Chapter 4

Open Channel Flows

4.1. Introduction

When the surface of flow is open to atmosphere, in other terms when there is only atmospheric pressure on the surface, the flow is named as *open channel flow*. The governing force for the open channel flow is the gravitational force component along the channel slope. Water flow in rivers and streams are obvious examples of open channel flow in natural channels. Other occurrences of open channel flow are flow in irrigation canals, sewer systems that flow partially full, storm drains, and street gutters.

4.2. Classification of Open Channel Flows

A channel in which the cross-sectional shape and size and also the bottom slope are constant is termed as a *prismatic channel*. Most of the man made (artificial) channels are prismatic channels over long stretches. The rectangle, trapezoid, triangle and circle are some of the commonly used shapes in made channels. All natural channels generally have varying cross-sections and consequently are *non-prismatic*.

a) **Steady and Unsteady Open Channel Flow:** If the flow depth or discharge at a cross-section of an open channel flow is not changing with time, then the flow is *steady flow*, otherwise it is called as *unsteady flow*.
   Flood flows in rivers and rapidly varying surges in canals are some examples of unsteady flows. Unsteady flows are considerably more difficult to analyze than steady flows.
b) **Uniform and Non-Uniform Open Channel Flow:** If the flow depth along the channel is not changing at every cross-section for a taken time, then the flow is *uniform flow*. If the flow depth changes at every cross-section along the flow direction for a taken time, then it is *non-uniform flow*. A prismatic channel carrying a certain discharge with a constant velocity is an example of uniform flow.
c) **Uniform Steady Flow:** The flow depth does not change with time at every cross section and at the same time is constant along the flow direction. The depth of flow will be constant along the channel length and hence the free surface will be parallel to the bed. (Figure 4.1).

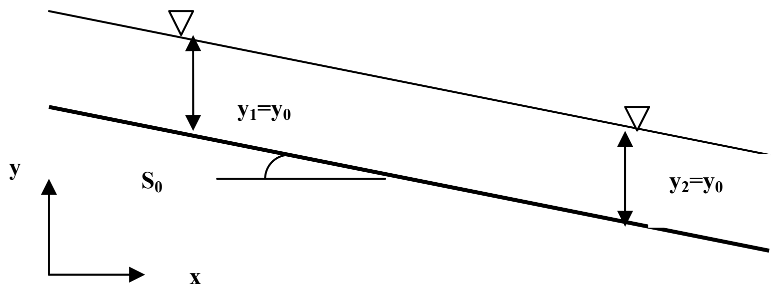


Figure 4.1

Mathematical definition of the Uniform Steady Flow is,

$$y_1 = y_2 = y_0$$
$$\frac{\partial y}{\partial t} = 0, \frac{\partial y}{\partial x} = 0 \qquad (4.1)$$
$$y_0 = Normal\ depth$$

d) **Non-Uniform Steady Flows:** The water depth changes along the channel cross-sections but does not change with time at each every cross section with time. A typical example of this kind of flow is the backwater water surface profile at the upstream of a dam.

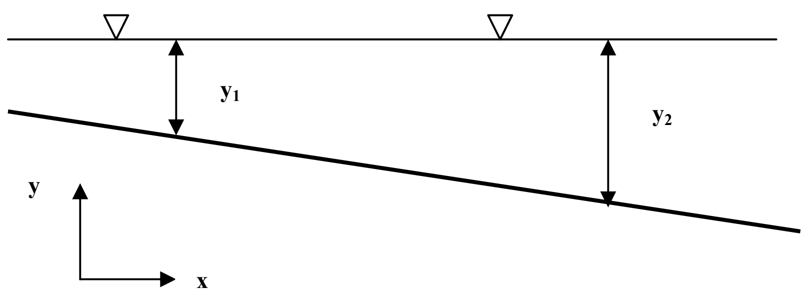


Figure 4.2. Non-Uniform steady Flow

Mathematical definition of the non-uniform steady flow is;

$$y_1 \neq y_2 \rightarrow \frac{\partial y}{\partial x} \neq 0 \quad \text{Non-Uniform Flow}$$
$$(4.2)$$
$$\frac{\partial y_1}{\partial t} = 0, \frac{\partial y_2}{\partial t} = 0 \quad \text{Steady Flow}$$

If the flow depth varies along the channel, these kinds of flows are called as *varied flows***.** If the depth variation is abrupt then the flow is called *abrupt varied flow* (flow under a sluice gate), or if the depth variation is gradual it is called *gradually varied flow* (flow at the upstream of a dam). Varied flows can be steady or unsteady.

Flood flows and waves are *unsteady varied flows* since the water depths vary at every cross-section and also at each cross-section it changes with time.

If the water depth in a flow varies at every cross-section along the channel but does not vary with time at each cross-section, it is *steady varied flow***.**

4.3. Types of Flow

The flow types are determined by relative magnitudes of the governing forces of the motion which are *inertia***,** *viscosity***,** and *gravity forces***.**

a) Viscosity Force Effect:

Viscosity effect in a fluid flow is examined by Reynolds number. As it was given in Chapter 1, Reynolds number was the ratio of the inertia force to the viscosity force.

$$\mathrm{Re} = \frac{InertiaForce}{Vis\cos ityForce}$$

For the pressured pipe flows,

$$\mathrm{Re} = \frac{VD}{\upsilon} < 2000 \rightarrow \text{ Laminar flow}$$

$$\mathrm{Re} = \frac{VD}{\upsilon} > 2500 \rightarrow \text{ Turbulent Flow}$$

Since D=4R, Reynolds number can be derived in open channel flows as,

$$\mathrm{Re} = \frac{VD}{\upsilon} = \frac{V4R}{\upsilon} = 2000 \rightarrow \mathrm{Re} = \frac{VR}{\upsilon} < 500 \rightarrow \text{Laminar Flow} \qquad (4.3)$$

$$\mathrm{Re} = \frac{VD}{\upsilon} = \frac{V4R}{\upsilon} = 2500 \rightarrow \mathrm{Re} = \frac{VR}{\upsilon} > 625 \rightarrow \text{Turbulent Flow} \qquad (4.4)$$

$$500 < \mathrm{Re} < 625 \rightarrow \text{ Transition zone}$$

R is the *Hydraulic Radius* of the open channel flow cross-section which can be taken as the flow depth y for wide channels.

Moody Charts can be used to find out the f friction coefficient by taking D=4R. Universal head loss equation for open channel flows can be derived as,

$$h_L = \frac{f}{D} \times \frac{V^2}{2g} \times L = \frac{f}{4R} \times \frac{V^2}{2g} \times L$$

$$S = \frac{h_L}{L} = \frac{fV^2}{8gR}$$

$$f = \frac{8gRS}{V^2}$$

Since *Friction Velocity* is,

$$u_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{gRS}$$

$$f = \frac{8u_*^2}{V^2} \qquad (4.5)$$

b) Gravity Force Effect:

The ratio of inertia force to gravity force is Froude Number.

$$Fr = \frac{InertiaForce}{GravityForce}$$

$$Fr = \frac{V}{\sqrt{g\frac{A}{L}}} \qquad (4.6)$$

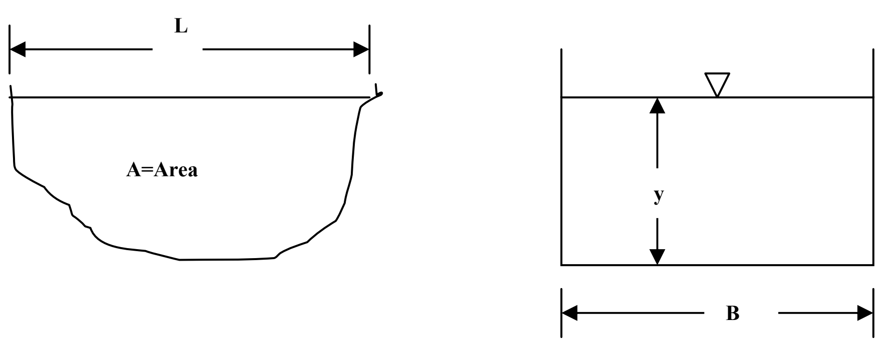


Figure 4.3.

Where ,

A = Wetted area
L= Free surface width

For rectangular channels,

$$A = By$$
$$L = B \qquad (4.7)$$
$$F_r = \frac{V}{\sqrt{g\frac{By}{B}}} = \frac{V}{\sqrt{gy}}$$

$$Fr = 1 \rightarrow \quad Critical\ Flow$$
$$Fr < 1 \rightarrow \quad Sub\ Critical \qquad (4.8)$$
$$Fr > 1 \rightarrow \quad Super\ Critical\ Flow$$

4.4. Energy Line Slope for Uniform Open Channel Flows

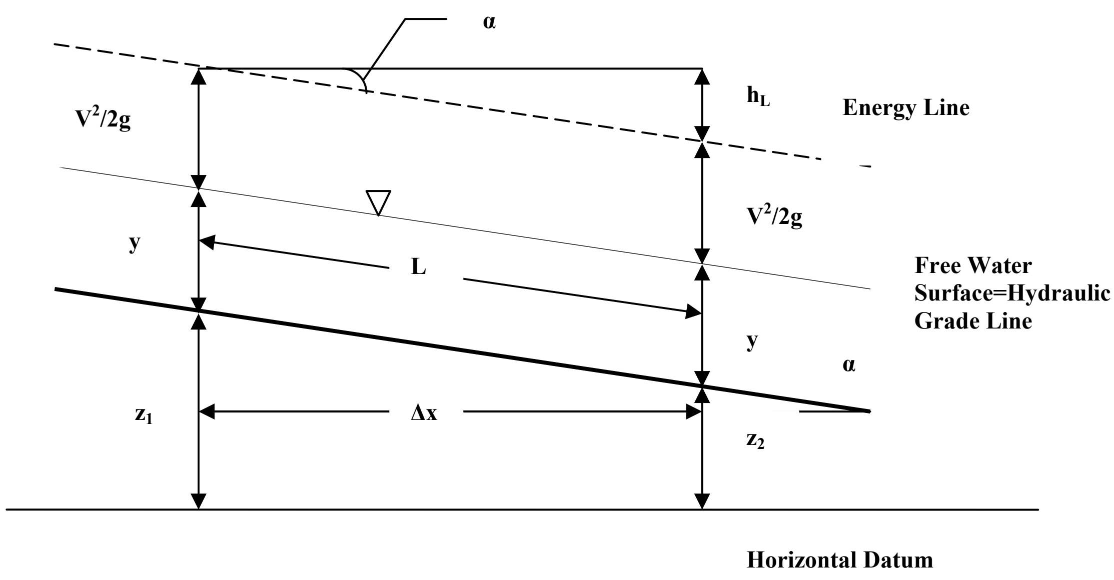


Figure 4.4

The head (energy) loss between cross-sections 1 and 2,

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

$$\frac{p_1}{\gamma} = \frac{p_2}{\gamma} = y$$

$$V_1 = V_2 = V$$

$$h_L = z_1 - z_2$$

The head loss for unit length of channel length is *energy line (hydraulic) slope*,

$$S_{ener} = \frac{h_L}{L} = \frac{z_1 - z_2}{L} = Sin\alpha$$

Since in open channel flows the channel slope is generally a small value,

$$\alpha < 5^0 - 10^0$$

$$Sin\alpha \cong Tan\alpha$$

$$Tan\alpha = \frac{h_L}{\Delta x} = S_0 \rightarrow \text{ (channel bottom slope)}$$

$$S_{ener} = S_0 \qquad (4.9)$$

Conclusion: *Hydraulic grade line coincides with water surface slope in every kind of open channel flows. Since the velocity will remain constant in every cross section at uniform flows, energy line slope, hydraulic grade line slope (water surface slope) and channel bottom slope are equal to each other and will be parallel as well.*

$$S = S_0 = S_{ener} \qquad (4.10)$$

Where S is the water surface slope.

4.5. Pressure Distribution

The intensity of pressure for a liquid at its free surface is equal to that of the surrounding atmosphere. Since the atmospheric pressure is commonly taken as a reference and of equal to zero, the free surface of the liquid is thus a surface of zero pressure. The pressure distribution in an open channel flow is governed by the acceleration of gravity g and other accelerations and is given by the Euler's equation as below:

In any arbitrary direction s,

$$-\frac{\partial(p+\gamma z)}{\partial s}=\rho a_s \qquad (4.11)$$

and in the direction normal to s direction, i.e. in the n direction,

$$-\frac{\partial}{\partial n}(p+\gamma z)=\rho a_n \qquad (4.12)$$

in which p = pressure, a_s = acceleration component in the s direction, a_n = acceleration in the n direction and z = geometric elevation measured above a datum.

Consider the s direction along the streamline and the n direction normal to it. The direction of the normal towards the centre of curvature is considered as positive. We are interested in studying the pressure distribution in the n-direction. The normal acceleration of any streamline at a cross-section is given by,

$$a_n=\frac{v^2}{r} \qquad (4.13)$$

where v = velocity of flow along the streamline of radius of curvature r.

4.5.1. Hydrostatic Pressure Distribution

The normal acceleration a_n will be zero,

1. if v = 0, i.e. when there is no motion, or
2. if $r \rightarrow \infty$, i.e. when the streamlines are straight lines.

Consider the case of no motion, i.e. the still water case, Fig.(4.5). From Equ. (4.12), since $a_n = 0$, taking n in the z direction and integrating,

$$\frac{p}{\gamma}+z=\text{constant}=\text{C} \qquad (4.14)$$

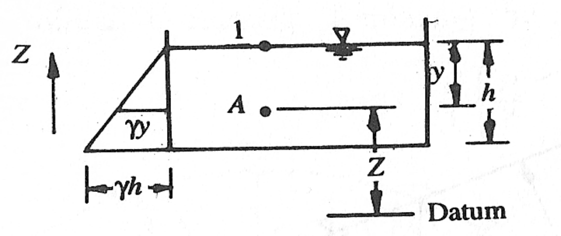


Figure. 4.4. Pressure distribution in still water

At the free surface [point 1 in Fig (4.5)] $p_1/\gamma = 0$ and $z = z_1$, giving $C = z_1$. At any point A at a depth y below the free surface,

$$\frac{p_A}{\gamma} = z_1 - z_A = y \qquad (4.15)$$

$$p_A = \gamma y$$

This linear variation of pressure with depth with the constant of proportionality equal to the specific weight of the liquid is known as *hydrostatic pressure distribution*.

4.5.2. Channels with Small Slope

Let us consider a channel with a very small value of the longitudinal slope θ. Let $\theta \approx \sin\theta \approx 1/1000$. For such channels the vertical section is practically the same as the normal section. If a flow takes place in this channel with the water surface parallel to the bed, the streamlines will be straight lines and as such in a vertical direction [section 0 – 1 in Fig. (4.6)] the normal acceleration $a_n = 0$. The pressure distribution at the section 0 – 1 will be hydrostatic. At any point A at a depth y below the water surface,

$$\frac{p}{\gamma} = y \quad \text{and} \quad \frac{p}{\gamma} + z = z_1 = \text{Elevation of water surface}$$

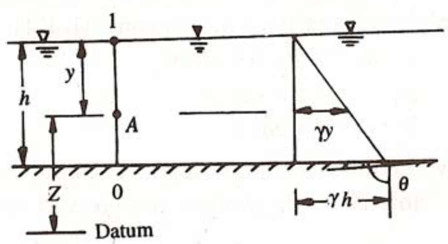


Figure 4. 6. Pressure distribution in a channel with small slope

Thus the piezometric head at any point in the channel will be equal to the water surface elevation. The hydraulic grade line will therefore lie (coincide) on the water surface.

4.5.3. Channels with Large Slope

Fig. (4.7) shows a uniform free surface flow in a channel with a large value of inclination θ. The flow is uniform, i.e. the water surface is parallel to the bed. An element of length ΔL is considered at the cross-section 0 – 1.

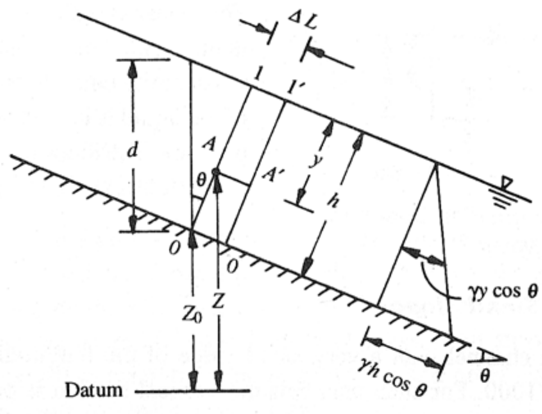


Figure 4.7. Pressure distribution in a channel with large slope

At any point A at a depth y measured normal to the water surface, the weight of column A11'A' = γΔLy and acts vertically downwards. The pressure at AA' supports the normal component of the column A11'A'. Thus,

$$p_A \Delta L = \gamma y \Delta L \cos\theta$$
$$p_A = \gamma y \cos\theta \qquad (4.16)$$
$$\frac{p_A}{\gamma} = \gamma \cos\theta$$

The pressure p_A varies linearly with the depth y but the constant of proportionality is $\gamma \cos\theta$. If h = normal depth of flow, the pressure on the bed at point O, $p_0 = \gamma h \cos\theta$. If d = vertical depth to water surface measured at point O, then $h = d\cos\theta$ and the pressure head at point O, on the bed is given by,

$$\frac{p_0}{\gamma} = h\cos\theta = d\cos^2\theta \qquad (4.17)$$

The piezometric head at any point A, $p_A = z + y\cos\theta = z_O + h\cos\theta$. Thus for channels with large values of the slope, the conventionally defined hydraulic gradient line does not lie on the water surface.

Channels of large slopes are encountered rather rarely in practice except, typically, in spillways and chutes. On the other hand, most of the canals, streams and rivers with which a hydraulic engineer is commonly associated will have slopes (sin θ) smaller than 1/100. For such cases cos θ ≈ 1.0. As such, the term cosθ in the expression for the pressure will be omitted assuming that the pressure distribution is hydrostatic.

4.5.4. Pressure Distribution in Curvilinear Flows.

Figure (4.8) shows a curvilinear flow in a vertical plane on an upward convex surface. For simplicity consider a section 01A2 in which the r direction and Z direction coincide. Replacing the n direction in Equ. (4.12) by (-r) direction,

$$\frac{\partial}{\partial r}\left(\frac{p}{\gamma}+z\right)=\frac{a_n}{g} \qquad (4.18)$$

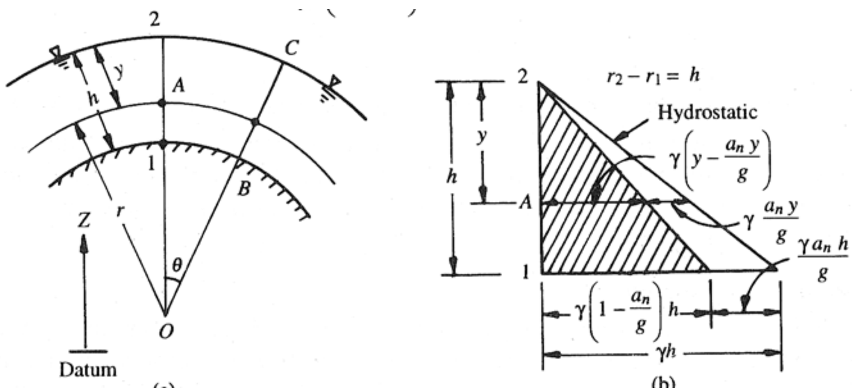


Figure 4.8. Convex curvilinear flow

Let us assume a simple case in which a_n = constant. Then the integration of Equ. (4.18) yields,

$$\frac{p}{\gamma}+z=\frac{a_n}{g}r+C \qquad (4.19)$$

in which C = constant. With the boundary conditions that at point 2 which lies on the surface, $r = r_2$ and $p/\gamma = 0$ and $z = z_2$.

$$C=\frac{p}{\gamma}+z-\frac{a_n}{g}r$$

$$C=0+z_2-\frac{a_n}{g}r_2$$

$$\frac{p}{\gamma}+z=\frac{a_n}{g}r+z_2-\frac{a_n}{g}r_2$$

$$\frac{p}{\gamma}=(z_2-z)-\frac{a_n}{g}(r_2-r) \qquad (4.20)$$

Let $z_2 - z = y =$ depth of flow the free surface of any point A in the section 01A2. Then for point A,

$$(r_2 - r) = y = (z_2 - z)$$

$$\frac{p}{\gamma} = y - \frac{a_n}{g} y \qquad (4.21)$$

Equ. (4.21) shows that the pressure is less than the pressure obtained by the hydrostatic distribution. Fig. (4.8).

For any normal direction OBC in Fig. (4.9), at point C, $(p/\gamma)_C = 0$, $r_C = r_2$, and for any point at a radial distance r from the origin O,

$$\frac{p}{\gamma} = (z_C - z) - \frac{a_n}{g}(r_2 - r)$$
$$z_C - z = (r_2 - r)\cos\theta$$

$$\frac{p}{\gamma} = (r_2 - r)\cos\theta - \frac{a_n}{g}(r_2 - r) \qquad (4.22)$$

If the curvature is convex downwards, (i.e. r direction is opposite to z direction) following the argument above, for constant a_n the pressure at any point A at a depth y below the free surface in a vertical section O1A2 [Fig. (4.9)] can be shown to be,

$$\frac{p}{\gamma} = y + \frac{a_n}{g} y \qquad (4.23)$$

The pressure distribution in vertical section is as shown in Fig. (4.9)

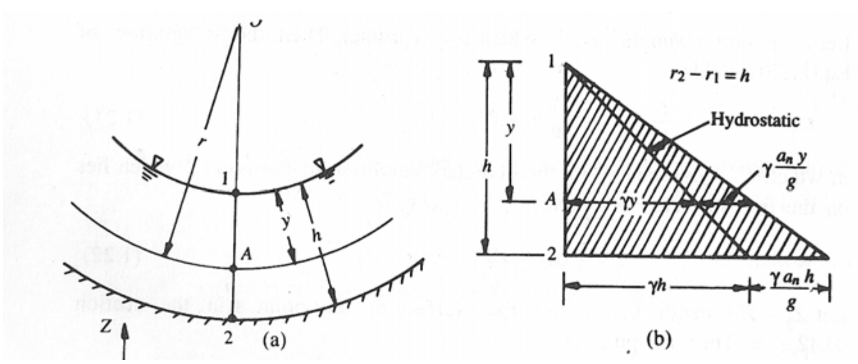


Figure 4.8. Concave curvilinear flow

Thus it is seen that for a curvilinear flow in a vertical plane, an additional pressure will be imposed on the hydrostatic pressure distribution. The extra pressure will be additive if the curvature is convex downwards and subtractive if it is convex upwards.

4.5.5. Normal Acceleration

In the previous section on curvilinear flows, the normal acceleration a_n was assumed to be constant. However, it is known that at any point in a curvilinear flow, $a_n = v^2 / r$, where v = velocity and r = radius of curvature of the streamline at that point.

In general, one can write v = f (r) and the pressure distribution can then be expressed by,

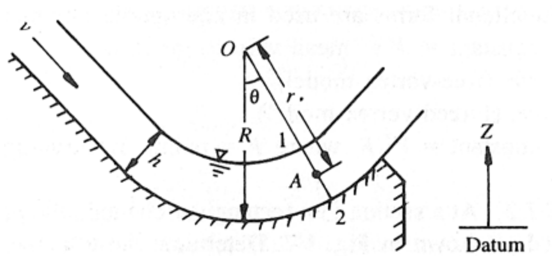
$$\left(\frac{p}{\gamma}+z\right)=\int\frac{v^2}{gr}dr+const \qquad (4.24)$$

This expression can be evaluated if v = f (r) is known. For simple analysis, the following functional forms are used in appropriate circumstances;

1. v = constant = V = mean velocity of flow
2. v = c / r, (free vortex)
3. v = cr, (forced vortex)
4. a_n = constant = V^2/R, where R = radius of curvature at mid depth.

Example 4.1: A spillway bucket has a radius of curvature R as shown in the Figure.

a) Obtain an expression for the pressure distribution at a radial section of inclination θ to the vertical. Assume the velocity at any radial section to be uniform and the depth of flow h to be constant,
b) What is the effective piezometric head for the above pressure distribution?



Solution:

a) Consider the section 012. Velocity = V = constant across 12. Depth of flow = h. From Equ. (4.24), since the curvature is convex downwards,

$$\left(\frac{p}{\gamma}+z\right)=\int\frac{v^2}{gr}dr+const$$

$$\frac{p}{\gamma}+z=\frac{V^2}{g}Lnr+C \qquad (A)$$

At point 1, $p/\gamma = 0$, $z = z_1$, $r = R - h$

$$C = z_1 - \frac{V^2}{g} Ln(R-h)$$

At any point A, at radial distance r from 0,

$$\frac{p}{\gamma} = (z_1 - z) + \frac{V^2}{g} Ln\left(\frac{r}{R-h}\right) \qquad \text{(B)}$$

$$z_1 - z = (r - R + h)\cos\theta$$

$$\frac{p}{\gamma} = (r - R + h)\cos\theta + \frac{V^2}{g} Ln\left(\frac{r}{R-h}\right) \qquad \text{(C)}$$

Equ. (C) represents the pressure distribution at any point (r, θ). At point 2, $r = R$, $p = p_2$.

b) Effective piezometric head, h_{ep}:

From Equ. (B), the piezometric head h_p at A is,

$$h_p = \left(\frac{p}{\gamma} + z\right)_A = z_1 + \frac{V^2}{g} Ln\left(\frac{r}{R-h}\right)$$

Noting that $z_1 = z_2 + h\cos\theta$ and expressing h_p in the form of Equ. (4.17),

$$h_p = z_2 + h\cos\theta + \Delta h$$

Where,

$$\Delta h = \frac{V^2}{g} Ln\frac{r}{R-h}$$

The effective piezometric head h_{ep} from Equ. (4.17),

$$h_{ep} = z_2 + h\cos\theta + \frac{1}{h\cos\theta}\int_{R-h}^{R}\left[\frac{V^2}{g} Ln\left(\frac{r}{R-h}\right)\right]dr$$

$$h_{ep} = z_2 + h\cos\theta + \frac{V^2}{gh\cos\theta}\left[-h + RLn\left(\frac{R}{R-h}\right)\right]$$

$$h_{ep} = z_2 + h\cos\theta + \frac{V^2\left[-\frac{h}{R} + Ln\left(\frac{1}{1-h/R}\right)\right]}{gR\left(\frac{h}{R}\right)\cos\theta} \qquad \text{(D)}$$

It may be noted that when R → ∞ and h/R → 0, $h_{ep} \rightarrow z_2 + h\cos\theta$

4.6. Uniform Open Channel Flow Velocity Equations

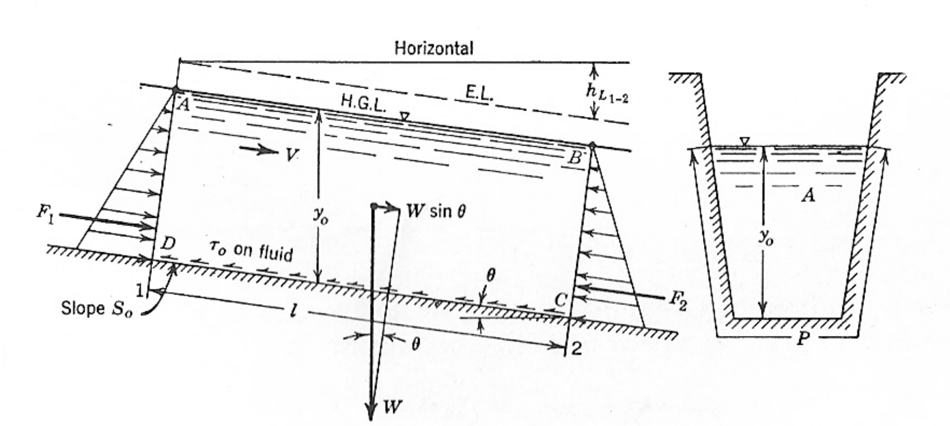


Figure 4.10.

The fundamental equation for uniform flow may be derived by applying the Impuls-Momentum equation to the control volume ABCD,

$$F = Ma$$

The external forces acting on the control volume are,

1) The forces of static pressure, F_1 and F_2 acting on the ends of the body,

$$\vec{F}_1 = \overleftarrow{F}_2 \rightarrow \text{ Uniform flow, flow depths are constant}$$

2) The weight, W, which has a component WSinθ in the direction of motion,

$$\vec{W}_x = WSin\theta = \gamma ALSin\theta$$

3) Change of momentum,

$$\vec{M}_1 = \overleftarrow{M}_2 = \rho QV \rightarrow \quad V_1 = V_2 = V$$

4) The force of resistance exerted by the bottom and sides of the channel cross-section,

$$\vec{T} = \tau_0 PL$$

Where P is wetted perimeter of the cross-section.

$$\vec{M}_1 + \vec{F}_1 + \vec{W}_x - \vec{M}_2 - \vec{F}_2 - \vec{T} = 0$$

$$\gamma ALSin\theta = \tau_0 PL$$

$$\tau_0 = \gamma \frac{A}{P} Sin\theta$$

$$\frac{A}{P} = R, Sin\theta = S_0$$

$$\tau_0 = \gamma RS_0 \qquad (4.25)$$

Using Equ. (4.5) and friction velocity equation,

$$\tau_0 = \rho U_*^2$$

$$U_*^2 = \frac{fV^2}{8}$$

$$\rho \frac{fV^2}{8} = \gamma RS_0$$

$$V^2 = \frac{8g}{f} \times RS_0$$

$$V = \sqrt{\frac{8g}{f}} \times \sqrt{RS_0} \qquad (4.26)$$

If we define,

$$C = \sqrt{\frac{8g}{f}} \qquad (4.27)$$

$$V = C\sqrt{RS_0}$$

This cross-sectional mean velocity equation for open channel flows is known as *Chezy equation*. The dimension of *Chezy coefficient C*,

$$[V] = [C][R]^{1/2}[S_0]^{1/2}$$

$$[LT^{-1}] = [C][L^{1/2}][F^0 L^0 T^0]$$

$$[C] = [L^{1/2} T^{-1}]$$

C Chezy coefficient has a dimension and there it is not a constant value. When using the Chezy equation to calculate the mean velocity, one should be careful since it takes different values for different unit systems.

The simplest relation and the most widely used equation for the mean velocity calculation is the *Manning equation* which has been derived by Robert Manning (1890) by analyzing the experimental data obtained from his own experiments and from those of others. His equation is,

$$V = \frac{1}{n} R^{2/3} S_0^{\ 1/2} \qquad (4.28)$$

Where n is the *Manning's roughness coefficient***.**

Equating Equs. (4.27) and (4.28),

$$CR^{1/2} S_0^{1/2} = \frac{1}{n} R^{2/3} S_0^{1/2}$$

$$C = \frac{R^{1/6}}{n} \qquad (4.29)$$

Equ. (4.28) was derived from metric data; hence, the unit of length is meter. Although n is often supposed to be a characteristic of channel roughness, it is convenient to consider n to be a dimensionless. Then, the values of n are the same in any measurement system.Some typical values of n are given in Table. (4.1)

Table 4.1. Typical values of the Manning's roughness coefficient n

	n
Artificial lined channels:	
Glass	0.010 ± 0.002
Brass	0.011 ± 0.002
Steel, smooth	0.012 ± 0.002
Painted	0.014 ± 0.003
Riveted	0.015 ± 0.002
Cast iron	0.013 ± 0.003
Concrete, finished	0.012 ± 0.002
Unfinished	0.014 ± 0.002
Planed wood	0.012 ± 0.002
Clay tile	0.014 ± 0.003
Brickwork	0.015 ± 0.002
Asphalt	0.016 ± 0.003
Corrugated metal	0.022 ± 0.005
Rubble masonry	0.025 ± 0.005
Excavated earth channels:	
Clean	0.022 ± 0.004
Gravelly	0.025 ± 0.005
Weedy	0.030 ± 0.005
Stony, cobbles	0.035 ± 0.010
Natural channels:	
Clean and straight	0.030 ± 0.005
Sluggish, deep pools	0.040 ± 0.010
Major rivers	0.035 ± 0.010
Floodplains:	
Pasture, farmland	0.035 ± 0.010
Light brush	0.05 ± 0.02
Heavy brush	0.075 ± 0.025
Trees	0.15 ± 0.05

4.6.1. Determination of Manning's Roughness Coefficient

In applying the Manning equation, the greatest difficulty lies in the determination of the roughness coefficient, n; there is no exact method of selecting the n value. Selecting a value of n actually means to estimate the resistance to flow in a given channel, which is really a matter of intangibles. (Chow, 1959) .To experienced engineers, this means the exercise of engineering judgment and experience; for a new engineer, it can be no more than a guess and different individuals will obtain different results.

4.6.2. Factors Affecting Manning's Roughness Coefficient

It is not uncommon for engineers to think of a channel as having a single value of n for all occasions. Actually, the value of n is highly variable and depends on a number of factors. The factors that exert the greatest influence upon the roughness coefficient in both artificial and natural channels are described below.

a) **Surface Roughness:** The surface roughness is represented by the size and shape of the grains of the material forming the wetted perimeter. This usually considered the only factor in selecting the roughness coefficient, but it is usually just one of the several factors. Generally, fine grains result in a relatively low value of n and coarse grains in a high value of n.
b) **Vegetation:** Vegetation may be regarded as a kind of surface roughness, but it also reduces the capacity of the channel. This effect depends mainly on height, density, and type of vegetation.
c) **Channel Irregularity:** Channel irregularity comprises irregularities in wetted perimeter and variations in cross-section, size, and shape along the channel length.
d) **Channel Alignment:** Smooth curvature with large radius will give a relatively low value of n, whereas sharp curvature with severe meandering will increase n.
e) **Silting and Scouring:** Generally speaking, silting may change a very irregular channel into a comparatively uniform one and decrease n, whereas scouring may do the reverse and increase n.
f) **Obstruction:** The presence of logjams, bridge piers, and the like tends to increase n.
g) **Size and Shape of the Channel:** There is no definite evidence about the size and shape of the channel as an important factor affecting the value of n.
h) **Stage and Discharge:** The n value in most streams decreases with increase in stage and discharge.
i) **Seasonal Change:** Owing to the seasonal growth of aquatic plants, the value of n may change from one season to another season.

4.6.3. Cowan Method

Taking into account primary factors affecting the roughness coefficient, Cowan (1956) developed a method for estimating the value of n. The value of n may be computed by,

$$n = (n_0 + n_1 + n_2 + n_3 + n_4) \times m \qquad (4.30)$$

Where n_0 is a basic value for straight, uniform, smooth channel in the natural materials involved, n_1 is a value added to n_0 to correct for the effect of surface irregularities, n_2 is a value for variations in shape and size of the channel cross-section, n_3 is a value of obstructions, n_4 is a value for vegetation and flow conditions, and m is a correction factor for meandering of channel. These coefficients are given in Table (4.2) depending on the channel characteristics. (French, 1994).

Example 4.2: A trapezoidal channel with width B=4 m, side slope m=2, bed slope S_0=0.0004, and water depth y=1 m is taken. This artificial channel has been excavated in soil. Velocity and discharge values will be estimated with the vegetation cover change in the channel. For newly excavated channel, n_0 value is taken as 0.02 from the Table (4.2).

Only n_0 and n_4 values will be taken in using Equ. (4.30), the effects of the other factors will be neglected.

$$n = n_0 + n_4 \qquad (A)$$

Table 4.2. Values for the Computation of the Roughness Coefficient

Channel conditions		Values	
Material involved	Earth	n_0	0.020
	Rock cut		0.025
	Fine gravel		0.024
	Coarse gravel		0.028
Degree of irregularity	Smooth	n_1	0.000
	Minor		0.005
	Moderate		0.010
	Severe		0.020
Variations of channel cross section	Gradual	n_2	0.000
	Alternating occasionally		0.005
	Alternating frequently		0.010–0.015
Relative effect of obstructions	Negligible	n_3	0.000
	Minor		0.010–0.015
	Appreciable		0.020–0.030
	Severe		0.040–0.060
Vegetation	Low	n_4	0.005–0.010
	Medium		0.010–0.025
	High		0.025–0.050
	Very high		0.050–0.100
	Minor		1.000

The calculated velocity and discharge values corresponding to the n values found by Equ. (A) have been given in Table. As can be seen from the Table, as the vegetation covers increases in the channel perimeter so the velocity and discharge decreases drastically. The discharge for the high vegetation cover is found to be 6 times less than the low vegetation cover. Vegetation cover also corresponds to the maintenance of the channel cross-section along the channel. High vegetation cover correspond low maintenance of the channel. It is obvious that the discharge decrease would be higher if we had taken the variations in other factors.

Table . Velocity and Discharge Variation with n

n	Velocity (m/sec)	Discharge (m^3/sec)
0.020	0.79	4.77
0.030	0.53	3.17
0.045	0.35	2.12
.070	0.23	1.36
0.120	0.13	0.79

The calculated velocity and discharge values corresponding to the n values found by Equ. (A) have been given in Table. As can be seen from the Table, as the vegetation covers increases in the channel perimeter so the velocity and discharge decreases drastically. The discharge for the high vegetation cover is found to be 6 times less than the low vegetation cover. Vegetation cover also corresponds to the maintenance of the channel cross-section along the channel. High vegetation cover correspond low maintenance of the channel. It is obvious that the discharge decrease would be higher if we had taken the variations in other factors.

4.6.4. Empirical Formulae for n

Many empirical formulae have been presented for estimating manning's coefficient n in natural streams. These relate n to the bed-particle size. (Subramanya, 1997). The most popular one under this type is the Strickler formula,

$$n = \frac{d_{50}^{1/6}}{21.1} \qquad (4.31)$$

Where d_{50} is in meters and represents the particle size in which 50 per cent of the bed material is finer. For mixtures of bed materials with considerable coarse-grained sizes,

$$n = \frac{d_{90}^{1/6}}{26} \qquad (4.32)$$

Where d_{90} = size in meters in which 90 per cent of the particles are finer than d_{90}. This equation is reported to be useful in predicting n in mountain streams paved with coarse gravel and cobbles.

4.7. Equivalent Roughness

In some channels different parts of the channel perimeter may have different roughnesses. Canals in which only the sides are lined, laboratory flumes with glass walls and rough beds, rivers with sand bed in deepwater portion and flood plains covered with vegetation, are some typical examples. For such channels it is necessary to determine an equivalent roughness coefficient that can be applied to the entire cross-sectional perimeter in using the Manning's formula. This *equivalent roughness*, also called the *composite roughness,* represents a weighted average value for the roughness coefficient, n.

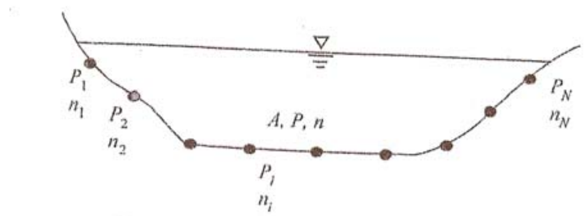


Figure 4.5. Multi-roughness type perimeter

Consider a channel having its perimeter composed of N types rough nesses. P_1, P_2,…., P_N are the lengths of these N parts and n_1, n_2,…….., n_N are the respective roughness coefficients (Fig. 4.5). Let each part P_i be associated with a partial area A_i such that,

$$\sum_{i-1}^{N} A_i = A_1 + A_2 + ... + A_i + ... + A_N = A = \text{Total area}$$

It is assumed that the mean velocity in each partial area is the mean velocity V for the entire area of flow,

$$V_1 = V_2 = ... = V_i = ... = V_N = V$$

By the Manning's equation,

$$S_0^{1/2} = \frac{V_1 n_1}{R_1^{2/3}} = \frac{V_2 n_2}{R_2^{2/3}} = ... = \frac{V_i n_i}{R_i^{2/3}} = ... = \frac{V_N n_N}{R_N^{2/3}} = \frac{Vn}{R^{2/3}} \qquad (4.33)$$

Where n = Equivalent roughness.

From Equ. (4.33),

$$\left(\frac{A_i}{A}\right)^{2/3} = \frac{n_i P_i^{2/3}}{n P^{2/3}}$$

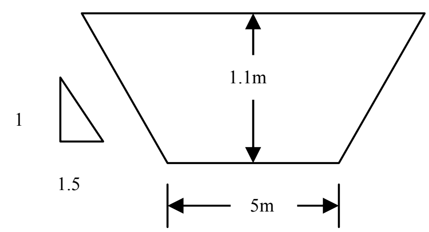
$$A_i = A \frac{n_i^{3/2} P_i}{n^{3/2} P} \qquad (4.34)$$

$$\sum A_i = A = A \frac{\sum \left(n_i^{3/2} P_i\right)}{n^{3/2} P}$$

$$n = \frac{\left(\sum n_i^{3/2} P_i\right)^{2/3}}{P^{2/3}} \qquad (4.35)$$

This equation gives a means of estimating the equivalent roughness of a channel having multiple roughness types in its perimeters.

Example 4.3: An earthen trapezoidal channel (n = 0.025) has a bottom width of 5.0 m, side slopes of 1.5 horizontal: 1 vertical and a uniform flow depth of 1.10 m. In an economic study to remedy excessive seepage from the canal two proposals, a) to line the sides only and, b) to line the bed only are considered. If the lining is of smooth concrete (n = 0.012), calculate the equivalent roughness in the above two cases.



Solution:

Case a): Lining on the sides only,

For the bed → $n_1 = 0.025$ and $P_1 = 5.0$ m.

For the sides → $n_2 = 0.012$ and $P_2 = 2 \times 1.10 \times \sqrt{1 + 1.5^2} = 3.97m$

$$P = P_1 + P_2 = 5.0 + 3.97 = 8.97m$$

Equivalent roughness, by Equ. (4.36),

$$n = \frac{\left[5.0 \times 0.025^{1.5} + 3.97 \times 0.012^{1.5}\right]^{2/3}}{8.97^{2/3}} = 0.020$$

Case b): Lining on the bottom only,

$$P_1 = 5.0 \text{ m} \rightarrow n_1 = 0.012$$
$$P_2 = 3.97 \text{ m} \rightarrow n_2 = 0.025 \rightarrow P = 8.97 \text{ m}$$

$$n = \frac{\left(5.0 \times 0.012^{1.5} + 3.97 \times 0.025^{1.5}\right)^{2/3}}{8.97^{2/3}} = 0.018$$

4.8. Hydraulic Radius

Hydraulic radius plays a prominent role in the equations of open-channel flow and therefore, the variation of hydraulic radius with depth and width of the channel becomes an important consideration. This is mainly a problem of section geometry.

Consider first the variation of hydraulic radius with depth in a rectangular channel of width B. (Fig. 4.11.a).

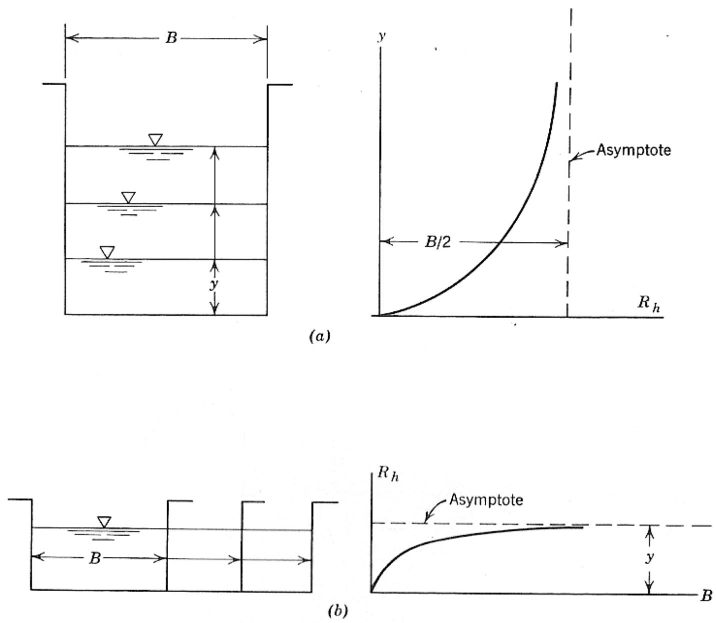


Figure 4.11.

$$A = By, P = B + 2y$$

$$R = \frac{A}{P} = \frac{By}{B + 2y}$$

$$R = \frac{B}{\frac{B}{y} + 2}$$

For $$y = 0 \rightarrow R = 0 \qquad y \rightarrow \infty, R = \frac{B}{2} \tag{4.36}$$

Therefore the variation of R with y is as shown in Fig (4.11.a). From this comes a useful engineering approximation: for *narrow deep cross-sections R≈ B/2***.** Since any (nonrectangular) section when deep and narrow approaches a rectangle, when a channel is deep and narrow, the hydraulic radius may be taken to be half of mean width for practical applications.

Consider the variation of hydraulic radius with width in a rectangular channel of with a constant water depth y. (Fig. 4.11.b).

$$R = \frac{By}{B+2y}$$

$$R = \frac{y}{1+\frac{2}{yB}}$$

$$B = 0 \rightarrow R = 0$$
$$B \rightarrow \infty, R \rightarrow y \qquad (4.37)$$

From this it may be concluded that for *wide shallow rectangular cross-sections R≈y* ; for rectangular sections the approximation is also valid if the section is wide and shallow, here the hydraulic radius approaches the mean depth.

4.9. Uniform Flow Depth

The equations that are used in uniform flow calculations are,

a) Continuity equation,

$$Q = VA$$

b) Manning velocity equation,

$$V = \frac{1}{n} R^{2/3} S_0^{1/2}$$

$$Q = AV = A\frac{1}{n} R^{2/3} S_0^{1/2}$$

The water depth in a channel for a given discharge Q and n= Manning coefficient, S_0= Channel slope, B= Channel width, is called as *Uniform Water Depth*.

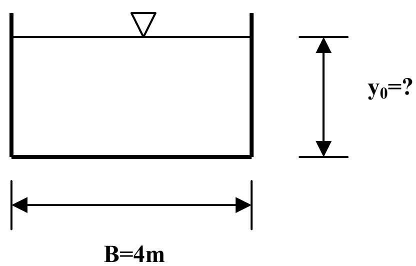
The basic variables in uniform flow problems can be the discharge Q, velocity of flow V, normal depth y_0, roughness coefficient n, channel slope S_0 and the geometric elements (e.g. B and side slope m for a trapezoidal channel). There can be many other derived variables accompanied by corresponding relationships. From among the above, the following five types of basic problems are recognized.

Problem Type	Given	Required
1	y_0, n, S_0, Geometric elements	Q and V
2	Q, y_0, n, Geometric elements	S_0
3	Q, y_0, S_0, Geometric elements	n
4	Q, n, S_0, Geometric elements	y_0
5	Q, y_0, n, S_0, Geometry	Geometric elements

Problems of the types 1, 2 and 3 normally have explicit solutions and hence do not represent any difficulty in their calculations. Problems of the types 4 and 5 usually do not have explicit solutions an as such may involve trial-and-error solution procedures.

Example 4.4: Calculate the uniform water depth of an open channel flow to convey Q=10 m^3/sec discharge with manning coefficient n=0.014, channel slope S_0=0.0004, and channel width B=4 m.

a) **Rectangular cross-section**



$$A = By_0 = 4y_0$$

$$P = B + 2y_0$$

$$R = \frac{A}{P} = \frac{4y_0}{4 + 2y_0}$$

$$Q = AV = A\frac{1}{n}R^{2/3}S_0^{1/2}$$

$$10 = 4 \times y_0 \times \frac{1}{0.014} \times \left(\frac{4y_0}{4 + 2y_0}\right)^{2/3} \times 0.0004^{1/2}$$

$$y_0 \times \left(\frac{y_0}{4 + 2y_0}\right)^{2/3} = \frac{10 \times 0.014}{4^{5/3} \times 0.02} = 0.694$$

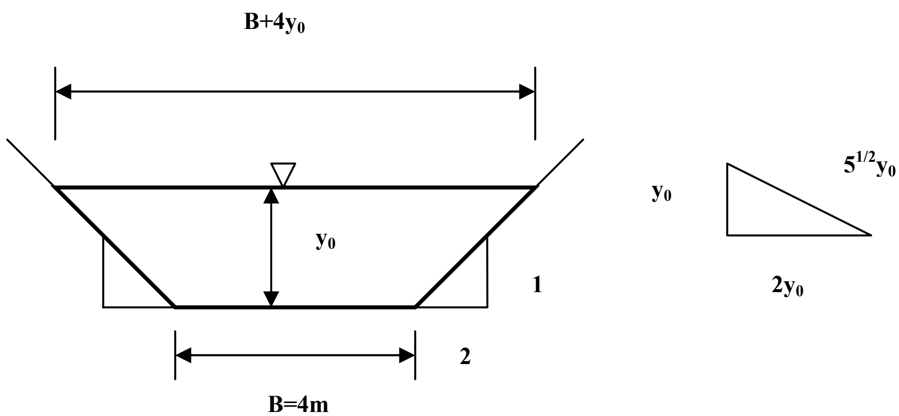
$$X = \frac{y_0^{5/3}}{(4 + 2y_0)^{2/3}}$$

$$y_0 = 2m \rightarrow X = 0.794 \neq 0.694$$
$$y_0 = 1.90m \rightarrow X = 0.741 \neq 0.694$$
$$y_0 = 1.80m \rightarrow X = 0.689 \neq 0.694$$
$$y_0 = 1.81m \rightarrow X = 0.694 \cong 0.694$$

The uniform water depth for this rectangular channel is y_0 = 1.81 m.

There is no implicit solution for calculation of water depths. Trial and error must be used in calculations.

b) Trapezoidal Cross-Section



$$A = \frac{B + B + 4y_0}{2} y_0 = (B + 2y_0)y_0 = (4 + 2y_0)y_0$$
$$P = B + 2 \times \sqrt{5} \times y_0 = 4 + 4.472 y_0$$
$$R = \frac{A}{P} = \frac{(4 + 2y_0)y_0}{4 + 4.472 y_0}$$

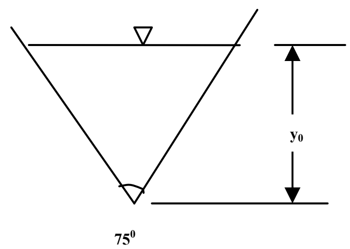
Trial and error method will be used to find the uniform water depth.

$$Q = AV = A\frac{1}{n} R^{2/3} S_o^{1/2}$$
$$10 = \left(4y_0 + 2y_o^2\right) \times \frac{1}{0.014} \times \left(\frac{4y_0 + 2y_0^2}{4 + 4.472 y_0}\right)^{2/3} \times 0.0004^{1/2}$$
$$\frac{10 \times 0.014}{0.02} = 7$$
$$X = \frac{\left(4y_0 + 2y_0^2\right)^{5/3}}{\left(4 + 4.472 y_0\right)^{2/3}}$$

$$y_0 = 1m \rightarrow X = 4.95 \neq 7$$
$$y_0 = 1.10m \rightarrow X = 5.70 \neq 7$$
$$y_0 = 1.20m \rightarrow X = 6.73 \neq 7$$
$$y_0 = 1.22m \rightarrow X = 6.94 \neq 7$$
$$y_0 = 1.23m \rightarrow X = 7.05 \cong 7$$

The uniform water depth for this trapezoidal channel is y_0 =1.23 m.

Example 4.5: A triangular channel with an apex angle of 75^0 carries a flow of 1.20 m^3/sec at a depth of 0.80 m. If the bed slope is S_0 = 0.009, find the roughness coefficient n of the channel.



Solution:

y_0 = Normal depth = 0.80 m

Referring to Figure,

Area $$A = \frac{1}{2} \times 0.80 \times 2 \times 0.80 \times \tan\left(\frac{75}{2}\right) = 0.491 m^2$$

Wetted perimeter $$P = 2 \times 0.80 \times \sec 37.5^0 = 2.02m$$

$$R = \frac{A}{P} = \frac{0.491}{2.02} = 0.243m$$

$$n = \frac{AR^{2/3}S_0^{1/2}}{Q} = \frac{0.491 \times 0.243^{2/3} \times 0.009^{0.5}}{1.20} = 0.0151$$

4.10. Best Hydraulic Cross-Section

a) The *best hydraulic (the most efficient)* cross-section for a given Q, n, and S_0 is the one with a minimum excavation and minimum lining cross-section. A = A_{min} and P = $P_{min.}$ The minimum cross-sectional area and the minimum lining area will reduce construction expenses and therefore that cross-section is economically the most efficient one.

$$V = \frac{Q}{A} \rightarrow \frac{Q}{A_{min}} = V_{max}$$

$$V = \frac{1}{n} S_0^{1/2} R^{2/3} = C \times R^{2/3}$$
$$V = V_{\max} \rightarrow R = R_{\max}$$

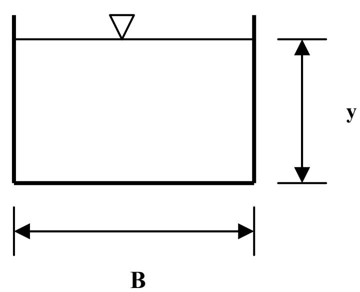
$$R = \frac{A}{P}$$
$$R = R_{\max} \rightarrow P = P_{\min}$$

b) The best hydraulic cross-section for a given A, n, and S_0 is the cross-section that conveys maximum discharge.

$$Q = A \frac{1}{n} R^{2/3} S_0^{1/2}$$
$$Q = C' \times R^{2/3}$$
$$C' = const$$
$$Q = Q_{\max} \rightarrow R = R_{\max}$$
$$R = \frac{A}{P}$$
$$R = R_{\max} \rightarrow P = P_{\min}$$

The cross-section with the minimum wetted perimeter is the best hydraulic cross-section within the cross-sections with the same area since lining and maintenance expenses will reduce substantially.

4.10.1. Rectangular Cross-Sections



$$A = By = \text{Constant}$$

$$B = \frac{A}{y}$$

$$P = B + 2y = \frac{A}{y} + 2y$$

Since $P = P_{min}$ for the best hydraulic cross-section, taking derivative of P with respect to y,

$$\frac{dP}{dy} = \frac{\frac{dA}{dy} \times y - A}{y^2} + 2 = 0$$

$$\frac{A}{y^2} = 2 \rightarrow A = 2y^2 = By \quad (4.38)$$

$$B = 2y$$

The best rectangular hydraulic cross-section for a constant area is the one with B = 2y. The hydraulic radius of this cross-section is,

$$R = \frac{A}{P} = \frac{2y^2}{4y} = \frac{y}{2} \quad (4.39)$$

For all best hydraulic cross-sections, the hydraulic radius should always be R = y/2 regardless of their shapes.

Example 4.6: Calculate the best hydraulic rectangular cross-section to convey Q=10 m^3/sec discharge with n= 0.02 and S_0= 0.0009 canal characteristics.

Solution: For the best rectangular hydraulic cross-section,

$$A = 2y^2$$

$$R = \frac{y}{2}$$

$$Q = A\frac{1}{n}R^{2/3}S_0^{1/2}$$

$$10 = 2y^2 \times \frac{1}{0.02} \times \left(\frac{y}{2}\right)^{2/3} \times 0.0009^{1/2}$$

$$y^{8/3} = \frac{10 \times 0.02 \times 2^{2/3}}{2 \times 0.03} = 5.29$$

$$y = 5.29^{3/8} = 1.87m$$

$$B = 2 \times y = 2 \times 1.87 = 3.74m$$

$$y = 1.87m$$

Implicit solutions are possible to calculate the water depths for the best hydraulic cross-sections.

4.10.2. Trapezoidal Cross-Sections

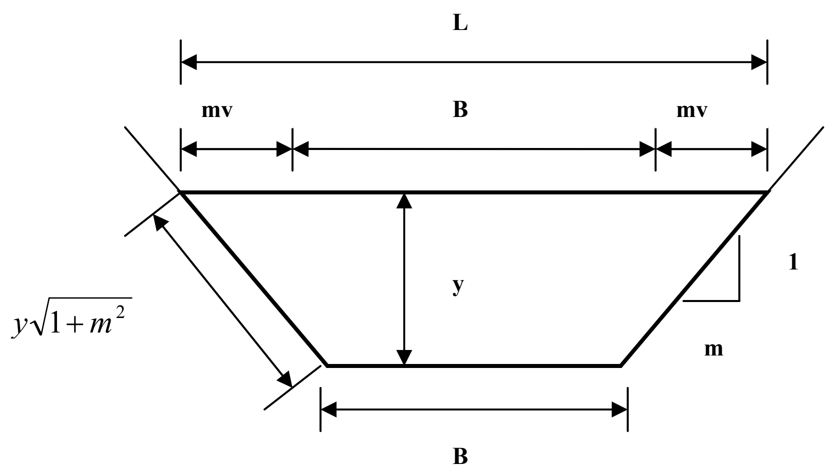


Figure 4.12.

$$A = \frac{B + B + 2my}{y} \times y = (B + my)y$$

$$P = B + 2y\sqrt{1+m^2}$$

$$B = \frac{A}{y} - my$$

$$P = \frac{A}{y} - my + 2y\sqrt{1+m^2}$$

As can be seen from equation, wetted perimeter is a function of side slope m and water depth y of the cross-section.

a) **For a given side slope m, what will be the water depth y for best hydraulic trapezoidal cross-section?**

For a given area A, $P = P_{\min}$,

$$\frac{dP}{dy} = 0, \frac{dA}{dy} = 0$$

$$\frac{dP}{dy} = \frac{\frac{dA}{dy}y - A}{y^2} - m + 2\sqrt{1+m^2} = 0$$

$$\frac{A}{y^2} = -m + 2\sqrt{1+m^2}$$

$$A = \left(2\sqrt{1+m^2} - m\right)y^2 \qquad (4.40)$$

$$P = \left(2\sqrt{1+m^2} - m\right)y - my + 2y\sqrt{1+m^2}$$
$$P = 2y\sqrt{1+m^2} - my - my + 2y\sqrt{1+m^2}$$

$$P = 2y\left(2\sqrt{1+m^2} - m\right) \qquad (4.41)$$

The hydraulic radius R, channel bottom width B, and free surface width L may be found as,

$$R = \frac{A}{P} = \frac{\left(2\sqrt{1+m^2} - m\right)y^2}{2y\left(2\sqrt{1+m^2} - m\right)}$$
$$R = \frac{y}{2}$$
$$B = \frac{A}{y} - my = \left(2\sqrt{1+m^2} - m\right)y - my$$

$$B = 2y\left(\sqrt{1+m^2} - m\right) \qquad (4.42)$$

$$L = B + 2my$$
$$L = 2y\left(\sqrt{1+m^2} - m\right) + 2my$$

$$L = 2y\sqrt{1+m^2} \qquad (4.43)$$

b) For a given water depth y, what will be the side slope m for best hydraulic trapezoidal cross-section?

$$P = P_{\min}$$
$$\frac{dP}{dm} = 0, \frac{dA}{dm} = 0$$

$$P = \frac{A}{y} - my + 2y\sqrt{1+m^2}$$

$$\frac{dP}{dm} = \frac{\frac{dA}{dm}y - 0}{y^2} - y + 2y\frac{2m}{2\sqrt{1+m^2}} = 0$$
$$y = \frac{2my}{\sqrt{1+m^2}} \rightarrow \frac{2m}{\sqrt{1+m^2}} = 1$$
$$4m^2 = 1 + m^2 \rightarrow 3m^2 = 1$$

$$m = \frac{1}{\sqrt{3}}$$

$$Tan\alpha = \frac{1}{m} = \sqrt{3} \rightarrow \alpha = 60^0 \qquad (4.44)$$

$$A = \left(2\sqrt{1+m^2} - m\right)y^2$$

$$A = \left(2\sqrt{1+\frac{1}{3}} - \frac{1}{\sqrt{3}}\right)y^2$$

$$A = \left(\frac{2\sqrt{4}-1}{\sqrt{3}}\right)y^2 = \frac{3}{\sqrt{3}}y^2$$

$$A = \sqrt{3}y^2 \qquad (4.45)$$

$$P = 2y\left(2\sqrt{1+m^2} - m\right)$$

$$P = 2y\left(2\sqrt{1+\frac{1}{3}} - \frac{1}{\sqrt{3}}\right)$$

$$P = 2y\left(\frac{4-1}{\sqrt{3}}\right) = \frac{6y}{\sqrt{3}}$$

$$P = 2y\sqrt{3} \qquad (4.46)$$

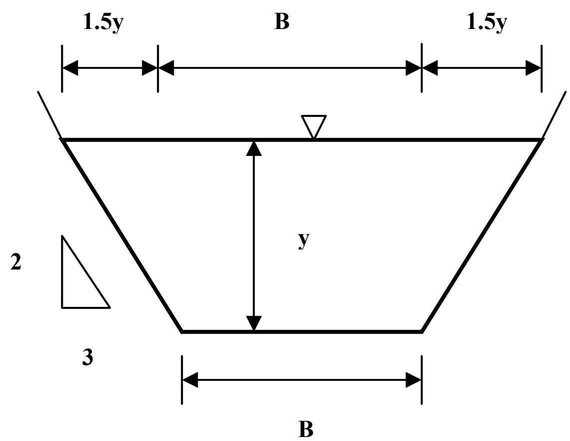
$$B = \frac{A}{y} - my = \frac{\sqrt{3}y^2}{y} - \frac{1}{\sqrt{3}}y = \frac{3-1}{\sqrt{3}}y$$

$$B = \frac{2\sqrt{3}}{3}y = \frac{P}{3} \qquad (4.47)$$

$$R = \frac{A}{P} = \frac{\sqrt{3}y^2}{2y\sqrt{3}} = \frac{y}{2}$$

The channel bottom width is equal one third of the wetted perimeter and therefore sides and channel width B are equal to each other at the best trapezoidal hydraulic cross-section. Since $\alpha = 60^0$, the cross-section is half of the hexagon.

Example 4.7: Design the trapezoidal channel as best hydraulic cross-section with Q= 10 m^3/sec, n= 0.014, S_0= 0.0004, and m= 3/2.



Solution:

$$A = \frac{B+B+3y}{2} \times y = (B+1.5y)y$$

$$P = B + 2y\sqrt{1+\frac{9}{4}} = B + 2y\sqrt{3.25} = B + 3.61y$$

$$B = \frac{A}{y} - 1.5y$$

$$P = \frac{A}{y} - 1.5y + 3.61y = \frac{A}{y} + 2.11y$$

$$\frac{dP}{dy} = \frac{\frac{dA}{dy}y - A}{y^2} + 2.11 = 0$$

$$A = 2.11y^2$$

$$B = 2.11y - 1.5y = 0.61y$$

$$P = 0.61y + 3.61y = 4.22y$$

$$R = \frac{A}{P} = \frac{2.11y^2}{4.22y} = \frac{y}{2}$$

$$Q = A\frac{1}{n}R^{2/3}S_0^{1/2}$$

$$10 = 2.11y^2 \times \frac{1}{0.014} \times \left(\frac{y}{2}\right)^{2/3} \times 0.0004^{1/2}$$

$$y^{8/3} = \frac{10 \times 0.014 \times 2^{2/3}}{2.11 \times 0.02} = 5.266$$

$$y = 5.266^{3/8} = 1.86m$$

$$A = 2.11y^2 = 2.11 \times 1.86^2 = 7.30m^2$$

$$B = 0.61y = 0.61 \times 1.86 = 1.13m$$

$$L = B + 3y = 1.13 + 3 \times 1.86 = 6.71m$$

$$P = 4.22y = 4.22 \times 1.86 = 7.85m$$

$$R = \frac{A}{P} = \frac{7.30}{7.85} = 0.93m \rightarrow R = \frac{y}{2} = \frac{1.86}{2} = 0.93m$$

Example 4.8: A slightly rough brick-lined trapezoidal channel (n = 0.017) carrying a discharge of Q = 25 m^3/sec is to have a longitudinal slope of S_0 = 0.0004. Analyze the proportions of,

a) An efficient trapezoidal channel section having a side of 1.5 horizontal: 1 vertical,

b) the most efficient-channel section of trapezoidal shape.

Solution:

Case a): m = 1.5

For an efficient trapezoidal channel section, by Equ. (4.40),

$$A = \left(2\sqrt{1+m^2} - m\right)y^2$$

$$A = \left(2\sqrt{1+1.5^2} - 1.5\right)y^2 = 2.106y^2$$

$$R = \frac{y}{2}, Q = 25m^3/\sec$$

$$25 = \frac{1}{0.017} \times 2.106y^2 \times \left(\frac{y}{2}\right)^{2/3} \times 0.0004^{1/2}$$

$$y = 2.83m$$

$$B = 2y\left(\sqrt{1+m^2} - m\right)$$
$$B = 2\times 2.83\times\left(\sqrt{1+1.5^2} - 1.5\right) = 1.72m$$

Case b): For the most-efficient trapezoidal channel section,

$$m = \frac{1}{\sqrt{3}} = 0.577$$
$$A = \sqrt{3}y^2 = 1.732y^2$$
$$R = \frac{y}{2}$$
$$25 = \frac{1}{0.017}\times 1.732y^2\times\left(\frac{y}{2}\right)^{2/3}\times 0.0004^{0.5}$$

$$y = 3.05m$$

$$B = \frac{2}{\sqrt{3}}\times 3.05 = 3.52m$$

4.10.3. Half a Circular Conduit

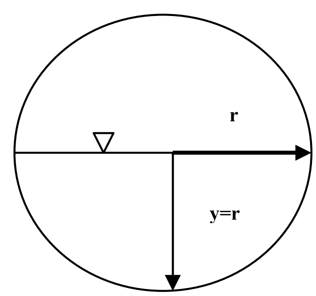


Figure 4.13.

Wetted area A, and wetted perimeter of the half a circular conduit,

$$A = \frac{\pi r^2}{2}$$
$$P = \pi r \qquad (4.48)$$
$$R = \frac{A}{P} = \frac{\frac{\pi r^2}{2}}{\pi r} = \frac{r}{2} = \frac{y}{2}$$

Half a circular conduit itself is a best hydraulic cross-section.

Example 4.9:

a) What are the best dimensions y and B for a rectangular brick channel designed to carry 5 m^3/sec of water in uniform flow with $S_0 = 0.001$, and n = 0.015?
b) Compare results with a half-hexagon and semi circle.

Solution:

a) For best rectangular cross-sections, the dimensions are,

$$A = 2y^2 \quad , \quad R = \frac{y}{2}$$

$$Q = A\frac{1}{n}R^{2/3}S_0^{1/2}$$

$$5 = 2y^2 \times \frac{1}{0.015} \times \left(\frac{y}{2}\right)^{2/3} \times 0.001^{0.5}$$

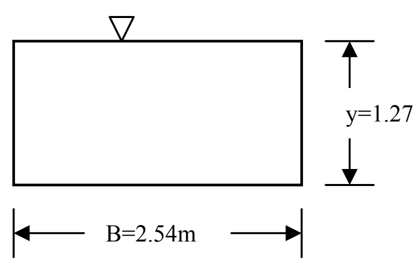
$$y^{8/3} = \frac{5 \times 0.015 \times 2^{2/3}}{2 \times 0.001^{0.5}} = 1.882$$

$$y = 1.27m$$

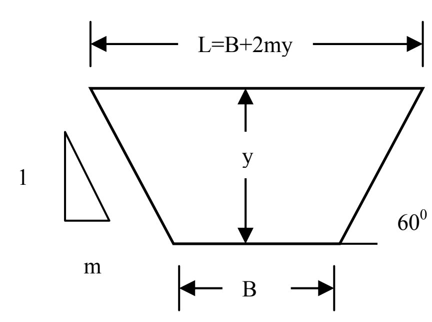
The proper area and width of the best rectangular cross-section are,

$$A = 2y^2 = 2 \times 1.27^2 = 3.23m^2$$

$$B = \frac{A}{y} = 2y = \frac{3.23}{1.27} = 2.54m$$



b) It is constructive to see what discharge a half-hexagon and semi circle would carry for the same area of A = 3.23 m^2.



$$m = \cot\alpha = \cot 60^0 = \frac{1}{\sqrt{3}} = 0.577$$

$$A = \frac{(B + B + 2my)y}{2} = (B + my)y \qquad \text{(A)}$$
$$A = (B + 0.577y)y$$

$$P = B + 2y\sqrt{1+m^2} \qquad \text{(B)}$$
$$P = B + 2y\sqrt{1+0.577^2} = B + 2.31y$$

Using Equs. (A) and (B),

$$B = \frac{A}{y} - 0.577y$$
$$P = \frac{A}{y} - 0.577y + 2.31y$$
$$P = \frac{3.23}{y} + 1.733y$$
$$\frac{dP}{dy} = -\frac{3.23}{y^2} + 1.733 = 0$$
$$y^2 = \frac{3.23}{1.733} \rightarrow y = 1.37m$$

$$B = \frac{3.23}{1.37} - 0.577 \times 1.37 = 1.57m$$
$$P = 1.57 + 2.31 \times 1.37 = 4.73m$$
$$R = \frac{A}{P} = \frac{3.23}{4.73} = 0.68m \rightarrow R = \frac{y}{2} = \frac{1.37}{2} \cong 0.68m$$

The discharge conveyed in this half a hexagon cross-section is,

$$Q = AV = 3.23 \times \frac{1}{0.015} \times 0.68^{2/3} \times 0.001^{0.5} = 5.27\, m^3/\sec$$
$$\frac{5.27 - 5.00}{5.00} = 0.054$$

Half of a hexagon cross-section with the same area of the rectangular cross-section will convey 5.4 per cent more discharge.

For a semicircle cross-section,

$$A = 3.23 = \frac{\pi D^2}{8}$$

$$D^2 = \frac{8 \times 3.23}{3.14} \rightarrow D = 2.87m$$

$$P = \frac{\pi D}{2} = \frac{3.14 \times 2.87}{2} = 4.51m$$

$$R = \frac{A}{P} = \frac{3.23}{4.51} = 0.716m = \frac{D}{4}$$

$$Q = 3.23 \times \frac{1}{0.015} \times 0.716^{2/3} \times 0.001^{0.5} = 5.45\, m^3/\sec$$

$$\frac{5.45 - 5.00}{5.00} = 0.09$$

$$\frac{5.45 - 5.27}{5.27} = 0.03$$

Semicircle carries more discharge comparing with the rectangular and trapezoidal cross-sections by 9 and 3 per cent respectively.

Table 4.3. Proportions of some most efficient sections

Channel Shape	**A**	**P**	**B**	**R**	**L**
Rectangle (half square)	$2y^2$	4y	2y	$\frac{y}{2}$	2y
Trapezoidal (half regular hexagon)	$\sqrt{3}y$	$2\sqrt{3}y$	$\frac{2}{\sqrt{3}}y$	$\frac{y}{2}$	$\frac{4}{\sqrt{3}}y$
Circular (semicircle)	$\frac{\pi}{2}y^2$	πy	-	$\frac{y}{2}$	2y
Triangle (vertex angle = 90^0)	y^2	$2\sqrt{3}y$	-	$\frac{y}{2\sqrt{2}}$	2y

(A = Area, P = Wetted perimeter, B = Base width,
R = Hydraulic radius, L = Water surface width)

4.10.4. Circular Conduit with a Free Surface

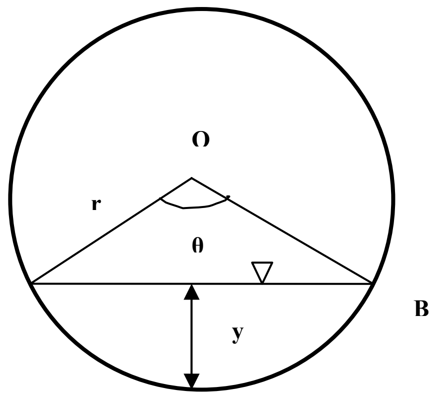


Figure 4.14.

A circular pipe with radius r is conveying water with depth y. The angle of the free water surface with the center of the circle is θ. Wetted area and wetted perimeter of the flow is,

$$360^0 = 2\pi radian \rightarrow A = \pi r^2 \rightarrow P = 2\pi r$$

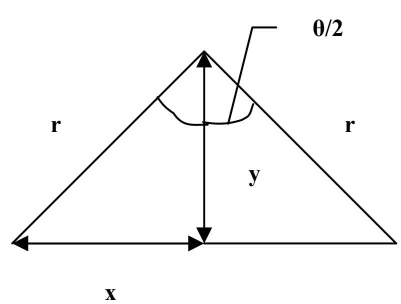
For θ radian, the area and the perimeter of the sector are,

$$A = \theta \frac{\pi r^2}{2\pi} = \frac{r^2\theta}{2} \qquad (4.49)$$

$$P_{AB} = \theta \frac{2\pi r}{2\pi} = r\theta \qquad (4.50)$$

Wetted area of the flow with the flow depth y is,

$$A = \frac{r^2\theta}{2} - A_{OAB} \qquad (4.51)$$



$$Sin\frac{\theta}{2} = \frac{x}{r} \rightarrow x = rSin\frac{\theta}{2}$$
$$Cos\frac{\theta}{2} = \frac{y}{r} \rightarrow y = rCos\frac{\theta}{2}$$

$$A_{OAB} = \frac{2rSin\frac{\theta}{2} \times rCos\frac{\theta}{2}}{2} = \frac{r^2}{2} \times 2Sin\frac{\theta}{2}Cos\frac{\theta}{2}$$
$$A_{OAB} = \frac{r^2}{2} \times Sin\frac{2\theta}{2} = \frac{r^2}{2}Sin\theta$$

Substituting A_{AOB} into the Equ. (4.51) give the wetted area of the flow,

$$A = \frac{r^2\theta}{2} - \frac{r^2}{2}Sin\theta \qquad (4.52)$$

a) What will be the θ angle for the maximum discharge?

$$Q = A\frac{1}{n}\left(\frac{A}{P}\right)^{2/3} S_0^{1/2} = \frac{1}{n}S_0^{1/2}\frac{A^{5/3}}{P^{2/3}}$$
$$C' = \frac{1}{n}S_0^{1/2} = cons$$
$$Q = C'\frac{A^{5/3}}{P^{2/3}}$$

Derivative of the discharge equation will be taken with respect to θ.

$$\frac{dQ}{d\theta} = \frac{\frac{5}{3}A^{2/3}\frac{dA}{d\theta}P^{2/3} - \frac{2}{3}P^{-1/3}\frac{dP}{d\theta}A^{5/3}}{P^{4/3}} \times C' = 0$$

$$\frac{5}{3} \times A^{2/3} \times \frac{dA}{d\theta} \times P^{2/3} = \frac{2}{3} \times \frac{dP}{d\theta} \times \frac{A^{5/3}}{P^{1/3}}$$
$$5 \times \frac{dA}{d\theta} \times P^{\frac{2}{3}+\frac{1}{3}} = 2 \times \frac{dP}{d\theta} \times A^{\frac{5}{3}-\frac{2}{3}}$$

$$5\frac{dA}{d\theta}P = 2\frac{dP}{d\theta}A \qquad (4.53)$$

Derivatives of the wetted area and the wetted perimeter are,

$$A = \frac{r^2}{2}\left(\theta - Sin\theta\right)$$

$$\frac{dA}{d\theta} = \frac{r^2}{2}\left(1 - Cos\theta\right)$$

$$P = r\theta$$

$$\frac{dP}{d\theta} = r$$

Substituting the derivates to the Equ. (4.53),

$$5\frac{r^2}{2}\left(1 - Cos\theta\right)r\theta = 2r\frac{r^2}{2}\left(\theta - Sin\theta\right)$$

$$5\theta\left(1 - Cos\theta\right) = 2\left(\theta - Sin\theta\right)$$

$$5\theta - 5\theta Cos\theta - 2\theta + 2Sin\theta = 0$$

$$3\theta - 5\theta Cos\theta + 2Sin\theta = 0 \qquad (4.54)$$

Solution of the Equ. (4.54) give the θ angle as,

$$\theta = 302^0 30' = 5.28 radian$$

The wetted area, wetted perimeter, and hydraulic radius of the flow are,

$$A = \frac{r^2}{2}\left(\theta - Sin\theta\right)$$

$$A = \frac{r^2}{2}\left(5.28 - Sin302.5^0\right) = \frac{r^2}{2}\left(5.28 + 0.84\right)$$

$$A = 3.06r^2$$

$$P = r\theta = 5.28r$$

$$R = \frac{A}{P} = \frac{3.06r^2}{5.28r} = 0.58r$$

The water depth in the circular conduit is,

$$y = r - rCos\frac{\theta}{2} = r - rCos\left(\frac{302.5^0}{2}\right) = r(1 + 0.88)$$
$$y = 1.88r = 0.94D$$

$$Q = A\frac{1}{n}R^{2/3}S_0^{1/2}$$
$$Q_{\max} = 3.06r^2 \times \frac{1}{n} \times (0.58r)^{2/3} S_0^{1/2}$$
$$Q_{\max} = 2.13r^{8/3}\frac{1}{n}S_0^{1/2}$$

b) What will be the θ angle for the maximum velocity?

$$V = \frac{1}{n}R^{2/3}S_0^{1/2}$$
$$V_{\max} \rightarrow R = \frac{A}{P} \rightarrow \max$$

$$dR = \frac{dA \times P - dP \times A}{P^2} = 0$$
$$\frac{dA}{A} = \frac{dP}{P}$$
$$\frac{\frac{r^2}{2}(1 - \cos\theta)}{\frac{r^2}{2}(\theta - \sin\theta)} = \frac{r}{r\theta}$$

$$(1 - \cos\theta)\theta = \theta - \sin\theta$$
$$\theta - \theta\cos\theta = \theta - \sin\theta$$

$$\theta = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\theta = 257^0 30' = 2.49 radian$$

$$y = r - r\cos\frac{\theta}{2} = r\left(1 - \cos\frac{257.5^0}{2}\right) = 1.626r = 0.813D$$

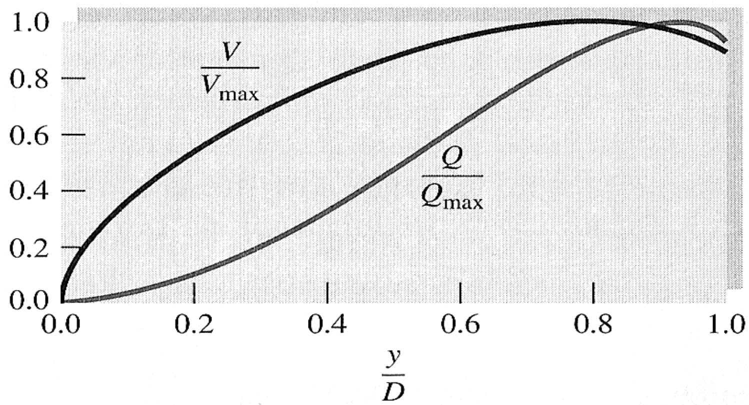


Figure 4.15.

4.11. Compound Sections

Some channel sections may be formed as a combination of elementary sections. Typically natural channels, such as rivers, have flood plains which are wide and shallow compared to the main channel. Fig. (4.16) represents a simplified section of a stream with flood banks.

Consider the compound section to be divided into subsections by arbitrary lines. These can be extensions of the deep channel boundaries as in Fig. (4.16). Assuming the longitudinal slope to be same for all subsections, it is easy to see that the subsections will have different mean velocities depending upon the depth and roughness of the boundaries. Generally, overbanks have larger size roughness than the deeper main channel. If the mean velocities V_i in the various subsections are known then the total discharge is $\sum V_i A_i$.

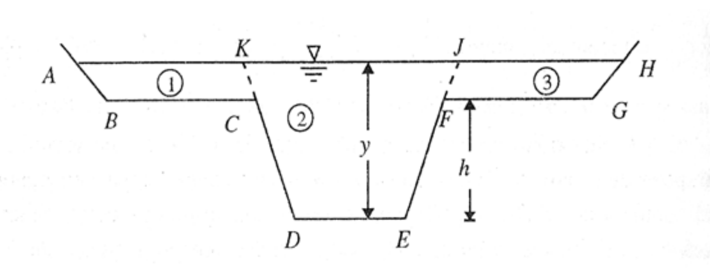


Figure 4.16. Compound section

If the depth of flow is confined to the deep channel only (y < h), calculation of discharge by using Manning's equation is very simple. However, when the flow spills over the flood plain (y > h), the problem of discharge calculation is complicated as the calculation may give a smaller hydraulic radius for the whole stream section and hence the discharge may be underestimated. The following method of discharge estimation can be used. In this method, while calculating the wetted perimeter for the sub-areas, the imaginary divisions (FJ and CK in the Figure) are considered as boundaries for the deeper portion only and neglected completely in the calculation relating to the shallower portion.

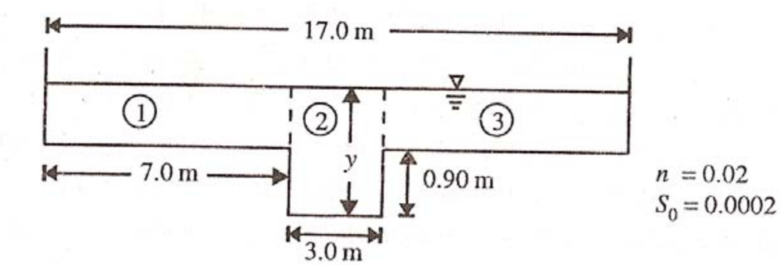
1. The discharge is calculated as the sum of the partial discharges in the sub-areas; for e.g. units 1, 2 and 3 in Fig. (4.16)

$$Q_p = \sum Q_i = \sum V_i A_i \qquad (4.55)$$

2. The discharge is also calculated by considering the whole section as one unit, (ABCDEFGH area in Fig.4.16), say Q_w.

3. The larger of the above discharges, Q_p and Q_w, is adopted as the discharge at the depth y.

Example 4.10: For the compound channel shown in the Figure, determine the discharge for a depth of flow 1.20 m. n = 0.02, S_0 = 0.0002.



Solution:

a): y = 1.20 m

Partial area discharge; Sub-area 1 and 3:

$$A_1 = 7.0 \times 0.3 = 2.1 m^2$$
$$P_1 = 0.3 + 7.0 = 7.3 m$$
$$R_1 = \frac{A_1}{P_1} = \frac{2.1}{7.3} = 0.288 m$$
$$Q_{p1} = \frac{1}{0.02} \times 2.1 \times 0.288^{2/3} \times 0.0002^{0.5}$$
$$Q_{p1} = 0.648 m^3/\sec = Q_{p3}$$

Sub-area 2:

$$A_2 = 3.0 \times 1.2 = 3.6 m^2$$
$$P_2 = 3.0 + 0.9 + 0.9 = 4.8 m$$
$$R_2 = \frac{A_2}{P_2} = \frac{3.6}{4.8} = 0.75 m$$
$$Q_{p2} = \frac{1}{0.02} \times 3.6 \times 0.75^{2/3} \times 0.0002^{0.5} = 2.10 m^3/\sec$$

Total discharge,

$$Q_p = Q_{p1} + Q_{p2} + Q_{p3}$$
$$Q_p = 0.648 + 2.10 + 0.6.48 = 3.396 m^3/\sec$$

b): By the total-section method:

$$A = 2.1 + 2.1 + 3.6 = 7.8 m^2$$
$$P = 0.3 + 7.0 + 0.9 + 3.0 + 0.9 + 7.0 + 0.3 = 19.4 m$$
$$R = \frac{A}{P} = \frac{7.8}{19.4} = 0.402$$
$$Q_w = \frac{1}{0.02} \times 7.8 \times 0.402^{2/3} \times 0.0002^{0.5} = 3.00 m^3/\sec$$

Since $Q_p > Q_w$, the discharge in the channel is,

$$Q = Q_p = 3.396 m^3/\sec$$

4.10 Design of Irrigation Channels

For a uniform flow in a canal,

$$Q = \frac{1}{n} A R^{2/3} S_0^{0.5}$$

Where A and R are in general, functions of the geometric elements of the canal. If the canal is of trapezoidal cross-section,

$$Q = f\left(n, y_0, S_0, B, m\right) \qquad (4.56)$$

Equ. (4.56) has six variables out of which one is a dependent variable and the rest five are independent ones. Similarly, for other channel shapes, the number of variables depends upon the channel geometry. In a channel design problem, the independent variables are known either explicitly or implicitly, or as inequalities, mostly in terms of empirical relationships. The canal-design practice given below is meant only for rigid-boundary channels, i.e. for lined an unlined non-erodible channels.

4.12.1. Canal Section

Normally a trapezoidal section is adopted. Rectangular cross-sections are also used in special situations, such as in rock cuts, steep chutes and in cross-drainage works.

The side slope, expressed as *m* horizontal: 1 vertical, depends on the type pf canal, i.e. lined or unlined, nature and type of soil through which the canal is laid. The slopes are designed to withstand seepage forces under critical conditions, such as;

1. A canal running full with banks saturated due to rainfall,
2. The sudden drawdown of canal supply.

Usually the slopes are steeper in cutting than in filling. For lined canals, the slopes roughly correspond to the angle of repose of the natural soil and the values of *m* range from 1.0 to 1.5 and rarely up to 2.0. The slopes recommended for unlined canals in cutting are given in Table (4.4).

Table 4.3. Side slopes for unlined canals in cutting

Type of soil	m
Very light loose sand to average sandy soil	1.5 – 2.0
Sandy loam, black cotton soil	1.0 – 1.5
Sandy to gravel soil	1.0 – 2.0
Murom, hard soil	0.75 – 1.5
Rock	0.25 – 0.5

4.12.2. Longitudinal Slope

The longitudinal slope is fixed on the basis of topography to command as much area as possible with the limiting velocities acting as constraints. Usually the slopes are of the order of 0.0001. For lined canals a velocity of about 2 m/sec is usually recommended.

4.12.3. Roughness coefficient n

Procedures for selecting *n* are discussed in Section (4.6.1). Values of n can be taken from Table (4.2).

4.12.4. Permissible Velocities

Since the cost for a given length of canal depends upon its size, if the available slope permits, it is economical to use highest safe velocities. High velocities may cause scour and erosion of the boundaries. As such, in unlined channels the maximum permissible velocities refer to the velocities that can be safely allowed in the channel without causing scour or erosion of the channel material.

In lined canals, where the material of lining can withstand very high velocities, the maximum permissible velocity is determined by the stability and durability of the lining and also on the erosive action of any abrasive material that may be carried in the stream. The permissible maximum velocities normally adopted for a few soil types and lining materials are given in Table (4.5).

Table 4.4. Permissible Maximum velocities

Nature of boundary	Permissible maximum velocity (m/sec)
Sandy soil	0.30 – 0.60
Black cotton soil	0.60 – 0.90
Hard soil	0.90 – 1.10
Firm clay and loam	0.90 – 1.15
Gravel	1.20
Disintegrated rock	1.50
Hard rock	4.00
Brick masonry with cement pointing	2.50
Brick masonry with cement plaster	4.00
Concrete	6.00
Steel lining	10.00

In addition to the maximum velocities, a minimum velocity in the channel is also an important constraint in the canal design. Too low velocity would cause deposition of suspended material, like silt, which cannot only impair the carrying capacity but also increase the maintenance costs. Also, in unlined canals, too low a velocity may encourage weed growth. The minimum velocity in irrigation channels is of the order of 0.30 m/sec.

4.12.5. Free Board

Free board for lined canals is the vertical distance between the full supply level to the top of lining (Fig. 4.17). For unlined canals, it is the vertical distance from the full supply level to the top of the bank.

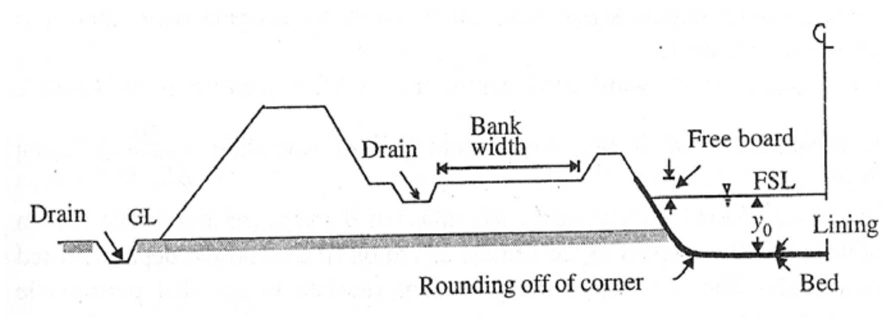


Figure 4.10. Typical cross-section of a lined canal

This distance should be sufficient to prevent overtopping of the canal lining or banks due to waves. The amount of free board provided depends on the canal size, location, velocity and depth of flow. Table (4.5) gives free board heights with respect to the maximum discharge of the canal.

Table 4.6.

Discharge (m^3/sec)	**Free board (m)**	
	Unlined	**Lined**
Q < 10.0	0.50	0.60
Q ≥ 10.0	0.75	0.75

4.10.6. Width to Depth Ratio

The relationship between width and depth varies widely depending upon the design practice. If the hydraulically most-efficient channel cross-section is adopted,

$$m = \frac{1}{\sqrt{3}} \rightarrow B = \frac{2y_0}{\sqrt{3}} = 1.155y_0 \rightarrow \frac{B}{y_0} = 1.155$$

If any other value of m is use, the corresponding value of B/y_0 for the efficient section would be from Equ. (4.18),

$$\frac{B}{y_0} = 2\left(2\sqrt{1+m^2} - m\right) \qquad (4.57)$$

In large channels it is necessary to limit the depth to avoid dangers of bank failure. Usually depths higher than about 4.0 m are applied only when it is absolutely necessary.

For selection of width and depth, the usual procedure is to adopt a recommended value

Example 4.11: A trapezoidal channel is to carry a discharge of 40 m^3/sec. The maximum slope that can be used is 0.0004. The soil is hard. Design the channel as, a) a lined canal with concrete lining, b) an unlined non-erodible channel.

Solution:

a) Lined canal

Choose side slope of 1 : 1, i.e., m = 1.0 (from Table 4.4)

n for concrete, n = 0.013 (from Table 4.2)

From Equ. (4.57),

$$\frac{B}{y_0} = 2\left(2\sqrt{1+m^2} - m\right)$$
$$\frac{B}{y_0} = 2 \times \left(2\sqrt{1+1^2} - 1\right)$$
$$\frac{B}{y_0} = 3.67$$

Since,

$$A = (B + my_0)y_0 \rightarrow \frac{A}{y_0} = \frac{B}{y_0} + m = 3.67 + 1 = 4.67$$

For the most-efficient hydraulic cross-sections, $R = \frac{y_0}{2} = 0.5y_0$

$$Q = \frac{1}{n} AR^{2/3} S_0^{0.5}$$
$$40 = \frac{1}{0.013} \times 4.67y_0 \times (0.5y_0)^{2/3} \times 0.0004^{0.5}$$
$$y_0^{5/3} = 8.838 \rightarrow y_0 = 3.70m$$

$$A = 4.67y_0 = 4.67 \times 3.70 = 17.28m^2$$
$$V = \frac{Q}{A} = \frac{40}{17.28} = 2.31 m/\sec$$

$$B = 3.67y_0 = 3.67 \times 3.70 = 13.58m$$

This velocity value is greater than the minimum velocity of 0.30 m/sec, and further is less than the maximum permissible velocity of 6.0 m/sec for concrete. Hence the selection of B and y_0 are all right. The recommended geometric parameters of the canal are therefore:

B = 13.58 m, m = 1.0, S_0 = 0.0004

Adopt a free board of 0.75 m. The normal depth for n = 0.013 will be 3.70 m.

b) Lined canal

From Table (4.), a side slope of 1 : 1 is chosen. From Table (4.2), take n for hard soil surface as n = 0.020.

From Equ. (4.57),

$$\frac{B}{y_0} = 2\left(2\sqrt{1+m^2} - m\right)$$
$$\frac{B}{y_0} = 2\times\left(2\sqrt{1+1^2} - 1\right)$$
$$\frac{B}{y_0} = 3.67$$

Since,

$$A = \left(B + my_0\right)y_0 \rightarrow \frac{A}{y_0} = \frac{B}{y_0} + m = 3.67 + 1 = 4.67$$

For the most-efficient hydraulic cross-sections, $R = \frac{y_0}{2} = 0.5y_0$

$$40 = \frac{1}{0.020}\times 4.67y_0 \times \left(0.5y_0\right)^{2/3} \times 0.0004^{0.5}$$
$$y_0^{5/3} = 13.60 \rightarrow y_0 = 4.79m$$

Since y_0 = 4.79 m > 4.0 m, y_0 = 4.0 m is chosen,

$$\frac{B}{y_0} = 3.67 \rightarrow B = 3.67\times 4.0 = 14.68m$$
$$A = 4.67\times 4 = 18.68m^2$$

$$V = \frac{Q}{A} = \frac{40}{18.68} = 2.14\, m/\sec$$

But this velocity is larger than the permissible velocity of 0.90 – 1.10 m/sec for hard soil (Table 4.5). In this case, therefore the maximum permissible velocity will control the channel dimensions.

Adopt V = 1.10 m/sec,

$$A = \frac{40}{1.10} = 36.36m^2$$
$$y_0 = 4.0$$

$$A = (B + my_0)y_0$$
$$B = \frac{A}{y_0} - my_0 = \frac{36.36}{4.0} - 1 \times 4.0 = 5.09m$$
$$P = B + 2\sqrt{m^2 + 1}y_0$$
$$P = 5.09 + 2\sqrt{1 + 1} \times 4.0 = 16.40m$$
$$Q = \frac{1}{0.020} \times 36.36 \times \left(\frac{36.36}{16.40}\right)^{2/3} \times 0.0004^{0.5}$$
$$Q = 61.82\, m^3/\sec \rangle 40\, m^3/\sec$$

The slope of the channel will be smoother,

$$Q = \frac{A}{n} R^{2/3} S_0^{0.5}$$
$$S_0 = \left(\frac{Qn}{R^{2/3} A}\right)^2$$

$$S_0 = \left[\frac{40 \times 0.02}{\left(\frac{36.36}{16.40}\right)^{2/3} \times 36.36}\right]^2$$
$$S_0 = 0.000167$$

Hence the recommended parameters pf the canal are B = 5.09 m, m = 1, and S_0 = 0.000167. Adopt a free board of 0.75 m. The normal depth for n = 0.02 will be y_0 = 4.0 m.