







CHAPTER FOUR

SYSTEMS OF NUMERATION

Exercise Set 4.1

1. A **number** is a quantity, and it answers the question, “How many?” A **numeral** is a symbol used to represent the number.
 2. $\cap, x, \dagger, \iota, \blacktriangleleft, 10$
 3. A **system of numeration** consists of a set of numerals and a scheme or rule for combining the numerals to represent numbers.
 4. $\oslash, c, \text{百}, \rho, 100$
 5. The Hindu-Arabic numeration system
 6. In an **additive system**, the sum of the values of the numerals equals the number.
 7. In a **multiplicative system**, there are numerals for each number less than the base and for powers of the base. Each numeral less than the base is multiplied by a numeral for the power of the base, and these products are added to obtain the number.
 8. In a **ciphred system**, the number represented by a particular set of numerals is the sum of the values of the numerals.
-
- | | |
|--|---|
| <ol style="list-style-type: none"> 9. $100 + 10 + 10 + 10 + 10 + 1 + 1 = 142$ 11. $1000 + 1000 + 100 + 100 + 100 + 100 + 10 + 10 + 1 + 1 = 2423$ 13. $100,000 + 100,000 + 100,000 + 10,000 + 10,000 + 10,000 + 1000 + 1000 + 1000 + 1000 + 100 + 100 + 10 + 1 + 1 + 1 + 1 = 334,214$ 15.  17.  19.  20.  21. $10 + (10 - 1) = 19$ 23. $500 + (50 - 10) + 5 + 1 + 1 = 547$ 25. $1000 + (500 - 100) + (100 - 10) + 1 + 1 = 1492$ 27. $1000 + 1000 + (1000 - 100) + (50 - 10) + 5 + 1 = 2946$ | <ol style="list-style-type: none"> 10. $100 + 100 + 10 + 10 + 1 + 1 = 222$ 12. $10,000 + 10,000 + 10,000 + 10,000 + 1000 + 100 + 100 + 10 = 41,210$ 14. $1,000,000 + 1,000,000 + 1,000,000 + 100,000 + 100,000 + 100,000 + 100 + 100 + 100 + 100 + 10 + 10 + 10 + 1 = 3,300,431$ 16.  18.  22. $10 + 5 + 1 = 16$ 24. $500 + 50 + 10 + 10 + 5 = 575$ 26. $1000 + (1000 - 100) + 10 + 5 + 1 + 1 + 1 = 1918$ 28. $1000 + 500 + 100 + 100 + (50 - 10) + 5 + 1 = 1746$ |
|--|---|

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29. $10(1000)+1000+1000+500+100+50+10$
 $+5+1=12,666$

31. $9(1000)+(500-100)+50+10+(5-1)=9464$

33. LIX

35. CXXXIV

37. MMV

39. $\overline{\text{IVDCCXCIII}}$

41. $\overline{\text{IXCMXCIX}}$

43. $\overline{\text{XXDCXLIV}}$

45. $7(10)+4=74$

47. $4(1000)+8(10)+1=4081$

49. $8(1000)+5(100)+5(10)=8550$

51. $4(1000)+3=4003$

53. 五
十
三

55. 三
百
七
十
八

57. 四
千
二
百
六
十

30. $50(1000)+1000+(1000-100)+(50-10)$
 $+ (5-1) = 51,944$

32. $5(1000)+1000+100+100+100+10+10$
 $+10+1+1+1=6333$

34. XCIV

36. CCLXIX

38. $\overline{\text{IVCCLXXXV}}$

40. $\overline{\text{VICCLXXIV}}$

42. $\overline{\text{XIVCCCXV}}$

44. $\overline{\text{XCIXCMXCIX}}$

46. $6(10)+2=62$

48. $3(1000)+2(10)+9=3029$

50. $3(1000)+4(100)+8(10)+7=3487$

52. $5(1000)+6(100)+2=5602$

54. 一
百
七
十
八

56. 二
千
零
一

58. 六
千
九
百
零
五

59.

七
千
零
五
十
六

60.

三
千
零
九

61. $300 + 40 + 1 = 341$

62. $700 + 30 + 6 = 736$

63. $20(1000) + 2(1000) + 500 + 5 = 22,505$

64. $100(1000) + 50(1000) + 800 + 10 + 3 = 150,813$

65. $9(1000) + 600 + 7 = 9607$

66. $4(1000) + 900 + 90 + 9 = 4999$

67. $\nu \theta$

68. $\rho o \eta$

69. $\psi \kappa \lambda$

70. $\beta' \alpha$

71. $\pi' \beta' \psi \delta$

72. $\chi' \rho' \phi \mu$

73. Advantage: You can write some numbers more compactly.

Disadvantage: There are more numerals to memorize.

74. Advantage: Numbers are written in a more compact form.

Disadvantage: There are more symbols to remember.

75. Advantage: You can write some numbers more compactly.

Disadvantage: There are more numerals to memorize.

The Hindu-Arabic system has fewer symbols, more compact notation, the inclusion of zero, and the capability of expressing decimal numbers and fractions.

76. $1000 + 10 + 10 + 1 = 1021$, MXXI,

 $\alpha' \kappa \alpha$ 一
千
零
二
十
一

77. $1000 + (1000 - 100) + 10 + 10 + 10 + 5 + 1 = 1936$,

 $\alpha' \pi \lambda \lambda$,一
千
九
百
三
十
六

78. $5(100) + 2(10) + 7 = 527$,

 , DXXVII, $\phi \kappa \zeta$

79. $400 + 20 + 2 = 422$, , CDXXII,

四
百
二
十
二

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80. $\overline{\text{CMXCIXCMXCIX}}$

82. a) – c) Answers will vary.

84. MM

81. $\pi'Q'\theta'\pi Q\theta$

83. Turn the book upside down.

85. 1888, MDCCCLXXXVIII

Exercise Set 4.2

1. A base 10 place-value system
2. Positional value system
3. $40 \rightarrow$ four tens, $400 \rightarrow$ four hundreds
4. Base 10, because we have 10 fingers.
5. A true positional-value system requires a base and a set of symbols, including a symbol for zero and one for each counting number less than the base.
6. a) 10
b) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
7. Write each digit times its corresponding positional value.
8. It lacked a symbol for zero.
9. a) There may be confusion because numbers could be interpreted in different ways. For example, ∇ could be interpreted to be either 1 or 60.

b) $\nabla\nabla \lll \nabla\nabla\nabla$ for both numbers; $133 = 2(60) + 13(1)$ and $7980 = 133(60)$

10. $(10+1)(1) = 11$ and $(10+1)(60) = 660$

11. $1, 20, 18 \times 20, 18 \times (20)^2, 18 \times (20)^3$

12. The Mayan system has a different base and the numbers are written vertically.

13. $(6 \times 10) + (3 \times 1)$

14. $(7 \times 10) + (5 \times 1)$

15. $(3 \times 100) + (5 \times 10) + (9 \times 1)$

16. $(5 \times 100) + (6 \times 10) + (2 \times 1)$

17. $(8 \times 100) + (9 \times 10) + (7 \times 1)$

18. $(3 \times 1000) + (7 \times 100) + (6 \times 10) + (9 \times 1)$

19. $(4 \times 1000) + (3 \times 100) + (8 \times 10) + (7 \times 1)$

20. $(2 \times 10,000) + (3 \times 1000) + (4 \times 100) + (6 \times 10) + (8 \times 1)$

21. $(1 \times 10,000) + (6 \times 1000) + (4 \times 100) + (0 \times 10) + (2 \times 1)$

22. $(1 \times 100,000) + (2 \times 10,000) + (5 \times 1000) + (6 \times 100) + (7 \times 10) + (8 \times 1)$

23. $(3 \times 100,000) + (4 \times 10,000) + (6 \times 1000) + (8 \times 100) + (6 \times 10) + (1 \times 1)$

24. $(3 \times 1,000,000) + (7 \times 100,000) + (6 \times 10,000) + (5 \times 1000) + (9 \times 100) + (3 \times 10) + (4 \times 1)$

25. $(10+10+10+10+1+1)(1) = 42$

26. $(10+10+10)(1) - (1+1+1+1)(1) = 30 - 4 = 26$


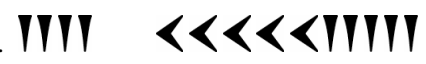

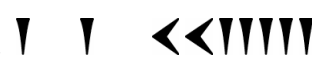

27. $(10+1+1+1)(60) + (1+1+1+1)(1) = 13(60) + 4(1) = 780 + 4 = 784$


28. $(10+1)(60) + ((10+10) - (1+1+1))(1) = 11(60) + (20-3)(1) = 660 + 17 = 677$

29. $1(60^2) + (10+10+1)(60) + (10 - (1+1))(1) = 3600 + 21(60) + (10-2)(1) = 3600 + 1260 + 8 = 4868$

30. $10(60^2) + ((10+10) - (1+1+1))(60) + (1+1)(1) = 10(3600) + (20-3)(60) + 2 = 36,000 + 17(60) + 2 = 36,000 + 1020 + 2 = 37,022$

31. 88 is 1 group of 60 and 28 units remaining. $\nabla \lll \nabla \nabla$

32. 97 is 1 group of 60 and 37 units remaining. 
33. 295 is 4 groups of 60 and 55 units remaining. 
34. 512 is 8 groups of 60 and 32 units remaining. 
35. 3685 is 1 group of 3600, 1 group of 60, and 25 units remaining. 
36. 3030 is 50 groups of 60 and 30 units remaining. 
37. $4(20)+12(1) = 80+12 = 92$
38. $10(20)+5(1) = 200+5 = 205$
39. $12(18\times 20)+0(20)+1(1) = 4320+0+1 = 4321$
40. $7(18\times 20)+9(20)+7(1) = 2520+180+7 = 2707$
41. $11(18\times 20)+2(20)+0(1) = 3960+40+0 = 4000$
42. $2(18\times 20)+10(20)+10(1) = 720+200+10 = 930$

43. 

44.
$$20 \overline{) 257} \begin{array}{r} 12 \\ \underline{20} \\ 57 \\ \underline{40} \\ 17 \end{array} \begin{array}{r} \bullet\bullet \\ \hline \bullet\bullet \\ \hline \hline \end{array}$$

$$257 = 12(20) + 17(1)$$

45.
$$20 \overline{) 297} \begin{array}{r} 14 \\ \underline{280} \\ 17 \end{array} \begin{array}{r} \bullet\bullet\bullet\bullet \\ \hline \bullet\bullet \\ \hline \hline \end{array}$$

$$297 = 14(20) + 17(1)$$

46.
$$360 \overline{) 406} \quad 20 \overline{) 46} \begin{array}{r} \bullet \\ \bullet \\ \bullet \\ \hline \end{array}$$

$$406 = 1(18\times 20) + 2(20) + 6(1)$$

47.
$$360 \overline{) 2163} \begin{array}{r} 6 \\ \underline{2160} \\ 3 \end{array} \begin{array}{r} \bullet \\ \hline \bullet\bullet \\ \hline \hline \end{array}$$

$$2163 = 6(360) + 0(20) + 3(1)$$

48.
$$360 \overline{) 1978} \quad 20 \overline{) 178} \begin{array}{r} \bullet\bullet\bullet \\ \bullet\bullet\bullet \\ \hline \hline \end{array}$$

$$1978 = 5(18\times 20) + 8(20) + 18(1)$$

49. Advantages: In general, a place-value system is more compact; large and small numbers can be written more easily; there are fewer symbols to memorize.
 Disadvantage: If many of the symbols in the numeral represent zero, then a place-value system may be less compact.

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50. Answers will vary.

51. Hindu-Arabic: $10+10+10+1+1+1=33$
 Mayan: $33=1(20)+13(1)$



52. Hindu-Arabic:

$$5(18 \times 20) + 7(20) + 4(1) = 1800 + 140 + 4 = 1944$$

Babylonian: $1944 = 32(60) + 24(1)$



53. $(\triangle \times \ominus^2) + (\square \times \ominus) + (\odot \times 1)$

54. $(\odot \times \ominus^3) + (\triangle \times \ominus^2) + (\odot \times \ominus) + (\square \times 1)$

55. a) No largest number; The positional values are ..., $(60)^3$, $(60)^2$, 60 , 1 .

b) $999,999 = 4(60)^3 + 37(60)^2 + 46(60) + 39(1)$



56. a) No largest number; The positional values above 18×20 are 18×20^2 , 18×20^3 , ...

b) $999,999 = 6(18 \times 20^3) + 18(18 \times 20^2) + 17(18 \times 20) + 13(20) + 19(1)$



57. $2(60) + 23(1) = 120 + 23 = 143$

23

$143 + 23 = 166$

$166 = 2(60) + 46(1)$



58. $3(60) + 33(1) = 180 + 33 = 213$

32

$213 - 32 = 181$

$181 = 3(60) + 1(1)$



$$59. \quad 7(18 \times 20) + 6(20) + 15(1) = 2520 + 120 + 15 = 2655$$

$$6(18 \times 20) + 7(20) + 13(1) = 2160 + 140 + 13 = 2313$$

$$2655 + 2313 = 4968$$

$$4968 = 13(18 \times 20) + 14(20) + 8(1)$$

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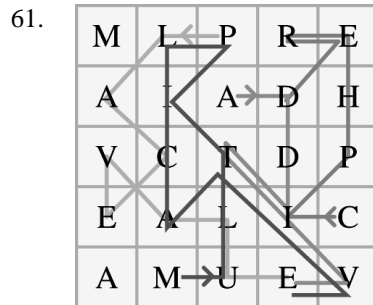
$$60. \quad 7(18 \times 20) + 6(20) + 15(1) = 2520 + 120 + 15 = 2655$$

$$6(18 \times 20) + 7(20) + 13(1) = 2160 + 140 + 13 = 2313$$

$$2655 - 2313 = 342$$

$$342 = 17(20) + 2(1)$$

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Exercise Set 4.3

1. Answers will vary.
2. Answers will vary.
3. $5_6 = 5(1) = 5$
4. $60_7 = 6(7) + 0(1) = 42 + 0 = 42$
5. $42_5 = 4(5) + 2(1) = 20 + 2 = 22$
6. $101_2 = 1(2^2) + 0(2) + 1(1) = 4 + 0 + 1 = 5$
7. $1011_2 = 1(2^3) + 0(2^2) + 1(2) + 1(1) = 8 + 0 + 2 + 1 = 11$
8. $1101_2 = 1(2^3) + 1(2^2) + 0(2) + 1(1) = 8 + 4 + 0 + 1 = 13$
9. $84_{12} = 8(12) + 4(1) = 96 + 4 = 100$
10. $21021_3 = 2(3^4) + 1(3^3) + 0(3^2) + 2(3) + 1(1) = 2(81) + 27 + 0(9) + 6 + 1 = 162 + 27 + 0 + 6 + 1 = 196$
11. $565_8 = 5(8^2) + 6(8) + 5(1) = 5(64) + 48 + 5 = 320 + 48 + 5 = 373$
12. $654_7 = 6(7^2) + 5(7) + 4(1) = 6(49) + 35 + 4 = 294 + 35 + 4 = 333$
13. $20432_5 = 2(5^4) + 0(5^3) + 4(5^2) + 3(5) + 2(1) = 2(625) + 0 + 4(25) + 15 + 2 = 1250 + 0 + 100 + 15 + 2 = 1367$
14. $101111_2 = 1(2^5) + 0(2^4) + 1(2^3) + 1(2^2) + 1(2) + 1(1) = 32 + 0 + 8 + 4 + 2 + 1 = 47$
15. $4003_6 = 4(6^3) + 0(6^2) + 0(6) + 3(1) = 4(216) + 0 + 0 + 3 = 864 + 0 + 0 + 3 = 867$
16. $123E_{12} = 1(12^3) + 2(12^2) + 3(12) + 11(1) = 1728 + 2(144) + 36 + 11 = 1728 + 288 + 36 + 11 = 2063$

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17. $123_8 = 1(8^2) + 2(8) + 3(1) = 64 + 16 + 3 = 83$

18. $2043_8 = 2(8^3) + 0(8^2) + 4(8) + 3(1) = 2(512) + 0 + 32 + 3 = 1024 + 0 + 32 + 3 = 1059$

19. $14705_8 = 1(8^4) + 4(8^3) + 7(8^2) + 0(8) + 5(1) = 4096 + 4(512) + 7(64) + 0 + 5 = 4096 + 2048 + 448 + 0 + 5 = 6597$

20. $67342_9 = 6(9^4) + 7(9^3) + 3(9^2) + 4(9) + 2(1) = 6(6561) + 7(729) + 3(81) + 36 + 2 = 39,366 + 5103 + 243 + 36 + 2 = 44,750$

21. To convert 8 to base 2

	...	16	8	4	2	1
8	4	2	1			
$\frac{1}{8}$	$\frac{0}{4}$	$\frac{0}{2}$	$\frac{0}{1}$			
<u>8</u>	<u>0</u>	<u>0</u>	<u>0</u>			
0	0	0	0			

$8 = 1000_2$

22. To convert 16 to base 2

	...	32	16	8	4	2	1
16	8	4	2	1			
$\frac{1}{16}$	$\frac{0}{8}$	$\frac{0}{4}$	$\frac{0}{2}$	$\frac{0}{1}$			
<u>16</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>			
0	0	0	0	0			

$16 = 10000_2$

23. To convert 23 to base 2

	...	32	16	8	4	2	1
16	8	4	2	1			
$\frac{1}{16}$	$\frac{0}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{1}$			
<u>16</u>	<u>0</u>	<u>4</u>	<u>2</u>	<u>1</u>			
7	7	3	1	0			

$23 = 10111_2$

24. To convert 243 to base 6

	...	1296	216	36	6	1
216	36	6	1			
$\frac{1}{216}$	$\frac{0}{36}$	$\frac{4}{6}$	$\frac{3}{1}$			
<u>216</u>	<u>0</u>	<u>24</u>	<u>3</u>			
27	27	3	0			

$243 = 1043_6$

25. To convert 635 to base 6

	...	1296	216	36	6	1
216	36	6	1			
$\frac{2}{216}$	$\frac{5}{36}$	$\frac{3}{6}$	$\frac{5}{1}$			
<u>432</u>	<u>180</u>	<u>18</u>	<u>5</u>			
203	23	5	0			

$635 = 2535_6$

26. To convert 908 to base 4

	...	1024	256	64	16	4	1
256	64	16	4	1			
$\frac{3}{256}$	$\frac{2}{64}$	$\frac{0}{16}$	$\frac{3}{4}$	$\frac{0}{1}$			
<u>768</u>	<u>128</u>	<u>0</u>	<u>12</u>	<u>0</u>			
140	12	12	0	0			

$908 = 32030_4$

27. To convert 2061 to base 12

	...	20,736	1728	144	12	1
1728	144	12	1			
$\frac{1}{1728}$	$\frac{2}{144}$	$\frac{3}{12}$	$\frac{9}{1}$			
<u>1728</u>	<u>288</u>	<u>36</u>	<u>9</u>			
333	45	9	0			

$2061 = 1239_{12}$

28. To convert 200 to base 4
- | | | | | | | | | | | |
|----|------------|----------|----------|----------|-----|----------|----|----|---|-------------------------|
| | 3 | 0 | 2 | 0 | ... | 256 | 64 | 16 | 4 | 1 |
| 64 | 200 | 16 | 8 | 4 | 1 | 0 | | | | |
| | <u>192</u> | <u>0</u> | <u>8</u> | <u>0</u> | | <u>0</u> | | | | |
| | 8 | 8 | 0 | 0 | | 0 | | | | 200 = 3020 ₄ |
-
29. To convert 529 to base 8
- | | | | | | | | | | | |
|-----|------------|----------|-----------|----------|-----|----------|-----|----|---|-------------------------|
| | 1 | 0 | 2 | 1 | ... | 4096 | 512 | 64 | 8 | 1 |
| 512 | 529 | 64 | 17 | 8 | 1 | 0 | | | | |
| | <u>512</u> | <u>0</u> | <u>16</u> | <u>1</u> | | <u>1</u> | | | | |
| | 17 | 17 | 1 | 0 | | 0 | | | | 529 = 1021 ₈ |
-
30. To convert 81 to base 3
- | | | | | | | | | | | | | |
|----|-----------|----------|----------|----------|----------|-----|----------|----------|----|---|---|-------------------------|
| | 1 | 0 | 0 | 0 | 0 | ... | 243 | 81 | 27 | 9 | 3 | 1 |
| 81 | 81 | 27 | 9 | 3 | 1 | 0 | | | | | | |
| | <u>81</u> | <u>0</u> | <u>0</u> | <u>0</u> | <u>0</u> | | <u>0</u> | <u>0</u> | | | | |
| | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | | | | 81 = 10000 ₃ |
-
31. To convert 2867 to base 12
- | | | | | | | | | | | |
|------|-------------|-------------|------------|-----------|-----|-----------|------|-----|----|---------------------------|
| | 1 | 7 | 10 | 11 | ... | 20,736 | 1728 | 144 | 12 | 1 |
| 1728 | 2867 | 144 | 1139 | 12 | 1 | 0 | | | | |
| | <u>1728</u> | <u>1008</u> | <u>120</u> | <u>11</u> | | <u>11</u> | | | | |
| | 1139 | 131 | 11 | 0 | | 0 | | | | 2867 = 17TE ₁₂ |
-
32. To convert 4312 to base 6
- | | | | | | | | | | | | | |
|------|-------------|------------|------------|-----------|----------|-----|-----------|----------|-----|----|---|---------------------------|
| | 3 | 1 | 5 | 4 | 4 | ... | 7776 | 1296 | 216 | 36 | 6 | 1 |
| 1296 | 4312 | 216 | 424 | 36 | 208 | 6 | 28 | 1 | 4 | 4 | | |
| | <u>3888</u> | <u>216</u> | <u>180</u> | <u>24</u> | <u>4</u> | | <u>24</u> | <u>4</u> | | | | |
| | 424 | 208 | 28 | 4 | 0 | | 4 | 0 | | | | 4312 = 31544 ₆ |
-
33. To convert 1011 to base 2
- | | | | | | | | | | | | | | | | | | | |
|-----|------------|------------|------------|-----------|-----------|-----|-----------|-----|-----|-----|----|----|----|---|---|---|---|--|
| | 1 | 1 | 1 | 1 | 1 | ... | 1024 | 512 | 256 | 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 | |
| 512 | 1011 | 256 | 499 | 128 | 243 | 64 | 115 | 32 | 51 | 19 | | | | | | | | |
| | <u>512</u> | <u>256</u> | <u>128</u> | <u>64</u> | <u>32</u> | | <u>19</u> | | | | | | | | | | | |
| | 499 | 243 | 115 | 51 | 19 | | | | | | | | | | | | | |
-
- | | | | | | | | | | | | | | | | | | | |
|----|-----------|----------|----------|----------|----------|---|----------|---|---|--|--|--|--|--|--|--|--|--------------------------------|
| | 1 | 0 | 0 | 1 | 1 | | | | | | | | | | | | | |
| 16 | 19 | 8 | 3 | 4 | 3 | 2 | 3 | 1 | 1 | | | | | | | | | |
| | <u>16</u> | <u>0</u> | <u>0</u> | <u>2</u> | <u>1</u> | | <u>1</u> | | | | | | | | | | | |
| | 3 | 3 | 3 | 1 | 0 | | 0 | | | | | | | | | | | 1011 = 1111110011 ₂ |
-
34. To convert 1589 to base 7
- | | | | | | | | | | | |
|-----|-------------|------------|-----------|----------|-----|----------|-----|----|---|--------------------------|
| | 4 | 4 | 3 | 0 | ... | 2401 | 343 | 49 | 7 | 1 |
| 343 | 1589 | 49 | 217 | 7 | 1 | 0 | | | | |
| | <u>1372</u> | <u>196</u> | <u>21</u> | <u>0</u> | | <u>0</u> | | | | |
| | 217 | 21 | 0 | 0 | | 0 | | | | 1589 = 4430 ₇ |
-
35. To convert 2307 to base 8
- | | | | | | | | | | | |
|-----|-------------|------------|----------|----------|-----|----------|-----|----|---|--------------------------|
| | 4 | 4 | 0 | 3 | ... | 4096 | 512 | 64 | 8 | 1 |
| 512 | 2307 | 64 | 259 | 8 | 1 | 0 | | | | |
| | <u>2048</u> | <u>256</u> | <u>0</u> | <u>3</u> | | <u>3</u> | | | | |
| | 259 | 3 | 3 | 0 | | 0 | | | | 2307 = 4403 ₈ |

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36. To convert 13,469 to base 8

...	32,768	4096	512	64	8	1			
	<u>3</u>	<u>2</u>	<u>2</u>	<u>3</u>	<u>5</u>				
4096	13,469	512	1181	64	157	8	29	1	5
	<u>12,288</u>	<u>1024</u>	<u>128</u>	<u>24</u>	<u>5</u>				
	1181	157	29	5	0				

$13,469 = 32235_8$

37. $735_{16} = 7(16^2) + 3(16) + 5(1) = 7(256) + 48 + 5 = 1792 + 48 + 5 = 1845$

38. $581_{16} = 5(16^2) + 8(16) + 1(1) = 5(256) + 128 + 1 = 1280 + 128 + 1 = 1409$

39. $6D3B7_{16} = 6(16^4) + 13(16^3) + 3(16^2) + 11(16) + 7(1) = 6(65,536) + 13(4096) + 3(256) + 176 + 7$
 $= 393,216 + 53,248 + 768 + 176 + 7 = 447,415$

40. $24FEA_{16} = 2(16^4) + 4(16^3) + 15(16^2) + 14(16) + 10(1) = 2(65,536) + 4(4096) + 15(256) + 224 + 10$
 $= 131,072 + 16,384 + 3840 + 224 + 10 = 151,530$

41. To convert 573 to base 16

...	4096	256	16	1	
	<u>2</u>	<u>3</u>	<u>13 = D</u>		
256	573	16	61	1	13
	<u>512</u>	<u>48</u>	<u>13</u>		
	61	13	0		

$573 = 23D_{16}$

42. To convert 349 to base 16

...	4096	256	16	1	
	<u>1</u>	<u>5</u>	<u>13 = D</u>		
256	349	16	93	1	13
	<u>256</u>	<u>80</u>	<u>13</u>		
	93	13	0		

$349 = 15D_{16}$

43. To convert 5478 to base 16

...	65,536	4096	256	16	1		
	<u>1</u>	<u>5</u>	<u>6</u>	<u>6</u>			
4096	5478	256	1382	16	6	1	6
	<u>4096</u>	<u>1280</u>	<u>96</u>	<u>6</u>			
	1382	102	6	0			

$5478 = 1566_{16}$

44. To convert 34,721 to base 16

...	65,536	4096	256	16	1		
	<u>8</u>	<u>7</u>	<u>10 = A</u>	<u>1</u>			
4096	34,721	256	1953	16	161	1	1
	<u>32,768</u>	<u>1792</u>	<u>160</u>				
	1953	161	1				

$34,721 = 87A1_{16}$

45. To convert 2005 to base 2

...	2048	1024	512	256	128	64	32	16	8	4	2	1
	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>						
1024	2005	512	981	256	469	128	213	64	85			
	<u>1024</u>	<u>512</u>	<u>256</u>	<u>213</u>	<u>85</u>	<u>64</u>	<u>21</u>					
	981	469	213	85	21							

32	<u>0</u>	<u>1</u>	<u>0</u>	<u>1</u>	<u>0</u>	<u>1</u>
	21	16	8	4	2	1
	<u>0</u>	<u>16</u>	<u>0</u>	<u>4</u>	<u>0</u>	<u>1</u>
	21	5	5	1	1	0

$2005 = 11111010101_2$

46. To convert 2005 to base 3

$$\begin{array}{r}
 \dots 2187 \quad 729 \quad 243 \quad 81 \quad 27 \quad 9 \quad 3 \quad 1 \\
 \begin{array}{r}
 2 \\
 729 \overline{) 2005} \\
 \underline{1458} \\
 547 \\
 \underline{486} \\
 61 \\
 \underline{61} \\
 0 \\
 \underline{54} \\
 7 \\
 \underline{7} \\
 0 \\
 \underline{6} \\
 1 \\
 \underline{1} \\
 0
 \end{array}
 \end{array}$$

$2005 = 2202021_3$

47. To convert 2005 to base 5

$$\begin{array}{r}
 \dots 3125 \quad 625 \quad 125 \quad 25 \quad 5 \quad 1 \\
 \begin{array}{r}
 3 \\
 625 \overline{) 2005} \\
 \underline{1875} \\
 130 \\
 \underline{125} \\
 5 \\
 \underline{5} \\
 0 \\
 \underline{0} \\
 0
 \end{array}
 \end{array}$$

$2005 = 31010_5$

48. To convert 2005 to base 7

$$\begin{array}{r}
 \dots 2401 \quad 343 \quad 49 \quad 7 \quad 1 \\
 \begin{array}{r}
 5 \\
 343 \overline{) 2005} \\
 \underline{1715} \\
 290 \\
 \underline{245} \\
 45 \\
 \underline{42} \\
 3 \\
 \underline{3} \\
 0
 \end{array}
 \end{array}$$

$2005 = 5563_7$

49. To convert 2005 to base 12

$$\begin{array}{r}
 \dots 20,736 \quad 1728 \quad 144 \quad 12 \quad 1 \\
 \begin{array}{r}
 1 \\
 1728 \overline{) 2005} \\
 \underline{1728} \\
 277 \\
 \underline{144} \\
 133 \\
 \underline{132} \\
 1 \\
 \underline{1} \\
 0
 \end{array}
 \end{array}$$

$2005 = 11E1_{12}$

50. To convert 2005 to base 16

$$\begin{array}{r}
 \dots 4096 \quad 256 \quad 16 \quad 1 \\
 \begin{array}{r}
 7 \\
 256 \overline{) 2005} \\
 \underline{1792} \\
 213 \\
 \underline{208} \\
 5 \\
 \underline{5} \\
 0
 \end{array}
 \end{array}$$

$2005 = 7D5_{16}$

51. Incorrect; there is no 5 in base 5.

53. Correct

55. Incorrect; there is no 8 in base 7.

57. $2(5) + 3(1) = 10 + 3 = 13$

59. $2(5^2) + 4(5) + 3(1) = 2(25) + 20 + 3$
 $= 50 + 20 + 3 = 73$

52. Incorrect; there is no 3 in base 3.

54. Correct

56. Correct

58. $4(5) + 3(1) = 20 + 3 = 23$

60. $3(5^2) + 0(5) + 3(1) = 3(25) + 0 + 3$
 $= 75 + 0 + 3 = 78$

61. To convert

$$\begin{array}{r}
 \dots 25 \quad 5 \quad 1 \\
 \begin{array}{r}
 3 = \ominus \\
 5 \overline{) 19} \\
 \underline{15} \\
 4
 \end{array}
 \quad
 \begin{array}{r}
 4 = \oplus \\
 1 \overline{) 4} \\
 \underline{4} \\
 0
 \end{array}
 \end{array}$$

$19 = \ominus \oplus_5$

62. To convert

$$\begin{array}{r}
 \dots 25 \quad 5 \quad 1 \\
 \begin{array}{r}
 4 = \oplus \\
 5 \overline{) 23} \\
 \underline{20} \\
 3
 \end{array}
 \quad
 \begin{array}{r}
 3 = \ominus \\
 1 \overline{) 3} \\
 \underline{3} \\
 0
 \end{array}
 \end{array}$$

$23 = \oplus \ominus_5$

73. a) Each remainder is multiplied by the proper power of 5.

$$\begin{array}{r} \text{b) } 5 \overline{) 683} \\ \underline{5} \\ 136 \\ \underline{125} \\ 27 \\ \underline{25} \\ 5 \\ \underline{5} \\ 0 \\ 1 \\ \underline{0} \\ 0 \end{array}$$

$$683 = 10213_5$$

$$\begin{array}{r} \text{c) } 8 \overline{) 763} \\ \underline{8} \\ 95 \\ \underline{96} \\ 11 \\ \underline{16} \\ 1 \\ \underline{0} \\ 0 \end{array}$$

$$763 = 1373_8$$

74. a) $1_3, 2_3, 10_3, 11_3, 12_3, 20_3, 21_3, 22_3, 100_3, 101_3, 102_3, 110_3, 111_3, 112_3, 120_3, 121_3, 122_3, 200_3, 201_3, 202_3$

b) 1000_3

75. Answers will vary.

76. $2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$

77. $1(b^2) + 1(b) + 1 = 43$

$$b^2 + b + 1 = 43$$

$$b^2 + b - 42 = 0$$

$$(b+7)(b-6) = 0$$

$$b+7 = 0 \text{ or } b-6 = 0$$

$$b = -7 \text{ or } b = 6$$

Since the base cannot be negative, $b = 6$.

78. $d(5^2) + d(5) + d(1) = 124$

$$25d + 5d + d = 124$$

$$\frac{31d}{31} = \frac{124}{31}$$

$$d = 4$$

79. a) $3(4^4) + 1(4^3) + 2(4^2) + 3(4) + 0(1) = 3(256) + 64 + 2(16) + 12 + 0 = 768 + 64 + 32 + 12 + 0 = 876$

b) To convert ... 256 64 16 4 1

$$64 \overline{) 177} = \text{(go)} \quad 16 \overline{) 49} = \text{(gr)} \quad 4 \overline{) 1} = \text{(b)} \quad 1 \overline{) 1} = \text{(r)}$$

$$\begin{array}{r} 64 \overline{) 177} \\ \underline{128} \\ 49 \end{array} \quad \begin{array}{r} 16 \overline{) 49} \\ \underline{48} \\ 1 \end{array} \quad \begin{array}{r} 4 \overline{) 1} \\ \underline{0} \\ 1 \end{array} \quad \begin{array}{r} 1 \overline{) 1} \\ \underline{1} \\ 0 \end{array}$$

$$177 = \text{(go)} \text{(gr)} \text{(b)} \text{(r)}_4$$

Exercise Set 4.4

1. a) $b^0 = 1, b^1 = b, b^2, b^3, b^4$

b) $6^0 = 1, 6^1 = 6, 6^2, 6^3, 6^4$

2. $8^0 = 1, 8^1 = 8, 8^2 = 64$ using base 8.

3. No; there is no 6 in base 5.

4. No; there is no 3 in base 3.

5. Answers will vary.

6. Answers will vary.

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- | | | | | | | | |
|-----|--|-----|---|-----|--|-----|--|
| 7. | $\begin{array}{r} 43_5 \\ \underline{41_5} \\ 134_5 \end{array}$ | 8. | $\begin{array}{r} 33_8 \\ \underline{65_8} \\ 120_8 \end{array}$ | 9. | $\begin{array}{r} 2303_4 \\ \underline{232_4} \\ 3201_4 \end{array}$ | 10. | $\begin{array}{r} 101_2 \\ \underline{11_2} \\ 1000_2 \end{array}$ |
| 11. | $\begin{array}{r} 799_{12} \\ \underline{218_{12}} \\ 9E5_{12} \end{array}$ | 12. | $\begin{array}{r} 222_3 \\ \underline{22_3} \\ 1021_3 \end{array}$ | 13. | $\begin{array}{r} 1112_3 \\ \underline{1011_3} \\ 2200_3 \end{array}$ | 14. | $\begin{array}{r} 470_{12} \\ \underline{347_{12}} \\ 7E7_{12} \end{array}$ |
| 15. | $\begin{array}{r} 14631_7 \\ \underline{6040_7} \\ 24001_7 \end{array}$ | 16. | $\begin{array}{r} 1341_8 \\ \underline{341_8} \\ 1702_8 \end{array}$ | 17. | $\begin{array}{r} 1110_2 \\ \underline{110_2} \\ 10100_2 \end{array}$ | 18. | $\begin{array}{r} 43A_{16} \\ \underline{496_{16}} \\ 8D0_{16} \end{array}$ |
| 19. | $\begin{array}{r} 322_4 \\ \underline{-103_4} \\ 213_4 \end{array}$ | 20. | $\begin{array}{r} 526_7 \\ \underline{-145_7} \\ 351_7 \end{array}$ | 21. | $\begin{array}{r} 2342_5 \\ \underline{-1442_5} \\ 400_5 \end{array}$ | 22. | $\begin{array}{r} 1011_2 \\ \underline{-101_2} \\ 110_2 \end{array}$ |
| 23. | $\begin{array}{r} 782_{12} \\ \underline{-13T_{12}} \\ 644_{12} \end{array}$ | 24. | $\begin{array}{r} 1221_3 \\ \underline{-202_3} \\ 1012_3 \end{array}$ | 25. | $\begin{array}{r} 1001_2 \\ \underline{-110_2} \\ 11_2 \end{array}$ | 26. | $\begin{array}{r} 2T34_{12} \\ \underline{-345_{12}} \\ 26TE_{12} \end{array}$ |
| 27. | $\begin{array}{r} 4223_7 \\ \underline{-304_7} \\ 3616_7 \end{array}$ | 28. | $\begin{array}{r} 4232_5 \\ \underline{-2341_5} \\ 1341_5 \end{array}$ | 29. | $\begin{array}{r} 2100_3 \\ \underline{-1012_3} \\ 1011_3 \end{array}$ | 30. | $\begin{array}{r} 4E7_{16} \\ \underline{-189_{16}} \\ 35E_{16} \end{array}$ |
| 31. | $\begin{array}{r} 33_5 \\ \times 2_5 \\ \hline 121_5 \end{array}$ | 32. | $\begin{array}{r} 323_6 \\ \times 4_6 \\ \hline 2140_6 \end{array}$ | 33. | $\begin{array}{r} 342_7 \\ \times 5_7 \\ \hline 2403_7 \end{array}$ | 34. | $\begin{array}{r} 101_2 \\ \times 11_2 \\ \hline 101 \\ \underline{101} \\ 1111_2 \end{array}$ |
| 35. | $\begin{array}{r} 512_6 \\ \times 23_6 \\ \hline 2340 \\ \underline{1424} \\ 21020_6 \end{array}$ | 36. | $\begin{array}{r} 124_{12} \\ \times 6_{12} \\ \hline 720_{12} \end{array}$ | 37. | $\begin{array}{r} 436_9 \\ \times 25_9 \\ \hline 2403 \\ \underline{873} \\ 12233_9 \end{array}$ | 38. | $\begin{array}{r} 6T3_{12} \\ \times 24_{12} \\ \hline 2350 \\ \underline{1186} \\ 13EE0_{12} \end{array}$ |
| 39. | $\begin{array}{r} 111_2 \\ \times 101_2 \\ \hline 111 \\ 000 \\ \underline{111} \\ 100011_2 \end{array}$ | 40. | $\begin{array}{r} 584_9 \\ \times 24_9 \\ \hline 2567 \\ \underline{1278} \\ 15457_9 \end{array}$ | 41. | $\begin{array}{r} 316_7 \\ \times 16_7 \\ \hline 2541 \\ \underline{316} \\ 6031_7 \end{array}$ | 42. | $\begin{array}{r} 8T_{12} \\ \times 2T_{12} \\ \hline 744 \\ \underline{158} \\ 2104_{12} \end{array}$ |

43. $1_2 \times 1_2 = 1_2$

$$1_2 \overline{) 110_2}$$

$$\begin{array}{r} 1 \\ \underline{01} \\ 1 \\ \underline{0} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

44. $4_6 \times 1_6 = 4_6$
 $4_6 \times 2_6 = 12_6$
 $4_6 \times 3_6 = 20_6$
 $4_6 \times 4_6 = 24_6$
 $4_6 \times 5_6 = 32_6$

$$4_6 \overline{) 231_6} \text{ R}3_6$$

$$\begin{array}{r} 20 \\ \underline{20} \\ 31 \\ \underline{24} \\ 3 \end{array}$$

45. $3_5 \times 1_5 = 3_5$
 $3_5 \times 2_5 = 11_5$
 $3_5 \times 3_5 = 14_5$
 $3_5 \times 4_5 = 22_5$

$$3_5 \overline{) 143_5}$$

$$\begin{array}{r} 14 \\ \underline{14} \\ 03 \\ \underline{3} \\ 0 \end{array}$$

46. $7_8 \times 1_8 = 7_8$
 $7_8 \times 2_8 = 16_8$
 $7_8 \times 3_8 = 25_8$
 $7_8 \times 4_8 = 34_8$
 $7_8 \times 5_8 = 43_8$
 $7_8 \times 6_8 = 52_8$
 $7_8 \times 7_8 = 61_8$

$$7_8 \overline{) 37_8} \text{ R}4_8$$

$$\begin{array}{r} 37_8 \\ \underline{25} \\ 65 \\ \underline{61} \\ 4 \end{array}$$

47. $2_4 \times 1_4 = 2_4$
 $2_4 \times 2_4 = 10_4$
 $2_4 \times 3_4 = 12_4$

$$2_4 \overline{) 123_4}$$

$$\begin{array}{r} 312_4 \\ \underline{2} \\ 11 \\ \underline{10} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

48. $6_{12} \times 1_{12} = 6_{12}$
 $6_{12} \times 2_{12} = 10_{12}$
 $6_{12} \times 3_{12} = 16_{12}$
 $6_{12} \times 4_{12} = 20_{12}$
 $6_{12} \times 5_{12} = 26_{12}$
 $6_{12} \times 6_{12} = 30_{12}$
 $6_{12} \times 7_{12} = 36_{12}$
 $6_{12} \times 8_{12} = 40_{12}$

$$6_{12} \overline{) 86_{12}} \text{ R}1_{12}$$

$$\begin{array}{r} 431_{12} \\ \underline{40} \\ 31 \\ \underline{30} \\ 1 \end{array}$$

49. $2_4 \times 1_4 = 2_4$
 $2_4 \times 2_4 = 10_4$
 $2_4 \times 3_4 = 12_4$

$$2_4 \overline{) 103_4} \text{ R}1_4$$

$$\begin{array}{r} 213_4 \\ \underline{2} \\ 01 \\ \underline{00} \\ 13 \\ \underline{12} \\ 1 \end{array}$$

50. $5_6 \times 1_6 = 5_6$
 $5_6 \times 2_6 = 14_6$
 $5_6 \times 3_6 = 23_6$
 $5_6 \times 4_6 = 32_6$
 $5_6 \times 5_6 = 41_6$

$$5_6 \overline{) 24_6} \text{ R}2_6$$

$$\begin{array}{r} 214_6 \\ \underline{14} \\ 34 \\ \underline{32} \\ 2 \end{array}$$

51. $3_5 \times 1_5 = 3_5$
 $3_5 \times 2_5 = 11_5$
 $3_5 \times 3_5 = 14_5$
 $3_5 \times 4_5 = 22_5$

$$3_5 \overline{) 41_5} \text{ R}1_5$$

$$\begin{array}{r} 224_5 \\ \underline{22} \\ 04 \\ \underline{3} \\ 1 \end{array}$$

52. $4_6 \times 1_6 = 4_6$
 $4_6 \times 2_6 = 12_6$
 $4_6 \times 3_6 = 20_6$
 $4_6 \times 4_6 = 24_6$
 $4_6 \times 5_6 = 32_6$

$$4_6 \overline{) 31_6} \text{ R}2_6$$

$$\begin{array}{r} 210_6 \\ \underline{20} \\ 10 \\ \underline{4} \\ 2 \end{array}$$

53. $6_7 \times 1_7 = 6_7$
 $6_7 \times 2_7 = 15_7$
 $6_7 \times 3_7 = 24_7$
 $6_7 \times 4_7 = 33_7$
 $6_7 \times 5_7 = 42_7$
 $6_7 \times 6_7 = 51_7$

$$6_7 \overline{) 45_7} \text{ R}2_7$$

$$\begin{array}{r} 404_7 \\ \underline{33} \\ 44 \\ \underline{42} \\ 2 \end{array}$$

54. $3_7 \times 1_7 = 3_7$
 $3_7 \times 2_7 = 6_7$
 $3_7 \times 3_7 = 12_7$
 $3_7 \times 4_7 = 15_7$
 $3_7 \times 5_7 = 21_7$
 $3_7 \times 6_7 = 24_7$

$$3_7 \overline{) 500_7} \text{ R}1_7$$

$$\begin{array}{r} 2101_7 \\ \underline{21} \\ 00 \\ \underline{00} \\ 01 \\ \underline{00} \\ 1 \end{array}$$

55. 2_5
 $\begin{array}{r} 2_5 \\ + 3_5 \\ \hline 10_5 = \ominus \ominus_5 \end{array}$

56. 3_5
 $\begin{array}{r} 3_5 \\ + 3_5 \\ \hline 11_5 = \ominus \ominus_5 \end{array}$

57. 21_5
 $\begin{array}{r} 21_5 \\ + 43_5 \\ \hline 114_5 = \ominus \ominus \ominus_5 \end{array}$

58. 23_5
 $\begin{array}{r} 23_5 \\ + 13_5 \\ \hline 41_5 = \ominus \ominus_5 \end{array}$

For #59-66, blue = 0 = b, red = 1 = r, gold = 2 = go, green = 3 = gr

59. 3_4
 $\begin{array}{r} 3_4 \\ + 2_4 \\ \hline 11_4 = \textcircled{r} \textcircled{r}_4 \end{array}$

60. 12_4
 $\begin{array}{r} 12_4 \\ + 30_4 \\ \hline 102_4 = \textcircled{r} \textcircled{b} \textcircled{go}_4 \end{array}$

61. 32_4
 $\begin{array}{r} 32_4 \\ + 11_4 \\ \hline 103_4 = \textcircled{r} \textcircled{b} \textcircled{gr}_4 \end{array}$

62. 130_4
 $\begin{array}{r} 130_4 \\ + 221_4 \\ \hline 1011_4 = \textcircled{r} \textcircled{b} \textcircled{r} \textcircled{r}_4 \end{array}$

63. 33_4
 $\begin{array}{r} 33_4 \\ - 12_4 \\ \hline 21_4 = \textcircled{go} \textcircled{r}_4 \end{array}$

64. 31_4
 $\begin{array}{r} 31_4 \\ - 13_4 \\ \hline 12_4 = \textcircled{r} \textcircled{go}_4 \end{array}$

65. 231_4
 $\begin{array}{r} 231_4 \\ - 103_4 \\ \hline 122_4 = \textcircled{r} \textcircled{go} \textcircled{go}_4 \end{array}$

66. 301_4
 $\begin{array}{r} 301_4 \\ - 120_4 \\ \hline 121_4 = \textcircled{r} \textcircled{go} \textcircled{r}_4 \end{array}$

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67. $2302_5 = 2(5^3) + 3(5^2) + 0(5) + 2(1) = 2(125) + 3(25) + 0 + 2 = 250 + 75 + 0 + 2 = 327$

68. To convert 327 to base 9

4	327
324	
3	

0	3
0	
3	

3	3
3	
0	

... 729 81 9 1

$327 = 403_9$

$9^2 = \dots$

$9^1 = \text{none}$

$9^0 = \dots$

69. $14_5 \times 1_5 = 14_5$ $14_5 \times 2_5 = 33_5$ $14_5 \times 3_5 = 102_5$ $14_5 \times 4_5 = 121_5$

13 ₅	242 ₅
14	
102	
102	
0	

70. $20_4 \times 1_4 = 20_4$ $20_4 \times 2_4 = 100_4$ $20_4 \times 3_4 = 120_4$

11 ₄	223 ₄	R3 ₄
20		
23		
20		
3		

71. a) 462_8

× 35 ₈
2772
1626
21252 ₈

b) $462_8 = 4(8^2) + 6(8) + 2(1) = 4(64) + 48 + 2 = 256 + 48 + 2 = 306$

$35_8 = 3(8) + 5(1) = 24 + 5 = 29$

c) $306 \times 29 = 8874$

d) $21252_8 = 2(8^4) + 1(8^3) + 2(8^2) + 5(8) + 2(1)$

$= 2(4096) + 512 + 2(64) + 40 + 2$

$= 8192 + 512 + 128 + 40 + 2 = 8874$

e) Yes, in part a), the numbers were multiplied in base 8 and then converted to base 10 in part d). In part b), the numbers were converted to base 10 first, then multiplied in part c).

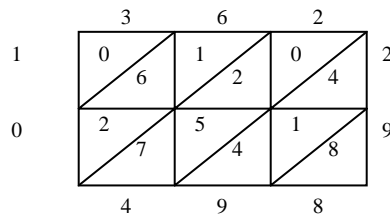
72. $b = 5$
73. Orange = 0; purple = 1; turquoise = 2; brown = 3

Exercise Set 4.5

1. Duplation and mediation, the galley method and Napier rods
2. a) Answers will vary.
- b) $267 - 193$
3. a) Answers will vary.
- b)

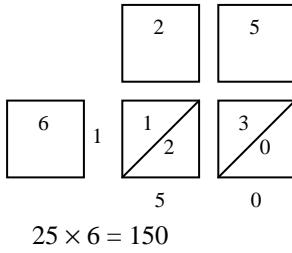
$133 - 386$

66	772
33	1544
16	3088
8	6176
4	12,352
2	24,704
1	49,408
51,531	



$362 \times 29 = 10,498$

4. a) Answers will vary.
b)



5.
$$\begin{array}{r} 23 - 31 \\ 11 - 62 \\ 5 - 124 \\ \underline{2 - 248} \\ 1 - 496 \\ \hline 713 \end{array}$$

6.
$$\begin{array}{r} 35 - 23 \\ 17 - 46 \\ \underline{8 - 92} \\ 4 - 184 \\ \underline{2 - 368} \\ 1 - 736 \\ \hline 805 \end{array}$$

7.
$$\begin{array}{r} 9 - 162 \\ \underline{4 - 324} \\ 8 - 648 \\ \underline{1 - 1296} \\ 1458 \end{array}$$

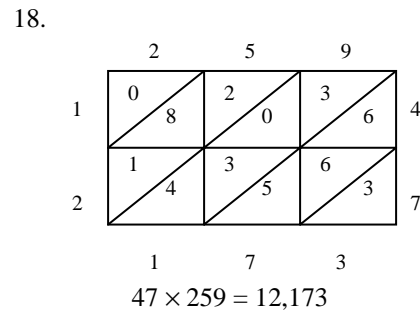
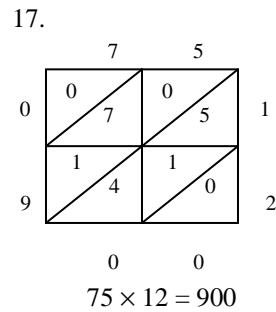
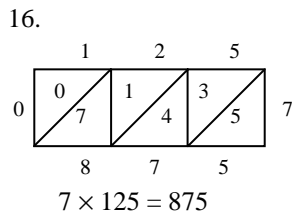
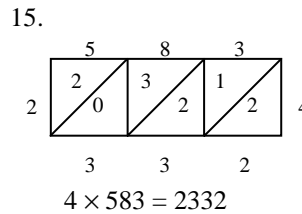
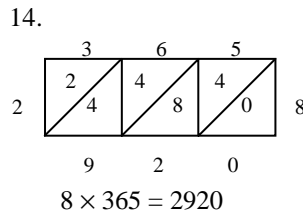
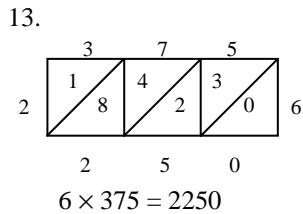
8.
$$\begin{array}{r} 175 - 86 \\ 87 - 172 \\ 43 - 344 \\ 21 - 688 \\ \underline{10 - 1376} \\ 5 - 2752 \\ \underline{2 - 5504} \\ 1 - 11,008 \\ \hline 15,050 \end{array}$$

9.
$$\begin{array}{r} 35 - 236 \\ 17 - 472 \\ \underline{8 - 944} \\ 4 - 1888 \\ \underline{2 - 3776} \\ 1 - 7552 \\ \hline 8260 \end{array}$$

10.
$$\begin{array}{r} 96 - 53 \\ 48 - 106 \\ \underline{24 - 212} \\ 12 - 424 \\ \underline{6 - 848} \\ 3 - 1696 \\ \underline{1 - 3392} \\ 5088 \end{array}$$

11.
$$\begin{array}{r} 93 - 93 \\ 46 - 186 \\ 23 - 372 \\ 11 - 744 \\ 5 - 1488 \\ \underline{2 - 2976} \\ 1 - 5952 \\ \hline 8649 \end{array}$$

12.
$$\begin{array}{r} 49 - 124 \\ 24 - 248 \\ \underline{12 - 496} \\ 6 - 992 \\ 3 - 1984 \\ \underline{1 - 3968} \\ 6076 \end{array}$$



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19.
$$\begin{array}{r} 314 \\ \times 652 \\ \hline 728 \\ 050 \\ 186 \\ \hline 204728 \end{array}$$

20.
$$\begin{array}{r} 634 \\ \times 832 \\ \hline 488 \\ 2168 \\ 5072 \\ \hline 527488 \end{array}$$

21.
$$\begin{array}{r} 63 \\ \times 8 \\ \hline 504 \end{array}$$

22.
$$\begin{array}{r} 63 \\ \times 7 \\ \hline 441 \end{array}$$

23.
$$\begin{array}{r} 58 \\ \times 7 \\ \hline 406 \end{array}$$

24.
$$\begin{array}{r} 125 \\ \times 7 \\ \hline 875 \end{array}$$

25.
$$\begin{array}{r} 125 \\ \times 5 \\ \hline 625 \end{array}$$

26. $75 \times 125 = (70 + 5)125 = 70(125) + 5(125)$
 From # 24, $70 \times 125 = 8750$
 From # 25, $5 \times 125 = \underline{625}$
 9375
 $75 \times 125 = 9375$

27.
$$\begin{array}{r} 6742 \\ \times 9 \\ \hline 60678 \end{array}$$

28.
$$\begin{array}{r} 3456 \\ \times 7 \\ \hline 24192 \end{array}$$

37.

	2	3	4	
1	0 4	1 1	1 3	2
2	1 3	2 2	3 1	4
	3	3	1	

$24_5 \times 234_5 = 12331_5$

38. a) $1000 + 500 + 100 + 100 + 50 + 10 + 10 + 5 + 1 = 1776$
 b) Answers will vary.

Review Exercises

1. $1000 + 1000 + 1000 + 100 + 1 + 1 + 1 = 3103$
2. $100 + 100 + 10 + 1000 + 1 = 1211$
3. $10 + 100 + 100 + 100 + 1 + 1000 = 1311$
4. $100 + 10 + 1000 + 1 + 1000 + 1 + 1 + 1 = 2114$
5. $1000 + 1000 + 100 + 100 + 100 + 10 + 1 + 1 + 1 + 1 = 2314$
6. $100 + 100 + 10 + 1 + 1000 + 1000 + 1 + 100 = 2312$
7. *bbbbbaaaaa*
8. *cbbaaaaa*
9. *ccbbbbbbbaaa*
10. *ddaaaaa*
11. *ddddccccccbbba*
12. *ddccbaaaa*
13. $4(10) + 3 = 40 + 3 = 43$
14. $2(10) + 7 = 20 + 7 = 27$
15. $7(100) + 4(10) + 9 = 700 + 40 + 9 = 749$
16. $4(1000) + 6(10) + 8 = 4000 + 60 + 8 = 4068$
17. $5(1000) + 6(100) + 4(10) + 8 = 5000 + 600 + 40 + 8 = 5648$
18. $6(1000) + 9(100) + 5 = 6000 + 900 + 5 = 6905$
19. *hxb*
20. *byixe*
21. *hyfxb*
22. *czixd*
23. *fzd*
24. *bza*
25. $4(10) + 5(1) = 40 + 5 = 45$
26. $3(100) + 8(1) = 300 + 8 = 308$
27. $5(100) + 6(10) + 8(1) = 500 + 60 + 8 = 568$
28. $4(10,000) + 6(1000) + 8(100) + 8(10) + 3(1) = 40,000 + 6000 + 800 + 80 + 3 = 46,883$
29. $6(10,000) + 4(1000) + 4(100) + 8(10) + 1 = 60,000 + 4000 + 400 + 80 + 1 = 64,481$
30. $6(10,000) + 5(100) + 2(10) + 9(1) = 60,000 + 500 + 20 + 9 = 60,529$
31. *qe*
32. *upb*
33. *vrc*
34. *BArg*
35. *ODvog*
36. *QFvrf*

37. 

38. MCDLXII

39. 一千四百六十二

40. $\alpha'v\xi\beta$

41.

24	◀◀◀◀◀	◀◀◀
60	1462	
	1440	
	22	

42.

4	◻◻◻◻◻	◻◻◻◻
360	1462	20
	1440	20
	22	2

$1462 = 24(60) + 22$

$1462 = 4(18 \times 20) + 1(20) + 2(1)$

43. $100,000 + 100,000 + 10,000 + 10,000 + 1000 + 1000 + 10 + 10 + 10 + 1 + 1 + 1 + 1 + 1 = 222,035$
 44. $8(1000) + 2(100) + 5(10) + 4 = 8000 + 200 + 50 + 4 = 8254$
 45. $600 + 80 + 5 = 685$
 46. $1000 + (1000 - 100) + (100 - 10) + 1 = 1000 + 900 + 90 + 1 = 1991$
 47. $21(60) + (20 - 3) = 1260 + 17 = 1277$
 48. $7(18 \times 20) + 8(20) + 10(1) = 7(360) + 160 + 10 = 2520 + 160 + 10 = 2690$
 49. $47_8 = 4(8) + 7(1) = 32 + 7 = 39$
 50. $101_2 = 1(2^2) + 0(2) + 1(1) = 4 + 0 + 1 = 5$
 51. $130_4 = 1(4^2) + 3(4) + 0(1) = 16 + 12 + 0 = 28$
 52. $3425_7 = 3(7^3) + 4(7^2) + 2(7) + 5(1) = 3(343) + 4(49) + 14 + 5 = 1029 + 196 + 14 + 5 = 1244$
 53. $10E_{12} = 10(12^2) + 0(12) + 11(1) = 10(144) + 0 + 11 = 1440 + 0 + 11 = 1451$
 54. $20220_3 = 2(3^4) + 0(3^3) + 2(3^2) + 2(3) + 0(1) = 2(81) + 0 + 2(9) + 6 + 0 = 162 + 0 + 18 + 6 + 0 = 186$

55. To convert 463 to base 4

	...	1024	256	64	16	4	1	
256	1	64	16	4	1			
	$\overline{463}$	$\overline{207}$	$\overline{15}$	$\overline{15}$	$\overline{3}$			
	$\underline{256}$	$\underline{192}$	$\underline{0}$	$\underline{12}$	$\underline{3}$			
	207	15	15	3	0			463 = 13033 ₄

56. To convert 463 to base 3

	...	729	243	81	27	9	3	1	
243	1	81	27	9	3	1			
	$\overline{463}$	$\overline{220}$	$\overline{58}$	$\overline{4}$	$\overline{4}$	$\overline{1}$	$\overline{1}$		
	$\underline{243}$	$\underline{162}$	$\underline{54}$	$\underline{0}$	$\underline{3}$	$\underline{1}$	$\underline{1}$		
	220	58	4	4	1	0	0		463 = 122011 ₃

57. To convert 463 to base 2

	...	512	256	128	64	32	16	8	4	2	1	
256	1	128	64	32	16	8	4	2	1	1		
	$\overline{463}$	$\overline{207}$	$\overline{79}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{7}$	$\overline{3}$	$\overline{1}$	$\overline{1}$		
	$\underline{256}$	$\underline{128}$	$\underline{64}$	$\underline{0}$	$\underline{0}$	$\underline{8}$	$\underline{4}$	$\underline{2}$	$\underline{1}$	$\underline{0}$		
	207	79	15	15	15	7	3	1	0	0		463 = 111001111 ₂

58. To convert 463 to base 5

	...	625	125	25	5	1	
125	3	25	5	1			
	$\overline{463}$	$\overline{88}$	$\overline{13}$	$\overline{3}$			
	$\underline{375}$	$\underline{75}$	$\underline{10}$	$\underline{3}$			
	88	13	3	0			463 = 3323 ₅

59. To convert 463 to base 12

	...	1728	144	12	1	
144	3	12	1			
	$\overline{463}$	$\overline{31}$	$\overline{7}$			
	$\underline{432}$	$\underline{24}$	$\underline{7}$			
	31	7	0			463 = 327 ₁₂

60. To convert 463 to base 8

	...	512	64	8	1	
64	7	8	1			
	$\overline{463}$	$\overline{15}$	$\overline{7}$			
	$\underline{448}$	$\underline{8}$	$\underline{7}$			
	15	7	0			463 = 717 ₈

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$$\begin{array}{r} 52_7 \\ \underline{55_7} \\ 140_7 \end{array}$$

$$\begin{array}{r} 10110_2 \\ \underline{11001_2} \\ 101111_2 \end{array}$$

$$\begin{array}{r} TE_{12} \\ \underline{87_{12}} \\ 176_{12} \end{array}$$

$$\begin{array}{r} 234_7 \\ \underline{456_7} \\ 1023_7 \end{array}$$

$$\begin{array}{r} 3024_5 \\ \underline{4023_5} \\ 12102_5 \end{array}$$

$$\begin{array}{r} 3407_8 \\ \underline{7014_8} \\ 12423_8 \end{array}$$

$$\begin{array}{r} 4032_7 \\ \underline{-321_7} \\ 3411_7 \end{array}$$

$$\begin{array}{r} 1001_2 \\ - \underline{101_2} \\ 100_2 \end{array}$$

$$\begin{array}{r} 3TT_{12} \\ \underline{-E7_{12}} \\ 2E3_{12} \end{array}$$

$$\begin{array}{r} 4321_5 \\ \underline{-442_5} \\ 3324_5 \end{array}$$

$$\begin{array}{r} 1713_8 \\ \underline{-1243_8} \\ 450_8 \end{array}$$

$$\begin{array}{r} 2021_3 \\ - \underline{212_3} \\ 1102_3 \end{array}$$

$$\begin{array}{r} 32_6 \\ \times 4_6 \\ \hline 212_6 \end{array}$$

$$\begin{array}{r} 34_5 \\ \times 21_5 \\ \hline 34 \\ \underline{123} \\ 1314_5 \end{array}$$

$$\begin{array}{r} 126_{12} \\ \times 47_{12} \\ \hline 856 \\ \underline{4T0} \\ 5656_{12} \end{array}$$

$$\begin{array}{r} 221_3 \\ \times 22_3 \\ \hline 1212 \\ \underline{1212} \\ 21102_3 \end{array}$$

$$\begin{array}{r} 1011_2 \\ \times 101_2 \\ \hline 1011 \\ 0000 \\ \underline{1011} \\ 110111_2 \end{array}$$

$$\begin{array}{r} 476_8 \\ \times 23_8 \\ \hline 1672 \\ \underline{1174} \\ 13632_8 \end{array}$$

$$79. \quad 1_2 \times 1_2 = 1_2$$

$$\begin{array}{r} 1011_2 \\ 1_2 \overline{)1011_2} \\ \underline{1} \\ 00 \\ \underline{00} \\ 01 \\ \underline{1} \\ 01 \\ \underline{1} \\ 0 \end{array}$$

$$80. \quad \begin{array}{l} 2_4 \times 1_4 = 2_4 \\ 2_4 \times 2_4 = 10_4 \\ 2_4 \times 3_4 = 12_4 \end{array}$$

$$\begin{array}{r} 130_4 \\ 2_4 \overline{)320_4} \\ \underline{2} \\ 12 \\ \underline{12} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

$$81. \quad \begin{array}{l} 3_5 \times 1_5 = 3_5 \\ 3_5 \times 2_5 = 11_5 \\ 3_5 \times 3_5 = 14_5 \\ 3_5 \times 4_5 = 22_5 \end{array}$$

$$\begin{array}{r} 23_5 \quad R1_5 \\ 3_5 \overline{)130_5} \\ \underline{11} \\ 20 \\ \underline{14} \\ 1 \end{array}$$

$$82. \quad \begin{array}{l} 4_6 \times 1_6 = 4_6 \\ 4_6 \times 2_6 = 12_6 \\ 4_6 \times 3_6 = 20_6 \\ 4_6 \times 4_6 = 24_6 \\ 4_6 \times 5_6 = 32_6 \end{array}$$

$$\begin{array}{r} 433_6 \\ 4_6 \overline{)3020_6} \\ \underline{24} \\ 22 \\ \underline{20} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

$$83. \quad \begin{array}{l} 3_6 \times 1_6 = 3_6 \\ 3_6 \times 2_6 = 10_6 \\ 3_6 \times 3_6 = 13_6 \\ 3_6 \times 4_6 = 20_6 \\ 3_6 \times 5_6 = 23_6 \end{array}$$

$$\begin{array}{r} 411_6 \quad R1_6 \\ 3_6 \overline{)2034_6} \\ \underline{20} \\ 03 \\ \underline{3} \\ 04 \\ \underline{3} \\ 1 \end{array}$$

$$84. \quad \begin{array}{l} 6_8 \times 1_8 = 6_8 \\ 6_8 \times 2_8 = 14_8 \\ 6_8 \times 3_8 = 22_8 \\ 6_8 \times 4_8 = 30_8 \\ 6_8 \times 5_8 = 36_8 \\ 6_8 \times 6_8 = 44_8 \\ 6_8 \times 7_8 = 52_8 \end{array}$$

$$\begin{array}{r} 664_8 \quad R2_8 \\ 6_8 \overline{)5072_8} \\ \underline{44} \\ 47 \\ \underline{44} \\ 32 \\ \underline{30} \\ 2 \end{array}$$

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16. In a place-value system, each number is multiplied by a power of the base. The position of the numeral indicates the power of the base by which it is multiplied.

17. $56_7 = 5(7) + 6(1) = 35 + 6 = 41$

18. $403_5 = 4(5^2) + 0(5) + 3(1) = 4(25) + 0 + 3 = 100 + 0 + 3 = 103$

19. $101101_2 = 1(2^5) + 0(2^4) + 1(2^3) + 1(2^2) + 0(2) + 1(1) = 32 + 0 + 8 + 4 + 0 + 1 = 45$

20. $368_9 = 3(9^2) + 6(9) + 8(1) = 3(81) + 54 + 8 = 243 + 54 + 8 = 305$

21. To convert 36 to base 2

	... 64	32	16	8	4	2	1
32	$\overline{)36}$	$\overline{)16}$	$\overline{)8}$	$\overline{)4}$	$\overline{)2}$	$\overline{)1}$	
	<u>32</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>	
	4	4	4	0	0	0	

$36 = 100100_2$

22. To convert 93 to base 5

	... 125	25	5	1
25	$\overline{)93}$	$\overline{)18}$	$\overline{)3}$	
	<u>75</u>	<u>15</u>	<u>3</u>	
	18	3	0	

$93 = 333_5$

23. To convert 2356 to base 12

	... 20,736	1728	144	12	1
1728	$\overline{)2356}$	$\overline{)628}$	$\overline{)52}$	$\overline{)4}$	
	<u>1728</u>	<u>576</u>	<u>48</u>	<u>4</u>	
	628	52	4	0	

$2356 = 1444_{12}$

24. To convert 2938 to base 7

	... 16,807	2401	343	49	7	1
2401	$\overline{)2938}$	$\overline{)537}$	$\overline{)194}$	$\overline{)47}$	$\overline{)5}$	
	<u>2401</u>	<u>343</u>	<u>147</u>	<u>42</u>	<u>5</u>	
	537	194	47	5	0	

$2938 = 11365_7$

25.
$$\begin{array}{r} 133_5 \\ + 434_5 \\ \hline 1122_5 \end{array}$$

26.
$$\begin{array}{r} 324_6 \\ - 142_6 \\ \hline 142_6 \end{array}$$

27.
$$\begin{array}{r} 45_6 \\ \times 23_6 \\ \hline 134 \\ + 223 \\ \hline 2003_6 \end{array}$$

28. $3_5 \times 1_5 = 3_5$
 $3_5 \times 2_5 = 11_5$
 $3_5 \times 3_5 = 14_5$
 $3_5 \times 4_5 = 22_5$

3_5	$\overline{)220_5}$
	<u>11</u>
	11
	<u>11</u>
	00
	<u>00</u>
	0

29.
$$\begin{array}{r} 35 - 28 \\ 17 - 56 \\ -8 - 112 \\ 4 - 224 \\ 2 - 448 \\ 1 - 896 \\ \hline 980 \end{array}$$

30.

	1	9	6	
0	0 4	3 6	2 4	4
8	0 3	2 7	1 8	3
	4	2	8	

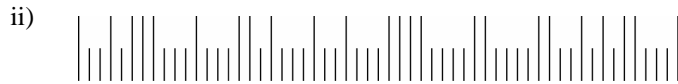
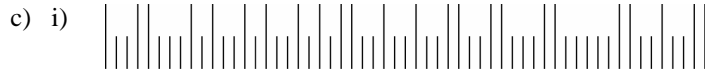
$43 \times 196 = 8428$

Group Projects

1. a) 06470-9869-1

b) i) 51593-4837-7

ii) 14527-8924-75-6



d) Answers will vary.