Chapter 4 **Derivatives of Sinusoidal Functions**

Chapter 4 Prerequisite Skills

Chapter 4 Prerequisite Skills **Question 1 Page 212**

a) $360° = 2\pi$ rad

b) $90° = \frac{\pi}{2}$ rad

c) $-45° = -\frac{\pi}{4}$ rad

d) $29.5° = \frac{59\pi}{360}$ rad

e) $115° = \frac{23\pi}{36}$ rad

f) $240° = \frac{4\pi}{3}$ rad

Chapter 4 Prerequisite Skills **Question 2 Page 212**

a) $a = 5\pi$
$\doteq 15.7$

The arc length is approximately 15.7 cm.

b) $a = 5(2.0)$
$= 10.0$

The arc length is 10.0 cm.

c) $a = 5\left(\frac{\pi}{3}\right)$
$\doteq 5.2$

The arc length is $\frac{5\pi}{3}$ cm or approximately 5.2 cm.

d) $a = 5\left(\frac{11.4}{360}\right)(2\pi)$
$\doteq 1.0$

The arc length is approximately 1.0 cm.

e) $a = \dfrac{5\pi}{2}$

The arc length is $\dfrac{5\pi}{2}$ cm or approximately 7.85 cm.

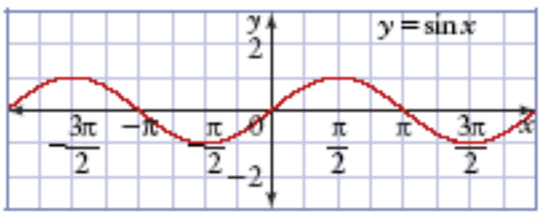
f) $a = 5\left(\dfrac{173}{360}\right)(2\pi)$

$= \dfrac{173\pi}{36}$

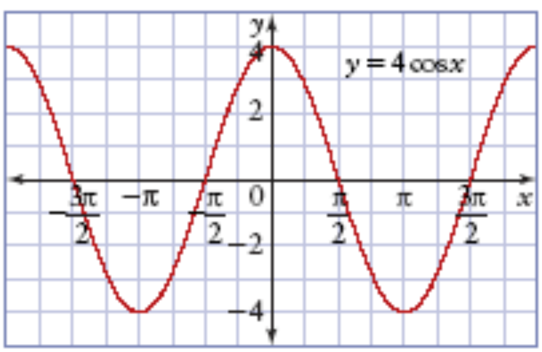
The arc length is $\dfrac{173\pi}{36}$ cm or approximately 15.10 cm.

Chapter 4 Prerequisite Skills **Question 3 Page 212**

a)



b)



Chapter 4 Prerequisite Skills **Question 4 Page 212**

a) amplitude: 1
period: 2π

b) amplitude: 4
period: 2π

Chapter 4 Prerequisite Skills **Question 5 Page 212**

a) The graph of $f(x) = \cos x$ is horizontally compressed by a factor of 2 and vertically stretched by a factor of 3 to obtain the graph of $y = 3f(2x)$.

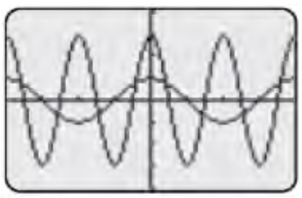
b) i) The minimum value is $3(-1) = -3$.

ii) The maximum value is $3(1) = 3$.

c) i) $\{x \mid x = k\pi, k \in \mathbb{Z}\}$

ii) $\{x \mid x = k\pi + \frac{\pi}{2}, k \in \mathbb{Z}\}$

d)



Chapter 4 Prerequisite Skills **Question 6 Page 212**

a) Graph A: maximum value 3, minimum value –3.
Graph B: maximum value 3, minimum value 1.

b) Answers will may vary. For example:

Graph A: $y = 3\sin\left(x - \frac{\pi}{2}\right)$

Graph B: $y = \cos\left(x - \frac{\pi}{2}\right) + 2$

c) Answers will vary. For example:

Sine and cosine functions are periodic, so there are many possible solutions.

Graph A: $y = 3\sin\left(x + \frac{(2k+1)\pi}{2}\right)$ for $k \in \mathbb{Z}$ has the same graph.

Graph B: $y = \cos\left(x + \frac{(2k+1)\pi}{2}\right) + 2$ for $k \in \mathbb{Z}$ has the same graph.

Chapter 4 Prerequisite Skills **Question 7 Page 212**

Use the $\left(1, 1, \sqrt{2}\right)$ and $\left(2, 1, \sqrt{3}\right)$ triangles.

a) $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

b) $\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

c) $\sin\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{3}\right) = 1 + \frac{1}{2}$

$= \frac{3}{2}$

d) $\sin^2\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{6}\right) = \left(\frac{1}{\sqrt{2}}\right)^2 - \frac{1}{2}$

$= 0$

e) $\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$

f) $\cot\left(\frac{\pi}{2}\right) = 0$

g) $\csc\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}$

h) $\sec^2\left(\frac{\pi}{4}\right) = 2$

Chapter 4 Prerequisite Skills **Question 8 Page 213**

a) $\frac{dy}{dx} = 5$

b) $\frac{dy}{dx} = -6x^2 + 8x$

c) $\frac{dy}{dx} = \frac{1}{2}(t^2 - 1)^{-\frac{1}{2}}(2t)$

d) $\dfrac{dy}{dx}=2\left[(x^{-2})\left(\dfrac{1}{2}(x-3)^{-\frac{1}{2}}(1)\right)+(x-3)^{\frac{1}{2}}(-2x^{-3})\right]$

Chapter 4 Prerequisite Skills Question 9 Page 213

a) $f(g(x))=(3x+4)^2$

$$\begin{aligned}\frac{d}{dx}[(f(g(x))]&=\frac{d}{dx}\left[(3x+4)^2\right]\\&=2(3x+4)(3)\end{aligned}$$

b) $g(f(x))=3x^2+4$

$$\begin{aligned}\frac{d}{dx}[g(f(x))]&=\frac{d}{dx}(3x^2+4)\\&=6x\end{aligned}$$

c) $f(f(x))=(x^2)^2$

$$\begin{aligned}\frac{d}{dx}[f(f(x))]&=\frac{d}{dx}(x^4)\\&=4x^3\end{aligned}$$

d) $$\begin{aligned}f(x)g(x)&=(x^2)(3x+4)\\&=3x^3+4x^2\end{aligned}$$

$$\frac{d}{dx}[f(x)g(x)]=9x^2+8x$$

Chapter 4 Prerequisite Skills Question 10 Page 213

To find the slope, differentiate the given function, $y=-3x^2+5x-11$, with respect to *x*.

$$\frac{dy}{dx}=-6x+5$$

The value of the derivative at $x = –4$, gives the slope of the function at that point.
$-6(-4)+5=29$

Therefore, the slope of the graph of $y=-3x^2+5x-11$ at $x=-4$ is. 29.

Chapter 4 Prerequisite Skills Question 11 Page 213

The slope of a line tangent to a curve, *y*, is given by $\dfrac{dy}{dx}$.

$$\frac{dy}{dx}=x+6$$

The value of the derivative at $x = –2$, gives the slope of the function at that point.

$-2+6=4$

At $x=-2$, slope = 4 and $y = -10$.
The equation of the tangent is of the form $y = mx + b$. Find b.

$$\begin{aligned} b &= -10 + 2(4) \\ &= -2 \end{aligned}$$

Therefore, the equation of the tangent is $y = 4x - 2$.

Chapter 4 Prerequisite Skills Question 12 Page 213

At local maxima and minima, $\frac{dy}{dx} = 0$.

$$\begin{aligned} \frac{dy}{dx} &= 3x^2 + 10x + 3 \\ 0 &= 3x^2 + 10x + 3 \\ x &= \frac{-10 \pm \sqrt{100-36}}{6} \\ x &= -3 \text{ or } x = -\frac{1}{3} \end{aligned}$$

At $x = -3$, $y = 6$ and at $x = -\frac{1}{3}$, $y = -\frac{94}{27}$.
Local maximum point: (–3, 6).
Local minimum point: $\left(-\frac{1}{3}, -\frac{94}{27}\right)$.

Since the y-values correspond to both critical points it is not necessary to use the second derivative test to determine if the points are local maxima or minima.

Chapter 4 Prerequisite Skills Question 13 Page 213

a) $\sin(a+b) = (\sin a)(\cos b) + (\cos a)(\sin b)$
Therefore, $x = \sin a$ and $y = \cos a$.

b) $\sin(a-b) = (\sin a)(\cos b) - (\cos a)(\sin b)$
Therefore, $x = \sin a$.

c) $\cos(a+b) = (\cos a)(\cos b) - (\sin a)(\sin b)$
Therefore, $x = \sin a$ and $y = \cos a$.

Chapter 4 Prerequisite Skills **Question 14 Page 213**

Answers may vary. For example:

a) Use the definitions of $\sin\ \theta = \dfrac{y}{r}$, $\cos\ \theta = \dfrac{x}{r}$, and the Pythagorean theorem.

$$\begin{aligned}\text{L.S.} &= \sin^2\theta \\ &= (\sin\ \theta)^2 \\ &= \left(\frac{y}{r}\right)^2 \\ &= \frac{y^2}{r^2}\end{aligned}$$

$$\begin{aligned}\text{R.S.} &= 1-\cos^2\theta \\ &= 1-(\cos\ \theta)^2 \\ &= 1-\left(\frac{x}{r}\right)^2 \\ &= \frac{r^2}{r^2}-\left(\frac{x}{r}\right)^2 \\ &= \frac{r^2}{r^2}-\frac{x^2}{r^2} \\ &= \frac{r^2-x^2}{r^2} \\ &= \frac{y^2}{r^2}\end{aligned}$$

Since L.S. = R.S., the identity has been proven.

b)

$$\begin{aligned}\text{L.S.} &= \tan(-\theta)\cos(-\theta) \\ &= (-\tan\ \theta)(\cos\ \theta) \\ &= \left(\frac{-\sin\ \theta}{\cos\ \theta}\right)(\cos\ \theta) \\ &= -\sin\ \theta\end{aligned}$$

$$\text{R.S.} = -\sin\ \theta$$

Since L.S. = R.S., the identity has been proven.

c)

$$\begin{aligned}\text{L.S.} &= \cot\ \theta \\ &= \frac{1}{\tan\ \theta} \\ &= \frac{1}{\left(\dfrac{\sin\ \theta}{\cos\ \theta}\right)} \\ &= \frac{\cos\ \theta}{\sin\ \theta}\end{aligned}$$

$$\text{R.S.} = \frac{\cos\theta}{\sin\theta}$$

Since L.S. = R.S., the identity has been proven.

d) $\text{L.S.} = \cot\ \theta$ R.S. = $\dfrac{\csc\ \theta}{\sec\ \theta}$

$= \dfrac{\cos\ \theta}{\sin\ \theta}$ Using the result of part c).

$$= \frac{\left(\dfrac{1}{\sec\ \theta}\right)}{\left(\dfrac{1}{\csc\ \theta}\right)}$$

$$= \frac{\csc\ \theta}{\sec\ \theta}$$

Since L.S. = R.S., the identity has been proven.

Chapter 4 Prerequisite Skills **Question 15 Page 213**

a) $$\frac{\sin x}{\tan x} = \frac{\sin x}{\left(\dfrac{\sin x}{\cos x}\right)}$$
$$= \cos x$$

b) $$\frac{1 - \sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta}$$
$$= 1$$

c) $$\frac{\sin x}{\sin^2 x} = \frac{1}{\sin x}$$
$$= \csc x$$

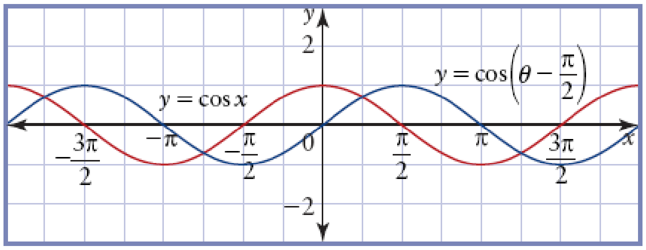
Chapter 4 Prerequisite Skills **Question 16 Page 213**

a) $\text{L.S.} = \cos\left(\theta - \dfrac{\pi}{2}\right)$ R.S. = $\sin\theta$

$$= \cos\theta\cos\frac{\pi}{2} + \sin\theta\sin\frac{\pi}{2}$$
$$= \cos\theta(0) + \sin\theta(1)$$
$$= 0 + \sin\theta$$
$$= \sin\theta$$

Since L.S. = R.S., the identity has been proven.

b)



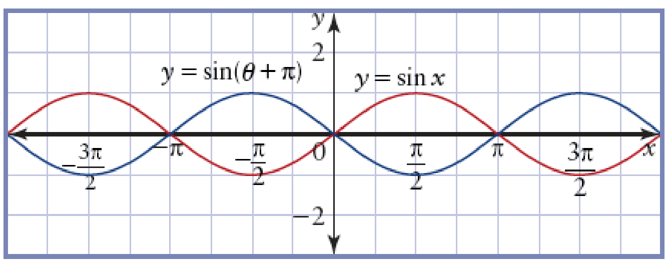
Chapter 4 Prerequisite Skills **Question 17 Page 213**

a) $\text{L.S.} = \sin\left(\theta + \pi\right)$ R.S. = $-\sin\theta$

$$\begin{aligned} &= \sin\theta\cos\pi + \cos\theta\sin\pi \\ &= \sin\theta(-1) + \cos\theta(0) \\ &= -\sin\theta + 0 \\ &= -\sin\theta \end{aligned}$$

Since L.S. = R.S., the identity has been proven.

b)



Chapter 4 Section 1 **Instantaneous Rates of Change of Sinusoidal Functions**

Chapter 4 Section 1 **Question 1 Page 217**

a) i) $\left\{ x \middle| x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \right\}$

ii) $\left\{ x \middle| x = \frac{3}{2}\pi + 2k\pi, k \in \mathbb{Z} \right\}$

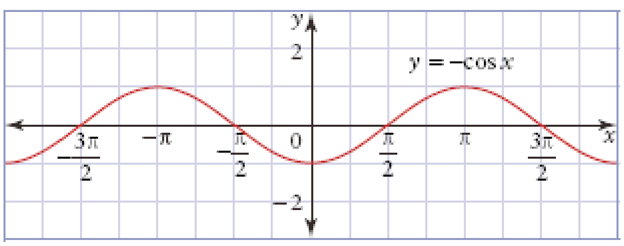
iii) $\left\{ x \middle| x = \frac{1}{2}\pi + 2k\pi, k \in \mathbb{Z} \right\}$

b) i) The curve is concave up for $\ldots\ (-2\pi \le x \le -\pi)$, $(0 \le x \le \pi) \ldots$

ii) The curve is concave down for $\ldots\ (-3\pi \le x \le -2\pi)$, $(-\pi \le x \le 0)$, $(\pi \le x \le 2\pi)$, …

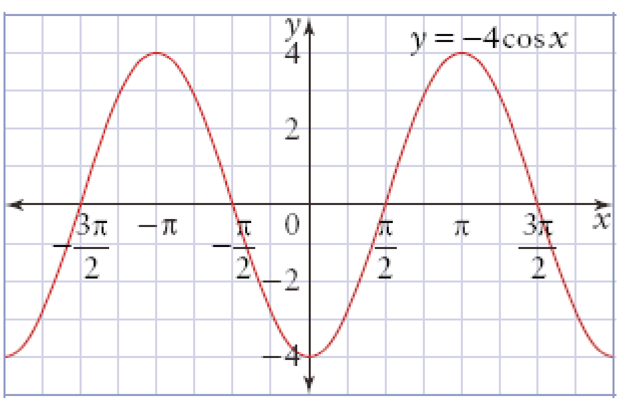
c) The maximum value of the slope is 1 at $(2k+1)\pi$.
The minimum value of the slope is −1 at $(2k)\pi$.

d)

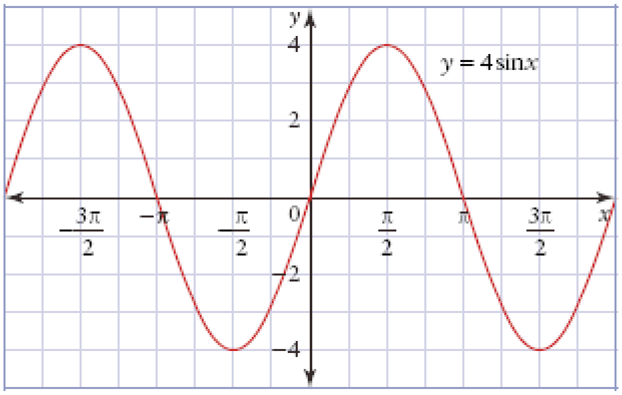


Chapter 4 Section 1 **Question 2 Page 217**

a)



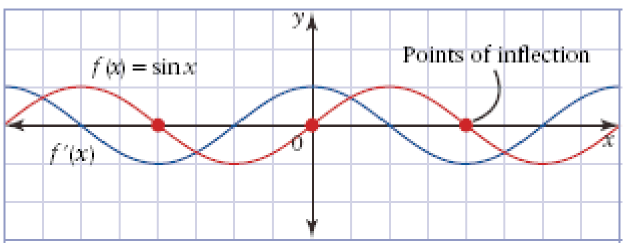
b)



Chapter 4 Section 1 Question 3 Page 217

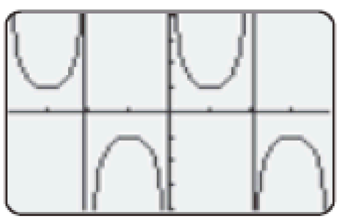
Answers may vary. For example:

Yes. A sinusoidal curve does have points of inflection. The points of inflection will occur at points where the first derivative is a local maximum or a local minimum.



Chapter 4 Section 1 Question 4 Page 217

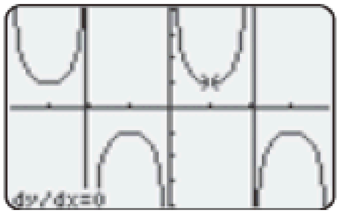
a)



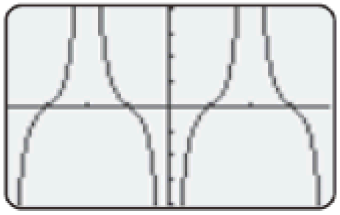
b) Answers may vary. For example:

Use a graphing calculator and the Value function.
The instantaneous rate of change is −2.140 787 at the point (−5.628 687, 1.642 679 6).
The instantaneous rate of change is 2.140 787 at the point (−2.487 094, −1.642 679 6).

The instantaneous rate of change is 0 at the point $\left(\frac{\pi}{2}, 1\right)$.



c)

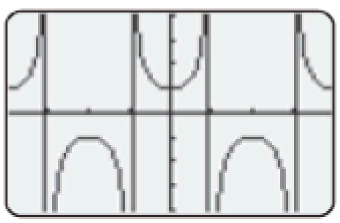


Answers may vary. For example:

The graph of the instantaneous rate of change of $y = \csc x$ as a function of *x* has points of inflection at the points where the graph of $y = \csc x$ has local maximum points and local minimum points. Both graphs have vertical asymptotes at the same *x*-values.

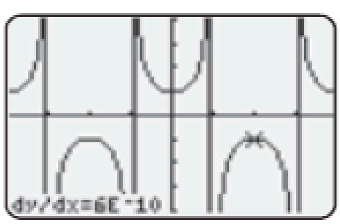
Chapter 4 Section 1 Question 5 Page 217

a)

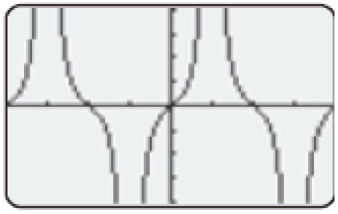


b) Answers may vary. For example:

Use a graphing calculator and the Value function.
The instantaneous rate of change is −1.414 216 9 at the point (−3.926 991, −1.414 214).
The instantaneous rate of change is −8.203 512 at the point (−1.916 297 9, −2.952 739).
The instantaneous rate of change is 0 at the point (π, −1).



c)

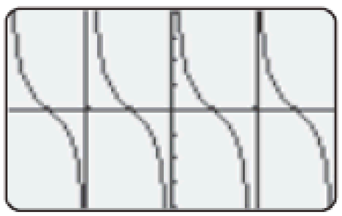


Answers may vary. For example:

The graph of the instantaneous rate of change of $y = \sec x$ as a function of x has points of inflection at the points where the graph of $y = \sec x$ has local maximum points and local minimum points. Both graphs have vertical asymptotes at the same x-values.

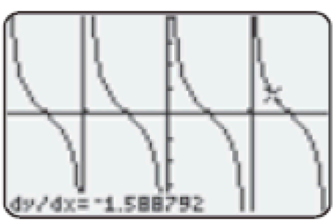
Chapter 4 Section 1 Question 6 Page 217

a)



b) Answers may vary. For example:

Use a graphing calculator and the Value function.
The instantaneous rate of change is $-4.000\ 015$ at the point $(-2.617\ 994, 1.732\ 051\ 3)$.
The instantaneous rate of change is $-2.000\ 003$ at the point $(2.356\ 194\ 5, -1)$.
The instantaneous rate of change is $-2.698\ 402$ at the point $(3.796\ 091\ 1, 1.303\ 225\ 4)$.



c)

Answers may vary. For example:

The graph of the instantaneous rate of change of $y = \cot x$ as a function of x has local minimum points where the graph of $y = \cot x$ has points of inflection. Both graphs have vertical asymptotes at the same x-values.

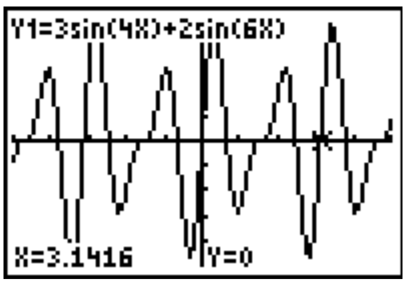
Chapter 4 Section 1 **Question 7 Page 217**

C is the correct answer.

The period of the function $y = \sin 4x$ is $\frac{2\pi}{4} = \frac{\pi}{2}$.

The period of the function $y = \sin 6x$ is $\frac{2\pi}{6} = \frac{\pi}{3}$.

Therefore, the period of the function $y = 3\sin 4x + 2\sin 6x$ is the LCM of $\frac{\pi}{2}$ and $\frac{\pi}{3}$, which is π.



The graph confirms that the period is π.

Chapter 4 Section 2 **Derivatives of the Sine and Cosine Functions**

Chapter 4 Section 2 **Question 1 Page 225**

a) $y = \sin x$; derivative: B $\frac{dy}{dx} = \cos x$

b) $y = \cos x$; derivative: C $\frac{dy}{dx} = -\sin x$

c) $y = -\sin x$; derivative: D $\frac{dy}{dx} = -\cos x$

d) $y = -\cos x$; derivative: A $\frac{dy}{dx} = \sin x$

Chapter 4 Section 2 **Question 2 Page 225**

a) $\frac{dy}{dx} = 4\cos x$

b) $\frac{dy}{dx} = -\pi \sin x$

c) $f'(x) = 3\sin x$

d) $g'(x) = \frac{1}{2}\cos x$

e) $f'(x) = 0.007\cos x$

Chapter 4 Section 2 **Question 3 Page 226**

a) $\frac{dy}{dx} = -\sin x - \cos x$

b) $\frac{dy}{dx} = \cos x - 2\sin x$

c) $\frac{dy}{dx} = 2x - 3\cos x$

d) $\frac{dy}{dx} = -\pi \sin x + 2 + 2\pi \cos x$

e) $\dfrac{dy}{dx} = 5\cos x - 15x^2$

f) $\dfrac{dy}{dx} = -\sin x + 7\pi\cos x - 3$

Chapter 4 Section 2 **Question 4 Page 226**

a) $f'(\theta) = 3\sin\theta - 2\cos\theta$

b) $f'(\theta) = \dfrac{\pi}{2}\cos\theta + \pi\sin\theta$

c) $f'(\theta) = -15\sin\theta + 1$

d) $f'(\theta) = -\dfrac{\pi}{4}\sin\theta - \dfrac{\pi}{3}\cos\theta$

Chapter 4 Section 2 **Question 5 Page 226**

a) To find the slope, differentiate the given function with respect to *x*.

$\dfrac{dy}{dx} = 5\cos x$

The value of the derivative at $x = \dfrac{\pi}{2}$, gives the slope of the function at that point.

Therefore, the slope of the graph of $y = 5\sin x$ at $x = \dfrac{\pi}{2}$ is $5\cos\dfrac{\pi}{2} = 0$.

b) Answers may vary. For example:

From part a), the slope of the graph of $y = 5\sin x$ at $x = \dfrac{\pi}{2}$ is $5\cos\dfrac{\pi}{2} = 0$. Similarly, the slope of the graph of $y = \sin x$ at $x = \dfrac{\pi}{2}$ is $\cos\dfrac{\pi}{2} = 0$. The derivative functions, $5\cos x$ and $\cos x$, of both these curves, $y = 5\sin x$ and $y = \sin x$, cross the *x*-axis at $x = \dfrac{\pi}{2}$.

Therefore, the derivatives (slopes) of both functions will be 0 at $x = \dfrac{\pi}{2}$.

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Chapter 4 Section 2 **Question 6 Page 226**

To find the slope, differentiate the given function, $y = 2\cos\theta$, with respect to θ.

$$\frac{dy}{d\theta} = -2\sin\theta$$

The value of the derivative at $\theta = \frac{\pi}{6}$, gives the slope of the function at that point.

Therefore, the slope of the graph of $y = 2\cos\theta$ at $\theta = \frac{\pi}{6}$ is $-2\sin\frac{\pi}{6} = -1$.

Chapter 4 Section 2 **Question 7 Page 226**

a) Answers may vary. For example:

Substitute $\frac{\pi}{3}$ for x in $y = \cos x$.

$$\begin{aligned} y &= \cos\left(\frac{\pi}{3}\right) \\ &= \frac{1}{2} \end{aligned}$$

Therefore, $\left(\frac{\pi}{3}, \frac{1}{2}\right)$ is a point on the curve $y = \cos x$.

b) To find the equation of the tangent line, you need its slope and a point on the line.
To find the slope, differentiate the given function, $y = \cos x$, with respect to x.

$$\frac{dy}{dx} = -\sin x$$

The value of the derivative at $x = \frac{\pi}{3}$, gives the slope of the function at that point.

Therefore, the slope of the graph of $y = \cos x$ at $x = \frac{\pi}{3}$ is $-\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$.

Front part a). $\left(\frac{\pi}{3}, \frac{1}{2}\right)$ is a point on the line.

Use the slope-point form of the line to find the equation of the tangent line.

$$y - y_1 = \left.\frac{dy}{dx}\right|_{x=\frac{\pi}{3}} (x - x_1)$$

$$y - \frac{1}{2} = -\frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right)$$

$$y = -\frac{\sqrt{3}}{2}x + \frac{\sqrt{3}}{6} + \frac{1}{2}$$

Therefore, the equation of the tangent is $y = -\frac{\sqrt{3}}{2}x + \frac{\sqrt{3}}{6} + \frac{1}{2}$.

Chapter 4 Section 2 **Question 8 Page 226**

Slope is $\frac{dy}{dx} = -4\cos x$.

If the equation of the tangent is $y = mx + b$ then, for $x = \frac{\pi}{4}$, $y = -2\sqrt{2}$, $m = -2\sqrt{2}$.

Therefore:

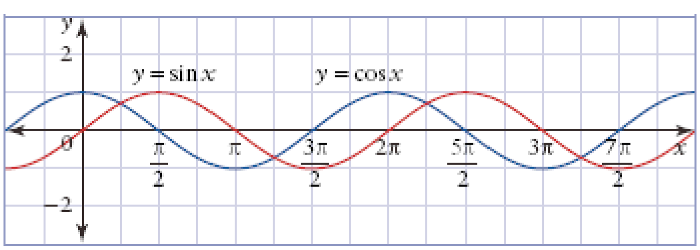
$$b = -2\sqrt{2} + 2\sqrt{2}\left(\frac{\pi}{4}\right)$$

$$y = -2\sqrt{2}x - 2\sqrt{2} + \frac{\sqrt{2}\pi}{2}$$

So, the equation of the tangent is $y = -2\sqrt{2}x + \frac{\sqrt{2}\pi}{2} - 2\sqrt{2}$.

Chapter 4 Section 2 **Question 9 Page 226**

a)

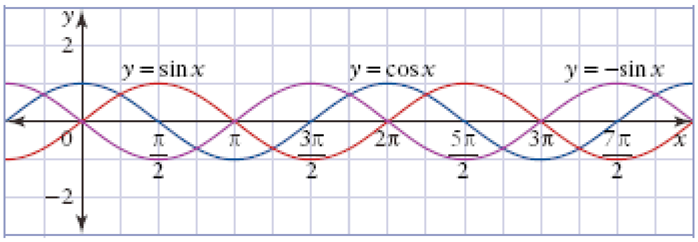


b) Answers may vary. For example:

The graph of $y = \cos x$ is the graph of $y = \sin x$ shifted horizontally $\frac{\pi}{2}$ units to the left.

c)

$$y = \sin x$$
$$\frac{dy}{dx} = \cos x$$
$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = -\sin x$$

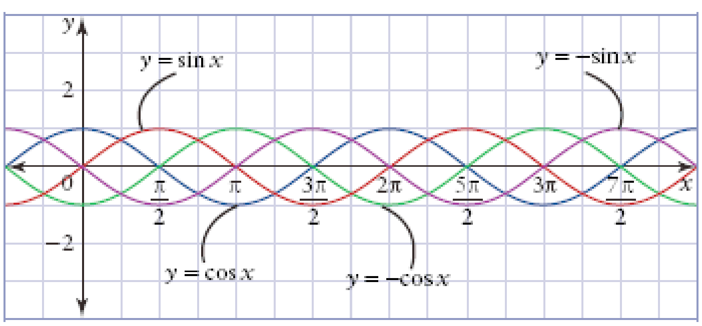


Answers may vary. For example:

The graph of $y = -\sin x$ is the graph of the first derivative, $y = \cos x$, shifted horizontally $\frac{\pi}{2}$ units to the left, and the graph of $y = \sin x$ shifted horizontally π units to the left.

d) Answers may vary. For example:

The graph of the third derivative is the graph of the second derivative, $y = -\sin x$, shifted horizontally $\frac{\pi}{2}$ units to the left; which is the same as the graph of the first derivative, $y = \cos x$, shifted horizontally ∂ units to the left, as well as the graph of $y = \sin x$ shifted horizontally $\frac{3\pi}{2}$ units to the left. The third derivative is $\frac{d^3y}{dx^3} = \frac{d}{dx}(-\sin x) = -\cos x$.



e) Answers may vary. For example:

i) The fourth derivative of $y = \sin x$ is the graph of $y = \sin x$ shifted horizontally $\frac{4\pi}{2}$ units to the left and will be the same as the graph of $y = \sin x$. The fourth derivative of $y = \sin x$ is $\frac{d^4y}{dx^4} = \sin x$.

ii) The tenth derivative of $y = \sin x$ is the graph of $y = \sin x$ shifted horizontally $\frac{10\pi}{2}$ units to the left and will be the same as the graph of the sixth derivative and the second derivative of $y = \sin x$.

The tenth derivative of $y = \sin x$ is $\frac{d^{10}y}{dx^{10}} = -\sin x$.

Chapter 4 Section 2 Question 10 Page 226

$$\frac{d^{15}y}{dx^{15}} = \sin x$$

Explanations may vary. For example:

The first five derivatives of $y = \cos x$ are:
$\frac{dy}{dx} = -\sin x$, $\frac{d^2y}{dx^2} = -\cos x$, $\frac{d^3y}{dx^3} = \sin x$, $\frac{d^4y}{dx^4} = \cos x$, and $\frac{d^5y}{dx^5} = -\sin x$.

The fourth, eighth, and twelfth derivatives will be the same as the original function, $y = \cos x$.

The fifth, ninth, and thirteenth derivatives will be the same as the first derivative, $\frac{dy}{dx} = -\sin x$.

The sixth, tenth, and fourteenth derivative will be the same as the second derivative, $\frac{d^2y}{dx^2} = -\cos x$.

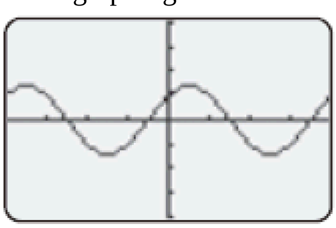
The seventh, eleventh, and fifteenth derivative will be the same as the third derivative, $\frac{d^3y}{dx^3} = \sin x$.

Therefore, the fifteenth derivative of $y = \cos x$ is $\frac{d^{15}y}{dx^{15}} = \sin x$.

Chapter 4 Section 2 Question 11 Page 226

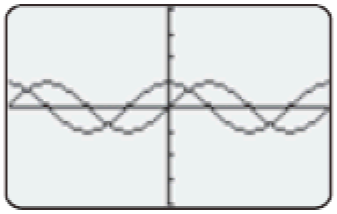
Answers may vary. For example:

Use a graphing calculator to graph the function $y = \sin x + \cos x$.



The derivative of $y = \sin x + \cos x$ is $\frac{dy}{dx} = \cos x - \sin x$.

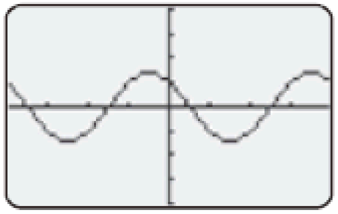
Use a graphing calculator to graph the functions: $y = \cos x$ and $y = -\sin x$ in the same viewing screen and display the table of values for the two functions.



X	Y_1	Y_2
-3	-.99	.14112
-2	-.4161	.9093
-1	.5403	.84147
0	1	0
1	.5403	-.8415
2	-.4161	-.9093
3	-.99	-.1411

X=-3

Graph the function $y = \cos x - \sin x$ and display the table of values for the function.



X	Y_1
-3	-.8489
-2	.49315
-1	1.3818
0	1
1	-.3012
2	-1.325
3	-1.131

X=-3

The y-values for the derivative function $\frac{dy}{dx} = \cos x - \sin x$ of the function $y = \sin x + \cos x$ are the sum of the y-values for the derivative of $y = \sin x$, $\frac{dy}{dx} = \cos x$ and the derivative of $y = \cos x$, $\frac{dy}{dx} = -\sin x$. This shows that the sum differentiation rule holds true for the sinusoidal function $y = \sin x + \cos x$. Using a similar method it can be shown that the difference differentiation rule will hold true for the sinusoidal function $y = \sin x - \cos x$.

Chapter 4 Section 2 Question 12 Page 226

a) Answers may vary. For example:

$$\begin{aligned} \text{Slope} &= \frac{dy}{dx} \\ &= \sin x \end{aligned}$$

If $\frac{dy}{dx} = -1$,

$$\sin x = -1$$

$$x = \left((2k-1)\pi + \frac{\pi}{2}\right), k \in \mathbb{Z}$$

For all $k \in \mathbb{Z}$, if $y = mx + b$ is the equation of a tangent, then at $x = \left((2k-1)\pi + \frac{\pi}{2}\right)$,

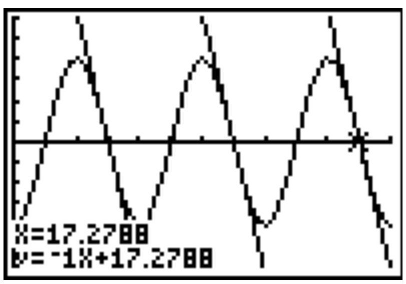
$$\begin{aligned} y &= -\cos\left((2k-1)\pi + \frac{\pi}{2}\right) \\ &= 0 \end{aligned} \qquad \begin{aligned} \frac{dy}{dx} &= m \\ &= -1 \end{aligned}$$

$$b = \left((2k-1)\pi + \frac{\pi}{2}\right)$$
$$y = -x + \left((2k-1)\pi + \frac{\pi}{2}\right)$$

If k = 1, this gives $y = -x + \frac{3\pi}{2}$.

b) Yes, there is more than one solution.

Since the function $y = -\cos x$ is periodic, there will be an infinite number of solutions, as shown from the tangent equation $y = -x + \left((2k-1)\pi + \frac{\pi}{2}\right)$, $k \in \mathbb{Z}$. Each value of k, where k is an integer, will provide a different equation of the tangent line. Examples are shown below.



Chapter 4 Section 2 **Question 13 Pages 226-227**

a) From the graph: maximum height: 18 m; minimum height: 2 m

b) Since the model has maxima at (0, 18) and (4π, 18) and minimum at (2π, 2) it has the form $y = 8\cos(bx - c) + 10$. (It is translated vertically by 10 m, and expanded vertically by a factor of 8.)

Use the maximum and minimum points.
$18 = 8\cos(-c) + 10$ ①
$2 = 8\cos\big(b(2\pi) - c\big) + 10$ ②

From ①:
$\cos(-c) = 1$
$c = -2k\pi,\ k \in \mathbb{Z}$

Need only one equation that relates vertical and horizontal positions, so let k = 0, c = 0.

From ②:

$$-8 = 8\cos\big(b(2\pi) - c\big)$$
$$-1 = \cos\big(b(2\pi)\big)$$
$$2b\pi = (2h+1)\pi$$
$$b = \frac{(2h+1)}{2},\ h \in \mathbb{Z}$$

Need only one equation that relates the vertical and horizontal positions, so let $h = 0,\ b = \frac{1}{2}$.

Therefore, an equation that models the vertical and the horizontal position is $y = 8\cos\left(\frac{1}{2}x\right) + 10$.

c) For $y = 8\cos\left(\frac{1}{2}x\right) + 10$:

$$\frac{dy}{dx} = -8\sin\left(\frac{1}{2}x\right)\left(\frac{1}{2}\right)$$
$$= -4\sin\left(\frac{1}{2}x\right)$$

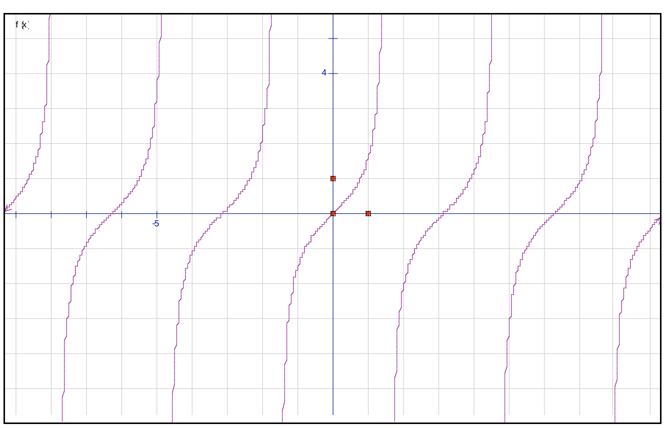
Since $-1 \le \sin\left(\frac{1}{2}x\right) \le 1$, $\frac{dy}{dx}$ has a maximum value of (–4)(–1) = 4 at $x = 3\pi$.

Chapter 4 Section 2 Question 14 Page 227

Solutions for Achievement Checks are shown in the Teacher Resource.

Chapter 4 Section 2 Question 15 Page 227

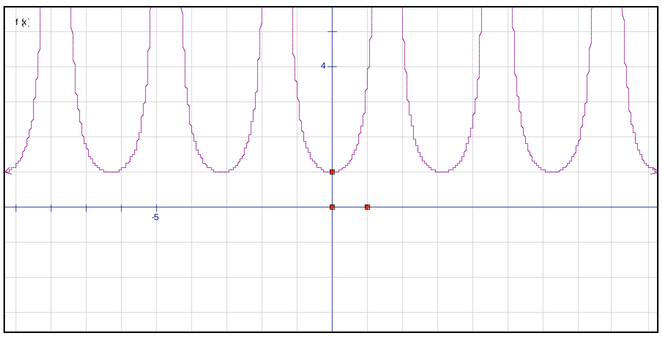
a) Yes the function $y = \tan x$ is periodic with period π.



b) Answers may vary. For example:

The graph of the derivative of $y = \tan x$ will have the same asymptotes as the graph of $y = \tan x$. The graph of the derivative of $y = \tan x$ will also have local minimum points for x-values where the function $y = \tan x$ crosses the x-axis and has points of inflection. For intervals where the graph of $y = \tan x$ is increasing and concave down, the derivative will be decreasing and concave up. For intervals where the graph of $y = \tan x$ is increasing and concave up, the derivative will be increasing and concave up.

c)



Answers may vary. For example:

Yes. The results were as I expected. The derivative of $y = \tan x$ is $y' = \sec^2 x$. The derivative function is positive for all values of x for which it is defined and will have local minimum values for values of x for which:

$$1 = \sec^2 x$$

$$\frac{1}{\cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\cos^2 x}$$

$$1 = 1 + \tan^2 x$$

$$0 = \tan^2 x$$

$$x = k\pi,\ k \in \mathbb{Z}$$

Chapter 4 Section 2 **Question 16 Page 227**

Answers may vary. For example:

a) i) As $x \to \frac{\pi}{2}$ from the left, the graph of the derivative of $y = \tan x$ becomes large and positive.

ii) As $x \to \frac{\pi}{2}$ from the right, the graph of the derivative of $y = \tan x$ becomes large and positive.

b) This implies that the value of the derivative of $y = \tan x$ at $x = \frac{\pi}{2}$ is not defined and there is discontinuity at $x = \frac{\pi}{2}$. Therefore, the derivative of $y = \tan x$ does not exist at $x = \frac{\pi}{2}$.

Chapter 4 Section 2 Question 17 Page 227

a) Answers may vary. For example:

This sketch illustrates that $y = \cos x$ is the derivative of $y = \sin x$. The slope of the tangent line at the point $(-7.43, -0.93)$ is 0.37. The equation of the tangent line to the function $y = \sin x$ at the point x_p is represented by $h(x) = y_p + f'(x_p) \cdot (x - x_p)$. The graph of $y = \cos x$ is the graph of $y = \sin x$ translated horizontally $\frac{\pi}{2}$ units to the left.

b) Answers may vary. For example:

If the **Animate P** button is pressed, the point P will move along the curve $y = \sin x$ from left to right and the green tangent line will move along the curve as well. The slope of the tangent line will increase to a local maximum value at the first point of inflection on the *x*-axis and then become 0 at the local maximum value where the line becomes horizontal. The slope will decrease to a local minimum value at the second point of inflection on the *x*-axis and then become 0 at the local minimum value where the tangent line becomes horizontal. As the point continues to travel to the right on the curve, the tangent line will continue in the same pattern.

c) The point P moves along the sine curve and the tangent to the curve at point P is shown.

d) Answers may vary.

e) Answers may vary.

Chapter 4 Section 2 Question 18 Page 227

Answers may vary. For example:

Consider the reciprocal trigonometric function $y = \csc x$. The derivative is $\frac{dy}{dx} = -\csc x \cot x$.

a) domain:
function: $x \in \mathbb{R}$, $x \neq n\pi$, $n \in \mathbb{Z}$
derivative: $x \in \mathbb{R}$, $x \neq n\pi$, $n \in \mathbb{Z}$

range:
function: $y \in (-\infty, -1]$ or $[1, \infty)$
derivative: $y \in (-\infty, \infty)$

b) No maximum or minimum values for the function or the derivative.
No local minimum or maximum values for the derivative.

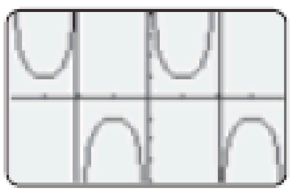
local maximum values:
function: $\left\{x \middle| x = \frac{3\pi}{2} + 2k\pi, k \in \mathbb{Z}\right\}$

local minimum values
function: $\left\{x \middle| x = \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}\right\}$

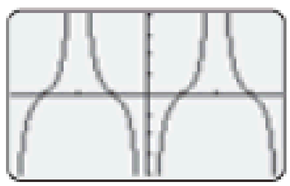
c) function: periodic with period 2π
derivative: periodic with period 2π

d) function: vertical asymptotes at $x = n\pi$, $n \in \mathbb{Z}$
derivative: vertical asymptotes at $x = n\pi$, $n \in \mathbb{Z}$

e) function



derivative



Chapter 4 Section 2 **Question 19 Page 227**

Answer may vary. For example:

Let $f(x) = \sin x$. Then $f'(x) = \cos x$.
$f'(x) \doteq \frac{f(y) - f(x)}{y - x}$ for $y \doteq x$ implies that $f(y) - f(x) \doteq f'(x) \times (y - x)$ for $y \doteq x$.

$$\begin{aligned} 37° - 36° &= 1° \\ &= \frac{\pi}{180} \text{ rad} \end{aligned}$$

Therefore,

$$\begin{aligned} \sin 37° - \sin 36° &\doteq \cos 36° \times \frac{\pi}{180} \\ &= \frac{1 + \sqrt{5}}{4} \times \frac{\pi}{180} \\ &= \frac{\left(1 + \sqrt{5}\right)\pi}{720} \end{aligned}$$

Chapter 4 Section 3 Differentiation Rules for Sinusoidal Functions

Chapter 4 Section 3 Question 1 Page 231

a) $\frac{dy}{dx} = 4\cos 4x$

b) $\frac{dy}{dx} = \pi\sin(-\pi x)$

c) $f'(x) = 2\cos(2x + \pi)$

d) $f'(x) = \sin(-x - \pi)$

Chapter 4 Section 3 Question 2 Page 231

a) $\frac{dy}{d\theta} = -6\cos(3\theta)$

b) $\frac{dy}{d\theta} = 5\sin\left(5\theta - \frac{\pi}{2}\right)$

c) $f'(\theta) = -\pi\sin(2\pi\theta)$

d) $f'(\theta) = -6\cos(2\theta - \pi)$

Chapter 4 Section 3 Question 3 Page 231

a) $\frac{dy}{dx} = 2\sin x\cos x$

b) $\frac{dy}{dx} = -\cos^2 x\sin x$

c) $f'(x) = -2\sin 2x$

d) $f'(x) = -6\cos^2 x\sin x - 4\cos^3 x\sin x$

Chapter 4 Section 3 Question 4 Page 231

a) $\frac{dy}{dt} = 12\sin(2t - 4)\cos(2t - 4) + 12\cos(3t + 1)\sin(3t + 1)$

b) $f'(t) = 2t\cos(t^2 + \pi)$

c) $\dfrac{dy}{dt} = \left[-\sin\left(\sin t\right)\right]\left[\cos t\right]$

d) $f'(t) = -2\left[\sin(\cos t)\right]\left[\cos(\cos t)\right]\left[\sin t\right]$

Chapter 4 Section 3 Question 5 Page 231

a) $\dfrac{dy}{dx} = -2x\sin 2x + \cos 2x$

b) $f'(x) = -3x^2\cos(3x - \pi) - 2x\sin(3x - \pi)$

c) $\dfrac{dy}{d\theta} = -2\sin^2\theta + 2\cos^2\theta$

d) $f'(\theta) = -2\sin^3\theta\cos\theta + 2\sin\theta\cos^3\theta$

e) $f'(t) = 18t(\sin^2(2t - \pi))(\cos(2t - \pi)) + 3(\sin^3(2t - \pi))$

f) $\dfrac{dy}{dx} = -2x^{-1}\cos x\sin x - x^{-2}\cos^2 x$

Chapter 4 Section 3 Question 6 Page 231

Answers may vary. For example:

a) The derivatives of each of the functions are the same: $\dfrac{dy}{dx} = \cos x$.

b) The equations of the three functions are $y = \sin x$ (middle), $y = \sin x + 3$ (top), and $y = \sin x - 2$ (bottom). The graph in the middle is a sinusoidal function with an amplitude of 1 and a period of 2π, a local maximum at $\left(\dfrac{\pi}{2},\ 1\right)$ and a local minimum at $\left(\dfrac{3\pi}{2},\ -1\right)$.
Therefore the equation of this function is $y = \sin x$.

The highest placed function is also a sinusoidal function with an amplitude of 1 and a period of 2π. The graph is congruent to $y = \sin x$ and has been vertically translated up 3 units. The equation of this function is $y = \sin x + 3$.

The lowest placed function is also a sinusoidal function with an amplitude of 1 and a period of 2π. The graph is congruent to $y = \sin x$ and has been vertically translated down 2 units. The equation of this function is $y = \sin x - 2$.

Chapter 4 Section 3 **Question 7 Page 231**

The slope of the function $y = 2\cos x \sin 2x$ is given by its derivative with respect to *x*.

$$y = 2\cos x \sin 2x$$
$$\frac{dy}{dx} = 2(-\sin x)(\sin 2x) + 2(\cos x)(\cos 2x)(2)$$
$$\frac{dy}{dx} = -2\sin x \sin 2x + 4\cos x \cos 2x$$

At $x = \frac{\pi}{2}$,

$$\begin{aligned}\frac{dy}{dx} &= -2\sin\frac{\pi}{2}\sin\pi + 4\cos\frac{\pi}{2}\cos\pi \\ &= (-2)(1)(0) + (4)(0)(-1) \\ &= 0\end{aligned}$$

Therefore, the slope of the function $y = 2\cos x \sin 2x$ at $x = \frac{\pi}{2}$ is 0.

Chapter 4 Section 3 **Question 8 Page 231**

To find the equation of the tangent line, you need its slope and a point on the line.
To find the slope, differentiate the given function, $y = x^2 \sin 2x$, with respect to *x*.

$$\frac{dy}{dx} = 2x\sin 2x + 2x^2\cos 2x$$

The value of the derivative at $x = -\partial$, gives the slope of the function at that point.

$$\begin{aligned}\left.\frac{dy}{dx}\right|_{x=-\pi} &= 2(-\pi)\sin(-2\pi) + 2(-\pi)^2\cos(-2\pi) \\ &= 0 + 2\pi^2 \\ &= 2\pi^2\end{aligned}$$

Substitute $x = -\pi$ into the original function to get $y = 0$.

Use the slope-point form of the line to get the equation for the tangent line.

$$\begin{aligned}y - y_1 &= \left.\frac{dy}{dx}\right|_{x=-\pi}(x - x_1) \\ y - 0 &= 2\pi^2(x - (-\pi)) \\ y &= 2\pi^2 x + 2\pi^3\end{aligned}$$

Therefore, the equation of the line tangent to $y = x^2 \sin 2x$ at $x = -\pi$ is $y = 2\pi^2 x + 2\pi^3$.

Chapter 4 Section 3 **Question 9 Page 231**

Answers may vary. For example:

a) An odd function is one for which satisfies $f(-x) = f(x)$.

$$\begin{aligned} f(x) &= \sin x \\ f(-x) &= \sin(-x) \\ &= -\sin x \\ &= -f(x) \end{aligned}$$

Since, $f(-x) = -f(x)$, the function $y = \sin x$ is an odd function.

b) Since $y = \sin x$ is an odd function, $\sin(-x) = -\sin(x)$.
Therefore,

$$\begin{aligned} \frac{dy}{dx}(\sin(-x)) &= \frac{dy}{dx}(-\sin x) \\ &= -\cos x \end{aligned}$$

Chapter 4 Section 3 **Question 10 Page 231**

Answers may vary. For example:

a) An even function is one which satisfies $f(-x) = -f(x)$.

$$\begin{aligned} f(x) &= \cos x \\ f(-x) &= \cos(-x) \\ &= -\cos(x) \\ &= -f(x) \end{aligned}$$

Since $f(-x) = f(x)$, the function $y = \cos x$ is an even function.

b) Since $y = \cos x$ is an even function, $\cos(-x) = \cos(x)$.

$$\begin{aligned} \frac{dy}{dx}(\cos(-x)) &= \frac{dy}{dx}(\cos x) \\ &= -\sin x \end{aligned}$$

Chapter 4 Section 3 Question 11 Page 231

a) Find the derivative of $y=\sin^2 x+\cos^2 x$.

$$\begin{aligned} y &= \sin^2 x+\cos^2 x \\ \frac{dy}{dx} &= 2\sin x\cos x+2\cos x(-\sin x) \\ &= 2\sin x\cos x-2\sin x\cos x \\ &= 0 \end{aligned}$$

Since $\frac{dy}{dx}=0$, $y=\sin^2 x+\cos^2 x$ is a constant function.

b) If $y=\sin^2 x+\cos^2 x$, then $y=1$, using the trigonometric identity $\sin^2 x+\cos^2 x=1$.
Therefore $y=\sin^2 x+\cos^2 x$ is a constant function.

Chapter 4 Section 3 Question 12 Page 232

$$\begin{aligned} \frac{dy}{dx} &= x^2(-\sin x)+2x\cos x \\ \frac{d^2y}{dx^2} &= -x^2\cos x-2x\sin x-2x\sin x+2\cos x \\ \frac{d^2y}{dx^2} &= -x^2\cos x-4x\sin x+2\cos x \end{aligned}$$

Chapter 4 Section 3 Question 13 Page 232

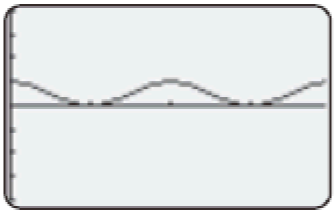
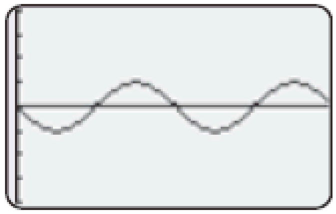
a) Answers may vary. For example:

For the function $f(x)=\cos^2 x$, all values in the range will be greater than or equal to zero. On the interval $0\le x<2\pi$, the zeros of this function are the same as the zeros of the function $f(x)=\cos x$, $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

The derivative of this function is $f'(x)=-2\sin x\cos x$. On the interval $0\le x<2\pi$, the zeros of the derivative function are the same as the zeros of the function $f(x)=\cos x$, i.e., $\frac{\pi}{2}$ and $\frac{3\pi}{2}$, and the zeros of the function $f(x)=\sin x$, i.e., 0 and π.

Therefore the function $f(x)=\cos^2 x$ will have half as many zeros as its derivative $f'(x)=-2\sin x\cos x$.

b)

Chapter 4 Section 3 **Question 14 Page 232**

Answers may vary. For example:

A composite function is $y = \sin(x^3)$.

First derivative: $\frac{dy}{dx} = \left[\cos(x^3)\right](3x^2)$.

Second derivative: $\frac{d^2y}{dx^2} = 6x\cos x^3 - 9x^4\sin x^3$.

Chapter 4 Section 3 **Question 15 Page 232**

a) $y = \frac{1}{\sin x}$

b) $y = (\sin x)^{-1}$

c)
$$\begin{aligned} y &= (\sin x)^{-1} \\ \frac{dy}{dx} &= -(\sin x)^{-2}\cos x \\ &= -\frac{\cos x}{(\sin x)^2} \\ &= -\csc x \cot x \end{aligned}$$

d) Domain of $y = \csc x$: $x \in \mathbb{R}$, $x \neq n\pi$, $n \in \mathbb{Z}$.

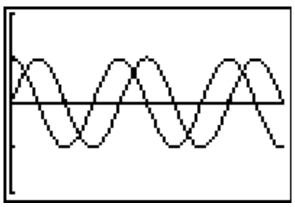
Domain of the derivative of $y = \csc x$, $\frac{dy}{dx} = -\csc x \cot x$: $x \in \mathbb{R}$, $x \neq n\pi$, $n \in \mathbb{Z}$.

Chapter 4 Section 3 **Question 16 Page 232**

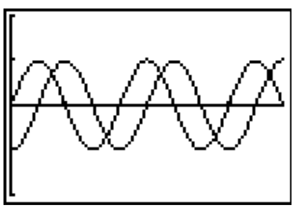
Answers may vary. For example:

A horizontal shift of a sinusoidal function will result in a similar shift of the derivative of that function. If the function $y = \cos x$ is shifted horizontally $\frac{3\pi}{2}$ units to the right then its derivative, $y = -\sin x$, will also shift $\frac{3\pi}{2}$ units to the right.

Here are graphs of $y = \cos x$ and $y = \cos\left(x + \frac{3\pi}{2}\right)$.



Here are graphs of the derivatives, $y = -\sin x$ and $y = -\sin\left(x + \frac{3\pi}{2}\right)$.



Chapter 4 Section 3 **Question 17 Page 232**

Answers may vary. For example:

a) I used a graphing calculator and systemic trial to determine that the function that models the roller coaster segment on the left is a piecewise sinusoidal function. On the interval $0 \le x \le \pi$, the function that models the roller coaster is $y = 0.25\sin^2 2x + 4$. On the interval $\pi < x \le 2\pi$, the function that models the roller coaster is $y = 2\sin x + 4$.

I also used a graphing calculator and systemic trial to determine that the function that models the roller coaster segment on the right is a piecewise sinusoidal function. On the interval $0 \le x \le \pi$, the function that models the roller coaster is $y = 3\sin 2x + 4$. On the interval $\pi < x \le 2\pi$, the function that models the roller coaster is $y = -3\sin 2x + 4$.

b) Maximum slope of the roller coaster segment on the left occurs when $x = \frac{7\pi}{4}$.

$$\begin{aligned} y' &= 2\cos\left(\frac{7\pi}{4}\right) \\ &= 2\frac{\sqrt{2}}{2} \\ &= \sqrt{2} \end{aligned}$$

When $x = \frac{7\pi}{4}$, the slope is $\sqrt{2}$.

Maximum slope of the roller coaster segment on the right occurs when $x = 1.5\pi$.

$$\begin{aligned} y' &= -3\left(\cos 2\left(\frac{3\pi}{2}\right)\right)(2) \\ &= -6(-1) \\ &= 6 \end{aligned}$$

When $x = 1.5\pi$, the slope is 6.

Chapter 4 Section 3 **Question 18 Page 232**

a)
$$\begin{aligned} y' &= \frac{d}{dx}(\sec x) \\ &= \frac{d}{dx}(\cos x)^{-1} \\ &= -(\cos x)^{-2}\sin x \\ &= \frac{1}{\cos x}\left(\frac{\sin x}{\cos x}\right) \\ &= \sec x \tan x \end{aligned}$$

b)
$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}\left(\frac{\sec x}{\cos^2 x}\right) \\ &= \frac{d}{dx}(\sec^3 x) \\ &= 3\sec^2 x(\sec x \tan x) \\ &= 3\tan x \cos^{-3} x \end{aligned}$$

Chapter 4 Section 3 Question 19 Page 232

Answers may vary. For example:

$$\begin{aligned}y &= \tan x\\ &= \frac{\sin x}{\cos x}\\ &= (\sin x)(\cos x)^{-1}\end{aligned}$$

$$\begin{aligned}\frac{dy}{dx} &= ((\sin x)(-(\cos x)^{-2})(\sin x)) + (\cos x)^{-1}(\cos x)\\ &= \frac{\sin^2 x}{\cos^2 x} + \frac{\cos x}{\cos x}\\ &= \tan^2 x + 1\end{aligned}$$

Therefore, $\frac{dy}{dx} = 1 + \tan^2 x$.

Chapter 4 Section 3 Question 20 Page 232

Answers may vary. For example:

$$\begin{aligned}y &= \cot x\\ &= \frac{\cos x}{\sin x}\\ &= (\cos x)(\sin x)^{-1}\end{aligned}$$

$$\begin{aligned}\frac{dy}{dx} &= ((\cos x)(-(\sin x)^{-2})(\cos x)) + (\sin x)^{-1}(-\sin x)\\ &= -\frac{\cos^2 x}{\sin^2 x} - \frac{\sin x}{\sin x}\\ &= -\frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x}\\ &= \frac{-(\cos^2 x + \sin^2 x)}{\sin^2 x}\\ &= \frac{-1}{\sin^2 x}\\ &= -\csc^2 x\end{aligned}$$

Therefore, $\frac{dy}{dx} = -\csc^2 x.$

Chapter 4 Section 3 **Question 21 Page 232**

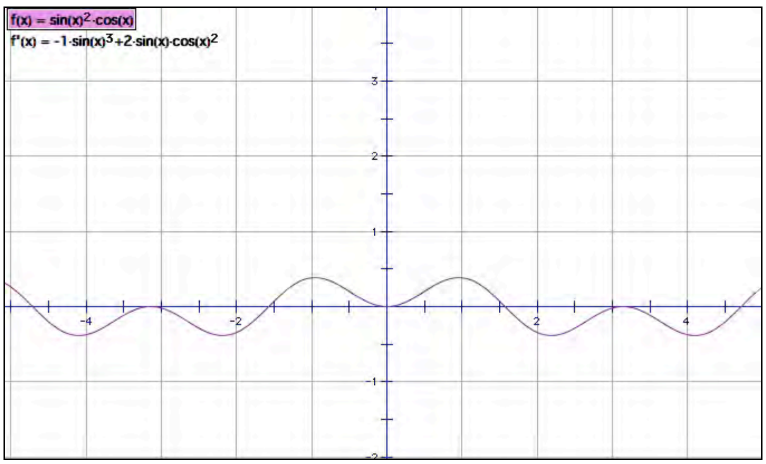
Answers may vary. For example:

$$y = \cos^3 5x$$
$$\frac{dy}{dx} = 3(\cos^2 5x)(-\sin 5x)(5)$$
$$= -15(\cos^2 5x)(-\sin 5x)$$

Chapter 4 Section 3 **Question 22 Page 232**

a) $f'(x) = (2\sin x)(\cos^2 x) - \sin^3 x$

b)



c) Yes. The software produced the same equation as the one in part a).

Chapter 4 Section 3 **Question 23 Page 232**

The correct answer is D.

The given infinite series is a geometric series with a common ratio $-\tan^2 x$. The sum of an infinite geometric series is given by Sum = $\frac{a}{1-r}$, where *a* is the first term and *r* is the common ratio.

$$S(x) = \frac{1}{1-(-\tan^2 x)}$$
$$= \frac{1}{\sec^2 x}$$
$$= \cos^2 x$$

Therefore:
$$S'(x) = -2\cos x \sin x$$
$$= -\sin 2x$$

Chapter 4 Section **Applications of Sinusoidal Functions and Their Derivatives**

Chapter 4 Section 4 **Question 1 Page 241**

a) $I(t) = 60\cos t + 25$ has a maximum value of 85 A when $\cos t = 1$ and a minimum value of –35 A when $\cos t = -1$.

Maximum current: 85 A at times *t*, in seconds, $\{t \mid t = 2k\pi,\ k \in \mathbb{Z},\ k \ge 0\}$
Minimum current: −35 A at times *t*, in seconds, $\{t \mid t = (2k+1)\pi,\ k \in \mathbb{Z},\ k \ge 0\}$

b) **i)** It is 2π since the function $I(t) = 60\cos t + 25$ has the same period as $\cos t$.
$T = 2\pi$ s

ii) The frequency is the reciprocal of the period.
Therefore, $f = \dfrac{1}{2\pi}$ Hz.

iii) The amplitude is the given by:
$$\begin{aligned} A &= \frac{85-(-35)}{2} \\ &= 60 \end{aligned}$$

The amplitude is 60 A.

Chapter 4 Section 4 **Question 2 Page 241**

a) $V(t) = 170\sin(120\pi t)$
$V'(t) = 170\big(\cos(120\pi t)\big)120\pi$

To find the maximum and minimum voltage, set the first derivative of voltage to zero. This will provide you with the complete set of critical points.

$V'(t) = 0$ when $\cos(120\pi t) = 0$.

$$120\pi t = \ldots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$$
$$t = \ldots, -\frac{3}{240}, -\frac{1}{240}, \frac{1}{240}, \frac{3}{240}, \ldots$$

Since time cannot be negative:
$$t = \left\{\frac{2k+1}{240}, k \ge 0, k \in \mathbb{Z}\right\}.$$

To determine the set of maxima and minima, consider $t = \dfrac{1}{240}$ and $t = \dfrac{3}{240}$.

At $t = \frac{1}{240}$,

$$\begin{aligned} V(t) &= 170\sin\left(120\pi \cdot \frac{1}{240}\right) \\ &= 170\sin\frac{\pi}{2} \\ &= 170 \end{aligned}$$

This value for $V(t)$ occurs at $t = \left\{\frac{4k+1}{240},\ k \in \mathbb{Z},\ k \geq 0\right\}$.

At $t = \frac{3}{240}$,

$$\begin{aligned} V(t) &= 170\sin\left(120\pi \cdot \frac{3}{240}\right) \\ &= 170\sin\frac{3\pi}{2} \\ &= -170 \end{aligned}$$

This value for $V(t)$ occurs at $t = \left\{\frac{4k+3}{240},\ k \in \mathbb{Z},\ k \geq 0\right\}$.

Maximum voltage: 170 V at times t, in seconds, $t = \left\{\frac{4k+1}{240},\ k \in \mathbb{Z},\ k \geq 0\right\}$.

Minimum voltage: −170 V at times t, in seconds, $t = \left\{\frac{4k+3}{240},\ k \in \mathbb{Z},\ k \geq 0\right\}$.

b) i) The period is $\frac{2\pi}{120\pi} = \frac{1}{60}$ s since the function $V(t) = 170\sin 120\pi t$ has the same period as $\sin 120\pi t$.

ii) $f = 60$ Hz

iii) The amplitude is given by:

$$\begin{aligned} A &= \frac{170 - (-170)}{2} \\ &= 170 \end{aligned}$$

The amplitude is 170 V.

Chapter 4 Section 4 **Question 3 Page 241**

a) Note that the length of the pendulum and the horizontal displacement are measured in centimetres and acceleration due to gravity is measured is meters per square seconds. Convert 50 cm and 8 cm into 0.5 m and 0.08 m respectively.

$$\begin{aligned} T &= 2\pi\sqrt{\frac{l}{g}} \\ &= 2\pi\sqrt{\frac{0.5}{9.8}} \\ &\doteq 1.42 \text{ s} \end{aligned}$$

b) $$\begin{aligned} h(t) &= A\cos\frac{2\pi t}{T} \\ &= 8\cos\frac{2\pi t}{1.42} \\ &\doteq 8\cos 1.4\pi t \end{aligned}$$

Based on this equation, the horizontal position is measured in centimetres.

c) $$\begin{aligned} v(t) &= h'(t) \\ &= 8(-1.4\pi)(\sin 1.4\pi) \\ &= -11.2\pi\sin 1.4\pi t \end{aligned}$$

Based on this equation, the velocity is measured in centimetres per second.

d) $$\begin{aligned} a(t) &= v'(t) \\ &= -11.2\pi(1.4\pi)\cos 1.4\pi t \\ &= -15.68\pi^2\cos 1.4\pi t \end{aligned}$$

Based on this equation, the acceleration is measured in centimetres per square seconds.

Chapter 4 Section 4 **Question 4 Page 241**

a) $v(t) = -11.2\pi\sin 1.4\pi t$ is maximised when $\sin 1.4\pi t$ is minimized. Since $-1 \le \sin 1.4\pi t \le 1$, the minimum possible value of $\sin 1.4\pi t$ is –1.

$$\begin{aligned} 1.4\pi t &= \frac{3\pi}{2} \\ t &= \frac{3}{2.8} \\ t &\doteq 1.1 \\ v(1.1) &= 35.19 \end{aligned}$$

Maximum velocity of the bob, 35.2 cm/s, first occurs at time t = 1.1 s.

b) $a(t) = -15.68\pi^2 \cos 1.4\pi t$ is maximised when $\cos 1.4\pi t$ is minimized. Since $-1 \le \cos 1.4\pi t \le 1$, the minimum possible value of $\cos 1.4\pi t$ is –1.

$$1.4\pi t = \pi$$
$$t = \frac{1}{1.4}$$
$$t \doteq 0.71$$
$$a(0.71) \doteq 154.8$$

Maximum acceleration of the bob, 154.8 cm/s^2, first occurs at time t = 0.71 s.

c) i) Answers may vary. For example:

Displacement equals zero when $h(t)$ = 0.

$$0 = A\cos\left(\frac{2\pi t}{T}\right)$$
$$\frac{2\pi t}{T} = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$
$$t = \frac{T}{4} \text{ or } \frac{3T}{4}$$

That is, at the $\frac{1}{4}$ and $\frac{3}{4}$ way point of each complete oscillation.

ii) The velocity equals zero when $v(t)$ = 0.

$$0 = -A\left(\frac{2\pi}{T}\right)\sin\left(\frac{2\pi t}{T}\right)$$
$$\frac{2\pi t}{T} = 0 \text{ or } \pi \text{ or } 2\pi$$
$$t = 0 \text{ or } \frac{T}{2} \text{ or } T$$

That is, at the beginning point, middle point, and end point of each complete oscillation.

iii) The acceleration equals zero when $a(t)$ = 0.

$$0 = -A\left(\frac{2\pi}{T}\right)^2 \cos\left(\frac{2\pi t}{T}\right)$$
$$\frac{2\pi t}{T} = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$
$$t = \frac{T}{4} \text{ or } \frac{3T}{4}$$

That is, the acceleration equals zero: at the $\frac{1}{4}$ and $\frac{3}{4}$ way point of each complete oscillation.

d) Answers may vary. For example:

The acceleration is equal to zero when the pendulum is in a vertical position and its displacement equals zero. The velocity will equal zero when the displacement of the pendulum is at a maximum.

Chapter 4 Section 4 **Question 5 Page 241**

a) The period of each oscillation is 1 s. Frequency of the oscillating spring is given by:

$$\begin{aligned} f &= \frac{1}{T} \\ &= 1 \end{aligned}$$

The frequency is 1 Hz.

b) To get a simplified expression for the position of the marble as a function of time, substitute the values of f and A into the function $h(t)$.

$$\begin{aligned} h(t) &= A\cos 2\pi ft \\ &= 10\cos 2\pi(1)t \\ &= 10\cos 2\pi t \end{aligned}$$

c) Velocity is defined as the rate of change of displacement.

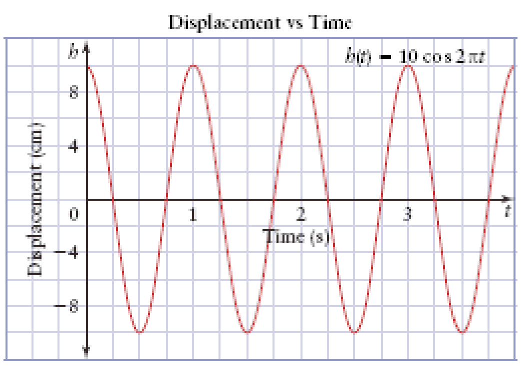
$$\begin{aligned} v(t) &= h'(t) \\ &= -20\pi \sin 2\pi t \end{aligned}$$

d) Acceleration is defined as the rate of change of velocity.

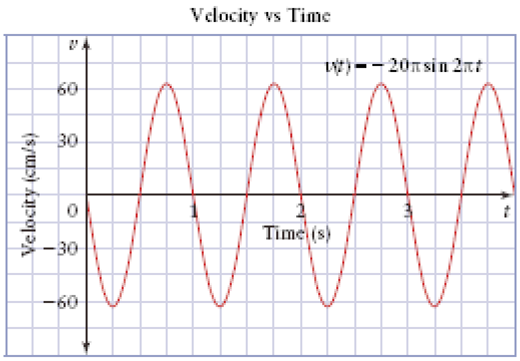
$$\begin{aligned} a(t) &= v'(t) \\ &= -40\pi^2 \cos 2\pi t \end{aligned}$$

Chapter 4 Section 4 **Question 6 Page 242**

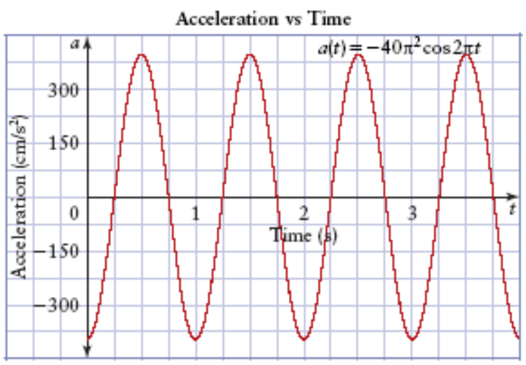
a) i)



ii)



iii)



b) Answers may vary. For example:

Similarities: The graphs of displacement versus time, velocity versus time, and acceleration versus time are all sinusoidal functions. The three graphs have the same period. The graphs of displacement versus time and acceleration versus time have the same zeros.

Differences: The three graphs have different amplitudes. The three graphs are graphs of a sine function shifted horizontally to the left or horizontally to the right.

c) Answers may vary. For example:

Maximum value(s) for displacement: The maximum displacement is 10 cm at the beginning point, middle point and end point of each complete oscillation.

Minimum value(s) for displacement: It is 0 cm at the $\frac{1}{4}$ way and $\frac{3}{4}$ way point of each complete oscillation.

These values make sense because when the bob is at its greatest displacement it will be at rest, but will quickly accelerate from rest.

Chapter 4 Section 4 **Question 7 Page 242**

a) Velocity of the piston head as a function of time:

$$\begin{aligned} v(t) &= h'(t) \\ &= -0.65\sin 13t \end{aligned}$$

b) First find all the critical points of the function *v*(*t*).

$$\begin{aligned} v'(t) &= a(t) \\ &= \frac{d}{dt}[-0.65\sin 13t] \\ &= -0.65(13)\cos 13t \end{aligned}$$

Solve $v'(t) = 0$.

$$\begin{aligned} v'(t) &= 0 \\ -0.65(13)\cos 13t &= 0 \\ \cos 13t &= 0 \\ 13t &= \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots \\ t &= \frac{\pi}{26}, \frac{3\pi}{26}, \frac{5\pi}{26}, \ldots \end{aligned}$$

Negative values of time have no meaning in this situation.

Evaluate *v*(*t*) at $t = \frac{\pi}{26}$ and $t = \frac{3\pi}{26}$.

$$\begin{aligned} v\left(\frac{\pi}{26}\right) &= -0.65\sin\left(13\frac{\pi}{26}\right) \\ &= -0.65\sin\left(\frac{\pi}{2}\right) \\ &= -0.65 \end{aligned}$$

$$\begin{aligned} v\left(\frac{3\pi}{26}\right) &= -0.65\sin\left(13\frac{3\pi}{26}\right) \\ &= -0.65\sin\left(\frac{3\pi}{2}\right) \\ &= 0.65 \end{aligned}$$

Maximum velocity: 0.65 m/s at time *t*, in seconds, $\left\{ t \mid t = \frac{(4k+3)\pi}{26},\ k \in \mathbb{Z},\ k \geq 0 \right\}$.

Minimum velocity: −0.65 m/s at time *t*, in seconds, $\left\{ t \mid t = \frac{(4k+1)\pi}{26},\ k \in \mathbb{Z},\ k \geq 0 \right\}$.

Chapter 4 Section 4 **Question 8 Page 242**

a) The AC component is:

$$V_{AC}(t) = A\sin 2\pi ft$$
$$= 380\sin 120\pi t$$

The DC component is: 120 kV.

Therefore, the total voltage: $V(t) = 380\sin(120\pi t) + 120$.

b) $V(t) = 380\sin(120\pi t) + 120$

$$V'(t) = 380(120\pi)\cos(120\pi t)$$

The critical points are given by $V'(t) = 0$.

$$0 = 380(120\pi)\cos(120\pi t)$$
$$0 = \cos(120\pi t)$$
$$120\pi t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$
$$t = \frac{1}{240}, \frac{3}{240}, \frac{5}{240}, \ldots$$

$$V\left(\frac{1}{240}\right) = 380 + 120$$
$$= 500 \text{ kV}$$

$$V\left(\frac{3}{240}\right) = -380 + 120$$
$$= -260 \text{ kV}$$

Maximum voltage: 500 kV at time $\left\{ t \middle| t = \frac{4k+1}{240},\ k \in \mathbb{Z},\ k \geq 0 \right\}$.

Minimum voltage: –260 kV at time $\left\{ t \middle| t = \frac{4k+3}{240},\ k \in \mathbb{Z},\ k \geq 0 \right\}$.

Chapter 4 Section 4 **Question 9 Page 242**

Answers may vary. For example:

$$y = \pi\sin\theta + 2\pi\cos\theta$$
$$\frac{dy}{d\theta} = \pi\cos\theta - 2\pi\sin\theta$$
$$\frac{d^2y}{d\theta^2} = \pi(-\sin\theta) - 2\pi\sin\theta$$
$$\frac{d^2y}{d\theta^2} = -\pi\sin\theta - 2\pi\sin\theta$$

Therefore:

$$\begin{aligned}\frac{d^2y}{d\theta^2}+y &= -\pi\sin\theta-2\pi\sin\theta + \pi\sin\theta+2\pi\cos\theta \\ &= 0\end{aligned}$$

The function $y=\pi\sin\theta+2\pi\cos\theta$ is a solution to the differential equation: $\frac{d^2y}{d\theta^2}+y=0$.

Chapter 4 Section 4 **Question 10 Page 242**

Answers may vary. For example:

a) A function that satisfies the differential equation $\frac{d^2y}{dx^2}=-4y$ is $y=\sin(2x)$.

b)

$$\begin{aligned}y &= \sin(2x) \\ \frac{dy}{dx} &= 2\cos(2x) \\ \frac{d^2y}{dx^2} &= (2)^2(-\sin(2x)) \\ &= -4\sin(2x) \\ &= -4y\end{aligned}$$

Chapter 4 Section 4 **Question 11 Page 242**

Answers will vary. For example:

a) A differential equation that is satisfied by a sinusoidal function is the function $\frac{d^2y}{d\theta^2}=-9y$. The sinusoidal function is $y=\cos 3\theta$.

$$\begin{aligned}y &= \cos 3\theta \\ \frac{dy}{d\theta} &= (-\sin 3\theta)(3) \\ \frac{d^2y}{d\theta^2} &= -3(\cos 3\theta)(3) \\ \frac{d^2y}{d\theta^2} &= -9(\cos 3\theta)\end{aligned}$$

Therefore, $\frac{d^2y}{d\theta^2}=-9y$.

b) Answers may vary. For example: I used the method of trial and error.

Chapter 4 Section 4 **Question 12 Page 242**

Solutions for Achievement Checks are shown in the Teacher Resource.

Chapter 4 Section 4 **Question 13 Page 242**

The displacement x in $U = \frac{1}{2}kx^2$ can be replaced by the expression for displacement in terms of t, i.e., $h(t) = A\cos 2\pi ft$ '

Therefore, $U = \frac{1}{2}k(A\cos 2\pi ft)^2$.

Chapter 4 Section 4 **Question 14 Page 242**

The velocity v in $K = \frac{kv^2T^2}{8\pi^2}$ can be replaced by the expression for velocity in terms of t, i.e.,

$$\begin{aligned} v(t) &= h'(t) \\ &= -A(2\pi f)\sin 2\pi ft \end{aligned}$$

$$\begin{aligned} K &= \frac{kv^2T^2}{8\pi^2} \\ &= \frac{k(-A(2\pi f)\sin 2\pi ft)^2T^2}{8\pi^2} \\ &= \frac{A^2 4\pi^2 f^2 k[\sin(2\pi t)]^2 T^2}{8\pi^2} \end{aligned}$$

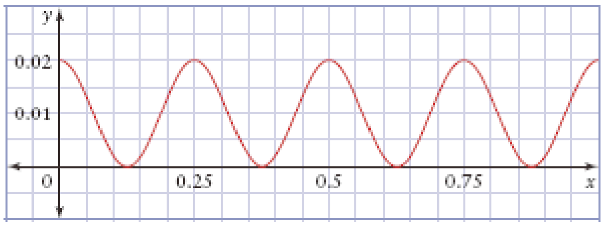
Therefore, $K = \frac{A^2k[\sin(2\pi t)]^2}{2}$.

Chapter 4 Section 4 **Question 15 Page 242**

a) $U = \frac{1}{2}k\left(A\cos 2\pi ft\right)^2$

$= 50(0.02)^2 \cos^2\left\{2\pi\left(\frac{1}{0.5}\right)t\right\}$

$= 0.02\cos^2 4\pi t$

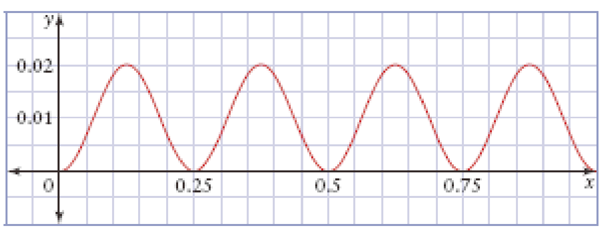


Maxima: 0.02 Nm at times t = 0, 0.25, 0.5,… s
Minima: 0 Nm at times t = 0.125, 0.375, 0.625,… s
Zeros: at times t = 0.125, 0.375, 0.625,… s

b) $K = \frac{A^2k[\sin(2\pi ft)]^2}{2}$

$= 50(0.02)^2\left\{\sin^2(2\pi\left(\frac{1}{0.5}\right)t)\right\}$

$= 0.02\sin^2 4\pi t$



Maxima: 0.02 Nm at time t = 0.125, 0.375, 0.625,… s
Minima: 0 Nm at time t = 0, 0.25, 0.5,… s
Zeros: at time t = 0, 0.25, 0.5,… s

c) Answers may vary. For example:

When the spring is either in a state of maximum extension or compression its potential energy is at a maximum and its kinetic energy is at a minimum. When the spring is in the same position as its resting position, its kinetic energy is at a maximum and its potential energy is at a minimum. The total energy is the sum of the potential energy and kinetic energy.

Chapter 4 Section 4 **Question 16 Page 242**

The correct answer is E.

Let $y = A\sin x + B\cos x$.

Then $\dfrac{dy}{dx} = A\cos x - B\sin x$ and $\dfrac{d^2y}{dx^2} = -A\sin x - B\cos x$.

If $\dfrac{dy}{dx} = 0$, $\tan x = \dfrac{A}{B}$.

Use $\sin x = \dfrac{A}{\sqrt{A^2 + B^2}}$ and $\cos x = \dfrac{B}{\sqrt{A^2 + B^2}}$,

$$\begin{aligned}\frac{d^2y}{dx^2} &= -\frac{A^2}{\sqrt{A^2+B^2}} - \frac{B^2}{\sqrt{A^2+B^2}}\\ &= -\sqrt{A^2+B^2}\\ &< 0\end{aligned}$$

Therefore the local maximum value of y is:

$$\frac{A^2}{\sqrt{A^2+B^2}} + \frac{B^2}{\sqrt{A^2+B^2}} = \sqrt{A^2+B^2}.$$

Chapter 4 Review

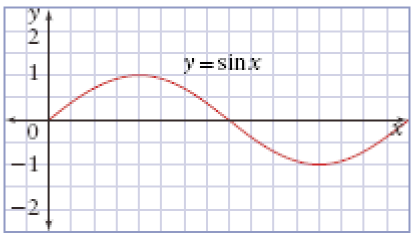
Chapter 4 Review **Question 1 Page 244**

a) i) $(0, -1)$, $(\pi, 1)$, $(2\pi, -1)$

ii) $(\pi, 1)$

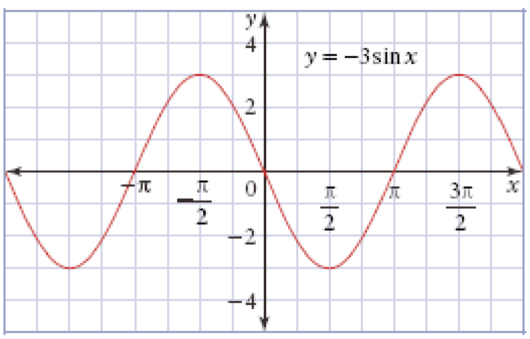
iii) $(2\pi, -1)$ or $(0, -1)$

b)

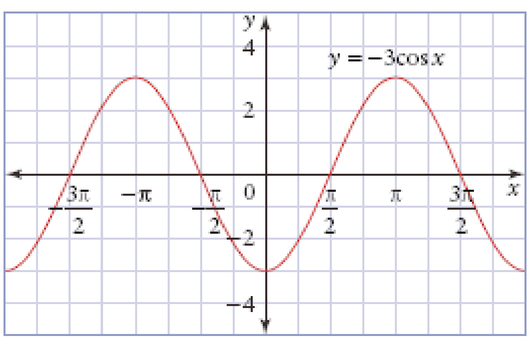


Chapter 4 Review **Question 2 Page 244**

a)



b)



Chapter 4 Review **Question 3 Page 244**

a) $\frac{dy}{dx} = -\sin x$

b) $f'(x) = -2\cos x$

c) $\frac{dy}{dx} = -\sin x - \cos x$

d) $f'(x) = 3\cos x + \pi \sin x$

Chapter 4 Review **Question 4 Page 244**

The slope of a function at a point is given by the value of its first derivative at that point.

$$\begin{aligned} y &= 4\sin x \\ \frac{dy}{dx} &= 4\cos x \\ \left.\frac{dy}{dx}\right|_{x=\frac{\pi}{3}} &= 4\cos\frac{\pi}{3} \\ &= 2 \end{aligned}$$

Therefore, the slope of the function $y = 4\sin x$ at $x = \frac{\pi}{3}$ is 2.

Chapter 4 Review **Question 5 Page 244**

a) To find the equation of the tangent line, find the slope and the *y*-coordinates at $\theta = \frac{\pi}{4}$.

$$\begin{aligned} y &= 2\sin\theta + 4\cos\theta \\ \frac{dy}{d\theta} &= 2\cos\theta - 4\sin\theta \\ \left.\frac{dy}{d\theta}\right|_{\theta=\frac{\pi}{4}} &= \frac{2}{\sqrt{2}} - \frac{4}{\sqrt{2}} \\ &= -\frac{2}{\sqrt{2}} \\ &= -\sqrt{2} \end{aligned}$$

At $\theta = \frac{\pi}{4}$, $y = 3\sqrt{2}$.

Use the slope-point form of a line to find the equation of the tangent line.

$$y-3\sqrt{2}=-\sqrt{2}\left(\theta-\frac{\pi}{4}\right)$$

Therefore, the equation of the tangent is $y=-\sqrt{2}\theta+\frac{\sqrt{2}}{4}\pi+3\sqrt{2}$.

b) Use the same approach as in part a).

$$\begin{aligned} y &= 2\cos\theta-\frac{1}{2}\sin\theta \\ \frac{dy}{d\theta} &= -2\sin\theta-\frac{1}{2}\cos\theta \\ \left.\frac{dy}{d\theta}\right|_{\theta=\frac{3\pi}{2}} &= -2(-1)-\frac{1}{2}(0) \\ &= 2 \end{aligned}$$

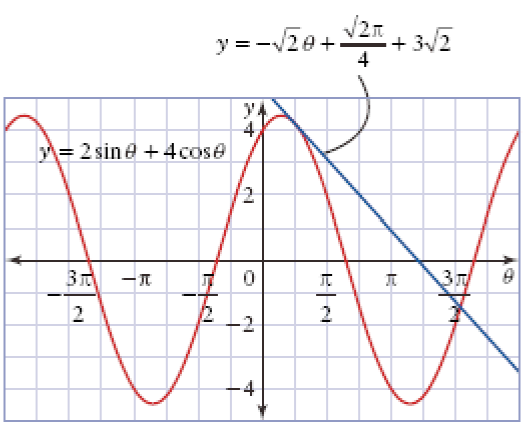
At $\theta=\frac{3\pi}{2}$, $y=\frac{1}{2}$.

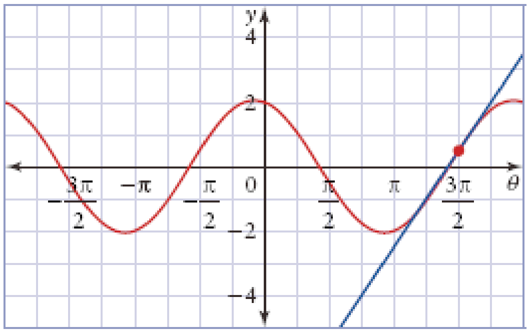
Use the slope-point form of a line to find the equation of the tangent.

$$y-\frac{1}{2}=2\left(\theta-\frac{3\partial}{2}\right)$$

Therefore, the equation of the tangent is $y=2\theta-3\pi+\frac{1}{2}$.

c)





Chapter 4 Review **Question 6 Page 244**

a) $\frac{dy}{dx} = -2\cos x(-\sin x)$
$= \sin 2x$

b) $\frac{dy}{d\theta} = 2\cos 2\theta + 4\sin 2\theta$

c) $f'(\theta) = -\pi\cos(2\theta - \pi)$

d) $f'(x) = (\cos(\sin x))(\cos x)$

e) $f'(x) = \left[-\sin(\cos x)\right](-\sin x)$
$= \left[\sin(\cos x)\right](\sin x)$

f) $f'(\theta) = -7\sin(7\theta) + 5\sin(5\theta)$

Chapter 4 Review **Question 7 Page 244**

a) $\frac{dy}{dy} = 3x\cos x + 3\sin x$

b) $f'(t) = -4t^2 \sin 2t + 4t\cos 2t$

c) $\frac{dy}{dt} = \pi^2 t(\cos(\pi t - 6)) + \pi\sin(\pi t - 6)$

d) $\frac{dy}{d\theta} = -\left[\sin(\sin\theta)\right](\cos\theta) - \left[\cos(\cos\theta)\right](\sin\theta)$

e) $f'(x) = -2\left[\cos(\sin x)\right]\left[\sin(\sin x)\right](\cos x)$

f) $f'(\theta) = -7(\sin 7\theta) + 10(\cos 5\theta)(\sin 5\theta)$

Chapter 4 Review **Question 8 Page 244**

a) Answers may vary. For example:

To find the slope, $f'(x)$, first find all the critical points at $f''(x)=0$.

$f(x)=2\cos 3x$

$f'(x)=-6\sin 3x$

$f''(x)=-18\cos 3x$

Let $f''(x)=0$

$\cos 3x = 0$

$$3x = \ldots,\ -\frac{3}{2}\pi,\ -\frac{\pi}{2},\ \frac{\pi}{2},\ \frac{3\pi}{2},\ \ldots$$

$$x = \ldots,\ -\frac{3}{6}\pi,\ -\frac{\pi}{6},\ \frac{\pi}{6},\ \frac{3\pi}{6},\ \ldots$$

$$x=\frac{(2k+1)\pi}{6},\ k\in\mathbb{Z}$$

At, $x=\ldots,\ -\frac{5\pi}{6},\ -\frac{\pi}{6},\frac{3\pi}{6},\ \frac{7\pi}{6},\ \ldots$, the maximum value of the slope is obtained $f'(x)=6$.

Need only one equation of the tangent, so let $x=\frac{\pi}{2}$.

$$f\left(\frac{\pi}{2}\right)=2\cos\frac{3\pi}{2}$$
$$=0$$

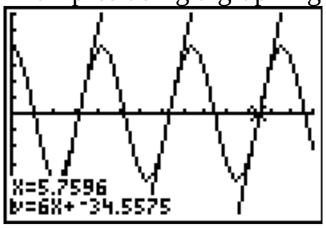
Use the slope-point form of a line to find the equation of the tangent line.

$$y-0=6\left(x-\frac{\pi}{2}\right)$$

Therefore, the equation of the tangent is $y=6x-3\pi$.

b) No, there are an infinite number of tangent lines to the curve $y=2\cos 3x$ whose slope is a maximum. From part a), at $x=\ldots,\ -\frac{5\pi}{6},\ -\frac{\pi}{6},\frac{3\pi}{6},\ \frac{7\pi}{6},\ \ldots$, the maximum value of the slope is obtained $f'(x)=6$. In more general terms, the set of all such x's can be expressed as $x=\frac{(4k+3)\pi}{6},\ k\in\mathbb{Z}$. For each value of k, you will obtain a different equation for the tangent line with maximised slope.

Examples using a graphing calculator and the tangent function are shown below.



Chapter 4 Section Review **Question 9 Page 244**

a) $V(t) = 130\sin(5t) + 18$

$V'(t) = 650\cos(5t)$

The critical points are given by $V'(t) = 0$.

$$\begin{aligned} 0 &= 650\cos(5t) \\ 0 &= \cos(5t) \\ 5t &= \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots \\ t &= \frac{\pi}{10}, \frac{3\pi}{10}, \frac{5\pi}{10}, \ldots \end{aligned}$$

$$\begin{aligned} V\left(\frac{\pi}{10}\right) &= 130\sin\left(5 \times \frac{\pi}{10}\right) + 18 \\ &= 130 + 18 \\ &= 148 \end{aligned}$$

$$\begin{aligned} V\left(\frac{3\pi}{10}\right) &= 130\sin\left(5 \times \frac{3\pi}{10}\right) + 18 \\ &= 130(-1) + 18 \\ &= -112 \end{aligned}$$

Maximum voltage: 148 V at time, in seconds, $\left\{ t \,\middle|\, t = \frac{(4k+1)\pi}{10},\ k \in \mathbb{Z},\ k \geq 0 \right\}$.

Minimum voltage: –112 V at time, in seconds, $\left\{ t \,\middle|\, t = \frac{(4k+3)\pi}{10},\ k \in \mathbb{Z},\ k \geq 0 \right\}$.

b) Period: $T = \frac{2\pi}{5}$ s

$$\begin{aligned} f &= \frac{1}{T} \\ &= \frac{5}{2\pi} \end{aligned}$$

Frequency: $\frac{5}{2\pi}$ Hz

$$A = \frac{1}{2}[148 - (-112)]$$
$$= 130$$

Amplitude: 130 V

Chapter 4 Section Review **Question 10 Page 245**

a) i) $\sin\theta = 1$

$$\theta = \frac{\pi}{2}$$

Maximum: The force $F = mg\sin\theta$ has a maximum value when $\theta = \frac{\pi}{2}$.

ii) $\sin\theta = 0$

$\theta = 0$

Minimum: The force $F = mg\sin\theta$ has a minimum value when $\theta = 0$.

b) Answers may vary. For example:

The formula for force is $F = mg\sin\theta$. The force will be a maximum at an angle where $\sin\theta$ is maximized i.e., $\frac{\pi}{2} = 90°$, since sine has a maximum value at 90°. The force will be a minimum at an angle where $\sin\theta$ is minimized i.e., 0°, since sine has a minimum value at 0°.

Chapter 4 Review **Question 11 Page 245**

a) Answers may vary. For example:

Given:

$p = mv$ ① (p is the momentum of the body.)

Differentiate ① with respect to time

$\frac{dp}{dt} = m\frac{dv}{dt}$ ②

$\frac{dv}{dt} = a$ ③ (Acceleration is the rate of change of velocity.)

Therefore, $\frac{dp}{dt} = ma$.

Combined with Newton's second law of motion: $F = \frac{dp}{dt}$, this gives $F = ma$.

b) $F = ma$

$$= m\frac{dv}{dt}$$

$$= m\frac{d}{dt}(2\cos 3t)$$

$$= -6m\sin 3t$$

Therefore $F = 0$ when $\sin 3t = 0$.

$t = 0,\ \frac{\pi}{3},\ \frac{2\pi}{3},\ \frac{3\pi}{3},\ \frac{4\pi}{3},\ \ldots$

$t = \frac{k\pi}{3},\ k \in \mathbb{Z},\ k \geq 0.$

c) $v(t) = 2\cos 3t$

$$v\left(\frac{k\pi}{3}\right) = 2\cos(k\pi)$$

$2\cos(k\pi)$ is 2 m/s when k is even and –2 m/s when k is odd.
Therefore the speed is $|v| = 2$ m/s.

Chapter 4 Practice Test

Chapter 4 Practice Test **Question 1 Page 246**

The correct answer is B.

Chapter 4 Practice Test **Question 2 Page 246**

The correct answer is C.

Chapter 4 Practice Test **Question 3 Page 246**

The correct answer is C.

Chapter 4 Practice Test **Question 4 Page 246**

The correct answer is D.

Chapter 4 Practice Test **Question 5 Page 246**

The correct answer is C.

Chapter 4 Practice Test **Question 6 Page 246**

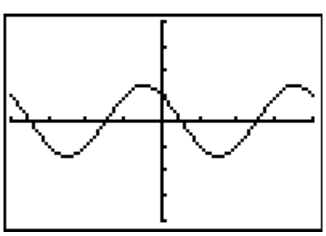
The correct answer is D.

$$\begin{aligned}\frac{dy}{dx} &= 2\left(\frac{1}{2}\right)\\ &= 1.\end{aligned}$$

Chapter 4 Practice Test **Question 7 Page 246**

The correct answer is B.

Use a graphing calculator to graph $\frac{dy}{dx} = \cos x - \sin x$ for the given window.



Chapter 4 Practice Test **Question 8 Page 246**

The correct answer is B.

Chapter 4 Practice Test **Question 9 Page 247**

a) $\frac{dy}{dx} = -\sin x - \cos x$

b) $\frac{dy}{d\theta} = 6\cos 2\theta$

c) $f'(x) = \pi \sin x \cos x$

d) $f'(t) = 3t^2 \cos t + 6t \sin t$

Chapter 4 Practice Test **Question 10 Page 247**

a) $\frac{dy}{dx} = \cos\left(\theta + \frac{\pi}{4}\right)$

b) $\frac{dy}{d\theta} = -\sin\left(\theta - \frac{\pi}{4}\right)$

c) $\frac{dy}{d\theta} = 4\sin^3\theta\left(\cos\theta\right)$

d) $\frac{dy}{d\theta} = 4\theta^3\cos\left(\theta^4\right)$

Chapter 4 Practice Test **Question 11 Page 247**

The slope of the tangent is the same as that of the curve, i.e., $\frac{dy}{dx}$ at $\frac{\pi}{4}$.

$$\frac{dy}{dx} = 2\cos^2 x - 2\sin^2 x$$

At $x = \frac{\pi}{4}$,

$$\begin{aligned}\frac{dy}{dx} &= 2\left(\frac{1}{2}\right) - 2\left(\frac{1}{2}\right) \\ &= 0\end{aligned}$$

Therefore, the slope of the line tangent to the curve $y = 2\sin x\cos x$ at $x = \frac{\pi}{4}$ is 0.

Chapter 4 Practice Test **Question 12 Page 247**

$$\frac{dy}{dx} = -6\cos^2 x \sin x$$

At $x = \frac{\pi}{3}$,

$$\begin{aligned}\frac{dy}{dx} &= -6\left(\frac{1}{4}\right)\left(\frac{\sqrt{3}}{2}\right) \\ &= -\frac{3\sqrt{3}}{4} \\ &= m\end{aligned}$$

Also at $x = \frac{\pi}{3}$,

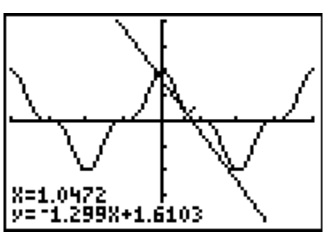
$$\begin{aligned}y &= 2\left(\frac{1}{2}\right)^3 \\ &= \frac{1}{4}\end{aligned}$$

Substituting for *m*, *y*, and *x* in *y* = *mx* + *b* gives:

$$\begin{aligned}b &= \frac{1}{4} + \frac{3\sqrt{3}}{4}\left(\frac{\pi}{3}\right) \\ &= \frac{1}{4} + \frac{\sqrt{3}\pi}{4}\end{aligned}$$

$y = -\frac{3\sqrt{3}}{4}x + \frac{\sqrt{3}\pi}{4} + \frac{1}{4}$ is the equation of the tangent line.

Use a graphing calculator and the tangent function to confirm this result.



a) $V(t) = 325\sin(100\pi t)$
$V'(t) = 325(100\pi)\cos(100\pi t)$

The critical points are given by $V'(t) = 0$.

$$0 = 325(100\pi)\cos(100\pi t)$$
$$0 = \cos(100\pi t)$$
$$100\pi t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$
$$t = \frac{1}{200}, \frac{3}{200}, \frac{5}{200}, \ldots$$

$$V\left(\frac{1}{200}\right) = 325\sin\left(100\pi\left(\frac{1}{200}\right)\right) = 325$$

$$V\left(\frac{3}{200}\right) = 325\sin\left(100\pi\left(\frac{3}{200}\right)\right) = -325$$

Maximum voltage: 325 V at time, in seconds, $\left\{t \,\middle|\, t = \frac{4k+1}{200},\ k \in \mathbb{Z},\ k \geq 0\right\}$.

Minimum voltage: –325 V at time, in seconds, $\left\{t \,\middle|\, t = \frac{4k+3}{200},\ k \in \mathbb{Z},\ k \geq 0\right\}$.

b) i)
$$T = \frac{2\pi}{100\pi} = \frac{1}{50}$$

The period is $\frac{1}{50}$ s.

ii)
$$f = \frac{1}{T} = 50 \text{ Hz}$$

The frequency is 50 Hz.

iii)
$$A = \frac{1}{2}[325 - (-325)] = 325$$

The amplitude is 325 V.

Chapter 4 Practice Test **Question 14 Page 247**

a) $V(t) = 170\sin(120\pi t)$

$V'(t) = 170(120\pi)\cos(120\pi t)$

The critical points are given by $V'(t) = 0$.

$0 = 170(120\pi)\cos(120\pi t)$

$0 = \cos(120\pi t)$

$120\pi t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$t = \frac{1}{240}, \frac{3}{240}, \frac{5}{240}, \ldots$

$$V\left(\frac{1}{240}\right) = 170\sin\left(120\pi\left(\frac{1}{240}\right)\right) = 170$$

$$V\left(\frac{3}{240}\right) = 170\sin\left(120\pi\left(\frac{3}{240}\right)\right) = -170$$

Maximum voltage: 170 V at time, in seconds, $\left\{t \middle| t = \frac{4k+1}{200},\ k \in \mathbb{Z},\ k \geq 0\right\}$.

Minimum voltage: –170 V at time, in seconds, $\left\{t \middle| t = \frac{4k+3}{200},\ k \in \mathbb{Z},\ k \geq 0\right\}$.

i) $T = \frac{2\partial}{120\partial}$

$= \frac{1}{60}$

The period is $\frac{1}{60}$ s.

ii) $f = \frac{1}{T}$

$= 60$ Hz

The frequency is 60 Hz.

iii) $A = \frac{1}{2}\left[170 - (-170)\right]$

$= 170$ V

The amplitude is 170 V.

b) Answers may vary. For example;

Similarities: Both functions are sinusoidal functions and both functions pass through the origin (0, 0).
Differences: The functions have different periods, frequencies, and amplitudes.

Chapter 4 Practice Test Question 15 Page 247

$$\begin{aligned}\text{L.S.} &= \frac{d}{dx}(\sin 2x) \\ &= 2\cos 2x\end{aligned} \qquad \begin{aligned}\text{R.S.} &= \frac{d}{dx}(2\sin x\cos x) \\ &= 2\cos^2 x - 2\sin^2 x\end{aligned}$$

Recall:

$$\sin 2x = 2\sin x\cos x$$

$$2\cos 2x = 2\cos^2 x - 2\sin^2 x$$

Therefore, differentiating both sides of the given equation gives the identity $\cos 2x = \cos^2 x - \sin^2 x$.

Chapter 4 Practice Test Question 16 Page 247

a) $f'(x) = -\sin^2 x + \cos^2 x$

$f''(x) = -4\sin x\cos x$

b) $f'''(x) = -4\left[-\sin^2 x + \cos^2 x\right]$

$f^{(4)}(x) = 16\sin x\cos x$

$f^{(5)}(x) = 16\left[-\sin^2 x + \cos^2 x\right]$

$f^{(6)}(x) = -64\sin x\cos x$

Answers may vary. For example:

The first, third, and fifth derivatives all have the expression $-\sin^2 x + \cos^2 x$ in the derivative.
The second, fourth, and sixth derivative all have the expression $\sin x \cos x$ in the derivative.
The second derivative is the original function multiplied by −4.
The third derivative is the first derivative multiplied by −4.
This pattern continues for fourth to sixth derivatives so that the n derivative is the $(n – 2)$ derivative multiplied by –4.

c) Answers may vary. For example:

My prediction for the seventh derivative is $f^{(7)}(x) = -64\left[-\sin^2 x + \cos^2 x\right]$.

My prediction for the eighth derivative is $f^{(8)}(x) = 256\sin x\cos x$.

When the sixth derivative is differentiated, the seventh derivative is $f^{(7)}(x) = -64\left[-\sin^2 x + \cos^2 x\right]$ as predicted. When the seventh derivative is differentiated, the eighth derivative is found to be $f^{(8)}(x) = 256\sin x\cos x$ as predicted.

d) Answers may vary. For example:

i) $f^{(2n)}(x) = (-4)^n \sin x \cos x$

ii) $f^{(2n+1)}(x) = (-4)^n(-\sin^2 x + \cos^2 x)$

e) i) $f^{(12)}(x) = 4096 \sin x \cos x$

ii) $f^{(15)}(x) = -16384(-\sin^2 x + \cos^2 x)$

Chapter 4 Practice Test **Question 17 Page 247**

Answers will vary. For example:

a)

$$y = \sin x$$
$$\frac{dy}{dx} = \cos x$$
$$\frac{d^2y}{dx^2} = -\sin x$$
$$\frac{d^3y}{dx^3} = -\cos x$$
$$\frac{d^4y}{dx^4} = \sin x$$
$$\frac{d^5y}{dx^5} = \cos x$$

b) $y = \cos x$, $y = -\cos x$

c) There are four functions that satisfy this differential equation. The fourth function is $y = -\sin x$. The functions are sinusoidal and the derivatives of each of the functions are shifted horizontally to the left, or to the right, $\frac{\pi}{2}$ units. The graph of the fifth derivative of each of the functions will be the same as the graph of the first derivative of each of the functions.