



# **Chapter 4**

## **Energy Balances**

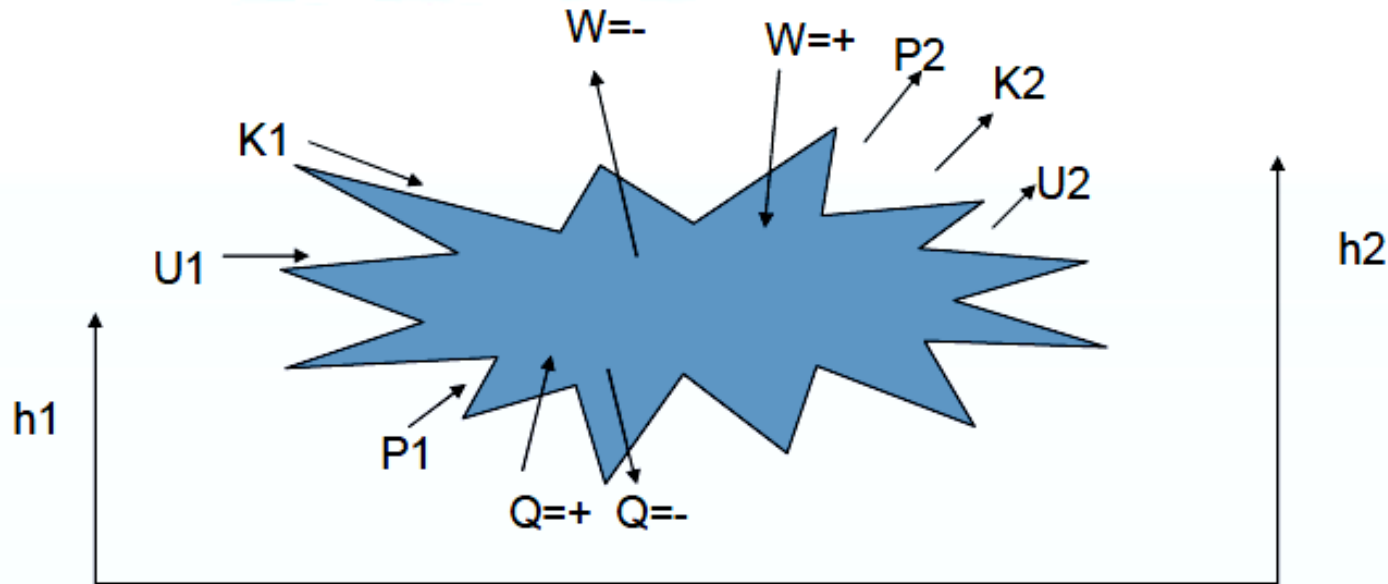
### **without reaction**



# General Energy Balance

- {accumulation of energy within the system} = {transfer of energy into system through system boundary} - {transfer of energy out of system through system boundary} + {energy generation within system} - {energy consumption within system}

# Energy Balance without reaction for closed system



$$\Delta E = E_{t2} - E_{t1} = \Delta U + \Delta P + \Delta K = Q + W$$

General process showing the system boundary and energy transport across the boundary.



# Energy Balances for open system (without chemical reaction)

- Accumulation:  $\Delta E = m_{t2}(u+k+p)_{t2} - m_{t1}(u+k+p)_{t1}$
- Energy transfer in with mass flow:  $(u_1+k_1+p_1)m_1$
- Energy transfer out with mass flow:  $(u_2+k_2+p_2)m_2$
- Net transfer by heat flow in:  $Q$
- Net transfer by mechanical or electrical work in:  $W$
- Net transfer by work to introduce and remove mass:  $p_1v_1m_1 - p_2v_2m_2$



# Application of the general energy balance without reaction

- 1) No mass transfer (closed or batch system) ( $m_1 = m_2 = 0$ ):  $\Delta E = Q + W$
- 2) No accumulation ( $\Delta E = 0$ ), no mass transfer ( $m_1 = m_2 = 0$ ):  $Q = -W$
- 3) No accumulation ( $\Delta E = 0$ ), but with mass flow:  $Q + W = \Delta(h + k + p)m$
- 4) No accumulation,  $Q = 0$ ,  $W = 0$ ,  $k = 0$ ,  $p = 0$ :  $\Delta H = 0$



# Special process names associated with energy balance

- **Isothermal** ( $dT=0$ ): constant-temperature process
- **Isobaric** ( $dp = 0$ ): constant pressure process
- **Isometric** or **isochoric** ( $dV = 0$ ): constant-volume process
- **Adiabatic** ( $Q = 0$ ): no heat interchange (i.e., an insulated system)
  - System is insulated
  - $Q$  is very small in relation to the other terms
  - The process takes place so fast that there is no time for heat to be transferred

# Energy Balances on Closed Systems

Open system ~ mass crosses the system boundary during the period of time covered by the energy balance → **semibatch and continuous process**

Closed system ~ mass does not cross the system boundary during the period of time covered by the energy balance → **batch process**

## For a closed system

**Note that energy can neither be created nor destroyed.**

$$\begin{array}{c}
 \text{Input} + \text{generation} - \text{output} - \text{consumption} = \text{accumulation} \\
 \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \\
 U_f + E_{kf} + E_{pf} \quad U_i + E_{ki} + E_{pi} \quad Q - W
 \end{array}$$

Subscripts i and f refer to the initial and final states of the system

U, E<sub>k</sub>, E<sub>p</sub>, Q and W represent internal energy, kinetic energy, potential energy, heat transferred to the system from its surroundings, and work done by the system on its surroundings

$$\longrightarrow (U_f - U_i) + (E_{kf} - E_{ki}) + (E_{pf} - E_{pi}) = Q - W \longrightarrow \Delta U + \Delta E_k + \Delta E_p = Q - W$$

$\Delta U + \Delta E_k + \Delta E_p = Q - W$  **Basic form of the first law of thermodynamics for a closed system**

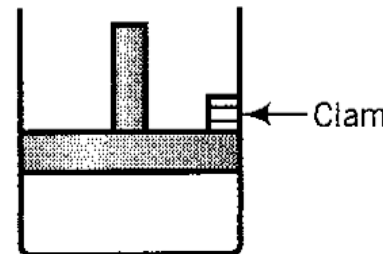
1. The internal energy of a system depends almost entirely on the **chemical composition, state of aggregation, and temperature of the system materials**. It is independent of pressure for ideal gases and nearly independent of pressure for liquids and solids. If no temperature changes, phase changes, or chemical reactions occur in a closed system, and if pressure changes are less than a few atmospheres, then  $\Delta U \approx 0$ .
2. If a system is not accelerating, then  $\Delta E_k = 0$ . If a system is not rising or falling, then  $\Delta E_p = 0$ .
3. If a system and its surroundings are at the same temperature or if the system is perfectly insulated, then  $Q = 0$ . The system is termed adiabatic.
4. If the system has no moving parts or electrical currents or radiation at the system boundary, then  $W = 0$ .



## Example:

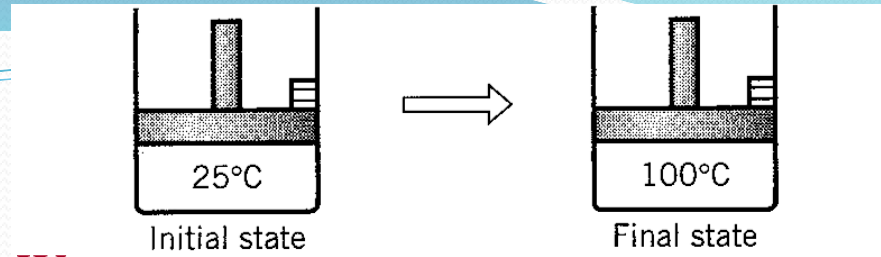


A gas is contained in a cylinder fitted with a movable piston. The initial gas temperature is  $25^{\circ}\text{C}$ . The cylinder is placed in boiling water with the piston held in a fixed position. Heat in the amount of 2.00 kcal is transferred to the gas, which equilibrates at  $100^{\circ}\text{C}$  and a higher pressure. The piston is then released, and the gas does 100 J of work in moving the piston to its new equilibrium position. The final gas temperature is  $100^{\circ}\text{C}$ . Write the energy balance for each of the two stages of the process, and in each case solve for the unknown energy term. In solving the problem, consider the gas to be the system, neglect the change in potential energy of the gas as the piston moves, and assume ideal gas behavior. Express all energies in joules.



## Solution

1.



$$Q = 2.00 \text{ kcal}$$

$$\Delta U + \Delta E_k + \Delta E_p = Q - W$$

$$\Delta U = Q$$

$$\Delta E_k = 0 \text{ (the system is stationary)}$$

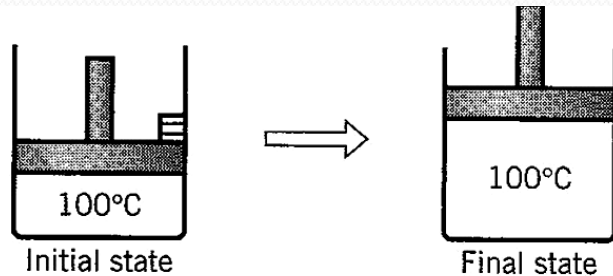
$$\Delta E_p = 0 \text{ (no vertical displacement)}$$

$$W = 0 \text{ (no moving boundaries)}$$

$$\Delta U = 2.00 \text{ kcal} \times \frac{10^3 \text{ cal}}{\text{kcal}} \times \frac{1 \text{ J}}{0.23901 \text{ cal}} = 8368 \text{ J}$$

→ The gas gains 8368 J of internal energy.

2.



$$\Delta U + \Delta E_k + \Delta E_p = Q - W$$

$$\Delta E_k = 0 \text{ (the system is stationary)}$$

$$\Delta E_p = 0 \text{ (assume negligible)}$$

$$U = 0 \text{ (depends only for an ideal gas)}$$

$$0 = Q - W \xrightarrow{W = 100 \text{ J}} Q = 100 \text{ J}$$

An additional 100 J of heat are absorbed by the gas as it expands and re-equilibrates at 100°C.



## Energy Balances for closed unsteady state systems



$$\left\{ \begin{array}{l} \text{accumulation of} \\ \text{mass within the} \\ \text{system boundary} \\ \text{from } t_1 \text{ to } t_2 \end{array} \right\} = \left\{ \begin{array}{l} \text{net transfer of mass into} \\ \text{the system through the} \\ \text{system boundary} \\ \text{from } t_1 \text{ to } t_2 \end{array} \right\} - \left\{ \begin{array}{l} \text{net transfer of mass out} \\ \text{of the system through} \\ \text{the system boundary} \\ \text{from } t_1 \text{ to } t_2 \end{array} \right\}$$

and in symbols is

$$\Delta m_{\text{system}} = m_{\text{in}} - m_{\text{out}}$$

## Example

## Application of an Energy Balance to a Closed, Unsteady-State System without Reaction



Alkaloids are chemical compounds containing nitrogen that can be produced by plant cells. In an experiment, an insulated closed vessel  $1.673 \text{ m}^3$  in volume was injected with a dilute water solution containing two alkaloids: ajmalicine and serpentine. The temperature of the solution was  $10^\circ\text{C}$ . To obtain an essentially dry residue of alkaloids, all of the water in the vessel was vaporized. Assume that the properties of water can be used in lieu of the properties of the solution. How much heat had to be transferred to the vessel if 1 kg of saturated liquid water initially at  $10^\circ\text{C}$  was completely vaporized to a final condition of  $100^\circ\text{C}$  and 1 atm

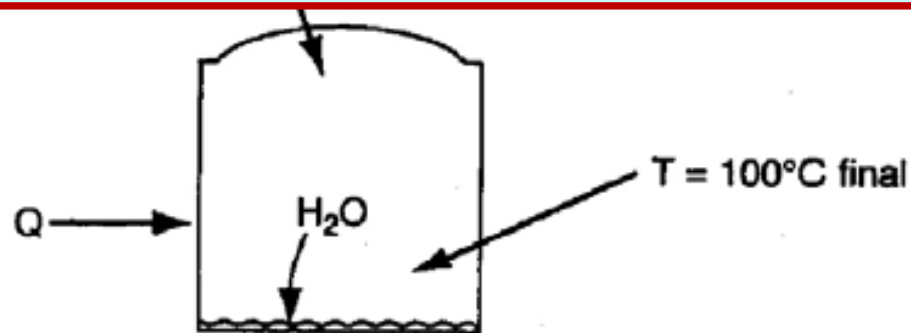


Figure E22.1





Sufficient data are given in the problem statement to fix the initial state and the final state of the water. You can look up the properties of water in the steam tables or the CD in the back of this book. Note that the specific volume of steam at 100°C and 1 atm is 1.673 m<sup>3</sup>/kg (!).

<b>Initial state (liquid)</b>	<b>Final state (gas)</b>
$p$ 1 atm	1 atm
$T$ 10.0°C	100°C
$\hat{U}$ 35 kJ/kg	2506.0 kJ/kg

You can look up additional properties of water such as  $\hat{V}$  and  $\hat{H}$ , but they are not needed for the problem.

The system is closed, unsteady state so that Equation (22.2) applies

$$\Delta E = \Delta U + \Delta PE + \Delta KE = Q + W$$

Because the system (the water) is at rest,  $\Delta KE = 0$ . Because the center of mass of the water changes so very slightly,  $\Delta PE = 0$ . No work is involved (fixed tank boundary). You can conclude using a basis of

Basis: 1 kg H<sub>2</sub>O evaporated



$$Q = \Delta U = m\Delta\check{U} = m(\check{U}_2 - \check{U}_1)$$

$$Q = \frac{1 \text{ kg H}_2\text{O} \quad | \quad (2506 - 35) \text{ kJ}}{\text{kg}} = 2471 \text{ kJ}$$



## EXAMPLE Calculation of $\Delta U$ Using American Engineering Units

Saturated liquid water is cooled from 80°F to 40°F still saturated. What are  $\Delta \hat{E}$ ,  $\Delta \hat{U}$ ,  $\Delta \hat{H}$ ,  $\Delta \hat{p}$ , and  $\Delta \hat{V}$ ?

### Solution

The system is closed and unsteady state, and thus analogous to Example 22.1. The properties listed below are from the steam tables for saturated liquid (at its vapor pressure).

	Initial Conditions	Final Conditions
$p^*$ (psia)	0.5067	0.1217
$\hat{V}$ (ft <sup>3</sup> /lb)	0.01607	0.01602
$\hat{H}$ (Btu/lb)	48.02	8.05

Basis: 1 lb water

Then

$$\Delta \hat{p} = (0.1217 - 0.5067) = -0.385 \text{ psia}$$

$$\Delta \hat{V} = (0.01602 - 0.01607) = \text{negligible value}$$

$$\Delta \hat{H} = (8.05 - 48.02) = -39.97 \text{ Btu/lb}$$

T °F	P psi	Spec. vol. ft <sup>3</sup> =lbm	Int. Ener. BTU/lbm	Enthalpy BTU/lbm		Entropy BTU=(lbm°R)			
		Sat. liq. v <sub>f</sub> X100	Sat. vap. v <sub>g</sub>	Sat. liq. u <sub>f</sub>	Sat. vap. u <sub>g</sub>	Sat. liq. h <sub>f</sub>	Sat. vap. h <sub>g</sub>	Sat. liq. s <sub>f</sub>	Sat. vap. s <sub>g</sub>
32.018	0.0887	1.5	3305	0	1021	0	1075	0.0000	2.187
35	0.0999	1.602	2948	3	1022	3	1077	0.0061	2.176
40	0.1217	1.602	2445	8.02	1024	8.02	1079	0.0162	2.159
45	0.1475	1.602	2037	13.04	1026	13.04	1081	0.0262	2.142
50	0.1781	1.602	1704	18.06	1027	18.06	1083	0.0361	2.126
60	0.2563	1.603	1207	28.08	1030	28.08	1088	0.0555	2.094
70	0.3632	1.605	867.6	38.09	1034	38.09	1092	0.0746	2.064
80	0.5073	1.607	632.7	48.08	1037	48.08	1096	0.0933	2.036

**Initial Conditions**

**Final Conditions**

$p^*$ (psia)

0.5067

0.1217

$\hat{V}$  (ft<sup>3</sup>/lb)

0.01607

0.01602

$\hat{H}$  (Btu/lb)

48.02

8.05

**Basis: 1 lb water**

Then

$$\Delta \hat{p} = (0.1217 - 0.5067) = -0.385 \text{ psia}$$

$$\Delta \hat{V} = (0.01602 - 0.01607) = \text{negligible value}$$

$$\Delta \hat{H} = (8.05 - 48.02) = -39.97 \text{ Btu/lb}$$





$$\hat{W} = 0$$

$$\Delta \widehat{KE} = 0$$

$$\Delta \widehat{PE} = 0$$

$$\hat{H} = \check{U} + P\tilde{V}$$

Now  $\Delta \hat{U} = \Delta \hat{H} - \Delta(p\hat{V})$

$$\Delta(p\hat{V}) = p_2\hat{V}_2 - p_1\hat{V}_1$$

	Initial Conditions	Final Conditions
$p^*$ (psia)	0.5067	0.1217
$\hat{V}$ (ft <sup>3</sup> /lb)	0.01607	0.01602
$\hat{H}$ (Btu/lb)	48.02	8.05

$$= \frac{0.1217 \text{ lb}_f}{\text{in.}^2} \left| \frac{144 \text{ in.}^2}{1 \text{ ft}^2} \right| \frac{0.01602 \text{ ft}^3}{\text{lb}_m} \left| \frac{1 \text{ Btu}}{778(\text{ft})(\text{lb}_f)} \right|$$

$$- \frac{0.5067 \text{ lb}_f}{\text{in.}^2} \left| \frac{144 \text{ in.}^2}{1 \text{ ft}^2} \right| \frac{0.01607 \text{ ft}^3}{\text{lb}_m} \left| \frac{1 \text{ Btu}}{778(\text{ft})(\text{lb}_f)} \right|$$

$$= -1.141 \times 10^{-3} \text{ Btu} \quad \text{a negligible quantity}$$

Thus

$$\Delta \hat{U} = \Delta \hat{H} = -39.97 \text{ Btu/lb}$$

For a change in state of a liquid to a vapor,  $\Delta(p\hat{V})$  may not be negligible.

## Energy Balances for closed, steady state systems



Recall that steady state means the accumulation in the system is zero, and that the flows of  $Q$  and  $W$  in and out of the system are constant. They can actually vary in the process, of course, but we are really interested only in their net cumulative values over a time interval, and look at only the final and initial conditions for  $\Delta E$ .

How should Equation (22.2) be modified to analyze steady-state systems?

$$\Delta (U + PE + KE)_{\text{inside}} \equiv \Delta E = Q + W \quad (22.2)$$

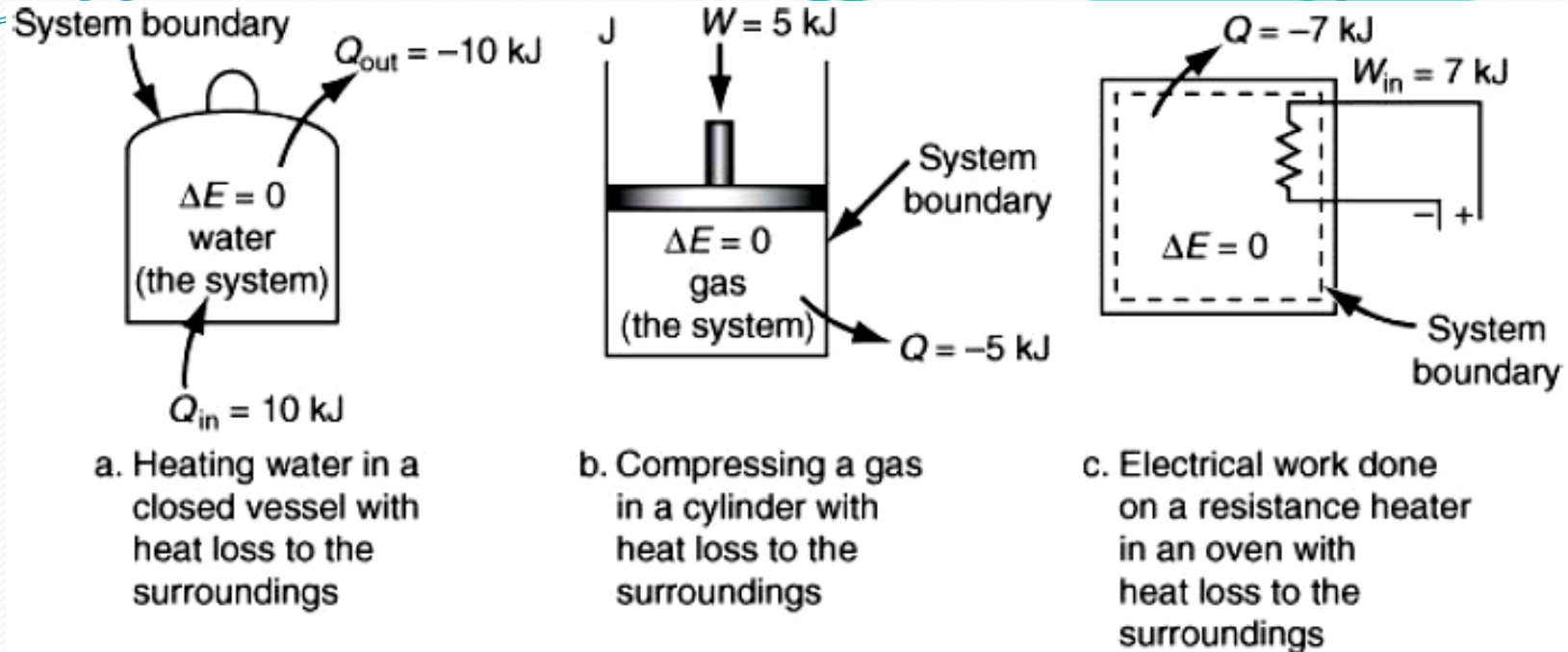
All you have to do is realize that inside the system

$$\begin{aligned} \Delta KE &= 0 & \Delta U &= 0 \\ \Delta PE &= 0 & \Delta E &= 0 \end{aligned}$$

hence 
$$Q + W = 0 \quad (22.4)$$

If you rearrange Equation (22.4), you get  $W = -Q$ , meaning that all of the work done on a closed, steady-state system must be transferred out as heat ( $-Q$ ).

# Energy Balances for closed, steady state systems



**Figure 22.3** Examples of closed, steady-state systems that involve energy changes.

Fig. a.  $W = 0$  and hence  $Q = 0$

( $Q$  is the cumulative *net* heat transfer in Equation (22.4).)

Fig. b.  $W = 5 \text{ kJ}$  and hence  $Q = -5 \text{ kJ}$

Fig. c.  $W = 7 \text{ kJ}$  and hence  $Q = -7 \text{ kJ}$

In summary, for a closed, steady-state system, the energy balance reduces Equation (22.4).



## Example Closed system

Ten pounds of  $\text{CO}_2$  at room temperature ( $80\text{ }^\circ\text{F}$ ) are stored in a fire extinguisher having a volume of  $4\text{ ft}^3$ . How much heat must be removed from the extinguisher so that 40% of the  $\text{CO}_2$  becomes liquid?

# Solution



Closed system without reaction

Use CO<sub>2</sub> chart in Appendix J to get properties

Specific volume of CO<sub>2</sub> is  $4/10 = 0.4 \text{ ft}^3/\text{lb}$ , hence CO<sub>2</sub> is a gas at the start, pressure is 300 psia and  $\Delta h = 160 \text{ Btu/lb}$

Basis: 10 lb CO<sub>2</sub>

In the energy balance:  $\Delta E = Q + W$

$W = 0$  (volume of the system is fixed,  $\Delta K = \Delta P = 0$ ),  $Q = \Delta U = \Delta h - \Delta(pv)$

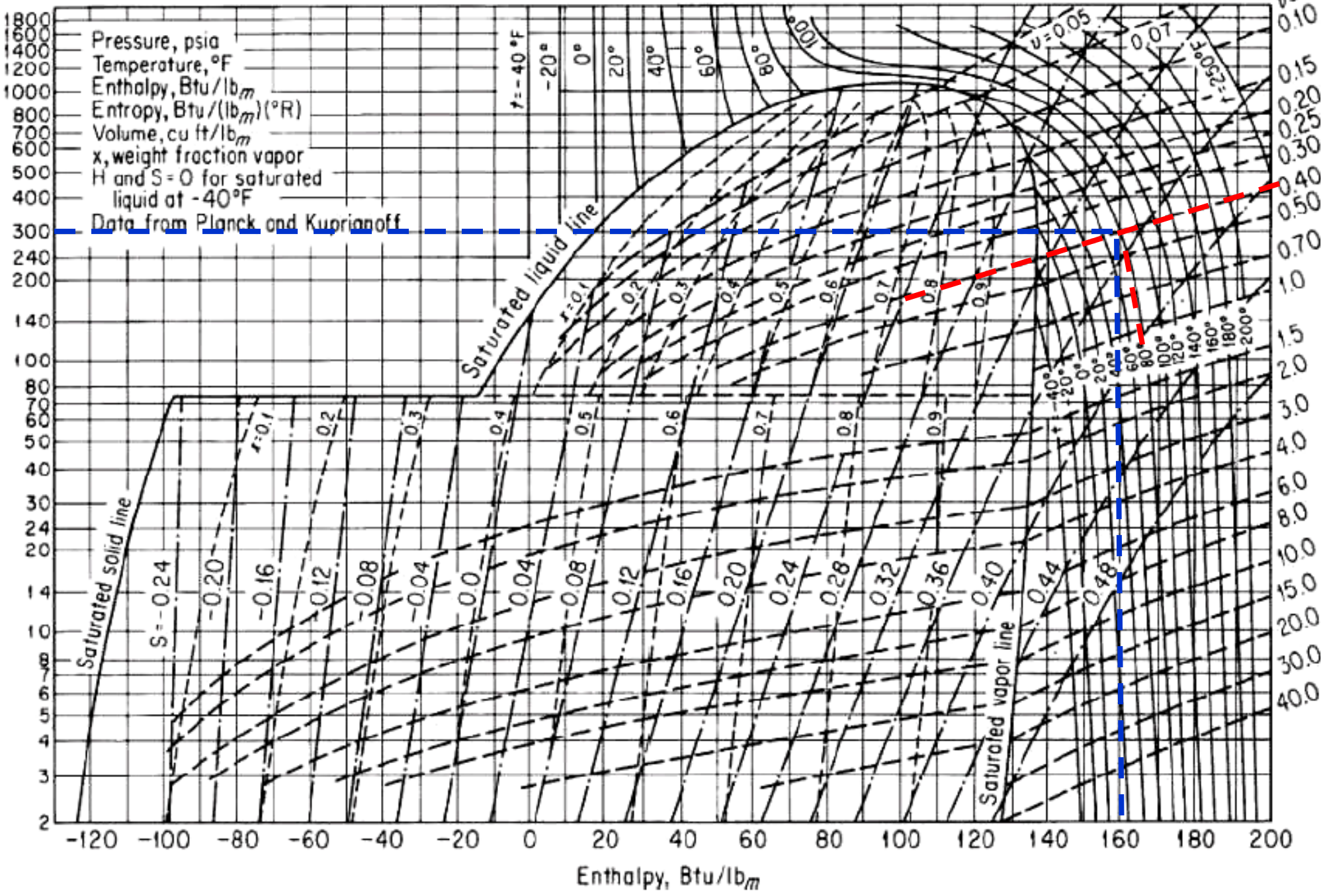
Find  $\Delta h_{\text{final}}$  from the CO<sub>2</sub> chart by following the constant-volume line of  $0.4 \text{ ft}^3/\text{lb}$  to the spot where the quality is 0.6.

$\Delta h_{\text{final}} = 81 \text{ Btu/lb}$ ,  $p_{\text{final}} = 140 \text{ psia}$

$Q = (81-160) - \left\{ \left[ \frac{(140)(144)(0.4)}{778.2} \right] - \left[ \frac{(300)(144)(0.4)}{778.2} \right] \right\} = -67.2 \text{ Btu/lb}$  (heat is removed)

$$778.2 \text{ ftlb}_f = 1 \text{ Btu}$$

Pressure, psia

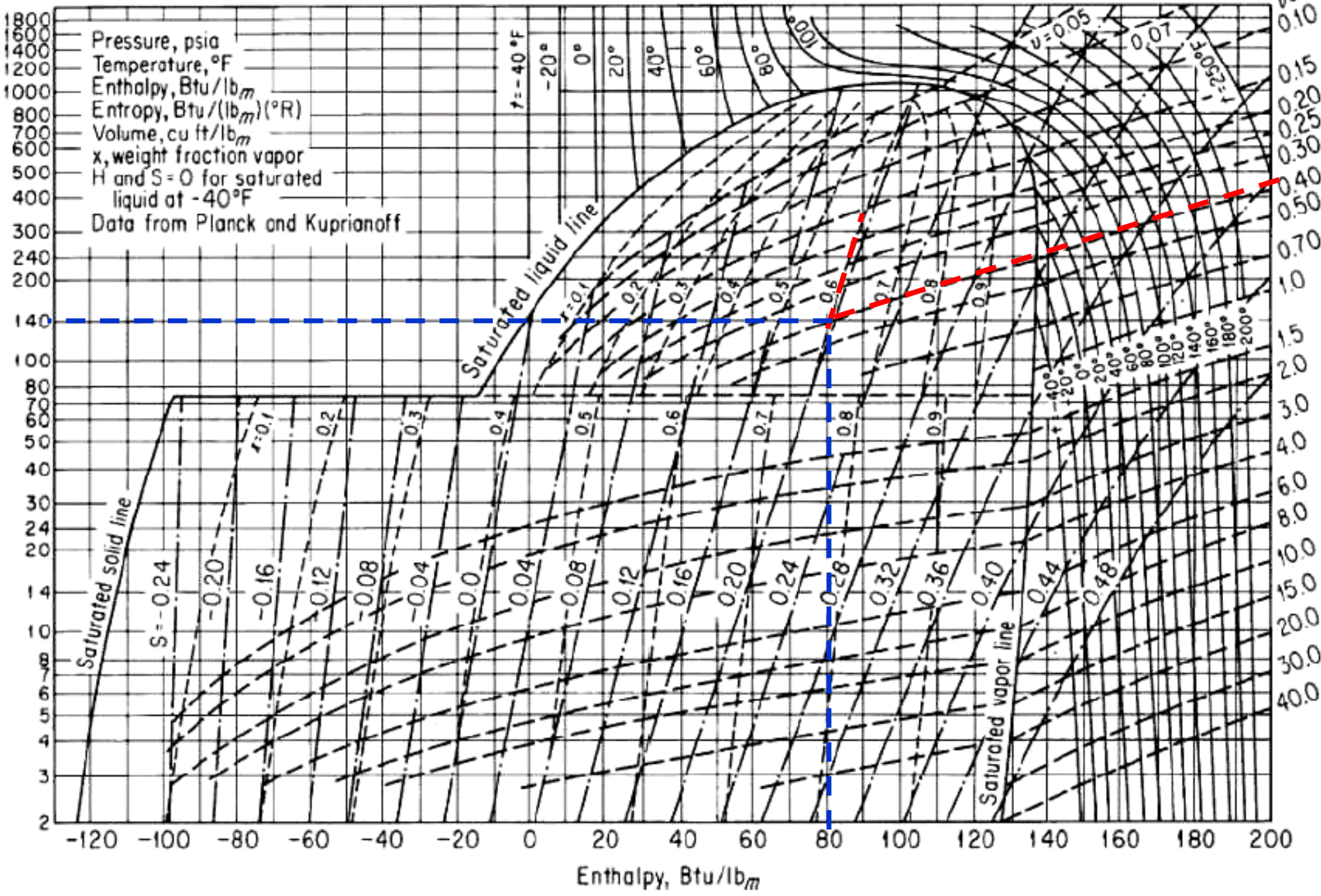


Pressure, psia  
Temperature, °F  
Enthalpy, Btu/lb<sub>m</sub>  
Entropy, Btu/(lb<sub>m</sub>)°R  
Volume, cu ft/lb<sub>m</sub>  
x, weight fraction vapor  
H and S = 0 for saturated liquid at -40°F

Data from Planck and Kuprianoff

Enthalpy, Btu/lb<sub>m</sub>

Pressure, psia



# Solution



Find  $\Delta h_{\text{final}}$  from the  $\text{CO}_2$  chart by following the constant-volume line of  $0.4 \text{ ft}^3/\text{lb}$  to the spot where the quality is 0.6.

$$\Delta h_{\text{final}} = 81 \text{ Btu/lb}, p_{\text{final}} = 140 \text{ psia}$$

$$Q = (81-160) - \left\{ \left[ \frac{(140)(144)(0.4)}{778.2} \right] - \left[ \frac{(300)(144)(0.4)}{778.2} \right] \right\} = -67.2 \text{ Btu/lb (heat is removed)}$$

$$Q = (81-160) \frac{\text{Btu}}{\text{lb}_m} - \left[ \frac{140 \text{ lb}_f}{\text{in.}^2} \left| \frac{144 \text{ in.}^2}{1 \text{ ft}^2} \right| \frac{0.4 \text{ ft}^3}{\text{lb}_m} \left| \frac{1 \text{ Btu}}{778(\text{ft})(\text{lb}_f)} \right. - \frac{300 \text{ lb}_f}{\text{in.}^2} \left| \frac{144 \text{ in.}^2}{1 \text{ ft}^2} \right| \frac{0.4 \text{ ft}^3}{\text{lb}_m} \left| \frac{1 \text{ Btu}}{778(\text{ft})(\text{lb}_f)} \right. \right]$$

$$778.2 \text{ ftlb} = 1 \text{ Btu}$$



# Example



Argon gas in an insulated plasma deposition chamber with a volume of 2 L is to be heated by an electric resistance heater. Initially the gas, which can be treated as an ideal gas, is at 1.5 Pa and 300 K. The 1000-ohm heater draws current at 40 V for 5 minutes (480 J of work is done by the surrounding).

What is the final gas temperature and pressure at equilibrium? The mass of the heater is 12 g and its heat capacity is 0.35 J/gK. Assume that the **heat** transfer to the chamber from the gas at this low pressure and in the short time period **is negligible**.



# Solution

Closed system:  $\Delta K = \Delta P = 0$

$$\Delta E = Q + W = \Delta U$$

$$Q = 0 \text{ so } \Delta E = W = \Delta U$$

Q is negligible

Basis: 5 minutes

Calculate mass of the gas:  $n = pV/RT$

$$n = \frac{1.5 \text{ Pa} \cdot 2 \text{ L} \cdot 10^{-3} \text{ m}^3}{1 \text{ L} \cdot 8.314 \text{ Pa m}^3 \cdot 300 \text{ K}} = 1.203 \cdot 10^{-6} \text{ gmol}$$

$$C_v = C_p - R \text{ and since } C_p = 5/2R, \text{ so } C_v = 3/2R$$

$$\Delta U = nC_v(T-300)$$

$$480 \text{ J} = (12 \text{ g})(0.35 \text{ J/g K})(T-300) + (1.203 \cdot 10^{-6}) (3/2)(8.214)(T-300)$$

$$T = 414.3 \text{ K}$$

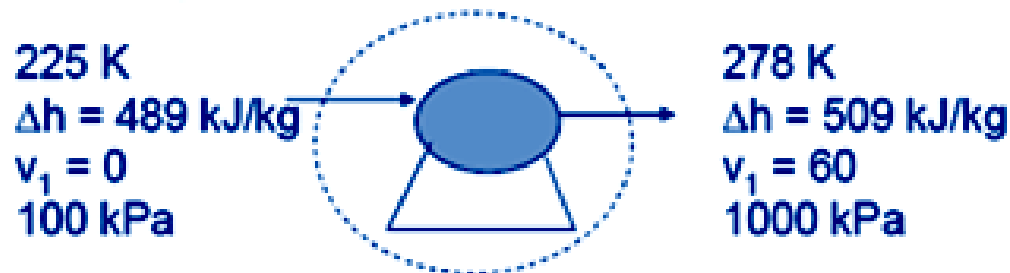
$$\text{Final pressure is } p_2 = p_1(T_2/T_1) = 1.5(414.3/300) = 2.07 \text{ Pa}$$

A monatomic gas  $C_v = (3/2)R$  . A diatomic gas  $C_v = (5/2)R$  . A tri-atomic gas  $C_v = 3R$



# Example

Air is being compressed from 100 kPa and 255 K (where it has an enthalpy of 489 kJ/kg) to 1000 kPa and 278 K (where it has an enthalpy of 509 kJ/kg). The exit velocity of the air from the compressor is 60 m/s. What is the power required (in kW) for the compressor if the load is 100 kg/hr of air?





# Solution

Basis: 100 kg of air = 1 hr

Simplify energy balance:  $\Delta E = Q + W - \Delta[(h+k+p)m]$

The process is in steady state:  $\Delta E = 0$

$$m_1 = m_2 = m$$

$$\Delta(pm) = 0$$

$Q = 0$  by assumption

$$W = \Delta H + \Delta K$$

$$\Delta H = (509 - 489) \text{ kJ/kg} \cdot 100 \text{ kg} = 2000 \text{ kJ}$$

$$\Delta K = \frac{1}{2}m(v_2^2 - v_1^2) = \frac{1}{2}(100 \text{ kg})(60^2 \text{ m}^2/\text{s}^2) = 180 \text{ kJ}$$

$$W = 2180 \text{ kJ}$$

Convert to power

$$\text{kW} = 2180 \text{ kJ}/1 \text{ hr} (=3600 \text{ sec}) = 0.61 \text{ kW}$$

# Example



Water is being pumped from the bottom of a well 15 feet deep at the rate of 200 gal/hr into a vented storage tank to maintain a level of water in a tank 165 ft above the ground. To prevent freezing in the winter a small heater puts 30,000 Btu/hr into the water during its transfer from the well to the storage tank. Heat is lost from the whole system at the constant rate of 25,000 Btu/hr. What is the temperature of the water as it enters the storage tank, assuming that the well water is at 35 F? A 2-hp pump is being used to pump the water. About 55% of the rated horsepower goes into the work of pumping and the rest is dissipated as heat to the atmosphere.





# Solution

Open system with flow in and out

Material balance is 200 gal enter and 200 gal leave in an hour

Energy balance is :  $\Delta E = Q + W - \Delta(h+k+p)m$

Process is in the steady state,  $\Delta E = 0$

$$m_2 = m_1 = m$$

$\Delta K = 0$  because will assume that  $v_1 = v_2 = 0$ , then  $0 = Q + W - \Delta(h + p)m$

At the top of the tank,  $\Delta H = m\Delta h = mC_p(T_2-35)$

Total mass of water pumped is  $(200 \text{ gal/hr})(8.33 \text{ lb/1gal}) = 1666 \text{ lb/hr}$

Potential energy change =  $(1666 \text{ lb/hr})(32.2 \text{ ft/s}^2)(180\text{ft}) / (32.2 \text{ ft lb}_m/\text{s}^2 \text{ lb}_f) / (778 \text{ ft lb}_f) = 385.5 \text{ Btu}$

$$\rho_{\text{H}_2\text{O}} = 8.33 \text{ lb}_m / \text{gal}$$

$$1.0 \text{ lb}_f = 32.2 \text{ ft lb}_m / \text{s}^2$$

$$778.2 \text{ ft lb}_f = 1 \text{ Btu}$$



Heat lost by system is 25,000 Btu while the heater puts 30,000 Btu into the system, hence the net heat exchange =  
 $Q = 30000 - 25000 = 5000 \text{ Btu}$

Rate of work being done on the water by the pump =  $w =$   
 $(2 \text{ hp})(0.55)(33,000 \text{ ft lb}_f/\text{min hp})(60 \text{ min/hr})(1 \text{ Btu}/778 \text{ ft lb}_f) = 2800 \text{ Btu/hr}$

$\Delta H$  can be calculated from:  $Q + W = \Delta H + \Delta P = 7414 \text{ Btu}$

$$\Delta H = Q + W - \Delta P$$

$$\Delta H = 5000 + 2800 - 385.5 = 7414 \text{ Btu}$$

Heat capacity of liquid water assumed to be constant and equal to 1.0 Btu/lb F

$$7414 = \Delta H = 1666(1.0)\Delta T$$

$$\Delta T = 4.5 \text{ F}, T = 39.5 \text{ F}$$

$$1 \text{ hp} = 33,000 \text{ ft}\cdot\text{lb}_f/\text{min}$$

**OR:**



The total mass of water pumped is

$$\frac{200 \text{ gal}}{\text{hr}} \left| \frac{8.33 \text{ lb}}{1 \text{ gal}} \right| = 1666 \text{ lb}$$

The potential energy change is

$$\begin{aligned} \Delta PE &= m \Delta \widehat{PE} = mg \Delta h = \frac{1666 \text{ lb}_m}{\text{hr}} \left| \frac{32.2 \text{ ft}}{\text{s}^2} \right| \left| \frac{180 \text{ ft}}{\text{hr}} \right| \left| \frac{(\text{s}^2)(\text{lb}_f)}{32.2(\text{ft})(\text{lb}_m)} \right| \left| \frac{1 \text{ Btu}}{778 (\text{ft})(\text{lb}_f)} \right| \\ &= 385.4 \text{ Btu} \end{aligned}$$

The heat lost by the system is 25,000 Btu while the heater puts 30,000 Btu into the system; hence the net heat exchange is

$$Q = 30,000 - 25,000 = 5000 \text{ Btu}$$





The rate of work being done on the water by the pump is

$$\dot{W} = \frac{2 \text{ hp}}{1} \left| \frac{0.55}{1} \right| \frac{33,000(\text{ft})(\text{lb}_f)}{(\text{min})(\text{hp})} \left| \frac{60 \text{ min}}{\text{hr}} \right| \frac{\text{Btu}}{778 (\text{ft})(\text{lb}_f)} = 2800 \text{ Btu/hr}$$

hence  $W = 2800 \text{ Btu}$

$\Delta H$  can be calculated from:  $Q + W = \Delta H + \Delta PE$

$$5000 + 2800 = \Delta H + 385$$

$$\Delta H = 7415 \text{ Btu}$$

Because the temperature range considered is small, the heat capacity of liquid water may be assumed to be constant and equal to  $1.0 \text{ Btu}/(\text{lb})(^\circ\text{F})$  for the problem.

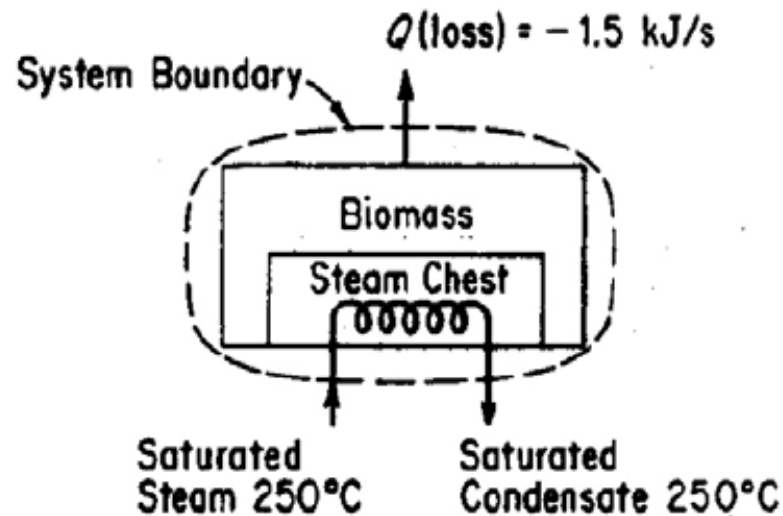
Thus,

$$7415 = \Delta H = mC_p \Delta T = 1666(1.0)(T_2 - 35)$$

$$\Delta T \cong 4.5^\circ\text{F} \text{ temperature rise, hence, } T_2 = 39.5^\circ\text{F.}$$

## Example : Application of the energy balance to Heating a Biomass

Steam at 250°C saturated (which is used to heat a fermentation broth) enters the steam chest of fermentor. The steam chest is segregated from the biomass in the fermentor. Assume that the steam is completely condensed in the steam chest. The rate of the heat loss from the surroundings is 1.5 kJ/s. The material to be heated is placed in the fermentor at 20°C and at the end of the heating is at 100°C. If the charge consists of 150 kg of material with an average heat capacity of  $C_p = 3.26 \text{ J/(g)(K)}$ , how many kilograms of steam are needed per kilogram of charge? The charge remains in the vessel for 1 hr.





**Basis: 1 hr of operation (150 kg of charge)**

The steam is the only material entering and leaving the system, and  $m_1 = m_2$  of the steam, hence the material balance is simple. The energy balance is

$$\Delta E = Q + W - \Delta[(\hat{H} + \hat{KE} + \hat{PE})m] \quad (a)$$

Let us simplify the energy balance

1. The process is not in the steady state, so  $\Delta E \neq 0$ .
2. We can safely assume that  $\Delta KE = 0$  and  $\Delta PE = 0$  inside the system.
3.  $W = 0$ .
4.  $\Delta KE$  and  $\Delta PE$  of the entering and exiting material are zero.

Consequently, Equation (a) becomes

$$\Delta E = \Delta U = Q - \Delta[(\hat{H})m] \quad (b)$$



where  $\Delta U$  can be calculated from just the change in state of the biomass, and does not include the water in the steam chest because we will assume that there was no water in the steam chest at the start of the hour and none in the steam chest at the end of the hour. (You could alternately assume the mass of water in the steam chest and its state were the same at  $t = 0$  and  $t = 1$  hour.) Let  $m_1 = m_2 = m_{\text{steam}}$  be the mass of steam that goes through the steam chest in 1 hour. Let the heat capacity of the biomass,  $C_{p, \text{biomass}}$ , be constant with respect to temperature. Then you can calculate the values of the terms in Equation (b):

(a)  $\Delta U = \Delta H - \Delta(pV) = \Delta H = m_{\text{biomass}} C_{p, \text{biomass}} (373 - 293)\text{K}$  [because we know that  $\Delta(pV)$  for the liquid or solid charge is negligible]. Thus

$$\Delta U = \Delta H_{\text{biomass}} = \frac{150 \text{ kg}}{(\text{kg})(\text{K})} \left| \frac{3.26 \text{ kJ}}{(\text{kg})(\text{K})} \right| \frac{(373 - 293)\text{K}}{(\text{kg})(\text{K})} = 39,120 \text{ kJ}$$

(b) The heat loss is given as  $Q = -1.50\text{kJ/s}$

$$\frac{-1.50 \text{ kJ}}{\text{s}} \left| \frac{3600 \text{ s}}{1 \text{ hr}} \right| \frac{1 \text{ hr}}{1 \text{ hr}} = -5400 \text{ kJ}$$



(c) The specific enthalpy change for the steam (if the changes consist only of the  $\Delta \hat{H}$  of condensation) can be determined from the steam tables. The  $\Delta \hat{H}_{vap}$  of saturated steam at 250°C is 1701 kJ/kg, so that

$$\Delta \hat{H}_{\text{steam}} = -1701 \text{ kJ/kg}$$

Introduction of all these values into Equation (b) gives

$$\Delta U = Q - \Delta[(H)m]$$

$$39,120 \text{ kJ} = -\left(-1701 \frac{\text{kJ}}{\text{kg steam}}\right)(m_{\text{steam}} \text{ kg}) - 5400 \text{ kJ} \quad (\text{c})$$

from which the kilograms of steam per hour,  $m_{\text{steam}}$ , can be calculated as

$$m_{\text{steam}} = \frac{44,520 \text{ kJ}}{1701 \text{ kJ}} \left| \frac{1 \text{ kg steam}}{1701 \text{ kJ}} \right. = 26.17 \text{ kg steam}$$

Thus

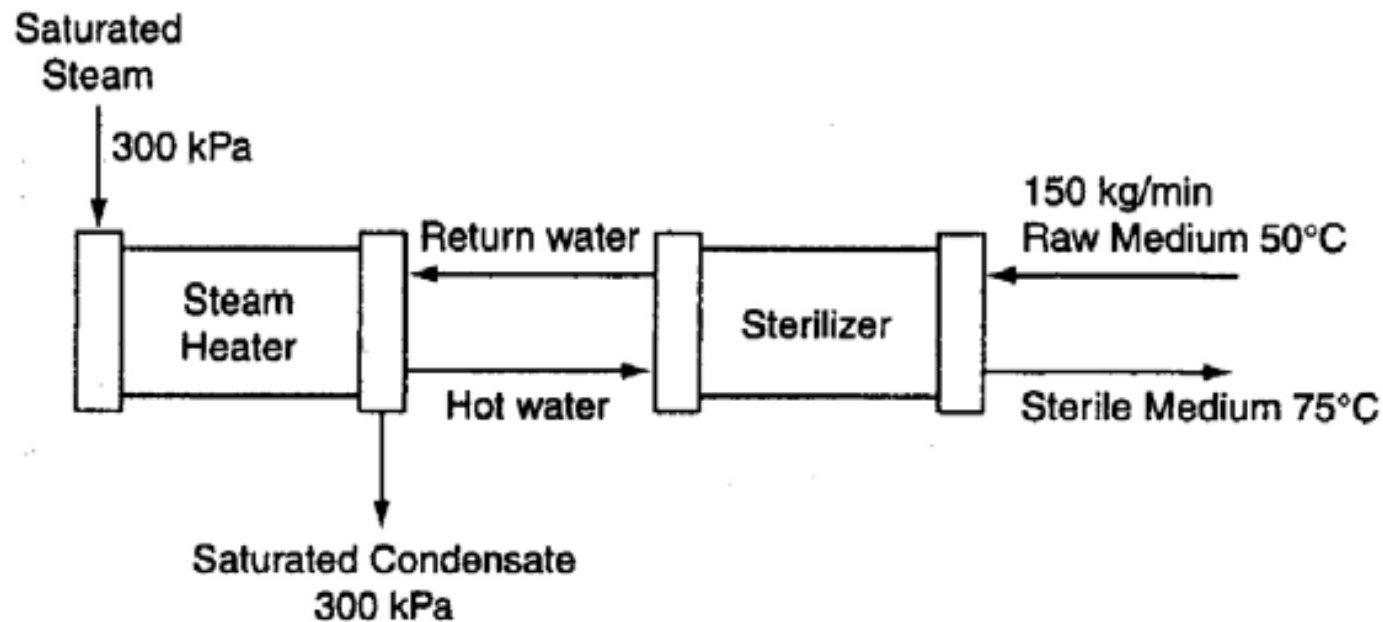
$$\frac{26.17 \text{ kg steam}}{150 \text{ kg charge}} = 0.174 \frac{\text{kg steam}}{\text{kg charge}}$$



## EXAMPLE Sterilization of a Fermentation Medium

For a biological media (or medical instruments) the primary objective of sterilization (pasteurization) is to destroy undesirable microorganisms. At the same time, you want to avoid nutrient degradation and minimize costs. Figure E24.8 shows a process in which a biological media passes through a sterilizer heated by hot water. The return hot water is heated by condensing steam. The total heat loss from the process is 1.63 kW. Figure E24.8 shows the conditions for the process flows. Calculate the flow rate of the steam entering the steam heater.

Assume that the biomedium has the properties of water. The system is in the steady state with no work, KE, PE involved.





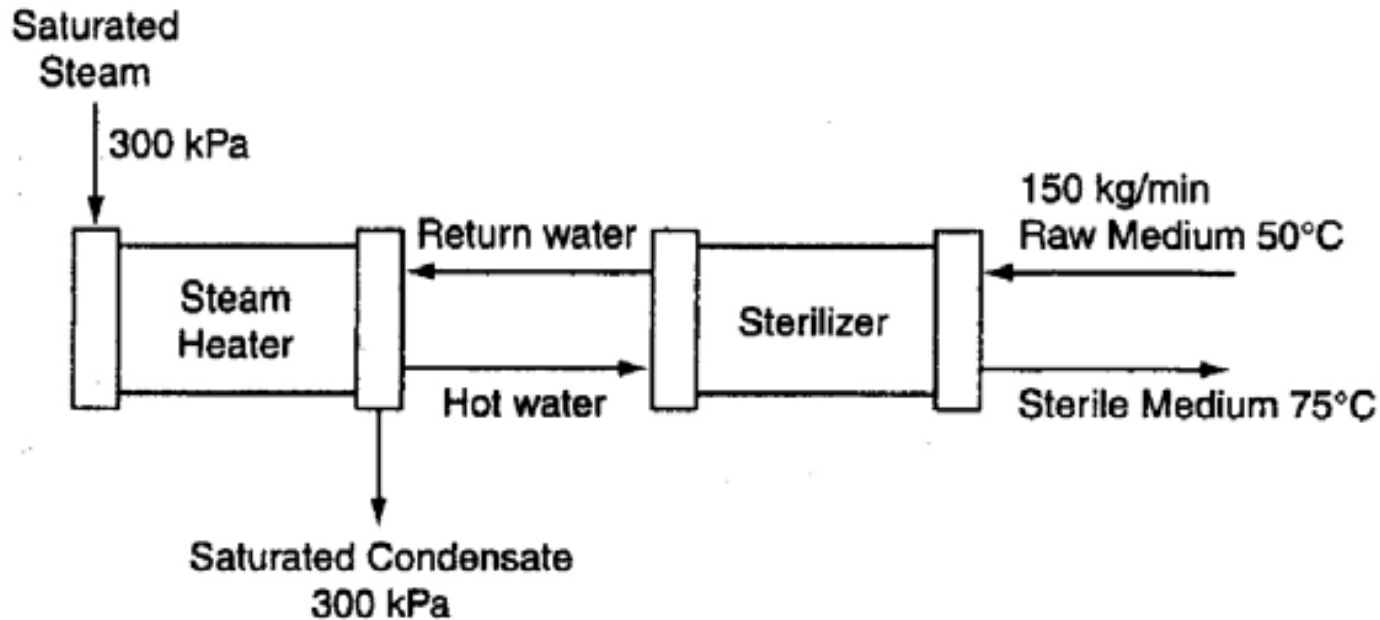
## Solution

Both material and energy balances are involved in the solution of this problem.

### Steps 1–4

The basic data are in Figure E24.8. From the steam tables (assuming that the biomedium has the properties of water), you can obtain the necessary enthalpies:

	$\Delta \hat{H}$ (kJ/kg)
Water, 50°C	207.5
Water, 75°C	310.3
Steam, saturated, 300 kPa	2724.9
Water, saturated, 300 kPa	561.2





## Solution

Both material and energy balances are involved in the solution of this problem.

### Steps 1–4

The basic data are in Figure E24.8. From the steam tables (assuming that the biomedica has the properties of water), you can obtain the necessary enthalpies:

	$\Delta \hat{H}$ (kJ/kg)
Water, 50°C	207.5
Water, 75°C	310.3
Steam, saturated, 300 kPa	2724.9
Water, saturated, 300 kPa	561.2

### Step 5

Basis: 150 kg biomedica (1 min)

### Steps 6 and 7

For the calculations you can pick as the designated system (a) the sterilizer, (b) the steam heater, or (c) the whole process. Let's pick (c) because the properties of the hot water in the streams between the two pieces of equipment are unknown. You can assume that the system is open and steady state.

The material balances are quite simple: what goes in the sterilizer comes out, namely 150 kg of biomedica, and what goes in the steam heater comes out, but we do not know the value. Let's call it  $m$  kg.





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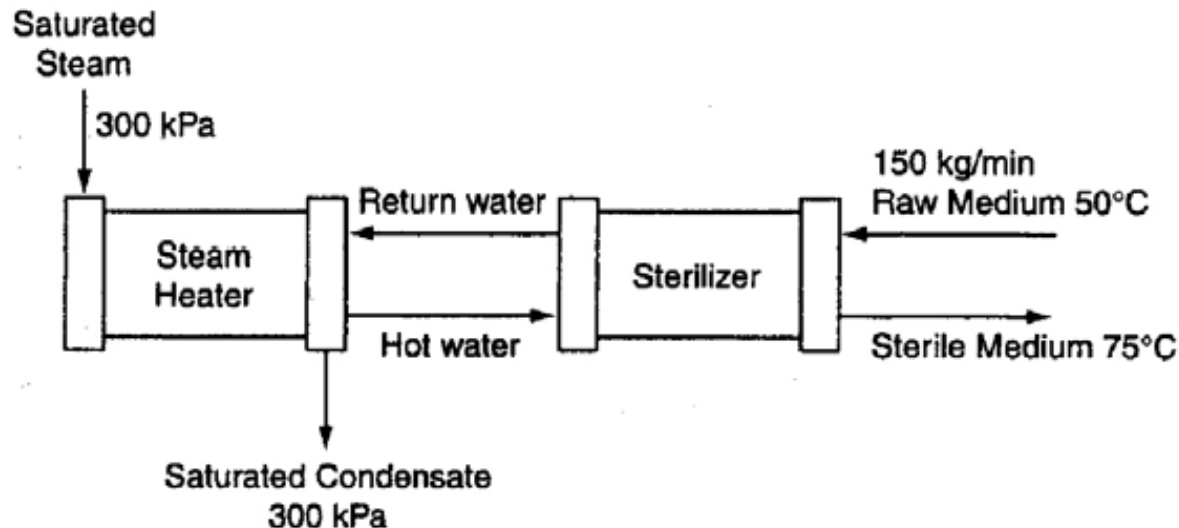
	$\Delta \hat{H}$ (kJ/kg)
Water, 50°C	207.5
Water, 75°C	310.3
Steam, saturated, 300 kPa	2724.9
Water, saturated, 300 kPa	561.2

$$Q = \Delta H = H_{out} - H_{in}$$

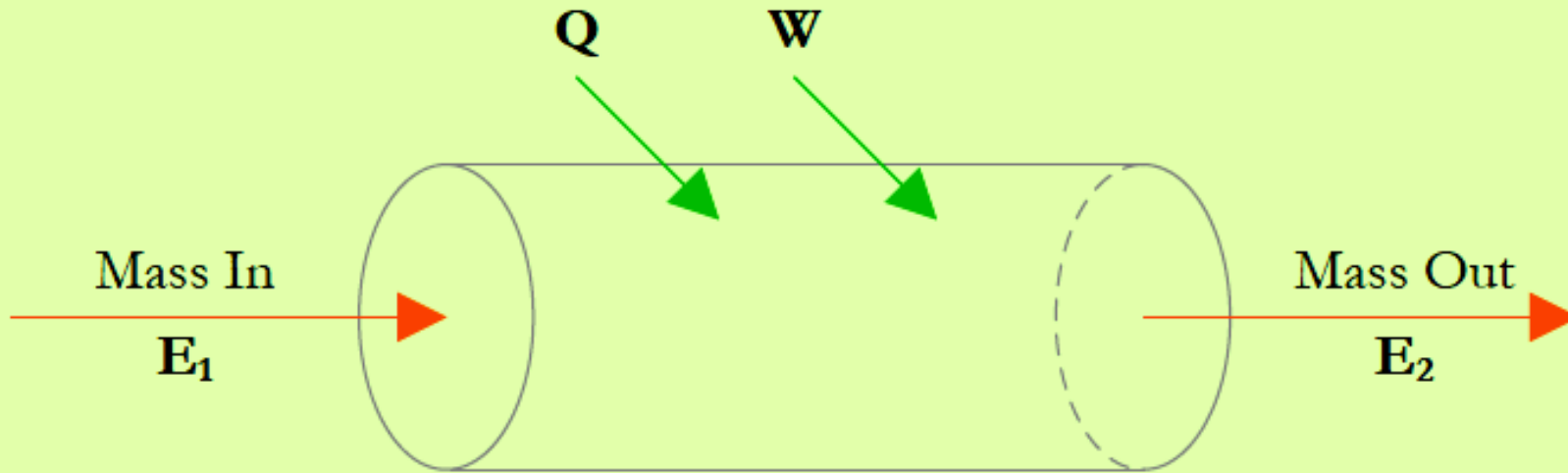
$$\frac{1.63 \text{ kW}}{(1 \text{ W})(s)} \left| \frac{1 \text{ J}}{1 \text{ min}} \right| \frac{60 \text{ s}}{1 \text{ min}} = \frac{150 \text{ kg } F}{\text{kg } F} \left| \frac{310.3 \text{ kJ}}{\text{kg } F} \right| + \frac{m \text{ kg water}}{\text{kg water}} \left| \frac{561.2 \text{ kJ}}{\text{kg water}} \right|$$

$$-\frac{150 \text{ kg } F}{\text{kg } F} \left| \frac{207.5 \text{ kJ}}{\text{kg } F} \right| - \frac{m \text{ kg steam}}{\text{kg steam}} \left| \frac{2724.9 \text{ kJ}}{\text{kg steam}} \right|$$

$$m = 7.08 \text{ kg of steam per minute}$$



# Energy Balance on an Open System



$$\Delta E = Q - W$$
$$E_2 - E_1 = Q - W$$

If there are multiple inlets and outlets,

$$\Sigma E_2 - \Sigma E_1 = Q - W$$

# Energy Balance on an Open System



The work appearing in the equation is the combined flow work and shaft work:

$$W = W_F + W_S$$

$W_F$  = flow work; work that is necessary to get mass into and out of the system

$W_S$  = shaft work; work produced or required beside getting mass into and out of the system.

Hence,

$$\Sigma E_2 - \Sigma E_1 = Q - (W_F + W_S)$$



The net flow work is determined as

$$\mathbf{W_F} = (\mathbf{W_F})_2 - (\mathbf{W_F})_1$$

The flow work is usually expressed in terms of pressure and volume:

$$\mathbf{W_F} = (\mathbf{PV})_2 - (\mathbf{PV})_1$$

For multiple inlets and outlets,

$$\mathbf{W_F} = \Sigma(\mathbf{PV})_2 - \Sigma(\mathbf{PV})_1$$



The energy balance becomes

$$\Sigma E_2 - \Sigma E_1 = Q - [(\Sigma(PV)_2 - \Sigma(PV)_1) + W_s]$$

Since  $E = K + P + U$ , then

$$\Sigma(K + P + U)_2 - \Sigma(K + P + U)_1 = Q - [(\Sigma(PV)_2 - \Sigma(PV)_1) + W_s]$$

Rearranging the terms,

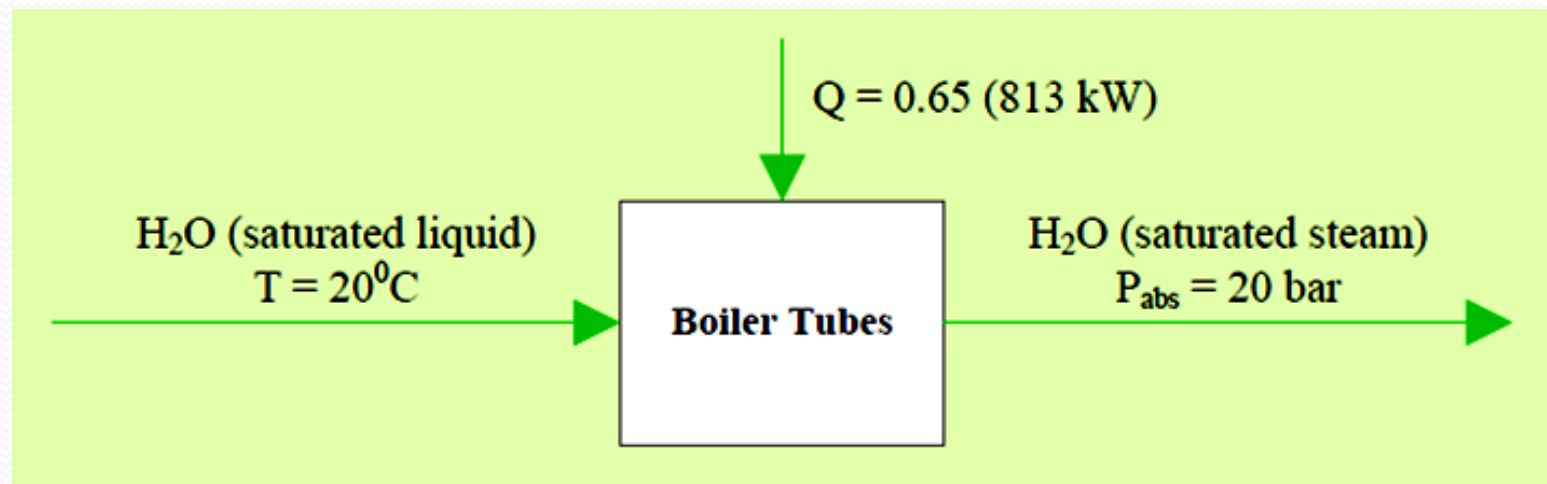
$$\begin{aligned}\Sigma(K + P + U + PV)_2 - \Sigma(K + P + U + PV)_1 &= Q - W_s \\ \Sigma(K + P + H)_2 - \Sigma(K + P + H)_1 &= Q - W_s\end{aligned}$$



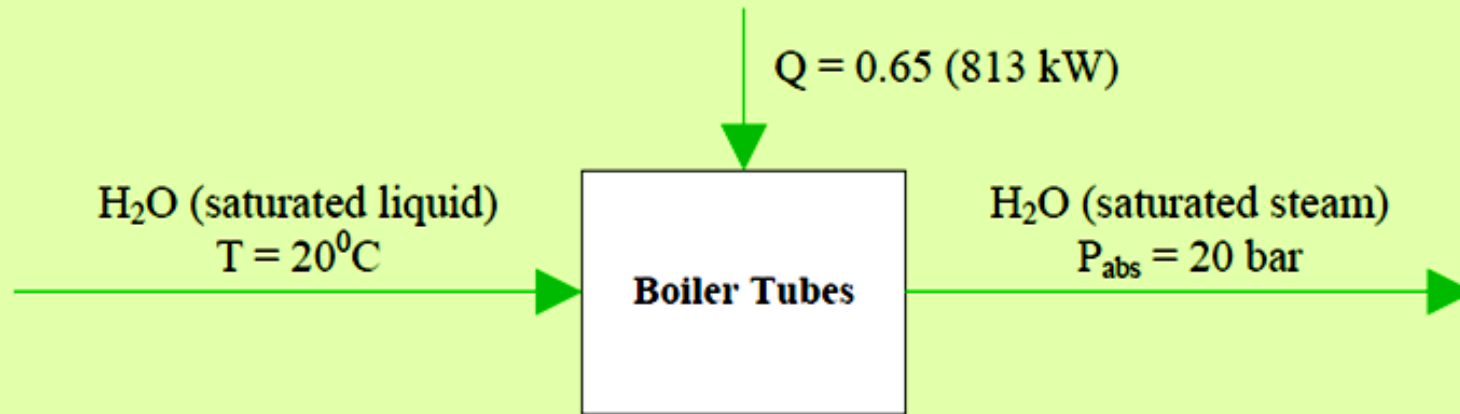
## Example. Heating of Water in Boiler Tubes

A fuel oil is burned with air in a boiler furnace. The combustion produces 813 kW of heat of which 65% is transferred as heat to boiler that pass through the furnace. Water enters the boiler tubes as a saturated liquid at  $20^{\circ}\text{C}$  and leaves the tubes as saturated steam at 20 bar absolute.

Calculate the mass flow rate (in kg/h) and volumetric flow rate (in  $\text{m}^3/\text{h}$ ) at which the saturated steam is produced.



## Example. Heating of Water in Boiler Tubes



Using the energy balance for an open system,

$$\Sigma(K + P + H)_2 - \Sigma(K + P + H)_1 = Q - W_s$$

Assuming  $\Delta K = \Delta P = W = 0$ , then

$$\Sigma H_2 - \Sigma H_1 = Q = 0.65 \text{ (813 kW)} = 528 \text{ kW}$$



## Example. Heating of Water in Boiler Tubes



In terms of specific enthalpy,  $\hat{H}$  (kJ/kg)

$$m_2\hat{H}_2 - m_1\hat{H}_1 = 528 \text{ kW}$$

where  $m$  = mass flow rate of water

If the process is under steady-state condition, then  $m_1 = m_2 = m$

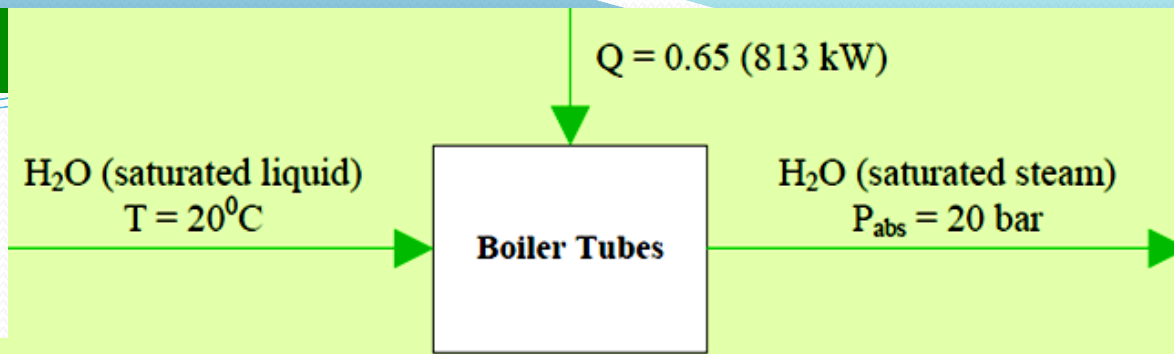
Hence,

$$m(\hat{H}_2 - \hat{H}_1) = 528 \text{ kW}$$

Solving for  $m$ :

$$m = \frac{528 \text{ kW}}{\hat{H}_2 - \hat{H}_1}$$





From saturated steam table,

$$\hat{H}_1 = 83.9 \text{ kJ/kg} \text{ and } \hat{H}_2 = 2797.2 \text{ kJ/kg}$$

And the mass flow rate is:

$$m = \frac{528 \text{ kJ/s}}{(2797.2 - 83.9) \text{ kJ/kg}} \left( \frac{3600 \text{ s}}{1 \text{ h}} \right) = 701 \text{ kg/h}$$

Calculating for the volume flow rate:

$$V = m\hat{V} = 701 \frac{\text{kg}}{\text{h}} \left( 0.0995 \frac{\text{m}^3}{\text{kg}} \right) = 69.7 \frac{\text{m}^3}{\text{h}}$$

↑  
from steam table

## Example . Compression of an Ideal Gas in a Cylinder



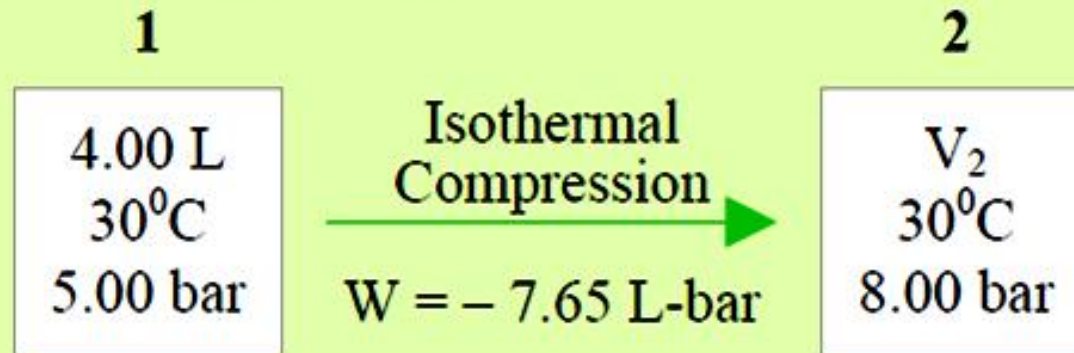
A cylinder with a movable piston contains 4.00 liters of a gas at  $30^{\circ}\text{C}$  and 5.00 bar. The piston is slowly moved to compress the gas to 8.00 bar.

- (a) If the compression is carried out isothermally, and the work done on the gas equals 7.65 L-bar, how much heat (in joules) is transferred to or from (state which) the surroundings.
- (b) Suppose instead that the process is adiabatic, what will happen to the temperature of the gas.

## Example . Compression of an Ideal Gas in a Cylinder



### (a) Isothermal Compression



This is a closed system and the energy balance is

$$\Delta K + \Delta P + \Delta U = Q - W$$

Since the system is stationary,  $\Delta K = \Delta P = 0$

Since the process is isothermal,  $\Delta T = 0$  and  $\Delta U = 0$

## Example . Compression of an Ideal Gas in a Cylinder



The energy balance is simplified to

$$0 = Q - W$$

Solving for Q:

$$Q = W = -7.65 \text{ L} \cdot \text{bar} \left( \frac{8.314 \text{ J}}{0.08314 \text{ L} \cdot \text{bar}} \right) = -765 \text{ J}$$

**(b) Adiabatic Compression**

$$\Delta K + \Delta P + \Delta U = Q - W$$

The energy balance reduces to  $\Delta U = -W = -(-765 \text{ J}) = 765 \text{ J}$

Since  $\Delta U$  is positive,  $\Delta T$  must also be positive. Hence, the temperature of the gas will increase.

## Example. Methane Flowing in a Pipe



Methane enters a 3-cm ID pipe at 30°C and 10 bar with an average velocity of 5.00 m/s and emerges at a point 200 m lower than the inlet at 30°C and 9 bar.

Calculate the  $\Delta K$  and  $\Delta P$  assuming the methane behaves as an ideal gas.

---

### **Solution:**

Mass flow must be the same at the inlet to attain steady-state condition.

$$\Delta K = m/2(v_2^2 - v_1^2) \text{ and } \Delta P = mg(h_2 - h_1)$$

## Example. Methane Flowing in a Pipe



Methane enters a 3-cm ID pipe at 30°C and 10 bar with an average velocity of 5.00 m/s and emerges at a point 200 m lower than the inlet at 30°C and 9 bar.

Determine the mass flow:

**Volumetric flow at the inlet =  $v_1A$**

If methane behaves as an ideal gas:

$$V = v_1A = \frac{mRT}{(MW)(P)}$$

Solving for mass flow:

$$m = \frac{(v_1A)(MW)(P_1)}{RT_1} = 0.0225 \text{ kg/s}$$

## Example. Methane Flowing in a Pipe



Methane enters a 3-cm ID pipe at 30°C and 10 bar with an average velocity of 5.00 m/s and emerges at a point 200 m lower than the inlet at 30°C and 9 bar.

Solving for  $\Delta P$ :

$$\Delta P = mg(h_2 - h_1) = \left(0.0225 \frac{\text{kg}}{\text{s}}\right) \left(9.81 \frac{\text{m}}{\text{s}^2}\right) (-200 \text{ m})$$

$$\Delta P = -44.1 \frac{\text{J}}{\text{s}} = -44 \text{ W}$$

Determine  $v_2$ :

$$P_1 V_1 = nRT_1$$

$$P_2 V_2 = nRT_2$$

$$T_1 = T_2$$

$$P_1 V_1 = P_2 V_2$$

$$P_1(v_1 A) = P_2(v_2 A)$$

## Example. Methane Flowing in a Pipe



Methane enters a 3-cm ID pipe at 30°C and 10 bar with an average velocity of 5.00 m/s and emerges at a point 200 m lower than the inlet at 30°C and 9 bar.

Solving for  $v_2$ :

$$v_2 = v_1 \left( \frac{P_1}{P_2} \right) = 5.00 \frac{\text{m}}{\text{s}} \left( \frac{10 \text{ bar}}{9 \text{ bar}} \right) = 5.555 \text{ m/s}$$

Solving for  $\Delta K$ :

$$\Delta K = \frac{m}{2} (v_2^2 - v_1^2) = \frac{1}{2} \left( 0.0225 \frac{\text{kg}}{\text{s}} \right) \left( 5.555^2 - 5.00^2 \frac{\text{m}^2}{\text{s}^2} \right)$$

$$\Delta K = 0.0659 \frac{\text{J}}{\text{s}} = 0.0659 \text{ W}$$



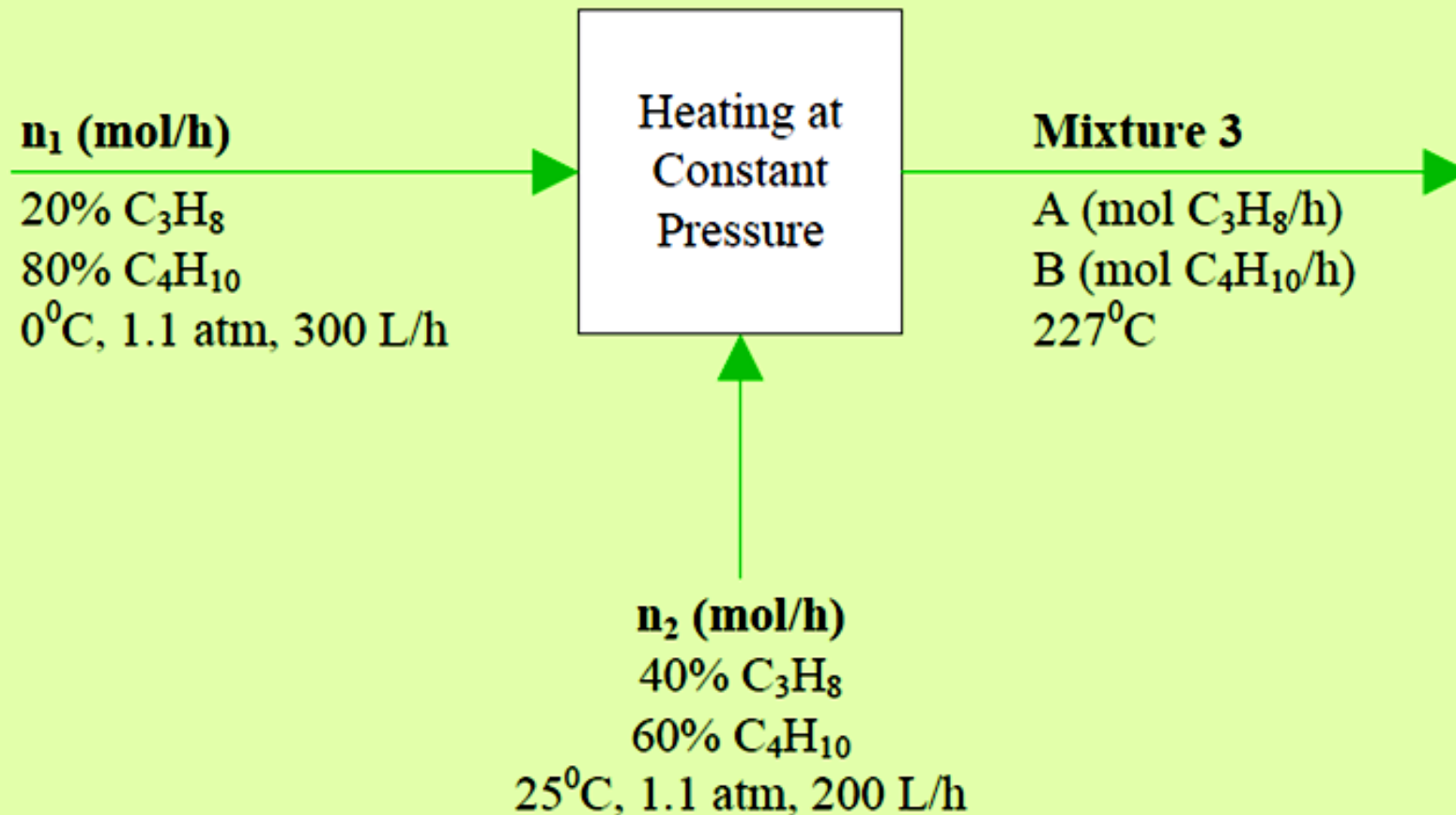


## Example . Mixing and Heating of Propane-Butane Mixtures

Three hundred L/h of 20 mole%  $C_3H_8$ -80% n- $C_4H_{10}$  gas mixture at  $0^\circ C$  and 1.1 atm and 200 L/h of a 40 mole%  $C_3H_8$ -60% n- $C_4H_{10}$  gas mixture at  $25^\circ C$  and 1.1 atm are mixed and heated to  $227^\circ C$  at constant pressure. Calculate the heat requirement of the process. Enthalpies of propane and n-butane are listed below. Assume ideal gas behaviour.

T ( $^\circ C$ )	Propane $\hat{H}$ (J/mol)	Butane $\hat{H}$ (J/mol)
0	0	0
25	1772	2394
227	20,685	27,442

## Example . Mixing and Heating of Propane-Butane Mixtures



# Example . Mixing and Heating of Propane-Butane Mixtures



Simplified energy balance for open system:

$$Q = \Sigma H_{out} - \Sigma H_{in}$$

$$Q = (H_{P3} + H_{B3}) - (H_{P2} + H_{B2} + H_{P1} + H_{B1})$$

Since mixture 1 is at 0°C, then

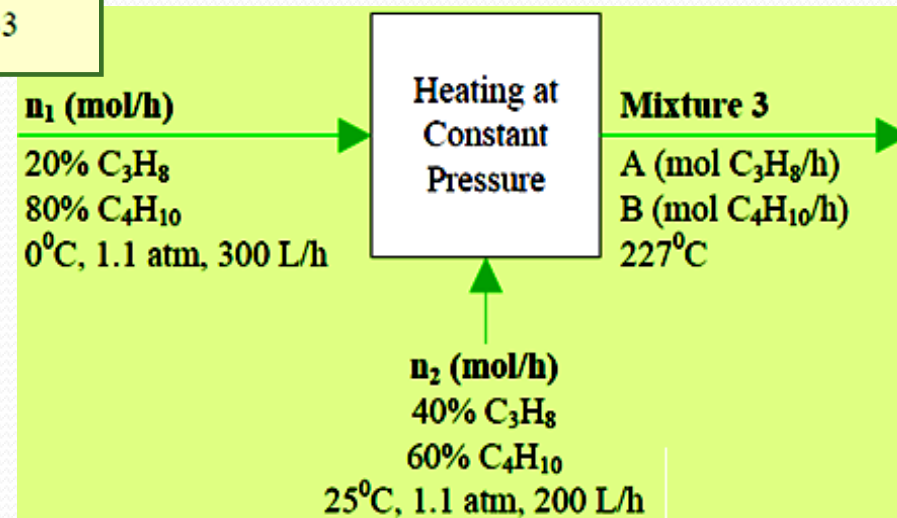
$$Q = (H_{P3} + H_{B3}) - (H_{P2} + H_{B2})$$

The total enthalpy of each component is determined as:

$$H_{P3} = A\hat{H}_{P3} \quad H_{B3} = B\hat{H}_{B3}$$

$$H_{P2} = 0.40n_2\hat{H}_{P2} \quad H_{B2} = 0.60n_2\hat{H}_{B2}$$

T (°C)	Propane $\hat{H}$ (J/mol)	Butane $\hat{H}$ (J/mol)
0	0	0
25	1772	2394
227	20,685	27,442



## Example . Mixing and Heating of Propane-Butane Mixtures



**Find  $n_1$ ,  $n_2$ , A, and B**

$n_1$  and  $n_2$  can be obtained from  $V_1$  and  $V_2$  using the ideal gas equation:

$$n_1 = \frac{P_1 V_1}{RT_1} = \frac{1.1 \text{ atm} \times 300 \text{ L/h}}{0.08205 \text{ L atm/mol K} \times 273 \text{ K}} = 14.7 \text{ mol/h}$$

$$n_1 = 14.7 \text{ mol/h} ; n_2 = 9.00 \text{ mol/h}$$

A and B can be obtained using material balances for propane and butane:

Propane:  $A = 0.20n_1 + 0.40n_2 = 6.54 \text{ mol C}_3\text{H}_8/\text{h}$

Butane:  $B = 0.80n_1 + 0.60n_2 = 17.16 \text{ mol C}_4\text{H}_{10}/\text{h}$



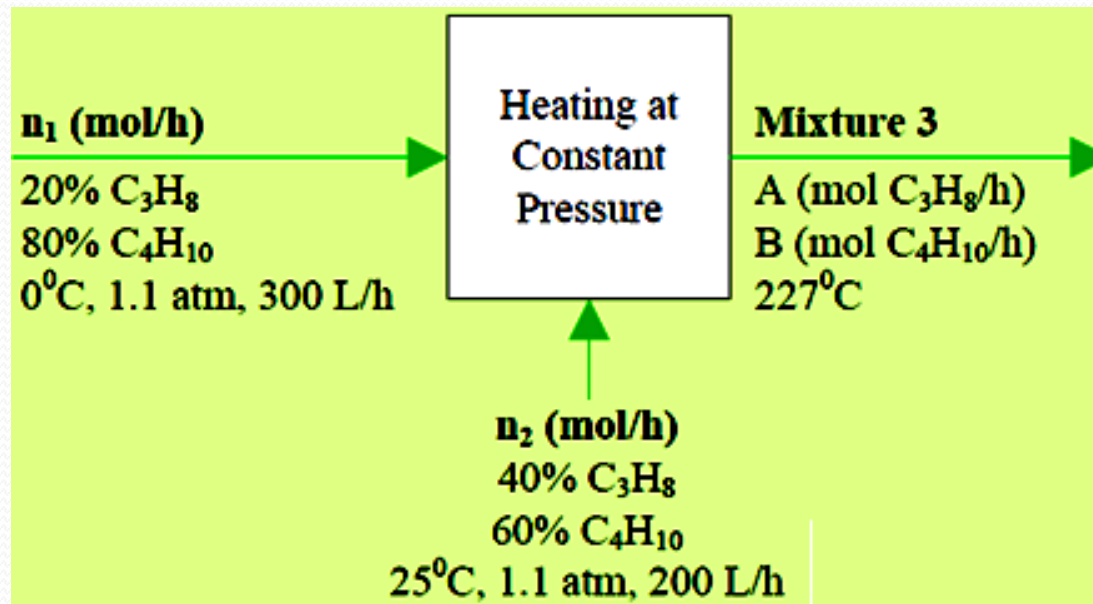
## Example . Mixing and Heating of Propane-Butane Mixtures

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## Example . Mixing and Heating of Propane-Butane Mixtures

Solving for the total enthalpies of the components:

$$\begin{aligned}
 H_{P3} &= A\hat{H}_{P3} & H_{B3} &= B\hat{H}_{B3} \\
 H_{P2} &= 0.40n_2\hat{H}_{P2} & H_{B2} &= 0.60n_2\hat{H}_{B3} \\
 H_{B3} &= (17.16 \text{ mol/h})(27.442 \text{ kJ/mol}) = 470.9 \text{ kJ/h}
 \end{aligned}$$

$$\begin{aligned}
 H_{P3} &= (6.54 \text{ mol/h})(20.865 \text{ kJ/mol}) = 136.5 \text{ kJ/h} \\
 H_{B3} &= (17.16 \text{ mol/h})(27.442 \text{ kJ/mol}) = 470.9 \text{ kJ/h} \\
 H_{P2} &= 0.40(9.00 \text{ mol/h})(1.772 \text{ kJ/mol}) = 6.38 \text{ kJ/h} \\
 H_{B2} &= 0.60(9.00 \text{ mol/h})(2.394 \text{ kJ/mol}) = 12.93 \text{ kJ/h}
 \end{aligned}$$

T (°C)	Propane $\hat{H}$ (J/mol)	Butane $\hat{H}$ (J/mol)
0	0	0
25	1772	2394
227	20,685	27,442

$$n_1 = 14.7 \text{ mol/h} ; n_2 = 9.00 \text{ mol/h}$$

$$\text{Propane: } A = 6.54 \text{ mol C}_3\text{H}_8/\text{h}$$

$$\text{Butane: } B = 17.16 \text{ mol C}_4\text{H}_{10}/\text{h}$$

$$Q = 5 \text{ kJ/h}$$

20% C<sub>3</sub>H<sub>8</sub>  
80% C<sub>4</sub>H<sub>10</sub>  
0°C, 1.1 atm, 300 L/h

Heating at  
Constant  
Pressure

**Mixture 3**

A (mol C<sub>3</sub>H<sub>8</sub>/h)  
B (mol C<sub>4</sub>H<sub>10</sub>/h)  
227°C

$n_2$  (mol/h)  
40% C<sub>3</sub>H<sub>8</sub>  
60% C<sub>4</sub>H<sub>10</sub>  
25°C, 1.1 atm, 200 L/h



## Example. Mixing and Heating of Propane-Butane Mixtures

Solving for the total enthalpies of the components:

$$H_{P3} = (6.54 \text{ mol/h})(20.865 \text{ kJ/mol}) = 136.5 \text{ kJ/h}$$

$$H_{B3} = (17.16 \text{ mol/h})(27.442 \text{ kJ/mol}) = 470.9 \text{ kJ/h}$$

$$H_{P2} = 0.40(9.00 \text{ mol/h})(1.772 \text{ kJ/mol}) = 6.38 \text{ kJ/h}$$

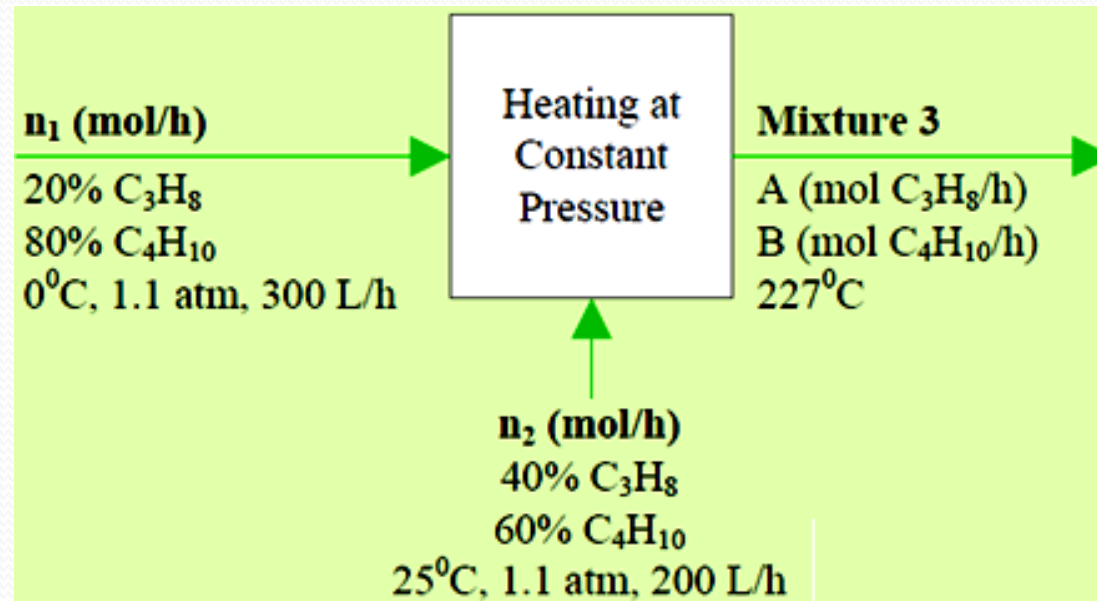
$$H_{B2} = 0.60(9.00 \text{ mol/h})(2.394 \text{ kJ/mol}) = 12.93 \text{ kJ/h}$$

T (°C)	Propane $\hat{H}$ (J/mol)	Butane $\hat{H}$ (J/mol)
0	0	0
25	1772	2394
227	20,685	27,442

$$n_1 = 14.7 \text{ mol/h} ; n_2 = 9.00 \text{ mol/h}$$

Propane: A = 6.54 mol C<sub>3</sub>H<sub>8</sub>/h

Butane: B = 17.16 mol C<sub>4</sub>H<sub>10</sub>/h



## Example . Mixing and Heating of Propane-Butane Mixtures



Solving for the total enthalpies of the components:

$$H_{P3} = (6.54 \text{ mol/h})(20.865 \text{ kJ/mol}) = 136.5 \text{ kJ/h}$$

$$H_{B3} = (17.16 \text{ mol/h})(27.442 \text{ kJ/mol}) = 470.9 \text{ kJ/h}$$

$$H_{P2} = 0.40(9.00 \text{ mol/h})(1.772 \text{ kJ/mol}) = 6.38 \text{ kJ/h}$$

$$H_{B2} = 0.60(9.00 \text{ mol/h})(2.394 \text{ kJ/mol}) = 12.93 \text{ kJ/h}$$

Solving for the heat requirement of the process:

$$Q = (H_{P3} + H_{B3}) - (H_{P2} + H_{B2})$$

$$Q = (136.5 + 470.9 - 6.38 - 12.93) \text{ kJ/h}$$

$$Q = \mathbf{587 \text{ kJ/h}}$$