Chapter 4

DYNAMICS OF FLUID FLOW

For the dynamics of fluid flow, or hydrodynamics, the fluid motion is studied including the force and energy considerations.

4-1 Types of Energy:

I- Potential Energy:

It is the energy possessed by a fluid particle due to its position with respect to an arbitrary datum.

II- Pressure Energy:

It is the energy possessed by a fluid particle due to its pressure.

III- Kinetic Energy:

It is the energy possessed by a fluid particle due to its motion or velocity.

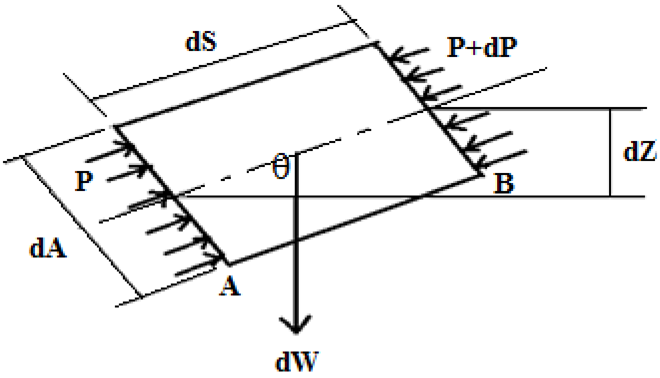
4-2 Euler's Equation:

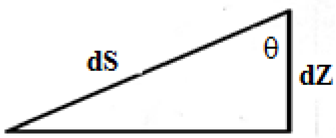
Assumptions and Limitations:

1- The fluid is ideal (non-viscous or no friction losses).
2- The fluid is incompressible (ρ is constant).
3- The flow is steady.
4- The velocity of flow is uniform over the section.
5- Only the gravity and pressure forces are considered.

The Equation:

For a steady flow of an ideal fluid, consider an element AB of the fluid, as shown in the figure.





$\cos\theta = dZ / dS$

$dZ = dS \cos\theta$

ds, dA: length and cross sectional area of the fluid element.
dW : Weight of the fluid element.
p : Pressure of the fluid element at A.
p + dp: Pressure of the fluid element at B.

Applying Newton's second law of motion in the direction of flow:

$$\sum F = M\,a$$

$$\sum F = P\,dA - (P + dP)\,dA - dW\cos\theta$$

$$dW = M\,g = \rho\,V\,g = \rho\,dA\,ds\,g$$

Thus, $$\sum F = -\,dP\,dA - \rho\,dA\,ds\,g\cos\theta \qquad \ldots\ldots\ldots\ldots \quad (1)$$

$$M\,a = \rho\,V\,a = \rho\,dA\,ds\,a$$

And, $$a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = \frac{dv}{ds}\,v$$

$$a\,ds = v\,dv$$

Then, $$M\,a = \rho\,dA\,v\,dv \qquad \ldots\ldots\ldots\ldots \quad (2)$$

(1) = (2), $$-\,dP\,dA - \rho\,dA\,ds\,g\cos\theta = \rho\,dA\,v\,dv$$

÷ dA, $$-\,dP - \rho\,g\,ds\cos\theta = \rho\,v\,dv$$

But, $$ds\cos\theta = dz$$

Thus, $$-\,dP - \rho\,g\,dz = \rho\,v\,dv$$

$$\rho\,g\,dz + dP + \rho\,v\,dv = 0$$

÷ (γ = ρ g), we get,

$$\mathbf{dz + \frac{dP}{\gamma} + \frac{v\,dv}{g} = 0}$$

4-3 Bernoulli's Equation:

Integrating Euler's equation, we get Bernoulli's equation.

$$Z + (P / \gamma) + (v^2 / 2g) = \text{Constant}$$

Where, Z : Potential energy per unit weight of fluid, or potential head with respect to an arbitrary datum.

P / γ : Pressure energy per unit weight of fluid, or pressure head.

$v^2/2g$: Kinetic energy per unit weight of fluid, or velocity head.

Applying Bernoulli's equation between two points along the flow of the fluid, *we get:*

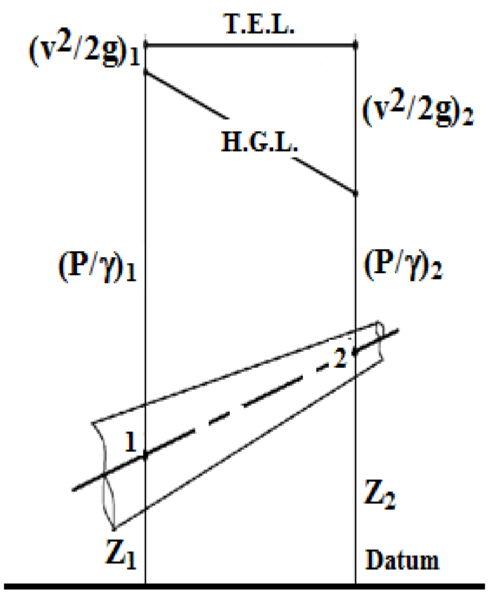
$$Z_1 + (P/\gamma)_1 + (v^2/2g)_1 = Z_2 + (P/\gamma)_2 + (v^2/2g)_2 = \text{Constant}$$

4-4 Total Energy Line (TEL) and Hydraulic Grade Line (HGL):

Total Energy = Potential Head + Pressure Head + Velocity Head

Piezometeric Head = Potential Head + Pressure Head

For different sections along the fluid flow, an arbitrary datum is chosen. The potential, pressure, and velocity heads are assigned on a vertical line through each section above the datum using adequate scale.



The line between points representing the total head is the total energy line (TEL).

The line between points representing the piezometeric head is the hydraulic grade line (HGL).

Notes:

1- Bernoulli's Equation for a Real Fluid Flow:

The real fluid has viscosity that resists the flow. So; a part of the total energy of flow is lost due to the friction. This head loss is denoted by h_L.

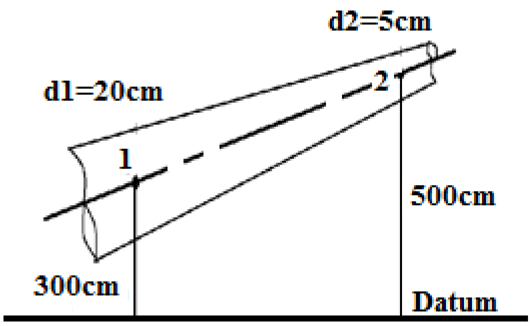
$$Z_1 + (P/\gamma)_1 + (v^2/2g)_1 = Z_2 + (P/\gamma)_2 + (v^2/2g)_2 + h_L$$

2- Direction of the Flow:

The fluid flows from the point of high total energy to that of low total energy.

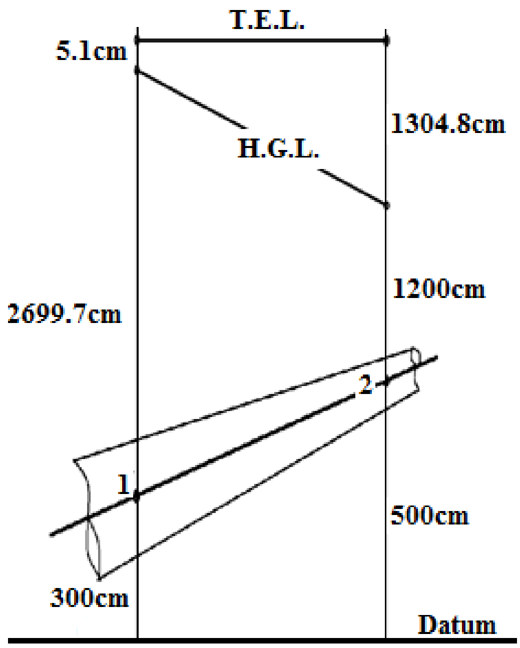
Example 1:

As shown in the figure, the diameter of a pipe changes from 20 cm at a section 3 m above datum, to 5 cm at another section 5 m above the datum. At the second section, the pressure of water is 1.2 kg/cm^2 and the velocity of flow is 16 m/sec.



Determine the pressure at the first section?

Solution



$A_1 = \pi\, d^2 / 4 = \pi\, (20)^2 / 4 = 314.16$ cm^2

$Z_1 = 3$ m $= 300$ cm

$A_2 = \pi\, d^2 / 4 = \pi\, (5)^2 / 4 = 19.63$ cm^2

$Z_2 = 5$ m $= 500$ cm

$P_2 = 1200$ g/cm^2

$V_2 = 16$ m/sec $= 1600$ cm/sec

$A_1\, v_1 = A_2\, v_2$

$V_1 = (19.63 \times 1600)/314.16 \sim 100$ cm/sec

$\gamma_w = 1$ gm/cm^3

Bernoulli's equation:

$Z_1 + (P/\gamma)_1 + (v^2/2g)_1 = Z_2 + (P/\gamma)_2 + (v^2/2g)_2$

$300 + (P_1/1) + [(100)^2 / (2 \times 981)] = 500 + (1200/1) + [(1600)^2 / (2 \times 981)]$

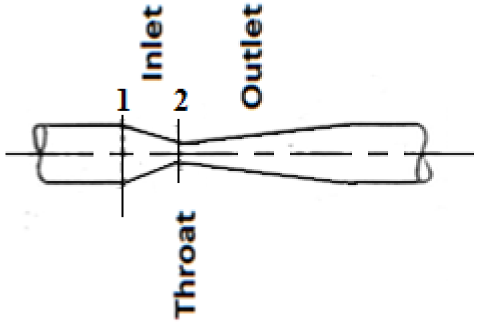
$P_1 = 2699.7$ gm/cm^2

4-5 Applications of Bernoulli's Equation:

There are many practical applications of Bernoulli's equation. We shall consider only three applications for flow measurements in pipes, using the three hydraulic devices: venturi meter, orifice meter, and pitot tube.

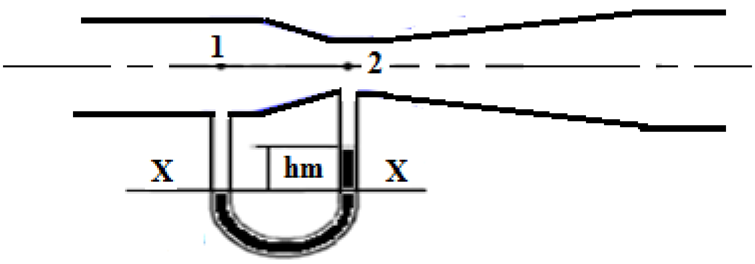
I- Venturi meter:

It is a device for measuring the discharge of a liquid flowing in a pipe. As shown in figure, it consists of three parts: convergent cone or inlet, throat, and divergent cone or outlet.



- The inlet is a short pipe that converges from the pipe diameter d_1 to a smaller diameter d_2. This convergent pipe converts pressure head into velocity head.
- The throat is a small circular pipe with constant diameter d_2.
- The outlet is a longer pipe that diverges from the throat diameter d_2 to the pipe diameter d_1. This divergent pipe converts velocity head into pressure head.

For ideal fluid, applying Bernoulli's equation between sections (1) and (2) representing the inlet and throat respectively, *as shown in the figure:*



$Z_1 + (P_1 / \gamma) + (v_1^2 /2g) = Z_2 + (P_2 / \gamma) + (v_2^2 /2g)$

$[Z_1 + (P_1 / \gamma)] - [Z_2 + (P_2 / \gamma)] = (v_2^2 - v_1^2) /2g$

<u>Let,</u> $[Z_1 + (P_1 / \gamma)] - [Z_2 + (P_2 / \gamma)] = H$

Where H is the change in piezometric head.

<u>Then,</u> $H = (v_2^2 - v_1^2) / 2g$

A manometer can be used to measure the change in piezometric head H.

At the centre line of the pipe, $P(1) = P(2)$

<u>Then,</u> <u>*at a datum x - x,*</u>

$P_1 + Z_1 \gamma = P_2 + (Z_2 - h_m) \gamma + h_m \gamma_m$

$(\div\ \gamma)$ $Z_1 + (P_1 / \gamma) = Z_2 + (P_2 / \gamma) - h_m + (\gamma_m / \gamma) h_m$

$[Z_1 + (P_1 / \gamma)] - [Z_2 + (P_2 / \gamma)] = h_m [(\gamma_m / \gamma) - 1]$

$H = h_m [(\gamma_m / \gamma) - 1]$

<u>Thus,</u> $H = h_m [(\gamma_m / \gamma) - 1] = (v_2^2 - v_1^2) / 2g$

$(v_2^2 - v_1^2) = 2 g H$

<u>But,</u> $Q = A_1 v_1 = A_2 v_2$ <u>So,</u> $v_1 = A_2 v_2 / A_1$

$v_2^2 - [(A_2^2 v_2^2) /A_1^2] = 2 g H$

$v_2^2 [1 - (A_2^2 /A_1^2)] = 2 g H$

$v_2^2 [(A_1^2 - A_2^2) / A_1^2] = 2 g H$

$v_2^2 = [A_1^2 / (A_1^2 - A_2^2)] 2 g H$

$v_2 = [A_1 / (A_1^2 - A_2^2)^{1/2}] (2 g H)^{1/2}$

<u>Thus,</u> $Q = A_2 v_2$

$$\mathbf{Q = [A_1 A_2 / (A_1^2 - A_2^2)^{1/2}] (2 g H)^{1/2}}$$

Or

$$Q = \frac{A_1 A_2 \sqrt{2gH}}{\sqrt{(A_1^2 - A_2^2)}}$$

This is the equation of venturi meter for measuring the discharge of ideal fluid flowing in a pipe.

Example 2:

A venturi meter of 15 cm inlet diameter and 10 cm throat is laid horizontally in a pipe to measure the flow of oil of 0.9 specific gravity. The reading of a mercury manometer is 20 cm.

Calculate the discharge in lit/min?

Solution

For inlet, $A_1 = (\pi\ d_1^2)/4 = (\pi \times 15^2)/4 = 176.7\ cm^2$

For throat, $A_2 = (\pi\ d_2^2)/4 = (\pi \times 10^2)/4 = 78.54\ cm^2$

$H = h_m\ [(\gamma_m / \gamma) - 1] = 20\ [(13.6 / 0.9) - 1] = 282.2$ cm of oil

$$Q = \frac{A_1 A_2 \sqrt{2gH}}{\sqrt{(A_1{}^2 - A_2{}^2)}}$$

$$Q = \frac{(176.7 x 78.54)\sqrt{2 x 981 x 282.2}}{\sqrt{(176.7)^2 - (78.54)^2}}$$

$Q = 65238.2\ cm^3/sec$

(x 60/1000) Thus, $Q = 3914.3$ lit/min

Example 3:

A 30 cm x 15 cm venturi meter is provided to vertical pipe line carrying oil with 0.9 specific gravity. The flow direction is upwards. The difference in elevation between inlet and throat is 30 cm. The reading of a mercury manometer is 25 cm.

1- Calculate the discharge?

2- Determine the pressure head between inlet and throat?

Solution

(1) For inlet, $A_1 = (\pi\ d_1^2)/4 = (\pi \times 30^2)/4 = 706.86\ cm^2$

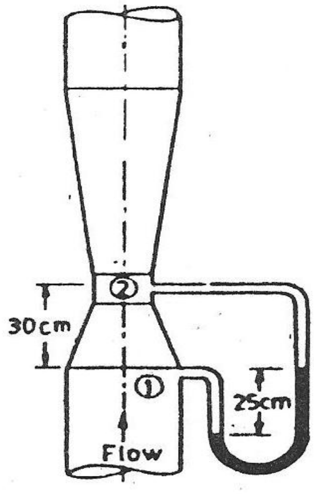
For throat, $A_2 = (\pi\, d_2^2)/4 = (\pi \times 15^2)/4 = 176.71\ cm^2$

$H = h_m\,[(\gamma_m / \gamma) - 1] = 25\,[(13.6 / 0.9) - 1] = 352.8$ cm of oil

$$Q = \frac{A_1 A_2 \sqrt{2gH}}{\sqrt{(A_1^2 - A_2^2)}}$$

$$Q = \frac{(706.86 x 176.71)\sqrt{2 x 981 x 352.8}}{\sqrt{(706.86)^2 - (176.71)^2}}$$

$Q = 151840.6\ cm^3/sec$



(2) $[Z_1 + (P_1 / \gamma)] - [Z_2 + (P_2 / \gamma)] = H$

$(P_1 / \gamma) - (P_2 / \gamma) + (Z_1 - Z_2) = 352.8$

$Z_1 - Z_2 = 0 - 30 = -30$

$(P_1 / \gamma) - (P_2 / \gamma) = 352.8 + 30 = 382.8$ cm of oil

Exercises:

(1) Resolve example (3) if the flow is downwards?

(2) Resolve examples (2) and (3) applying only Bernoulli's equation and without using equation of venturi meter?

Important Notes:

1- The reading of the manometer attached to a venturi meter is constant. It does not depend on the position of the venturi meter.

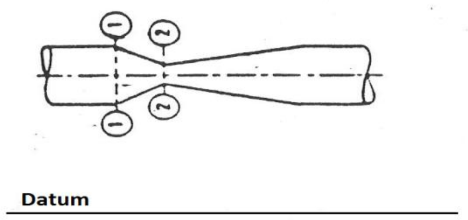
$[Z_1 + (P_1 / \gamma)] - [Z_2 + (P_2 / \gamma)] = H = (v_2^2 - v_1^2) / 2g$

H = Constant, as the velocities v_1 & v_2 are constant for continuous flow.

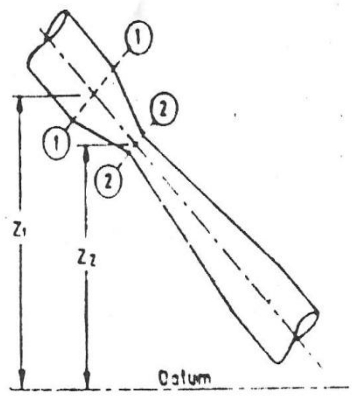
* <u>For horizontal venturi meter:</u>

The reading of manometer h_m
is employed to get H, as discussed before.

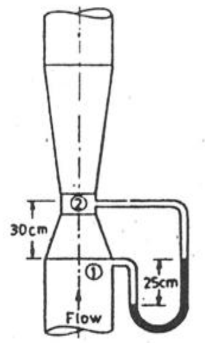
H = Potential Head + Pressure Head



* <u>For inclined venturi meter:</u>
When the venturi meter is inclined, the potential head changes. Consequently, pressure head changes also in a manner such that H is still constant. That is to say, if potential head decreases, then pressure head increases such that their sum H is still constant.



* <u>For vertical venturi meter:</u>
When the venturi meter is fixed vertically, potential head changes according to direction of flow (upwards or downwards). Consequently, pressure head changes such that their sum H is still constant.



2- Venturi meter for a real fluid flow.

For the case of a real fluid, there is energy loss between any sections decreasing the values of velocity. Thus, a coefficient of venturi meter (or coefficient of discharge) C_d is employed.

$$\mathbf{Q_R = C_d\, Q_I}$$

$C_d = 0.95 - 0.99$ (It is designed to minimize energy losses.)

<u>It may be noted that,</u>

$$C_d = Q_R / Q_I = (v_R\, A) / (v_I\, A) = (v_R / v_I)$$

$$\mathbf{V_R = C_d\, v_I}$$

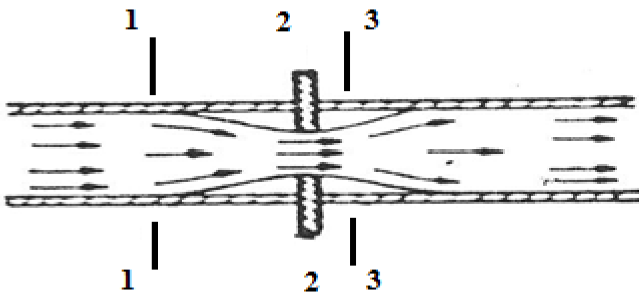
3- Negative pressure at the throat of a venturi meter.

At the throat of a venturi meter, the velocity is maximum because it has minimum cross sectional area and consequently the pressure is minimum. Thus, the pressure may be zero or even negative. When this negative pressure reaches the value of vapor pressure of the liquid flowing in the pipe, the liquid evaporates. So, the flow becomes discontinuous due to the existed vapor. Cavitation takes place as the liquid evaporates and the vapor condenses to a liquid and so on.

To avoid cavitation, the pressure at the throat of a venturi meter has not to reach the value of vapor pressure of the flowing liquid.

II- Orifice Meter:

It is a device for measuring the discharge of a liquid flowing in a pipe. As shown in figure, it consists of a plate having a sharp edged circular orifice (hole).



For ideal fluid, applying Bernoulli's equation between sections (1) and (2) representing the pipe and orifice respectively,

$$Z_1 + (P_1 / \gamma) + (v_1^2 / 2g) = Z_2 + (P_2 / \gamma) + (v_2^2 / 2g)$$
$$[Z_1 + (P_1 / \gamma)] - [Z_2 + (P_2 / \gamma)] = H = (v_2^2 - v_1^2) / 2g$$

Where, H is the change in piezometric head.

Then, $(v_2^2 - v_1^2) = 2\,g\,H$

$$(Q^2 / A_2^2) - (Q^2 / A_1^2) = 2\,g\,H$$

$$Q^2 [(1/A_2^2) - (1/A_1^2)] = 2 g H$$
$$Q^2 = [(A_1^2 A_2^2) / (A_1^2 - A_2^2)] (2 g H)$$

$$Q = \frac{A_1 A_2 \sqrt{2gH}}{\sqrt{(A_1^2 - A_2^2)}}$$

This is the equation of orifice for measuring discharge of ideal fluid flowing in a pipe.

For the case of a real fluid, there is energy loss between any sections decreasing the values of velocity. Thus, a coefficient of discharge C_d is employed, that accounts for both velocity and contraction.

$$\mathbf{Q_R = C_d\, Q_I}$$

$C_d = 0.6$ (It is simple and cheap with high energy losses.)

In fact, there is a vena contraction downstream the orifice at a distance = (1/2) diameter of orifice, say at section (3).
Introducing the effect of this contraction, the equation (1) is applied for sections (1) and (3) obtaining another equation of orifice.

Example 4:
An orifice meter has an orifice of 10 cm diameter and a coefficient of discharge of 0.65. It is fixed in a pipe of 25 cm diameter with flowing oil of 0.8 specific gravity. The pressure difference between pipe and orifice is measured by a mercury manometer that gives a reading of 80 cm.
Determine the discharge?

Solution

For pipe, $A_1 = (\pi\, d_1^2)/4 = (\pi \times 25^2)/4 = 490.87\ cm^2$

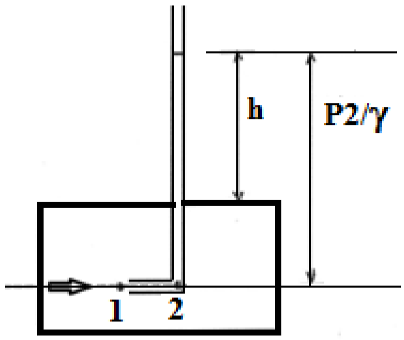
For orifice, $A_2 = (\pi\, d_2^2)/4 = (\pi \times 10^2)/4 = 78.54\ cm^2$

$H = h_m [(\gamma_m / \gamma) - 1] = 80 [(13.6 / 0.8) - 1] = 1280$ cm of oil

$$Q = \frac{490.87x78.54\sqrt{2x981x1280}}{\sqrt{(490.87)^2-(78.54)^2}} \quad x\ 0.65$$

$Q = 81957.8\ cm^3/sec$

III- Pitot Tube:



It is a device for measuring the velocity of a liquid flowing in a pipe or an open channel. As shown in figure, it consists of a glass tube bent at 90º with short length. The lower short end of pitot tube is put to face the direction of flow. The liquid rises in the tube due to the pressure of flowing liquid. The rise of liquid is measured to calculate the velocity of flowing liquid.

For ideal fluid, applying Bernoulli's equation between sections (1) and (2) as shown in figure,

$$Z_1 + (P_1 / \gamma) + (v_1^2 / 2g) = Z_2 + (P_2 / \gamma) + 0$$

At the same level, $Z_1 = Z_2 = 0$

<u>Thus,</u> $(P_1 / \gamma) + (v_1^2 /2g) = (P_2 / \gamma)$

<u>But,</u> $(P_2 /\gamma) = (P_1 / \gamma) + h$

Where h is the rise of flowing liquid in pitot tube above the surface.

<u>Then,</u> $(P_1 / \gamma) + (v_1^2 /2g) = (P_1 / \gamma) + h$

$(v_1^2 /2g) = h$

$\mathbf{v = (2\ g\ h)^{1/2}}$

This is the equation of pitot tube for measuring velocity of ideal fluid flowing in a pipe or an open channel.

For the case of a real fluid, $\mathbf{Q_R = C_d\ Q_I}$

$C_d \sim 1$(The velocity becomes zero rapidly upon entry to the tube with negligible energy losses.)

Chapter 5

FLOW THROUGH AN ORIFICE

5-1 The Orifice:

It is a small opening in any vessel through which liquid flows out. An orifice may be located in a vertical side of the vessel or in its base. It may be rounded or sharp edged. The main function of an orifice is measuring the discharge.

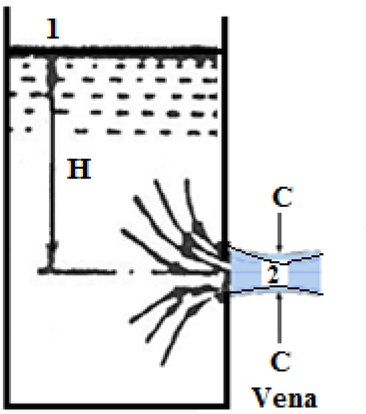
5-2 Orifice under a Constant Head:

Jet of Liquid:

It is the continuous stream of liquid that flows out the orifice.

Vena Contraction:

As shown in the figure, fluid particles take a turn to enter the orifice from all directions. This consumes some energy of the flowing liquid. The liquid flowing out the orifice (liquid jet) contracts as it is unable to make sharp turns. The maximum contraction (minimum cross sectional area of the liquid jet) is found to be slightly downstream the orifice. The section of maximum contraction is called vena contraction (section C-C in the figure).



Applying Bernoulli's equation between the point (1) on the liquid surface and the point (2) at centre line of vena contraction (section C-C in the figure),

$$H + 0 + 0 = 0 + 0 + (v^2/2g)$$

Where, H: Elevation of liquid surface above vena contraction.

v :Velocity at vena contraction.

$$\mathbf{v = (2gH)^{1/2}}$$

Coefficient of Contraction C_c:

$$C_c = \frac{\text{Actual Area}}{\text{Theoretical Area}} = \frac{\text{Area of Jet at Vena Contraction}}{\text{Area of Orifice}} = \frac{A_c}{A_o}$$

$A_c = C_c\, A_o$ & $C_c = 0.61 - 0.69$

Coefficient of Velocity C_v:

Considering the friction loss of energy between the two points, *thus:*

$$C_v = \frac{\text{Actual Velocity}}{\text{Theoretical Velocity}} = \frac{v_a}{v}$$

$v_a = C_v\, v$ & $C_v = 0.95 - 0.99$

Coefficient of Discharge C_d:

$$C_d = \frac{\text{Actual Discharge}}{\text{Theoretical Discharge}} = \frac{Q_a}{Q}$$

$Q_a = C_d\, Q$

$Q = A_o\, v$

$Q_a = A_c\, v_a = (C_c\, A_o)\,(C_v\, v) = (C_c\, C_v)\,(A_o\, v)$

Thus, $\mathbf{C_d = C_c\, C_v}$

Example 1:

Water of head 9 m is flowing through an orifice of 60 mm diameter. The coefficients of discharge and velocity are 0.6 and 0.9 respectively.

1- Calculate the actual discharge through the orifice?

2- Determine the actual velocity at vena contraction?

Solution

1- $A_o = \pi\, d^2 / 4 = (\pi\, 6^2) / 4 = 28.27\ cm^2$

$Q = A_o\,(2gH)^{1/2} = 28.27\,[2\,(981)\,(900)]^{1/2} = 37566.14\ cm^3/sec$

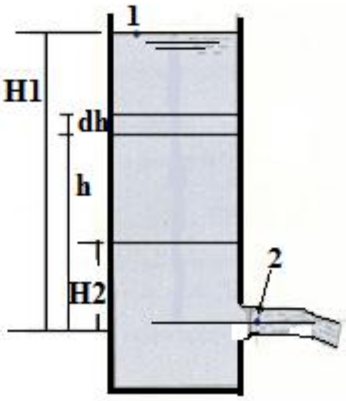
$Q_a = C_d\, Q = 0.6\ (37566.14) = 22539.68\ cm^3/sec$

2- $v = (2gH)^{1/2} = [2\ (981)\ (900)]^{1/2} = 1328.83\ cm/sec$

$v_a = C_v\, v = 0.9\ (1328.83) = 1195.95\ cm/sec$

5-3 Orifice under a Varying Head:

Assume a tank, as shown in the figure, of a cross sectional area A_T contains a liquid that is flowing through an orifice. During a period of time T, the liquid level decreased from H_1 to H_2.



At some instant, the liquid height is (h+dh) above the orifice. During a small interval of time dt, the head decreases by a small amount dh.

The actual discharge of liquid flowing out the orifice is dQ, $dQ = C_d\, A_o\, (2\, g\, h)^{1/2}$

The volume of liquid flowing out the orifice is dV,

$$dV = dQ\, dt = C_d\, A_o\, (2\, g\, h)^{1/2}\, dt$$

Also, with respect to the tank, the liquid volume in it decreased during the time interval dt by the amount dV, $dV = -\, A_T\, dh$

Thus, $C_d\, A_o\, (2\, g\, h)^{1/2}\, dt = -\, A_T\, dh$

$$dt = -\, A_T\, (h)^{-1/2}\, dh / C_d\, A_o\, (2\, g)^{1/2}$$

The total time T for decreasing liquid level from H_1 to H_2,

$$T = [-\, A_T / C_d\, A_o\, (2\, g)^{1/2}]\, (h)^{-1/2}\, dh$$

$$\mathbf{T = -\, 2\, A_T\, (H_2^{1/2} - H_1^{1/2}) / C_d\, A_o\, (2\, g)^{1/2}}$$

Minus sign may be eliminated by using (H_1 - H_2), where $H_1 > H_2$.

$\mathbf{T = 2\, A_T\, (H_1^{1/2} - H_2^{1/2}) / C_d\, A_o\, (2\, g)^{1/2}}$ $\Longrightarrow$ $T = \dfrac{2A_T(\sqrt{H1} - \sqrt{H2})}{C_d A_o \sqrt{2g}}$

Applications:

This equation can be used to determine the time T required to reduce the liquid surface from one level to another.

Also, it can be applied to detect the time T required to empty any tank containing a liquid. For this case, the liquid level decreases from H_1 to zero.

Example 2:

A swimming pool 10 m long and 6 m wide holds water to a depth of 1.25 m. The water is discharged from the pool through an orifice at its bottom. The area of the orifice is 0.23 m^2, and the coefficient of discharge is 0.62.

Determine the time required to completely empty the pool?

Solution

A_T = 10 x 6 = 60 m^2

A_o = 0.23 m^2

H_1 = 1.25 m & H_2 = 0

$$T = \frac{2A_T(\sqrt{H1}-\sqrt{H2})}{C_d A_o \sqrt{2g}}$$

$$T = \frac{2*60(\sqrt{1.25}-0)}{0.62*0.23\sqrt{2*9.81}}$$

T = 212 sec

Chapter 6

MOMENTUM EQUATION

6-1 Momentum:

Momentum of any moving body (such as a flowing fluid particle) is the quantity of motion. It is the product of its mass and velocity.

Momentum = Mass x Velocity **= M * v**

Units: *SI system:* kg.m/s

Examples:

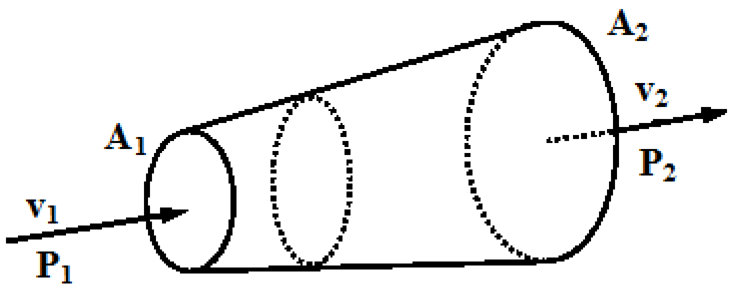
- The lift force on an aircraft is exerted by the air moving over the wing.
- A jet of water from a hose exerts a force on whatever it hits.

As the velocity, the momentum is a vector quantity.

According to Newton's second law, the rate of change of momentum of a moving body (or its acceleration) is equal to the resultant force acting on the body, and takes place in the direction of the force.

Momentum for One Dimensional Flow:

For a steady and non-uniform flow in the shown streamtube,



Mass entering the streamtube = Volume * Density = $A_1(v_1t)*\rho$

Momentum entering the streamtube = Mass*Velocity = $A_1(v_1t)\rho*v_1$

Similarly,

Mass leaving the streamtube = Volume * Density = $A_2(v_2t)*\rho$

Momentum leaving the streamtube = Mass*Velocity = $A_2(v_2t)\rho*v_2$

Resultant force on the fluid = Rate of change of momentum

$F = [A_2(v_2t)\rho*v_2 - A_1(v_1t)\rho*v_1] / t$ & $Q = A_2\, v_2 = A_1\, v_1$

$\therefore$ $F = \rho\, Q\, (v2 - v1)$

The resultant force is acting in the direction of the flow of the fluid.

<u>Momentum for Two Dimensional Flow:</u>

For a steady and non-uniform flow in the shown streamtube,

<u>In x – direction,</u>

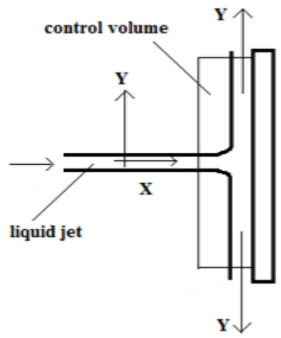
F_x = Rate of change of momentum in x - direction

$F_x = \rho\, Q\, (v_{2x} - v_{1x})$

<u>In y – direction,</u>

F_y = Rate of change of momentum in y - direction

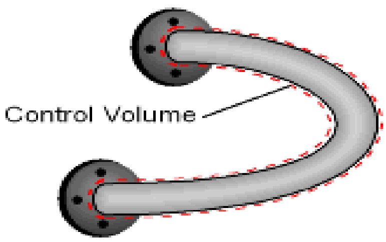
$F_y = \rho\, Q\, (v_{2y} - v_{1y})$



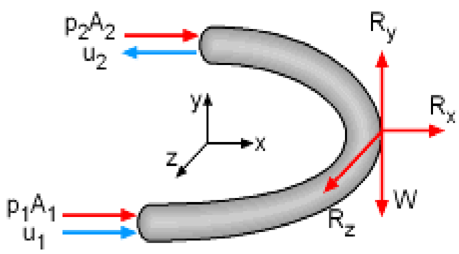
The resultant force: $F = \sqrt{Fx^2 + Fy^2}$

F is inclined with angle $\propto$ to the x – axis: $\propto = \tan^{-1}(F_y / F_x)$

<u>Momentum for Three Dimensional Flow:</u>



Pipe Elbow



<u>In general,</u> $F_T = F_B + F_W + F_P = \rho\, Q\, (v_{out} - v_{in})$

F_T : The total force.

F_B : The force exerted on the fluid by the surrounding boundary.

F_W : The force exerted on the fluid by the gravity (the weight).

F_P : The force exerted on the fluid by the pressure.

According to Newton's third law, the fluid will exert an equal and opposite reaction.

The reaction or the force exerted by the fluid on the surrounding boundary is equal and opposite to (F_B).

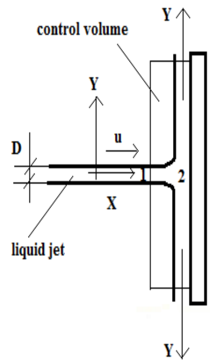
$R = -F_B$

6-2 Force Applied by a Liquid Jet on a Flat Plate:

Example 1:

A flat fixed plate is hit normally by a jet of water 25 mm diameter with a velocity of 18 m/s, as shown in figure.

Find the force on the plate?



Solution

In x – direction:

$F_{Tx} = F_B + F_W + F_P = \rho\, Q\, (v_{out} - v_{in})_x$

F_B : The force exerted on the fluid by the surrounding boundary (the plate).

F_W : The gravity force. $F_W = 0$ (Negligible weight of water)

F_P : The pressure force. $F_P = 0$ (Atmospheric pressure)

$F_B + 0 + 0 = \rho\, Q\, (v_{out} - v_{in})_x$

$F_B = 1{,}000 * 18 * (\pi 0.025^2/4) * (0 - 18) = -162$ N *in – ve x - direction*

$\therefore$ The force on the plate $R = -F_B = 162$ N *in + ve x - direction*

In y – direction:

$F_{Ty} = F_B + F_W + F_P = \rho\, Q\, (v_{out} - v_{in})_y$

$(v_{out} = v_{in})_y$

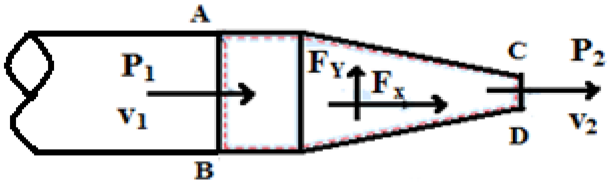
$\therefore$ No force in y – direction.

Example 2:

A nozzle is connected to a hose. At the entry section, the pressure is 250 kPa and the diameter is 25 mm. At the exit section, the diameter is 10 mm. The flow rate is 1 lit/s.

Determine the force required to hold the nozzle?

Solution



$q = 1$ lit/s $= 0.001\ m^3/s$

At entry: $P_1 = 250$ k Pa $d_1 = 25$ mm
At exit: $P_2 = 1$ atm $= 0$ $d_2 = 10$ mm

In x – direction:

$v_1 = Q / A_1 = 0.001 / (\pi\,(0.025)^2/4) = 2.04$ m/s
$v_2 = Q / A_2 = 0.001 / (\pi\,(0.01)^2/4) = 12.73$ m/s

$F_{Tx} = F_B + F_W + F_P = \rho\, Q\, (v_{out} - v_{in})_x$

$F_B + (P_1\, A_1 + 0) + 0 = \rho\, Q\, (v_2 - v_1)_x$
$F_B = -\,[(250{,}000 * (\pi\, 0.025^2/4)] + 1{,}000 * 0.001 * (12.73 - 2.04)$

$\therefore\ F_B = -\,112$ N *in – ve x - direction*

Thus, the required holding force is $R = -\,F_B = 112$ N *in + ve x - direction*

In y – direction:

$v_{out\ Y} = v_{in\ Y} = 0$

$\therefore$ No force in y – direction.