Chapter 4: Products and Factors

Section 4.1: Products of Polynomials

Terminology:

- **Polynomial:**
  Any one term or the sum of terms whose variables have whole-number exponents.
  Ex: $x^2 + 3xy - 2y^2 + 5x$

- **Variable:**
  Any letter of symbol that takes the place of a value.

- **Coefficient:**
  Any number that is multiplied to a variable.

- **Constant:**
  Any number in an expression that has no variable multiplied to it.

- **Like-Terms:**
  Any terms that contain the same variable(s) with the same exponents.
  Ex: $2x$ *and* $5x$, $\;-3y^2$ *and* $6y^2$, $\;5x^3y^2$ *and* $6x^3y^2$

- **Unlike-Terms:**
  Any terms that contain either different variable(s), different exponents, or both.
  Ex: $2x$ *and* $5x^2$, $-3y$ *and* $6x$, $\;5x^2y^3$ *and* $6x^3y^2$

Multiplying Monomials

To multiply monomials, simply multiply coefficients with coefficients and variables with variables

(a) $(3x)(5x)$ (b) $(-2x^3)(4x^2)$ (c) $(6x^2y^3)(7xy^4)$

(d) $(5y)(4y)$ (e) $(3z^4)(-5z^3)$ (f) $(3m^4n^5)(-2m^2n^3)$

Multiplying Monomial by Binomials

The best method for multiplying a monomial by a binomial is to use the distributive property. In this method, we multiply each term of the binomial inside the brackets by the monomial outside the brackets.

Expand:

(a) $5x(2x^2 + 5x)$

(b) $3x^2y(5x - 3y)$

(c) $6n^2(-2n + 3n^4)$

(d) $4x^3y^2(4x^2 + 2y^2)$

Multiplying Binomials

There are multiple ways to multiply binomials. Several are shown below:

Method 1: Alge-Tiles

Create the alge-tile representations for each binomial. One along the top of a rectangle and one down the left hand side. We then create lines that will complete the rectangle. The solution will be the rectangle that is produced.

Remember: x^2-tiles are large squares, x-tiles are narrow rectangles, and ones are small rectangles:

Ex1: $(x + 5)(x - 2)$

Ex2: $(2x - 1)(x - 6)$

Ex3: $(x - 3)(x + 4)$

Method 2: Double Distributive Method

The double distributive method apples the distributive law such that you multiply each term in the second binomial by each term in the first. Then add together any like terms.

This method can be remembered in several different ways including:

a) The Anagram **FOIL**: First – Outside – Inside – Last
b) Bill, The Binomial:

$(x+2)(x-5)$

c) The Crescent Moon

$(x+2)(x-5)$

Expand Each:

Ex 1: $(x+5)(x-2)$

Ex 2: $(x-1)(x+5)$

Ex 3: $(3x-5)(5x+1)$

Ex 4: $(2x+1)(2x-4)$

Ex 5: $(7x-2y)(8x+10y)$

Ex 6: $(5m-3n)(5m+3n)$

Method 3: The Box Area Method

Create a $2x2$ box. Write the terms of the first binomial along the top, one to each square, and write the second binomial down the left side, one term to each square. Multiply the monomials on the outside of each box and add up each area to find the solution.

Expand Each:

Ex 1: $(x + 4)(x + 9)$

Ex 2: $(x - 5)(x + 3)$

Ex 3: $(3x - 1)(2x + 3)$

Ex 4: $(5x + 2)(3x - 4)$

Perfect Square Polynomials

Expand

(a) $(2x - 5)^2$

(b) $(4x + 3)^2$

Multiplying Trinomials and Other Polynomials

To multiply any polynomial by another, including trinomials, we simply make sure to multiply each term from the first polynomial by each term in the second polynomial.

Expand Each

(a) $(5x + 3)(6x^2 - 3x + 7)$

(b) $(8x - 1)(2x^2 + 5x - 2)$

(c) $(3y^2 + 5y - 2)(7y^2 - 10y + 11)$

(d) $(2j^2 - 3j + 4)(6j^2 - 2j - 1)$

(e) $(3a^2 + 5ab - 2b^2)(4a^2 - ab + 4b^2)$

(f) $(7f^2 - 3fg + g^2)(2f^2 + 7fg - 2g^2)$

Section 4.2: Factoring Polynomials

There are four major ways of factoring a polynomial:

1. **Factoring Using GCF:**
   In some situations, all terms of expression have common factors. In such cases, the gcf can be factored out.

   EXAMPLES: Solve The following

   a. $5x - 10$

   b. $12g + 18$

   c. $9d^5 - 7d^2$

   d. $3d + 4d^4$

   e. $6x^3 + 4x^2 - 12x$

   f. $3x^5 + 15x^3 + 9x^2$

   g. $-12x^3y - 20xy^2 - 16x^2y^2$

   h. $-20c^4d - 30c^3d^2 - 25c^2d$

NOTE: When determining the gcf for an expression, you must determine the gcf for the coefficients as well as any variables that they share. Note, we always choose the lowest power amongst the common variables to be included in the gcf.

Once the gcf is removed, each term remaining is the value of the original term divided by the gcf.

2. **Factoring Using Product and Sum**:
Product and Sum can only be used to factor a trinomial of the form $y = ax^2 + bx + c$ in situations where "a=1." In such cases you must determine your factors by concluding what possible combination of two numbers can multiply to "c" and add to "b"
EXAMPLES: Factor The following

a. $x^2 + 6x + 8$

b. $k^2 - 7k - 30$

c. $j^2 + 11j - 42$

d. $f^2 - 9f + 20$

e. $k^2 + 19k + 70$

f. $j^2 + 3j - 54$

g. $m^2 + 6mn + 9n^2$

h. $x^2 - 7xy + 10y^2$

i. $2x^3 + 10x^2 - 28x$

j. $5x^3 + 15x^2 + 10x$

3. **<u>Factoring Using Decomposition</u>:**
Decomposition can be used in situations where "a≠1" and a GCF cannot be removed. In such cases you must determine your factors by following these steps:
STEP1: Conclude what possible combination of two numbers can multiply to "a×c" and add to "b."
STEP2: Decompose your middle term into those two numbers.
STEP3: Group the first set and second set of terms. Pull out the GCF (*Greatest Common Factor*) of each group.
STEP4: Then factor out the common bracketed term.

EXAMPLES: Factor The following

a. $5x^2 - 7x - 6$

b. $3k^2 - 13k - 10$

c. $8j^2 + 18j - 5$

d. $15f^2 - 7f - 2$

e. $6j^2 + 17j - 14$

f. $4k^2 - 21k + 20$

g. $6n^2 - 7n - 3$

h. $10j^2 - 3j - 7$

i. $2x^2 + 7xy + 3y^2$

j. $12a^2 + 5ab - 2b^2$

k. $10g^2 - 5g - 5$

l. $9z^2 - 3z - 6$

4. Difference of Squares:

Difference of squares can only be used in situations where "b=0." In such cases both the "a" and "c" values will be perfect squares and there is a subtraction symbol between them. Your resulting factors will be the square root of each term with a different sign between them.

EXAMPLES: Factor The following

a. $x^2 - 9$

b. $9k^2 - 25$

c. $144j^2 - 100$

d. $12f^2 - 75$

e. $25z^2 - 81y^2$

f. $49v^2 - 100w^2$

g. $8m^2 - 18n^2$

h. $5q^2 - 80r^2$

i. $121h^2 - 256j^2$

j. $196a^2 - 169b^2$

Special Difference of Squares

Sometimes when a difference of squares involves an exponent of 4, it will result in a special factoring case where the difference of squares can be applied twice in the factoring process.

Factor:

(a) $x^4 - 81$

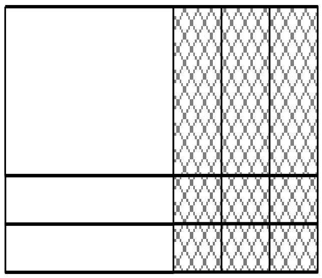
(b) $z^4 - 16$

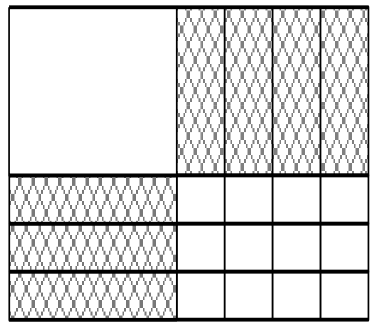
(c) $16y^4 - 1$

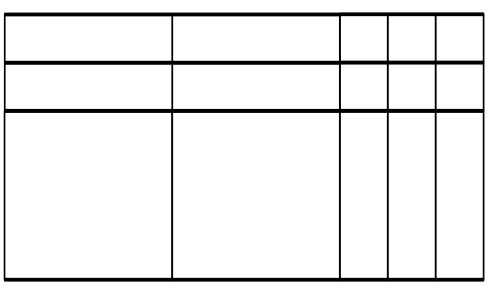
(d) $81w^4 - 256$

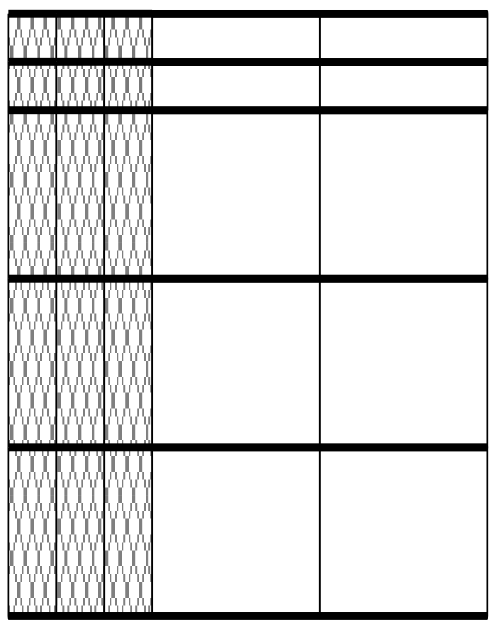
Factoring From Alge-Tile Models

In each example below, state the polynomial that is depicted and the factors that produce it.







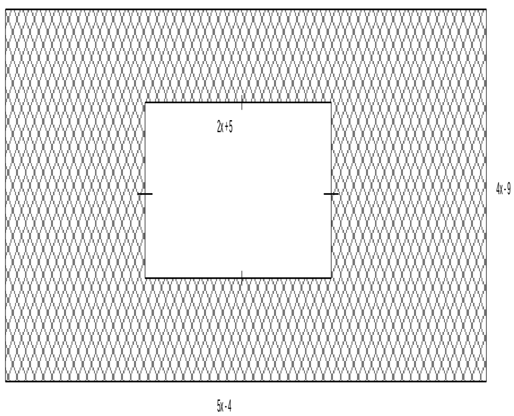


Area Questions

Much like questions that you encountered in grade 9, these polynomial questions are still common place in this course.

In each example below, determine the area of the shaded region.

(a)



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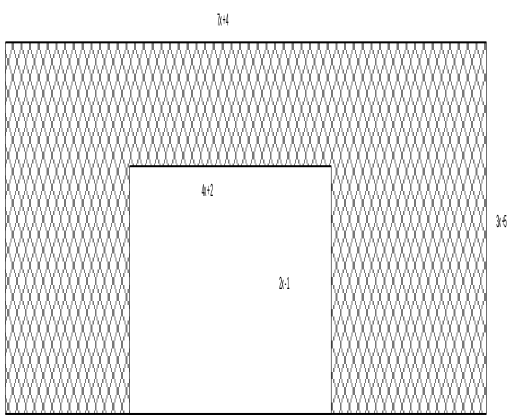
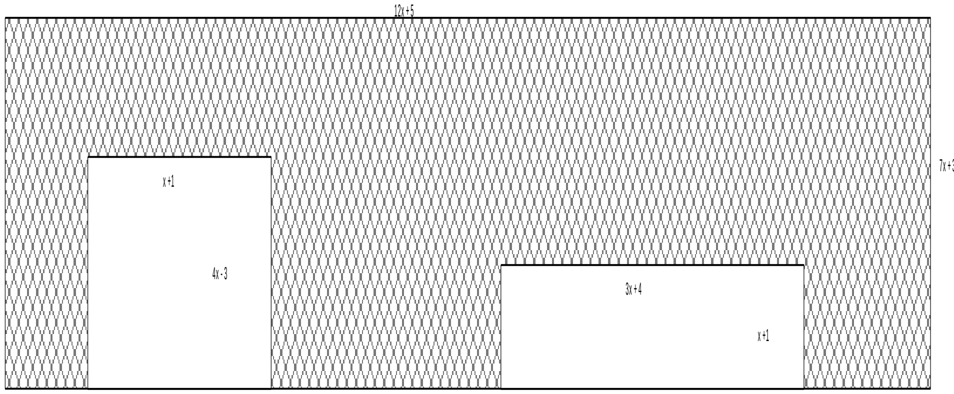


(c)



(d)