Chapter 4

- Series Decay and Equilibrium
- Neutron Activation

SERIES DECAY AND EQUILIBRIUM

Objectives

- Explain the concept of radioactive series decay
- Calculate the ingrowth of activity of a radioactive decay product from a parent radionuclide, given elapsed time and initial amount of parent
- Define the terms <u>secular</u> equilibrium and <u>transient</u> equilibrium

Example of Serial Decay

$^{90}Sr \rightarrow {}^{90}Y \rightarrow {}^{90}Zr$

^{90}Y: $T_{1/2} = 64\ h$

^{90}Sr: $T_{1/2} = 28\ y$

Note: Also see MISC-31

Serial Decay Equation

Rate of Change of Daughter =
(Production of Daughter) – (Decay of Daughter)

$$\frac{dN_D}{dt} = \lambda_P N_P - \lambda_D N_D$$

Assumes the branching ratio is 1

Serial Decay Equation

$$N_D(t) = \frac{\lambda_P N_P^0}{\lambda_D - \lambda_P} \left(e^{-\lambda_P t} - e^{-\lambda_D t}\right)$$

Recall that $\lambda_P N^0_P = A^0_P$ = initial activity of Parent at time t = 0.

The General Equation for Radioactive Series Decay

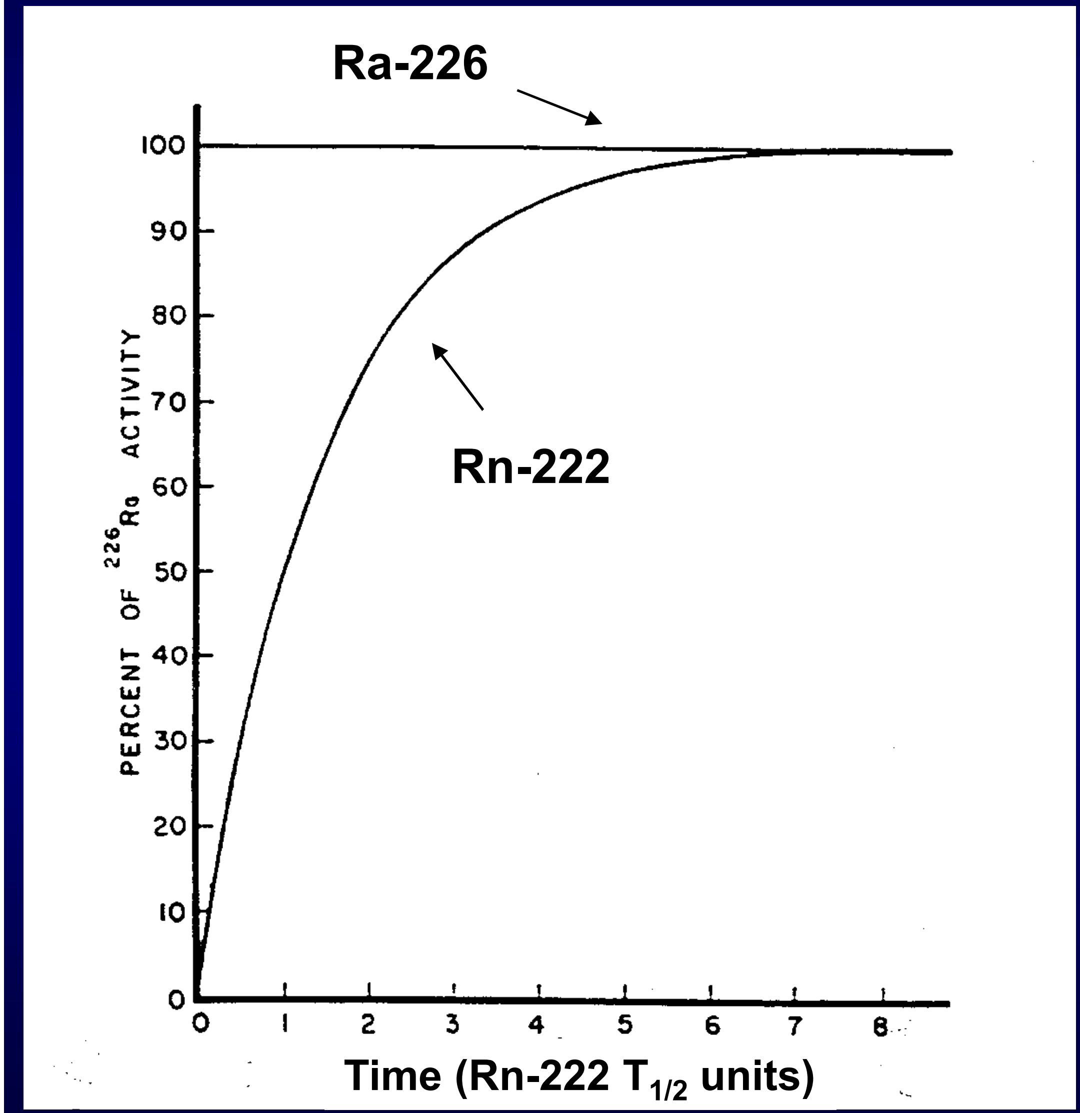
$$\lambda_D N_D(t) = \frac{\lambda_D \lambda_P N_P^0}{\lambda_D - \lambda_P} \left(e^{-\lambda_P t} - e^{-\lambda_D t}\right)$$

Activity of Daughter at time t

Types of Radioactive Equilibrium

Secular	Half-life of parent much greater than (> 100 times) that of decay product
Transient	Half-life of parent only greater than that of decay product (> 10 times)
No Equilibrium	Half-life of parent less than that of decay product

Secular Equilibrium



$T_{1/2}$ = 3.8 d

$$A_{Rn}(t) = A^0_{Ra}(1 - e^{-\lambda t})$$

$T_{1/2}$ = 1,600 y

Buildup of a Decay Product under Secular Equilibrium Conditions

$$A_D(t) = A_P^0 \left(1 - e^{-\lambda_D t}\right)$$

Behavior of the Secular Equilibrium Equation

Time t (in half-lives of daughter, D)	A_P (dps) (constant)	$(1 - e^{-\lambda_D t})$	A_D (t) (dps)
0	1000	0	0
1	1000	0.5	500
2	1000	0.75	750
3	1000	0.875	875
4	1000	0.937	937
5	1000	0.969	969
6	1000	0.984	984
7	1000	0.992	992
8	1000	0.996	996

Secular Equilibrium

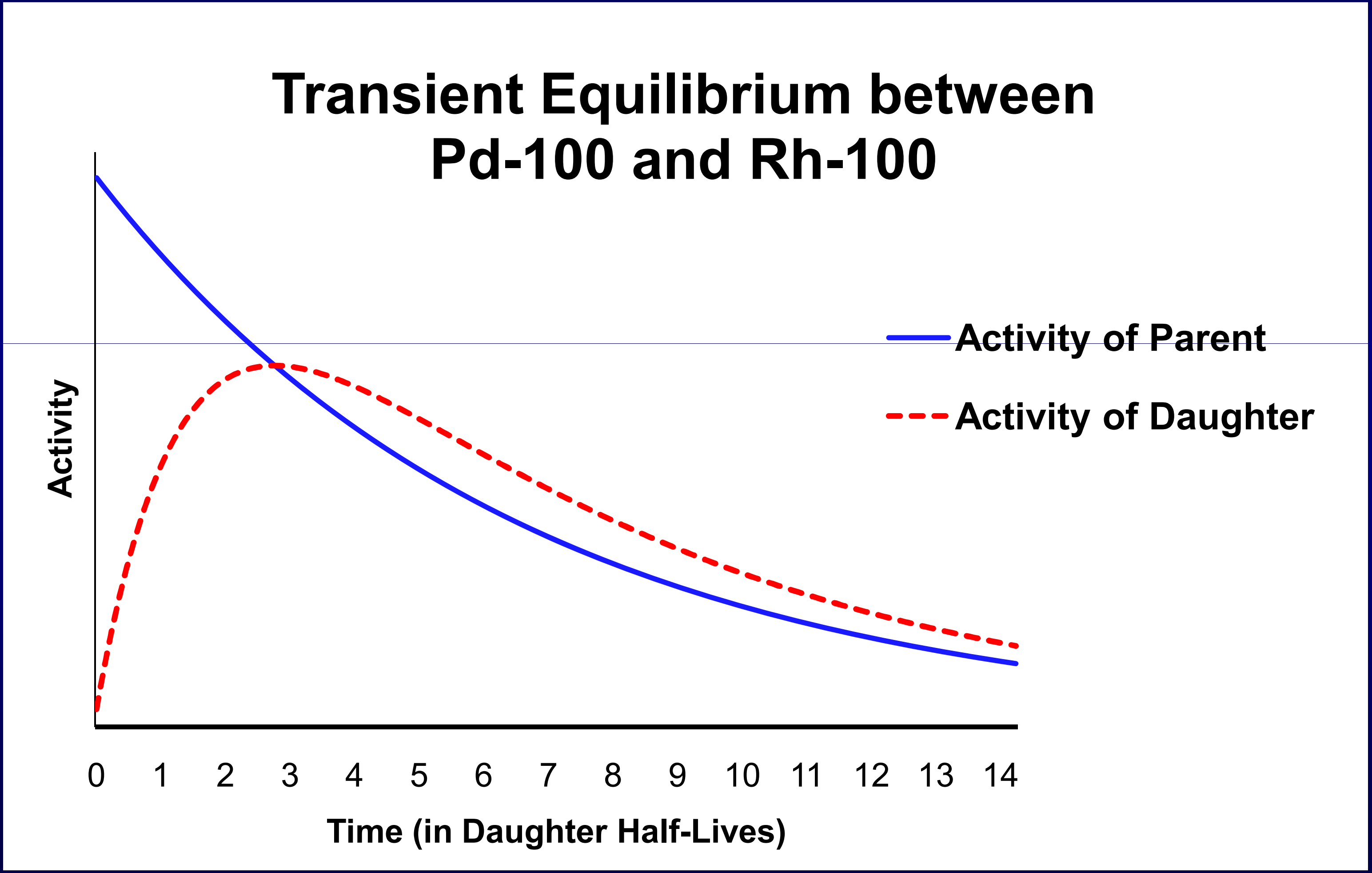
$$\lambda_P N_P = \lambda_D N_D$$

$$A_P = A_D$$

Note: Total activity in the sample =

$$A_T = A_P + A_D = 2\,A_P$$

Transient Equilibrium



Equation for Transient Equilibrium

$$\lambda_D N_D = \frac{\lambda_D \lambda_P N_P}{\lambda_D - \lambda_P}$$

Note: This equation is only valid once transient equilibrium is reached

Transient Equilibrium

$$A_D = \frac{A_P \lambda_D}{\lambda_D - \lambda_P}$$

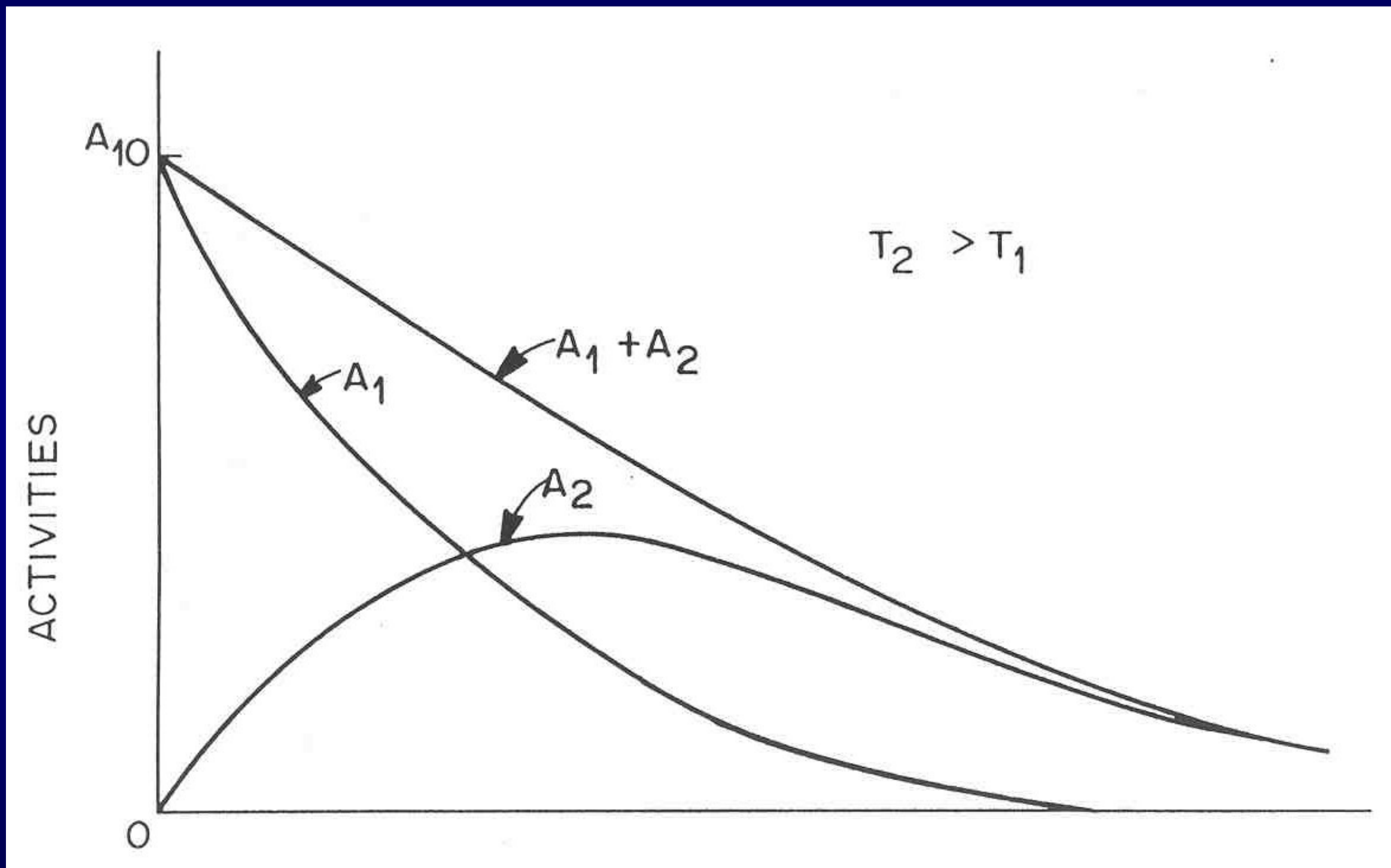
Note: This equation is only valid once transient equilibrium is reached

Transient Equilibrium

Time for decay product (D) to reach maximum activity

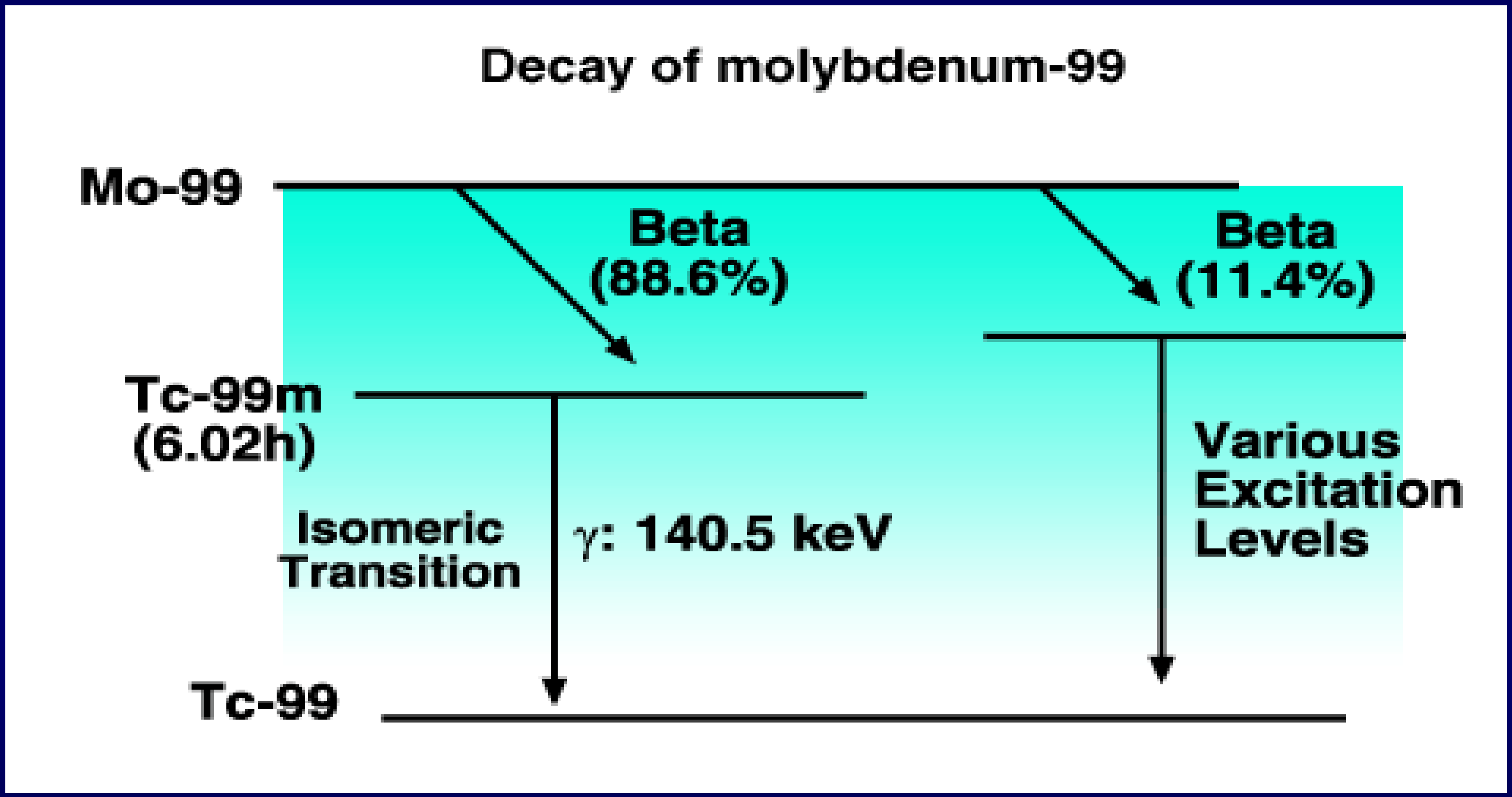
$$t_{mD} = \frac{\ln\left(\frac{\lambda_D}{\lambda_P}\right)}{\lambda_D - \lambda_P}$$

No Equilibrium



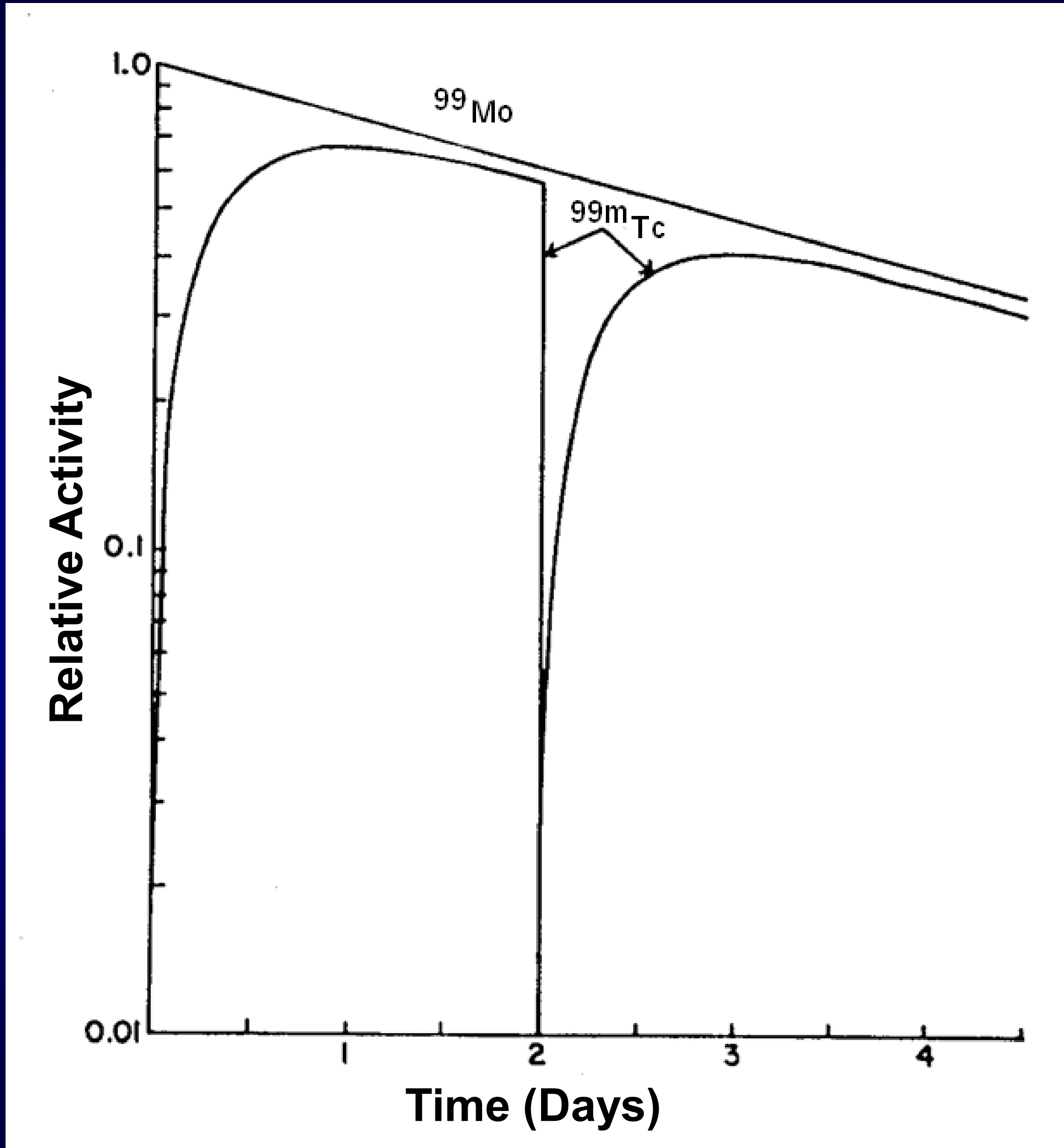
No Equilibrium (e.g. Ce-146 → Pr-146)

Moly-Tech Production



Tc-99 $T_{1/2}$ = 212,000 yrs

Moly-Tech Production



Problem

A laboratory chemically separates out 50 mCi of pure Sr-90 to use for research purposes. The Sr-90 is then stored in a vial for 48 hours prior to being used. What is the activity of Y-90 in the sample vial at that time?

END OF SERIES DECAY AND EQUILIBRIUM

Parent $T_{1/2}$ = 60 min — Daughter $T_{1/2}$ = 1 min

Time (min)	Residual Parent atoms	Parent atoms transformed in 1 min	Daughter atoms received from parent	Total Daughter atoms (remaining after 1 min + new ones from parent)
0	1000	0	0	0
1	990	10	10	0 + 10 = 10
2	980	10	20	5 + 10 = 15
3	970	10	30	7.5 + 10 = 17.5
4	960	10	40	8.7 + 10 = 18.7
5	950	10	50	9.3 + 10 = 19.3
6	940	10	60	9.7 + 10 = 19.7
7	930	10	70	9.8 + 10 = 19.8
8	920	10	80	9.9 + 10 = 19.9
9	910	10	90	9.9 + 10 = 19.9
10	900	10	100	10 + 10 = 20*
11	890	10	110	10 + 10 = 20
⇓	⇓	⇓	⇓	⇓
60	500	10	500	10 + 10 = 20

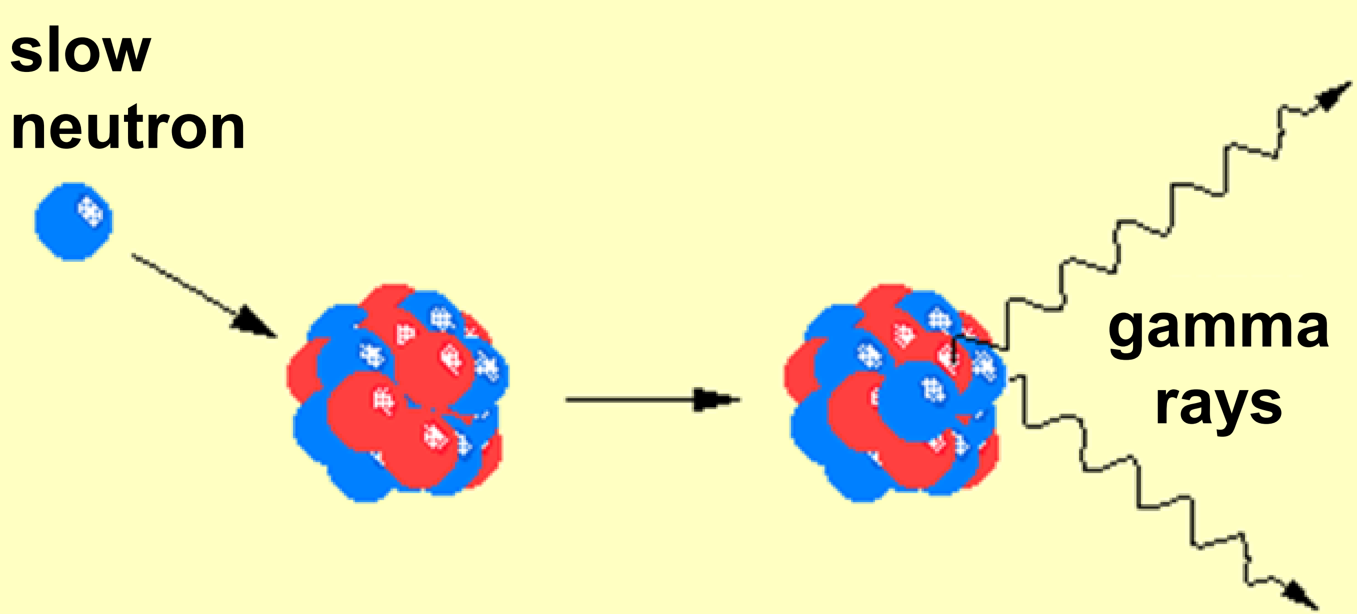
* at this point both the daughter and parent are decaying at the same rate (10 atoms per minute) although the daughter only has 20 total atoms while the parent has hundreds. This will continue until the parent and the daughter both decay to 0 together.

NEUTRON ACTIVATION

Objectives

- Explain neutron activation and discuss real world applications of the phenomenon
- Explain each term given the activation equation and give the equation for maximum (saturation) activity
- Calculate the amount of radioactivity produced in a specified time period given the activation equation and the maximum activity that could be produced

Neutron Capture



$$n + {}^{23}Na \rightarrow {}^{24}Na + \gamma$$

Neutron capture in sodium-23:

$^{23}Na(n,\gamma)^{24}Na$ Reaction

Applications of Neutron Activation

- Production of medical and industrial isotopes (e.g. Co-60, Ir-192, Mo-99, etc.)
- Accident dosimetry (e.g. Na-24 in blood)
- Forensic medicine (Napoleon's hair)
- Activation analysis to measure trace elements
- Activated cobalt (Co-60) in reactor coolant system components is the primary source of radiation exposure to plant workers. Activation products at reactors can also increase public exposure (e.g. N-16 in steam in BWRs)

Applications of Neutron Activation

Determination of fast neutron radiation component at Hiroshima:

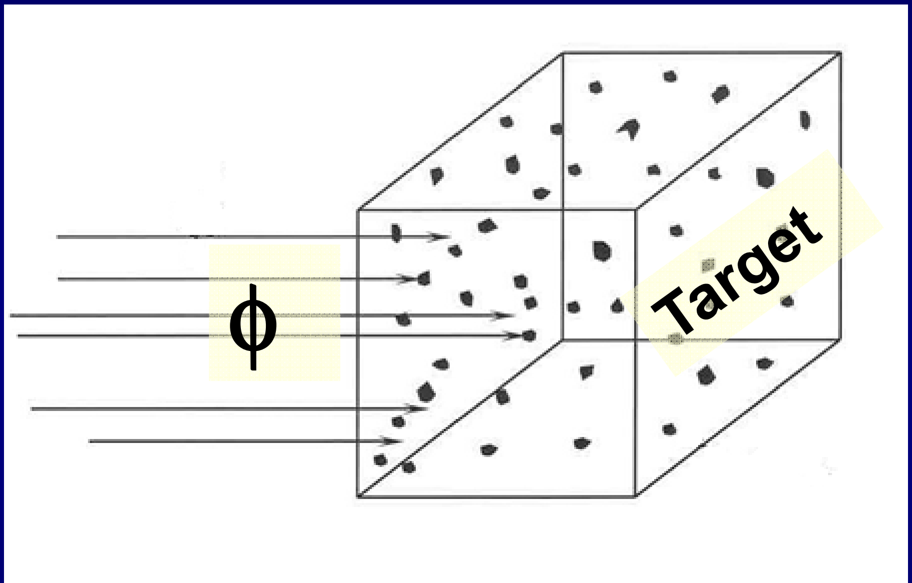
- **Accelerator mass spectrometry of ^{63}Ni (half-life = 100 years) produced by fast neutron activation of copper in building materials**

- **Reaction is $^{63}Cu(n, p)^{63}Ni$**

Terminology

- **Cross section** - probability of neutron interacting with a target atom, denoted by σ, given in units of barns (1 barn = 10^{-24} cm^2)
- **Target** - number of atoms being irradiated by neutrons, denoted by n_0
- **Neutron flux ϕ** - neutrons incident on a unit area per unit time (e.g. n cm^{-2} s^{-1})

Activation Rate



Activation Rate = $\phi \sigma n_0$

ϕ = neutrons/cm^2 · s

σ = cm^2/neutron · atom

n_0 = number of target atoms

Assumptions:

ϕ remains constant

n_0 remains constant

Activation Equation

$$\frac{dN}{dt} = \phi\sigma n_0 - \lambda N$$

Activation Equation

$$N(t) = \phi\sigma n_0 \frac{(1 - e^{-\lambda t})}{\lambda}$$

Activation Equation

$$\lambda N(t) = A(t) = \phi\sigma n_0 (1 - e^{-\lambda t})$$

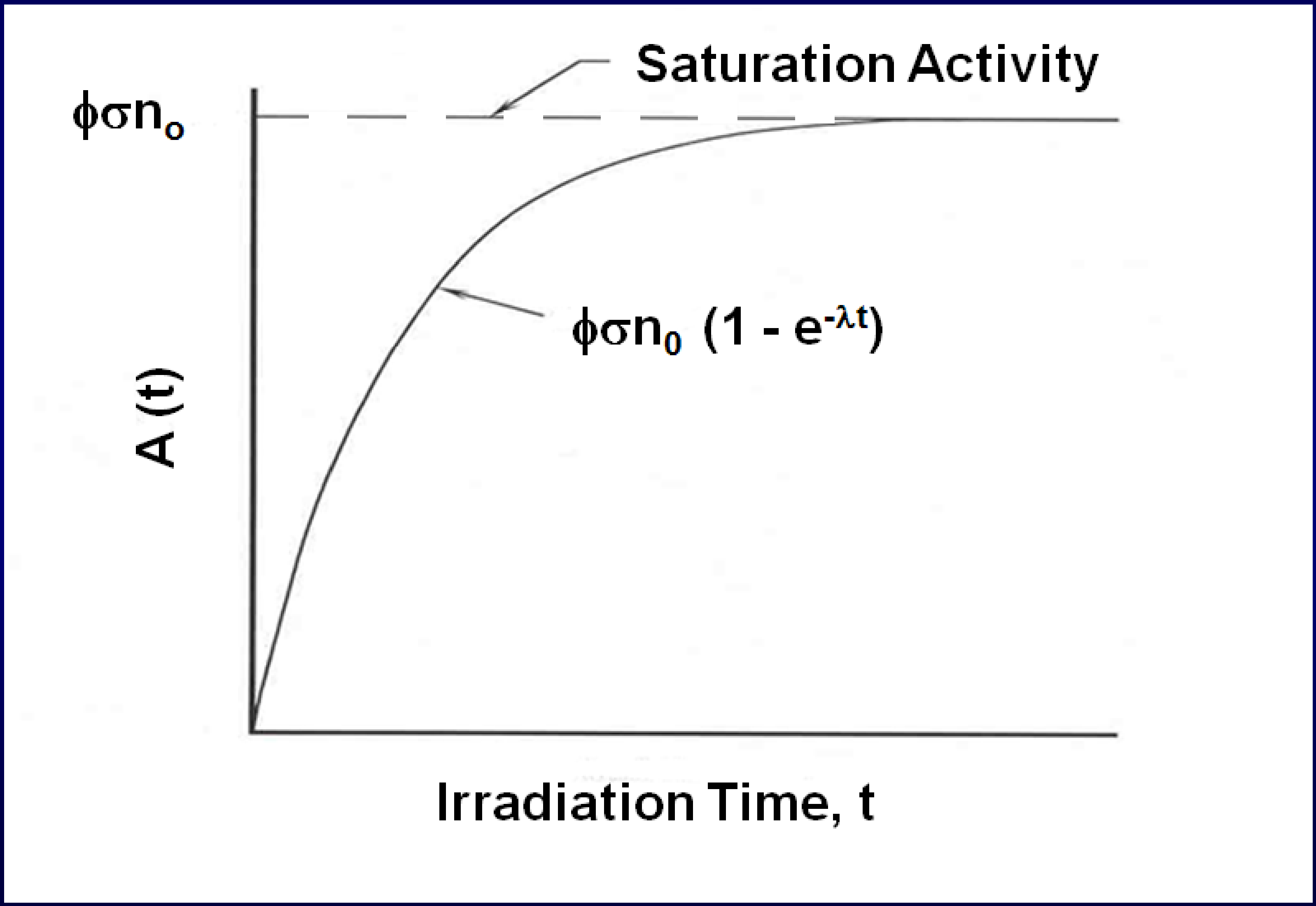
Buildup to Maximum (i.e. Saturation) Activity as a Function of Neutron Bombardment Time

Number of Elapsed Half-Lives of Activation Product	$(1 - e^{-\lambda t})$
1	0.50
2	0.75
3	0.87
4	0.94
5	0.97
6	0.98
7	0.99

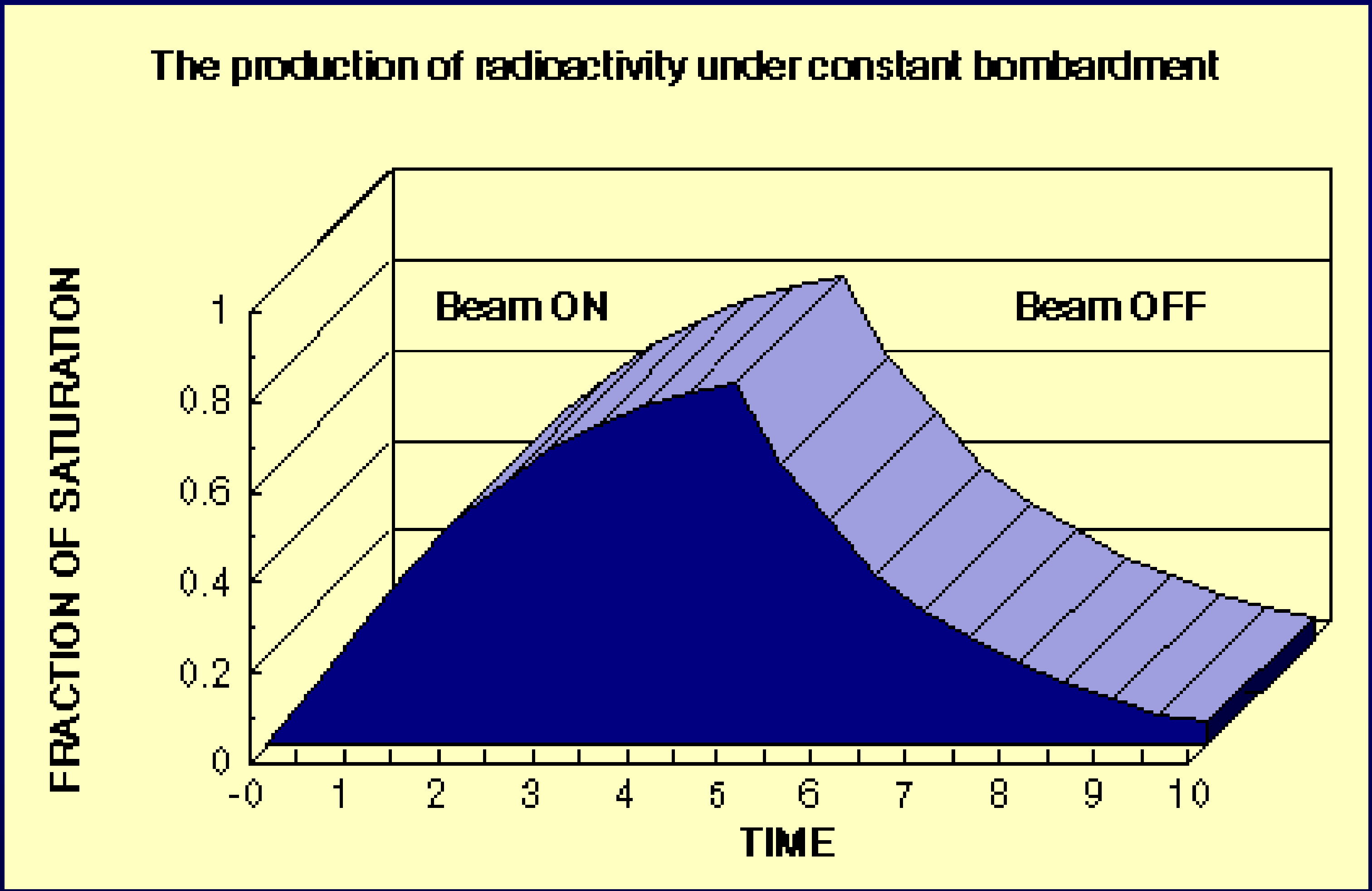
Saturation Activity

$$A(\infty) = \phi \sigma n_0$$

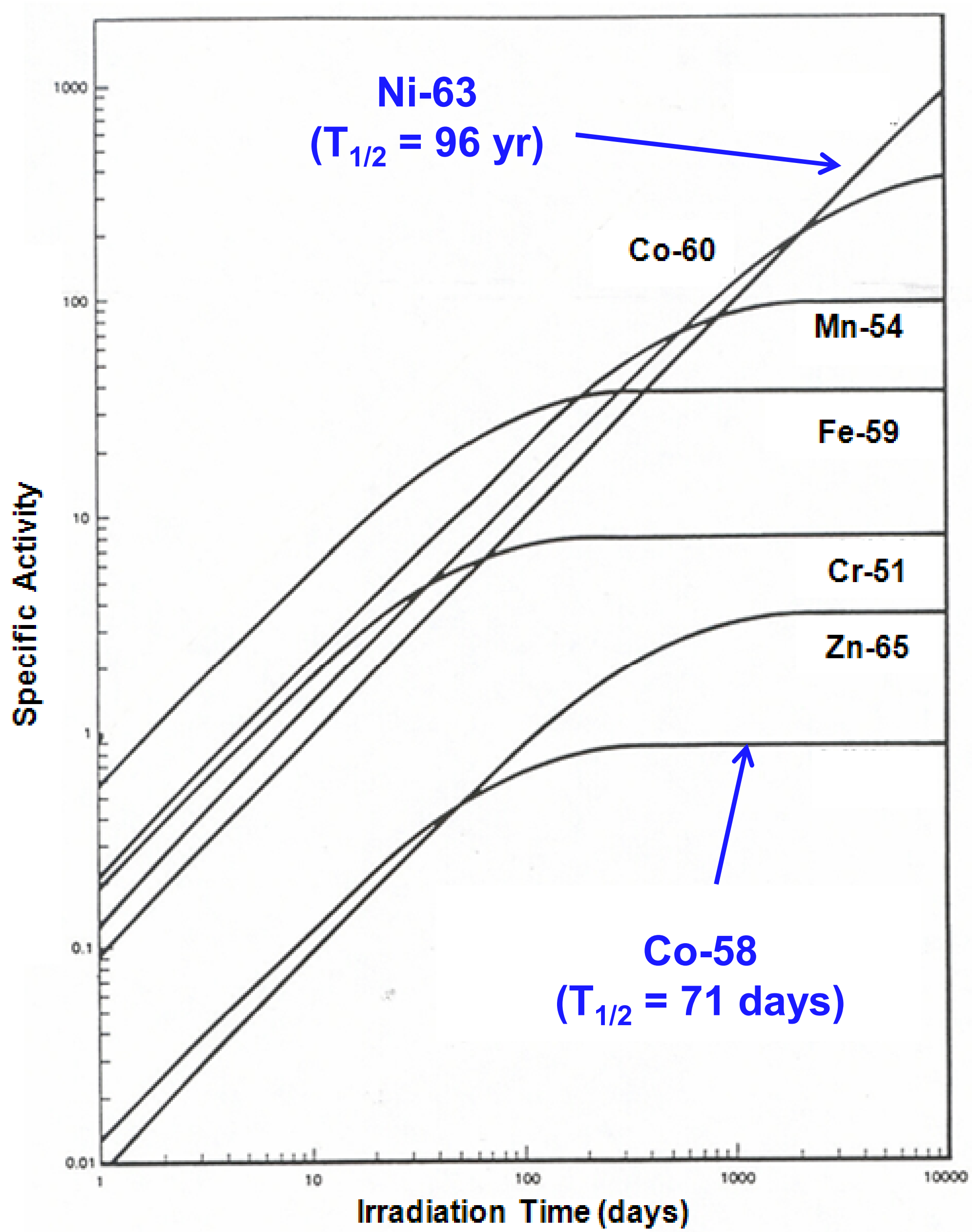
Buildup of Activation Products in a Target Material



Simultaneous Production and Decay



Saturation



Specific Activity of Major Corrosion Products as a Function of Irradiation Time

- Saturation activity is a function of neutron flux, cross section, and number of target atoms
- Time to reach saturation activity is a function of product half-life

END OF NEUTRON ACTIVATION

END OF CHAPTER 4