power} \\ k_a \leq 1 \end{cases}$$

$$\text{Figure of merit} \quad \left.\frac{(SNR)_O}{(SNR)_C}\right|_{AM} \cong \frac{k_a^2 P}{1 + k_a^2 P} < 1$$

Threshold effect

- The ***threshold is a value of carrier-to-noise ratio below which*** *the* noise performance of a demodulator deteriorates much more rapidly than proportionately to the carrier-to-noise ratio.

- **Every noncoherent detector** exhibits a threshold effect, below the threshold the restored message signal becomes practically useless.

Threshold effect

Physical explanation:

- **If the carrier-to-noise ratio is high enough** then the signal dominates and the noise causes only a small unwanted AM and PM.

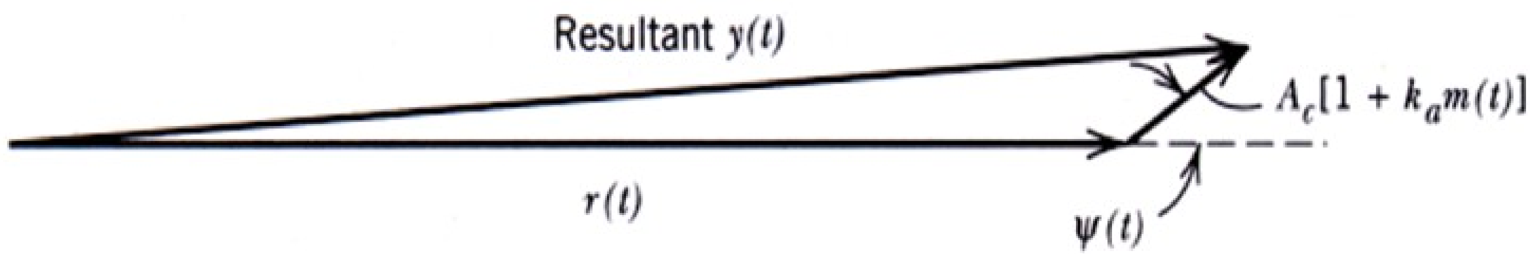
- However, **if the carrier-to-noise ratio is small** then the noise dominates which results in a complete loss of information.

- As a result, **the demodulator output does not contain the message signal at all.**

$$n(t) = r(t)\cos[2\pi f_C(t) + \psi(t)]$$

where $r(t)$ is envelope, $\psi(t)$ is phase

$$x(t) = s(t) + n(t)$$
$$= A_C[1 + k_a m(t)]\cos(2\pi f_C t) + r(t)\cos(2\pi f_C t + \psi(t))$$



$$y(t) \cong r(t) + A_C[1 + k_a m(t)]\cos[\psi(t)]$$
$$\cong r(t) + A_C\cos[\psi(t)] + A_C k_a m(t)\cos[\psi(t)]$$

where $\psi(t)$ is uniformly distributed over $[0, 2\pi]$

$\Rightarrow$ complete loss of information

Threshold Effect : loss of message in an envelope detector that operates at a *low CNR.*

- Figure of merit for DSB modulation:

$$\text{Figure of merit} = \frac{(\text{SNR})_O}{(\text{SNR})_C} = \frac{k_a^2 P}{1 + k_a^2 P}$$

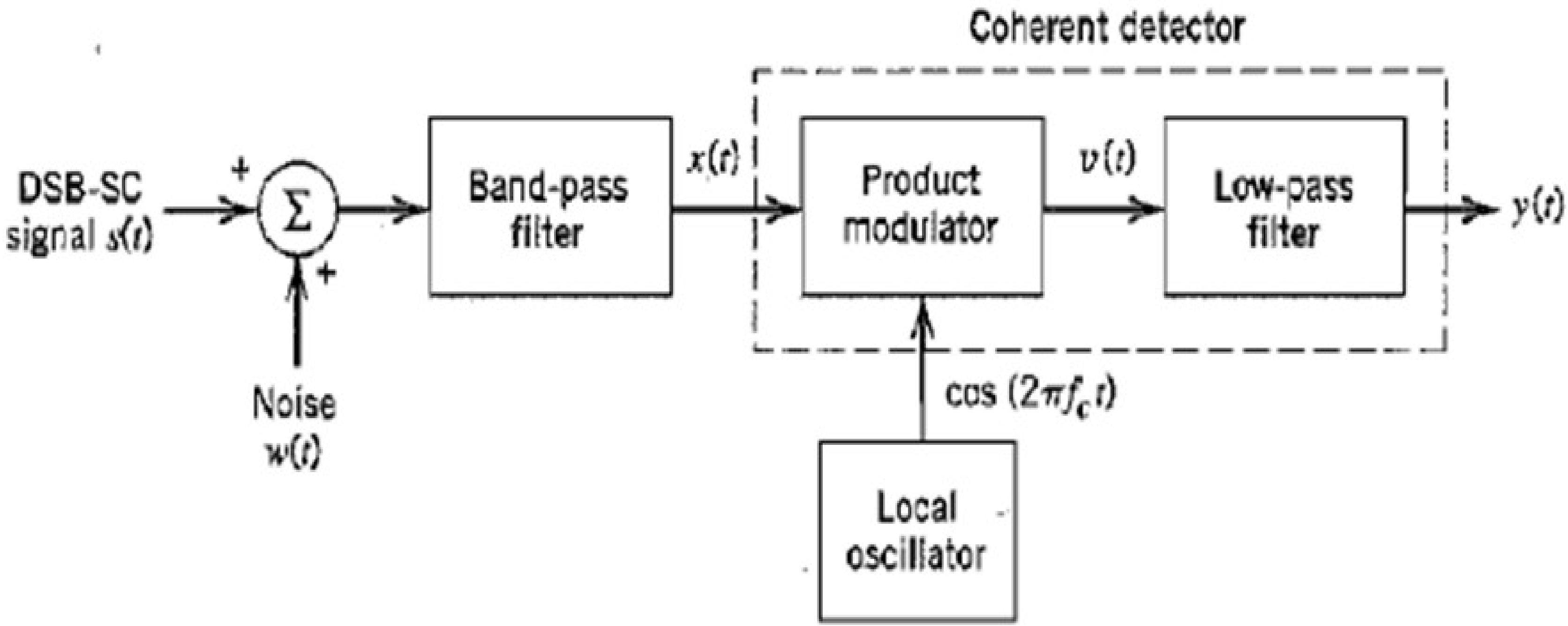
where ***P denotes the average power*** of message signal *m(t) and* ***ka is the amplitude sensitivity*** of AM modulator.

- The best figure of merit is achieved if the modulation factor is $\boldsymbol{\mu = k_a A_m = 1}$

$$\text{Figure of merit} = \frac{\mu^2}{2 + \mu^2} = \frac{1}{3}$$

- DSB sy e detection must transmit three times as much average power as a suppressed-carrier system

Noise in AM DSB-SC Receivers



- $s(t) = CA_C \cos(2\pi f_C t)m(t)$

  where C : scaling factor

  Power spectral density of m(t) : $S_M(f)$

  W : message bandwidth

- Average signal power

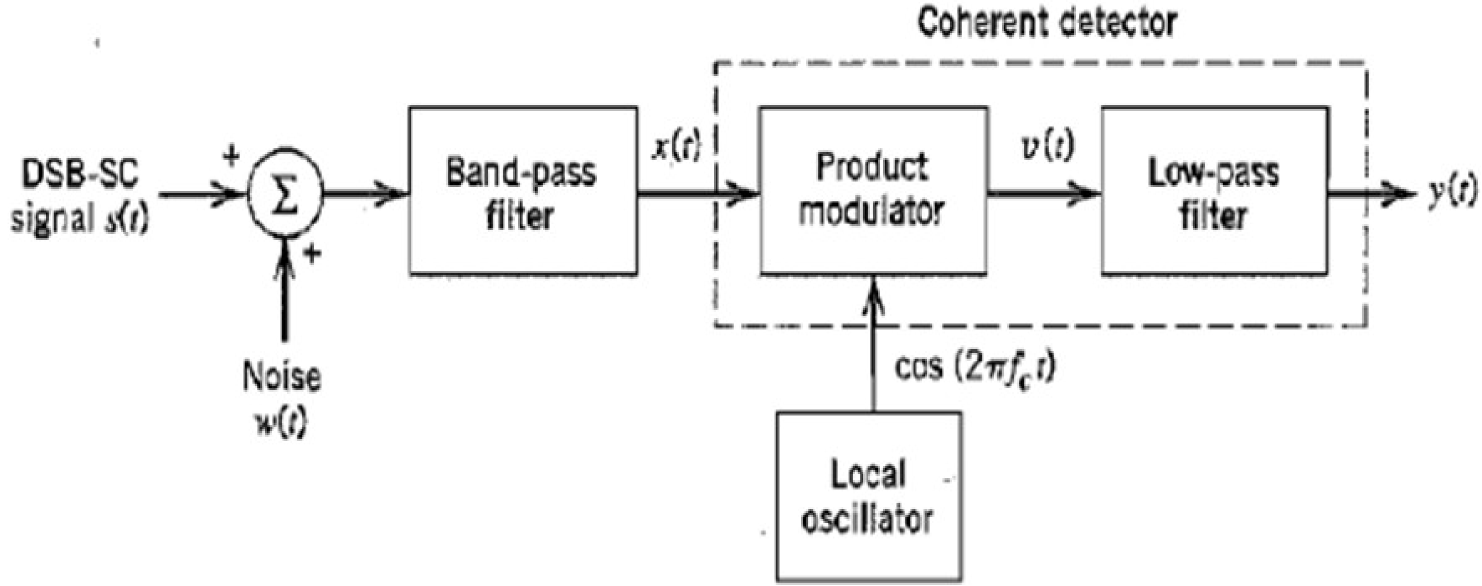
  $$P = \int_{-W}^{W} S_M(f)\,df$$

- Average power of $s(t) = \dfrac{C^2 A_C^2 P}{2}$

- Average noise power $= 2W \times \dfrac{N_0}{2} = WN_0$

  (baseband)

- $$(SNR)_{C,DSB} = \frac{C^2 A_C^2 P}{2WN_0}$$



Finding $(SNR)_O$

- $x(t) = s(t) + n(t)$

$$= CA_C \cos(2\pi f_C t) m(t) + n_I(t)\cos(2\pi f_C t) - n_Q(t)\sin(2\pi f_C t)$$

- $v(t) = x(t)\cos(2\pi f_C t)$

$$= \frac{1}{2} CA_C m(t) + \frac{1}{2} n_I(t) + \frac{1}{2}\left[CA_C m(t) + n_I(t)\right]\cos(4\pi f_C t) - \frac{1}{2} A_C n_Q(t)\sin(4\pi f_C t)$$

$$\therefore\ y(t) = \frac{1}{2} CA_C m(t) + \frac{1}{2} n_I(t)$$

- Average signal power $= \dfrac{C^2 A_C^2 P}{4}$

- Average noise power $= \dfrac{1}{4}(2W)N_0 = \dfrac{1}{2} WN_0$ (passband)

$\because$ Power$(n_I(t)) =$ Power of band pass filtered noise $n(t) = 2WN_0$

- $\therefore$ $(SNR)_O = \dfrac{C^2 A_C^2 P/4}{WN_0/2} = \dfrac{C^2 A_C^2 P}{2WN_0}$

$\therefore$ Figure of merit

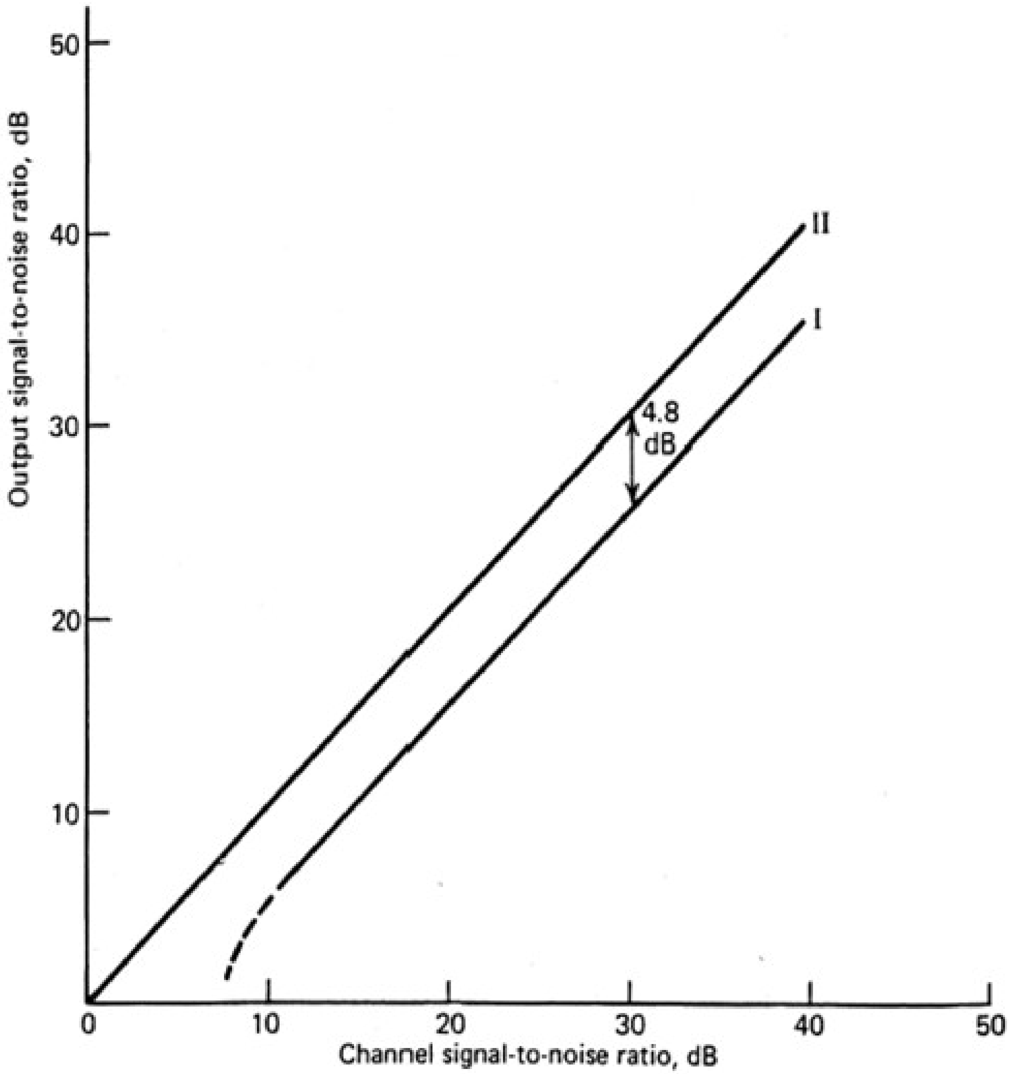
$$\left.\frac{(SNR)_O}{(SNR)_C}\right|_{DSB-SC} = 1$$

Noise performance of AM receivers

		Type of demodulator			
		Coherent		Noncoherent	
		Figure of merit	Threshold effect	Figure of merit	Threshold effect
Type of CW modulation	DSB	$\frac{k_a^2 P}{1+k_a^2 P} \leq \frac{1}{3}$	no	$\frac{k_a^2 P}{1+k_a^2 P} \leq \frac{1}{3}$	yes
	DSB–SC	1	no	$\times$	$\times$
	SSB	1	no	$\times$	$\times$

Note: For high value of (SNR)$_C$, *the noise performance of coherent and* noncoherent DSB are identical. But noncoherent DSB has a *threshold effect*. Coherent AM detectors have no threshold effect!

Comparison of noise performance of AM modulation schemes



Remarks

- ***Curve I:*** *DSB modulation and envelope detector with modulation factor μ = 1*
- ***Curve II:*** *DSB–SC and SSB with coherent demodulator*
- ***Note the threshold effect that appears at about 10 dB***