GLENCOE MATHEMATICS

Pre-Algebra

Chapter 4 Resource Masters

Glencoe McGraw-Hill

New York, New York
Columbus, Ohio
Chicago, Illinois
Peoria, Illinois
Woodland Hills, California

Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks in both English and Spanish.

Study Guide and Intervention Workbook	0-07-827794-9
Study Guide and Intervention Workbook (Spanish)	0-07-827795-7
Skills Practice Workbook	0-07-827788-4
Skills Practice Workbook (Spanish)	0-07-827790-6
Practice Workbook	0-07-827789-2
Practice Workbook (Spanish)	0-07-827791-4

Answers for Workbooks The answers for Chapter 4 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

Spanish Assessment Masters Spanish versions of forms 2A and 2C of the Chapter 4 Test are available in the *Pre-Algebra Spanish Assessment Masters* (0-07-830412-1).

Glencoe/McGraw-Hill
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Send all inquiries to:
Glencoe/McGraw-Hill
8787 Orion Place
Columbus, OH 43240

ISBN: 0-07-827770-1

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Teacher s Guide to Using the *Chapter 4 Resource Masters*

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 4 Resource Masters* includes the core materials needed for Chapter 4. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Pre-Algebra TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii-viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

When to Use Give these pages to students before beginning Lesson 4-1. Encourage them to add these pages to their Pre-Algebra Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Study Guide and Intervention

Each lesson in Pre-Algebra addresses one or two objectives. There is one Study Guide and Intervention master for each lesson.

When to Use Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

When to Use These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

When to Use These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics

One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

When to Use This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

When to Use These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 4 Resource Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

Chapter Tests

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

  All of the above tests include a free-response Bonus question.

- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Pre-Algebra. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of pre-algebra concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and open-ended questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 196–197. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

NAME _____ DATE _____ PERIOD _____

4 Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of key vocabulary terms you will learn in Chapter 4. As you study this chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Pre-Algebra Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
algebraic fraction		
base		
composite number		
divisible		
expanded form		
exponent		
factor		
factor tree		

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
greatest common factor (GCF)		
monomial		
power		
prime factorization		
prime number		
scientific notation		
simplest form		
standard form		
Venn Diagram		

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4-1

Study Guide and Intervention

Factors and Monomials

Finding Factors Two or more numbers that are multiplied to form a product are called factors. Any number is divisible by its factors. The following rules can be used to determine mentally whether a number is divisible by 2, 3, 5, 6, or 10.

A number is divisible by:

- 2 if the ones digit is divisible by 2.
- 3 if the sum of the digits is divisible by 3.
- 5 if the ones digit is 0 or 5.
- 6 if the number is divisible by 2 and by 3.
- 10 if the ones digit is 0.

Example **Determine whether 108 is divisible by 2, 3, 5, 6, or 10.**

Number	Divisible?	Reason
2	yes	The ones digit is 8, and 8 is divisible by 2.
3	yes	The sum of the digits is 9, and 9 is divisible by 3.
5	no	The ones digit is 8, not 0 or 5.
6	yes	108 is divisible by 2 and by 3.
10	no	The ones digit is not 0.

108 is divisible by 2, 3, and 6.

A **monomial** is a number, a variable, or a product of numbers and/or variables. So, 108 is a monomial. The expression $5q$ is also a monomial since it is the product of a number and a variable, $5 \cdot q$. However, $2x + 1$ is not a monomial since it is the sum of two terms.

Exercises

Use divisibility rules to determine whether each number is divisible by 2, 3, 5, 6, or 10.

1. 105

2. 600

3. 462

4. 197

List all the factors of each number.

5. 76

6. 42

7. 182

8. 80

Determine whether each expression is a monomial. Explain why or why not.

9. 13

10. $x + y$

11. $3(x - 1)$

12. $5st$

NAME _____ DATE _____ PERIOD _____

4-1 Skills Practice

Factors and Monomials

Use divisibility rules to determine whether each number is divisible by 2, 3, 5, 6, or 10.

1. 100

2. 66

3. 88

4. 123

5. 240

6. 280

7. 255

8. 165

9. 318

10. 1000

List all the factors of each number.

11. 36

12. 29

13. 45

14. 81

15. 125

16. 117

17. 16

18. 63

Determine whether each expression is a monomial. Explain why or why not.

19. p

20. 73

21. $2 + n$

22. $h - w$

23. $3(a + 6)$

24. $-3k$

25. $q + r$

26. $4y - 6$

27. $3(x - 3)$

28. $6s \cdot 4p$

29. SEATING Can 132 graduates be seated in rows of 6 at the graduation ceremony? Explain.

30. SCHOOL SUPPLIES When Alex's mother buys pencils for school, she divides them equally among Alex and his sister. Should she buy the pencils in packages of 15 or 30? Explain.

NAME _____ DATE _____ PERIOD _____

4-1

Practice

Factors and Monomials

Use divisibility rules to determine whether each number is divisible by 2, 3, 5, 6, or 10.

1. 476

2. 117

3. 426

4. 29

5. 735

6. 276

7. 1200

8. 2370

9. 700

10. 4200

List all the factors of each number.

11. 48

12. 24

13. 121

14. 82

15. 37

16. 196

17. 95

18. 110

19. 96

20. 200

Determine whether each expression is a monomial. Explain why or why not.

21. 82

22. $4(-m)$

23. m

24. rv

25. $6(x - 6)$

26. $8n - 8$

27. $(-12)(-8)x$

28. $w \cdot \ell$

29. $2\ell + 2w$

30. $2s - t$

NEWSPAPERS **For Exercises 31 and 32, refer to the following information.**

Brandon delivers newspapers in his neighborhood. On Sunday, he must deliver 112 papers. Since he rides his bike, he separates the papers into smaller stacks and delivers one stack at a time.

31. What size stacks can he make?

32. If Brandon can carry no more than 30 papers at a time and can return home to restock no more than 5 times, how can he organize the 112 papers?

Lesson 4-1

NAME _____ DATE _____ PERIOD _____

4-1

Reading to Learn Mathematics

Factors and Monomials

Pre-Activity

How are side lengths of rectangles related to factors?

Do the activity at the top of page 148 in your textbook. Write your answers below.

a. Use grid paper to draw as many other rectangles as possible with an area of 36 square units. Label the length and width of each rectangle.

b. Did you draw a rectangle with a length of 5 units? Why or why not?

c. List all of the pairs of whole numbers whose product is 36. Compare this list to the lengths and widths of all the rectangles that have an area of 36 square units. What do you observe?

d. Predict the number of rectangles that can be drawn with an area of 64 square units. Explain how you can predict without actually drawing them.

Reading the Lesson

Write a definition and give an example of each new vocabulary word.

Vocabulary	Definition	Example
1. factors		
2. divisible		
3. monomial		

4. Is the expression $2x - 1$ a monomial? Explain.

Helping You Remember

5. Explain in your own words how to determine whether an expression is a monomial.

NAME _____ DATE _____ PERIOD _____

4-1 Enrichment

Divisibility

Divisibility rule for 7

Determine whether 4032 is divisible by 7.

403~~2~~	**Cross out the ones digit.**
− 4	**Subtract twice the value of the ones digit from the rest of the number.**
39~~8~~	**If the difference is a number that you know is divisible by 7, stop. If not, repeat.**
− 18	
21	Since 21 is divisible by 7, 4032 is divisible by 7.

Divisibility rule for 11

Determine whether 5159 is divisible by 11.

Method 1

515~~9~~	**Cross out the ones digit.**
− 9	**Subtract the value of the ones digit from the rest of the number.**
50~~6~~	**If the difference is a number that you know is divisible by 11, stop. If not, repeat.**
− 6	
44	Since 44 is divisible by 11, 5159 is divisible by 11.

Method 2

5159	
5 + 5 = 10	**Add the odd-numbered digits (first and third).**
1 + 9 = 10	**Add the even-numbered digits (second and fourth).**
0	**Subtract the sums. If the difference is divisible by 11, the number is divisible by 11.**

Since 0 is divisible by 11, 5159 is divisible by 11.

Determine whether 62,382 is divisible by 11.

6 + 3 + 2 = 11	**Add the odd-numbered digits.**
2 + 8 = 10	**Add the even-numbered digits.**
1	**Subtract the sums.**

Since 1 is not divisible by 11, 62,382 is not divisible by 11.

Determine whether each number is divisible by 7 or 11.

1. 266 **2.** 4312 **3.** 8976

4. 936 **5.** 13,293 **6.** 7085

7. 2957 **8.** 3124 **9.** 6545

NAME _____ DATE _____ PERIOD _____

4-2 Study Guide and Intervention

Powers and Exponents

A number that is expressed using an exponent is called a **power**. The **base** is the number that is multiplied. The **exponent** tells how many times the base is used as a factor. So, 4^3 has a base of 4 and an exponent of 3, and $4^3 = 4 \cdot 4 \cdot 4 = 64$.

Example 1 **Write each expression using exponents.**

a. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

The base is 10. It is a factor 5 times, so the exponent is 5.

$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^5$

b. $(p + 2)(p + 2)(p + 2)$

The base is $p + 2$. It is a factor 3 times, so the exponent is 3.

$(p + 2)(p + 2)(p + 2) = (p + 2)^3$

Expressions involving powers are evaluated using order of operations. Powers are repeated multiplications. They are evaluated after any grouping symbols and before other multiplication or division operations.

Example 2 **Evaluate $x^2 - 4$ if $x = -6$.**

$x^2 - 4 = (-6)^2 - 4$ Replace x with -6.

$= (-6)(-6) - 4$ -6 is a factor 2 times.

$= 36 - 4$ Multiply.

$= 32$ Subtract.

Exercises

Write each expression using exponents.

1. $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

2. $(-7)(-7)(-7)$

3. $d \cdot d \cdot d \cdot d$

4. $x \cdot x \cdot y \cdot y$

5. $(z - 4)(z - 4)$

6. $3(-t)(-t)(-t)$

Evaluate each expression if $g = 3$, $h = -1$, and $m = 9$.

7. g^5

8. $5g^2$

9. $g^2 - m$

10. hm^2

11. $g^3 + 2h$

12. $m + hg^3$

NAME _____ DATE _____ PERIOD _____

4-2 Skills Practice

Powers and Exponents

Write each expression using exponents.

1. $7 \cdot 7$

2. $(-3)(-3)(-3)(-3)(-3)$

3. 4

4. $(k \cdot k)(k \cdot k)(k \cdot k)$

5. $p \cdot p \cdot p \cdot p \cdot p \cdot p$

6. $3 \cdot 3$

7. $(-a)(-a)(-a)(-a)$

8. $6 \cdot 6 \cdot 6 \cdot 6$

9. $9 \cdot 9 \cdot 9$

10. $4 \cdot y \cdot z \cdot z \cdot z$

11. $s \cdot s \cdot s \cdot s \cdot t \cdot u \cdot u$

12. $5 \cdot 5 \cdot 5 \cdot q \cdot q$

Express each number in expanded form.

13. 135

14. 8732

15. 1005

16. 989

Evaluate each expression if $b = 8$, $c = 2$, and $d = -3$.

17. 4^c

18. c^0

19. b^3

20. $c^3 \cdot 3^c$

21. 3^c

22. c^4

23. $c^2 + d$

24. $2b^2$

25. $b^2 + c^3$

26. d^2

27. d^3

28. $b^2 + d^3$

29. b^2d

30. $(b - c)^2$

Lesson 4-2

NAME _____ DATE _____ PERIOD _____

4-2

Practice

Powers and Exponents

Write each expression using exponents.

1. $11 \cdot 11 \cdot 11$

2. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

3. 5

4. $(-4)(-4)$

5. $a \cdot a \cdot a \cdot a$

6. $n \cdot n \cdot n \cdot n \cdot n$

7. $4 \cdot 4 \cdot 4$

8. $(b \cdot b)(b \cdot b)(b \cdot b)$

9. $(-v)(-v)(-v)(-v)$

10. $x \cdot x \cdot z \cdot z \cdot z$

11. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot t \cdot t$

12. $m \cdot m \cdot m \cdot n \cdot p \cdot p$

Express each number in expanded form.

13. 13

14. 1006

15. 17,629

16. 897

Evaluate each expression if $x = 3$, $y = -2$, and $z = 4$.

17. y^x

18. 51^0

19. z^2

20. x^2

21. 9^x

22. $z^2 \cdot 2^2$

23. y^5

24. $z^2 - y^4$

25. $x^2 + y^2 + z^2$

26. $z^2 - x^2$

FAMILY TREE For Exercises 27 and 28, refer to the following information.

When examining a family tree, the branches are many. You are generation "now." One generation ago, your 2 parents were born. Two generations ago your 4 grandparents were born.

27. How many great-grandparents were born three generations ago?

28. How many "great" grandparents were born ten generations ago?

NAME _____ DATE _____ PERIOD _____

4-2

Reading to Learn Mathematics

Powers and Exponents

Pre-Activity *Why are exponents important in comparing computer data?*

Do the activity at the top of page 153 in your textbook. Write your answers below.

a. Write 16 as a product of factors of 2. How many factors are there?

b. How many factors of 2 form the product 128?

c. One megabyte is 1024 kilobytes. How many factors of 2 form the product 1024?

Reading the Lesson

Write a definition and give an example of each new vocabulary word or phrase.

Vocabulary	Definition	Example
1. base		
2. exponent		
3. power		
4. standard form		
5. expanded form		

6. Write each expression using exponents.

a. $4 \cdot 4 \cdot 4 \cdot 4$ **b.** $x \cdot x \cdot x \cdot y \cdot y$

c. $(-2)(-2)(-2)$ **d.** $5 \cdot r \cdot r \cdot m \cdot m \cdot m$

7. The number $(3 \times 10^3) + (5 \times 10^2) + (0 \times 10^1) + (2 \times 10^0)$ is written in _____ form, while 3502 is written in _____ form.

Helping You Remember

8. Explain how the terms *base*, *power*, and *exponent* are related. Provide an example.

Lesson 4-2

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4-2 Enrichment

Exponents

Numbers can be expressed in several ways. Some numbers are expressed as sums. Some numbers are expressed as products of factors, while other numbers are expressed as powers.

Two ways to express 27 are $3 \cdot 3 \cdot 3$ and 3^3.

The number 1 million can be expressed in the following ways.

1,000,000	$1000 \cdot 1000$	$100 \cdot 100 \cdot 100$	$10^2 \cdot 10^2 \cdot 10^2$
$1{,}000{,}000^1$	1000^2	100^3	10^6

Write names for each number below using the given exponents.

1. 16; exponents: 2 and 4

2. 81; exponents: 2 and 4

3. 64; exponents: 2 and 6

4. 256; exponents: 2 and 8

5. 625; exponents: 2 and 4

6. 729; exponents: 2 and 6

7. 2401; exponents: 2 and 4

8. 4096; exponents: 2 and 12

9. 6561; exponents: 2 and 8

10. 390,625; exponents: 2 and 8

Numbers that can be named as powers with like bases can be multiplied by adding the exponents.

$8 \cdot 8 = 2^3 \cdot 2^3$

$= 2^{3+3}$

$= 2^6$

Write the product of each pair of factors in exponential form.

11. $9 \cdot 9$

12. $4 \cdot 4$

13. $16 \cdot 8$

14. $125 \cdot 25$

15. $27 \cdot 9$

16. $81 \cdot 27$

17. $49 \cdot 49$

18. $121 \cdot 121$

NAME _____ DATE _____ PERIOD _____

4-3 Study Guide and Intervention

Prime Factorization

A **prime number** is a whole number that has exactly two factors, 1 and itself. A **composite number** is a whole number that has more than two factors. Zero and 1 are neither prime nor composite.

Example 1 **Determine whether 29 is prime or composite.**

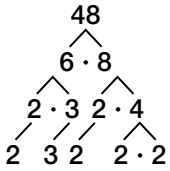
Find the factors of 29.

$29 = 1 \cdot 29$

The only factors of 29 are 1 and 29, therefore 29 is a prime number.

Any composite number can be written as a product of prime numbers. A factor tree can be used to find the prime factorization.

Example 2 **Find the prime factorization of 48.**



48 is the number to be factored.

Find any pair of whole number factors of 48.

Continue to factor any number that is not prime.

The factor tree is complete when there is a row of prime numbers.

The prime factorization of 48 is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$ or $2^4 \cdot 3$.

In algebra, monomials can be factored as a product of prime numbers and variables with no exponent greater than 1. So, $8x^2$ factors as $2 \cdot 2 \cdot 2 \cdot x \cdot x$.

Exercises

Determine whether each number is *prime* or *composite*.

1. 27

2. 151

3. 77

4. 25

Write the prime factorization for each number. Use exponents for repeated factors.

5. 16

6. 45

7. 78

8. 70

Factor each monomial.

9. $6m^3$

10. $-20xy^2$

11. $a^2b^2c^3$

12. $25h$

Lesson 4-3

NAME _____ DATE _____ PERIOD _____

4-3

Skills Practice

Prime Factorization

Determine whether each number is *prime* or *composite*.

1. 41

2. 29

3. 87

4. 36

5. 57

6. 61

7. 71

8. 103

9. 39

10. 91

11. 47

12. 67

Write the prime factorization of each number. Use exponents for repeated factors.

13. 20

14. 40

15. 32

16. 44

17. 90

18. 121

19. 46

20. 30

21. 65

22. 80

Factor each monomial.

23. $15t$

24. $16r^2$

25. $-11m^2$

26. $-49y^3$

27. $21ab$

28. $-42xyz$

29. $45j^2k$

30. $17u^2v^2$

31. $27d^4$

32. $-16cd^2$

NAME _____ DATE _____ PERIOD _____

4-3

Practice

Prime Factorization

Determine whether each number is *prime* or *composite*.

1. 11

2. 63

3. 73

4. 75

5. 49

6. 69

7. 53

8. 83

Write the prime factorization of each number. Use exponents for repeated factors.

9. 33

10. 24

11. 72

12. 276

13. 85

14. 1024

15. 95

16. 200

17. 243

18. 735

Factor each monomial.

19. $35v$

20. $49c^2$

21. $-14b^3$

22. $-81h^2$

23. $33wz$

24. $-56ghj$

25. NUMBER THEORY *Twin primes* are a pair of consecutive odd primes, which differ by 2. For example, 3 and 5 are twin primes. Find the twin primes less than 100. (*Hint*: There are 8 pairs of twins less than 100.)

Lesson 4-3

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4-3

Reading to Learn Mathematics

Prime Factorization

Pre-Activity *How can models be used to determine whether numbers are prime?*

Do the activity at the top of page 159 in your textbook. Write your answers below.

a. Use grid paper to draw as many different rectangular arrangements of 2, 3, 4, 5, 6, 7, 8, and 9 squares as possible.

b. Which numbers of squares can be arranged in more than one way?

c. Which numbers of squares can only be arranged one way?

d. What do all rectangles that you listed in part **c** have in common? Explain.

Reading the Lesson

Write a definition and give an example of each new vocabulary word or phrase.

Vocabulary	Definition	Example
1. composite number		
2. factor		
3. factor tree		
4. prime factorization		
5. prime number		

Helping You Remember

6. *Composite* is a word used in everyday English.

a. Find the definition of *composite* in the dictionary. Write the definition.

b. Explain how the English definition can help you remember how composite is used in mathematics.

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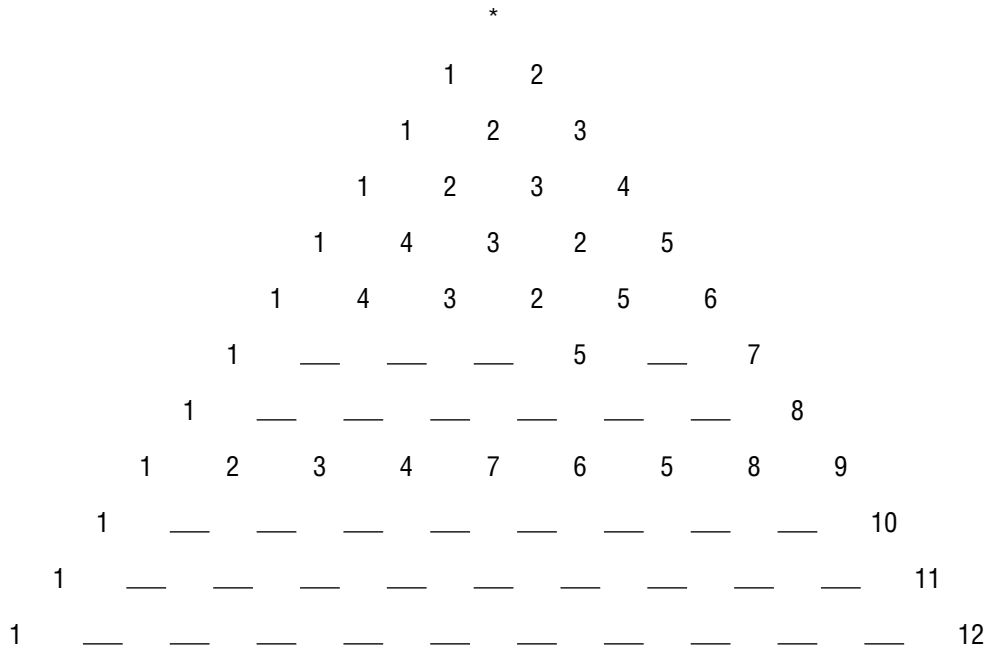
4-3 Enrichment

Prime Pyramid

A **prime number** is a whole number that has exactly two factors—itself and 1. The pyramid below is called a *prime pyramid*. Each row begins with 1 and ends with the number of that row. So, row 2 begins with 1 and ends with 2, row 3 begins with 1 and ends with 3, and so on. In each row, the numbers from 1 to the row number are arranged such that the sum of any two adjacent numbers is a prime number.

For example, look at row 4:

- It must contain the numbers 1, 2, 3, and 4.
- It must begin with 1 and end with 4.
- The sum of adjacent pairs must be a prime number:
  $1 + 2 = 3,\ 2 + 3 = 5,\ 3 + 4 = 7$



1. Complete the pyramid by filling in the missing numbers.

2. Extend the pyramid to row 13.

3. Explain the patterns you see in the completed pyramid.

Lesson 4-3

NAME _____ DATE _____ PERIOD _____

4-4 Study Guide and Intervention

Greatest Common Factor (GCF)

The greatest number that is a factor of two or more numbers is the **greatest common factor (GCF)**. Two ways to find the GCF are shown below.

Example 1 **Find the GCF of 24 and 32.**

Method 1 List the factors.

factors of 24: 1, 2, 3, 4, 6, 8, 12, 24 — Look for factors common to both lists, 1, 2, 4, and 8.

factors of 32: 1, 2, 4, 8, 16, 32

The greatest common factor of 24 and 32 is 8.

Method 2 Use prime factorization.

$24 = (2) \cdot (2) \cdot (2) \cdot 3$ — Find the common prime factors of 24 and 32.

$32 = (2) \cdot (2) \cdot (2) \cdot 2 \cdot 2$

Multiply the common prime factors. The greatest common factor of 24 and 32 is $2 \cdot 2 \cdot 2$ or 8.

In algebra, greatest common factors are used to factor expressions.

Example 2 **Factor $5x + 10$.**

First, find the GCF of $5x$ and 10.

$5x = 5 \cdot x$

$10 = 2 \cdot 5$ — The GCF is 5.

Now write each term as a product of the GCF and its remaining factors.

$5x + 10 = 5(x) + 5(2)$

$= 5(x + 2)$ — Distributive Property

So, $5x + 10 = 5(x + 2)$.

Exercises

Find the GCF of each set of numbers.

1. 30, 42 **2.** 15, 33 **3.** 44, 110 **4.** 16, 48

Factor each expression.

5. $4g + 16$ **6.** $2d - 6$ **7.** $8a + 24$

8. $f^2 + 2f$ **9.** $6 - 3j$ **10.** $16n^2 - 40n$

NAME _____ DATE _____ PERIOD _____

4-4

Skills Practice

Greatest Common Factor (GCF)

Find the GCF of each set of numbers or monomials.

1. 15, 50

2. 24, 81

3. 18, 27

4. 36, 64

5. 88, 40

6. 54, 63

7. 11, 22

8. 14, 25

9. 20, 30

10. 16, 18

11. 64, 80

12. 16, 24

13. $30t$, $40t^2$

14. 6, $9t$

15. $16k^2$, $40k$

16. $9m$, $15n$

17. $7pq$, $8q$

18. $18p$, 45

Factor each expression.

19. $5b + 15$

20. $7t + 49$

21. $6w + 18$

22. $100 + 50x$

23. $7x + 7$

24. $12n + 60$

25. $24 + 8g$

26. $50 + 5f$

27. $3n + 24$

28. $9\ell + 63$

29. $6u + 36$

30. $70 - 7c$

31. $42 - 21x$

32. $12y + 16$

33. $6p - 12$

34. $9r - 81$

35. $6 + 8q$

36. $21x + 33$

Lesson 4-4

NAME _____ DATE _____ PERIOD _____

4-4

Practice

Greatest Common Factor (GCF)

Find the GCF of each set of numbers or monomials.

1. 9, 36

2. 42, 60

3. 16, 60

4. 29, 58

5. 18, 35

6. 90, 480

7. 80, 45

8. 700, 200

9. 17, 85

10. 24, 84, 168

11. 55, 105

12. 252, 126

13. $5p$, $20p^2$

14. $28a$, $49ab$

15. $8b$, $5c$

16. $6a^2$, $18b^2$

17. $88s^2t$, $40st^2$

18. $42a^2b$, $60ab^2$

Factor each expression.

19. $10x + 40$

20. $8v + 56$

21. $9t + 9$

22. $13m + 39$

23. $90 + 45n$

24. $15p + 60$

25. $48 + 8r$

26. $11z - 55$

27. $18q - 54$

28. $125 - 25h$

29. $42a - 77$

30. $30 + 45s$

31. $50n + 30$

32. $18 + 12d$

33. $27m + 105$

34. $65 - 39b$

35. $21d - 63$

36. $48 + 84m$

37. SCHOOL TRIP Thirty-two seventh graders, 48 eighth graders, and 60 ninth graders are taking a ski trip. In order to help students get better acquainted, students from each grade level are to ride each bus. What is the greatest number of buses that can be used if students from each grade level are divided equally among the buses?

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4-4

Reading to Learn Mathematics

Greatest Common Factor (GCF)

Pre-Activity

How can a diagram be used to find the greatest common factor?

Do the activity at the top of page 164 in your textbook. Write your answers below.

a. Which numbers are in both circles?

b. Find the product of the numbers that are in both circles.

c. Is the product also a factor of 12 and 20?

d. Make a Venn diagram showing the prime factors of 16 and 28. Then use it to find the common factors of the numbers.

Reading the Lesson

Write a definition and give an example of each new vocabulary word or phrase.

Vocabulary	Definition	Example
1. Venn diagram		
2. greatest common factor		

Helping You Remember

3. Summarize in your own words how to find the greatest common factor of two numbers using each method.

a. prime factorization

b. lists of factors

c. a Venn diagram

Lesson 4-4

NAME _____ DATE _____ PERIOD _____

4-4 # Enrichment

GCFs by Successive Division

Another way to find the greatest common factor (GCF) of two numbers is to use **successive division**. This method works well for large numbers.

Find the GCF of 848 and 1325.

Step 1 Divide the smaller number into the greater number.

$$\begin{array}{r} 1 \text{ R}477 \\ 848\overline{)1325} \\ \underline{848} \\ 477 \end{array}$$

Step 2 Divide the remainder into the divisor. Repeat this step until you get a remainder of 0.

$$\begin{array}{r} 1 \text{ R}371 \\ 477\overline{)848} \\ \underline{477} \\ 371 \end{array} \qquad \begin{array}{r} 1 \text{ R}106 \\ 371\overline{)477} \\ \underline{371} \\ 106 \end{array} \qquad \begin{array}{r} 3 \text{ R}53 \\ 106\overline{)371} \\ \underline{318} \\ 53 \end{array} \qquad \begin{array}{r} 2 \text{ R}0 \\ 53\overline{)106} \\ \underline{106} \\ 0 \end{array}$$

The last divisor is the GCF of the two original numbers. So the GCF of 848 and 1325 is 53.

Use the method above to find the GCF of each pair of numbers.

1. 187; 578

2. 161; 943

3. 215; 1849

4. 453; 484

5. 432; 588

6. 279; 403

7. 1325; 3498

8. 9840; 1751

9. 3484; 5963

10. 1802; 106

11. 45,787; 69,875

12. 35,811; 102,070

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4-5 Study Guide and Intervention

Simplifying Algebraic Fractions

A fraction is in **simplest form** when the GCF of the numerator and the denominator is 1. One way to write a fraction in simplest form is to write the prime factorization of the numerator and the denominator. Then divide the numerator and denominator by the GCF.

Example 1 **Write $\frac{18}{24}$ in simplest form.**

Write the prime factorization of the numerator and the denominator.

$$\frac{18}{24} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}} \cdot 3}{\underset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{3}}}$$ Divide the numerator and denominator by the GCF, $2 \cdot 3$.

$$= \frac{3}{2 \cdot 2} \text{ or } \frac{3}{4}$$ Simplify.

Algebraic fractions can also be written in simplest form. Again, you can write the prime factorization of the numerator and the denominator, then divide by the GCF.

Example 2 **Simplify $\frac{24ab^2}{36a^2}$.**

$$\frac{24ab^2}{36a^2} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{a}} \cdot b \cdot b}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 3 \cdot \underset{1}{\cancel{a}} \cdot a}$$ Divide the numerator and denominator by the GCF, $2 \cdot 2 \cdot 3 \cdot a$.

$$= \frac{2 \cdot b \cdot b}{3 \cdot a} \text{ or } \frac{2b^2}{3a}$$ Simplify.

Exercises

Simplify each fraction. If the fraction is already in simplest form, write *simplified*.

1. $\frac{12}{20}$ **2.** $\frac{16}{36}$ **3.** $\frac{75}{100}$

4. $\frac{6}{15}$ **5.** $\frac{8}{24}$ **6.** $\frac{3}{8}$

7. $\frac{c}{c^3}$ **8.** $\frac{r^4}{r^2}$ **9.** $\frac{14b}{21b}$

10. $\frac{24w}{26w}$ **11.** $\frac{5s}{12t}$ **12.** $\frac{d}{3d^2}$

Lesson 4-5

NAME _____ DATE _____ PERIOD _____

4-5 Skills Practice

Simplifying Algebraic Fractions

Write each fraction in simplest form. If the fraction is already in simplest form, write *simplified*.

1. $\frac{10}{70}$ **2.** $\frac{12}{18}$ **3.** $\frac{30}{45}$

4. $\frac{8}{24}$ **5.** $\frac{4}{6}$ **6.** $\frac{56}{63}$

7. $\frac{18}{24}$ **8.** $\frac{7}{49}$ **9.** $\frac{13}{39}$

10. $\frac{21}{36}$ **11.** $\frac{32}{40}$ **12.** $\frac{4}{36}$

13. $\frac{44}{55}$ **14.** $\frac{4}{14}$ **15.** $\frac{36}{48}$

16. $\frac{81}{90}$ **17.** $\frac{5}{25}$ **18.** $\frac{56}{74}$

19. $\frac{22}{42}$ **20.** $\frac{7}{18}$ **21.** $\frac{d^3}{d^4}$

22. $\frac{y}{y^3}$ **23.** $\frac{q^3}{q}$ **24.** $\frac{s^4}{s^2}$

25. $\frac{x^2}{y}$ **26.** $\frac{9a}{12a}$ **27.** $\frac{8t}{16t}$

28. $\frac{14g}{24g}$ **29.** $\frac{35j}{40}$ **30.** $\frac{100p}{200p^2}$

31. $\frac{75n}{100n^3}$ **32.** $\frac{6k^5}{21k^2}$ **33.** $\frac{3a}{4b}$

34. $\frac{16b}{24d}$ **35.** $\frac{8a}{24a}$ **36.** $\frac{5t^3}{35t^2}$

NAME _____ DATE _____ PERIOD _____

4-5

Practice

Simplifying Algebraic Fractions

Write each fraction in simplest form. If the fraction is already in simplest form, write *simplified*.

1. $\frac{9}{36}$ **2.** $\frac{6}{10}$ **3.** $\frac{19}{57}$

4. $\frac{21}{24}$ **5.** $\frac{6}{39}$ **6.** $\frac{85}{100}$

7. $\frac{16}{72}$ **8.** $\frac{32}{136}$ **9.** $\frac{45}{72}$

10. $\frac{46}{92}$ **11.** $\frac{35}{55}$ **12.** $\frac{64}{80}$

13. $\frac{57}{60}$ **14.** $\frac{17}{56}$ **15.** $\frac{33}{63}$

16. $\frac{34}{60}$ **17.** $\frac{24}{52}$ **18.** $\frac{96}{108}$

19. $\frac{45}{48}$ **20.** $\frac{14}{29}$ **21.** $\frac{x^3}{x^7}$

22. $\frac{m^4}{m^5}$ **23.** $\frac{a^7}{a^4}$ **24.** $\frac{u^5}{u}$

25. $\frac{21y}{24y}$ **26.** $\frac{4q^2}{14q^2}$ **27.** $\frac{15x^2}{18x^2}$

28. $\frac{63c}{126c}$ **29.** $\frac{11v^2}{121v}$ **30.** $\frac{42b^2}{49}$

31. $\frac{e^2f^2}{e^3f}$ **32.** $\frac{m^2}{p^3}$ **33.** $\frac{2a^3b^4}{10a^5b}$

34. SKI RESORT A local ski resort is open for business 13 weeks in the winter. Write a fraction in simplest form that represents the fraction of a year the resort is open.

Lesson 4-5

NAME _____ DATE _____ PERIOD _____

4-5

Reading to Learn Mathematics

Simplifying Algebraic Fractions

Pre-Activity

How are simplified fractions useful in representing measurements?

Do the activity at the top of page 169 in your textbook. Write your answers below.

a. Are the three fractions equivalent? Explain your reasoning.

b. Which figure is divided into the least number of parts?

c. Which fraction would you say is written in simplest form? Why?

Reading the Lesson

Write a definition and give an example of each new vocabulary phrase.

Vocabulary	Definition	Example
1. simplest form		
2. algebraic fraction		

3. Use a Venn diagram to explain how to simplify $\frac{18}{45}$.

Helping You Remember

4. Explain the similarities and differences between simplifying a numerical fraction and simplifying an algebraic fraction.

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4-5 Enrichment

Matching Equivalent Fractions

Cut out the pieces below and match the edges so that equivalent fractions meet. The pieces form a 4 × 6 rectangle. The outer edges of the rectangle formed will have no fractions.

$\frac{9}{10}$ $\frac{15}{35}$ $\frac{5}{9}$ $\frac{3}{24}$	$\frac{1}{7}$ $\frac{12}{42}$ $\frac{3}{8}$ $\frac{2}{10}$	$\frac{11}{12}$ $\frac{16}{28}$ $\frac{5}{8}$ $\frac{45}{50}$	$\frac{12}{13}$ $\frac{8}{20}$ $\frac{4}{7}$ $\frac{40}{45}$	$\frac{8}{9}$ $\frac{5}{15}$ $\frac{3}{7}$ $\frac{3}{21}$
$\frac{9}{11}$ $\frac{15}{21}$ $\frac{40}{44}$	$\frac{3}{10}$ $\frac{1}{2}$ $\frac{52}{56}$	$\frac{1}{9}$ $\frac{35}{77}$ $\frac{7}{13}$	$\frac{1}{8}$ $\frac{18}{48}$ $\frac{4}{9}$ $\frac{2}{18}$	$\frac{8}{10}$ $\frac{27}{33}$
$\frac{1}{12}$ $\frac{24}{54}$ $\frac{2}{20}$	$\frac{1}{10}$ $\frac{49}{91}$	$\frac{6}{8}$ $\frac{4}{5}$ $\frac{21}{33}$	$\frac{7}{8}$ $\frac{1}{3}$ $\frac{3}{18}$	$\frac{7}{11}$ $\frac{9}{15}$ $\frac{5}{7}$ $\frac{44}{48}$
$\frac{4}{6}$ $\frac{3}{4}$ $\frac{9}{33}$	$\frac{1}{5}$ $\frac{14}{56}$ $\frac{5}{11}$	$\frac{1}{4}$ $\frac{2}{8}$	$\frac{1}{6}$ $\frac{2}{7}$ $\frac{2}{8}$	$\frac{13}{14}$ $\frac{2}{5}$ $\frac{35}{40}$
$\frac{2}{3}$ $\frac{6}{20}$	$\frac{6}{7}$ $\frac{25}{45}$ $\frac{3}{36}$		$\frac{3}{11}$ $\frac{3}{6}$ $\frac{3}{5}$ $\frac{48}{52}$	$\frac{10}{11}$ $\frac{20}{32}$ $\frac{30}{35}$

Lesson 4-5

NAME _____ DATE _____ PERIOD _____

4-6 Study Guide and Intervention

Multiplying and Dividing Monomials

When multiplying powers with the same base, add the exponents.

Symbols	Example
$a^m \cdot a^n = a^{m+n}$	$4^2 \cdot 4^5 = 4^{2+5}$ or 4^7

When dividing powers with the same base, subtract the exponents.

Symbols	Example
$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$	$\frac{5^6}{5^2} = 5^{6-2}$ or 5^4

Example 1 **Find $2a^2(3a)$. Express your answer using exponents.**

$2a^2(3a) = (2 \cdot 3)(a^2 \cdot a)$ Use the Commutative and Associative Properties.

$= (6)(a^{2+1})$ The common base is a.

$= 6a^3$ Add the exponents.

Example 2 **Find $\frac{(-8)^4}{(-8)^2}$. Express your answer using exponents.**

$\frac{(-8)^4}{(-8)^2} = (-8)^{4-2}$ The common base is -8.

$= (-8)^2$ Subtract the exponents.

Exercises

Find each product or quotient. Express your answer using exponents.

1. $4^7 \cdot 4^6$

2. $v^5 \cdot v^4$

3. $(f^3)(f^9)$

4. $22^5 \cdot 22^5$

5. $7h(5h^3)$

6. $-10x^2(7x^3)$

7. $\frac{7^5}{7^2}$

8. $\frac{1^8}{1^6}$

9. $\frac{(-12)^3}{(-12)^3}$

10. $3^8 \cdot 3^3$

11. $\frac{c^{20}}{c^{13}}$

12. $\frac{(-p)^{18}}{(-p)^{12}}$

13. $-7u^6(-6u^5)$

14. $\frac{2w^3}{2w}$

15. $-5m^3(4m^6)$

16. the product of two cubed and two squared

17. the quotient of six to the eighth power and six squared

NAME _____ DATE _____ PERIOD _____

4-6

Skills Practice

Multiplying and Dividing Monomials

Find each product or quotient. Express your answer using exponents.

1. $2^3 \cdot 2^5$

2. $10^2 \cdot 10^7$

3. $1^4 \cdot 1$

4. $6^3 \cdot 6^3$

5. $(-3)^2(-3)^3$

6. $(-9)^2(-9)^2$

7. $a^2 \cdot a^3$

8. $n^8 \cdot n^3$

9. $(p^4)(p^4)$

10. $(z^6)(z^7)$

11. $(6b^3)(3b^4)$

12. $(-v)^3(-v)^7$

13. $11a^2 \cdot 3a^6$

14. $10t^2 \cdot 4t^{10}$

15. $(8c^2)(9c)$

16. $(4f^8)(5f^6)$

17. $\frac{5^{10}}{5^2}$

18. $\frac{10^6}{10^2}$

19. $\frac{7^9}{7^6}$

20. $\frac{12^8}{12^3}$

21. $\frac{100^9}{100^8}$

22. $\frac{(-2)^3}{-2}$

23. $\frac{r^8}{r^7}$

24. $\frac{z^{10}}{z^8}$

25. $\frac{q^8}{q^4}$

26. $\frac{g^{12}}{g^8}$

27. $\frac{(-y)^7}{(-y)^2}$

28. $\frac{(-z)^{12}}{(-z)^5}$

29. the product of two squared and two to the sixth power

30. the quotient of ten to the seventh power and ten cubed

31. the product of y squared and y cubed

32. the quotient of a to the twentieth power and a to the tenth power

NAME _____ DATE _____ PERIOD _____

4-6

Practice

Multiplying and Dividing Monomials

Find each product or quotient. Express your answer using exponents.

1. $4^2 \cdot 4^3$

2. $9^8 \cdot 9^6$

3. $7^4 \cdot 7^2$

4. $13^2 \cdot 13^4$

5. $(-8)^5(-8)^3$

6. $(-21)^9(-21)^5$

7. $t^9 \cdot t^3$

8. $h^4 \cdot h^{13}$

9. $(m^6)(m^6)$

10. $(u^{11})(u^{10})$

11. $(-r)^7(-r)^{20}$

12. $(-w)(-w)^9$

13. $4d^5 \cdot 8d^6$

14. $7j^{50} \cdot 6j^{50}$

15. $-5b^9 \cdot 6b^2$

16. $12^1 \cdot 12^2$

17. $\frac{6^{11}}{6^3}$

18. $\frac{15^3}{15^2}$

19. $\frac{9^9}{9^7}$

20. $\frac{18^4}{18^4}$

21. $\frac{(-7)^6}{(-7)^5}$

22. $\frac{95^{21}}{95^{18}}$

23. $\frac{v^{30}}{v^{20}}$

24. $\frac{n^{19}}{n^{11}}$

25. the product of five cubed and five to the fourth power

26. the quotient of eighteen to the ninth power and eighteen squared

27. the product of z cubed and z cubed

28. the quotient of x to the fifth power and x cubed

29. **SOUND** Decibels are units used to measure sound. The softest sound that can be heard is rated as 0 decibels (or a relative loudness of 1). Ordinary conversation is rated at about 60 decibels (or a relative loudness of 10^6). A rock concert is rated at about 120 decibels (or a relative loudness of 10^{12}). How many times greater is the relative loudness of a rock concert than the relative loudness of ordinary conversation?

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4-6 Reading to Learn Mathematics

Multiplying and Dividing Monomials

Pre-Activity

How are powers of monomials useful in comparing earthquake magnitudes?

Do the activity at the top of page 175 in your textbook. Write your answers below.

a. Examine the exponents of the factors and the exponents of the products in the last column. What do you observe?

b. Make a conjecture about a rule for determining the exponent of the product when you multiply powers with the same base. Test your rule by multiplying $2^2 \cdot 2^4$ using a calculator.

Reading the Lesson

1. When multiplying powers with like bases, _____ the exponents.

2. When dividing powers with like bases, _____ the exponents.

3. Write a division expression whose quotient is 7^2.

4. Write a multiplication expression whose product is v^5.

5. Find each product.

a. $4 \cdot 4^3$

b. $y^7 \cdot y^5$

c. $(-2x^2)(5x^2)$

d. $-3r^2 \cdot r$

6. Find each quotient.

a. $\frac{7^4}{7^2}$

b. $\frac{v^9}{v^3}$

c. $\frac{6^7}{6^6}$

d. $\frac{a^2b^2}{b^2}$

Helping You Remember

7. Explain how dividing powers is related to simplifying fractions. Provide an example as part of your explanation.

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4-6 Enrichment

Dividing Powers with Different Bases

Some powers with different bases can be divided. First, you must be able to write both as powers of the same base. An example is shown below.

$$\frac{2^5}{8^2} = \frac{2^5}{(2^3)^2}$$ To find the power of a power, multiply the exponents.

$$= \frac{2^5}{2^6}$$

$$= 2^{-1} \text{ or } \frac{1}{2}$$

This method could not have been used to divide $\frac{2^5}{9^2}$, since 9 cannot be written as a power of 2 using integers.

Simplify each fraction using the method shown above. Express the solution without exponents.

1. $\frac{8^2}{2^2}$

2. $\frac{16^4}{8^3}$

3. $\frac{9^3}{3^3}$

4. $\frac{81^4}{3^4}$

5. $\frac{3^9}{81^2}$

6. $\frac{32^4}{16^4}$

7. $\frac{125^2}{25^3}$

8. $\frac{6^6}{216^2}$

9. $\frac{10^6}{1000^3}$

10. $\frac{64^3}{8^5}$

11. $\frac{27^5}{9^4}$

12. $\frac{343^3}{7^5}$

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4-7 Study Guide and Intervention

Negative Exponents

Extending the pattern below shows that $4^{-1} = \frac{1}{4}$ or $\frac{1}{4^1}$.

$4^2 = 16$ ÷ 4

$4^1 = 4$ ÷ 4

$4^0 = 1$ ÷ 4

$4^{-1} = \frac{1}{4}$

This suggests the following definition.

$a^{-n} = \frac{1}{a^n}$, for $a \neq 0$ and any integer n.

Example 1 **Write each expression using a positive exponent.**

a. 3^{-4}

$3^{-4} = \frac{1}{3^4}$

b. y^{-2}

$y^{-2} = \frac{1}{y^2}$

We can evaluate algebraic expressions with negative exponents using the definition of negative exponents.

Example 2 **Evaluate b^{-2} if $b = 3$.**

$b^{-2} = 3^{-2}$ Replace b with 3.

$= \frac{1}{3^2}$ Definition of negative exponent

$= \frac{1}{9}$ Find 3^2.

Exercises

Write each expression using a positive exponent.

1. 6^{-4} **2.** $(-7)^{-8}$ **3.** b^{-6} **4.** n^{-1}

Write each fraction as an expression using a negative exponent other than −1.

5. $\frac{1}{2^2}$ **6.** $\frac{1}{13^4}$ **7.** $\frac{1}{25}$ **8.** $\frac{1}{49}$

Evaluate each expression if $m = -4$, $n = 1$, and $p = 6$.

9. p^{-2} **10.** m^{-3} **11.** $(np)^{-1}$ **12.** 3^m

Lesson 4-7

NAME _____ DATE _____ PERIOD _____

4-7 Skills Practice

Negative Exponents

Write each expression using a positive exponent.

1. 3^{-4} **2.** 8^{-7} **3.** 10^{-4}

4. $(-2)^{-6}$ **5.** $(-40)^{-3}$ **6.** $(-17)^{-12}$

7. n^{-10} **8.** b^{-8} **9.** q^{-5}

10. m^{-4} **11.** v^{-11} **12.** p^{-2}

Write each fraction as an expression using a negative exponent other than −1.

13. $\frac{1}{8^2}$ **14.** $\frac{1}{10^5}$ **15.** $\frac{1}{2^3}$

16. $\frac{1}{6^7}$ **17.** $\frac{1}{17^4}$ **18.** $\frac{1}{21^2}$

19. $\frac{1}{3^7}$ **20.** $\frac{1}{9^2}$ **21.** $\frac{1}{3^2}$

22. $\frac{1}{121}$ **23.** $\frac{1}{25}$ **24.** $\frac{1}{36}$

Evaluate each expression if $x = 1$, $y = 2$, and $z = -3$.

25. y^{-z} **26.** z^{-2} **27.** x^{-8}

28. y^{-5} **29.** z^{-3} **30.** y^{-1}

31. z^{-4} **32.** 5^{z} **33.** x^{-99}

34. 1^{z} **35.** 4^{z} **36.** y^{z}

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4-7

Practice

Negative Exponents

Write each expression using a positive exponent.

1. 7^{-8} **2.** 10^{-6} **3.** 23^{-1}

4. $(-5)^{-2}$ **5.** $(-18)^{-10}$ **6.** m^{-99}

7. $(-1)^{-12}$ **8.** c^{-6} **9.** p^{-5}

10. g^{-17} **11.** $5z^{-4}$ **12.** $3t^{-1}$

Write each fraction as an expression using a negative exponent.

13. $\frac{1}{2^{10}}$ **14.** $\frac{1}{29^3}$ **15.** $\frac{1}{4^4}$

16. $\frac{1}{39}$ **17.** $\frac{1}{81^7}$ **18.** $\frac{1}{m^4}$

19. $\frac{1}{x^3}$ **20.** $\frac{1}{a^2}$ **21.** $\frac{1}{49}$

22. $\frac{1}{8}$ **23.** $\frac{1}{144}$ **24.** $\frac{1}{169}$

Evaluate each expression if $x = 3$, $y = -2$, and $z = 4$.

25. x^{-4} **26.** y^{-2} **27.** y^{-5}

28. z^{-4} **29.** 5^y **30.** 10^y

31. $3z^{-1}$ **32.** z^y **33.** $(xz)^{-2}$

34. HAIR Hair grows at a rate of $\frac{1}{64}$ inch per day. Write this number using negative exponents.

Lesson 4-7

NAME _____ DATE _____ PERIOD _____

4-7 Reading to Learn Mathematics

Negative Exponents

Pre-Activity *How do negative exponents represent repeated division?*

Do the activity at the top of page 181 in your textbook. Write your answers below.

a. Describe the pattern of powers in the first column. Continue the pattern by writing the next two powers in the table.

b. Describe the pattern of values in the second column. Then complete the second column.

c. Verify that the powers you wrote in part **a** are equal to the values that you found in part **b**.

d. Determine how 3^{-1} should be defined.

Reading the Lesson

1. Explain the value of 5^{-3} using a pattern.

Power	Value
5^1	5
5^0	1
5^{-1}	$\frac{1}{5}$
5^{-2}	$\frac{1}{25}$
5^{-3}	

2. Using what you know about the Quotient of Powers rule, fill in the missing number.

$5^{-3} = \frac{?}{5^5}$

Helping You Remember

3. Are $-x^2$ and x^{-2} equivalent? Explain.

NAME _____ DATE _____ PERIOD _____

4-7 Enrichment

Proving Definitions of Exponents

Recall the rules for multiplying and dividing powers with the same base. Use these rules, along with other properties you have learned, to justify each definition. Abbreviations for some properties you may wish to use are listed below.

Associative Property of Multiplication (APM)
Multiplicative Identity Property (MIP)
Inverse Property of Multiplication (IPM)
Additive Identity Property (AIP)
Inverse Property of Addition (IPA)

Write the reason for each statement.

1. Prove: $a^0 = 1$

Statement	**Reason**
Let m be an integer, and let a be any nonzero number.	**a.** Given
$a^m \cdot a^0 = a^{m+0}$	**b.** _____
$a^m \cdot a^0 = a^m$	**c.** _____
$\frac{1}{a^m} \cdot (a^m \cdot a^0) = \frac{1}{a^m} \cdot a^m$	**d.** _____
$\left(\frac{1}{a^m} \cdot a^m\right) \cdot a^0 = \frac{1}{a^m} \cdot a^m$	**e.** _____
$1 \cdot a^0 = 1$	**f.** _____
$a^0 = 1$	**g.** _____

2. Prove: $a^{-n} = \frac{1}{a^n}$

Statement	**Reason**
Let n be an integer, and let a be any nonzero number.	**a.** Given
$a^{-n} \cdot a^n = a^{-n+n}$	**b.** _____
$a^{-n} \cdot a^n = a^0$	**c.** _____
$a^{-n} \cdot a^n = 1$	**d.** _____
$(a^{-n} \cdot a^n) \cdot \frac{1}{a^n} = 1 \cdot \frac{1}{a^n}$	**e.** _____
$a^{-n} \cdot \left(a^n \cdot \frac{1}{a^n}\right) = 1 \cdot \frac{1}{a^n}$	**f.** _____
$a^{-n} \cdot 1 = 1 \cdot \frac{1}{a^n}$	**g.** _____
$a^{-n} = \frac{1}{a^n}$	**h.** _____

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4-8 Study Guide and Intervention

Scientific Notation

When you deal with very large numbers like 5,000,000 or very small numbers like 0.0005, it is difficult to keep track of place value. Numbers such as these can be written in **scientific notation**. A number is expressed in scientific notation when it is written as a product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10.

By definition, a number in scientific notation is written as $a \cdot 10^n$, where $1 \leq a < 10$ and n is an integer.

Example 1

Express each number in standard form.

a. 6.32×10^5

$6.32 \times 10^5 = 632{,}000$ Move the decimal point 5 places to the right.

b. 7.8×10^{-6}

$7.8 \times 10^{-6} = 0.0000078$ Move the decimal point 6 places to the left.

Example 2

Express each number in scientific notation.

a. 62,000,00

To write in scientific notation, place the decimal point after the first nonzero digit, then find the power of 10.

$62{,}000{,}00 = 6.2 \times 10^7$ The decimal point moves 7 places. The power of 10 is 7.

b. 0.00025

$0.00025 = 2.5 \times 10^{-4}$ Place the decimal point after the first nonzero digit. The power of 10 is -4.

Exercises

1. 4.12×10^6 **2.** 5.8×10^2 **3.** 9.01×10^{-3}

4. 6.72×10^{-7} **5.** 8.72×10^4 **6.** 4.44×10^{-5}

Express each number in scientific notation.

7. 12,000,000,000 **8.** 5000 **9.** 0.00475

10. 0.00007463 **11.** 235,000 **12.** 0.000377

Choose the greater number in each pair.

13. 4.9×10^4, 9.9×10^{-4} **14.** 2.004×10^3, 2.005×10^{-2}

15. 3.2×10^2, 700 **16.** 0.002, 3.6×10^{-4}

NAME _____ DATE _____ PERIOD _____

4-8 Skills Practice

Scientific Notation

Express each number in standard form.

1. 1.5×10^3

2. 4.01×10^4

3. 6.78×10^2

4. 5.925×10^6

5. 7.0×10^8

6. 9.99×10^7

7. 3.0005×10^5

8. 2.54×10^5

9. 1.75×10^4

10. 1.2×10^{-6}

11. 7.0×10^{-1}

12. 6.3×10^{-3}

13. 5.83×10^{-2}

14. 8.075×10^{-4}

15. 1.1×10^{-5}

16. 7.3458×10^7

Express each number in scientific notation.

17. 1,000,000

18. 17,400

19. 500

20. 803,000

21. 0.00027

22. 5300

23. 18

24. 0.125

25. 17,000,000,000

26. 0.01

27. 21,800

28. 2,450,000

29. 0.0054

30. 0.000099

31. 8,888,800

32. 0.00912

Choose the greater number in each pair.

33. 8.8×10^3, 9.1×10^{-4}

34. 5.01×10^2, 5.02×10^{-1}

35. 6.4×10^3, 900

36. 1.9×10^{-2}, 0.02

37. 2.2×10^{-3}, 2.1×10^2

38. 8.4×10^2, 839

Lesson 4-8

NAME _____ DATE _____ PERIOD _____

4-8

Practice

Scientific Notation

Express each number in standard form.

1. 2.4×10^4

2. 9.0×10^3

3. 4.385×10^7

4. 1.03×10^8

5. 3.05×10^2

6. 5.11×10^{10}

7. 6.000032×10^6

8. 1.0×10^1

9. 8.75×10^5

10. 8.49×10^{-2}

11. 7.1×10^{-6}

12. 1.0×10^{-3}

13. 4.39×10^{-7}

14. 1.25×10^{-4}

Express each number in scientific notation.

15. 40,000

16. 16

17. 876,000,000

18. 4500

19. 151

20. 0.00037

21. 83,000,000

22. 919,100

23. 5,000,000,000,000

24. 0.13

25. 0.0000007

26. 0.0067

NIAGARA FALLS **For Exercises 27 and 28, use the following information.**

Every minute, 840,000,000,000 drops of water flow over Niagara Falls.

27. Write this number in scientific notation.

28. How many drops flow over the falls in a day?

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4-8

Reading to Learn Mathematics

Scientific Notation

Pre-Activity ***Why is scientific notation an important tool in comparing real-world data?***

Do the activity at the top of page 186 in your textbook. Write your answers below.

a. Write the track length in millimeters.

b. Write the track width in millimeters. (1 micron = 0.001 millimeter)

Reading the Lesson

Write a definition and give an example of the new vocabulary phrase.

	Vocabulary	Definition	Example
1.	slope intercept form		

2. To multiply by a power of 10, move the decimal point to the _____ if the exponent is positive.

3. Which is larger, -2.1×10^4 or -2.1×10^{-4}? Explain.

Helping You Remember

4. Explain how to express each number in scientific notation.

a. a number greater than 1

b. a number less than one

c. the number 1

Lesson 4-8

NAME _____ DATE _____ PERIOD _____

4-8 Enrichment

Scientific Notation

It is sometimes necessary to multiply and divide very large or very small numbers using scientific notation.

To multiply numbers in scientific notation, use the following rule.

For any numbers a and b, and any numbers c and d,
$(c \times 10^a)(d \times 10^b) = (c \times d) \times 10^{a+b}$.

Example 1 $(3.0 \times 10^4)(-5.0 \times 10^{-2}) = [3.0 \times (-5.0)] \times 10^{4 + (-2)}$

$= -15.0 \times 10^2$

$= -1.5 \times 10^3$ or -1500

To divide numbers in scientific notation, use the following rule.

For any numbers a and b, and any numbers c and d, $(d \neq 0)$
$(c \times 10^a) \div (d \times 10^b) = (c \div d) \times 10^{a-b}$.

Example 2 $(24 \times 10^{-4}) \div (1.5 \times 10^2) = (24 \div 1.5) \times 10^{-4-2}$

$= 16 \times 10^{-6}$

$= 1.6 \times 10^{-5}$ or 0.000016

Multiply or divide. Express each product or quotient in scientific notation.

1. $(2.7 \times 10^9) \times (3.1 \times 10^2)$

2. $(6.1 \times 10^{-2}) \times (1.3 \times 10^5)$

3. $(5.4 \times 10^{-3}) \div (1.8 \times 10^2)$

4. $(6.9 \times 10^{-3}) \div (3.0 \times 10^{-8})$

5. $(1.1 \times 10^{-5}) \times (9.9 \times 10^{-1})$

6. $(4.0 \times 10^0) \div (1.0 \times 10^{-2})$

Solve. Write your answers in standard form.

7. The distance from Earth to the Moon is about 2.0×10^5 miles. The distance from Earth to the Sun is about 9.3×10^7 miles. How many times farther is it to the Sun than to the Moon?

8. If each of the 3.0×10^4 people employed by Sunny Motors earned 4.0×10^4 dollars last year, how much money did the company pay out to its employees?

NAME _____ DATE _____ PERIOD _____

4

Chapter 4 Test, Form 1

SCORE _____

Write the letter for the correct answer in the blank at the right of each question.

1. Use divisibility rules to determine which number is a factor of 39.

A. 3 **B.** 5 **C.** 6 **D.** 10 **1.** _____

2. Determine which expression is not a monomial.

A. 36 **B.** x **C.** $x - 5$ **D.** $4k$ **2.** _____

3. Write (6)(6)(6) using exponents.

A. 3^6 **B.** 6^6 **C.** 1^3 **D.** 6^3 **3.** _____

4. Evaluate k^3 if $k = 2$.

A. 2 **B.** 6 **C.** 8 **D.** 27 **4.** _____

5. Write the prime factorization of 18.

A. $2 \cdot 9$ **B.** $2 \cdot 3 \cdot 3$ **C.** $2 \cdot 2 \cdot 3$ **D.** $3 \cdot 6$ **5.** _____

6. Factor $35x^2y$ completely.

A. $5 \cdot 7 \cdot x \cdot x \cdot y$ **B.** $35 \cdot 1 \cdot x \cdot x \cdot y$

C. $5 \cdot 7 \cdot x \cdot y \cdot y$ **D.** $5 \cdot 7 \cdot x^2 \cdot y$ **6.** _____

For Questions 7 and 8, find the GCF of each set of numbers or monomials.

7. 12, 20

A. 2 **B.** 32 **C.** 240 **D.** 4 **7.** _____

8. 6, 8

A. 2 **B.** 3 **C.** 4 **D.** 48 **8.** _____

9. Factor $3b + 12$.

A. $12(3b + 1)$ **B.** $4(b + 3)$ **C.** $3(b + 4)$ **D.** $b(3 + 12)$ **9.** _____

For Questions 10 and 11, write each fraction in simplest form.

10. $\frac{12}{21}$

A. $\frac{6}{7}$ **B.** $\frac{7}{3}$ **C.** $\frac{4}{7}$ **D.** $\frac{1}{2}$ **10.** _____

11. $\frac{15a}{25a}$

A. $\frac{3}{5}$ **B.** $\frac{3a}{5}$ **C.** $\frac{3a}{5a}$ **D.** $\frac{3}{5a}$ **11.** _____

Assessment

4 Chapter 4 Test, Form 1 *(continued)*

12. Eight inches is what part of 1 foot?

A. $\frac{2}{9}$ **B.** $\frac{2}{3}$ **C.** $\frac{2}{25}$ **D.** $\frac{1}{2}$ **12.** _____

Find each product.

13. $4^5 \cdot 4^2$

A. 16^7 **B.** 4^{52} **C.** 4^{10} **D.** 4^7 **13.** _____

14. $m^4 \cdot m$

A. 4^m **B.** m^5 **C.** m^4 **D.** $2m^4$ **14.** _____

For Questions 15 and 16, find each quotient.

15. $\frac{5^4}{5^2}$

A. 1^2 **B.** 5^6 **C.** 5^2 **D.** 5^8 **15.** _____

16. $\frac{t^3}{t}$

A. 1^3 **B.** t^3 **C.** t^4 **D.** t^2 **16.** _____

17. Write $\frac{1}{3^5}$ using a negative exponent.

A. 3^{-5} **B.** 5^{-3} **C.** -3^5 **D.** -5^3 **17.** _____

18. Evaluate y^{-2} if $y = 4$.

A. $-\frac{1}{8}$ **B.** $\frac{1}{16}$ **C.** 8 **D.** $-\frac{1}{16}$ **18.** _____

19. The speed of light is 300,000,000 meters per second. Express this number in scientific notation.

A. 300×10^3 **B.** 30.0×10^8 **C.** 3.0×10^8 **D.** 0.03×10^7 **19.** _____

20. Choose the true statement.

A. $3.1 \times 10^5 < 2.7 \times 10^5$ **B.** $1.8 \times 10^{-1} > 1.1 \times 10^1$

C. $5.4 \times 10^4 > 3.7 \times 10^7$ **D.** $3.7 \times 10^{-4} < 3.4 \times 10^{-1}$ **20.** _____

Bonus Dyenitha is renting tables for her wedding reception. She can choose from tables that seat 6 or 8. If she is expecting 176 people and wants the same number of people at each table, which size table should she order? **B:** _____

NAME _____ DATE _____ PERIOD _____

4

Chapter 4 Test, Form 2A

SCORE _____

Write the letter for the correct answer in the blank at the right of each question.

1. Use divisibility rules to determine which number is a factor of 126.

A. 6 **B.** 10 **C.** 4 **D.** 5 **1.** _____

2. Determine which expression is not a monomial.

A. $-16xyz$ **B.** $7(x + y)$ **C.** 12 **D.** u **2.** _____

3. Write $(y)(y)(y)(y)(y)$ using exponents.

A. $5 + y$ **B.** $5y$ **C.** 5^y **D.** y^5 **3.** _____

4. Evaluate $3m^2$ if $m = 5$.

A. 21 **B.** 30 **C.** 75 **D.** 225 **4.** _____

5. Write the prime factorization of 36.

A. $2 \cdot 2 \cdot 9$ **B.** $2 \cdot 2 \cdot 2 \cdot 2$ **C.** $6 \cdot 6$ **D.** $2 \cdot 2 \cdot 3 \cdot 3$ **5.** _____

6. Factor $20x^2y$ completely.

A. $2 \cdot 2 \cdot 5 \cdot x \cdot x \cdot y$ **B.** $2 \cdot 25x^2y$

C. $4 \cdot 5 \cdot x \cdot x \cdot y$ **D.** $2 \cdot 10 \cdot x \cdot x \cdot y$ **6.** _____

7. Find the GCF of $14x$ and $35x^2$.

A. $490x$ **B.** $7x$ **C.** $490x^2$ **D.** $14x^3$ **7.** _____

8. Factor $5 + 10y$.

A. $5y(1 + 2)$ **B.** $5(1 + 10y)$ **C.** $5(1 + 2y)$ **D.** $5y(1 + 2y)$ **8.** _____

For Questions 9 and 10, write each fraction in simplest form.

9. $\frac{35}{77}$

A. $\frac{5}{11}$ **B.** $\frac{1}{2}$ **C.** $\frac{14}{33}$ **D.** $\frac{1}{3}$ **9.** _____

10. $\frac{20ab^2}{50ab}$

A. $\frac{2ab}{5}$ **B.** $\frac{2b}{5a}$ **C.** $\frac{2ab^2}{5ab}$ **D.** $\frac{2b}{5}$ **10.** _____

11. Twenty centimeters is what part of a meter?

A. $\frac{1}{5}$ **B.** $\frac{1}{50}$ **C.** $\frac{1}{10}$ **D.** $\frac{4}{9}$ **11.** _____

Assessment

4 Chapter 4 Test, Form 2A *(continued)*

Find each product.

12. $8^4 \cdot 8^4$

A. 16^4 **B.** 8^8 **C.** 8^{16} **D.** 64^8 **12.** _____

13. $4x^2 \cdot 5x^4$

A. $9x^6$ **B.** $9x^8$ **C.** $20x^6$ **D.** $20x^8$ **13.** _____

For Questions 14 and 15, find each quotient.

14. $\frac{m^5}{m}$

A. m^4 **B.** 5 **C.** m^6 **D.** 1^5 **14.** _____

15. $\frac{4^6}{4^2}$

A. 1^4 **B.** 4^3 **C.** 4^4 **D.** 4^8 **15.** _____

16. Write $\frac{1}{5^6}$ as an expression using a negative exponent.

A. -5^{-6} **B.** 5^{-6} **C.** 6^{-5} **D.** -6 **16.** _____

For Questions 17 and 18, evaluate each expression if $a = -2$ and $b = 3$.

17. b^{-3}

A. $\frac{1}{27}$ **B.** -9 **C.** $-\frac{1}{27}$ **D.** 27 **17.** _____

18. 2^a

A. $-\frac{1}{2}$ **B.** $\frac{1}{4}$ **C.** $-\frac{1}{4}$ **D.** 4 **18.** _____

19. A red blood cell is about 7.5×10^{-4} centimeter long. Express this number in standard form.

A. 0.0075 **B.** 0.07500 **C.** 7500 **D.** 0.00075 **19.** _____

20. Choose the number that is greater than 2.7×10^4.

A. 26,000 **B.** 1.4×10^5 **C.** 3.1×10^{-6} **D.** 2.5×10^4 **20.** _____

Bonus Order the planets in the table at the right from least to greatest diameter.

Planet	Diameter (mi)
Venus	7.52×10^3
Uranus	3.18×10^4
Neptune	3.08×10^4
Earth	7.93×10^3

B: _____

NAME _____ DATE _____ PERIOD _____

4

Chapter 4 Test, Form 2B

SCORE _____

Write the letter for the correct answer in the blank at the right of each question.

1. Use divisibility rules to determine which number is a factor of 105.

A. 4 **B.** 5 **C.** 6 **D.** 10 **1.** _____

2. Determine which expression is not a monomial.

A. $3(x - y)$ **B.** $-13xyz$ **C.** 16 **D.** w **2.** _____

3. Write $(m)(m)(m)(m)$ using exponents.

A. $4 + m$ **B.** $4m$ **C.** 4^m **D.** m^4 **3.** _____

4. Evaluate $3m^3$ if $m = 6$.

A. 54 **B.** 648 **C.** 216 **D.** 5832 **4.** _____

5. Write the prime factorization of 24.

A. $2 \cdot 2 \cdot 3$ **B.** $2 \cdot 2 \cdot 2 \cdot 3$ **C.** $4 \cdot 6$ **D.** $2 \cdot 2 \cdot 3 \cdot 3$ **5.** _____

6. Factor $28xy^2$ completely.

A. $4 \cdot 7 \cdot x \cdot y \cdot y$ **B.** $2 \cdot 2 \cdot 7 \cdot x \cdot y \cdot y$

C. $1 \cdot 28 \cdot x \cdot y \cdot y$ **D.** $2 \cdot 2 \cdot 7 \cdot x \cdot y^2$ **6.** _____

7. Find the GCF of $16x$ and $64x^2$.

A. $4x$ **B.** $8x^2$ **C.** $16x$ **D.** $32x^3$ **7.** _____

8. Factor $12 + 6y$.

A. $12(1 + 6y)$ **B.** $6(2 + y)$ **C.** $2(6 + y)$ **D.** $6(3y)$ **8.** _____

For Questions 9 and 10, write each fraction in simplest form.

9. $\frac{36}{66}$

A. $\frac{1}{2}$ **B.** $\frac{18}{33}$ **C.** $\frac{6}{11}$ **D.** $\frac{12}{22}$ **9.** _____

10. $\frac{30a^2}{50a}$

A. $\frac{3}{5}$ **B.** $\frac{3a}{5}$ **C.** $\frac{30a}{50}$ **D.** $\frac{6}{5}$ **10.** _____

11. Twenty-five centimeters is what part of a meter?

A. $\frac{1}{4}$ **B.** $\frac{25}{36}$ **C.** $\frac{1}{2}$ **D.** $\frac{1}{40}$ **11.** _____

Assessment

Chapter 4 Test, Form 2B *(continued)*

Find each product.

12. $10^3 \cdot 10^6$

A. 100^9 **B.** 10^{36} **C.** 10^{18} **D.** 10^9 **12.** _____

13. $(3y^2)(6y^3)$

A. $18y^5$ **B.** $9y^4$ **C.** $18y^6$ **D.** $9y^5$ **13.** _____

For Questions 14 and 15, find each quotient.

14. $\frac{x^6}{x}$

A. 1^6 **B.** 6 **C.** x^7 **D.** x^5 **14.** _____

15. $\frac{(-3)^4}{(-3)^1}$

A. 1^3 **B.** 4 **C.** $(-3)^3$ **D.** $(-3)^5$ **15.** _____

16. Write $\frac{1}{4^7}$ as an expression using a negative exponent.

A. -4^{-7} **B.** 4^{-7} **C.** -7^4 **D.** -7 **16.** _____

For Questions 17 and 18, evaluate each expression if $s = -2$ and $t = 3$.

17. t^{-2}

A. $-\frac{3}{2}$ **B.** -9 **C.** $\frac{1}{9}$ **D.** $-\frac{1}{9}$ **17.** _____

18. 4^{-1}

A. $\frac{1}{4}$ **B.** -4 **C.** $-\frac{1}{4}$ **D.** 4 **18.** _____

19. Bacteria are among the smallest living things. Some of the largest bacteria measure 7.87×10^{-5} inch across. Express this number in standard form.

A. 0.0000787 **B.** 787,000 **C.** 0.00787 **D.** 0.000787 **19.** _____

20. Choose the number that is less than 3.4×10^{-4}.

A. 2.1×10^6 **B.** 2.1×10^2 **C.** 43,000 **D.** 5.4×10^{-6} **20.** _____

Bonus Order the planets in the table at the right from least to greatest diameter.

Planet	Diameter (mi)
Saturn	1.21×10^5
Uranus	5.11×10^4
Jupiter	1.43×10^5
Neptune	4.95×10^4

B: _____

NAME _____ DATE _____ PERIOD _____

4

Chapter 4 Test, Form 2C

SCORE _____

Use divisibility rules to determine whether each number is divisible by 2, 3, 5, 6, or 10.

1. 1000 **1.** _____

2. 324 **2.** _____

For Questions 3 and 4, determine whether each expression is a monomial.

3. $10m(n - 8)$ **3.** _____

4. $9b(7e)$ **4.** _____

5. Write (2)(2)(2)(2)(2) using exponents. **5.** _____

6. Evaluate $7a^3$ if $a = 2$. **6.** _____

Write the prime factorization of each number or monomial.

7. 88 **7.** _____

8. $42a^3x$ **8.** _____

For Questions 9 and 10, find the GCF of each set of numbers or monomials.

9. 20, 36, 48 **9.** _____

10. $20y$, $30y^2$ **10.** _____

11. Factor $18b + 9$. **11.** _____

For Questions 12 and 13, write each fraction in simplest form.

12. $\frac{10}{65}$ **12.** _____

13. $\frac{25r^2}{40r}$ **13.** _____

14. Forty minutes is what part of an hour? **14.** _____

Assessment

NAME _____ DATE _____ PERIOD _____

4

Chapter 4 Test, Form 2C *(continued)*

Find each product. Express using exponents.

15. $m^4 \cdot m^3$ **15.** _____

16. $(11x^3y)(5y)$ **16.** _____

Find each quotient. Express using exponents.

17. $\frac{(-4)^7}{(-4)^2}$ **17.** _____

18. $\frac{b^{11}}{b}$ **18.** _____

For Questions 19 and 20, write each expression using a negative exponent other than −1.

19. $\frac{1}{3^9}$ **19.** _____

20. 0.0001 **20.** _____

21. Evaluate b^{-3} if $b = 5$. **21.** _____

22. Express 5.09×10^{-4} in standard form. **22.** _____

23. Scientists have discovered that many mammals can expect to live for 1.5 billion heartbeats. Write this number in scientific notation. **23.** _____

24. The distance between Saturn and Mars is 7.53×10^8 miles. The distance between Mars and Mercury is 8.37×10^7 miles. Is Mars closer to Saturn or to Mercury? **24.** _____

25. David is packing bundles of 3-inch-by-5-inch cards, face up, into a square box. He places the cards side-by-side so that there is no wasted space. Find the smallest possible measure for the edge of the box. **25.** _____

Bonus Write the prime factorization of 1188. Use exponents for repeated factors. **B:** _____

NAME _____ DATE _____ PERIOD _____

4

Chapter 4 Test, Form 2D

SCORE _____

Use divisibility rules to determine whether each number is divisible by 2, 3, 5, 6, or 10.

1. 98 **1.** _____

2. 360 **2.** _____

For Questions 3 and 4, determine whether each expression is a monomial.

3. $9k(p - 3)$ **3.** _____

4. $6a(4c)$ **4.** _____

5. Write (3)(3)(3)(3)(3)(3) using exponents. **5.** _____

6. Evaluate $5y^4$ if $y = 3$. **6.** _____

Write the prime factorization of each number or monomial.

7. 68 **7.** _____

8. $32p^2x$ **8.** _____

For Questions 9 and 10, find the GCF of each set of numbers or monomials.

9. 20, 30, 45 **9.** _____

10. $16x^2$, $18x$ **10.** _____

11. Factor $20a + 4$. **11.** _____

Write each fraction in simplest form.

12. $\frac{18}{64}$ **12.** _____

13. $\frac{16a^2}{20a}$ **13.** _____

14. Forty-five seconds is what part of a minute? **14.** _____

Assessment

4

Chapter 4 Test, Form 2D *(continued)*

Find each product. Express using exponents.

15. $d^3 \cdot d^2$ **15.** _____

16. $(12x^2y)(3y)$ **16.** _____

Find each quotient. Express using exponents.

17. $\frac{(-3)^5}{(-3)^2}$ **17.** _____

18. $\frac{a^{16}}{a}$ **18.** _____

For Questions 19 and 20, write each expression using a negative exponent other than −1.

19. $\frac{1}{4^7}$ **19.** _____

20. 0.001 **20.** _____

21. Evaluate a^{-4} if $a = 2$. **21.** _____

22. Express 4.68×10^{-4} in standard form. **22.** _____

23. The number of different hands possible in the game of bridge is about 635 billion. Write this number in scientific notation. **23.** _____

24. The number of neurons in the neocortex of the human brain is 3.0×10^{10}. The neocortex of a gorilla contains 7.5×10^8 neurons. Which mammal has more neurons? **24.** _____

25. Scott is packing bundles of 4-inch-by-6-inch cards, face up, into a square box. He places the cards side-by-side so that there is no wasted space. Find the smallest possible measure for the edge of the box. **25.** _____

Bonus Write the prime factorization of 1584. Use exponents for repeated factors. **B:** _____

NAME _____ DATE _____ PERIOD _____

4

Chapter 4 Test, Form 3

SCORE _____

Use divisibility rules to determine whether each number is divisible by 2, 3, 5, 6, or 10.

1. 6036 **1.** _____

2. 12,420 **2.** _____

3. 22,523 **3.** _____

Determine whether each expression is a monomial. Explain why or why not.

4. $6a + 2b$ **4.** _____

5. $(-12x)(13y)$ **5.** _____

6. $14k(2p - 3)$ **6.** _____

7. $-29c^2d(3c \cdot 4)$ **7.** _____

Write each expression using exponents.

8. $3 \cdot 3 \cdot x \cdot x \cdot y$ **8.** _____

9. $7 \cdot (2 - d) \cdot (2 - d)$ **9.** _____

Evaluate each expression if $a = -2$, $b = 3$, and $c = 5$.

10. $4a^4 + 3^b$ **10.** _____

11. $2(3a + c)^4$ **11.** _____

Determine whether each number is *prime* or *composite*.

12. 211 **12.** _____

13. 57 **13.** _____

Factor each number or monomial completely.

14. 99 **14.** _____

15. $-45qr^2s^3$ **15.** _____

Find the GCF of each set of numbers or monomials.

16. 26, 65, 91 **16.** _____

17. $12a^2c^2$, $30a^3b^2$ **17.** _____

Assessment

NAME _____ DATE _____ PERIOD _____

4

Chapter 4 Test, Form 3 *(continued)*

Factor each expression.

18. $x^2 + 3x$ **18.** _____

19. $18 - 6y$ **19.** _____

For Questions 20 and 21, write each fraction in simplest form. If already in simplest form, write *simplified*.

20. $\frac{91}{156}$ **20.** _____

21. $\frac{23p^2}{69pq}$ **21.** _____

22. Forty-four feet is what part of a mile? **22.** _____

23. Twelve inches is what part of one yard? **23.** _____

For Questions 24–27, find each product or quotient. Express using exponents.

24. $(-5x^3)(3x^2)$ **24.** _____

25. $(4s^4t)(st^2)$ **25.** _____

26. $\frac{a^3b}{b^5a^2}$ **26.** _____

27. $\left(\frac{m^4}{3m}\right)\left(\frac{12}{m^2}\right)$ **27.** _____

28. Write $\frac{1}{36}$ using a negative exponent other than -1. **28.** _____

Evaluate each expression if $a = 4$, $b = -3$, and $c = -1$.

29. $(b^3)(3^c)$ **29.** _____

30. $(bc)^{-5}$ **30.** _____

31. Express 1.057×10^{-4} in standard form. **31.** _____

32. To find how many seconds it takes light to travel from the Sun to Earth, divide the total distance, 93,000,000 miles, by the distance light travels in one second, 186,000 miles. Write the result in scientific notation. **32.** _____

33. Order 3.13×10^{-4}, 0.0313, 3.03×10^{-4}, 0.00303, and 3.0×10^{-4} from least to greatest. **33.** _____

Bonus Write all of the prime numbers between 1 and 50. **B:** _____

NAME _____ DATE _____ PERIOD _____

4 Chapter 4 Open-Ended Assessment

SCORE _____

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Methods of finding prime numbers have intrigued mathematicians for years. For example, *Goldbach's conjecture* states that every even number greater than 2 can be written as the sum of two prime numbers. Choose three even numbers greater than 20 and less than 100. Write each as the sum of two prime numbers.

2. Write an argument or counterexample to support your answers to the following questions.

a. Are all numbers that are divisible by 9 also divisible by 3?

b. Are all numbers that are divisible by 3 also divisible by 9?

3. The Warrior High School band and drill team have been invited to participate in the Thanksgiving parade. There are 210 members in the band and 40 members in the drill team.

a. Can the band march in a rectangular formation having 8 band members in each row? Why or why not? If not, give an example of a rectangular formation that would be possible.

b. Twenty-six drill team members and 140 band members turned in their permission slips for the Thanksgiving trip. Explain how to tell whether a greater fractional part of the drill team or band turned in their slips. Find which part is greater.

4. To understand mathematics, you must understand the language or symbols used.

a. Explain the difference in the meanings of $3a$ and a^3.

b. Explain the difference in the meanings of $-3b$ and b^{-3}.

Assessment

NAME _____ DATE _____ PERIOD _____

4

Chapter 4 Vocabulary Test/Review

SCORE _____

algebraic fraction	expanded form	power
base	exponent	prime number
base two	factor	scientific notation
binary	greatest common factor (GCF)	simplest form
divisible	monomial	Venn diagram

Write the letter of the term that best matches each statement or phrase.

_____ **1.** shows relationships among sets of numbers or objects using overlapping circles in a rectangle

_____ **2.** a whole number with exactly two factors, 1 and itself

_____ **3.** tells how many times a number is used as a factor

_____ **4.** GCF of the numerator and denominator is 1

_____ **5.** $(1 \times 10^3) + (3 \times 10^2) + (2 \times 10^1) + (7 \times 10^0)$

_____ **6.** language that uses a base two system of numbers

_____ **7.** a number, variable, or product of numbers and/or variables

_____ **8.** the greatest number that is a factor of two or more numbers

_____ **9.** a number that is expressed using an exponent

a. power

b. binary

c. exponent

d. Venn diagram

e. monomial

f. GCF

g. expanded form

h. simplest form

i. prime number

In your own words—
Define each term.

10. scientific notation

11. base

12. algebraic fraction

NAME _____ DATE _____ PERIOD _____

4

Chapter 4 Quiz

(Lessons 4—1 and 4—2)

SCORE _____

Use divisibility rules to determine whether each number is divisible by 2, 3, 5, 6, or 10.

1. 52 **1.** _____

2. 90 **2.** _____

3. 711 **3.** _____

4. 435 **4.** _____

Determine whether each expression is a monomial. Explain why or why not.

5. xy **5.** _____

6. $x + y$ **6.** _____

Write each expression using exponents.

7. $x \cdot x \cdot x$ **7.** _____

8. $(9 \cdot 9 \cdot 9) \cdot (9 \cdot 9)$ **8.** _____

Evaluate each expression if $x = 2$, $a = 3$, and $b = 2$.

9. $12x^4$ **9.** _____

10. a^2b^3 **10.** _____

NAME _____ DATE _____ PERIOD _____

4

Chapter 4 Quiz

(Lessons 4—3 and 4—4)

SCORE _____

Write the prime factorization of each number. Use exponents for repeated factors.

1. 27 **2.** 63 **3.** 112 **1.** _____ **2.** _____ **3.** _____

Factor each monomial completely.

4. $21b$ **4.** _____

5. $30x^2y$ **5.** _____

Find the GCF of each set of numbers or monomials.

6. 12, 16 **6.** _____

7. 120, 130, 140 **7.** _____

8. $28x^4$, $35x^6$ **8.** _____

Factor each expression.

9. $4a + 14$ **9.** _____

10. $30 + 5y$ **10.** _____

Assessment

NAME _____ DATE _____ PERIOD _____

4

Chapter 4 Quiz

SCORE _____

Write each fraction in simplest form. If the fraction is already in simplest form, write *simplified*.

1. $\frac{24}{36}$ **2.** $\frac{30}{42}$ **3.** $\frac{28x^6}{40x}$

4. Fifteen centimeters is what part of a meter?

5. Standardized Test Practice Which fraction is $\frac{5r}{r^2}$ written in simplest form?

A. $\frac{1}{5r}$ **B.** $5r$ **C.** $\frac{r}{5}$ **D.** $\frac{5}{r}$

Find each product or quotient. Express using exponents.

6. $3^6 \cdot 3^4$ **7.** $x^4 \cdot x^7 \cdot x^2$ **8.** $(2x^3)(5x^2)$

9. $\frac{(-4)^6}{(-4)^6}$ **10.** $\frac{x^{13}}{x^9}$

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____

NAME _____ DATE _____ PERIOD _____

4

Chapter 4 Quiz

SCORE _____

Write each expression using a positive exponent.

1. 7^{-5} **2.** y^{-3}

Evaluate each expression if $x = 3$ and $y = 2$.

3. x^{-4} **4.** $(xy)^{-3}$

Write each number in scientific notation.

5. 0.0000001602 **6.** 200,000,000 **7.** 3,000,000,000

Choose the greater number in each pair.

8. 4.62×10^{-3}, 0.000462 **9.** 3.06×10^{-5}, 3.60×10^{-5}

10. The table at the right shows the masses of three subatomic particles. Write the names of the particles in order from least to greatest mass.

Particle	Mass
electron	9.10×10^{-31}
neutron	1.68×10^{-27}
proton	1.67×10^{-27}

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____

NAME _____ DATE _____ PERIOD _____

4

Chapter 4 Mid-Chapter Test

SCORE _____

(Lessons 4—1 through 4—8)

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

1. Determine which expression is not a monomial.

A. $9(a + b)$ **B.** $-29x^2yz$ **C.** 112 **D.** k **1.** _____

2. Evaluate $2x^2$ if $x = 3$.

A. 36 **B.** 8 **C.** 18 **D.** 12 **2.** _____

3. Write the prime factorization of 84.

A. $2 \cdot 2 \cdot 21$ **B.** $2 \cdot 2 \cdot 2 \cdot 3 \cdot 7$

C. $4 \cdot 21$ **D.** $2 \cdot 2 \cdot 3 \cdot 7$ **3.** _____

4. Factor $21b + 35$.

A. $3(7b + 5)$ **B.** $7(3b + 5)$

C. $b(21 + 35)$ **D.** $7(3 + 5b)$ **4.** _____

5. Factor $15 + 45y$.

A. $15 \cdot 3y$ **B.** $15(1 + 3y)$

C. $60y$ **D.** $y(15 + 45)$ **5.** _____

6. Choose the number that is prime.

A. 21 **B.** 55 **C.** 37 **D.** 49 **6.** _____

Part II

7. Determine whether 252 is divisible by 2, 3, 5, 6, or 10. **7.** _____

8. Write (3)(3)(3)(2)(2) using exponents. **8.** _____

Factor each number or monomial completely.

9. 66 **9.** _____

10. $24x^3y^2$ **10.** _____

In Questions 11 and 12, find the GCF of each set of numbers or monomials.

11. 32, 56 **11.** _____

12. $20x^2$, $8x$ **12.** _____

13. Amy raised chickens for her 4-H project. The chickens have laid 70 eggs. Does Amy have enough eggs to completely fill cartons that contain 12 eggs each? Why or why not? **13.** _____

Assessment

NAME _____ DATE _____ PERIOD _____

4

Chapter 4 Cumulative Review

(Chapters 1—4)

1. State the domain and range of the relation {(2.3, 4), (5, 3.2), (4.6, 3.3)}. (Lesson 1–6) **1.** _____

2. Determine whether a scatter plot of the outside temperatures and the corresponding air conditioning bills might show a *positive*, *negative*, or *no* relationship. Explain your answer. (Lesson 1–7) **2.** _____

For Questions 3 and 4, simplify each expression. **3.** _____

3. $-7m + (-15m)$ (Lesson 3–2)

4. $9x - (-23x)$ (Lesson 3–2) **4.** _____

5. Evaluate $\frac{-4h}{3k}$ if $h = 6$ and $k = -2$. (Lesson 2–3) **5.** _____

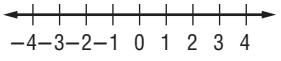
6. Find the average (mean) of -24, 16, 21, 9, -12. (Lesson 2–5) **6.** _____

7. Name the quadrant in which the graph of $(6, -5)$ lies. (Lesson 2–6) **7.** _____

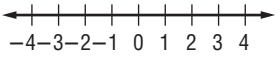
8. When you divide a number by -9, the result is 18. Write and solve an equation to find the number. (Lesson 3–4) **8.** _____

Graph the solution of each equation on a number line.

9. $2 + y = -1$ (Lesson 3–3) **9.**



10. $-4 = z - 2$ (Lesson 3–3) **10.**



For Questions 11 and 12, evaluate each expression if $n = 6$ and $r = 4$.

11. n^4 (Lesson 4–2) **11.** _____

12. $15r^3$ (Lesson 4–2) **12.** _____

13. Write the prime factorization of 78. (Lesson 4–3) **13.** _____

14. Factor $810abc^3$ completely. (Lesson 1–6) **14.** _____

For Questions 15 and 16, find the GCF of each set of numbers or monomials.

15. 45, 75, 90 (Lesson 4–4) **15.** _____

16. $36r^2s^4t$, $81r^3t^2$ (Lesson 4–2) **16.** _____

17. Factor $18 + 42y$. (Lesson 4–4) **17.** _____

NAME _____ DATE _____ PERIOD _____

4 Standardized Test Practice

(Chapters 1—4)

Part 1: Multiple Choice

Instructions: Fill in the appropriate oval for the best answer.

1. Lenora wants to buy a CD player that costs \$169 (including tax). She has \$108 in the bank, she earns \$30 babysitting, \$18 taking care of a neighbor's pet, and receives \$12 in allowance. How much more money does she need to buy the CD player? (Lesson 1–1)

A. \$7 **B.** \$1 **C.** \$4 **D.** none **1.** Ⓐ Ⓑ Ⓒ Ⓓ

2. Simplify $7(k + 5) + 9k$. (Lesson 1–4)

E. $16k + 35$ **F.** $10k + 12$ **G.** $16k + 5$ **H.** $17k + 5$ **2.** Ⓔ Ⓕ Ⓖ Ⓗ

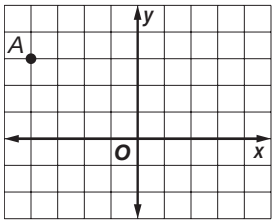
3. Simplify $(-4)(3a)(-4b)$. (Lesson 2–4)

A. $48ab$ **B.** $-48ab$ **C.** $-11ab$ **D.** $-5ab$ **3.** Ⓐ Ⓑ Ⓒ Ⓓ

4. Evaluate the expression $\frac{x}{y}$ if $x = -60$ and $y = 5$. (Lesson 2–5)

E. 12 **F.** -12 **G.** $\frac{1}{12}$ **H.** $-\frac{1}{12}$ **4.** Ⓔ Ⓕ Ⓖ Ⓗ

5. Name the ordered pair for the point A graphed on the coordinate plane at the right. (Lesson 2–6)



A. (3, 4) **B.** (3, −4)
C. (−4, 3) **D.** (−4, −3) **5.** Ⓐ Ⓑ Ⓒ Ⓓ

6. In the school cafeteria, an apple costs a cents and a carton of milk costs 40 cents. Which expression represents the total cost of an apple and 2 cartons of milk for s students? (Lesson 3–1)

E. $a + 80 + s$ **F.** $s(a + 80)$ **G.** $sa + 80$ **H.** $s(a + 40)$ **6.** Ⓔ Ⓕ Ⓖ Ⓗ

7. Wanda drove for w hours on a trip. Her husband drove 3 hours more than Wanda. Which expression represents the total time they spent driving? (Lesson 3–2)

A. $w + 6$ **B.** $2w + 3$ **C.** $w - 6$ **D.** $2w - 3$ **7.** Ⓐ Ⓑ Ⓒ Ⓓ

8. If $z - 5 = -9$, find the numerical value of $-2z - 5$. (Lesson 3–3)

E. 23 **F.** -33 **G.** -13 **H.** 3 **8.** Ⓔ Ⓕ Ⓖ Ⓗ

9. Nine more than eight times a number is -47. Translate this sentence into an equation. (Lesson 3–6)

A. $8n + 9 = -47$ **B.** $9n + 8 = -47$
C. $8(n + 9) = -47$. **D.** $9(8 + n) = -47$ **9.** Ⓐ Ⓑ Ⓒ Ⓓ

10. The formula $d = rt$ can be rewritten as (Lesson 3–7)

E. $dr = t$. **F.** $r = \frac{t}{d}$. **G.** $t = \frac{d}{r}$. **H.** $r = dt$. **10.** Ⓔ Ⓕ Ⓖ Ⓗ

Assessment

NAME _____ DATE _____ PERIOD _____

4 Standardized Test Practice *(continued)*

11. Choose the expression that is *not* a monomial. (Lesson 4–1)

A. $(4k^2)(9m)$ **B.** $\frac{3r - s}{4s}$ **C.** $-29xy^2$ **D.** $\frac{17c^2}{3}$ **11.** Ⓐ Ⓑ Ⓒ Ⓓ

12. Write $(a)(a)(a)(b)(b)$ using exponents. (Lesson 4–2)

E. a^3b^2 **F.** $a^{-3}b^{-2}$ **G.** 3^a2^b **H.** $3a^32b^2$ **12.** Ⓔ Ⓕ Ⓖ Ⓗ

13. Write the prime factorization of 42. (Lesson 4–3)

A. $1 \cdot 42$ **B.** $2 \cdot 21$ **C.** $2 \cdot 3 \cdot 7$ **D.** $6 \cdot 7$ **13.** Ⓐ Ⓑ Ⓒ Ⓓ

14. Find the GCF of 36 and 54. (Lesson 4–4)

E. 2 **F.** 3 **G.** 6 **H.** 18 **14.** Ⓔ Ⓕ Ⓖ Ⓗ

15. Simplify $\frac{112c^2}{16c^4}$. (Lesson 4–5)

A. $\frac{7c^2}{14}$ **B.** $\frac{14}{c^2}$ **C.** $\frac{7}{c^2}$ **D.** $7c^4$ **15.** Ⓐ Ⓑ Ⓒ Ⓓ

16. Which is $\frac{1}{144}$ written with a negative exponent? (Lesson 4–7)

E. 14^{-2} **F.** $\frac{1}{12^2}$ **G.** 144^{-2} **H.** 12^{-2} **16.** Ⓔ Ⓕ Ⓖ Ⓗ

Part 2: Grid In

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate oval that corresponds to that entry.

17. The end zone of a football field is 30 feet wide and 160 feet long. What is its area in square feet? (Lesson 3–7)

18. What is the least 3-digit number that is divisible by 4 and 7? (Lesson 4–1)

17.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

18.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Part 3: Short Response

Instructions: Write your answer in the blank at the right of each question.

19. Order the integers {32, −18, 2, 7, 0, −5, −11} from least to greatest. (Lesson 2–1) **19.** _____

20. Saturn is about 799,800,000 miles from Earth. Write this number in scientific notation. (Lesson 4–8) **20.** _____

NAME _____ DATE _____ PERIOD _____

4 Standardized Test Practice

Student Record Sheet *(Use with pages 196—197 of the Student Edition.)*

Part 1 Multiple Choice

Select the best answer from the choices given and fill in the corresponding oval.

1 Ⓐ Ⓑ Ⓒ Ⓓ	**4** Ⓐ Ⓑ Ⓒ Ⓓ	**7** Ⓐ Ⓑ Ⓒ Ⓓ	**10** Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ	**5** Ⓐ Ⓑ Ⓒ Ⓓ	**8** Ⓐ Ⓑ Ⓒ Ⓓ	**11** Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ	**6** Ⓐ Ⓑ Ⓒ Ⓓ	**9** Ⓐ Ⓑ Ⓒ Ⓓ	

Part 2 Short Response/Grid In

Solve the problem and write your answer in the blank.

For Questions 15, 17, and 19, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

12 _____

13 _____

14 _____

15 _____ (grid in)

16 _____

17 _____ (grid in)

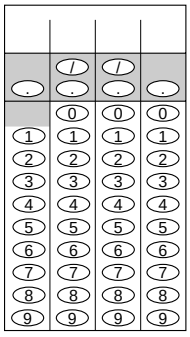
18 _____

19 _____ (grid in)

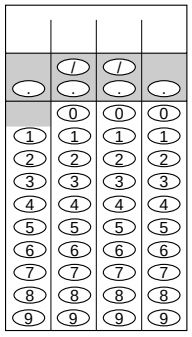
20 _____

21 _____

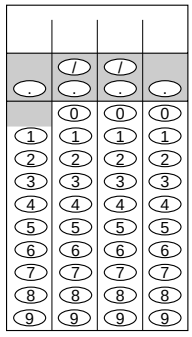
15



17



19



Part 3 Extended Response

Record your answers for Question 22 on the back of this paper.

Answers

NAME _____ DATE _____ PERIOD ____

4-1 Study Guide and Intervention

Factors and Monomials

Finding Factors Two or more numbers that are multiplied to form a product are called factors. Any number is divisible by its factors. The following rules can be used to determine mentally whether a number is divisible by 2, 3, 5, 6, or 10.

A number is divisible by:

- 2 if the ones digit is divisible by 2.
- 3 if the sum of the digits is divisible by 3.
- 5 if the ones digit is 0 or 5.
- 6 if the number is divisible by 2 and by 3.
- 10 if the ones digit is 0.

Example **Determine whether 108 is divisible by 2, 3, 5, 6, or 10.**

Number	Divisible?	Reason
2	yes	The ones digit is 8, and 8 is divisible by 2.
3	yes	The sum of the digits is 9, and 9 is divisible by 3.
5	no	The ones digit is 8, not 0 or 5.
6	yes	108 is divisible by 2 and by 3.
10	no	The ones digit is not 0.

108 is divisible by 2, 3, and 6.

A **monomial** is a number, a variable, or a product of numbers and/or variables. So, 108 is a monomial. The expression $5q$ is also a monomial since it is the product of a number and a variable, $5 \cdot q$. However, $2x + 1$ is not a monomial since it is the sum of two terms.

Exercises

Use divisibility rules to determine whether each number is divisible by 2, 3, 5, 6, or 10.

1. 105 3, 5
2. 600 2, 3, 5, 6, 10
3. 462 2, 3, 6
4. 197 none

List all the factors of each number.

5. 76 1, 2, 4, 19, 38, 76
6. 42 1, 2, 3, 6, 7, 14, 21, 42
7. 182 1, 2, 7, 13, 14, 26, 91, 182
8. 80 1, 2, 4, 5, 8, 10, 16, 20, 40, 80

Determine whether each expression is a monomial. Explain why or why not.

9. 13 yes; a number
10. $x + y$ no; sum of two terms
11. $3(x - 1)$ no; difference of two terms
12. $5st$ yes; product of a number and variables

NAME _____ DATE _____ PERIOD ____

4-1 Skills Practice

Factors and Monomials

Use divisibility rules to determine whether each number is divisible by 2, 3, 5, 6, or 10.

1. 100 2, 5, 10
2. 66 2, 3, 6
3. 88 2
4. 123 3
5. 240 2, 3, 5, 6, 10
6. 280 2, 5, 10
7. 255 3, 5
8. 165 3, 5
9. 318 2, 3, 6
10. 1000 2, 5, 10

List all the factors of each number.

11. 36 1, 2, 3, 4, 6, 9, 12, 18, 36
12. 29 1, 29
13. 45 1, 3, 5, 9, 15, 45
14. 81 1, 3, 9, 27, 81
15. 125 1, 5, 25, 125
16. 117 1, 3, 9, 13, 39, 117
17. 16 1, 2, 4, 8, 16
18. 63 1, 3, 7, 9, 21, 63

Determine whether each expression is a monomial. Explain why or why not.

19. p yes; a variable
20. 73 yes; a number
21. $2 + n$ no; sum of two terms
22. $h - w$ no; difference of two terms
23. $3(a + 6)$ no; sum of two terms
24. $-3k$ yes; product of a number and a variable
25. $q + r$ no; sum of two terms
26. $4y - 6$ no; difference of two terms
27. $3(x - 3)$ no; difference of two terms
28. $6s \cdot 4p$ yes; product of numbers and variables

29. **SEATING** Can 132 graduates be seated in rows of 6 at the graduation ceremony? Explain. Yes. Since 132 is divisible by 2 and by 3, it is divisible by 6. So the graduates can be seated in rows of 6 with no extra people or empty chairs.

30. **SCHOOL SUPPLIES** When Alex's mother buys pencils for school, she divides them equally among Alex and his sister. Should she buy the pencils in packages of 15 or 30? Explain. She should buy packages of 30 since 30 is divisible by 2, but 15 is not.

NAME _____ DATE _____ PERIOD ____

4-1

Practice

Factors and Monomials

Use divisibility rules to determine whether each number is divisible by 2, 3, 5, 6, or 10.

1. 476 2
2. 117 3
3. 426 2, 3, 6
4. 29 none
5. 735 3, 5
6. 276 2, 3, 6
7. 1200 2, 3, 5, 6, 10
8. 2370 2, 3, 5, 6, 10
9. 700 2, 5, 10
10. 4200 2, 3, 5, 6, 10

List all the factors of each number.

11. 48 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
12. 24 1, 2, 3, 4, 6, 8, 12, 24
13. 121 1, 11, 121
14. 82 1, 2, 41, 82
15. 37 1, 37
16. 196 1, 2, 4, 7, 14, 28, 49, 98, 196
17. 95 1, 5, 19, 95
18. 110 1, 2, 5, 10, 11, 22, 55, 110
19. 96 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96
20. 200 1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200

Determine whether each expression is a monomial. Explain why or why not.

21. 82 yes; a number
22. $4(-m)$ yes; product of a number and a variablle
23. m yes; a variable
24. rv yes; product of variables
25. $6(x - 6)$ no; difference of two terms
26. $8n - 8$ no; difference of two terms
27. $(-12)(-8)x$ yes; product of numbers and a variable
28. $w \cdot \ell$ yes; product of variables
29. $2\ell + 2w$ no; sum of two terms
30. $2s - t$ no; difference of two terms

NEWSPAPERS **For Exercises 31 and 32, refer to the following information.**

Brandon delivers newspapers in his neighborhood. On Sunday, he must deliver 112 papers. Since he rides his bike, he separates the papers into smaller stacks and delivers one stack at a time.

31. What size stacks can he make? 2 stacks of 56 (or 56 stacks of 2), 4 stacks of 28 (or 28 stacks of 4), 7 stacks of 16 (or 16 stacks of 7), 8 stacks of 14 (or 14 stacks of 8)
32. If Brandon can carry no more than 30 papers at a time and can return home to restock no more than 5 times, how can he organize the 112 papers? 4 stacks of 28 papers

Lesson 4-1

NAME _____ DATE _____ PERIOD ____

4-1

Reading to Learn Mathematics

Factors and Monomials

Pre-Activity *How are side lengths of rectangles related to factors?*

Do the activity at the top of page 148 in your textbook. Write your answers below.

a. Use grid paper to draw as many other rectangles as possible with an area of 36 square units. Label the length and width of each rectangle.
Students should draw rectangles with dimensions 1×36, 2×18, 3×12, and 6×6.

b. Did you draw a rectangle with a length of 5 units? Why or why not?
No, if the length were 5, there is no whole number width that would give an area of 36.

c. List all of the pairs of whole numbers whose product is 36. Compare this list to the lengths and widths of all the rectangles that have an area of 36 square units. What do you observe? 1 and 36, 2 and 18, 3 and 12, 4 and 9, 6 and 6; they are the same.

d. Predict the number of rectangles that can be drawn with an area of 64 square units. Explain how you can predict without actually drawing them. 4 rectangles; find the factor pairs whose product is 64: 1×64, 2×32, 4×16, 8×8.

Reading the Lesson 1–3. See students' work.

Write a definition and give an example of each new vocabulary word.

Vocabulary	Definition	Example
1. factors		
2. divisible		
3. monomial		

4. Is the expression $2x - 1$ a monomial? Explain. $2x - 1$ is not a monomial because it is the difference of two terms.

Helping You Remember

5. Explain in your own words how to determine whether an expression is a monomial.
Sample answer: A monomial is a number, a variable, or the product of numbers and/or variables.

Answers

NAME _____ DATE _____ PERIOD ____

4-1 Enrichment

Divisibility

Divisibility rule for 7

Determine whether 4032 is divisible by 7.

403~~2~~	**Cross out the ones digit.**
$-\ 4$	**Subtract twice the value of the ones digit from the rest of the number.**
39~~8~~	**If the difference is a number that you know is divisible by 7, stop. If not, repeat.**
$-\ 18$	
21	Since 21 is divisible by 7, 4032 is divisible by 7.

Divisibility rule for 11

Determine whether 5159 is divisible by 11.

Method 1

515~~9~~	**Cross out the ones digit.**
$-\ 9$	**Subtract the value of the ones digit from the rest of the number.**
50~~6~~	**If the difference is a number that you know is divisible by 11, stop. If not, repeat.**
$-\ 6$	
44	Since 44 is divisible by 11, 5159 is divisible by 11.

Method 2

5159	
$5 + 5 = 10$	**Add the odd-numbered digits (first and third).**
$1 + 9 = \underline{10}$	**Add the even-numbered digits (second and fourth).**
0	**Subtract the sums. If the difference is divisible by 11, the number is divisible by 11.**

Since 0 is divisible by 11, 5159 is divisible by 11.

Determine whether 62,382 is divisible by 11.

$6 + 3 + 2 = 11$	**Add the odd-numbered digits.**
$2 + 8 = \underline{10}$	**Add the even-numbered digits.**
1	**Subtract the sums.**

Since 1 is not divisible by 11, 62,382 is not divisible by 11.

Determine whether each number is divisible by 7 or 11.

1. 266 **7**
2. 4312 **7 and 11**
3. 8976 **11**
4. 936 **neither**
5. 13,293 **7**
6. 7085 **neither**
7. 2957 **neither**
8. 3124 **11**
9. 6545 **7 and 11**

Lesson 4-1

NAME _____ DATE _____ PERIOD ____

4-2 Study Guide and Intervention

Powers and Exponents

A number that is expressed using an exponent is called a **power**. The **base** is the number that is multiplied. The **exponent** tells how many times the base is used as a factor. So, 4^3 has a base of 4 and an exponent of 3, and $4^3 = 4 \cdot 4 \cdot 4 = 64$.

Example 1 **Write each expression using exponents.**

a. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

The base is 10. It is a factor 5 times, so the exponent is 5.

$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^5$

b. $(p + 2)(p + 2)(p + 2)$

The base is $p + 2$. It is a factor 3 times, so the exponent is 3.

$(p + 2)(p + 2)(p + 2) = (p + 2)^3$

Expressions involving powers are evaluated using order of operations. Powers are repeated multiplications. They are evaluated after any grouping symbols and before other multiplication or division operations.

Example 2 **Evaluate $x^2 - 4$ if $x = -6$.**

$x^2 - 4 = (-6)^2 - 4$	Replace x with -6.
$= (-6)(-6) - 4$	-6 is a factor 2 times.
$= 36 - 4$	Multiply.
$= 32$	Subtract.

Exercises

Write each expression using exponents.

1. $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ **5^7**
2. $(-7)(-7)(-7)$ **$(-7)^3$**
3. $d \cdot d \cdot d \cdot d$ **d^4**
4. $x \cdot x \cdot y \cdot y$ **x^2y^2**
5. $(z - 4)(z - 4)$ **$(z - 4)^2$**
6. $3(-t)(-t)(-t)$ **$3(-t)^3$**

Evaluate each expression if $g = 3$, $h = -1$, and $m = 9$.

7. g^5 **243**
8. $5g^2$ **45**
9. $g^2 - m$ **0**
10. hm^2 **−81**
11. $g^3 + 2h$ **25**
12. $m + hg^3$ **−18**

NAME _____ DATE _____ PERIOD ____

4-2 Skills Practice

Powers and Exponents

Write each expression using exponents.

1. $7 \cdot 7$ 7^2
2. $(-3)(-3)(-3)(-3)(-3)$ $(-3)^5$
3. 4 4^1
4. $(k \cdot k)(k \cdot k)(k \cdot k)$ k^6
5. $p \cdot p \cdot p \cdot p \cdot p \cdot p$ p^6
6. $3 \cdot 3$ 3^2
7. $(-a)(-a)(-a)(-a)$ $(-a)^4$
8. $6 \cdot 6 \cdot 6 \cdot 6$ 6^4
9. $9 \cdot 9 \cdot 9$ 9^3
10. $4 \cdot y \cdot z \cdot z \cdot z$ $4yz^3$
11. $s \cdot s \cdot s \cdot s \cdot t \cdot u \cdot u$ s^4tu^2
12. $5 \cdot 5 \cdot 5 \cdot q \cdot q$ 5^3q^2

Express each number in expanded form.

13. 135
$(1 \times 10^2) + (3 \times 10^1) + (5 \times 10^0)$
14. 8732
$(8 \times 10^3) + (7 \times 10^2) + (3 \times 10^1) + (2 \times 10^0)$
15. 1005
$(1 \times 10^3) + (0 \times 10^2) + (0 \times 10^1) + (5 \times 10^0)$
16. 989
$(9 \times 10^2) + (8 \times 10^1) + (9 \times 10^0)$

Evaluate each expression if $b = 8$, $c = 2$, and $d = -3$.

17. 4^c 16
18. c^0 1
19. b^3 512
20. $c^3 \cdot 3^c$ 72
21. 3^c 9
22. c^4 16
23. $c^2 + d$ 1
24. $2b^2$ 128
25. $b^2 + c^3$ 72
26. d^2 9
27. d^3 –27
28. $b^2 + d^3$ 37
29. b^2d –192
30. $(b - c)^2$ 36

Lesson 4-2

NAME _____ DATE _____ PERIOD ____

4-2 Practice

Powers and Exponents

Write each expression using exponents.

1. $11 \cdot 11 \cdot 11$ 11^3
2. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ 2^8
3. 5 5^1
4. $(-4)(-4)$ $(-4)^2$
5. $a \cdot a \cdot a \cdot a$ a^4
6. $n \cdot n \cdot n \cdot n \cdot n$ n^5
7. $4 \cdot 4 \cdot 4$ 4^3
8. $(b \cdot b)(b \cdot b)(b \cdot b)$ b^6
9. $(-v)(-v)(-v)(-v)$ $(-v)^4$
10. $x \cdot x \cdot z \cdot z \cdot z$ x^2z^3
11. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot t \cdot t$ 2^5t^2
12. $m \cdot m \cdot m \cdot n \cdot p \cdot p$ m^3np^2

Express each number in expanded form.

13. 13
$(1 \times 10^1) + (3 \times 10^0)$
14. 1006
$(1 \times 10^3) + (0 \times 10^2) + (0 \times 10^1) + (6 \times 10^0)$
15. 17,629 $(1 \times 10^4) + (7 \times 10^3) + (6 \times 10^2) + (2 \times 10^1) + (9 \times 10^0)$
16. 897 $(8 \times 10^2) + (9 \times 10^1) + (7 \times 10^0)$

Evaluate each expression if $x = 3$, $y = -2$, and $z = 4$.

17. y^x -8
18. 51^0 1
19. z^2 16
20. x^2 9
21. 9^x 729
22. $z^2 \cdot 2^2$ 64
23. y^5 -32
24. $z^2 - y^4$ 0
25. $x^2 + y^2 + z^2$ 29
26. $z^2 - x^2$ 7

FAMILY TREE **For Exercises 27 and 28, refer to the following information.**

When examining a family tree, the branches are many. You are generation "now." One generation ago, your 2 parents were born. Two generations ago your 4 grandparents were born.

27. How many great-grandparents were born three generations ago? 2^3 or 8
28. How many "great" grandparents were born ten generations ago? 2^{10} or 1024

Answers

NAME _____ DATE _____ PERIOD ____

4-2 Reading to Learn Mathematics

Powers and Exponents

Pre-Activity ***Why are exponents important in comparing computer data?***

Do the activity at the top of page 153 in your textbook. Write your answers below.

a. Write 16 as a product of factors of 2. How many factors are there?
$2 \times 2 \times 2 \times 2$; 4 factors

b. How many factors of 2 form the product 128? 7 factors

c. One megabyte is 1024 kilobytes. How many factors of 2 form the product 1024? 10 factors

Reading the Lesson 1–5. See students' work.

Write a definition and give an example of each new vocabulary word or phrase.

Vocabulary	Definition	Example
1. base		
2. exponent		
3. power		
4. standard form		
5. expanded form		

6. Write each expression using exponents.

a. $4 \cdot 4 \cdot 4 \cdot 4$ 4^4 **b.** $x \cdot x \cdot x \cdot y \cdot y$ x^3y^2

c. $(-2)(-2)(-2)$ $(-2)^3$ **d.** $5 \cdot r \cdot r \cdot m \cdot m \cdot m$ $5r^2m^3$

7. The number $(3 \times 10^3) + (5 \times 10^2) + (0 \times 10^1) + (2 \times 10^0)$ is written in expanded form, while 3502 is written in standard form.

Helping You Remember

8. Explain how the terms *base*, *power*, and *exponent* are related. Provide an example.
Sample answer: A power is an expression with two parts—a base and an exponent. For example, the power 2^3 has a base of 2 and an exponent of 3.

Lesson 4-2

NAME _____ DATE _____ PERIOD ____

4-2 Enrichment

Exponents

Numbers can be expressed in several ways. Some numbers are expressed as sums. Some numbers are expressed as products of factors, while other numbers are expressed as powers.

Two ways to express 27 are $3 \cdot 3 \cdot 3$ and 3^3.

The number 1 million can be expressed in the following ways.

1,000,000	$1000 \cdot 1000$	$100 \cdot 100 \cdot 100$	$10^2 \cdot 10^2 \cdot 10^2$
$1{,}000{,}000^1$	1000^2	100^3	10^6

Write names for each number below using the given exponents.

1. 16; exponents: 2 and 4 $4^2, 2^4$

2. 81; exponents: 2 and 4 $9^2, 3^4$

3. 64; exponents: 2 and 6 $8^2, 2^6$

4. 256; exponents: 2 and 8 $16^2, 2^8$

5. 625; exponents: 2 and 4 $25^2, 5^4$

6. 729; exponents: 2 and 6 $27^2, 3^6$

7. 2401; exponents: 2 and 4 $49^2, 7^4$

8. 4096; exponents: 2 and 12 $64^2, 2^{12}$

9. 6561; exponents: 2 and 8 $81^2, 3^8$

10. 390,625; exponents: 2 and 8 $625^2, 5^8$

Numbers that can be named as powers with like bases can be multiplied by adding the exponents.

$8 \cdot 8 = 2^3 \cdot 2^3$

$= 2^{3+3}$

$= 2^6$

Write the product of each pair of factors in exponential form.

11. $9 \cdot 9$ $3^2 \cdot 3^2 = 3^4$

12. $4 \cdot 4$ $2^2 \cdot 2^2 = 2^4$

13. $16 \cdot 8$ $2^4 \cdot 2^3 = 2^7$

14. $125 \cdot 25$ $5^3 \cdot 5^2 = 5^5$

15. $27 \cdot 9$ $3^3 \cdot 3^2 = 3^5$

16. $81 \cdot 27$ $3^4 \cdot 3^3 = 3^7$

17. $49 \cdot 49$ $7^2 \cdot 7^2 = 7^4$

18. $121 \cdot 121$ $11^2 \cdot 11^2 = 11^4$

NAME _____ DATE _____ PERIOD ____

4-3 Study Guide and Intervention

Prime Factorization

A **prime number** is a whole number that has exactly two factors, 1 and itself. A **composite number** is a whole number that has more than two factors. Zero and 1 are neither prime nor composite.

Example 1 **Determine whether 29 is prime or composite.**

Find the factors of 29.

$29 = 1 \cdot 29$

The only factors of 29 are 1 and 29, therefore 29 is a prime number.

Any composite number can be written as a product of prime numbers. A factor tree can be used to find the prime factorization.

Example 2 **Find the prime factorization of 48.**

48 is the number to be factored.

Find any pair of whole number factors of 48.

Continue to factor any number that is not prime.

The factor tree is complete when there is a row of prime numbers.

The prime factorization of 48 is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$ or $2^4 \cdot 3$.

In algebra, monomials can be factored as a product of prime numbers and variables with no exponent greater than 1. So, $8x^2$ factors as $2 \cdot 2 \cdot 2 \cdot x \cdot x$.

Exercises

Determine whether each number is *prime* or *composite*.

1. 27 composite
2. 151 prime
3. 77 composite
4. 25 composite

Write the prime factorization for each number. Use exponents for repeated factors.

5. 16 2^4
6. 45 $3^2 \cdot 5$
7. 78 $2 \cdot 3 \cdot 13$
8. 70 $2 \cdot 5 \cdot 7$

Factor each monomial.

9. $6m^3$ $2 \cdot 3 \cdot m \cdot m \cdot m$
10. $-20xy^2$ $-1 \cdot 2 \cdot 2 \cdot 5 \cdot x \cdot y \cdot y$
11. $a^2b^2c^3$ $a \cdot a \cdot b \cdot b \cdot c \cdot c \cdot c$
12. $25h$ $5 \cdot 5 \cdot h$

Lesson 4-3

NAME _____ DATE _____ PERIOD ____

4-3 Skills Practice

Prime Factorization

Determine whether each number is *prime* or *composite*.

1. 41 prime
2. 29 prime
3. 87 composite
4. 36 composite
5. 57 composite
6. 61 prime
7. 71 prime
8. 103 prime
9. 39 composite
10. 91 composite
11. 47 prime
12. 67 prime

Write the prime factorization of each number. Use exponents for repeated factors.

13. 20 $2^2 \cdot 5$
14. 40 $2^3 \cdot 5$
15. 32 2^5
16. 44 $2^2 \cdot 11$
17. 90 $2 \cdot 3^2 \cdot 5$
18. 121 11^2
19. 46 $2 \cdot 23$
20. 30 $2 \cdot 3 \cdot 5$
21. 65 $5 \cdot 13$
22. 80 $2^4 \cdot 5$

Factor each monomial.

23. $15t$ $3 \cdot 5 \cdot t$
24. $16r^2$ $2 \cdot 2 \cdot 2 \cdot 2 \cdot r \cdot r$
25. $-11m^2$ $-1 \cdot 11 \cdot m \cdot m$
26. $-49y^3$ $-1 \cdot 7 \cdot 7 \cdot y \cdot y \cdot y$
27. $21ab$ $3 \cdot 7 \cdot a \cdot b$
28. $-42xyz$ $-1 \cdot 2 \cdot 3 \cdot 7 \cdot x \cdot y \cdot z$
29. $45j^2k$ $3 \cdot 3 \cdot 5 \cdot j \cdot j \cdot k$
30. $17u^2v^2$ $17 \cdot u \cdot u \cdot v \cdot v$
31. $27d^4$ $3 \cdot 3 \cdot 3 \cdot d \cdot d \cdot d \cdot d$
32. $-16cd^2$ $-1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot c \cdot d \cdot d$

Answers

NAME _____ DATE _____ PERIOD ____

4-3 Practice

Prime Factorization

Determine whether each number is *prime* or *composite*.

1. 11 prime
2. 63 composite
3. 73 prime
4. 75 composite
5. 49 composite
6. 69 composite
7. 53 prime
8. 83 prime

Write the prime factorization of each number. Use exponents for repeated factors.

9. 33 $3 \cdot 11$
10. 24 $2^3 \cdot 3$
11. 72 $2^3 \cdot 3^2$
12. 276 $2^2 \cdot 3 \cdot 23$
13. 85 $5 \cdot 17$
14. 1024 2^{10}
15. 95 $5 \cdot 19$
16. 200 $2^3 \cdot 5^2$
17. 243 3^5
18. 735 $3 \cdot 5 \cdot 7^2$

Factor each monomial.

19. $35v$ $5 \cdot 7 \cdot v$
20. $49c^2$ $7 \cdot 7 \cdot c \cdot c$
21. $-14b^3$ $-1 \cdot 2 \cdot 7 \cdot b \cdot b \cdot b$
22. $-81h^2$ $-1 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot h \cdot h$
23. $33wz$ $3 \cdot 11 \cdot w \cdot z$
24. $-56ghj$ $-1 \cdot 2 \cdot 2 \cdot 2 \cdot 7 \cdot g \cdot h \cdot j$

25. **NUMBER THEORY** *Twin primes* are a pair of consecutive odd primes, which differ by 2. For example, 3 and 5 are twin primes. Find the twin primes less than 100. (*Hint*: There are 8 pairs of twins less than 100.) 3, 5; 5, 7; 11, 13; 17, 19; 29, 31; 41, 43; 59, 61; 71, 73

Lesson 4-3

NAME _____ DATE _____ PERIOD ____

4-3 Reading to Learn Mathematics

Prime Factorization

Pre-Activity ***How can models be used to determine whether numbers are prime?***

Do the activity at the top of page 159 in your textbook. Write your answers below.

a. Use grid paper to draw as many different rectangular arrangements of 2, 3, 4, 5, 6, 7, 8, and 9 squares as possible. See students' answers.

b. Which numbers of squares can be arranged in more than one way? 4, 6, 8, 9

c. Which numbers of squares can only be arranged one way? 2, 3, 5, 7

d. What do all rectangles that you listed in part **c** have in common? Explain. They all have a width of 1 because no other pair of factors can be found.

Reading the Lesson 1–5. See students' work.

Write a definition and give an example of each new vocabulary word or phrase.

Vocabulary	Definition	Example
1. composite number		
2. factor		
3. factor tree		
4. prime factorization		
5. prime number		

Helping You Remember

6. *Composite* is a word used in everyday English.

a. Find the definition of *composite* in the dictionary. Write the definition. made up of distinct parts

b. Explain how the English definition can help you remember how composite is used in mathematics. Sample answer: Composite numbers are made up of many distinct parts, or factors.

NAME _____ DATE _____ PERIOD ____

4-3 Enrichment

Prime Pyramid

A **prime number** is a whole number that has exactly two factors—itself and 1. The pyramid below is called a *prime pyramid*. Each row begins with 1 and ends with the number of that row. So, row 2 begins with 1 and ends with 2, row 3 begins with 1 and ends with 3, and so on. In each row, the numbers from 1 to the row number are arranged such that the sum of any two adjacent numbers is a prime number.

For example, look at row 4:

- It must contain the numbers 1, 2, 3, and 4.
- It must begin with 1 and end with 4.
- The sum of adjacent pairs must be a prime number:
  $1 + 2 = 3,\ 2 + 3 = 5,\ 3 + 4 = 7$

*
1 2
1 2 3
1 2 3 4
1 4 3 2 5
1 4 3 2 5 6
1 4 3 2 5 6 7
1 2 3 4 7 6 5 8
1 2 3 4 7 6 5 8 9
1 2 3 4 7 6 5 8 9 10
1 4 3 2 5 6 7 10 9 8 11
1 4 3 2 5 6 7 10 9 8 11 12

1. Complete the pyramid by filling in the missing numbers.

2. Extend the pyramid to row 13.

1, 4, 3, 2, 5, 6, 7, 10, 9, 8, 11, 12, 13

3. Explain the patterns you see in the completed pyramid.

Sample answer: Each row alternates odd and even numbers. Multiples of 3 form diagonals that are constant.

Lesson 4-3

NAME _____ DATE _____ PERIOD ____

4-4 Study Guide and Intervention

Greatest Common Factor (GCF)

The greatest number that is a factor of two or more numbers is the **greatest common factor (GCF)**. Two ways to find the GCF are shown below.

Example 1 **Find the GCF of 24 and 32.**

Method 1 List the factors.

factors of 24: 1, 2, 3, 4, 6, 8, 12, 24 — Look for factors common to both lists, 1, 2, 4, and 8.

factors of 32: 1, 2, 4, 8, 16, 32

The greatest common factor of 24 and 32 is 8.

Method 2 Use prime factorization.

$24 = \boxed{2} \cdot \boxed{2} \cdot \boxed{2} \cdot 3$ — Find the common prime factors of 24 and 32.

$32 = \boxed{2} \cdot \boxed{2} \cdot \boxed{2} \cdot 2 \cdot 2$

Multiply the common prime factors. The greatest common factor of 24 and 32 is $2 \cdot 2 \cdot 2$ or 8.

In algebra, greatest common factors are used to factor expressions.

Example 2 **Factor $5x + 10$.**

First, find the GCF of $5x$ and 10.

$5x = 5 \cdot x$

$10 = 2 \cdot 5$ — The GCF is 5.

Now write each term as a product of the GCF and its remaining factors.

$5x + 10 = 5(x) + 5(2)$

$= 5(x + 2)$ — Distributive Property

So, $5x + 10 = 5(x + 2)$.

Exercises

Find the GCF of each set of numbers.

1. 30, 42 6 **2.** 15, 33 3 **3.** 44, 110 22 **4.** 16, 48 16

Factor each expression.

5. $4g + 16$ $4(g + 4)$ **6.** $2d - 6$ $2(d - 3)$ **7.** $8a + 24$ $8(a + 3)$

8. $f^2 + 2f$ $f(f + 2)$ **9.** $6 - 3j$ $3(2 - j)$ **10.** $16n^2 - 40n$ $8n(2n - 5)$

Answers (Lessons 4-3 and 4-4)

Answers

NAME _____ DATE _____ PERIOD ____

4-4 Skills Practice

Greatest Common Factor (GCF)

Find the GCF of each set of numbers or monomials.

1. 15, 50 **5**
2. 24, 81 **3**
3. 18, 27 **9**
4. 36, 64 **4**
5. 88, 40 **8**
6. 54, 63 **9**
7. 11, 22 **11**
8. 14, 25 **1**
9. 20, 30 **10**
10. 16, 18 **2**
11. 64, 80 **16**
12. 16, 24 **8**
13. $30t$, $40t^2$ **$10t$**
14. 6, $9t$ **3**
15. $16k^2$, $40k$ **$8k$**
16. $9m$, $15n$ **3**
17. $7pq$, $8q$ **q**
18. $18p$, 45 **9**

Factor each expression.

19. $5b + 15$ **$5(b + 3)$**
20. $7t + 49$ **$7(t + 7)$**
21. $6w + 18$ **$6(w + 3)$**
22. $100 + 50x$ **$50(2 + x)$**
23. $7x + 7$ **$7(x + 1)$**
24. $12n + 60$ **$12(n + 5)$**
25. $24 + 8g$ **$8(3 + g)$**
26. $50 + 5f$ **$5(10 + f)$**
27. $3n + 24$ **$3(n + 8)$**
28. $9\ell + 63$ **$9(l + 7)$**
29. $6u + 36$ **$6(u + 6)$**
30. $70 - 7c$ **$7(10 - c)$**
31. $42 - 21x$ **$21(2 - x)$**
32. $12y + 16$ **$4(3y + 4)$**
33. $6p - 12$ **$6(p - 2)$**
34. $9r - 81$ **$9(r - 9)$**
35. $6 + 8q$ **$2(3 + 4q)$**
36. $21x + 33$ **$3(7x + 11)$**

NAME _____ DATE _____ PERIOD ____

4-4 Practice

Greatest Common Factor (GCF)

Find the GCF of each set of numbers or monomials.

1. 9, 36 **9**
2. 42, 60 **6**
3. 16, 60 **4**
4. 29, 58 **29**
5. 18, 35 **1**
6. 90, 480 **30**
7. 80, 45 **5**
8. 700, 200 **100**
9. 17, 85 **17**
10. 24, 84, 168 **12**
11. 55, 105 **5**
12. 252, 126 **126**
13. $5p$, $20p^2$ **$5p$**
14. $28a$, $49ab$ **$7a$**
15. $8b$, $5c$ **1**
16. $6a^2$, $18b^2$ **6**
17. $88s^2t$, $40st^2$ **$8st$**
18. $42a^2b$, $60ab^2$ **$6ab$**

Factor each expression.

19. $10x + 40$ **$10(x + 4)$**
20. $8v + 56$ **$8(v + 7)$**
21. $9t + 9$ **$9(t + 1)$**
22. $13m + 39$ **$13(m + 3)$**
23. $90 + 45n$ **$45(2 + n)$**
24. $15p + 60$ **$15(p + 4)$**
25. $48 + 8r$ **$8(6 + r)$**
26. $11z - 55$ **$11(z - 5)$**
27. $18q - 54$ **$18(q - 3)$**
28. $125 - 25h$ **$25(5 - h)$**
29. $42a - 77$ **$7(6a - 11)$**
30. $30 + 45s$ **$15(2 + 3s)$**
31. $50n + 30$ **$10(5n + 3)$**
32. $18 + 12d$ **$6(3 + 2d)$**
33. $27m + 105$ **$3(9m + 35)$**
34. $65 - 39b$ **$13(5 - 3b)$**
35. $21d - 63$ **$7(3d - 9)$**
36. $48 + 84m$ **$12(4 + 7m)$**

37. **SCHOOL TRIP** Thirty-two seventh graders, 48 eighth graders, and 60 ninth graders are taking a ski trip. In order to help students get better acquainted, students from each grade level are to ride each bus. What is the greatest number of buses that can be used if students from each grade level are divided equally among the buses? **4 buses**

NAME _____ DATE _____ PERIOD ____

4-4 Reading to Learn Mathematics

Greatest Common Factor (GCF)

Pre-Activity ***How can a diagram be used to find the greatest common factor?***

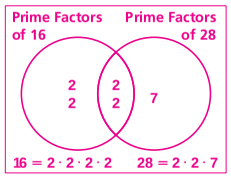
Do the activity at the top of page 164 in your textbook. Write your answers below.

a. Which numbers are in both circles? 2, 2

b. Find the product of the numbers that are in both circles. 4

c. Is the product also a factor of 12 and 20? yes

d. Make a Venn diagram showing the prime factors of 16 and 28. Then use it to find the common factors of the numbers. 2, 2



Reading the Lesson 1–2. See students' work.

Write a definition and give an example of each new vocabulary word or phrase.

Vocabulary	Definition	Example
1. Venn diagram		
2. greatest common factor		

Helping You Remember

3. Summarize in your own words how to find the greatest common factor of two numbers using each method.

a. prime factorization Write the prime factorization of each number, then look for the prime factors common to both numbers. The greatest common factor is the product of the common factors.

b. lists of factors Make a list of all factors of both numbers. The largest factor that is in both lists is the greatest common factor.

c. a Venn diagram Make a Venn diagram with the prime factors of each number in a circle. The GCF is the product of the factors in the overlapping section of the diagram.

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4-4 Enrichment

GCFs by Successive Division

Another way to find the greatest common factor (GCF) of two numbers is to use **successive division**. This method works well for large numbers.

Find the GCF of 848 and 1325.

Step 1 Divide the smaller number into the greater number.

$$\begin{array}{r} 1 \text{ R}477 \\ 848\overline{)1325} \\ \underline{848} \\ 477 \end{array}$$

Step 2 Divide the remainder into the divisor. Repeat this step until you get a remainder of 0.

$$\begin{array}{r} 1 \text{ R}371 \\ 477\overline{)848} \\ \underline{477} \\ 371 \end{array} \qquad \begin{array}{r} 1 \text{ R}106 \\ 371\overline{)477} \\ \underline{371} \\ 106 \end{array} \qquad \begin{array}{r} 3 \text{ R}53 \\ 106\overline{)371} \\ \underline{318} \\ 53 \end{array} \qquad \begin{array}{r} 2 \text{ R}0 \\ 53\overline{)106} \\ \underline{106} \\ 0 \end{array}$$

The last divisor is the GCF of the two original numbers. So the GCF of 848 and 1325 is 53.

Use the method above to find the GCF of each pair of numbers.

1. 187; 578 17

2. 161; 943 23

3. 215; 1849 43

4. 453; 484 1

5. 432; 588 12

6. 279; 403 31

7. 1325; 3498 53

8. 9840; 1751 1

9. 3484; 5963 67

10. 1802; 106 106

11. 45,787; 69,875 1

12. 35,811; 102,070 173

Answers

NAME _____ DATE _____ PERIOD ____

4-5 Study Guide and Intervention

Simplifying Algebraic Fractions

A fraction is in **simplest form** when the GCF of the numerator and the denominator is 1. One way to write a fraction in simplest form is to write the prime factorization of the numerator and the denominator. Then divide the numerator and denominator by the GCF.

Example 1 **Write $\frac{18}{24}$ in simplest form.**

Write the prime factorization of the numerator and the denominator.

$\frac{18}{24} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}} \cdot 3}{\underset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{3}}}$ Divide the numerator and denominator by the GCF, 2 · 3.

$= \frac{3}{2 \cdot 2}$ or $\frac{3}{4}$ Simplify.

Algebraic fractions can also be written in simplest form. Again, you can write the prime factorization of the numerator and the denominator, then divide by the GCF.

Example 2 **Simplify $\frac{24ab^2}{36a^2}$.**

$\frac{24ab^2}{36a^2} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{a}} \cdot b \cdot b}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 3 \cdot \underset{1}{\cancel{a}} \cdot a}$ Divide the numerator and denominator by the GCF, 2 · 2 · 3 · a.

$= \frac{2 \cdot b \cdot b}{3 \cdot a}$ or $\frac{2b^2}{3a}$ Simplify.

Exercises

Simplify each fraction. If the fraction is already in simplest form, write *simplified*.

1. $\frac{12}{20}$ $\frac{3}{5}$
2. $\frac{16}{36}$ $\frac{4}{9}$
3. $\frac{75}{100}$ $\frac{3}{4}$
4. $\frac{6}{15}$ $\frac{2}{5}$
5. $\frac{8}{24}$ $\frac{1}{3}$
6. $\frac{3}{8}$ simplified
7. $\frac{c}{c^3}$ $\frac{1}{c^2}$
8. $\frac{r^4}{r^2}$ r^2
9. $\frac{14b}{21b}$ $\frac{2}{3}$
10. $\frac{24w}{26w}$ $\frac{12}{13}$
11. $\frac{5s}{12t}$ simplified
12. $\frac{d}{3d^2}$ $\frac{1}{3d}$

Lesson 4-5

NAME _____ DATE _____ PERIOD ____

4-5 Skills Practice

Simplifying Algebraic Fractions

Write each fraction in simplest form. If the fraction is already in simplest form, write *simplified*.

1. $\frac{10}{70}$ $\frac{1}{7}$
2. $\frac{12}{18}$ $\frac{2}{3}$
3. $\frac{30}{45}$ $\frac{2}{3}$
4. $\frac{8}{24}$ $\frac{1}{3}$
5. $\frac{4}{6}$ $\frac{2}{3}$
6. $\frac{56}{63}$ $\frac{8}{9}$
7. $\frac{18}{24}$ $\frac{3}{4}$
8. $\frac{7}{49}$ $\frac{1}{7}$
9. $\frac{13}{39}$ $\frac{1}{3}$
10. $\frac{21}{36}$ $\frac{7}{12}$
11. $\frac{32}{40}$ $\frac{4}{5}$
12. $\frac{4}{36}$ $\frac{1}{9}$
13. $\frac{44}{55}$ $\frac{4}{5}$
14. $\frac{4}{14}$ $\frac{2}{7}$
15. $\frac{36}{48}$ $\frac{3}{4}$
16. $\frac{81}{90}$ $\frac{9}{10}$
17. $\frac{5}{25}$ $\frac{1}{5}$
18. $\frac{56}{74}$ $\frac{28}{37}$
19. $\frac{22}{42}$ $\frac{11}{21}$
20. $\frac{7}{18}$ simplified
21. $\frac{d^3}{d^4}$ $\frac{1}{d}$
22. $\frac{y}{y^3}$ $\frac{1}{y^2}$
23. $\frac{q^3}{q}$ q^2
24. $\frac{s^4}{s^2}$ s^2
25. $\frac{x^2}{y}$ simplified
26. $\frac{9a}{12a}$ $\frac{3}{4}$
27. $\frac{8t}{16t}$ $\frac{1}{2}$
28. $\frac{14g}{24g}$ $\frac{7}{12}$
29. $\frac{35j}{40}$ $\frac{7j}{8}$
30. $\frac{100p}{200p^2}$ $\frac{1}{2p}$
31. $\frac{75n}{100n^3}$ $\frac{3}{4n^2}$
32. $\frac{6k^5}{21k^2}$ $\frac{2k^3}{7}$
33. $\frac{3a}{4b}$ simplified
34. $\frac{16b}{24d}$ $\frac{2b}{3d}$
35. $\frac{8a}{24a}$ $\frac{1}{3}$
36. $\frac{5t^3}{35t^2}$ $\frac{t}{7}$

NAME _____ DATE _____ PERIOD ____

4-5 Practice

Simplifying Algebraic Fractions

Write each fraction in simplest form. If the fraction is already in simplest form, write *simplified*.

1. $\frac{9}{36}$ $\frac{1}{4}$
2. $\frac{6}{10}$ $\frac{3}{5}$
3. $\frac{19}{57}$ $\frac{1}{3}$
4. $\frac{21}{24}$ $\frac{7}{8}$
5. $\frac{6}{39}$ $\frac{2}{13}$
6. $\frac{85}{100}$ $\frac{17}{20}$
7. $\frac{16}{72}$ $\frac{2}{9}$
8. $\frac{32}{136}$ $\frac{4}{17}$
9. $\frac{45}{72}$ $\frac{5}{8}$
10. $\frac{46}{92}$ $\frac{1}{2}$
11. $\frac{35}{55}$ $\frac{7}{11}$
12. $\frac{64}{80}$ $\frac{4}{5}$
13. $\frac{57}{60}$ $\frac{19}{20}$
14. $\frac{17}{56}$ simplified
15. $\frac{33}{63}$ $\frac{11}{21}$
16. $\frac{34}{60}$ $\frac{17}{30}$
17. $\frac{24}{52}$ $\frac{6}{13}$
18. $\frac{96}{108}$ $\frac{8}{9}$
19. $\frac{45}{48}$ $\frac{15}{16}$
20. $\frac{14}{29}$ simplified
21. $\frac{x^3}{x^7}$ $\frac{1}{x^4}$
22. $\frac{m^4}{m^5}$ $\frac{1}{m}$
23. $\frac{a^7}{a^4}$ a^3
24. $\frac{u^5}{u}$ u^4
25. $\frac{21y}{24y}$ $\frac{7}{8}$
26. $\frac{4q^2}{14q^2}$ $\frac{2}{7}$
27. $\frac{15x^2}{18x^2}$ $\frac{5}{6}$
28. $\frac{63c}{126c}$ $\frac{1}{2}$
29. $\frac{11v^2}{121v}$ $\frac{v}{11}$
30. $\frac{42b^2}{49}$ $\frac{6b^2}{7}$
31. $\frac{e^2f^2}{e^3f}$ $\frac{f}{e}$
32. $\frac{m^2}{p^3}$ simplified
33. $\frac{2a^3b^4}{10a^5b}$ $\frac{b^3}{5a^2}$

34. **SKI RESORT** A local ski resort is open for business 13 weeks in the winter. Write a fraction in simplest form that represents the fraction of a year the resort is open. $\frac{1}{4}$

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4-5 Reading to Learn Mathematics

Simplifying Algebraic Fractions

Pre-Activity ***How are simplified fractions useful in representing measurements?***

Do the activity at the top of page 169 in your textbook. Write your answers below.

a. Are the three fractions equivalent? Explain your reasoning.
Yes; the same portion of each circle is shaded.

b. Which figure is divided into the least number of parts?
the third figure

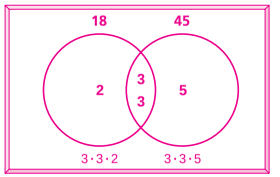
c. Which fraction would you say is written in simplest form? Why? $\frac{1}{4}$; The shaded part is not divided into smaller parts.

Reading the Lesson 1–2. See students' work.

Write a definition and give an example of each new vocabulary phrase.

Vocabulary	Definition	Example
1. simplest form		
2. algebraic fraction		

3. Use a Venn diagram to explain how to simplify $\frac{18}{45}$. The prime factors of the numerator are 3, 3, and 2. The prime factors of the denominator are 3, 3, and 5. The GCF of the numbers, 9, is shown by the intersection. So, the simplified fraction is $\frac{2}{5}$.



Helping You Remember

4. Explain the similarities and differences between simplifying a numerical fraction and simplifying an algebraic fraction. Both fractions are simplified by dividing the numerator and denominator by the GCF. The only difference is that an algebraic fraction has variables as factors in the numerator and/or the denominator.

Answers

NAME _____ DATE _____ PERIOD ____

4-5 Enrichment

Matching Equivalent Fractions

Cut out the pieces below and match the edges so that equivalent fractions meet. The pieces form a 4 × 6 rectangle. The outer edges of the rectangle formed will have no fractions.

$\frac{9}{10}$ $\frac{15}{35}$ **15** $\frac{5}{9}$ $\frac{2}{24}$	$\frac{1}{7}$ $\frac{12}{42}$ **18** $\frac{3}{8}$ $\frac{2}{10}$	$\frac{11}{12}$ $\frac{16}{28}$ **11** $\frac{5}{8}$ $\frac{45}{50}$	$\frac{12}{13}$ $\frac{8}{20}$ **10** $\frac{4}{7}$ $\frac{45}{49}$	$\frac{8}{9}$ $\frac{5}{15}$ **14** $\frac{3}{7}$ $\frac{3}{27}$
$\frac{6}{11}$ $\frac{15}{21}$ **8** $\frac{44}{49}$	$\frac{3}{10}$ **5** $\frac{1}{2}$ $\frac{25}{25}$	$\frac{1}{9}$ $\frac{35}{77}$ **23** $\frac{7}{13}$	$\frac{1}{8}$ $\frac{18}{48}$ **19** $\frac{4}{9}$ $\frac{2}{18}$	$\frac{8}{10}$ **4** $\frac{27}{33}$
$\frac{1}{12}$ $\frac{24}{54}$ **20** $\frac{2}{20}$	$\frac{1}{10}$ $\frac{49}{91}$ **24**	$\frac{6}{8}$ **3** $\frac{4}{5}$ $\frac{21}{21}$	$\frac{7}{8}$ **13** $\frac{1}{3}$ $\frac{3}{18}$	$\frac{11}{7}$ $\frac{9}{15}$ **7** $\frac{5}{7}$ $\frac{44}{48}$
$\frac{4}{6}$ **2** $\frac{3}{4}$ $\frac{23}{23}$	$\frac{1}{5}$ $\frac{14}{56}$ **22** $\frac{5}{11}$	$\frac{1}{4}$ **21** $\frac{2}{8}$	$\frac{1}{6}$ **17** $\frac{2}{7}$ $\frac{2}{8}$	$\frac{13}{14}$ **9** $\frac{2}{5}$ $\frac{40}{40}$
1 $\frac{2}{3}$ $\frac{9}{20}$	$\frac{9}{7}$ $\frac{25}{45}$ **16** $\frac{3}{36}$		$\frac{11}{3}$ $\frac{3}{6}$ **6** $\frac{3}{5}$ $\frac{49}{49}$	$\frac{10}{11}$ $\frac{20}{32}$ **12** $\frac{35}{35}$

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20
21	22	23	24

Lesson 4-5

NAME _____ DATE _____ PERIOD ____

4-6 Study Guide and Intervention

Multiplying and Dividing Monomials

When multiplying powers with the same base, add the exponents.

Symbols	Example
$a^m \cdot a^n = a^{m+n}$	$4^2 \cdot 4^5 = 4^{2+5}$ or 4^7

When dividing powers with the same base, subtract the exponents.

Symbols	Example
$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$	$\frac{5^6}{5^2} = 5^{6-2}$ or 5^4

Example 1 **Find $2a^2(3a)$. Express your answer using exponents.**

$2a^2(3a) = (2 \cdot 3)(a^2 \cdot a)$ Use the Commutative and Associative Properties.

$= (6)(a^{2+1})$ The common base is a.

$= 6a^3$ Add the exponents.

Example 2 **Find $\frac{(-8)^4}{(-8)^2}$. Express your answer using exponents.**

$\frac{(-8)^4}{(-8)^2} = (-8)^{4-2}$ The common base is -8.

$= (-8)^2$ Subtract the exponents.

Exercises

Find each product or quotient. Express your answer using exponents.

1. $4^7 \cdot 4^6$ 4^{13}
2. $v^5 \cdot v^4$ v^9
3. $(f^3)(f^9)$ f^{12}
4. $22^5 \cdot 22^5$ 22^{10}
5. $7h(5h^3)$ $35h^4$
6. $-10x^2(7x^3)$ $-70x^5$
7. $\frac{7^5}{7^2}$ 7^3
8. $\frac{1^8}{1^6}$ 1^2
9. $\frac{(-12)^3}{(-12)^3}$ $(-12)^0$ or 1
10. $3^8 \cdot 3^3$ 3^{11}
11. $\frac{c^{20}}{c^{13}}$ c^7
12. $\frac{(-p)^{18}}{(-p)^{12}}$ $(-p)^6$
13. $-7u^6(-6u^5)$ $42u^{11}$
14. $\frac{2w^3}{2w}$ w^2
15. $-5m^3(4m^6)$ $-20m^9$
16. the product of two cubed and two squared 2^5
17. the quotient of six to the eighth power and six squared 6^6

NAME _____ DATE _____ PERIOD ____

4-6

Skills Practice

Multiplying and Dividing Monomials

Find each product or quotient. Express your answer using exponents.

1. $2^3 \cdot 2^5$ 2^8
2. $10^2 \cdot 10^7$ 10^9
3. $1^4 \cdot 1$ 1^5
4. $6^3 \cdot 6^3$ 6^6
5. $(-3)^2(-3)^3$ $(-3)^5$
6. $(-9)^2(-9)^2$ $(-9)^4$
7. $a^2 \cdot a^3$ a^5
8. $n^8 \cdot n^3$ n^{11}
9. $(p^4)(p^4)$ p^8
10. $(z^6)(z^7)$ z^{13}
11. $(6b^3)(3b^4)$ $18b^7$
12. $(-v)^3(-v)^7$ $(-v)^{10}$
13. $11a^2 \cdot 3a^6$ $33a^8$
14. $10t^2 \cdot 4t^{10}$ $40t^{12}$
15. $(8c^2)(9c)$ $72c^3$
16. $(4f^8)(5f^6)$ $20f^{14}$
17. $\frac{5^{10}}{5^2}$ 5^8
18. $\frac{10^6}{10^2}$ 10^4
19. $\frac{7^9}{7^6}$ 7^3
20. $\frac{12^8}{12^3}$ 12^5
21. $\frac{100^9}{100^8}$ 100^1 or 100
22. $\frac{(-2)^3}{-2}$ $(-2)^2$
23. $\frac{r^8}{r^7}$ r^1 or r
24. $\frac{z^{10}}{z^8}$ z^2
25. $\frac{q^8}{q^4}$ q^4
26. $\frac{g^{12}}{g^8}$ g^4
27. $\frac{(-y)^7}{(-y)^2}$ $(-y)^5$
28. $\frac{(-z)^{12}}{(-z)^5}$ $(-z)^7$
29. the product of two squared and two to the sixth power 2^8
30. the quotient of ten to the seventh power and ten cubed 10^4
31. the product of y squared and y cubed y^5
32. the quotient of a to the twentieth power and a to the tenth power a^{10}

Lesson 4-6

NAME _____ DATE _____ PERIOD ____

4-6

Practice

Multiplying and Dividing Monomials

Find each product or quotient. Express your answer using exponents.

1. $4^2 \cdot 4^3$ 4^5
2. $9^8 \cdot 9^6$ 9^{14}
3. $7^4 \cdot 7^2$ 7^6
4. $13^2 \cdot 13^4$ 13^6
5. $(-8)^5(-8)^3$ $(-8)^8$
6. $(-21)^9(-21)^5$ $(-21)^{14}$
7. $t^9 \cdot t^3$ t^{12}
8. $h^4 \cdot h^{13}$ h^{17}
9. $(m^6)(m^6)$ m^{12}
10. $(u^{11})(u^{10})$ u^{21}
11. $(-r)^7(-r)^{20}$ $(-r)^{27}$
12. $(-w)(-w)^9$ $(-w)^{10}$
13. $4d^5 \cdot 8d^6$ $32d^{11}$
14. $7j^{50} \cdot 6j^{50}$ $42j^{100}$
15. $-5b^9 \cdot 6b^2$ $-30b^{11}$
16. $12^1 \cdot 12^2$ 12^3
17. $\frac{6^{11}}{6^3}$ 6^8
18. $\frac{15^3}{15^2}$ 15^1 or 15
19. $\frac{9^9}{9^7}$ 9^2
20. $\frac{18^4}{18^4}$ 18^0 or 1
21. $\frac{(-7)^6}{(-7)^5}$ $(-7)^1$ or -7
22. $\frac{95^{21}}{95^{18}}$ 95^3
23. $\frac{v^{30}}{v^{20}}$ v^{10}
24. $\frac{n^{19}}{n^{11}}$ n^8
25. the product of five cubed and five to the fourth power 5^7
26. the quotient of eighteen to the ninth power and eighteen squared 18^7
27. the product of z cubed and z cubed z^6
28. the quotient of x to the fifth power and x cubed x^2
29. **SOUND** Decibels are units used to measure sound. The softest sound that can be heard is rated as 0 decibels (or a relative loudness of 1). Ordinary conversation is rated at about 60 decibels (or a relative loudness of 10^6). A rock concert is rated at about 120 decibels (or a relative loudness of 10^{12}). How many times greater is the relative loudness of a rock concert than the relative loudness of ordinary conversation? 10^6 or 1,000,000 times

Answers

NAME _____ DATE _____ PERIOD ____

4-6

Reading to Learn Mathematics

Multiplying and Dividing Monomials

Pre-Activity ***How are powers of monomials useful in comparing earthquake magnitudes?***

Do the activity at the top of page 175 in your textbook. Write your answers below.

a. Examine the exponents of the factors and the exponents of the products in the last column. What do you observe? **The exponents of the factors are added to get the exponent of the product.**

b. Make a conjecture about a rule for determining the exponent of the product when you multiply powers with the same base. Test your rule by multiplying $2^2 \cdot 2^4$ using a calculator. **Sample answer: Add the exponents.**

Reading the Lesson

1. When multiplying powers with like bases, **add** the exponents.
2. When dividing powers with like bases, **subtract** the exponents.
3. Write a division expression whose quotient is 7^2. **Sample answer:** $\frac{7^3}{7}$
4. Write a multiplication expression whose product is v^5. **Sample answer:** $v^2 \cdot v^3$
5. Find each product.
   - **a.** $4 \cdot 4^3$ **4^4**
   - **b.** $y^7 \cdot y^5$ **y^{12}**
   - **c.** $(-2x^2)(5x^2)$ **$-10x^4$**
   - **d.** $-3r^2 \cdot r$ **$-3r^3$**
6. Find each quotient.
   - **a.** $\frac{7^4}{7^2}$ **7^2**
   - **b.** $\frac{v^9}{v^3}$ **v^6**
   - **c.** $\frac{6^7}{6^6}$ **6^1 or 6**
   - **d.** $\frac{a^2b^2}{b^2}$ **a^2**

Helping You Remember

7. Explain how dividing powers is related to simplifying fractions. Provide an example as part of your explanation. **Sample answer: When dividing powers with like bases, subtracting the exponents is equivalent to simplifying fractions. For example, $3^4 \div 3^2 = 3^{4-2}$ or 3^2 by the Quotient of Powers rule. When simplifying $\frac{3^4}{3^2}$, divide the numerator and denominator by the GCF, 3^2, to get $\frac{3^2}{1}$ or 3^2.**

NAME _____ DATE _____ PERIOD ____

4-6

Enrichment

Dividing Powers with Different Bases

Some powers with different bases can be divided. First, you must be able to write both as powers of the same base. An example is shown below.

$$\frac{2^5}{8^2} = \frac{2^5}{(2^3)^2}$$ To find the power of a power, multiply the exponents.

$$= \frac{2^5}{2^6}$$

$$= 2^{-1} \text{ or } \frac{1}{2}$$

This method could not have been used to divide $\frac{2^5}{9^2}$, since 9 cannot be written as a power of 2 using integers.

Simplify each fraction using the method shown above. Express the solution without exponents.

1. $\frac{8^2}{2^2}$ **16**
2. $\frac{16^4}{8^3}$ **128**
3. $\frac{9^3}{3^3}$ **27**
4. $\frac{81^4}{3^4}$ **531,441**
5. $\frac{3^9}{81^2}$ **3**
6. $\frac{32^4}{16^4}$ **16**
7. $\frac{125^2}{25^3}$ **1**
8. $\frac{6^6}{216^2}$ **1**
9. $\frac{10^6}{1000^3}$ **0.001**
10. $\frac{64^3}{8^5}$ **8**
11. $\frac{27^5}{9^4}$ **2187**
12. $\frac{343^3}{7^5}$ **2401**

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4-7 Study Guide and Intervention

Negative Exponents

Extending the pattern below shows that $4^{-1} = \frac{1}{4}$ or $\frac{1}{4^1}$.

$4^2 = 16$ ÷ 4

$4^1 = 4$ ÷ 4

$4^0 = 1$ ÷ 4

$4^{-1} = \frac{1}{4}$

This suggests the following definition.

$a^{-n} = \frac{1}{a^n}$, for $a \neq 0$ and any integer n.

Example 1 **Write each expression using a positive exponent.**

a. 3^{-4}

$3^{-4} = \frac{1}{3^4}$

b. y^{-2}

$y^{-2} = \frac{1}{y^2}$

We can evaluate algebraic expressions with negative exponents using the definition of negative exponents.

Example 2 **Evaluate b^{-2} if $b = 3$.**

$b^{-2} = 3^{-2}$ Replace b with 3.

$= \frac{1}{3^2}$ Definition of negative exponent

$= \frac{1}{9}$ Find 3^2.

Exercises

Write each expression using a positive exponent.

1. 6^{-4} $\frac{1}{6^4}$
2. $(-7)^{-8}$ $\frac{1}{(-7)^8}$
3. b^{-6} $\frac{1}{b^6}$
4. n^{-1} $\frac{1}{n^1}$ or $\frac{1}{n}$

Write each fraction as an expression using a negative exponent other than −1.

5. $\frac{1}{2^2}$ 2^{-2}
6. $\frac{1}{13^4}$ 13^{-4}
7. $\frac{1}{25}$ 5^{-2}
8. $\frac{1}{49}$ 7^{-2}

Evaluate each expression if $m = -4$, $n = 1$, and $p = 6$.

9. p^{-2} $\frac{1}{36}$
10. m^{-3} $-\frac{1}{64}$
11. $(np)^{-1}$ $\frac{1}{6}$
12. 3^m $\frac{1}{81}$

NAME _____ DATE _____ PERIOD ____

4-7 Skills Practice

Negative Exponents

Write each expression using a positive exponent.

1. 3^{-4} $\frac{1}{3^4}$
2. 8^{-7} $\frac{1}{8^7}$
3. 10^{-4} $\frac{1}{10^4}$
4. $(-2)^{-6}$ $\frac{1}{(-2)^6}$
5. $(-40)^{-3}$ $\frac{1}{(-40)^3}$
6. $(-17)^{-12}$ $\frac{1}{(-17)^{12}}$
7. n^{-10} $\frac{1}{n^{10}}$
8. b^{-8} $\frac{1}{b^8}$
9. q^{-5} $\frac{1}{q^5}$
10. m^{-4} $\frac{1}{m^4}$
11. v^{-11} $\frac{1}{v^{11}}$
12. p^{-2} $\frac{1}{p^2}$

Write each fraction as an expression using a negative exponent other than −1.

13. $\frac{1}{8^2}$ 8^{-2}
14. $\frac{1}{10^5}$ 10^{-5}
15. $\frac{1}{2^3}$ 2^{-3}
16. $\frac{1}{6^7}$ 6^{-7}
17. $\frac{1}{17^4}$ 17^{-4}
18. $\frac{1}{21^2}$ 21^{-2}
19. $\frac{1}{3^7}$ 3^{-7}
20. $\frac{1}{9^2}$ 9^{-2}
21. $\frac{1}{3^2}$ 3^{-2}
22. $\frac{1}{121}$ 11^{-2}
23. $\frac{1}{25}$ 5^{-2}
24. $\frac{1}{36}$ 6^{-2}

Evaluate each expression if $x = 1$, $y = 2$, and $z = -3$.

25. y^{-z} $\frac{1}{8}$
26. z^{-2} $\frac{1}{9}$
27. x^{-8} 1
28. y^{-5} $\frac{1}{32}$
29. z^{-3} $-\frac{1}{27}$
30. y^{-1} $\frac{1}{2}$
31. z^{-4} $\frac{1}{81}$
32. 5^z $\frac{1}{125}$
33. x^{-99} 1
34. 1^z 1
35. 4^z $\frac{1}{64}$
36. y^z $\frac{1}{8}$

Answers

NAME _____ DATE _____ PERIOD ____

4-7

Practice

Negative Exponents

Write each expression using a positive exponent.

1. 7^{-8} $\frac{1}{7^8}$
2. 10^{-6} $\frac{1}{10^6}$
3. 23^{-1} $\frac{1}{23^1}$
4. $(-5)^{-2}$ $\frac{1}{(-5)^2}$
5. $(-18)^{-10}$ $\frac{1}{(-18)^{10}}$
6. m^{-99} $\frac{1}{m^{99}}$
7. $(-1)^{-12}$ $\frac{1}{(-1)^{12}}$
8. c^{-6} $\frac{1}{c^6}$
9. p^{-5} $\frac{1}{p^5}$
10. g^{-17} $\frac{1}{g^{17}}$
11. $5z^{-4}$ $5\left(\frac{1}{z^4}\right)$
12. $3t^{-1}$ $3\left(\frac{1}{t}\right)$

Write each fraction as an expression using a negative exponent.

13. $\frac{1}{2^{10}}$ 2^{-10}
14. $\frac{1}{29^3}$ 29^{-3}
15. $\frac{1}{4^4}$ 4^{-4}
16. $\frac{1}{39}$ 39^{-1}
17. $\frac{1}{81^7}$ 81^{-7}
18. $\frac{1}{m^4}$ m^{-4}
19. $\frac{1}{x^3}$ x^{-3}
20. $\frac{1}{a^2}$ a^{-2}
21. $\frac{1}{49}$ 7^{-2}
22. $\frac{1}{8}$ 2^{-3}
23. $\frac{1}{144}$ 12^{-2}
24. $\frac{1}{169}$ 13^{-2}

Evaluate each expression if $x = 3$, $y = -2$, and $z = 4$.

25. x^{-4} $\frac{1}{81}$
26. y^{-2} $\frac{1}{4}$
27. y^{-5} $-\frac{1}{32}$
28. z^{-4} $\frac{1}{256}$
29. 5^y $\frac{1}{25}$
30. 10^y $\frac{1}{100}$
31. $3z^{-1}$ $\frac{3}{4}$
32. z^y $\frac{1}{16}$
33. $(xz)^{-2}$ $\frac{1}{144}$

34. **HAIR** Hair grows at a rate of $\frac{1}{64}$ inch per day. Write this number using negative exponents. 8^{-2} or 4^{-3} or 2^{-6}

Lesson 4-7

NAME _____ DATE _____ PERIOD ____

4-7

Reading to Learn Mathematics

Negative Exponents

Pre-Activity *How do negative exponents represent repeated division?*

Do the activity at the top of page 181 in your textbook. Write your answers below.

a. Describe the pattern of powers in the first column. Continue the pattern by writing the next two powers in the table. The exponents decrease by 1; 2^0, 2^{-1}.

b. Describe the pattern of values in the second column. Then complete the second column. Each number is divided by 2; 1, $\frac{1}{2}$.

c. Verify that the powers you wrote in part **a** are equal to the values that you found in part **b**. See students' work.

d. Determine how 3^{-1} should be defined. $\frac{1}{3}$

Reading the Lesson

1. Explain the value of 5^{-3} using a pattern. Sample answer: The value in each row is the previous value divided by 5, so 5^{-3} is $\frac{1}{125}$.

Power	Value
5^1	5
5^0	1
5^{-1}	$\frac{1}{5}$
5^{-2}	$\frac{1}{25}$
5^{-3}	$\frac{1}{125}$

2. Using what you know about the Quotient of Powers rule, fill in the missing number.
$5^{-3} = \frac{?}{5^5}$ $5^{-3} = \frac{5^2}{5^5}$

Helping You Remember

3. Are $-x^2$ and x^{-2} equivalent? Explain. Sample answer: No; let $x = 3$. $-3^2 = -9$, but $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$.

NAME _____ DATE _____ PERIOD ____

4-7 Enrichment

Proving Definitions of Exponents

Recall the rules for multiplying and dividing powers with the same base. Use these rules, along with other properties you have learned, to justify each definition. Abbreviations for some properties you may wish to use are listed below.

Associative Property of Multiplication (APM)
Multiplicative Identity Property (MIP)
Inverse Property of Multiplication (IPM)
Additive Identity Property (AIP)
Inverse Property of Addition (IPA)

Write the reason for each statement.

1. Prove: $a^0 = 1$

Statement	Reason
Let m be an integer, and let a be any nonzero number.	**a.** Given
$a^m \cdot a^0 = a^{m+0}$	**b.** Product of Powers
$a^m \cdot a^0 = a^m$	**c.** IPA
$\frac{1}{a^m} \cdot (a^m \cdot a^0) = \frac{1}{a^m} \cdot a^m$	**d.** Mult. Prop. Equality
$\left(\frac{1}{a^m} \cdot a^m\right) \cdot a^0 = \frac{1}{a^m} \cdot a^m$	**e.** APM
$1 \cdot a^0 = 1$	**f.** IPM
$a^0 = 1$	**g.** MIP

2. Prove: $a^{-n} = \frac{1}{a^n}$

Statement	Reason
Let n be an integer, and let a be any nonzero number.	**a.** Given
$a^{-n} \cdot a^n = a^{-n+n}$	**b.** Product of Powers
$a^{-n} \cdot a^n = a^0$	**c.** IPA
$a^{-n} \cdot a^n = 1$	**d.** Def. Zero Exponents
$(a^{-n} \cdot a^n) \cdot \frac{1}{a^n} = 1 \cdot \frac{1}{a^n}$	**e.** Mult. Prop. Equality
$a^{-n} \cdot \left(a^n \cdot \frac{1}{a^n}\right) = 1 \cdot \frac{1}{a^n}$	**f.** APM
$a^{-n} \cdot 1 = 1 \cdot \frac{1}{a^n}$	**g.** IPM
$a^{-n} = \frac{1}{a^n}$	**h.** MIP

Lesson 4-7

NAME _____ DATE _____ PERIOD ____

4-8 Study Guide and Intervention

Scientific Notation

When you deal with very large numbers like 5,000,000 or very small numbers like 0.0005, it is difficult to keep track of place value. Numbers such as these can be written in **scientific notation**. A number is expressed in scientific notation when it is written as a product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10.

By definition, a number in scientific notation is written as $a \cdot 10^n$, where $1 \leq a < 10$ and n is an integer.

Example 1 **Express each number in standard form.**

a. 6.32×10^5

$6.32 \times 10^5 = 632000$ Move the decimal point 5 places to the right.

b. 7.8×10^{-6}

$7.8 \times 10^{-6} = 0.0000078$ Move the decimal point 6 places to the left.

Example 2 **Express each number in scientific notation.**

a. 62,000,00

To write in scientific notation, place the decimal point after the first nonzero digit, then find the power of 10.

$62{,}000{,}000 = 6.2 \times 10^7$ The decimal point moves 7 places. The power of 10 is 7.

b. 0.00025

$0.00025 = 2.5 \times 10^{-4}$ Place the decimal point after the first nonzero digit. The power of 10 is -4.

Exercises

1. 4.12×10^6 4,120,000 **2.** 5.8×10^2 580 **3.** 9.01×10^{-3} 0.00901

4. 6.72×10^{-7} 0.000000672 **5.** 8.72×10^4 87,200 **6.** 4.44×10^{-5} 0.0000444

Express each number in scientific notation.

7. 12,000,000,000 1.2×10^{10} **8.** 5000 5.0×10^3 **9.** 0.00475 4.75×10^{-3}

10. 0.00007463 7.463×10^{-5} **11.** 235,000 2.35×10^5 **12.** 0.000377 3.77×10^{-4}

Choose the greater number in each pair.

13. 4.9×10^4, 9.9×10^{-4}

14. 2.004×10^3, 2.005×10^{-2}

15. 3.2×10^2, 700

16. 0.002, 3.6×10^{-4}

Answers

Answers (Lessons 4-7 and 4-8)

NAME _____ DATE _____ PERIOD ____

4-8 Skills Practice

Scientific Notation

Express each number in standard form.

1. 1.5×10^3 **1500**
2. 4.01×10^4 **40,100**
3. 6.78×10^2 **678**
4. 5.925×10^6 **5,925,000**
5. 7.0×10^8 **700,000,000**
6. 9.99×10^7 **99,900,000**
7. 3.0005×10^5 **300,050**
8. 2.54×10^5 **254,000**
9. 1.75×10^4 **17,500**
10. 1.2×10^{-6} **0.0000012**
11. 7.0×10^{-1} **0.7**
12. 6.3×10^{-3} **0.0063**
13. 5.83×10^{-2} **0.0583**
14. 8.075×10^{-4} **0.0008075**
15. 1.1×10^{-5} **0.000011**
16. 7.3458×10^7 **73,458,000**

Express each number in scientific notation.

17. 1,000,000 **1.0×10^6**
18. 17,400 **1.74×10^4**
19. 500 **5.0×10^2**
20. 803,000 **8.03×10^5**
21. 0.00027 **2.7×10^{-4}**
22. 5300 **5.3×10^3**
23. 18 **1.8×10^1**
24. 0.125 **1.25×10^{-1}**
25. 17,000,000,000 **1.7×10^{10}**
26. 0.01 **1.0×10^{-2}**
27. 21,800 **2.18×10^4**
28. 2,450,000 **2.45×10^6**
29. 0.0054 **5.4×10^{-3}**
30. 0.000099 **9.9×10^{-5}**
31. 8,888,800 **8.8888×10^6**
32. 0.00912 **9.12×10^{-3}**

Choose the greater number in each pair.

33. 8.8×10^3, 9.1×10^{-4}
34. 5.01×10^2, 5.02×10^{-1}
35. 6.4×10^3, 900
36. 1.9×10^{-2}, 0.02
37. 2.2×10^{-3}, 2.1×10^2
38. 8.4×10^2, 839

Lesson 4-8

NAME _____ DATE _____ PERIOD ____

4-8 Practice

Scientific Notation

Express each number in standard form.

1. 2.4×10^4 **24,000**
2. 9.0×10^3 **9000**
3. 4.385×10^7 **43,850,000**
4. 1.03×10^8 **103,000,000**
5. 3.05×10^2 **305**
6. 5.11×10^{10} **51,100,000,000**
7. 6.000032×10^6 **6,000,032**
8. 1.0×10^1 **10**
9. 8.75×10^5 **875,000**
10. 8.49×10^{-2} **0.0849**
11. 7.1×10^{-6} **0.0000071**
12. 1.0×10^{-3} **0.001**
13. 4.39×10^{-7} **0.000000439**
14. 1.25×10^{-4} **0.000125**

Express each number in scientific notation.

15. 40,000 **4.0×10^4**
16. 16 **1.6×10^1**
17. 876,000,000 **8.76×10^8**
18. 4500 **4.5×10^3**
19. 151 **1.51×10^2**
20. 0.00037 **3.7×10^{-4}**
21. 83,000,000 **8.3×10^7**
22. 919,100 **9.191×10^5**
23. 5,000,000,000,000 **5.0×10^{12}**
24. 0.13 **1.3×10^{-1}**
25. 0.0000007 **7.0×10^{-7}**
26. 0.0067 **6.7×10^{-3}**

NIAGARA FALLS **For Exercises 27 and 28, use the following information.**

Every minute, 840,000,000,000 drops of water flow over Niagara Falls.

27. Write this number in scientific notation. **8.4×10^{11}**
28. How many drops flow over the falls in a day? **1.2096×10^{15}**

NAME _____ DATE _____ PERIOD ____

4-8 Reading to Learn Mathematics

Scientific Notation

Pre-Activity ***Why is scientific notation an important tool in comparing real-world data?***

Do the activity at the top of page 186 in your textbook. Write your answers below.

a. Write the track length in millimeters. 5,000,000 mm

b. Write the track width in millimeters. (1 micron = 0.001 millimeter) 0.0005 mm

Reading the Lesson

Write a definition and give an example of the new vocabulary phrase.

	Vocabulary	Definition	Example
1.	slope intercept form	See students' work.	

2. To multiply by a power of 10, move the decimal point to the right if the exponent is positive.

3. Which is larger, -2.1×10^4 or -2.1×10^{-4}? Explain. -2.1×10^{-4} is larger, since the numbers are negative and it is closer to zero.

Helping You Remember

4. Explain how to express each number in scientific notation.

a. a number greater than 1 Place the decimal point after the first nonzero digit, then multiply by the appropriate positive power of 10.

b. a number less than one Place the decimal point after the first nonzero digit, then multiply by the appropriate negative power of 10.

c. the number 1 Place the decimal point after the 1, then multiply by 10^0 (1.0×10^0).

NAME _____ DATE _____ PERIOD ____

4-8 Enrichment

Scientific Notation

It is sometimes necessary to multiply and divide very large or very small numbers using scientific notation.

To multiply numbers in scientific notation, use the following rule.

For any numbers a and b, and any numbers c and d,
$(c \times 10^a)(d \times 10^b) = (c \times d) \times 10^{a+b}$.

Example 1 $(3.0 \times 10^4)(-5.0 \times 10^{-2}) = [3.0 \times (-5.0)] \times 10^{4+(-2)}$
$= -15.0 \times 10^2$
$= -1.5 \times 10^3$ or -1500

To divide numbers in scientific notation, use the following rule.

For any numbers a and b, and any numbers c and d, $(d \neq 0)$
$(c \times 10^a) \div (d \times 10^b) = (c \div d) \times 10^{a-b}$.

Example 2 $(24 \times 10^{-4}) \div (1.5 \times 10^2) = (24 \div 1.5) \times 10^{-4-2}$
$= 16 \times 10^{-6}$
$= 1.6 \times 10^{-5}$ or 0.000016

Multiply or divide. Express each product or quotient in scientific notation.

1. $(2.7 \times 10^9) \times (3.1 \times 10^2)$ 8.37×10^{11}

2. $(6.1 \times 10^{-2}) \times (1.3 \times 10^5)$ 7.93×10^3

3. $(5.4 \times 10^{-3}) \div (1.8 \times 10^2)$ 3.0×10^{-5}

4. $(6.9 \times 10^{-3}) \div (3.0 \times 10^{-8})$ 2.3×10^5

5. $(1.1 \times 10^{-5}) \times (9.9 \times 10^{-1})$ 1.089×10^{-5}

6. $(4.0 \times 10^0) \div (1.0 \times 10^{-2})$ 4.0×10^2

Solve. Write your answers in standard form.

7. The distance from Earth to the Moon is about 2.0×10^5 miles. The distance from Earth to the Sun is about 9.3×10^7 miles. How many times farther is it to the Sun than to the Moon? 465

8. If each of the 3.0×10^4 people employed by Sunny Motors earned 4.0×10^4 dollars last year, how much money did the company pay out to its employees? $1,200,000,000

Answers

Chapter 4 Assessment Answer Key

Form 1

Page 205

1. A
2. C
3. D
4. C
5. B
6. A
7. D
8. A
9. C
10. C
11. A

Page 206

12. B
13. D
14. B
15. C
16. D
17. A
18. B
19. C
20. D

B: Tables that seat 8 people

Form 2A

Page 207

1. A
2. B
3. D
4. C
5. D
6. A
7. B
8. C
9. A
10. D
11. A

(continued on the next page)

Chapter 4 Assessment Answer Key

Form 2A *(continued)*
Page 208

12. B

13. C

14. A

15. C

16. B

17. A

18. B

19. D

20. B

B: Venus, Earth, Neptune, Uranus

Form 2B
Page 209

1. B

2. A

3. D

4. B

5. B

6. B

7. C

8. B

9. C

10. B

11. A

Page 210

12. D

13. A

14. D

15. C

16. B

17. C

18. A

19. A

20. D

B: Neptune, Uranus, Saturn, Jupiter

Answers

Chapter 4 Assessment Answer Key

Form 2C

Page 211

1. 2, 5, 10
2. 2, 3, 6
3. no
4. yes
5. 2^5
6. 56
7. $2 \cdot 2 \cdot 2 \cdot 11$
8. $2 \cdot 3 \cdot 7 \cdot a \cdot a \cdot a \cdot x$
9. 4
10. $10y$
11. $9(2b + 1)$
12. $\frac{2}{13}$
13. $\frac{5r}{8}$
14. $\frac{2}{3}$

Page 212

15. m^7
16. $55x^3y^2$
17. $(-4)^5$
18. b^{10}
19. 3^{-9}
20. 10^{-4}
21. $\frac{1}{125}$
22. 0.000509
23. 1.5×10^9
24. Mercury
25. 15 in.

B: $2^2 \cdot 3^3 \cdot 11$

Chapter 4 Assessment Answer Key

Form 2D

Page 213

1. 2
2. 2, 3, 5, 6, 10
3. no
4. yes
5. 3^6
6. 405
7. $2 \cdot 2 \cdot 17$
8. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot p \cdot p \cdot x$
9. 5
10. $2x$
11. $4(5a + 1)$
12. $\frac{9}{32}$
13. $\frac{4a}{5}$
14. $\frac{3}{4}$

Page 214

15. d^5
16. $36x^2y^2$
17. $(-3)^3$
18. a^{15}
19. 4^{-7}
20. 10^{-3}
21. $\frac{1}{16}$
22. 0.000468
23. 6.35×10^{11}
24. human being
25. 12 in.

B: $2^4 \cdot 3^2 \cdot 11$

Answers

Chapter 4 Assessment Answer Key

Form 3

Page 215

1. 2, 3, 6
2. 2, 3, 5, 6, 10
3. none
4. No; it has two terms involving addition.
5. Yes; it is the product of numbers and variables.
6. No; it has two terms involving subtraction.
7. Yes; it is the product of numbers and variables.
8. 3^2x^2y
9. $7(2 - d)^2$
10. 91
11. 2
12. prime
13. composite
14. $3 \cdot 3 \cdot 11$
15. $-1 \cdot 3 \cdot 3 \cdot 5 \cdot q \cdot r \cdot r \cdot s \cdot s \cdot s$
16. 13
17. $6a^2$

Page 216

18. $x(x + 3)$
19. $6(3 - y)$
20. $\frac{7}{12}$
21. $\frac{p}{3q}$
22. $\frac{1}{120}$
23. $\frac{1}{3}$
24. $-15x^5$
25. $4s^5t^3$
26. $\frac{a}{b^4}$
27. $4m$
28. 6^{-2}
29. -9
30. $\frac{1}{243}$
31. 0.0001057
32. 5×10^2 s
33. 3.0×10^{-4}, 3.03×10^{-4}, 3.13×10^{-4}, 0.00303, 0.0313

B: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47

Chapter 4 Assessment Answer Key

Page 217, Open-Ended Assessment
Scoring Rubric

Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts *factors, fractions, exponent, coefficient, prime,* and *composite.* • Uses appropriate strategies to compare fractions. • Computations are correct. • Written explanations are exemplary. • Diagram is accurate. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows an understanding of the concepts *factors, fractions, exponent, coefficient, prime,* and *composite.* • Uses appropriate strategies to compare fractions. • Computations to compare fractions and identify prime numbers are mostly correct. • Written explanations are effective. • Diagram is mostly accurate. • Satisfies all requirements of problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows an understanding of most of the concepts *factors, fractions, exponent, coefficient, prime,* and *composite.* • May not use appropriate strategies to compare fractions. • Computations to compare fractions and identify prime numbers are mostly correct. • Written explanations are satisfactory. • Diagram is mostly accurate. • Satisfies most requirements of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work is shown to substantiate the final computation. • Diagrams may be accurate but lack detail or explanation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts *factors, fractions, exponent, coefficient, prime,* and *composite.* • May not use appropriate strategies to compare fractions. • Computations to compare fractions and identify prime numbers are incorrect. • Written explanations are not satisfactory. • Diagram is not accurate. • Does not satisfy requirements of problems.

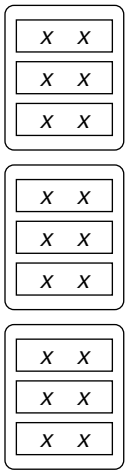
Chapter 4 Assessment Answer Key

Page 217, Open-Ended Assessment
Sample Answers

In addition to the scoring rubric found on page A27, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1. Sample answer: $50 = 3 + 47$, $74 = 13 + 61$, $98 = 37 + 61$

2a. Yes. If a number is divisible by 9, this means you can separate that many objects into nine equal groups, with none remaining. You can then separate each of these nine groups into three equal groups. Thus, the original number is divisible by 3. Following is an example of 18 objects.



2b. No. Consider 12. It is divisible by 3, but not by 9.

3a. No, because 210 is not divisible by 8. Students should give pairs of numbers that are factors of 210, such as 2 by 105, 105 by 2, 3 by 70, 70 by 3, 5 by 42, 42 by 5, 6 by 35, 35 by 6, 7 by 30, 30 by 7, 10 by 21, 21 by 10, 14 by 15, or 15 by 14.

3b. Find a common denominator to compare fractions.

$\frac{26}{40} = \frac{273}{420}$ and $\frac{140}{210} = \frac{280}{420}$

A greater part of the band turned in their slips.

4a. $3a$ means $3 \cdot a$ or $a + a + a$.
a^3 means $a \cdot a \cdot a$.

4b. $-3b$ means $-3 \cdot b$ or $-(b + b + b)$.

b^{-3} means $\frac{1}{b^3}$ or $\frac{1}{b \cdot b \cdot b}$.

Chapter 4 Assessment Answer Key

Vocabulary Test/Review
Page 218

1. d
2. i
3. c
4. h
5. g
6. b
7. e
8. f
9. a
10. Sample answer: A number expressed in scientific notation is written as the product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10.
11. Sample answer: In a power, the base is the number that is multiplied.
12. Sample answer: An algebraic fraction is a fraction with variables in the numerator or denominator.

Quiz (Lessons 4-1 and 4-2)
Page 219

1. 2
2. 2, 3, 5, 6, 10
3. 3
4. 3, 5
5. Yes; it is the product of two variables.
6. No; two variables are added.
7. x^3
8. 9^5
9. 192
10. 72

Quiz (Lessons 4-3 and 4-4)
Page 219

1. 3^3
2. $3^2 \cdot 7$
3. $2^4 \cdot 7$
4. $3 \cdot 7 \cdot b$
5. $2 \cdot 3 \cdot 5 \cdot x \cdot x \cdot y$
6. 4
7. 10
8. $7x^4$
9. $2(2a + 7)$
10. $5(6 + y)$

Quiz (Lessons 4-5 and 4-6)
Page 220

1. $\frac{2}{3}$
2. $\frac{5}{7}$
3. $\frac{7x^5}{10}$
4. $\frac{3}{20}$
5. D
6. 3^{10}
7. x^{13}
8. $10x^5$
9. 1
10. x^4

Quiz (Lessons 4-7 and 4-8)
Page 220

1. $\frac{1}{7^5}$
2. $\frac{1}{y^3}$
3. $\frac{1}{81}$
4. $\frac{1}{216}$
5. 1.602×10^{-7}
6. 2.0×10^8
7. 3.0×10^9
8. 4.62×10^{-3}
9. 3.60×10^{-5}
10. electron, proton, neutron

Answers

Chapter 4 Assessment Answer Key

Mid-Chapter Test
Page 221

1. A

2. C

3. D

4. B

5. B

6. C

7. 2, 3, 6

8. $3^3 \cdot 2^2$

9. $2 \cdot 3 \cdot 11$

10. $2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \cdot y \cdot y$

11. 8

12. $4x$

13. No; 12 is not a factor of 70.

Cumulative Review
Page 222

1. domain = {2.3, 4.6, 5}
range = {3.2, 3.3, 4}

2. Positive; as the outside temperature increases, so does the air conditioning bill.

3. $-22m$

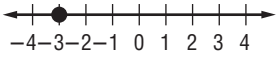
4. $32x$

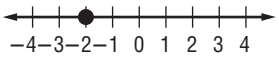
5. 4

6. 2

7. IV

8. $\frac{x}{-9} = 18$; -162

9. 

10. 

11. 1296

12. 960

13. $2 \cdot 3 \cdot 13$

14. $2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot a \cdot b \cdot c \cdot c \cdot c$

15. 15

16. $9r^2t$

17. $6(3 + 7y)$

Chapter 4 Assessment Answer Key

Standardized Test Practice

Page 223

1. A (B) C D

2. (E) F G H

3. (A) B C D

4. E (F) G H

5. A B (C) D

6. E F G (H)

7. A (B) C D

8. E F G (H)

9. (A) B C D

10. E F (G) H

Page 224

11. A (B) C D

12. (E) F G H

13. A B (C) D

14. E F G (H)

15. A B (C) D

16. E F G (H)

17.

4	8	0	0
.	/ .	/ .	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	**8**	8	8
9	9	9	9

18.

1	1	2	
.	/ .	/ .	.
0	0	0	0
1	**1**	1	1
2	2	**2**	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

19. $\{-18, -11, -5, 0, 2, 7, 32\}$

20. 7.998×10^8

Answers