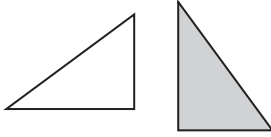## Chapter 4

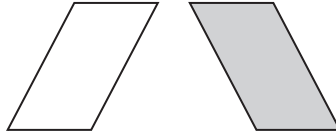# Maintaining Mathematical Proficiency

Tell whether the shaded figure is a translation, reflection, rotation, or dilation of the nonshaded figure.
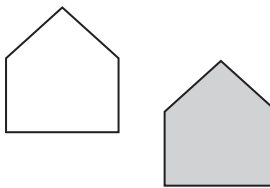
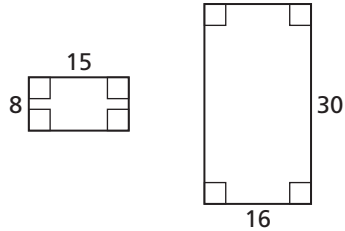**1.**
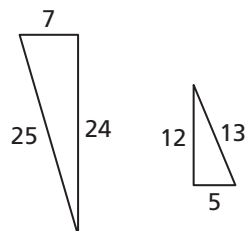
2.

3.

4.

Tell whether the two figures are similar. Explain your reasoning.

5.

15

8

30

16

6.

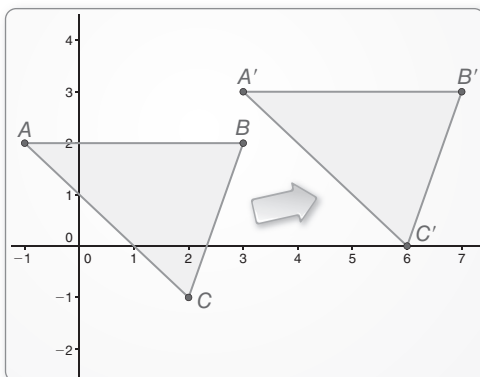7

25

24

12

13

5

4.1 Translations
For use with Exploration 4.1

Essential Question How can you translate a figure in a coordinate plane?

---

1 **EXPLORATION: Translating a Triangle in a Coordinate Plane**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

    a. Use dynamic geometry software to draw any triangle and label it $\triangle ABC$.

    b. Copy the triangle and *translate* (or slide) it to form a new figure, called an *image*, $\triangle A'B'C'$. (read as "triangle A prime, B prime, C prime").

    c. What is the relationship between the coordinates of the vertices of $\triangle ABC$ and those of $\triangle A'B'C'$?

    d. What do you observe about the side lengths and angle measures of the two triangles?

Sample

---

2 **EXPLORATION: Translating a Triangle in a Coordinate Plane**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.
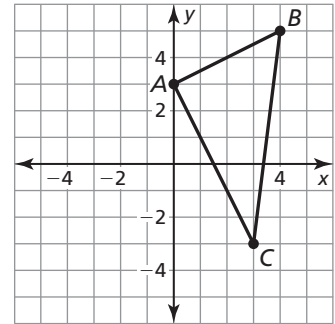
Work with a partner.

    a. The point (x, y) is translated a units horizontally and b units vertically. Write a rule to determine the coordinates of the image of (x, y).

$$(x, y) \rightarrow (\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$$

4.1 **Translations** (continued)

b. Use the rule you wrote in part (a) to translate $\triangle ABC$ 4 units left and 3 units down. What are the coordinates of the vertices of the image, $\triangle A'B'C'$?

c. Draw $\triangle A'B'C'$. Are its side lengths the same as those of $\triangle ABC$? Justify your answer.

3 **EXPLORATION:** Comparing Angles of Translations

Work with a partner.

a. In Exploration 2, is $\triangle ABC$ a right triangle? Justify your answer.

b. In Exploration 2, is $\triangle A'B'C'$ a right triangle? Justify your answer.

c. Do you think translations always preserve angle measures? Explain your reasoning.

Communicate Your Answer

4. How can you translate a figure in a coordinate plane?

5. In Exploration 2, translate $\triangle A'B'C'$ 3 units right and 4 units up. What are the coordinates of the vertices of the image, $\triangle A''B''C''$? How are these coordinates related to the coordinates of the vertices of the original triangle, $\triangle ABC$?

4.1 Notetaking with Vocabulary
For use after Lesson 4.1

In your own words, write the meaning of each vocabulary term.

vector

initial point

terminal point

horizontal component

vertical component

component form

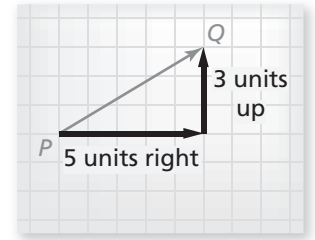transformation

image

preimage

translation

rigid motion

composition of transformations

4.1 **Notetaking with Vocabulary** (continued)

Core Concepts

Vectors

The diagram shows a vector. The **initial point**, or starting point, of the vector is P, and the **terminal point**, or ending point, is Q. The vector is named $\overrightarrow{PQ}$, which is read as "vector PQ." The **horizontal component** of $\overrightarrow{PQ}$ is 5, and the **vertical component** is 3. The **component form** of a vector combines the horizontal and vertical components. So, the component form of $\overrightarrow{PQ}$ is $\langle 5, 3 \rangle$.
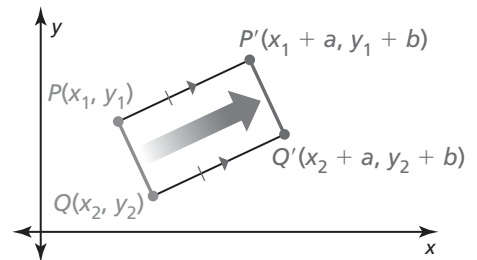
Notes:

Translations

A translation moves every point of a figure the same distance in the same direction. More specifically, a translation *maps*, or moves the points P and Q of a plane figure along a vector $\langle a, b \rangle$ to the points P' and Q', so that one of the following statements is true.
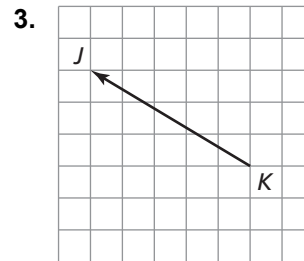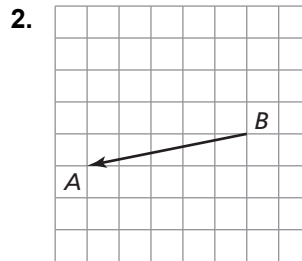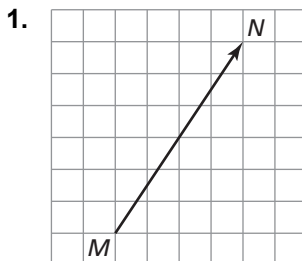
- $PP' = QQ'$ and $\overline{PP'} \parallel \overline{QQ'}$, or

- $PP' = QQ'$ and $\overline{PP'}$ and $\overline{QQ'}$ are collinear.

Notes:

Extra Practice

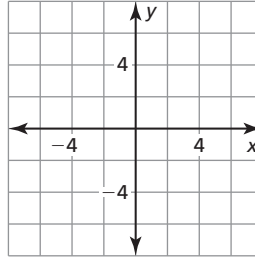**In Exercises 1–3, name the vector and write its component form.**

1.

2.

3.

Name _____ Date _____

In Exercises 4–7, the vertices of △ABC are A(1, 2), B(5, 1), C(5, 4).

Translate △ABC using the given vector. Graph △ABC and its image.

4. $\langle -4, 0 \rangle$

5. $\langle -2, -4 \rangle$

6. $\langle 0, -5 \rangle$
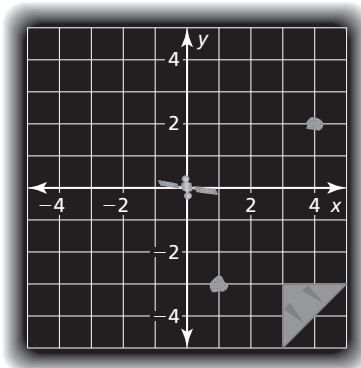
7. $\langle 1, -3 \rangle$

In Exercises 8 and 9, write a rule for the translation of quadrilateral PQRS to quadrilateral P'Q'R'S'.

8.

9.

In Exercises 10 and 11, use the translation.

$(x, y) \rightarrow (x + 6, y - 3)$

10. What is the image of $J(4, 5)$?

11. What is the image of $R'(0, -5)$?

12. In a video game, you move a spaceship 1 unit left and 4 units up. Then, you move the spaceship 2 units left. Rewrite the composition as a single transformation.
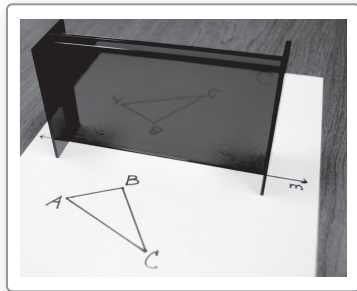
Name_____ Date _____

4.2 Reflections
For use with Exploration 4.2

Essential Question   How can you reflect a figure in a coordinate plane?

Work with a partner. Use a straightedge to draw any triangle on paper. Label it $\triangle ABC$.

a. Use the straightedge to draw a line that does not pass through the triangle. Label it m.

b. Place a reflective device on line m.

c. Use the reflective device to plot the images of the vertices of $\triangle ABC$. Label the images of vertices A, B, and C as A', B', and C', respectively.

d. Use a straightedge to draw $\triangle A'B'C'$ by connecting the vertices.

4.2 **Reflections** (continued)

2 **EXPLORATION:** Reflecting a Triangle in a Coordinate Plane

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw any triangle and label it $\triangle ABC$.

   a. *Reflect* $\triangle ABC$ in the *y*-axis to form $\triangle A'B'C'$.

   b. What is the relationship between the coordinates of the vertices of $\triangle ABC$ and those of $\triangle A'B'C'$?

   c. What do you observe about the side lengths and angle measures of the two triangles?

   d. *Reflect* $\triangle ABC$ in the *x*-axis to form $\triangle A'B'C'$. Then repeat parts (b) and (c).

Communicate Your Answer

   3. How can you reflect a figure in a coordinate plane?

Name_____ Date_____

4.2 Notetaking with Vocabulary
For use after Lesson 4.2

In your own words, write the meaning of each vocabulary term.

reflection

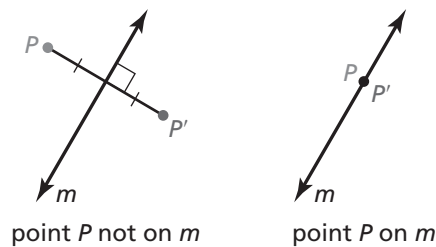line of reflection

glide reflection

line symmetry

line of symmetry

Core Concepts

Reflections

A **reflection** is a transformation that uses a line like a mirror to reflect a figure. The mirror line is called the **line of reflection**.

A reflection in a line m maps every point P in the plane to a point P', so that for each point on of the following properties is true.

point P not on m point P on m

- If P is not m, then m is the perpendicular bisector of $\overline{PP'}$, or

- If P is on m, then $P = P'$.

Notes:

Name _____ Date _____

Core Concepts

Coordinate Rules for Reflections

- If (a, b) is reflected in the *x*-axis, then its image is the point $(a, -b)$.

- If (a, b) is reflected in the *y*-axis, then its image is the point $(-a, b)$.

- If (a, b) is reflected in the line $y = x$, then its image is the point (b, a).

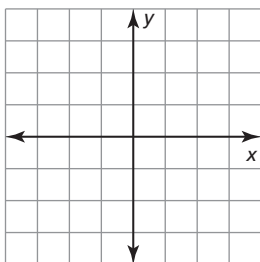- If (a, b) is reflected in the line $y = -x$, then its image is the point $(-b, -a)$.

Notes:

Postulate 4.2 Reflection Postulate
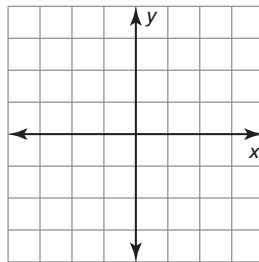
A reflection is a rigid motion.

Extra Practice

In Exercises 1–4, graph △ABC and its image after a reflection in the given line.
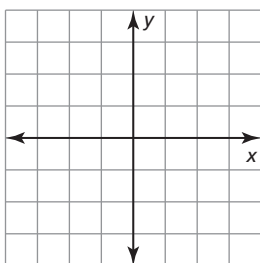
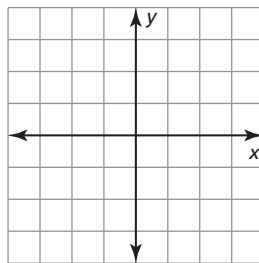**1.** $A(-1, 5)$, $B(-4, 4)$, $C(-3, 1)$; *y*-axis

2. $A(0, 2)$, $B(4, 5)$, $C(5, 2)$; *x*-axis

3. $A(2, -1)$, $B(-4, -2)$, $C(-1, -3)$; $y = 1$
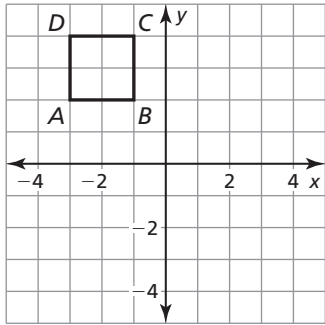

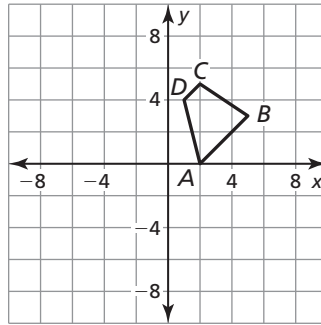**4.** $A(-2, 3)$, $B(-2, -2)$, $C(0, -2)$; $x = -3$

4.2   **Notetaking with Vocabulary** (continued)

In Exercises 5 and 6, graph the polygon's image after a reflection in the given line.
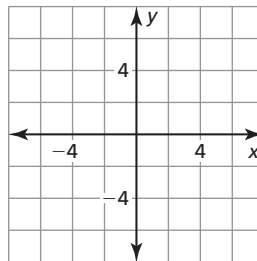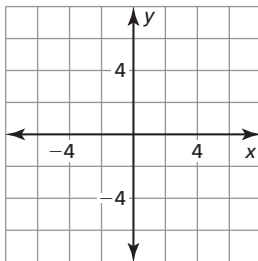
5. $y = x$

6. $y = -x$

In Exercises 7 and 8, graph $\triangle JKL$ **with vertices** $J(3, 1)$, $K(4, 2)$, **and** $L(1, 3)$ **and its image after the glide reflection.**

7. Translation: $(x, y) \rightarrow (x - 6, y - 1)$

Reflection: in the line $y = -x$

8. Translation: $(x, y) \rightarrow (x, y - 4)$

Reflection: in the line $x = 1$

In Exercises 9–12, identify the line symmetry (if any) of the word.
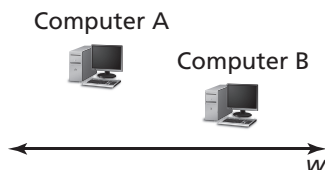
9. MOON

10. WOW

11. KID

12. DOCK

13. You are placing a power strip along wall w that connects to two computers. Where should you place the power strip to minimize the length of the connecting cables?

Computer A

Computer B

w

Name _____ Date _____

Essential Question How can you rotate a figure in a coordinate plane?

1 EXPLORATION: Rotating a Triangle in a Coordinate Plane

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.
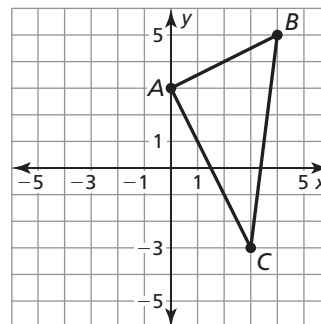
Work with a partner.

a. Use dynamic geometry software to draw any triangle and label it $\triangle ABC$.

b. *Rotate* the triangle 90° counterclockwise about the origin to form $\triangle A'B'C'$.

c. What is the relationship between the coordinates of the vertices of $\triangle ABC$ and those of $\triangle A'B'C'$?

d. What do you observe about the side lengths and angle measures of the two triangles?

2 EXPLORATION: Rotating a Triangle in a Coordinate Plane

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. The point (x, y) is rotated 90° counterclockwise about the origin. Write a rule to determine the coordinates of the image of (x, y).

b. Use the rule you wrote in part (a) to rotate $\triangle ABC$ 90° counterclockwise about the origin. What are the coordinates of the vertices of the image, $\triangle A'B'C'$?

c. Draw $\triangle A'B'C'$. Are its side lengths the same as those of $\triangle ABC$? Justify your answer.

Name_____ Date _____

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3 **EXPLORATION:** Rotating a Triangle in a Coordinate Plane

Work with a partner.

a. The point (x, y) is rotated 180° counterclockwise about the origin. Write a rule to determine the coordinates of the image of (x, y). Explain how you found the rule.

b. Use the rule you wrote in part (a) to rotate $\triangle ABC$ 180° counterclockwise about the origin. What are the coordinates of the vertices of the image, $\triangle A'B'C'$?
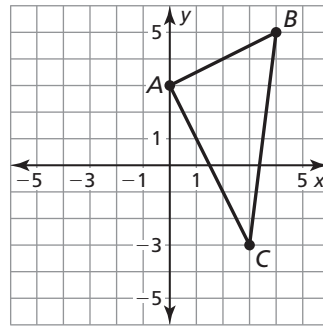

Communicate Your Answer

4. How can you rotate a figure in a coordinate plane?

5. In Exploration 3, rotate $\triangle A'B'C'$ 180° counterclockwise about the origin. What are the coordinates of the vertices of the image, $\triangle A''B''C''$? How are these coordinates related to the coordinates of the vertices of the original triangle, $\triangle ABC$?

Name _____ Date _____

4.3   Notetaking with Vocabulary
For use after Lesson 4.3

In your own words, write the meaning of each vocabulary term.

rotation

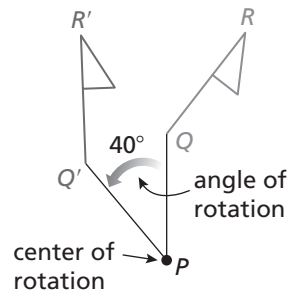center of rotation

angle of rotation

rotational symmetry

center of symmetry

Core Concepts

Rotations

A **rotation** is a transformation is which a figure is turned about a fixed
point called the **center of rotation**. Rays drawn from the center of rotation
to a point and its image form the **angle of rotation**.

A rotation about a point P through an angle of $x°$ maps every point Q in
the plane to a point Q', so that one of the following properties is true.

- If Q is not the center of rotation P, then $QP = Q'P$ and
  $m\angle QPQ' = x°$, or

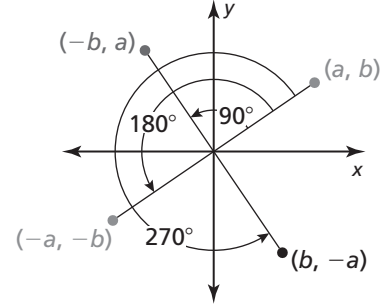- If Q is the center of rotation P, then $Q = Q'$.

Notes:

Name_____ Date _____

Coordinate Rules for Rotations about the Origin

When a point (a, b) is rotated counterclockwise about the origin, the following are true.

- For a rotation of 90°, $(a, b) \rightarrow (-b, a)$.
- For a rotation of 180°, $(a, b) \rightarrow (-a, -b)$.
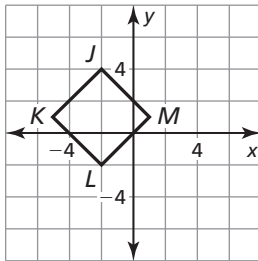- For a rotation of 270°, $(a, b) \rightarrow (b, -a)$.

Notes:

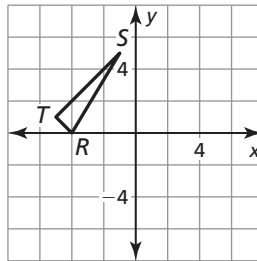## Postulate 4.3 Rotation Postulate

A rotation is a rigid motion.

Extra Practice

In Exercises 1–3, graph the image of the polygon after a rotation of the given number of degrees about the origin.
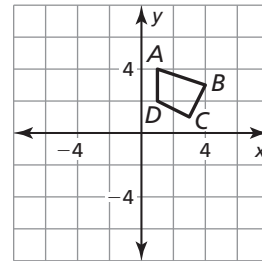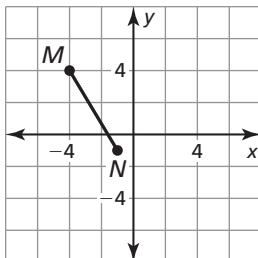
1. 180°

2. 90°

3. 270°

In Exercises 4–7, graph the image of $\overline{MN}$ after the composition.
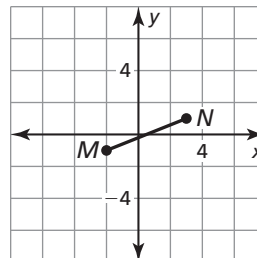
4. Reflection: x-axis

Rotation: 180° about the origin
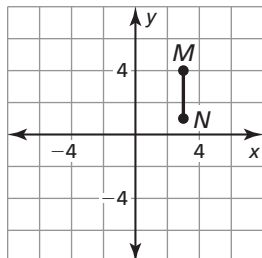
5. Rotation: 90° about the origin

Translation: $(x, y) \rightarrow (x + 2, y - 3)$
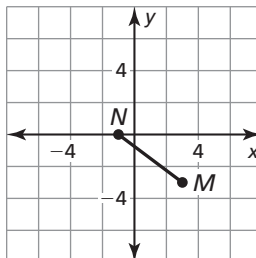
4.3 **Notetaking with Vocabulary** (continued)

6. Rotation: $270°$ about the origin

Reflection: in the line $y = x$

7. Rotation: $90°$ about the origin

Translation: $(x, y) \rightarrow (x - 5, y)$

In Exercises 8 and 9, graph $\triangle JKL$ **with vertices** $J(2, 3)$, $K(1, -1)$, **and** $L(-1, 0)$ **and its image after the composition.**

8. Rotation: $180°$ about the origin

Reflection: $x = 2$

9. Translation: $(x, y) \rightarrow (x - 4, y - 4)$

Rotation: $270°$ about the origin

In Exercises 10 and 11, determine whether the figure has rotational symmetry. If so, describe any rotations that map the figure onto itself.

10.

11.

Name_____ Date _____

4.4 Congruence and Transformations
For use with Exploration 4.4

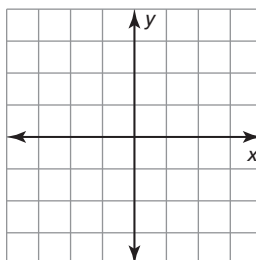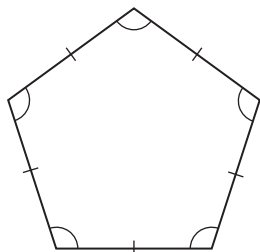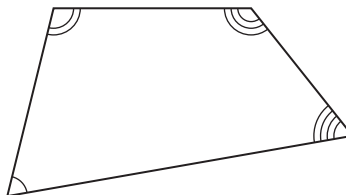**Essential Question**   What conjectures can you make about a figure reflected in two lines?

---

1   EXPLORATION: Reflections in Parallel Lines

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw any scalene triangle and label it $\triangle ABC$.

   a. Draw any line $\overleftrightarrow{DE}$. Reflect $\triangle ABC$ in $\overleftrightarrow{DE}$ to form $\triangle A'B'C'$.

   b. Draw a line parallel to $\overleftrightarrow{DE}$. Reflect $\triangle A'B'C'$ in the new line to form $\triangle A''B''C''$.

   c. Draw the line through point A that is perpendicular to $\overleftrightarrow{DE}$. What do you notice?

   d. Find the distance between points A and A''. Find the distance between the two parallel lines. What do you notice?

   e. Hide $\triangle A'B'C'$. Is there a single transformation that maps $\triangle ABC$ to $\triangle A''B''C''$. Explain.

   f. Make conjectures based on your answers in parts (c)−(e). Test your conjectures by changing $\triangle ABC$ and the parallel lines.

4.4 **Congruence and Transformations** (continued)

2 **EXPLORATION:** Reflections in Intersecting Lines

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw any scalene triangle and label it $\triangle ABC$.

  a. Draw any line $\overrightarrow{DE}$. Reflect $\triangle ABC$ in $\overrightarrow{DE}$ to form $\triangle A'B'C'$.

  b. Draw any line $\overrightarrow{DF}$ so that $\angle EDF$ is less than or equal to 90°. Reflect $\triangle A'B'C'$ in $\overrightarrow{DF}$ to form $\triangle A''B''C''$.

  c. Find the measure of $\angle EDF$. Rotate $\triangle ABC$ counterclockwise about point D twice using the measure of $\angle EDF$.

  d. Make a conjecture about a figure reflected in two intersecting lines. Test your conjecture by changing $\triangle ABC$ and the lines.

Communicate Your Answer

  3. What conjectures can you make about a figure reflected in two lines?

  4. Point Q is reflected in two parallel lines, $\overrightarrow{GH}$ and $\overrightarrow{JK}$, to form Q' and Q''. The distance from $\overrightarrow{GH}$ to $\overrightarrow{JK}$ is 3.2 inches. What is the distance QQ''?

4.4 Notetaking with Vocabulary
For use after Lesson 4.4

In your own words, write the meaning of each vocabulary term.
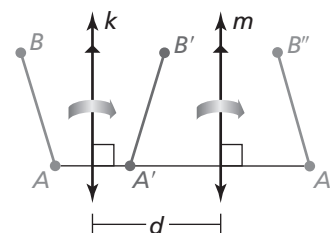
congruent figures

congruence transformation

Theorems

Theorem 4.2   Reflections in Parallel Lines Theorem

If lines k and m are parallel, then a reflection in line k followed
by a reflection in line m is the same as a translation.

If A'' is the image of A, then

   1. AA'' is perpendicular to k and m, and

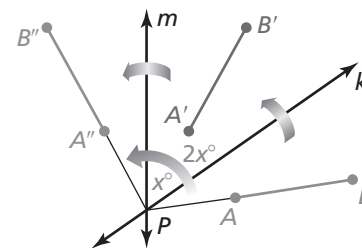   2. $AA'' = 2d$, where d is the distance between k and m.

Proof Ex. 31. p. 206

Notes:

Theorem 4.3   Reflections in Intersecting Lines Theorem

If lines k and m intersect at point P, then a reflection in line k followed
by a reflection in line m is the same as a rotation about point P.

The angle of rotation is $2x°$, where $x°$ is the measure of the acute or
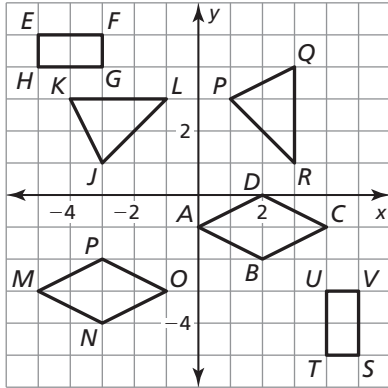right angle formed by lines k and m.
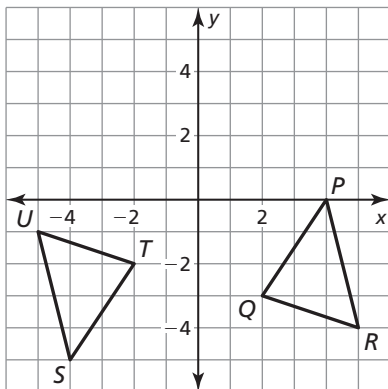
Proof Ex. 31. p. 206

Notes:

4.4   **Notetaking with Vocabulary (continued)**
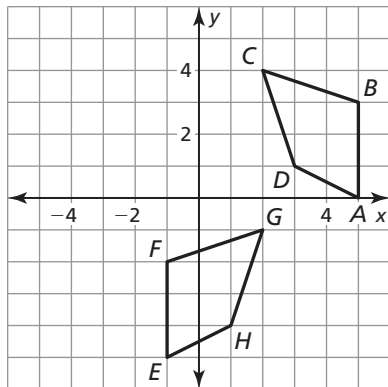
Extra Practice

1. Identify any congruent figures in the coordinate plane. Explain.

2. Describe a congruence transformation that maps $\triangle PQR$ to $\triangle STU$.
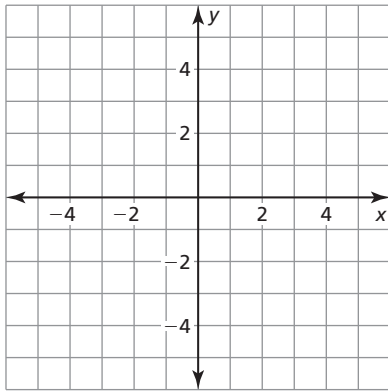

3. Describe a congruence transformation that maps polygon $ABCD$ to polygon $EFGH$.
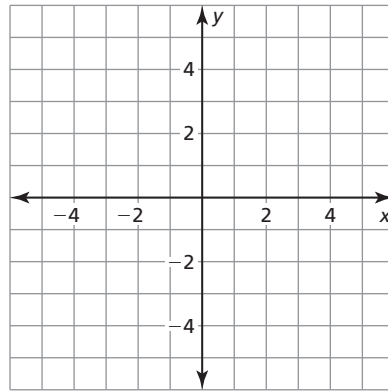
4.4 Notetaking with Vocabulary (continued)

In Exercises 4 and 5, determine whether the polygons with the given vertices are congruent. Use transformations to explain your reasoning.

4. $A(2, 2)$, $B(3, 1)$, $C(1, 1)$ and
   $D(2, -2)$, $E(3, -1)$, $F(1, -1)$

5. $G(3, 3)$, $H(2, 1)$, $I(6, 2)$, $J(6, 3)$ and
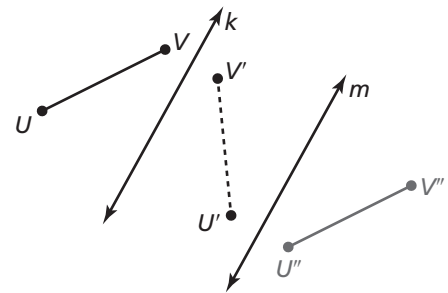 $K(2, -1)$, $L(-3, -3)$, $M(2, -2)$, $N(2, -1)$

In Exercises 6–9, $k \parallel m$, $\overline{UV}$ is reflected in line k, and $\overline{U'V'}$ is reflected in line m.

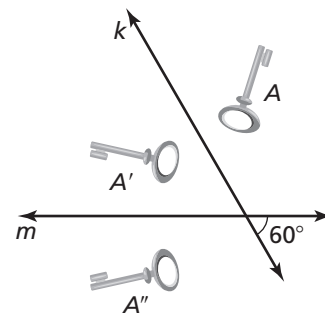6. A translation maps $\overline{UV}$ onto which segment?

7. Which lines are perpendicular to $\overline{UU''}$?

8. Why is V'' the image of V? Explain your reasoning.

9. If the distance between k and m is 5 inches, what is the length of $\overline{VV''}$?

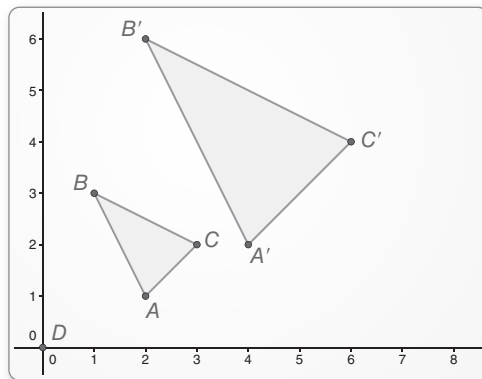10. What is the angle of rotation that maps A onto A''?

4.5 Dilations

For use with Exploration 4.5

Essential Question What does it mean to dilate a figure?

---

1 **EXPLORATION:** Dilating a Triangle in a Coordinate Plane

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw any triangle and label it △*ABC*.

a. *Dilate* △*ABC* using a *scale factor* of 2 and a *center of dilation* at the origin to form △*A′B′C′*. Compare the coordinates, side lengths, and angle measures of △*ABC* and △*A′B′C′*.

Sample

b. Repeat part (a) using a *scale factor* of $\frac{1}{2}$.

c. What do the results of parts (a) and (b) suggest about the coordinates, side lengths, and angle measures of the image of △*ABC* after a dilation with a scale factor of *k*?
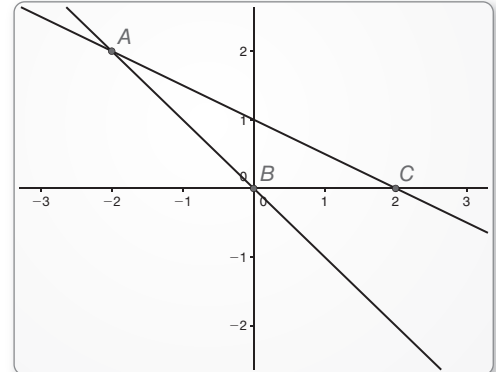
4.5 **Dilations** (continued)

2 **EXPLORATION:** Dilating Lines in a Coordinate Plane

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw $\overrightarrow{AB}$ that passes through the origin and $\overrightarrow{AC}$ that does not pass through the origin.

a. *Dilate* $\overrightarrow{AB}$ *using a* scale factor *of 3 and a* center *of dilation at the origin. Describe the image.*

b. *Dilate* $\overrightarrow{AC}$ *using a* scale factor *of 3 and a* center *of dilation at the origin. Describe the image.*

c. Repeat parts (a) and (b) using a scale factor of $\dfrac{1}{4}$.

d. What do you notice about dilations of lines passing through the center of dilation and dilations of lines not passing through the center of dilation?

Communicate Your Answer

3. What does it mean to dilate a figure?

4. Repeat Exploration 1 using a center of dilation at a point other than the origin.

4.5 Notetaking with Vocabulary
For use after Lesson 4.5

In your own words, write the meaning of each vocabulary term.

dilation

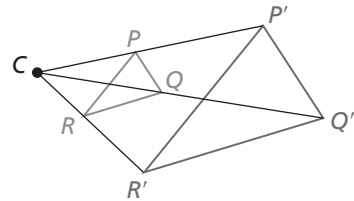center of dilation

scale factor

enlargement

reduction

Core Concepts

Dilations

A **dilation** is a transformation in which a figure is enlarged or reduced with respect to a fixed point C called the **center of dilation** and a **scale factor** k, which is the ratio of the lengths of the corresponding sides of the image and the preimage.

A dilation with center of dilation C and scale factor k maps every point P in a figure to a point P' so that the following are true.
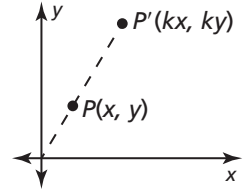
- If P is the center point C, then $P = P'$.

- If P is not the center point C, then the image point P' lies on $\overrightarrow{CP}$.
  The scale factor k is a positive number such that $k = \dfrac{CP'}{CP}$.

- Angle measures are preserved.

Notes:

Name_____ Date_____

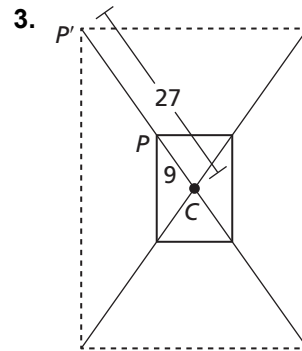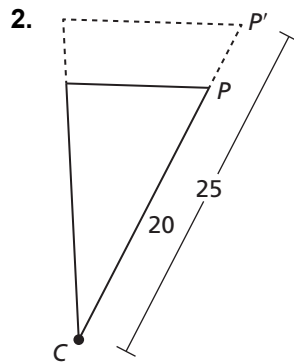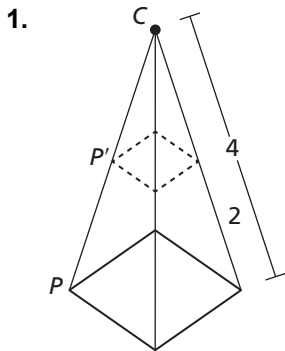## Coordinate Rule for Dilations

If $P(x, y)$ is the preimage of a point, then its image after a dilation centered at the origin $(0, 0)$ with scale factor k is the point $P'(kx, ky)$.
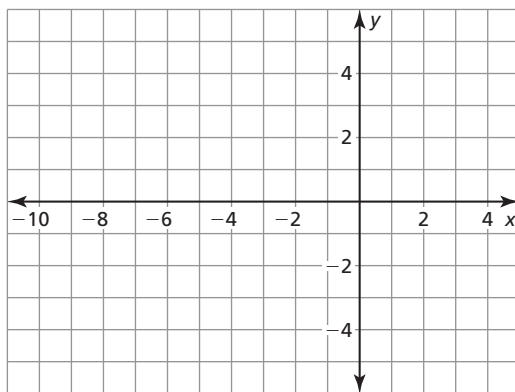
Notes:

Extra Practice

In Exercises 1–3, find the scale factor of the dilation. Then tell whether the dilation is a _reduction_ or an _enlargement_.
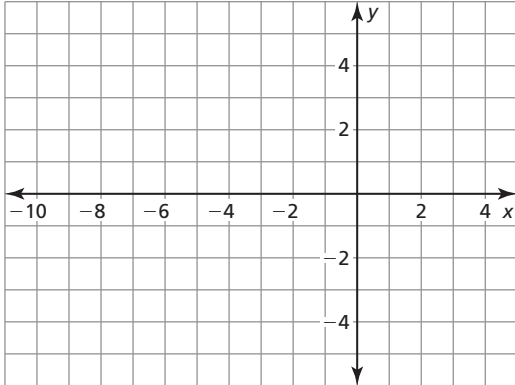
1.

2.

3.

In Exercises 4 and 5, graph the polygon and its image after a dilation with scale factor k.

4. $A(-3, 1), B(-4, -1), C(-2, -1); k = 2$

4.5 **Notetaking with Vocabulary** (continued)

5. $P(-10, 0), Q(-5, 0), R(0, 5), S(-5, 5); k = \frac{1}{5}$

In Exercises 6 and 7, find the coordinates of the image of the polygon after a dilation with scale factor k.

6. $A(-3, 1), B(-4, -1), C(-2, -1); k = -6$

7. $P(-8, 4), Q(20, -8), R(16, 4), S(0, 12); k = -0.25$

8. You design a poster on an 8.5-inch by 11-inch paper for a contest at your school. The poster of the winner will be printed on a 34-inch by 44-inch canvas to be displayed. What is the scale factor of this dilation?

9. A biology book shows the image of an insect that is 10 times its actual size. The image of the insect is 8 centimeters long. What is the actual length of the insect?

4.6 Similarity and Transformations
For use with Exploration 4.6

Essential Question When a figure is translated, reflected, rotated, or dilated in the plane, is the image always similar to the original figure?
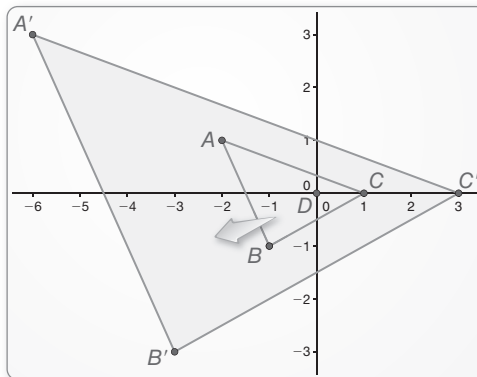
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1 EXPLORATION: Dilations and Similarity

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. Use dynamic geometry software to draw any triangle and label it $\triangle ABC$.

b. Dilate the triangle using a scale factor of 3. Is the image similar to the original triangle? Justify your answer.

Sample

4.6   **Similarity and Transformations** (continued)

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2   **EXPLORATION:** Rigid Motions and Similarity

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

  a. Use dynamic geometry software to draw any triangle.

  b. Copy the triangle and translate it 3 units left and 4 units up. Is the image similar to the original triangle? Justify your answer.

  c. Reflect the triangle in the *y*-axis. Is the image similar to the original triangle? Justify your answer.

  d. Rotate the original triangle 90° counterclockwise about the origin. Is the image similar to the original triangle? Justify your answer.

Communicate Your Answer

  3. When a figure is translated, reflected, rotated, or dilated in the plane, is the image always similar to the original figure? Explain your reasoning.

  4. A figure undergoes a composition of transformations, which includes translations, reflections, rotations, and dilations. Is the image similar to the original figure? Explain your reasoning.

4.6 Notetaking with Vocabulary
For use after Lesson 4.6

In your own words, write the meaning of each vocabulary term.

similarity transformation

similar figures

Notes:

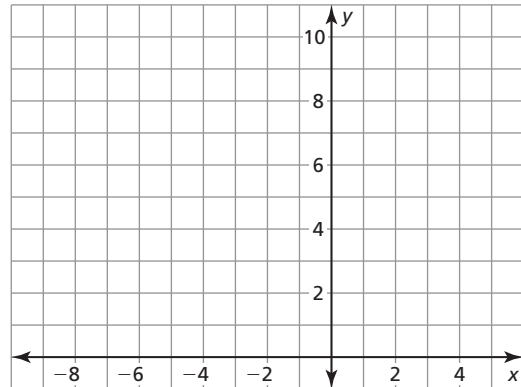**4.6** **Notetaking with Vocabulary** (continued)

Extra Practice

In Exercises 1–3, graph the polygon with the given vertices and its image after the similarity transformation.

1. $A(3, 6)$, $B(2, 5)$, $C(4, 3)$, $D(5, 5)$

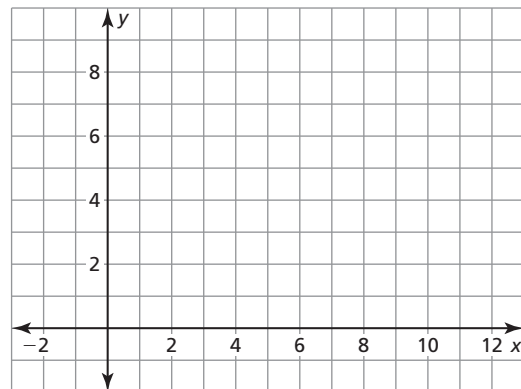   **Translation:** $(x, y) \rightarrow (x - 5, y - 3)$

   Dilation: $(x, y) \rightarrow (3x, 3y)$

2. $R(12, 8)$, $S(8, 0)$, $T(0, 4)$

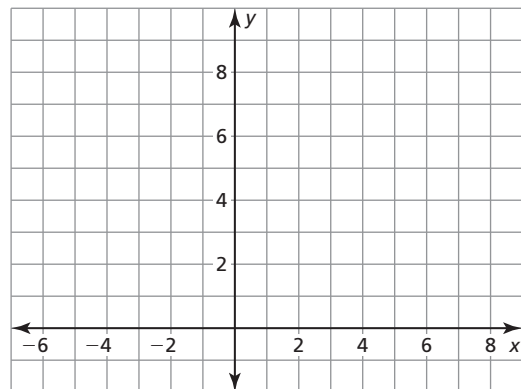   **Dilation:** $(x, y) \rightarrow \left(\frac{1}{4}x, \frac{1}{4}y\right)$

   Reflection: in the y-axis

3. $X(9, 6)$, $Y(3, 3)$, $Z(-3, 6)$

   Rotation: 90° about the origin

   Dilation: $(x, y) \rightarrow \left(\frac{2}{3}x, \frac{2}{3}y\right)$

4.6   **Notetaking with Vocabulary** (continued)

In Exercises 4–6, describe the similarity transformation that maps the preimage to the image.

4.

5.

6.