

CHAPTER.3: Transistor at Low Frequencies

- Introduction
- Amplification in the AC domain
- BJT transistor modeling
- The re Transistor Model
- The Hybrid equivalent Model

Introduction

- There are three models commonly used in the small – signal ac analysis of transistor networks:
- The re model
- The hybrid π model
- The hybrid equivalent model

Amplification in the AC domain

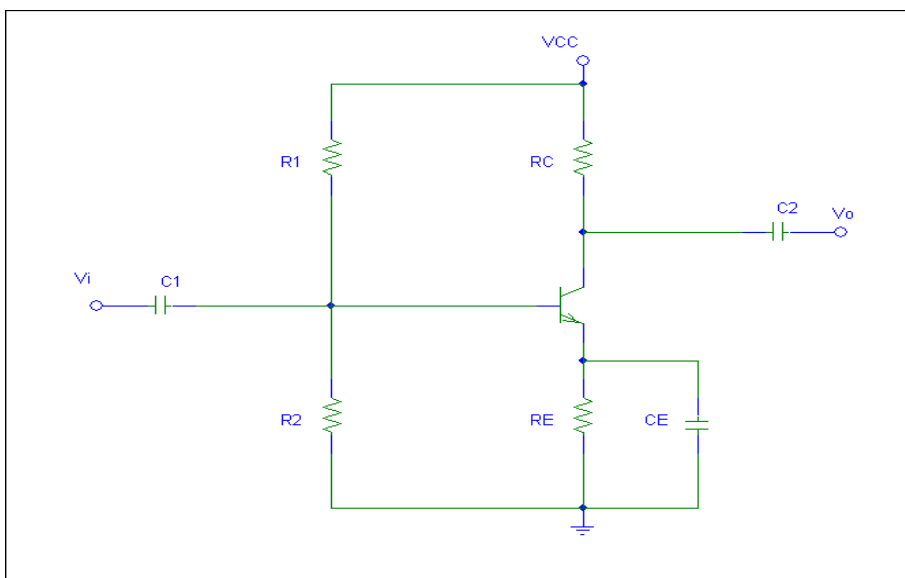
The transistor can be employed as an amplifying device, that is, the output ac power is greater than the input ac power. The factor that permits an ac power output greater than the input ac power is the applied DC power. The amplifier is initially biased for the required DC voltages and currents. Then the ac to be amplified is given as input to the amplifier. If the applied ac exceeds the limit set by dc level, clipping of the peak region will result in the output. Thus, proper (faithful) amplification design requires that the dc and ac components be sensitive to each other's requirements and limitations. The superposition theorem is applicable for the analysis and design of the dc and ac components of a BJT network, permitting the separation of the analysis of the dc and ac responses of the system.

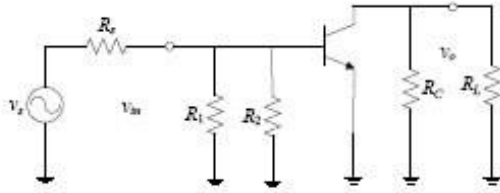
BJT Transistor modeling

- The key to transistor small-signal analysis is the use of the equivalent circuits (models). A MODEL IS A COMBINATION OF CIRCUIT ELEMENTS LIKE VOLTAGE OR CURRENT SOURCES, RESISTORS, CAPACITORS etc. that best approximates the behavior of a device under specific operating conditions. Once the model (ac equivalent circuit) is determined, the schematic symbol for the device can be replaced by the equivalent circuit and the basic methods of circuit analysis applied to determine the desired quantities of the network.
- Hybrid equivalent network – employed initially. Drawback – It is defined for a set of operating conditions that might not match the actual operating conditions.
- re model: desirable, but does not include feedback term
- Hybrid π model: model of choice.

AC equivalent of a network

- AC equivalent of a network is obtained by:
- Setting all dc sources to zero and replacing them by a short – circuit equivalent
- Replacing all capacitors by short – circuit equivalent
- Removing all elements bypassed by the short – circuit equivalents
- Redrawing the network in a more convenient and logical form.



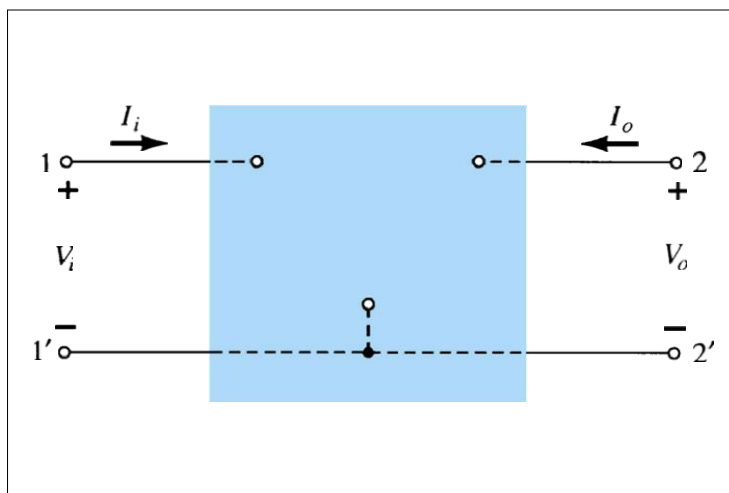


re model

- In re model, the transistor action has been replaced by a single diode between emitter and base terminals and a controlled current source between base and collector terminals.
- This is rather a simple equivalent circuit for a device

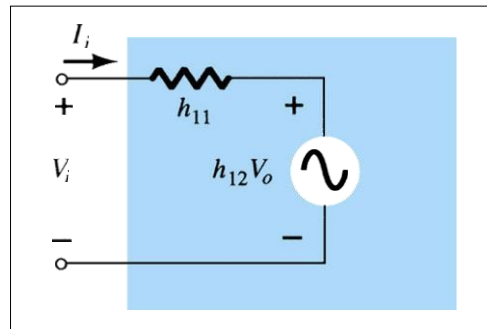
The Hybrid equivalent model

- For the hybrid equivalent model, the parameters are defined at an operating point.
- The quantities h_{ie} , h_{re} , h_{fe} , and h_{oe} are called hybrid parameters and are the components of a small – signal equivalent circuit.
- The description of the hybrid equivalent model will begin with the general two port system.

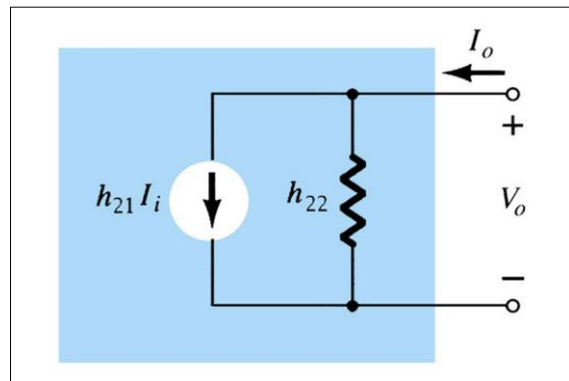


- The set of equations in which the four variables can be related are:
- $V_i = h_{11}I_i + h_{12}V_o$
- $I_o = h_{21}I_i + h_{22}V_o$
- The four variables h_{11} , h_{12} , h_{21} and h_{22} are called hybrid parameters (the mixture of variables in each equation results in a “ hybrid” set of units of measurement for the h – parameters.
- Set $V_o = 0$, solving for h_{11} , $h_{11} = V_i / I_i$ Ohms
- This is the ratio of input voltage to the input current with the output terminals shorted. It is called Short circuit input impedance parameter.
- If I_i is set equal to zero by opening the input leads, we get expression for h_{12} :
 $h_{12} = V_i / V_o$, This is called open circuit reverse voltage ratio.
- Again by setting V_o to zero by shorting the output terminals, we get
 $h_{21} = I_o / I_i$ known as short circuit forward transfer current ratio.
- Again by setting $I_i = 0$ by opening the input leads, $h_{22} = I_o / V_o$. This is known as open – circuit output admittance. This is represented as resistor ($1/h_{22}$)
- $h_{11} = h_i =$ input resistance
- $h_{12} = h_r =$ reverse transfer voltage ratio
- $h_{21} = h_f =$ forward transfer current ratio
- $h_{22} = h_o =$ Output conductance

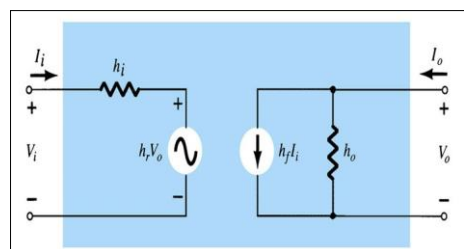
Hybrid Input equivalent circuit



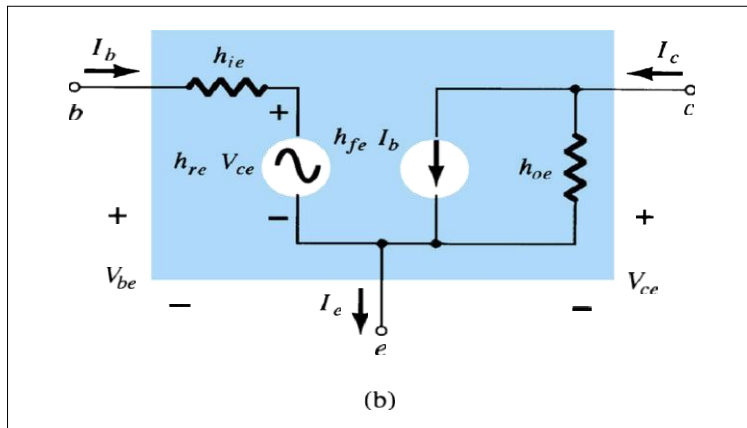
Hybrid output equivalent circuit



Complete hybrid equivalent circuit

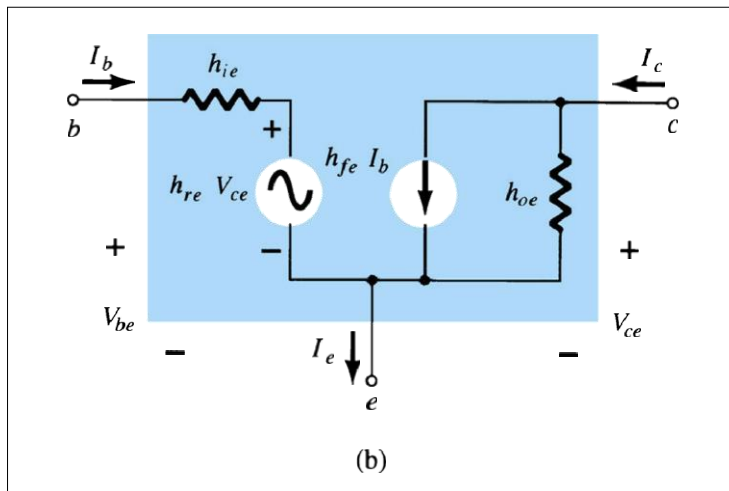


Common Emitter Configuration - hybrid equivalent circuit



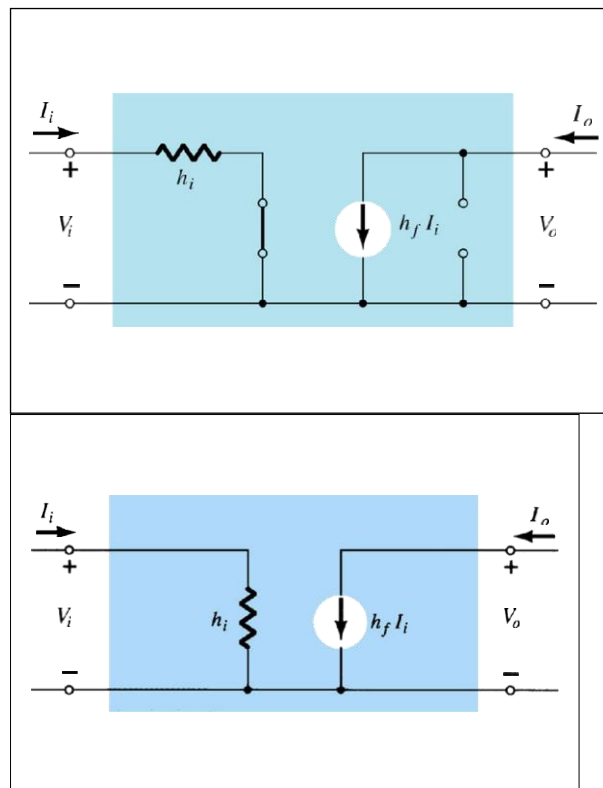
- Essentially, the transistor model is a three terminal two – port system.
- The h – parameters, however, will change with each configuration.
- To distinguish which parameter has been used or which is available, a second subscript has been added to the h – parameter notation.
- For the common – base configuration, the lowercase letter b is added, and for common emitter and common collector configurations, the letters e and c are used respectively.

Common Base configuration - hybrid equivalent circuit

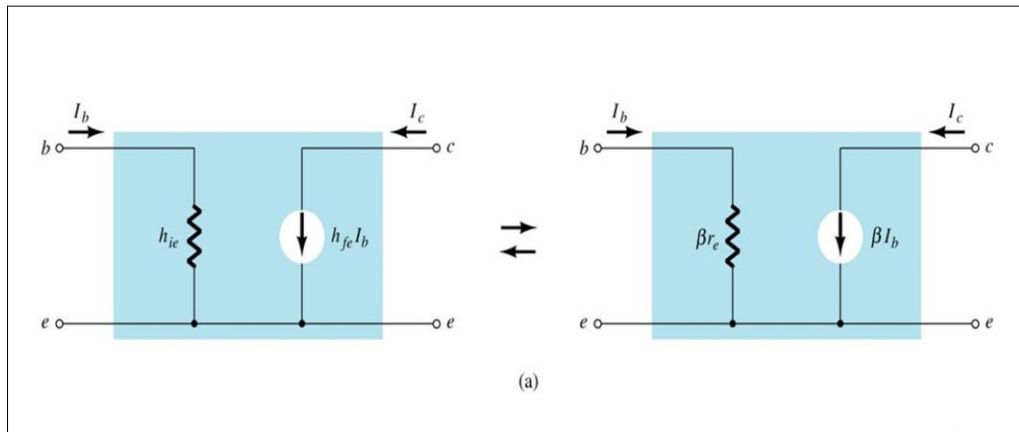


Configuration	I_i	I_o	V_i	V_o
Common emitter	I_b	I_c	V_{be}	V_{ce}
Common base	I_e	I_c	V_{eb}	V_{cb}
Common Collector	I_b	I_e	V_{be}	V_{ec}

- Normally h_r is a relatively small quantity, its removal is approximated by $h_r \cong 0$ and $h_r V_o = 0$, resulting in a short – circuit equivalent.
- The resistance determined by $1/h_o$ is often large enough to be ignored in comparison to a parallel load, permitting its replacement by an open – circuit equivalent.



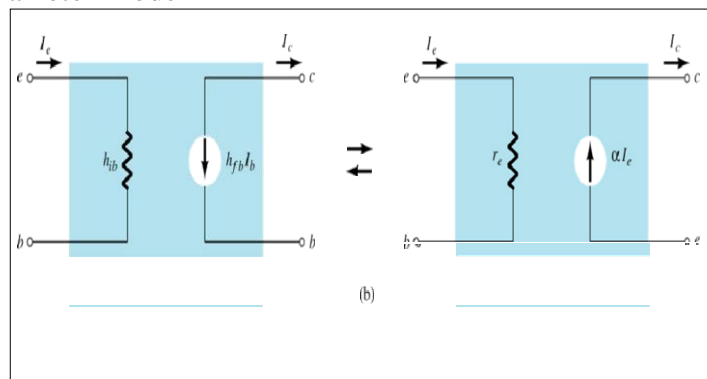
h-Parameter Model v/s. re Model



$$h_{ie} = \beta r_e$$

$$h_{fe} = \beta a_c$$

Common Base: re v/s. h-Parameter Model



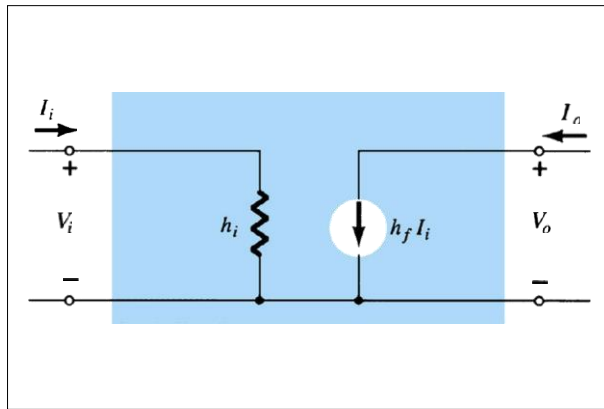
Common-Base configurations - h-Parameters

$$h_{ib} = r_e$$

$$h_{fb} = -\alpha = -1$$

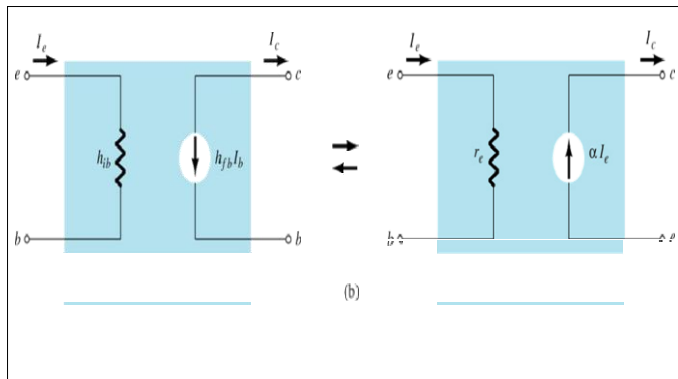
Problem

- Given $I_E = 3.2\text{mA}$, $h_{fe} = 150$, $h_{oe} = 25\mu\text{S}$ and $h_{ob} = 0.5 \mu\text{S}$. Determine
- The common – emitter hybrid equivalent
- The common – base re model



Solution:

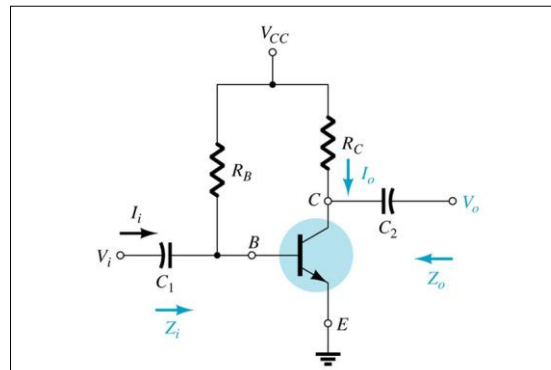
- We know that, $h_{ie} = \beta r_e$ and $r_e = 26\text{mV}/I_E = 26\text{mV}/3.2\text{mA} = 8.125\Omega$
- $\beta r_e = (150)(8.125) = 1218.75\text{k}\Omega$
- $r_o = 1/h_{oe} = 1/25\mu\text{S} = 40\text{k}\Omega$



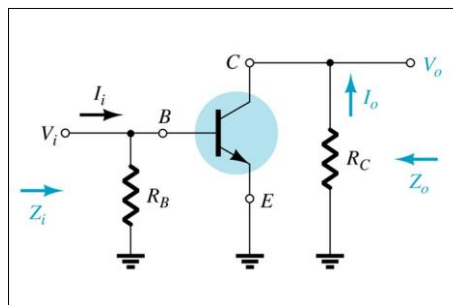
- $r_e = 8.125\Omega$
- $r_o = 1/h_{ob} = 1/0.5\mu\text{S} = 2\text{M}\Omega$
- $\alpha \cong 1$

- Small signal ac analysis includes determining the expressions for the following parameters in terms of Z_i , Z_o and A_V in terms of
 - β
 - r_e
 - r_o and
 - R_B, R_C
- Also, finding the phase relation between input and output
- The values of β , r_o are found in datasheet
- The value of r_e must be determined in dc condition as $r_e = 26\text{mV} / I_E$

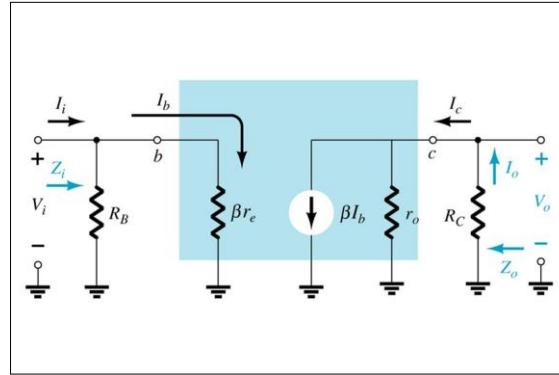
Common Emitter - Fixed bias configuration



Removing DC effects of VCC and Capacitors



re model



Small signal analysis – fixed bias

- From the above re model,

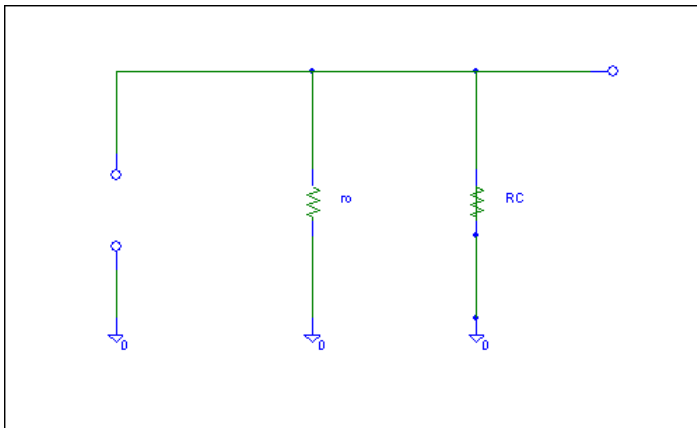
$$Z_i = [R_B \parallel \beta r_e] \text{ ohms}$$

If $R_B > 10 \beta r_e$, then,

$$[R_B \parallel \beta r_e] \cong \beta r_e$$

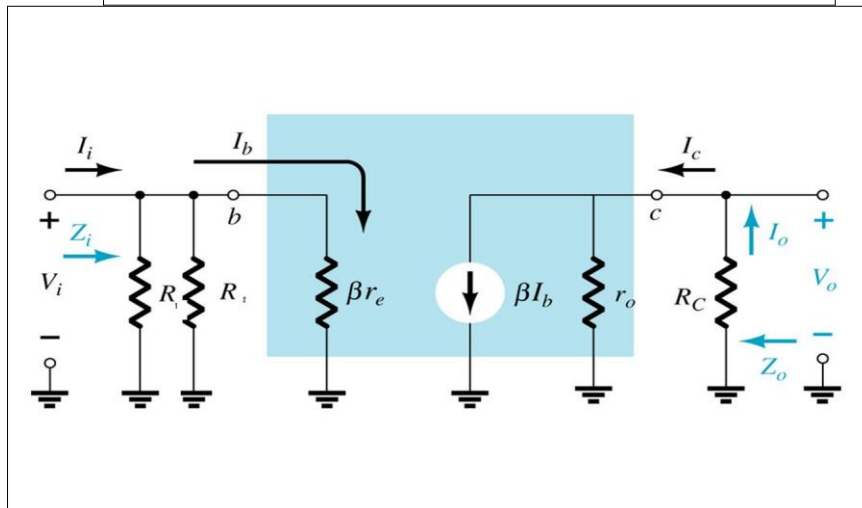
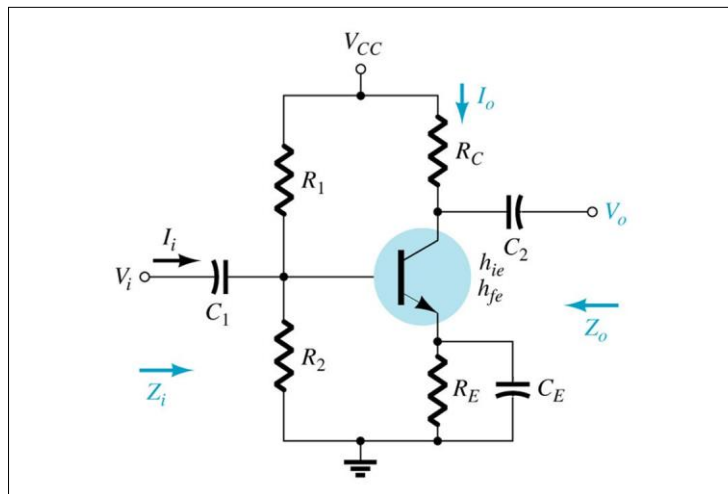
Then, $Z_i \cong \beta r_e$

- Z_o is the output impedance when $V_i = 0$. When $V_i = 0$, $i_b = 0$, resulting in open circuit equivalence for the current source.



- $Z_o = [RC \parallel r_o]$ ohms
- A_V
- $V_o = -\beta I_b (RC \parallel r_o)$
- From the r_e model, $I_b = V_i / \beta r_e$
- thus,
- $V_o = -\beta (V_i / \beta r_e) (RC \parallel r_o)$
- $A_V = V_o / V_i = - (RC \parallel r_o) / r_e$
- If $r_o > 10RC$,
- $A_V = - (RC / r_e)$
- The negative sign in the gain expression indicates that there exists 180° phase shift between the input and output.

Common Emitter - Voltage-Divider Configuration



- The re model is very similar to the fixed bias circuit except for R_B is $R_1 \parallel R_2$ in the case of voltage divider bias.

- Expression for A_V remains the same.

$$Z_i = R_1 \parallel R_2 \parallel \beta r_e$$

$$Z_o = R_C$$

- From the re model, $I_b = V_i / \beta r_e$

- thus,

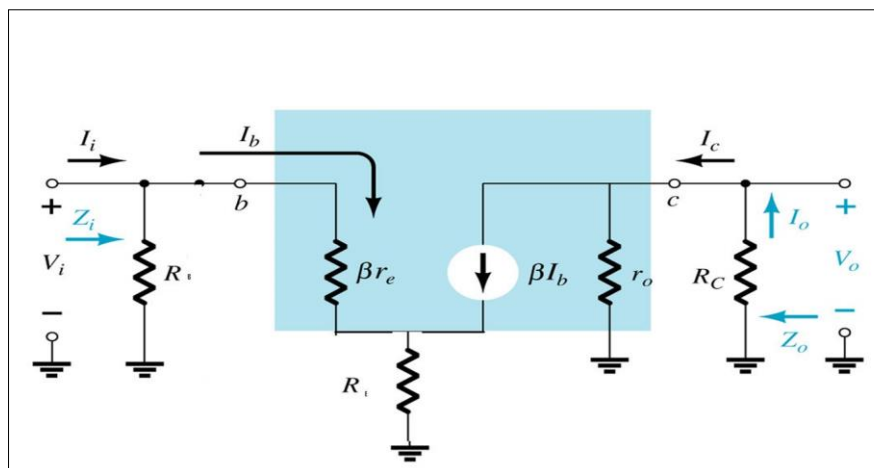
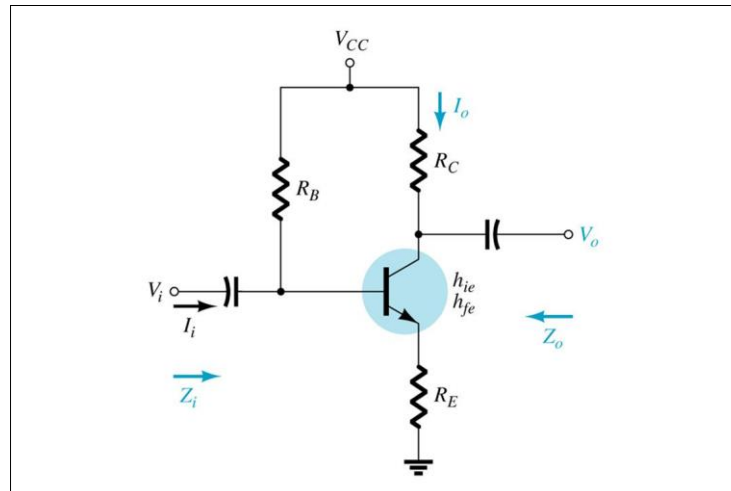
$$V_o = - \beta (V_i / \beta r_e) (R_C \parallel r_o)$$

- $AV = V_o / V_i = - (R_C \parallel r_o) / r_e$

If $r_o > 10R_C$,

$$AV = - (R_C / r_e)$$

Common Emitter - Unbypassed Emitter-Bias Configuration



- Applying KVL to the input side:

$$V_i = I_b \beta r_e + I_e R_E$$

$$V_i = I_b \beta r_e + (\beta + 1) I_b R_E$$

Input impedance looking into the network to the right of R_B is

$$Z_b = V_i / I_b = \beta r_e + (\beta + 1) R_E$$

Since $\beta \gg 1$, $(\beta + 1) r_e = \beta r_e$

Thus,

$$Z_b = V_i / I_b = \beta (r_e + R_E)$$

- Since R_E is often much greater than r_e , $Z_b = \beta R_E$,

$$Z_i = R_B || Z_b$$

• Z_o is determined by setting V_i to zero, $I_b = 0$ and βI_b can be replaced by open circuit equivalent. The result is,

- $Z_o = R_C$

• A_V : We know that, $V_o = - I_o R_C$

$$= - \beta I_b R_C$$

$$= - \beta (V_i / Z_b) R_C$$

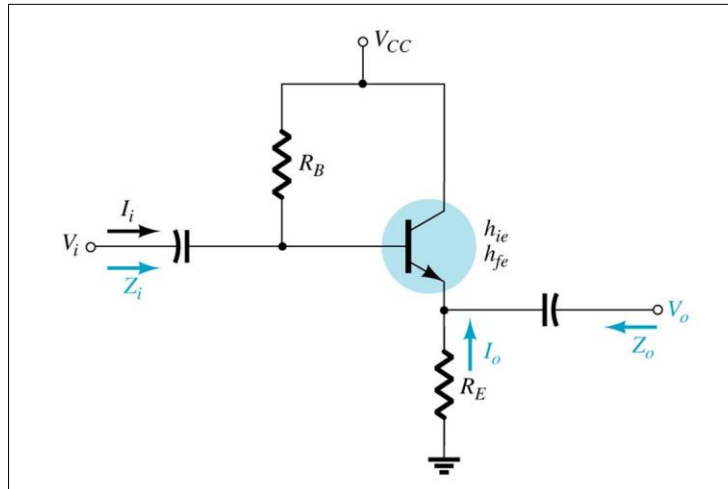
$$A_V = V_o / V_i = - \beta (R_C / Z_b)$$

Substituting, $Z_b = \beta (r_e + R_E)$

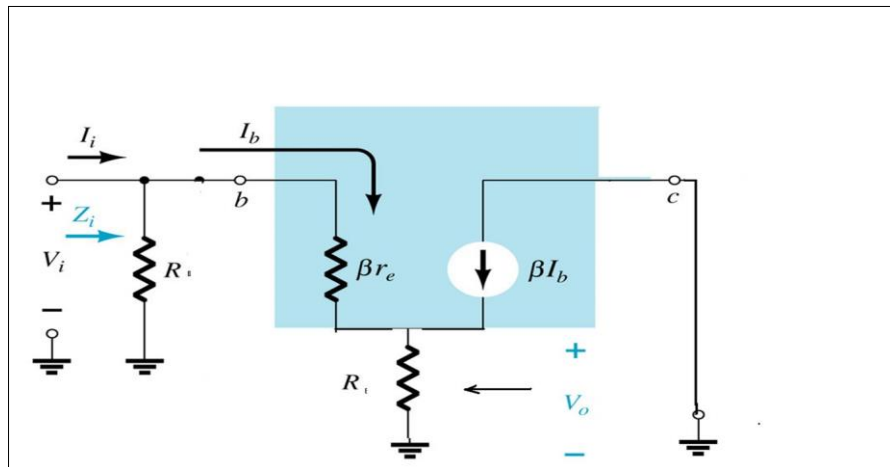
$$A_V = V_o / V_i = - \beta [R_C / (r_e + R_E)] \quad R_E \gg r_e, A_V = V_o / V_i = - \beta [R_C / R_E]$$

- Phase relation: The negative sign in the gain equation reveals a 180° phase shift between input and output.

Emitter – follower



r_e model



- $Z_i = R_B \parallel Z_b$
- $Z_b = \beta r_e + (\beta + 1)R_E$
- $Z_b = \beta(r_e + R_E)$
- Since R_E is often much greater than r_e , $Z_b = \beta R_E$

- To find Z_o , it is required to find output equivalent circuit of the emitter follower at its input terminal.

- This can be done by writing the equation for the current I_b .

$$I_b = V_i / Z_b$$

$$I_e = (\beta + 1)I_b$$

$$= (\beta + 1) (V_i / Z_b)$$

- We know that, $Z_b = \beta r_e + (\beta + 1)R_E$ substituting this in the equation for I_e we get,

$$I_e = (\beta + 1) (V_i / Z_b)$$

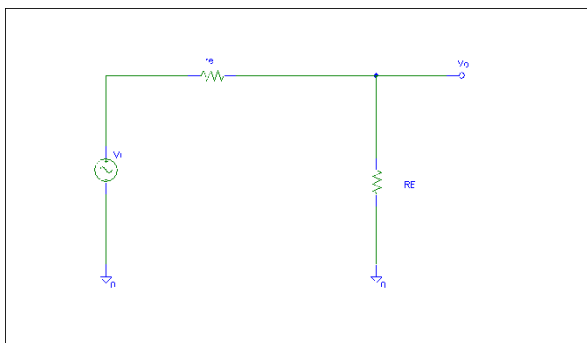
$$= (\beta + 1) (V_i / \beta r_e + (\beta + 1)R_E)$$

$$I_e = V_i / [\beta r_e / (\beta + 1)] + R_E$$

- Since $(\beta + 1) = \beta$,

$$I_e = V_i / [r_e + R_E]$$

- Using the equation $I_e = V_i / [r_e + R_E]$, we can write the output equivalent circuit as,



-

- As per the equivalent circuit,

$$Z_o = R_E || r_e$$

- Since R_E is typically much greater than r_e , $Z_o \cong r_e$

- A_V – Voltage gain:

- Using voltage divider rule for the equivalent circuit, $V_o = V_i R_E / (R_E + r_e)$

$$A_V = V_o / V_i = [R_E / (R_E + r_e)]$$

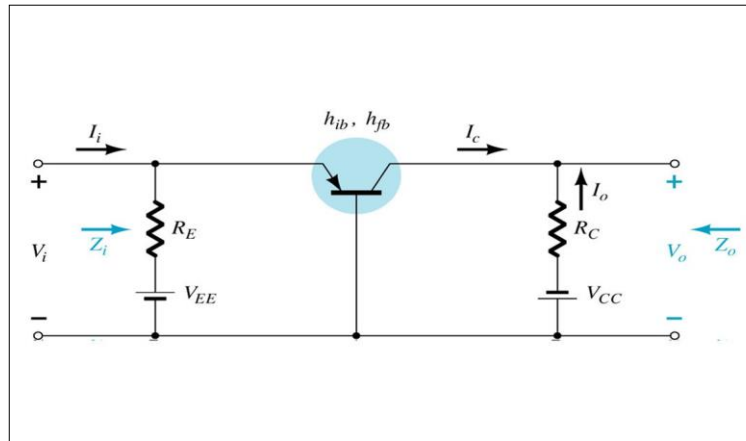
- Since $(R_E + r_e) \cong R_E$,

$$A_V \cong [R_E / R_E] \cong 1$$

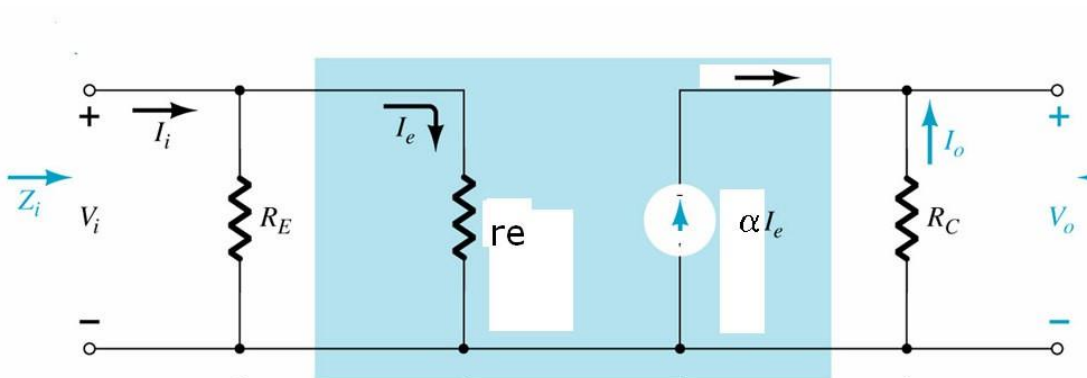
- Phase relationship

As seen in the gain equation, output and input are in phase.

Common base configuration



re model



Small signal analysis

- Input Impedance: $Z_i = R_E || r_e$
- Output Impedance: $Z_o = RC$
- To find, Output voltage, $V_o = - I_o RC$

$$V_o = - (-I_C)RC = \alpha I_e RC$$

o $I_e = V_i / r_e$, substituting this in the above equation, $V_o = \alpha (V_i / r_e) RC$

$$V_o = \alpha (V_i / r_e) RC$$

Voltage Gain: A_V :

$$A_V = V_o / V_i = \alpha (RC / r_e)$$

$$\alpha \cong 1; A_V = (RC / r_e)$$

Current gain

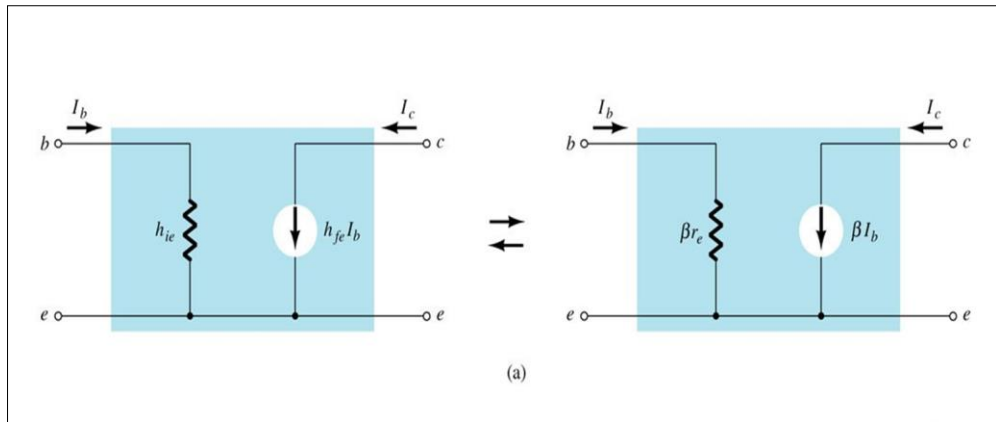
$$A_i = I_o / I_i$$

$$I_o = - \alpha I_e = - \alpha I_i$$

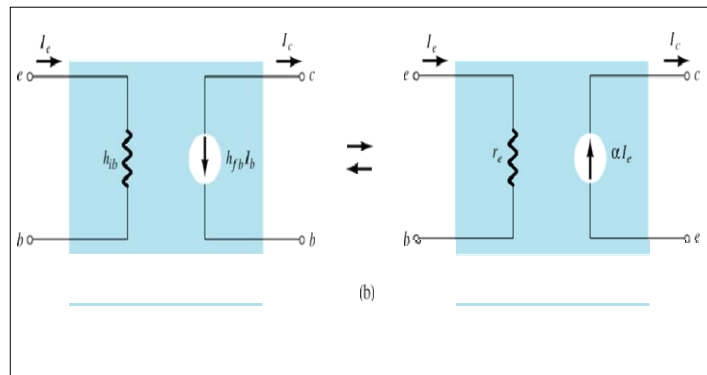
$$I_o / I_i = - \alpha \cong -1$$

Phase relation: Output and input are in phase.

h-Parameter Model vs. re Model



- CB re vs. h-Parameter Model



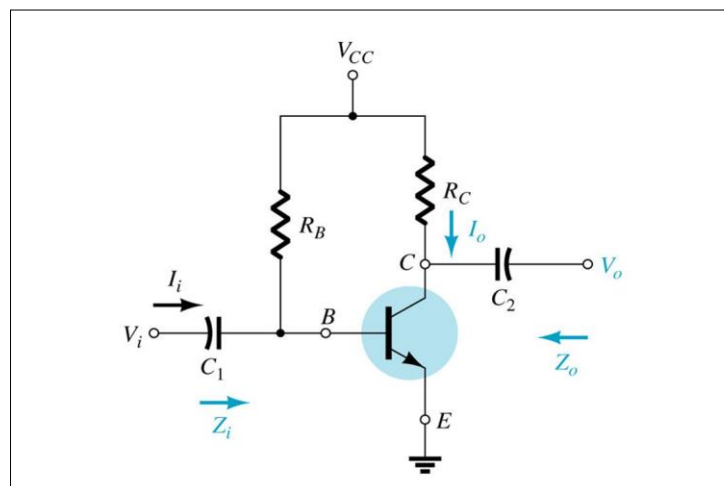
Common-Base h-Parameters

$$h_{ib} = r_e$$

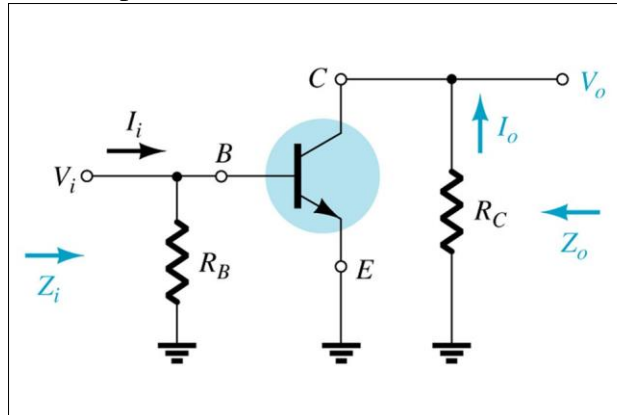
$$h_{fb} = -\alpha \cong -1$$

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- Also, finding the phase relation between input and output
- The values of β , r_o are found in datasheet
- The value of r_e must be determined in dc condition as $r_e = 26\text{mV} / I_E$

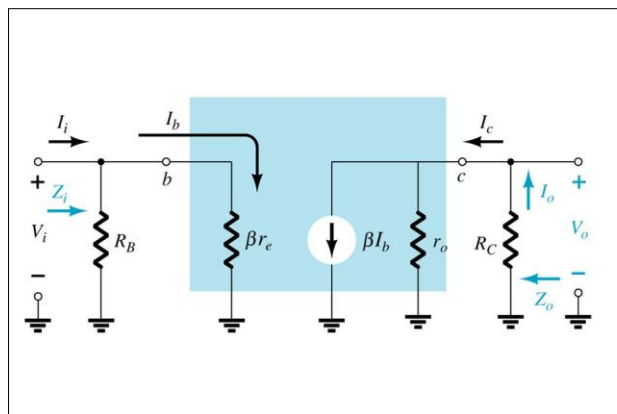
Common Emitter Fixed bias configuration



Removing DC effects of VCC and Capacitors



re model



Small signal analysis – fixed bias

Input impedance Z_i :

From the above re model, is,

$$Z_i = [R_B \parallel \beta r_e] \text{ ohms}$$

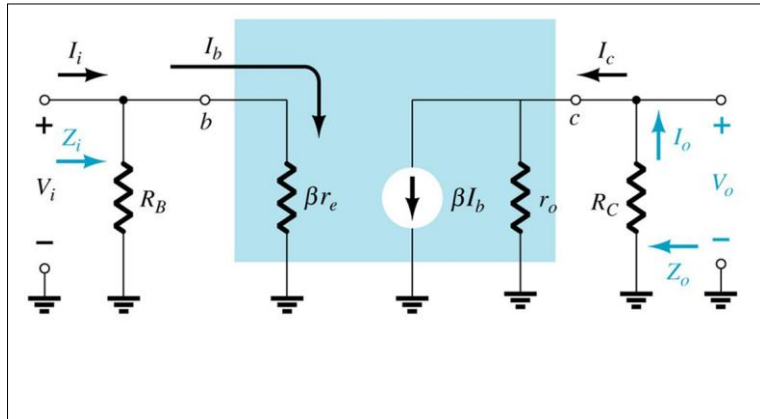
If $R_B > 10 \beta r_e$, then,

$$[R_B \parallel \beta r_e] \cong \beta r_e$$

Then, $Z_i \cong \beta r_e$

Output impedance Z_o :

Z_o is the output impedance when $V_i = 0$. When $V_i = 0$, $I_b = 0$, resulting in open circuit equivalence for the current source.



$$Z_o = [R_C \parallel r_o] \text{ ohms}$$

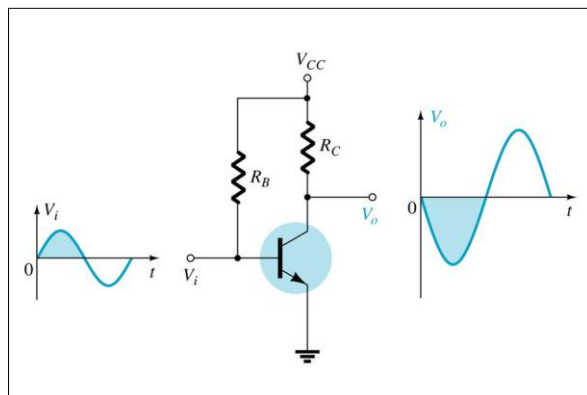
Voltage Gain A_v :

$$V_o = -\beta I_b (R_C \parallel r_o) \text{ From the re model, } I_b = V_i / \beta r_e$$

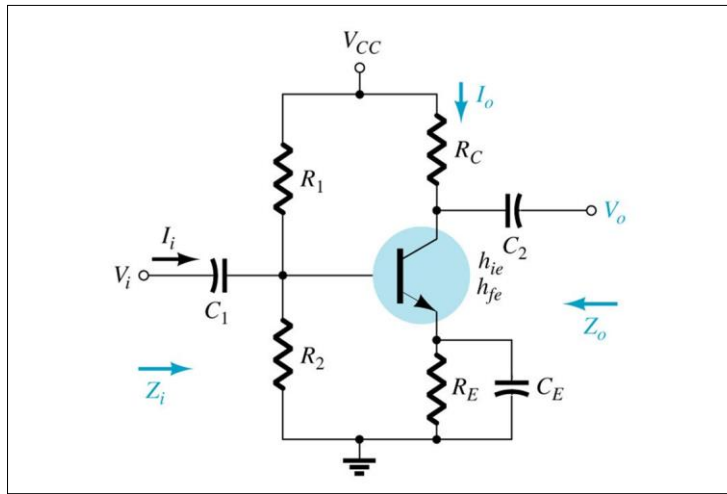
$$\text{thus, } V_o = -\beta (V_i / \beta r_e) (R_C \parallel r_o) \quad A_v = V_o / V_i = - (R_C \parallel r_o) / r_e$$

If $r_o \gg 10R_C$, $A_v = - (R_C / r_e)$ Phase Shift:

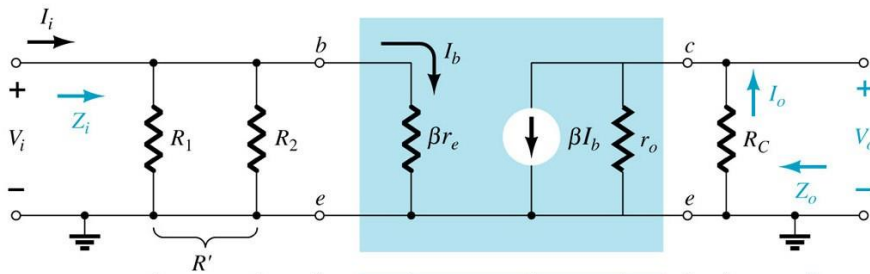
The negative sign in the gain expression indicates that there exists 180° phase shift between the input and output.



Common Emitter - Voltage-Divider Configuration



Equivalent Circuit:



The r_e model is very similar to the fixed bias circuit except for R_B is $R_1 \parallel R_2$ in the case of voltage divider bias.

Expression for AV remains the same. $Z_i = R_1 \parallel R_2 \parallel \beta r_e$

$$Z_o = R_C$$

:

Voltage Gain, AV:

From the r_e model,

$$I_b = V_i / \beta r_e$$

$$V_o = - I_o (RC \parallel r_o),$$

$$I_o = \beta I_b$$

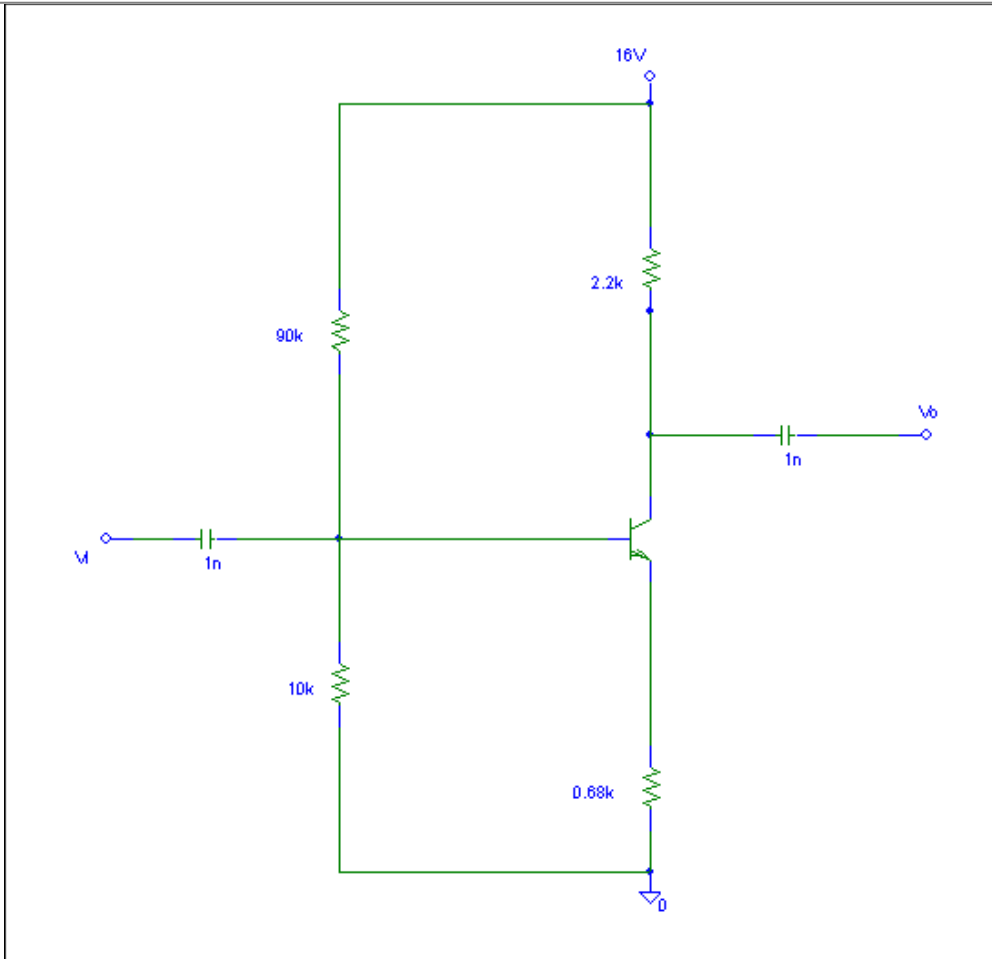
thus, $V_o = - \beta (V_i / \beta r_e) (RC \parallel r_o)$ $AV = V_o / V_i = - (RC \parallel r_o) / r_e$

If $r_o > 10RC$, $AV = - (RC / r_e)$

Problem:

Given: $\beta = 210$, $r_o = 50k\Omega$.

Determine: r_e , Z_i , Z_o , AV . For the network given:



To perform DC analysis, we need to find out whether to choose exact analysis or approximate analysis.

This is done by checking whether $\beta R_E > 10R_2$, if so, approximate analysis can be chosen. Here, $\beta R_E = (210)(0.68k) = 142.8k\Omega$.

$10R_2 = (10)(10k) = 100k$. Thus, $\beta R_E > 10R_2$.

Therefore using approximate analysis,

$$V_B = V_{cc}R_2 / (R_1+R_2)$$

$$= (16)(10k) / (90k+10k) = 1.6V \quad V_E = V_B - 0.7 = 1.6 - 0.7 = 0.9V$$

$$I_E = V_E / R_E = 1.324mA$$

$$r_e = 26mV / 1.324mA = 19.64\Omega$$

Effect of r_o can be neglected if $r_o \geq 10(R_C)$. In the given circuit, $10R_C$ is 22k, r_o is 50K.

Thus effect of r_o can be neglected.

$$Z_i = (R_1 \parallel R_2 \parallel \beta R_E)$$

$$= [90k \parallel 10k \parallel (210)(0.68k)] = 8.47k\Omega$$

$$Z_o = R_C = 2.2 k\Omega$$

$$A_V = - R_C / R_E = - 3.24$$

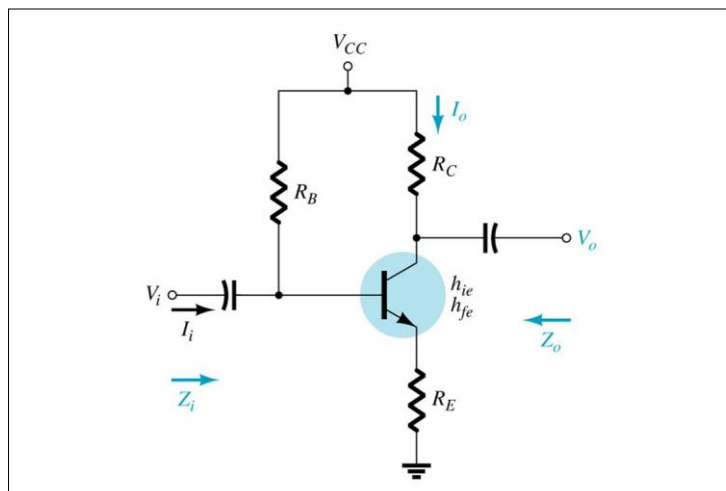
If the same circuit is with emitter resistor bypassed, Then value of r_e remains same.

$$Z_i = (R_1 \parallel R_2 \parallel \beta r_e) = 2.83 k\Omega$$

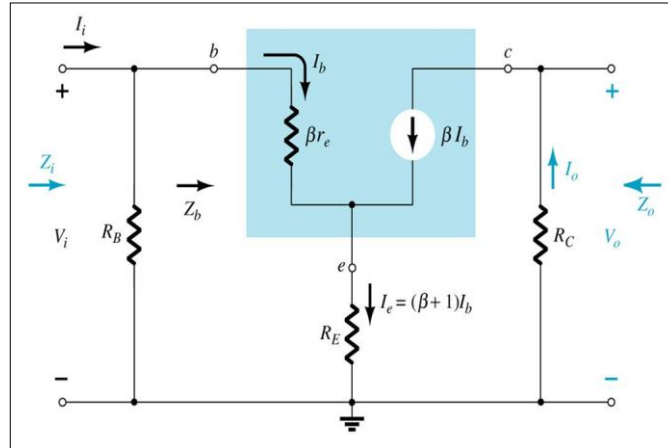
$$Z_o = R_C = 2.2 k\Omega$$

$$A_V = - R_C / r_e = - 112.02$$

Common Emitter Un bypassed Emitter - Fixed Bias Configuration



Equivalent Circuit:



Applying KVL to the input side:

$$V_i = I_b \beta r_e + I_e R_E$$

$$V_i = I_b \beta r_e + (\beta + 1) I_b R_E$$

Input impedance looking into the network to the right of R_B is

$$Z_b = V_i / I_b = \beta r_e + (\beta + 1) R_E$$

$$\text{Since } \beta \gg 1, \quad (\beta + 1) = \beta$$

$$\text{Thus, } Z_b = V_i / I_b = \beta (r_e + R_E)$$

Since R_E is often much greater than r_e , $Z_b = \beta R_E$, $Z_i = R_B \parallel Z_b$

Z_o is determined by setting V_i to zero, $I_b = 0$ and βI_b can be replaced by open circuit equivalent.

The result is, $Z_o = R_C$

We know that, $V_o = - I_o R_C$

$$= - \beta I_b R_C$$

$$= - \beta (V_i / Z_b) R_C$$

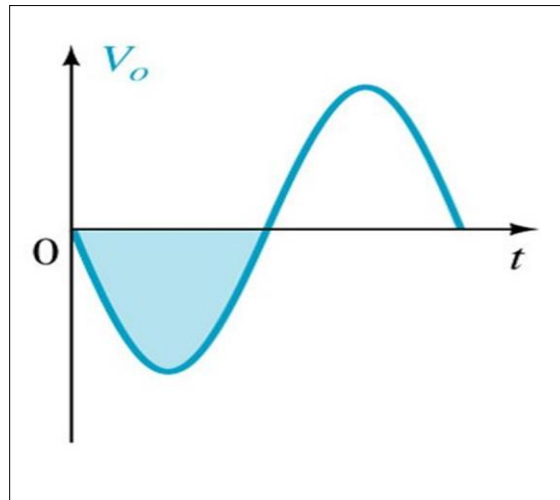
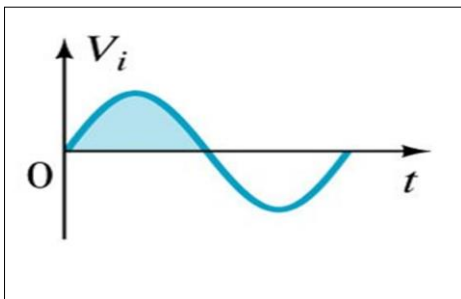
$$AV = V_o / V_i = - \beta(RC/Z_b)$$

Substituting $Z_b = \beta(re + RE)$

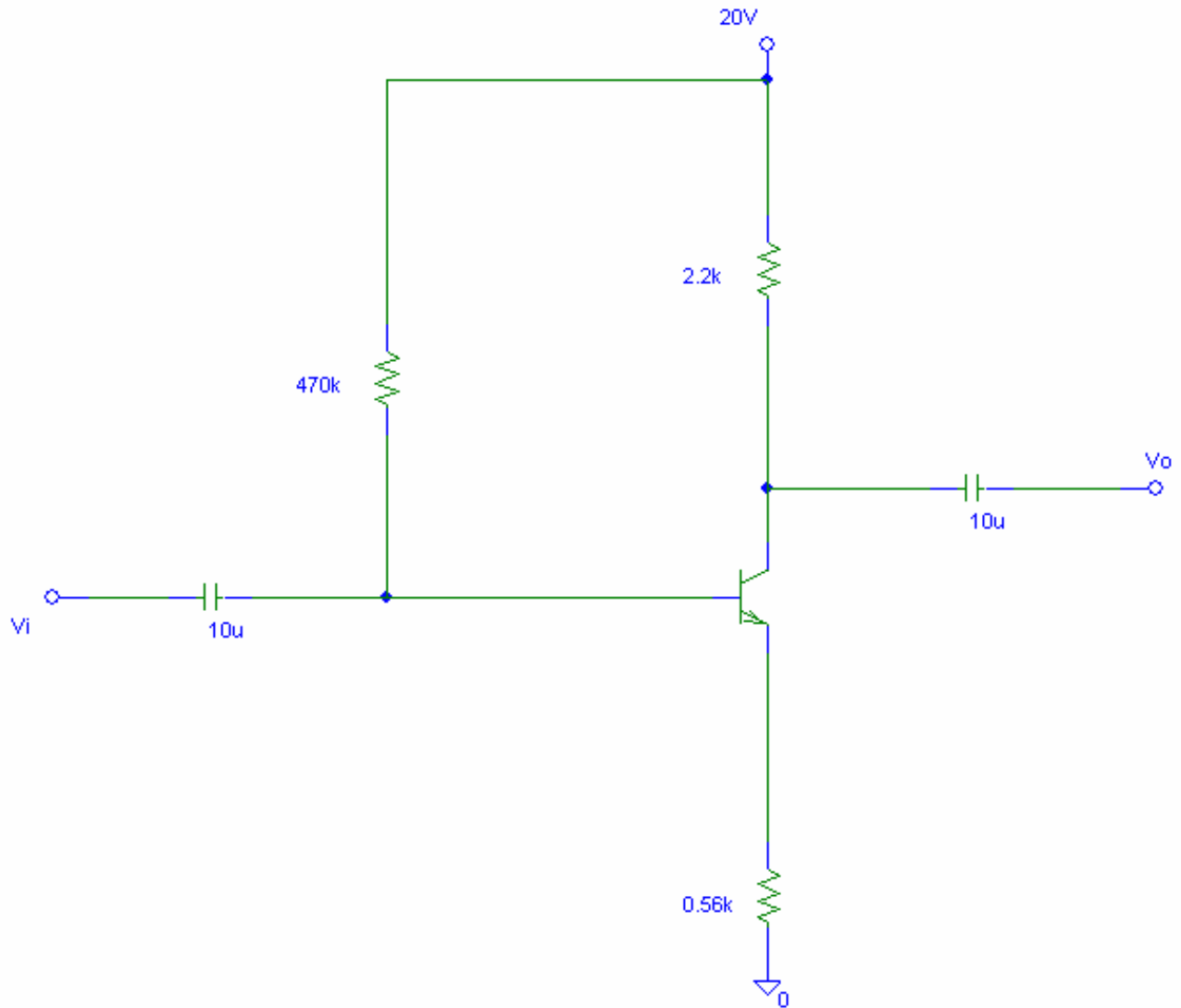
$$AV = V_o / V_i = - \beta[RC / (re + RE)]$$

$RE \gg re,$ $AV = V_o / V_i = - \beta[RC / RE]$

Phase relation: The negative sign in the gain equation reveals a 180° phase shift between input and output.



Problem:



Given: $\beta = 120$, $r_o = 40\text{k}\Omega$.

Determine: r_e , Z_i , Z_o , A_V .

To find r_e , it is required to perform DC analysis and find I_E as $r_e = 26\text{mV} / I_E$

To find I_E , it is required to find I_B .

We know that,

$$I_B = (V_{CC} - V_{BE}) / [R_B + (\beta+1)R_E]$$

$$I_B = (20 - 0.7) / [470k + (120+1)0.56k] = 35.89\mu A$$

$$I_E = (\beta+1)I_B = 4.34mA$$

$$r_e = 26mV / I_E = 5.99\Omega$$

Effect of r_o can be neglected, if $r_o \geq 10(R_C + R_E)$

$$10(R_C + R_E) = 10(2.2 k\Omega + 0.56k)$$

$$= 27.6 k\Omega$$

and given that r_o is 40 k Ω , thus effect of r_o can be ignored.

$$Z_i = R_B \parallel [\beta (r_e + R_E)]$$

$$= 470k \parallel [120 (5.99 + 560)] = 59.34\Omega$$

$$Z_o = R_C = 2.2 k\Omega$$

$$A_V = - \beta R_C / [\beta (r_e + R_E)]$$

$$= - 3.89$$

Analyzing the above circuit with Emitter resistor bypassed i.e., Common Emitter

$$I_B = (V_{CC} - V_{BE}) / [R_B + (\beta+1)R_E]$$

$$I_B = (20 - 0.7) / [470k + (120+1)0.56k]$$

$$= 35.89\mu A$$

$$I_E = (\beta+1)I_B = 4.34mA$$

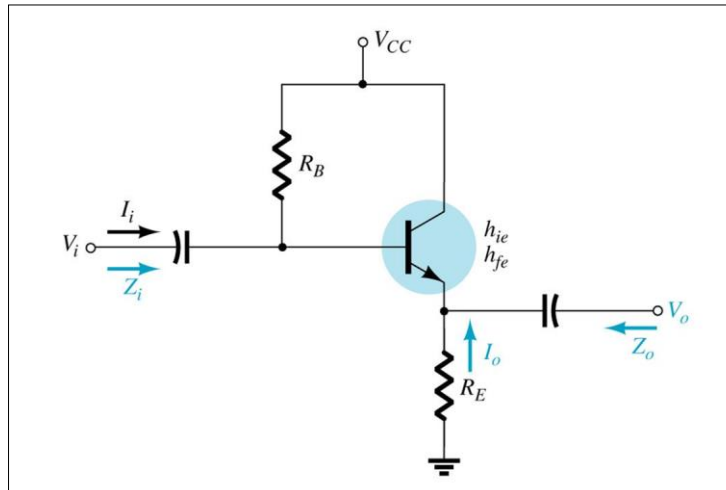
$$r_e = 26mV / I_E = 5.99\Omega$$

$$Z_i = R_B \parallel [\beta r_e] = 717.70\Omega$$

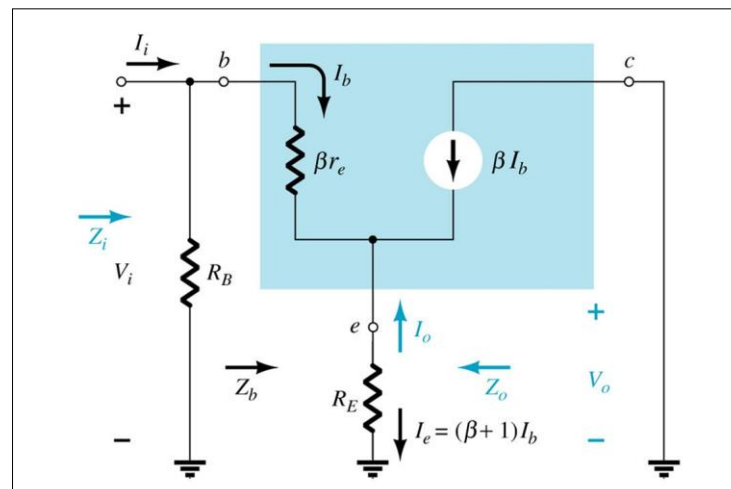
$$Z_o = R_C = 2.2\text{ k}\Omega$$

$$A_V = - R_C / r_e = - 367.28 \text{ (a significant increase)}$$

Emitter – follower



re model



$$Z_i = R_B \parallel Z_b$$

$$Z_b = \beta r_e + (\beta + 1)R_E$$

$$Z_b = \beta(r_e + R_E)$$

Since R_E is often much greater than r_e , $Z_b = \beta R_E$

To find Z_o , it is required to find output equivalent circuit of the emitter follower at its input terminal.

This can be done by writing the equation for the current I_b .

$$I_b = V_i / Z_b$$

$$I_e = (\beta + 1)I_b$$

$$= (\beta + 1) (V_i / Z_b)$$

We know that, $Z_b = \beta r_e + (\beta + 1)R_E$

substituting this in the equation for I_e we get,

$$I_e = (\beta + 1) (V_i / Z_b)$$

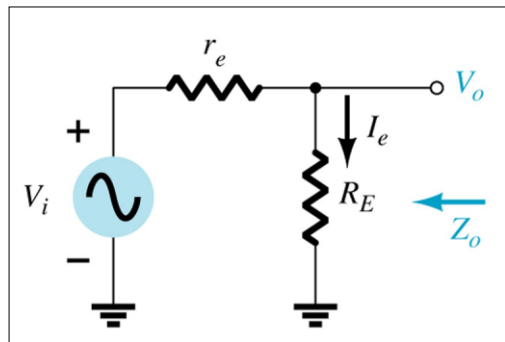
$$= (\beta + 1) (V_i / \beta r_e + (\beta + 1)R_E)$$

dividing by $(\beta + 1)$, we get, Since $(\beta + 1) = \beta$,

$$I_e = V_i / [\beta r_e / (\beta + 1)] + R_E$$

$$I_e = V_i / [r_e + R_E]$$

Using the equation $I_e = V_i / [r_e + R_E]$, we can write the output equivalent circuit as,



As per the equivalent circuit,

$$Z_o = R_E || r_e$$

Since R_E is typically much greater than r_e , $Z_o \cong r_e$

AV – Voltage gain:

Using voltage divider rule for the equivalent circuit, $V_o = V_i R_E / (R_E + r_e)$

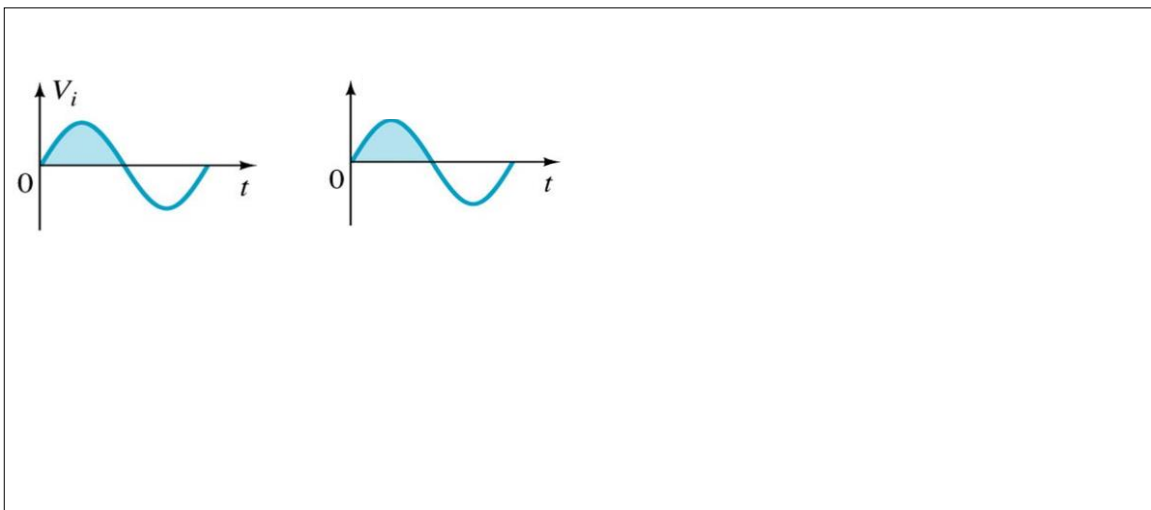
$$AV = V_o / V_i = [R_E / (R_E + r_e)]$$

Since $(R_E + r_e) \cong R_E$,

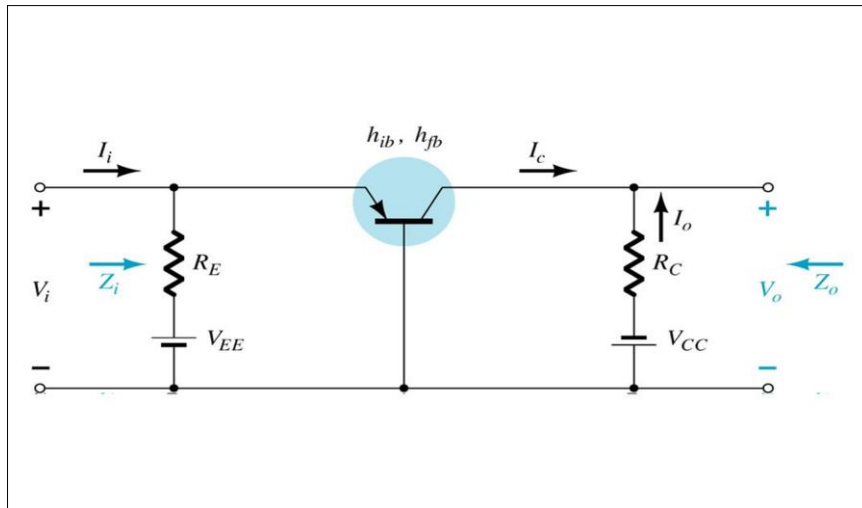
$$AV \cong [R_E / (R_E)] \cong 1$$

Phase relationship

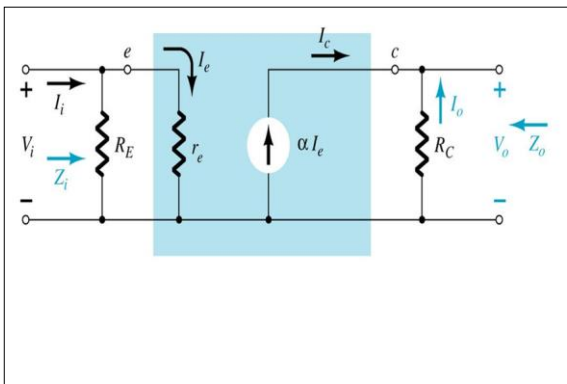
As seen in the gain equation, output and input are in phase.



Common base configuration



re model



Small signal analysis

$$Z_i = R_E || r_e$$

$$Z_o = R_C$$

To find

$$V_o = -I_o R_C$$

$$V_o = -(-I_C)R_C = \alpha I_e R_C$$

Substituting this in the above equation, $I_e = V_i / r_e$, $V_o = \alpha (V_i / r_e) RC$

$$V_o = \alpha (V_i / r_e) RC$$

$$AV = V_o / V_i = \alpha (RC / r_e)$$

$$\alpha \cong 1; AV = (RC / r_e)$$

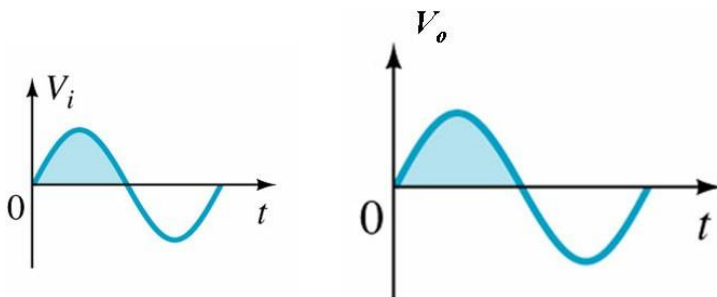
Current gain A_i :

$$A_i = I_o / I_i$$

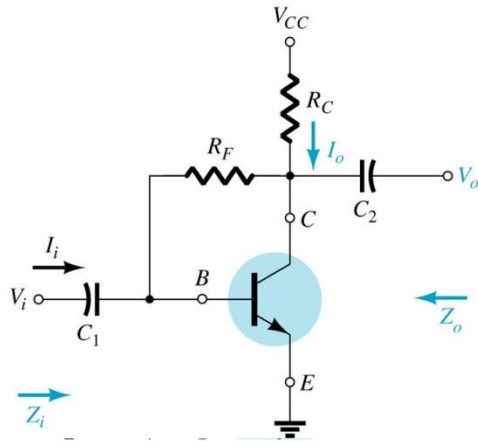
$$I_o = -\alpha I_e = -\alpha I_i$$

$$I_o / I_i = -\alpha \cong -1$$

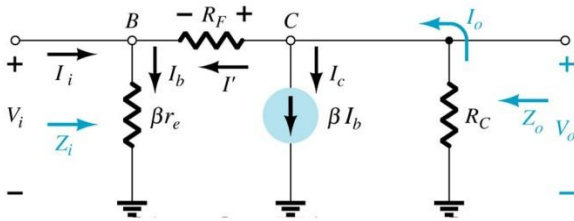
Phase relation: Output and input are in phase.



Common Emitter - Collector Feedback Configuration



re Model



Input Impedance: Z_i

$$Z_i = V_i / I_i, I_i = I_b - I'$$

thus it is required to find expression for I' in terms of known resistors.

$$I' = (V_o - V_i) / R_F \quad (1)$$

$$V_o = -I_o R_C$$

$$I_o = \beta I_b + I' \text{ Normally, } I' \ll \beta I_b \text{ thus, } I_o = \beta I_b$$

$$V_o = -I_o R_C$$

$$V_o = -\beta I_b R_C$$

Replacing I_b by $V_i / \beta r_e$

Thus,

Substituting (2) in (1):

$$\begin{aligned}V_o &= -\beta (V_i RC) / \beta r_e \\ &= - (V_i RC) / r_e \quad (2)\end{aligned}$$

$$\begin{aligned}I' &= (V_o - V_i) / R_F \\ &= (V_o / R_F) - (V_i / R_F) \\ &= - [(V_i RC) / R_F r_e] - (V_i / R_F)\end{aligned}$$

$$I' = - V_i / R_F [(RC / r_e) + 1]$$

We know that, $V_i = I_b \beta r_e$,

$$I_b = I_i + I'$$

$$\text{and, } I' = - V_i / R_F [(RC / r_e) + 1]$$

$$\begin{aligned}\text{Thus, } V_i &= (I_i + I') \beta r_e = I_i \beta r_e + I' \beta r_e \\ &= I_i \beta r_e - (V_i \beta r_e) (1/R_F) [(RC / r_e) + 1]\end{aligned}$$

Taking V_i terms on left side:

$$V_i + (V_i \beta r_e) (1/R_F) [(RC / r_e) + 1] = I_i \beta r_e$$

$$V_i [1 + (\beta r_e) (1/R_F) [(RC / r_e) + 1]] = I_i \beta r_e$$

$$V_i / I_i = \beta r_e / [1 + (\beta r_e) (1/R_F) [(RC / r_e) + 1]]$$

$$\text{But, } [(RC / r_e) + 1] \cong RC / r_e$$

(because $RC \gg r_e$)

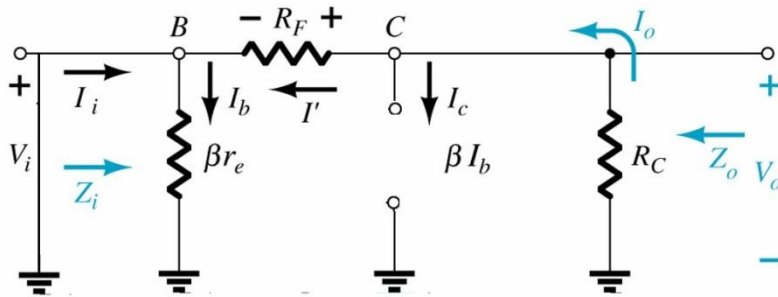
$$\text{Thus, } Z_i = V_i / I_i$$

$$= \beta r_e / [1 + (\beta r_e) (1/R_F) [(RC / r_e)]]$$

$$= \beta r_e / [1 + (\beta)(RC/RF)]$$

Thus, $Z_i = r_e / [(1/\beta) + (RC/RF)]$

To find Output Impedance Z_o :



$Z_o = RC \parallel RF$ (Note that $i_b = 0$, thus no effect of βr_e on Z_o)

Voltage Gain AV:

$$V_o = - I_o RC$$

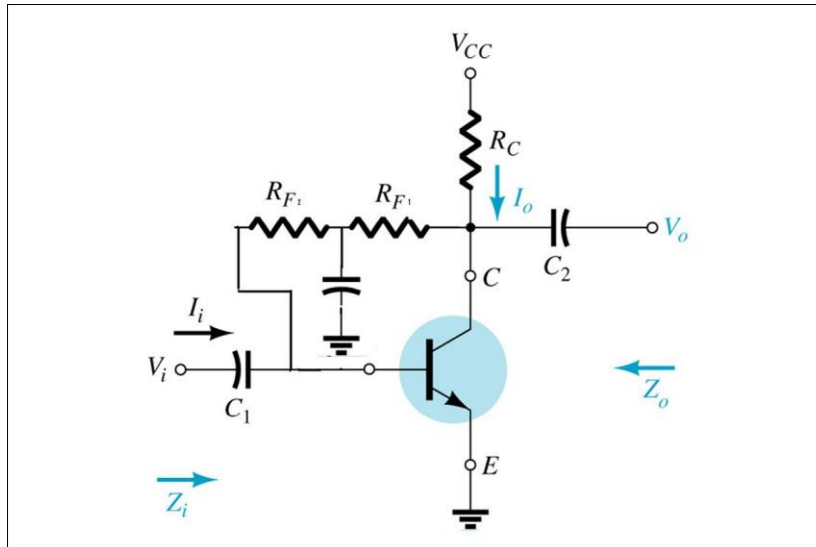
$$= - \beta I_b RC \text{ (neglecting the value of } I' \text{)}$$

$$= - \beta (V_i / \beta r_e) RC$$

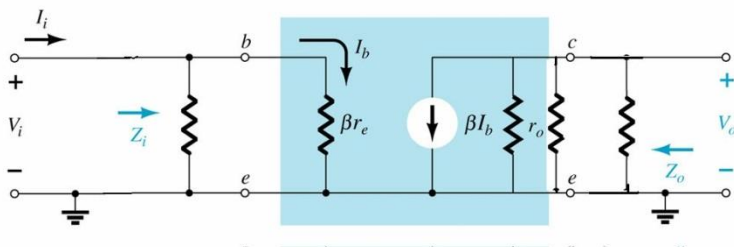
$$AV = V_o / V_i = - (RC/r_e)$$

Phase relation: - sign in AV indicates phase shift of 180° between input and output.

Collector DC feedback configuration



re model



$$Z_i = R_{F1} \parallel \beta r_e$$

$$Z_o = R_C \parallel R_{F2} \parallel r_o, \text{ for } r_o \geq 10R_C, \quad Z_o = R_C \parallel R_{F2}$$

To find Voltage Gain A_V :

$$V_o = -\beta I_b (R_{F2} \parallel R_C \parallel r_o), \quad I_b = V_i / \beta r_e$$

$$V_o = -\beta (V_i / \beta r_e) (R_{F2} \parallel R_C \parallel r_o)$$

$$V_o / V_i = - (R_{F2} \parallel R_C \parallel r_o) / r_e,$$

for $r_o \geq 10R_C$,

$$A_V = V_o / V_i = - (R_{F2} \parallel R_C) / r_e$$

Determining the current gain

For each transistor configuration, the current gain can be determined directly from the voltage gain, the defined load, and the input impedance.

We know that, current gain (A_i) = I_o / I_i

$$I_o = (V_o / R_L) \text{ and } I_i = V_i / Z_i$$

$$\text{Thus, } A_i = - (V_o / R_L) / (V_i / Z_i)$$

$$= - (V_o Z_i / V_i R_L) \quad A_i = - A_V Z_i / R_L$$

Example:

For a voltage divider network, we have found that, $Z_i = \beta r_e$

$$A_V = - R_C / r_e \text{ and } R_L = R_C$$

$$\text{Thus, } A_i = - A_V Z_i / R_L$$

$$= - (- R_C / r_e)(\beta r_e) / R_C$$

$$A_i = \beta$$

For a Common Base amplifier, $Z_i = r_e$, $A_V = R_C / r_e$, $R_L = R_C$

$$A_i = - A_V Z_i / R_L$$

$$= - (R_C / r_e)(r_e) / R_C$$

$$= - 1$$

Effect of R_L and R_S :

Voltage gain of an amplifier without considering load resistance (R_L) and source resistance (R_S) is A_{VNL} .

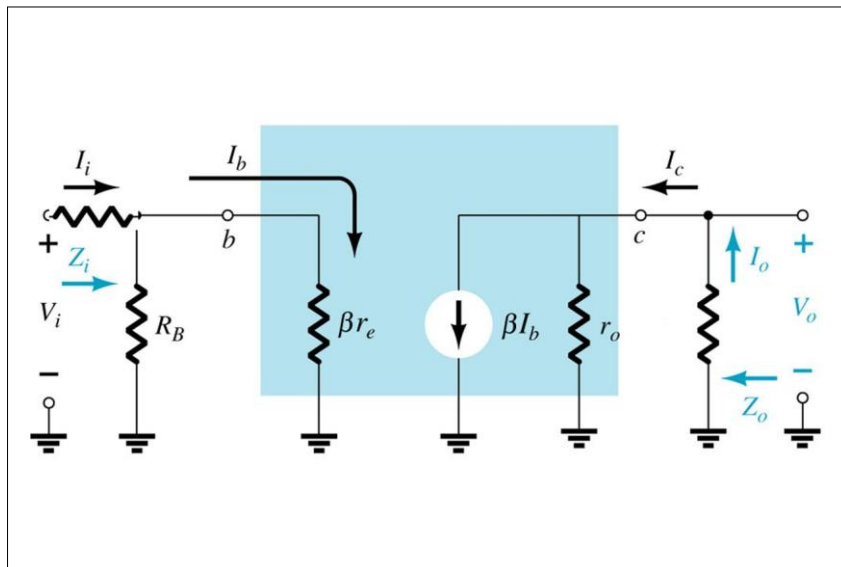
Voltage gain considering load resistance (R_L) is $A_V < A_{VNL}$

Voltage gain considering R_L and R_S is A_{VS} , where $A_{VS} < A_{VNL} < A_V$

For a particular design, the larger the level of R_L , the greater is the level of ac gain.

Also, for a particular amplifier, the smaller the internal resistance of the signal source, the greater is the overall gain.

Fixed bias with R_S and R_L :



$$A_V = - (R_C || R_L) / r_e$$

$$Z_i = R_B || \beta r_e$$

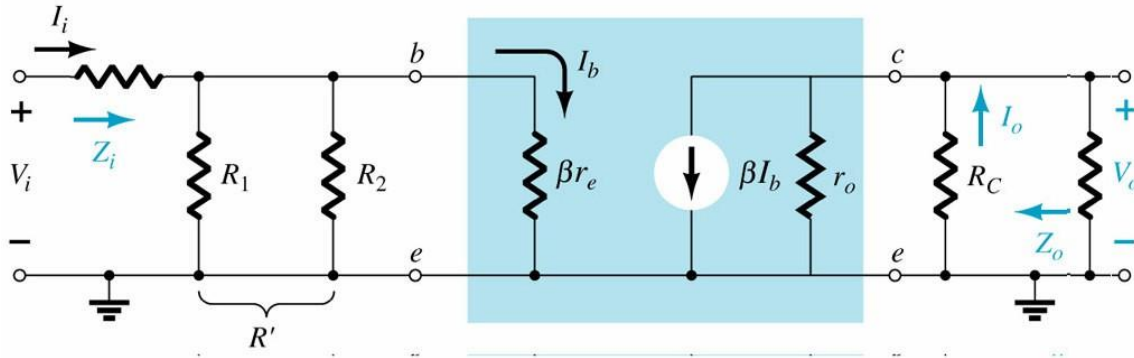
$$Z_o = R_C || r_o$$

To find the gain A_{VS} , (Z_i and R_S are in series and applying voltage divider rule) $V_i = V_S Z_i / (Z_i + R_S)$

$$V_i / V_S = Z_i / (Z_i + R_S)$$

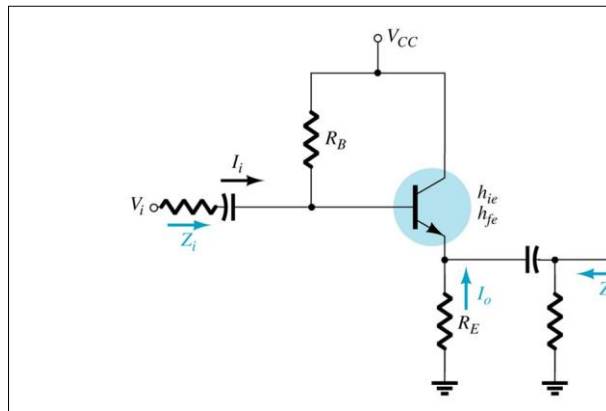
$$A_{VS} = V_o / V_S = (V_o / V_i) (V_i / V_S) \quad A_{VS} = A_V [Z_i / (Z_i + R_S)]$$

Voltage divider with RS and RL

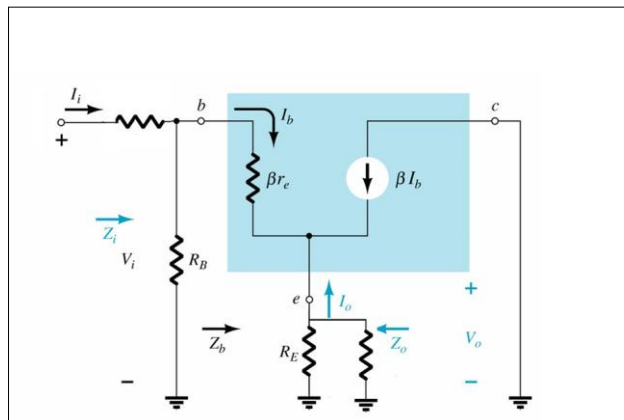


Voltage gain: $A_V = - [R_C || R_L] / r_e$ Input Impedance: $Z_i = R_1 || R_2 || \beta r_e$ Output Impedance: $Z_o = R_C || R_L || r_o$

Emitter follower with RS and RL



re model:



Voltage Gain: $A_V = (R_E \parallel R_L) / [R_E \parallel R_L + r_e]$ Input Impedance: $Z_i = R_B \parallel Z_b$

Input Impedance seen at Base: $Z_b = \beta(R_E \parallel R_L)$

Output Impedance $Z_o = r_e$

Two – port systems approach

This is an alternative approach to the analysis of an amplifier.

This is important where the designer works with packaged with packaged products rather than individual elements.

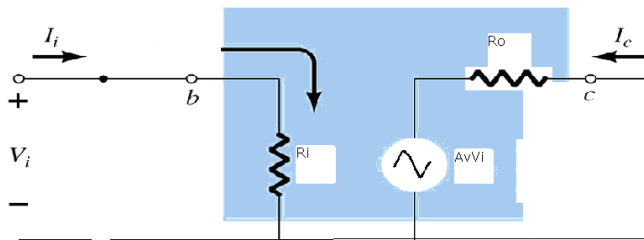
An amplifier may be housed in a package along with the values of gain, input and output impedances.

But those values are no load values and by using these values, it is required to find out the gain and various impedances under loaded conditions.

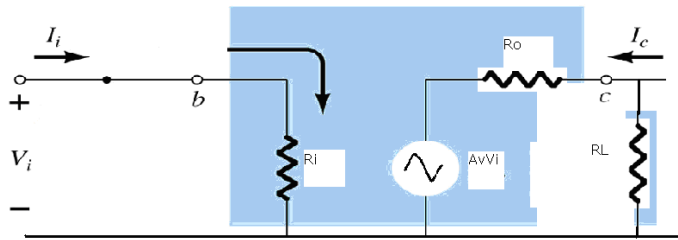
This analysis assumes the output port of the amplifier to be seen as a voltage source. The value of this output voltage is obtained by Thevinising the output port of the amplifier.

$$E_{th} = A_{VNL} V_i$$

Model of two port system



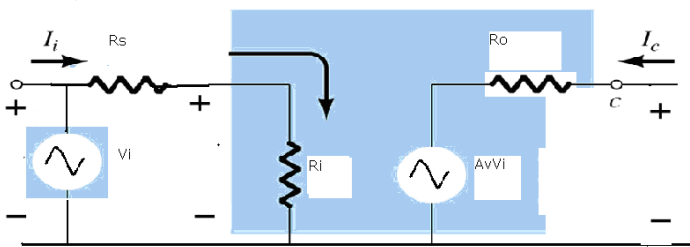
Applying the load to the two port system



Applying voltage divider in the above system: $V_o = AVNLV_i R_L / [R_L + R_o]$

Including the effects of source resistance R_S

Applying voltage divider at the input side, we get:



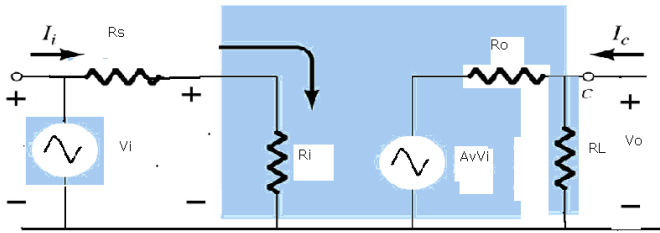
$$V_i = V_s R_i / [R_s + R_i] \quad V_o = AVNL V_i$$

$$V_i = V_s R_i / [R_s + R_i]$$

$$V_o = AVNL V_s R_i / [R_s + R_i]$$

$$V_o / V_s = AVS = AVNL R_i / [R_s + R_i]$$

Two port system with R_S and R_L



We know that, at the input side

$$V_i = V_s R_i / [R_s + R_i]$$

$$V_i / V_s = R_i / [R_s + R_i]$$

At the output side,

$$V_o = A_{vNL} V_i R_L / [R_L + R_o] \quad V_o / V_i = A_{vNL} R_L / [R_L + R_o]$$

Thus, considering both R_s and R_L : $A_V = V_o / V_s$

$$= [V_o / V_i] [V_i / V_s]$$

$$A_V = (A_{vNL} R_L / [R_L + R_o]) (R_i / [R_s + R_i])$$

Example:

Given an amplifier with the following details:

$$R_s = 0.2 \text{ k}\Omega, \quad A_{vNL} = -480, \quad Z_i = 4 \text{ k}\Omega, \quad Z_o = 2 \text{ k}\Omega$$

Determine:

$$A_V \text{ with } R_L = 1.2 \text{ k}\Omega$$

$$A_V \text{ and } A_i \text{ with } R_L = 5.6 \text{ k}\Omega, \quad A_{VS} \text{ with } R_L = 1.2$$

Solution:

$$AV = AVNLRL / (RL + Ro)$$

$$= (-480)1.2k / (1.2k+2k)$$

$$= -180$$

$$\text{With } RL = 5.6k, AV = -353.76$$

This shows that, larger the value of load resistor, the better is the gain.

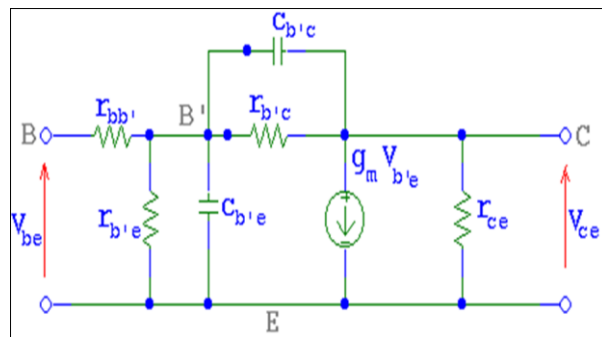
$$AVS = [Ri / (Ri+RS)] [RL / (RL+Ro)] AVNL$$

$$= -171.36$$

$Ai = -AVZi/RL$, here AV is the voltage gain when $RL = 5.6k$. $Ai = -AVZi/RL$

$$= -(-353.76)(4k/5.6k) = 252.6$$

Hybrid π model



This is more accurate model for high frequency effects. The capacitors that appear are stray parasitic capacitors between the various junctions of the device. These capacitances come into picture only at high frequencies.

- C_{bc} or C_u is usually few pico farads to few tens of pico farads.
- r_{bb} includes the base contact, base bulk and base spreading resistances.
- r_{be} (r_{π}), r_{bc} , r_{ce} are the resistances between the indicated terminals.

-
- r_{be} (r_{π}) is simply βr_e introduced for the CE r_e model.
 - r_{bc} is a large resistance that provides feedback between the output and the input.
 - $r_{\pi} = \beta r_e$
 - $g_m = 1/r_e$
 - $r_o = 1/h_{oe}$
 - $h_{re} = r_{\pi} / (r_{\pi} + r_{bc})$