GLENCOE MATHEMATICS

Geometry

Chapter 4
Resource Masters

McGraw-Hill Glencoe

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Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks.

Study Guide and Intervention Workbook	0-07-860191-6
Skills Practice Workbook	0-07-860192-4
Practice Workbook	0-07-860193-2
Reading to Learn Mathematics Workbook	0-07-861061-3

ANSWERS FOR WORKBOOKS The answers for Chapter 4 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

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Geometry
Chapter 4 Resource Masters

1 2 3 4 5 6 7 8 9 10 009 11 10 09 08 07 06 05 04 03

Contents

Teacher's Guide to Using the Chapter 4 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 4 Resource Masters* includes the core materials needed for Chapter 4. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Geometry TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 4-1. Encourage them to add these pages to their Geometry Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Vocabulary Builder Pages ix–x include another student study tool that presents up to fourteen of the key theorems and postulates from the chapter. Students are to write each theorem or postulate in their own words, including illustrations if they choose to do so. You may suggest that students highlight or star the theorems or postulates with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 4-1. Encourage them to add these pages to their Geometry Study Notebook. Remind them to update it as they complete each lesson.

Study Guide and Intervention
Each lesson in *Geometry* addresses two objectives. There is one Study Guide and Intervention master for each objective.

WHEN TO USE Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

WHEN TO USE These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics
One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 4 Resources Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

  All of the above tests include a free-response Bonus question.
- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Geometry. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of geometry concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and short-response questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 232–233. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

NAME ____ DATE ____________ PERIOD _____

4 Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 4. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Geometry Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
acute triangle		
base angles		
congruence transformation kuhn·GROO·uhns		
congruent triangles		
coordinate proof		
corollary		
equiangular triangle		
equilateral triangle		
exterior angle		

(continued on the next page)

4 Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
flow proof		
included angle		
included side		
isosceles triangle		
obtuse triangle		
remote interior angles		
right triangle		
scalene triangle SKAY·leen		
vertex angle		

NAME ____ DATE ____________ PERIOD _____

4 Learning to Read Mathematics

Proof Builder

Proof Builder

This is a list of key theorems and postulates you will learn in Chapter 4. As you study the chapter, write each theorem or postulate in your own words. Include illustrations as appropriate. Remember to include the page number where you found the theorem or postulate. Add this page to your Geometry Study Notebook so you can review the theorems and postulates at the end of the chapter.

Theorem or Postulate	Found on Page	Description/Illustration/Abbreviation
Theorem 4.1 *Angle Sum Theorem*		
Theorem 4.2 *Third Angle Theorem*		
Theorem 4.3 *Exterior Angle Theorem*		
Theorem 4.4		
Theorem 4.5 *Angle-Angle-Side Congruence (AAS)*		
Theorem 4.6 *Leg-Leg Congruence (LL)*		
Theorem 4.7 *Hypotenuse-Angle Congruence (HA)*		

(continued on the next page)

4

Learning to Read Mathematics

Proof Builder *(continued)*

Theorem or Postulate	Found on Page	Description/Illustration/Abbreviation
Theorem 4.8 *Leg-Angle Congruence (LA)*		
Theorem 4.9 *Isosceles Triangle Theorem*		
Theorem 4.10		
Postulate 4.1 *Side-Side-Side Congruence (SSS)*		
Postulate 4.2 *Side-Angle-Side Congruence (SAS)*		
Postulate 4.3 *Angle-Side-Angle Congruence (ASA)*		
Postulate 3.4 *Hypotenuse-Leg Congruence (HL)*		

NAME ______________________________ DATE ____________ PERIOD _____

4-1 Study Guide and Intervention

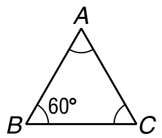
Classifying Triangles

Classify Triangles by Angles One way to classify a triangle is by the measures of its angles.

• If *one* of the angles of a triangle is an obtuse angle, then the triangle is an **obtuse triangle**.
• If *one* of the angles of a triangle is a right angle, then the triangle is a **right triangle**.
• If *all three* of the angles of a triangle are acute angles, then the triangle is an **acute triangle**.
• If all three angles of an acute triangle are congruent, then the triangle is an **equiangular triangle**.

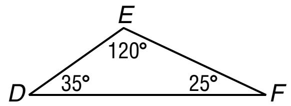
Example **Classify each triangle.**

a.



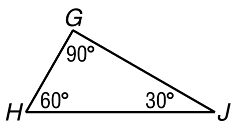
All three angles are congruent, so all three angles have measure 60°. The triangle is an equiangular triangle.

b.



The triangle has one angle that is obtuse. It is an obtuse triangle.

c.

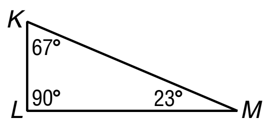


The triangle has one right angle. It is a right triangle.

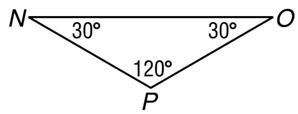
Exercises

Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

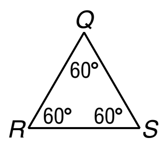
1.



2.



3.



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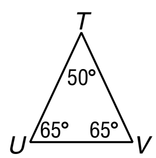

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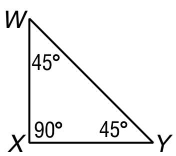
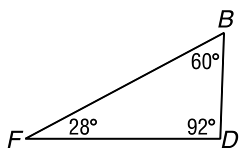


6.



4-1 Study Guide and Intervention *(continued)*

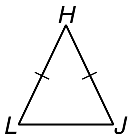
Classifying Triangles

Classify Triangles by Sides You can classify a triangle by the measures of its sides. Equal numbers of hash marks indicate congruent sides.

• If *all three* sides of a triangle are congruent, then the triangle is an **equilateral triangle**.
• If *at least two* sides of a triangle are congruent, then the triangle is an **isosceles triangle**.
• If *no two* sides of a triangle are congruent, then the triangle is a **scalene triangle**.

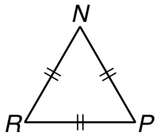
Example **Classify each triangle.**

a.



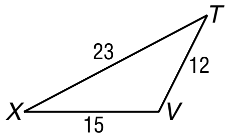
Two sides are congruent. The triangle is an isosceles triangle.

b.



All three sides are congruent. The triangle is an equilateral triangle.

c.

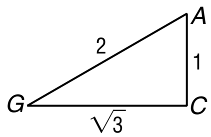


The triangle has no pair of congruent sides. It is a scalene triangle.

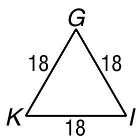
Exercises

Classify each triangle as *equilateral*, *isosceles*, or *scalene*.

1.



2.



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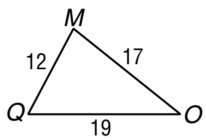

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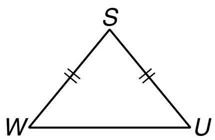

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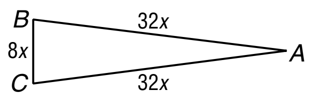

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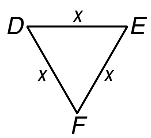


7. Find the measure of each side of equilateral $\triangle RST$ with $RS = 2x + 2$, $ST = 3x$, and $TR = 5x - 4$.

8. Find the measure of each side of isosceles $\triangle ABC$ with $AB = BC$ if $AB = 4y$, $BC = 3y + 2$, and $AC = 3y$.

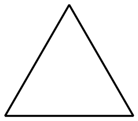
9. Find the measure of each side of $\triangle ABC$ with vertices $A(-1, 5)$, $B(6, 1)$, and $C(2, -6)$. Classify the triangle.

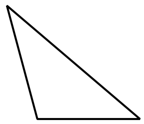
NAME ____ DATE ____________ PERIOD _____

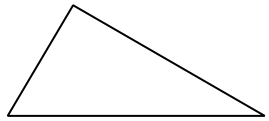
4-1 Skills Practice

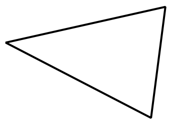
Classifying Triangles

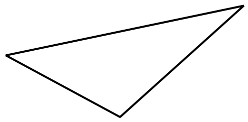
Use a protractor to classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

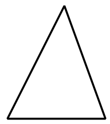
1. 

2. 

3. 

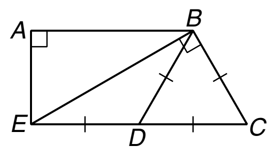
4. 

5. 

6. 

Lesson 4-1

Identify the indicated type of triangles.



7. right

8. isosceles

9. scalene

10. obtuse

ALGEBRA Find *x* and the measure of each side of the triangle.

11. $\triangle ABC$ is equilateral with $AB = 3x - 2$, $BC = 2x + 4$, and $CA = x + 10$.

12. $\triangle DEF$ is isosceles, $\angle D$ is the vertex angle, $DE = x + 7$, $DF = 3x - 1$, and $EF = 2x + 5$.

Find the measures of the sides of $\triangle RST$ and classify each triangle by its sides.

13. $R(0, 2)$, $S(2, 5)$, $T(4, 2)$

14. $R(1, 3)$, $S(4, 7)$, $T(5, 4)$

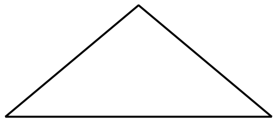
NAME ____ DATE ____________ PERIOD _____

4-1 Practice

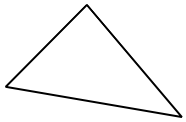
Classifying Triangles

Use a protractor to classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

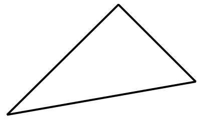
1.



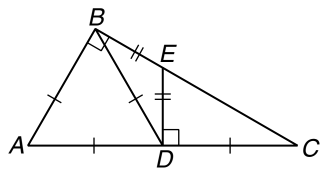
2.



3.



Identify the indicated type of triangles if $\overline{AB} \cong \overline{AD} \cong \overline{BD} \cong \overline{DC}$, $\overline{BE} \cong \overline{ED}$, $\overline{AB} \perp \overline{BC}$, and $\overline{ED} \perp \overline{DC}$.



4. right

5. obtuse

6. scalene

7. isosceles

ALGEBRA Find x and the measure of each side of the triangle.

8. $\triangle FGH$ is equilateral with $FG = x + 5$, $GH = 3x - 9$, and $FH = 2x - 2$.

9. $\triangle LMN$ is isosceles, $\angle L$ is the vertex angle, $LM = 3x - 2$, $LN = 2x + 1$, and $MN = 5x - 2$.

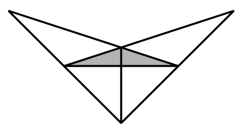
Find the measures of the sides of $\triangle KPL$ and classify each triangle by its sides.

10. $K(-3, 2)$ $P(2, 1)$, $L(-2, -3)$

11. $K(5, -3)$, $P(3, 4)$, $L(-1, 1)$

12. $K(-2, -6)$, $P(-4, 0)$, $L(3, -1)$

13. DESIGN Diana entered the design at the right in a logo contest sponsored by a wildlife environmental group. Use a protractor. How many right angles are there?



NAME ______________________________ DATE ____________ PERIOD _____

4-1 Reading to Learn Mathematics

Classifying Triangles

Pre-Activity **Why are triangles important in construction?**

Read the introduction to Lesson 4-1 at the top of page 178 in your textbook.

- Why are triangles used for braces in construction rather than other shapes?
- Why do you think that isosceles triangles are used more often than scalene triangles in construction?

Lesson 4-1

Reading the Lesson

1. Supply the correct numbers to complete each sentence.

a. In an obtuse triangle, there are ____ acute angle(s), ____ right angle(s), and ____ obtuse angle(s).

b. In an acute triangle, there are ____ acute angle(s), ____ right angle(s), and ____ obtuse angle(s).

c. In a right triangle, there are ____ acute angle(s), ____ right angle(s), and ____ obtuse angle(s).

2. Determine whether each statement is *always*, *sometimes*, or *never* true.

a. A right triangle is scalene.

b. An obtuse triangle is isosceles.

c. An equilateral triangle is a right triangle.

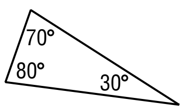
d. An equilateral triangle is isosceles.

e. An acute triangle is isosceles.

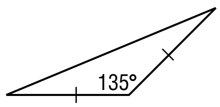
f. A scalene triangle is obtuse.

3. Describe each triangle by as many of the following words as apply: *acute*, *obtuse*, *right*, *scalene*, *isosceles*, or *equilateral*.

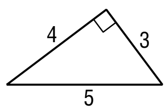
a.



b.



c.



Helping You Remember

4. A good way to remember a new mathematical term is to relate it to a nonmathematical definition of the same word. How is the use of the word *acute*, when used to describe *acute pain*, related to the use of the word *acute* when used to describe an *acute angle* or an *acute triangle*?

NAME ____ DATE ____________ PERIOD _____

4-1 Enrichment

Reading Mathematics

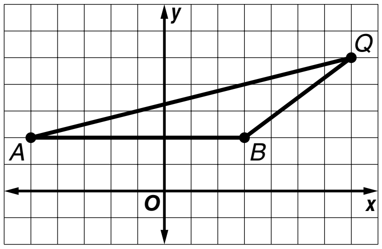
When you read geometry, you may need to draw a diagram to make the text easier to understand.

Example **Consider three points, *A*, *B*, and *C* on a coordinate grid. The *y*-coordinates of *A* and *B* are the same. The *x*-coordinate of *B* is greater than the *x*-coordinate of *A*. Both coordinates of *C* are greater than the corresponding coordinates of *B*. Is triangle *ABC* acute, right, or obtuse?**

To answer this question, first draw a sample triangle that fits the description.

Side *AB* must be a horizontal segment because the *y*-coordinates are the same. Point *C* must be located to the right and up from point *B*.

From the diagram you can see that triangle *ABC* must be obtuse.



Answer each question. Draw a simple triangle on the grid above to help you.

1. Consider three points, *R*, *S*, and *T* on a coordinate grid. The *x*-coordinates of *R* and *S* are the same. The *y*-coordinate of *T* is between the *y*-coordinates of *R* and *S*. The *x*-coordinate of *T* is less than the *x*-coordinate of *R*. Is angle *R* of triangle *RST* acute, right, or obtuse?

2. Consider three noncollinear points, *J*, *K*, and *L* on a coordinate grid. The *y*-coordinates of *J* and *K* are the same. The *x*-coordinates of *K* and *L* are the same. Is triangle *JKL* acute, right, or obtuse?

3. Consider three noncollinear points, *D*, *E*, and *F* on a coordinate grid. The *x*-coordinates of *D* and *E* are opposites. The *y*-coordinates of *D* and *E* are the same. The *x*-coordinate of *F* is 0. What kind of triangle must $\triangle DEF$ be: scalene, isosceles, or equilateral?

4. Consider three points, *G*, *H*, and *I* on a coordinate grid. Points *G* and *H* are on the positive *y*-axis, and the *y*-coordinate of *G* is twice the *y*-coordinate of *H*. Point *I* is on the positive *x*-axis, and the *x*-coordinate of *I* is greater than the *y*-coordinate of *G*. Is triangle *GHI* scalene, isosceles, or equilateral?

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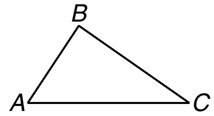
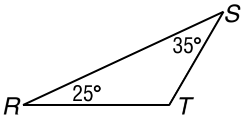
4-2 Study Guide and Intervention

Angles of Triangles

Angle Sum Theorem If the measures of two angles of a triangle are known, the measure of the third angle can always be found.

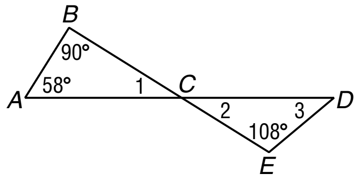
Angle Sum Theorem	The sum of the measures of the angles of a triangle is 180. In the figure at the right, $m\angle A + m\angle B + m\angle C = 180$. 

Example 1 **Find $m\angle T$.**



$m\angle R + m\angle S + m\angle T = 180$	Angle Sum Theorem
$25 + 35 + m\angle T = 180$	Substitution
$60 + m\angle T = 180$	Add.
$m\angle T = 120$	Subtract 60 from each side.

Example 2 **Find the missing angle measures.**

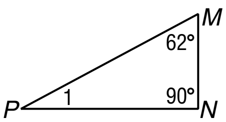


$m\angle 1 + m\angle A + m\angle B = 180$	Angle Sum Theorem
$m\angle 1 + 58 + 90 = 180$	Substitution
$m\angle 1 + 148 = 180$	Add.
$m\angle 1 = 32$	Subtract 148 from each side.
$m\angle 2 = 32$	Vertical angles are congruent.
$m\angle 3 + m\angle 2 + m\angle E = 180$	Angle Sum Theorem
$m\angle 3 + 32 + 108 = 180$	Substitution
$m\angle 3 + 140 = 180$	Add.
$m\angle 3 = 40$	Subtract 140 from each side.

Exercises

Find the measure of each numbered angle.

1.



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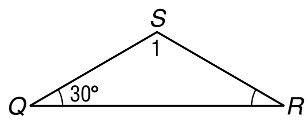

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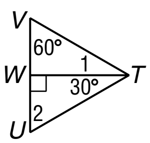
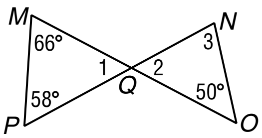


4.



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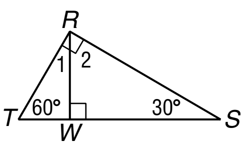
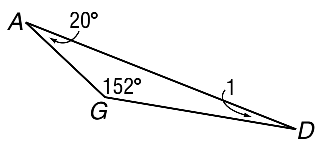


6.



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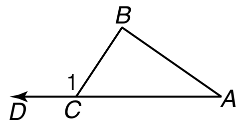
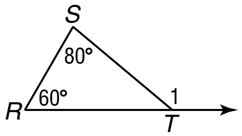
4-2 Study Guide and Intervention *(continued)*

Angles of Triangles

Exterior Angle Theorem At each vertex of a triangle, the angle formed by one side and an extension of the other side is called an **exterior angle** of the triangle. For each exterior angle of a triangle, the **remote interior angles** are the interior angles that are not adjacent to that exterior angle. In the diagram below, $\angle B$ and $\angle A$ are the remote interior angles for exterior $\angle DCB$.

Exterior Angle Theorem	The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles. $m\angle 1 = m\angle A + m\angle B$ 

Example 1 **Find $m\angle 1$.**

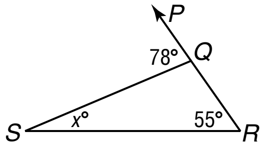


$m\angle 1 = m\angle R + m\angle S$ Exterior Angle Theorem

$= 60 + 80$ Substitution

$= 140$ Add.

Example 2 **Find x.**



$m\angle PQS = m\angle R + m\angle S$ Exterior Angle Theorem

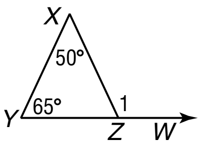
$78 = 55 + x$ Substitution

$23 = x$ Subtract 55 from each side.

Exercises

Find the measure of each numbered angle.

1.



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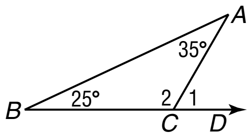

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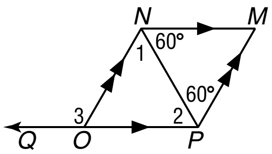

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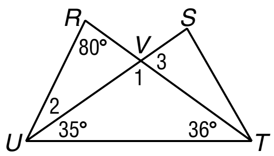
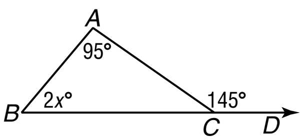


Find x.

5.



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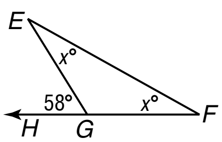

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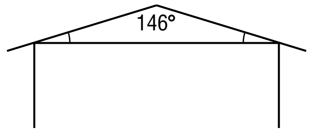
4-2 Skills Practice

Angles of Triangles

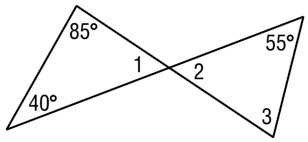
Find the missing angle measures.

1.

2.



Find the measure of each angle.

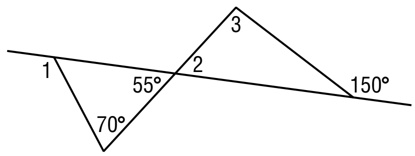


3. $m\angle 1$

4. $m\angle 2$

5. $m\angle 3$

Find the measure of each angle.

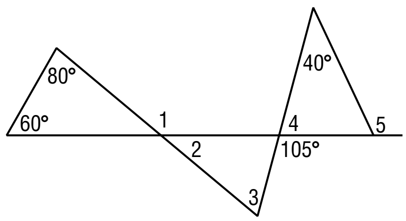


6. $m\angle 1$

7. $m\angle 2$

8. $m\angle 3$

Find the measure of each angle.



9. $m\angle 1$

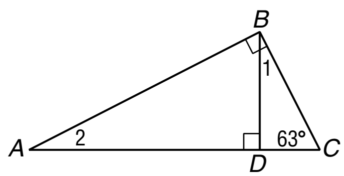
10. $m\angle 2$

11. $m\angle 3$

12. $m\angle 4$

13. $m\angle 5$

Find the measure of each angle.



14. $m\angle 1$

15. $m\angle 2$

Lesson 4-2

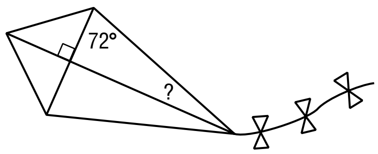
NAME ____ DATE ____________ PERIOD _____

4-2 Practice

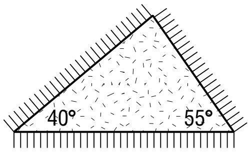
Angles of Triangles

Find the missing angle measures.

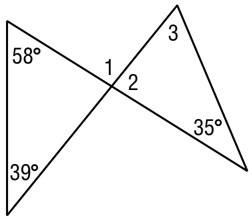
1.



2.



Find the measure of each angle.

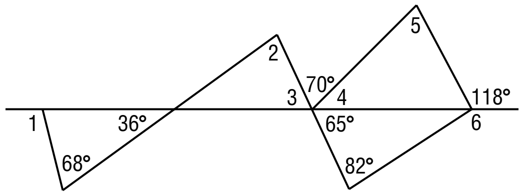


3. $m\angle 1$

4. $m\angle 2$

5. $m\angle 3$

Find the measure of each angle.



6. $m\angle 1$

7. $m\angle 4$

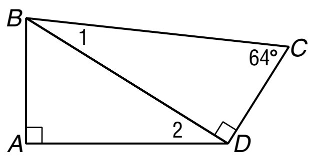
8. $m\angle 3$

9. $m\angle 2$

10. $m\angle 5$

11. $m\angle 6$

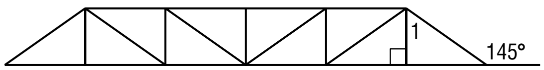
Find the measure of each angle if $\angle BAD$ and $\angle BDC$ are right angles and $m\angle ABC = 84$.



12. $m\angle 1$

13. $m\angle 2$

14. CONSTRUCTION The diagram shows an example of the Pratt Truss used in bridge construction. Use the diagram to find $m\angle 1$.



4-2 Reading to Learn Mathematics

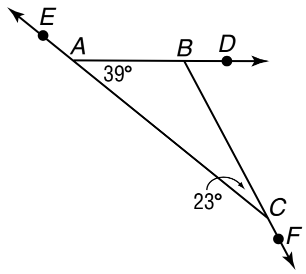
Angles of Triangles

Pre-Activity **How are the angles of triangles used to make kites?**

Read the introduction to Lesson 4-2 at the top of page 185 in your textbook.

The frame of the simplest kind of kite divides the kite into four triangles. Describe these four triangles and how they are related to each other.

Reading the Lesson



1. Refer to the figure.

a. Name the three interior angles of the triangle. (Use three letters to name each angle.)

b. Name three exterior angles of the triangle. (Use three letters to name each angle.)

c. Name the remote interior angles of $\angle EAB$.

d. Find the measure of each angle without using a protractor.

i. $\angle DBC$ **ii.** $\angle ABC$ **iii.** $\angle ACF$ **iv.** $\angle EAB$

2. Indicate whether each statement is *true* or *false*. If the statement is false, replace the underlined word or number with a word or number that will make the statement true.

a. The acute angles of a right triangle are <u>supplementary</u>.

b. The sum of the measures of the angles of any triangle is <u>100</u>.

c. A triangle can have at most one right angle or <u>acute</u> angle.

d. If two angles of one triangle are congruent to two angles of another triangle, then the third angles of the triangles are <u>congruent</u>.

e. The measure of an exterior angle of a triangle is equal to the <u>difference</u> of the measures of the two remote interior angles.

f. If the measures of two angles of a triangle are 62 and 93, then the measure of the third angle is <u>35</u>.

g. An <u>exterior</u> angle of a triangle forms a linear pair with an interior angle of the triangle.

Helping You Remember

3. Many students remember mathematical ideas and facts more easily if they see them demonstrated visually rather than having them stated in words. Describe a visual way to demonstrate the Angle Sum Theorem.

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4-2 Enrichment

Finding Angle Measures in Triangles

You can use algebra to solve problems involving triangles.

Example **In triangle *ABC*, $m\angle A$, is twice $m\angle B$, and $m\angle C$ is 8 more than $m\angle B$. What is the measure of each angle?**

Write and solve an equation. Let $x = m\angle B$.

$$\begin{aligned} m\angle A + m\angle B + m\angle C &= 180 \\ 2x + x + (x + 8) &= 180 \\ 4x + 8 &= 180 \\ 4x &= 172 \\ x &= 43 \end{aligned}$$

So, $m\angle A = 2(43)$ or 86, $m\angle B = 43$, and $m\angle C = 43 + 8$ or 51.

Solve each problem.

1. In triangle *DEF*, $m\angle E$ is three times $m\angle D$, and $m\angle F$ is 9 less than $m\angle E$. What is the measure of each angle?

2. In triangle *RST*, $m\angle T$ is 5 more than $m\angle R$, and $m\angle S$ is 10 less than $m\angle T$. What is the measure of each angle?

3. In triangle *JKL*, $m\angle K$ is four times $m\angle J$, and $m\angle L$ is five times $m\angle J$. What is the measure of each angle?

4. In triangle *XYZ*, $m\angle Z$ is 2 more than twice $m\angle X$, and $m\angle Y$ is 7 less than twice $m\angle X$. What is the measure of each angle?

5. In triangle *GHI*, $m\angle H$ is 20 more than $m\angle G$, and $m\angle G$ is 8 more than $m\angle I$. What is the measure of each angle?

6. In triangle *MNO*, $m\angle M$ is equal to $m\angle N$, and $m\angle O$ is 5 more than three times $m\angle N$. What is the measure of each angle?

7. In triangle *STU*, $m\angle U$ is half $m\angle T$, and $m\angle S$ is 30 more than $m\angle T$. What is the measure of each angle?

8. In triangle *PQR*, $m\angle P$ is equal to $m\angle Q$, and $m\angle R$ is 24 less than $m\angle P$. What is the measure of each angle?

9. Write your own problems about measures of triangles.

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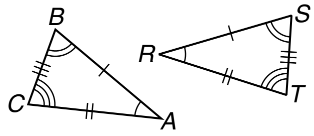
4-3 Study Guide and Intervention

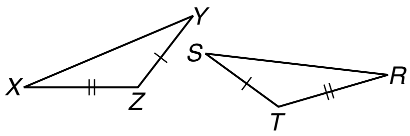
Congruent Triangles

Corresponding Parts of Congruent Triangles



Triangles that have the same size and same shape are **congruent triangles**. Two triangles are congruent if and only if all three pairs of corresponding angles are congruent and all three pairs of corresponding sides are congruent. In the figure, $\triangle ABC \cong \triangle RST$.

Example **If $\triangle XYZ \cong \triangle RST$, name the pairs of congruent angles and congruent sides.**



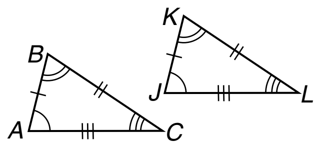
$\angle X \cong \angle R$, $\angle Y \cong \angle S$, $\angle Z \cong \angle T$

$\overline{XY} \cong \overline{RS}$, $\overline{XZ} \cong \overline{RT}$, $\overline{YZ} \cong \overline{ST}$

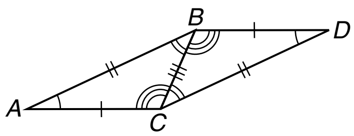
Exercises

Identify the congruent triangles in each figure.

1.



2.



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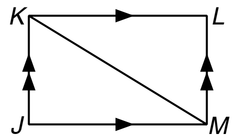
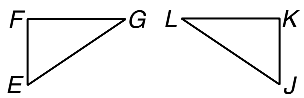


Name the corresponding congruent angles and sides for the congruent triangles.

4.



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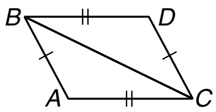

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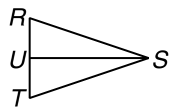


Lesson 4-3

4-3 Study Guide and Intervention *(continued)*

Congruent Triangles

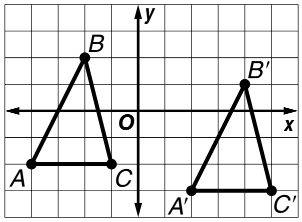
Identify Congruence Transformations If two triangles are congruent, you can slide, flip, or turn one of the triangles and they will still be congruent. These are called **congruence transformations** because they do not change the size or shape of the figure. It is common to use prime symbols to distinguish between an original $\triangle ABC$ and a transformed $\triangle A'B'C'$.

Example **Name the congruence transformation that produces $\triangle A'B'C'$ from $\triangle ABC$.**

The congruence transformation is a slide.

$\angle A \cong \angle A'$; $\angle B \cong \angle B'$; $\angle C \cong \angle C'$;

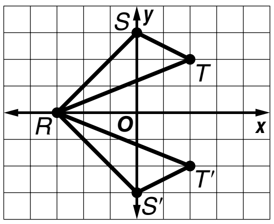
$\overline{AB} \cong \overline{A'B'}$; $\overline{AC} \cong \overline{A'C'}$; $\overline{BC} \cong \overline{B'C'}$



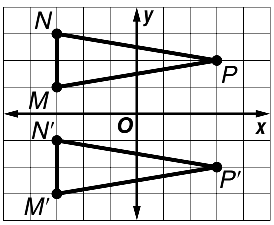
Exercises

Describe the congruence transformation between the two triangles as a *slide*, a *flip*, or a *turn*. Then name the congruent triangles.

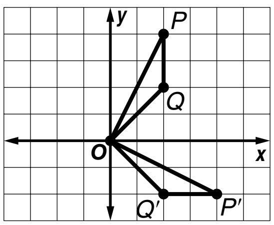
1.



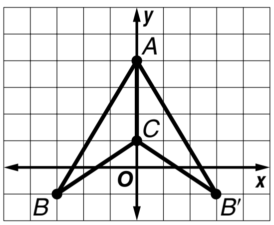
2.



3.



4.



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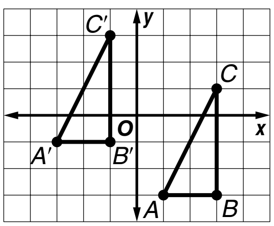

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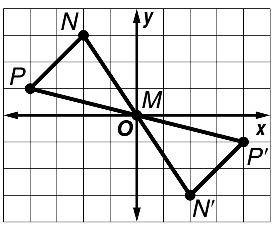


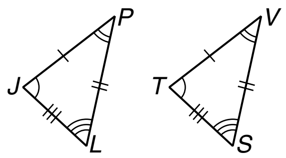
NAME ____ DATE ____________ PERIOD _____

4-3 Skills Practice

Congruent Triangles

Identify the congruent triangles in each figure.

1.



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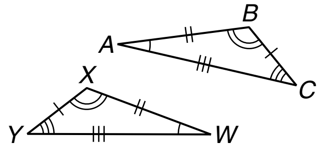

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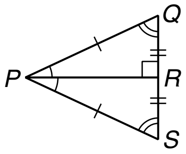
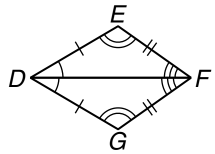


4.



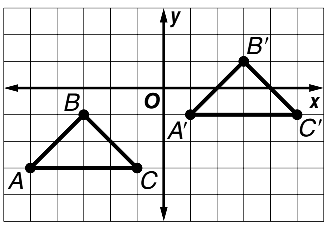
Name the congruent angles and sides for each pair of congruent triangles.

5. $\triangle ABC \cong \triangle FGH$

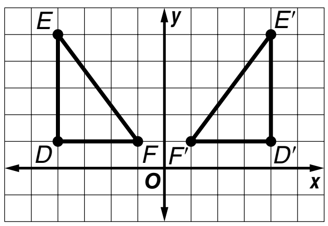
6. $\triangle PQR \cong \triangle STU$

Verify that each of the following transformations preserves congruence, and name the congruence transformation.

7. $\triangle ABC \cong \triangle A'B'C'$



8. $\triangle DEF \cong \triangle D'E'F'$



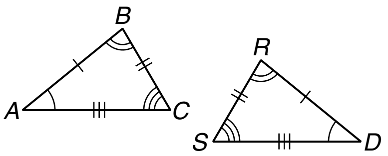
Lesson 4-3

4-3 Practice

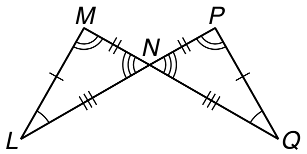
Congruent Triangles

Identify the congruent triangles in each figure.

1.



2.



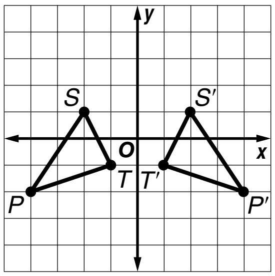
Name the congruent angles and sides for each pair of congruent triangles.

3. $\triangle GKP \cong \triangle LMN$

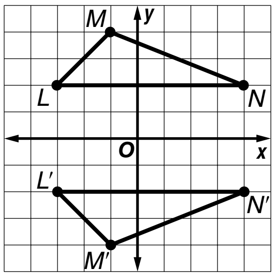
4. $\triangle ANC \cong \triangle RBV$

Verify that each of the following transformations preserves congruence, and name the congruence transformation.

5. $\triangle PST \cong \triangle P'S'T'$



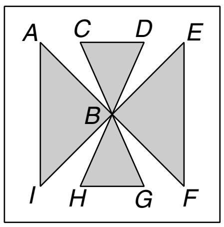
6. $\triangle LMN \cong \triangle L'M'N'$



QUILTING For Exercises 7 and 8, refer to the quilt design.

7. Indicate the triangles that appear to be congruent.

8. Name the congruent angles and congruent sides of a pair of congruent triangles.



NAME ____ DATE ____________ PERIOD _____

4-3 Reading to Learn Mathematics

Congruent Triangles

Pre-Activity **Why are triangles used in bridges?**

Read the introduction to Lesson 4-3 at the top of page 192 in your textbook.

In the bridge shown in the photograph in your textbook, diagonal braces were used to divide squares into two isosceles right triangles. Why do you think these braces are used on the bridge?

Reading the Lesson

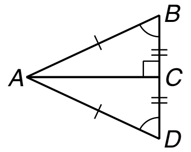
1. If $\triangle RST \cong \triangle UWV$, complete each pair of congruent parts.

$\angle R \cong$ ______ ______ $\cong \angle W$ $\angle T \cong$ ______

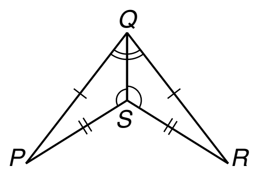
$\overline{RT} \cong$ ______ ______ $\cong \overline{UW}$ ______ $\cong \overline{WV}$

2. Identify the congruent triangles in each diagram.

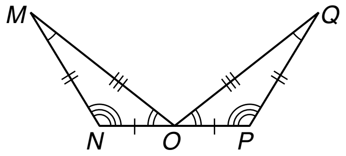
a.



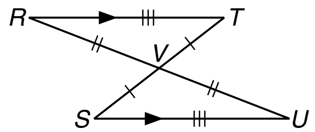
b.



c.



d.



3. Determine whether each statement says that congruence of triangles is *reflexive*, *symmetric*, or *transitive*.

a. If the first of two triangles is congruent to the second triangle, then the second triangle is congruent to the first.

b. If there are three triangles for which the first is congruent to the second and the second is congruent to the third, then the first triangle is congruent to the third.

c. Every triangle is congruent to itself.

Helping You Remember

4. A good way to remember something is to explain it to someone else. Your classmate Ben is having trouble writing congruence statements for triangles because he thinks he has to match up three pairs of sides and three pairs of angles. How can you help him understand how to write correct congruence statements more easily?

4-3 Enrichment

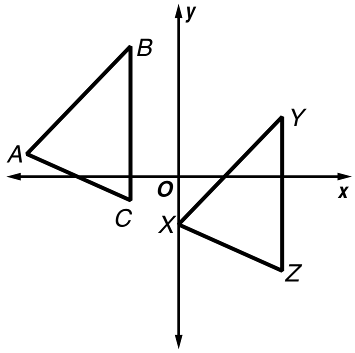
Transformations in The Coordinate Plane

The following statement tells one way to map preimage points to image points in the coordinate plane.

$(x, y) \rightarrow (x + 6, y - 3)$

This can be read, "The point with coordinates (x, y) is mapped to the point with coordinates $(x + 6, y - 3)$." With this transformation, for example, (3, 5) is mapped to $(3 + 6, 5 - 3)$ or (9, 2). The figure shows how the triangle *ABC* is mapped to triangle *XYZ*.

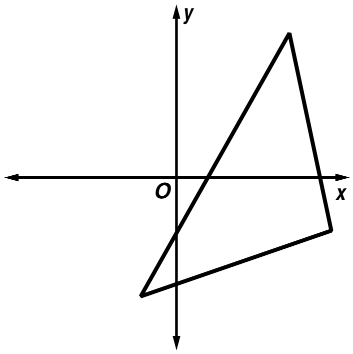
$(x, y) \rightarrow (x + 6, y - 3)$



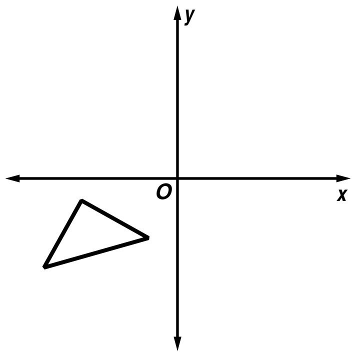
1. Does the transformation above appear to be a congruence transformation? Explain your answer.

Draw the transformation image for each figure. Then tell whether the transformation is or is not a congruence transformation.

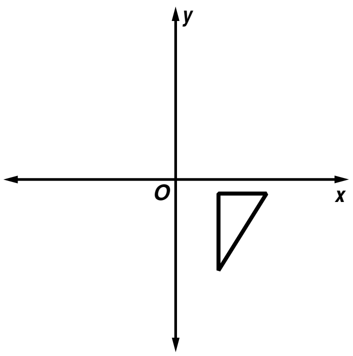
2. $(x, y) \rightarrow (x - 4, y)$



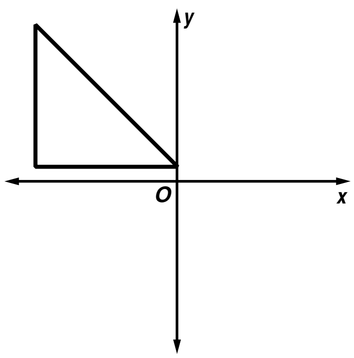
3. $(x, y) \rightarrow (x + 8, y + 7)$



4. $(x, y) \rightarrow (-x, -y)$



5. $(x, y) \rightarrow \left(-\frac{1}{2}x, y\right)$



4-4 Study Guide and Intervention

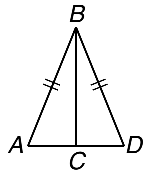
Proving Congruence—SSS, SAS

SSS Postulate You know that two triangles are congruent if corresponding sides are congruent and corresponding angles are congruent. The Side-Side-Side (SSS) Postulate lets you show that two triangles are congruent if you know only that the sides of one triangle are congruent to the sides of the second triangle.

SSS Postulate	If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent.

Example **Write a two-column proof.**

Given: $\overline{AB} \cong \overline{DB}$ and C is the midpoint of $\overline{AD}$.
Prove: $\triangle ABC \cong \triangle DBC$

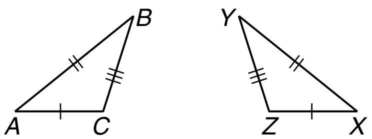


Statements	**Reasons**
1. $\overline{AB} \cong \overline{DB}$	**1.** Given
2. C is the midpoint of $\overline{AD}$.	**2.** Given
3. $\overline{AC} \cong \overline{DC}$	**3.** Definition of midpoint
4. $\overline{BC} \cong \overline{BC}$	**4.** Reflexive Property of $\cong$
5. $\triangle ABC \cong \triangle DBC$	**5.** SSS Postulate

Exercises

Write a two-column proof.

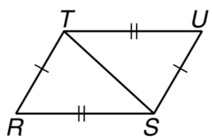
1.



Given: $\overline{AB} \cong \overline{XY}$, $\overline{AC} \cong \overline{XZ}$, $\overline{BC} \cong \overline{YZ}$
Prove: $\triangle ABC \cong \triangle XYZ$

Statements	**Reasons**

2.



Given: $\overline{RS} \cong \overline{UT}$, $\overline{RT} \cong \overline{US}$
Prove: $\triangle RST \cong \triangle UTS$

Statements	**Reasons**

Lesson 4-4

4-4 Study Guide and Intervention *(continued)*

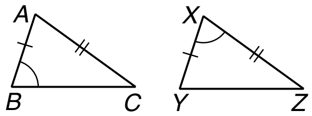
Proving Congruence—SSS, SAS

SAS Postulate Another way to show that two triangles are congruent is to use the Side-Angle-Side (SAS) Postulate.

SAS Postulate	If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

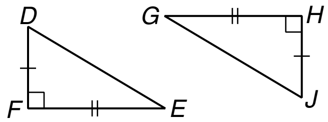
Example **For each diagram, determine which pairs of triangles can be proved congruent by the SAS Postulate.**

a.



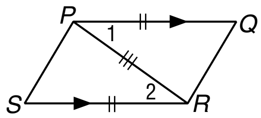
In $\triangle ABC$, the angle is not "included" by the sides $\overline{AB}$ and $\overline{AC}$. So the triangles cannot be proved congruent by the SAS Postulate.

b.



The right angles are congruent and they are the included angles for the congruent sides. $\triangle DEF \cong \triangle JGH$ by the SAS Postulate.

c.

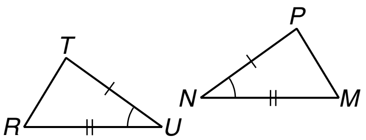


The included angles, $\angle 1$ and $\angle 2$, are congruent because they are alternate interior angles for two parallel lines. $\triangle PSR \cong \triangle RQP$ by the SAS Postulate.

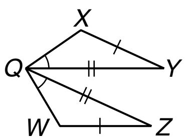
Exercises

For each figure, determine which pairs of triangles can be proved congruent by the SAS Postulate.

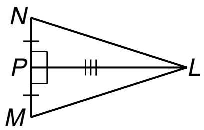
1.



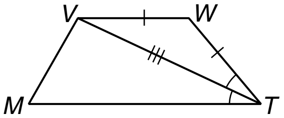
2.



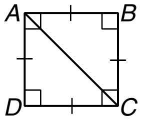
3.



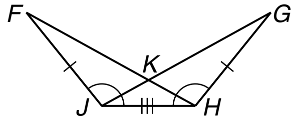
4.



5.



6.



4-4 Skills Practice

Proving Congruence—SSS, SAS

Determine whether $\triangle ABC \cong \triangle KLM$ given the coordinates of the vertices. Explain.

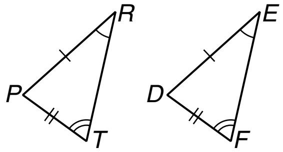
1. $A(-3, 3)$, $B(-1, 3)$, $C(-3, 1)$, $K(1, 4)$, $L(3, 4)$, $M(1, 6)$

2. $A(-4, -2)$, $B(-4, 1)$, $C(-1, -1)$, $K(0, -2)$, $L(0, 1)$, $M(4, 1)$

3. Write a flow proof.

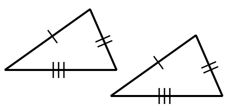
Given: $\overline{PR} \cong \overline{DE}$, $\overline{PT} \cong \overline{DF}$
$\angle R \cong \angle E$, $\angle T \cong \angle F$

Prove: $\triangle PRT \cong \triangle DEF$

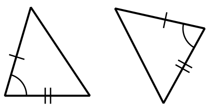


Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

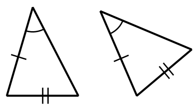
4.



5.



6.



Lesson 4-4

NAME ____ DATE ____________ PERIOD _____

4-4 Practice

Proving Congruence—SSS, SAS

Determine whether $\triangle DEF \cong \triangle PQR$ given the coordinates of the vertices. Explain.

1. $D(-6, 1)$, $E(1, 2)$, $F(-1, -4)$, $P(0, 5)$, $Q(7, 6)$, $R(5, 0)$

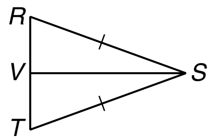
2. $D(-7, -3)$, $E(-4, -1)$, $F(-2, -5)$, $P(2, -2)$, $Q(5, -4)$, $R(0, -5)$

3. Write a flow proof.

Given: $\overline{RS} \cong \overline{TS}$

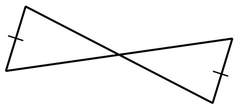
V is the midpoint of $\overline{RT}$.

Prove: $\triangle RSV \cong \triangle TSV$

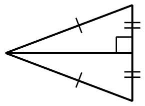


Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

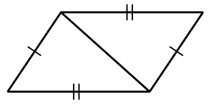
4.



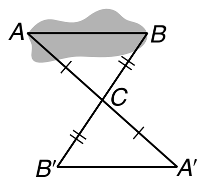
5.



6.



7. INDIRECT MEASUREMENT To measure the width of a sinkhole on his property, Harmon marked off congruent triangles as shown in the diagram. How does he know that the lengths $A'B'$ and AB are equal?



NAME ____ DATE ____________ PERIOD _____

4-4 Reading to Learn Mathematics

Proving Congruence—SSS, SAS

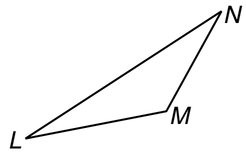
Pre-Activity **How do land surveyors use congruent triangles?**

Read the introduction to Lesson 4-4 at the top of page 200 in your textbook.

Why do you think that land surveyors would use congruent right triangles rather than other congruent triangles to establish property boundaries?

Reading the Lesson

1. Refer to the figure.



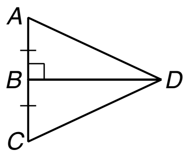
a. Name the sides of $\triangle LMN$ for which $\angle L$ is the included angle.

b. Name the sides of $\triangle LMN$ for which $\angle N$ is the included angle.

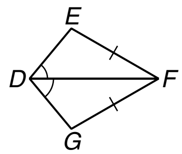
c. Name the sides of $\triangle LMN$ for which $\angle M$ is the included angle.

2. Determine whether you have enough information to prove that the two triangles in each figure are congruent. If so, write a congruence statement and name the congruence postulate that you would use. If not, write *not possible*.

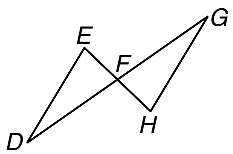
a.



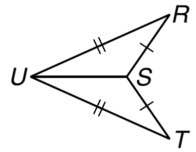
b.



c. $\overline{EH}$ and $\overline{DG}$ bisect each other.



d.



Lesson 4-4

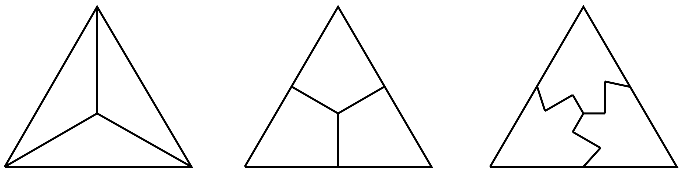
Helping You Remember

3. Find three words that explain what it means to say that two triangles are congruent and that can help you recall the meaning of the SSS Postulate.

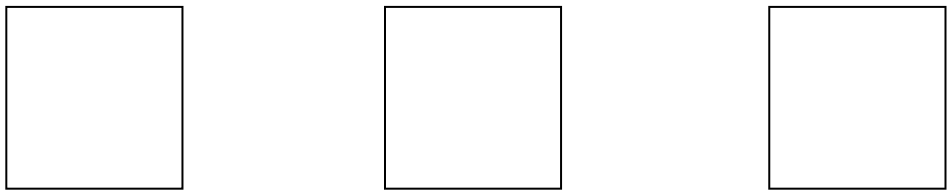
4-4 Enrichment

Congruent Parts of Regular Polygonal Regions

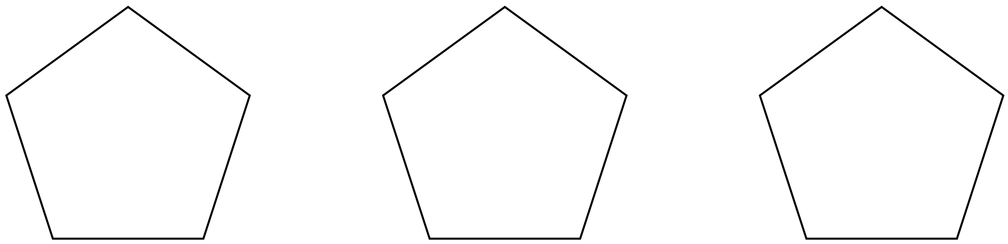
Congruent figures are figures that have exactly the same size and shape. There are many ways to divide regular polygonal regions into congruent parts. Three ways to divide an equilateral triangular region are shown. You can verify that the parts are congruent by tracing one part, then rotating, sliding, or reflecting that part on top of the other parts.



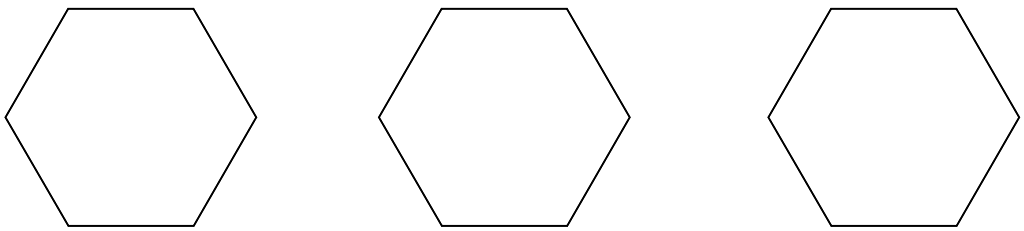
1. Divide each square into four congruent parts. Use three different ways.



2. Divide each pentagon into five congruent parts. Use three different ways.



3. Divide each hexagon into six congruent parts. Use three different ways.



4. What hints might you give another student who is trying to divide figures like those into congruent parts?

NAME ____ DATE ____________ PERIOD _____

4-5 Study Guide and Intervention

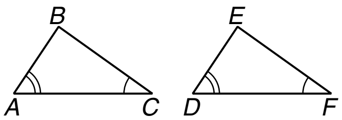
Proving Congruence—ASA, AAS

ASA Postulate The Angle-Side-Angle (ASA) Postulate lets you show that two triangles are congruent.

ASA Postulate	If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

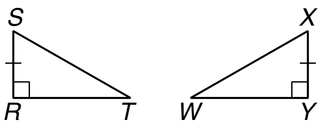
Example **Find the missing congruent parts so that the triangles can be proved congruent by the ASA Postulate. Then write the triangle congruence.**

a.



Two pairs of corresponding angles are congruent, $\angle A \cong \angle D$ and $\angle C \cong \angle F$. If the included sides $\overline{AC}$ and $\overline{DF}$ are congruent, then $\triangle ABC \cong \triangle DEF$ by the ASA Postulate.

b.

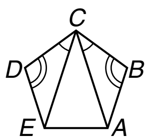


$\angle R \cong \angle Y$ and $\overline{SR} \cong \overline{XY}$. If $\angle S \cong \angle X$, then $\triangle RST \cong \triangle YXW$ by the ASA Postulate.

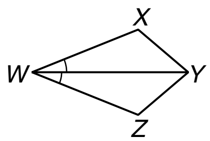
Exercises

What corresponding parts must be congruent in order to prove that the triangles are congruent by the ASA Postulate? Write the triangle congruence statement.

1.



2.



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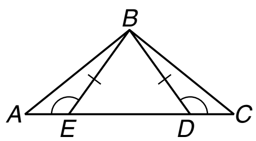

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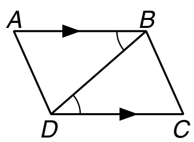

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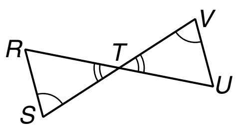

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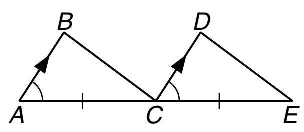


Lesson 4-5

4-5 Study Guide and Intervention *(continued)*

Proving Congruence—ASA, AAS

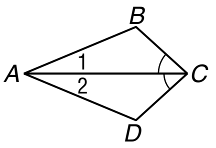
AAS Theorem Another way to show that two triangles are congruent is the Angle-Angle-Side (AAS) Theorem.

AAS Theorem	If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.

You now have five ways to show that two triangles are congruent.

- definition of triangle congruence
- SSS Postulate
- SAS Postulate
- ASA Postulate
- AAS Theorem

Example **In the diagram, $\angle BCA \cong \angle DCA$. Which sides are congruent? Which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the AAS Postulate?**



$\overline{AC} \cong \overline{AC}$ by the Reflexive Property of congruence. The congruent angles cannot be $\angle 1$ and $\angle 2$, because $\overline{AC}$ would be the included side. If $\angle B \cong \angle D$, then $\triangle ABC \cong \triangle ADC$ by the AAS Theorem.

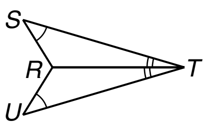
Exercises

In Exercises 1 and 2, draw and label $\triangle ABC$ and $\triangle DEF$. Indicate which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the AAS Theorem.

1. $\angle A \cong \angle D$; $\angle B \cong \angle E$

2. $BC \cong EF$; $\angle A \cong \angle D$

3. Write a flow proof.
Given: $\angle S \cong \angle U$; $\overline{TR}$ bisects $\angle STU$.
Prove: $\angle SRT \cong \angle URT$

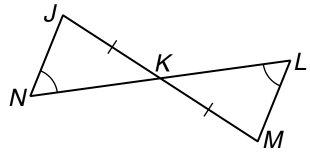


4-5 Skills Practice

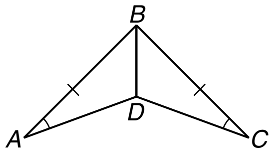
Proving Congruence—ASA, AAS

Write a flow proof.

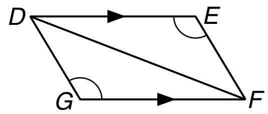
1. Given: $\angle N \cong \angle L$
$\overline{JK} \cong \overline{MK}$
Prove: $\triangle JKN \cong \triangle MKL$



2. Given: $\overline{AB} \cong \overline{CB}$
$\angle A \cong \angle C$
$\overline{DB}$ bisects $\angle ABC$.
Prove: $\overline{AD} \cong \overline{CD}$



3. Write a paragraph proof.
Given: $\overline{DE} \parallel \overline{FG}$
$\angle E \cong \angle G$
Prove: $\triangle DFG \cong \triangle FDE$



Lesson 4-5

NAME ____ DATE ____________ PERIOD _____

4-5 Practice

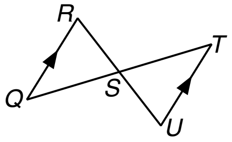
Proving Congruence—ASA, AAS

1. Write a flow proof.

Given: S is the midpoint of $\overline{QT}$.

$\overline{QR} \parallel \overline{TU}$

Prove: $\triangle QSR \cong \triangle TSU$

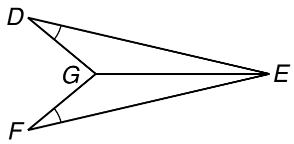


2. Write a paragraph proof.

Given: $\angle D \cong \angle F$

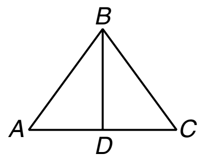
$\overline{GE}$ bisects $\angle DEF$.

Prove: $\overline{DG} \cong \overline{FG}$



ARCHITECTURE For Exercises 3 and 4, use the following information.

An architect used the window design in the diagram when remodeling an art studio. $\overline{AB}$ and $\overline{CB}$ each measure 3 feet.



3. Suppose D is the midpoint of $\overline{AC}$. Determine whether $\triangle ABD \cong \triangle CBD$. Justify your answer.

4. Suppose $\angle A \cong \angle C$. Determine whether $\triangle ABD \cong \triangle CBD$. Justify your answer.

NAME ____ DATE ____________ PERIOD _____

4-5 Reading to Learn Mathematics

Proving Congruence—ASA, AAS

Pre-Activity **How are congruent triangles used in construction?**

Read the introduction to Lesson 4-5 at the top of page 207 in your textbook.

Which of the triangles in the photograph in your textbook appear to be congruent?

Reading the Lesson

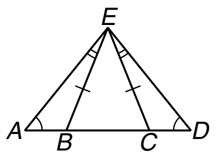
1. Explain in your own words the difference between how the ASA Postulate and the AAS Theorem are used to prove that two triangles are congruent.

2. Which of the following conditions are sufficient to prove that two triangles are congruent?

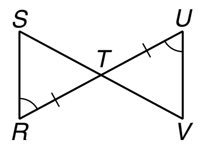
A. Two sides of one triangle are congruent to two sides of the other triangle.
B. The three sides of one triangles are congruent to the three sides of the other triangle.
C. The three angles of one triangle are congruent to the three angles of the other triangle.
D. All six corresponding parts of two triangles are congruent.
E. Two angles and the included side of one triangle are congruent to two sides and the included angle of the other triangle.
F. Two sides and a nonincluded angle of one triangle are congruent to two sides and a nonincluded angle of the other triangle.
G. Two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of the other triangle.
H. Two sides and the included angle of one triangle are congruent to two sides and the included angle of the other triangle.
I. Two angles and a nonincluded side of one triangle are congruent to two angles and a nonincluded side of the other triangle.

3. Determine whether you have enough information to prove that the two triangles in each figure are congruent. If so, write a congruence statement and name the congruence postulate or theorem that you would use. If not, write *not possible*.

a.



b. T is the midpoint of $\overline{RU}$.



Helping You Remember

4. A good way to remember mathematical ideas is to summarize them in a general statement. If you want to prove triangles congruent by using three pairs of corresponding parts, what is a good way to remember which combinations of parts will work?

4-5 Enrichment

Congruent Triangles in the Coordinate Plane

If you know the coordinates of the vertices of two triangles in the coordinate plane, you can often decide whether the two triangles are congruent. There may be more than one way to do this.

1. Consider $\triangle ABD$ and $\triangle CDB$ whose vertices have coordinates $A(0, 0)$, $B(2, 5)$, $C(9, 5)$, and $D(7, 0)$. Briefly describe how you can use what you know about congruent triangles and the coordinate plane to show that $\triangle ABD \cong \triangle CDB$. You may wish to make a sketch to help get you started.

2. Consider $\triangle PQR$ and $\triangle KLM$ whose vertices are the following points.

$P(1, 2)$	$Q(3, 6)$	$R(6, 5)$
$K(-2, 1)$	$L(-6, 3)$	$M(-5, 6)$

Briefly describe how you can show that $\triangle PQR \cong \triangle KLM$.

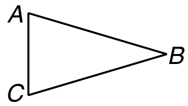
3. If you know the coordinates of all the vertices of two triangles, is it *always* possible to tell whether the triangles are congruent? Explain.

4-6 Study Guide and Intervention

Isosceles Triangles

Properties of Isosceles Triangles An **isosceles triangle** has two congruent sides. The angle formed by these sides is called the **vertex angle**. The other two angles are called **base angles**. You can prove a theorem and its converse about isosceles triangles.

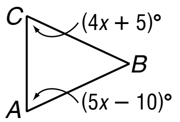
- If two sides of a triangle are congruent, then the angles opposite those sides are congruent. **(Isosceles Triangle Theorem)**
- If two angles of a triangle are congruent, then the sides opposite those angles are congruent.



If $\overline{AB} \cong \overline{CB}$, then $\angle A \cong \angle C$.

If $\angle A \cong \angle C$, then $\overline{AB} \cong \overline{CB}$.

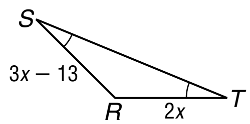
Example 1 **Find x.**



$BC = BA$, so

$m\angle A = m\angle C$.	Isos. Triangle Theorem
$5x - 10 = 4x + 5$	Substitution
$x - 10 = 5$	Subtract $4x$ from each side.
$x = 15$	Add 10 to each side.

Example 2 **Find x.**



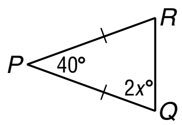
$m\angle S = m\angle T$, so

$SR = TR$.	Converse of Isos. △ Thm.
$3x - 13 = 2x$	Substitution
$3x = 2x + 13$	Add 13 to each side.
$x = 13$	Subtract $2x$ from each side.

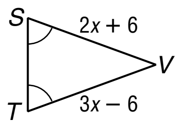
Exercises

Find x.

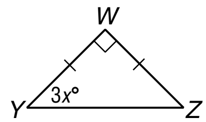
1.



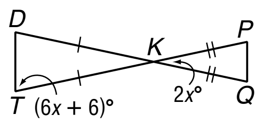
2.



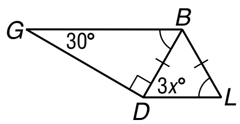
3.



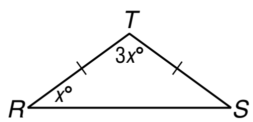
4.



5.



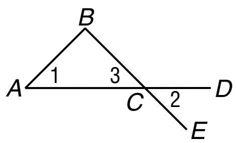
6.



7. Write a two-column proof.

Given: $\angle 1 \cong \angle 2$

Prove: $\overline{AB} \cong \overline{CB}$



Statements	Reasons

NAME ______________________________ DATE ____________ PERIOD _____

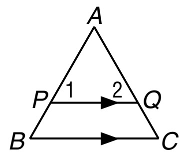
4-6 Study Guide and Intervention *(continued)*

Isosceles Triangles

Properties of Equilateral Triangles An **equilateral triangle** has three congruent sides. The Isosceles Triangle Theorem can be used to prove two properties of equilateral triangles.

> **1.** A triangle is equilateral if and only if it is equiangular.
> **2.** Each angle of an equilateral triangle measures 60°.

Example **Prove that if a line is parallel to one side of an equilateral triangle, then it forms another equilateral triangle.**



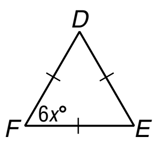
Proof:

Statements	**Reasons**
1. $\triangle ABC$ is equilateral; $\overline{PQ} \parallel \overline{BC}$.	**1.** Given
2. $m\angle A = m\angle B = m\angle C = 60$	**2.** Each $\angle$ of an equilateral $\triangle$ measures 60°.
3. $\angle 1 \cong \angle B$, $\angle 2 \cong \angle C$	**3.** If $\parallel$ lines, then corres. $\angle$s are $\cong$.
4. $m\angle 1 = 60$, $m\angle 2 = 60$	**4.** Substitution
5. $\triangle APQ$ is equilateral.	**5.** If a $\triangle$ is equiangular, then it is equilateral.

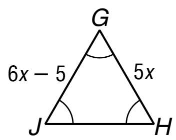
Exercises

Find *x*.

1.



2.



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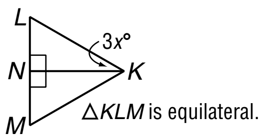

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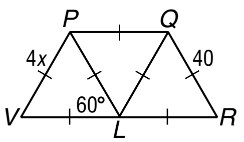

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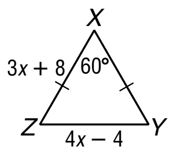
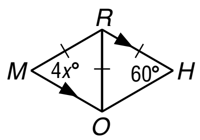
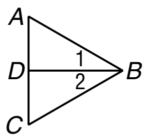


6.



7. Write a two-column proof.
Given: $\triangle ABC$ is equilateral; $\angle 1 \cong \angle 2$.
Prove: $\angle ADB \cong \angle CDB$



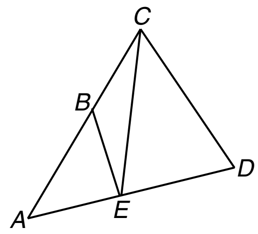
Proof:

Statements	**Reasons**

4-6 Skills Practice

Isosceles Triangles

Refer to the figure.



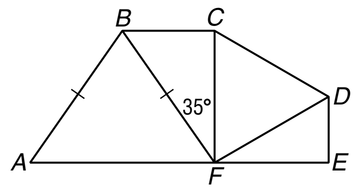
1. If $\overline{AC} \cong \overline{AD}$, name two congruent angles.

2. If $\overline{BE} \cong \overline{BC}$, name two congruent angles.

3. If $\angle EBA \cong \angle EAB$, name two congruent segments.

4. If $\angle CED \cong \angle CDE$, name two congruent segments.

$\triangle ABF$ is isosceles, $\triangle CDF$ is equilateral, and $m\angle AFD = 150$. Find each measure.



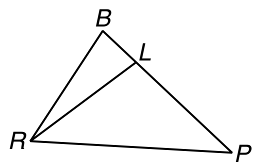
5. $m\angle CFD$

6. $m\angle AFB$

7. $m\angle ABF$

8. $m\angle A$

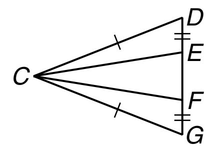
In the figure, $\overline{PL} \cong \overline{RL}$ and $\overline{LR} \cong \overline{BR}$.



9. If $m\angle RLP = 100$, find $m\angle BRL$.

10. If $m\angle LPR = 34$, find $m\angle B$.

11. Write a two-column proof.



Given: $\overline{CD} \cong \overline{CG}$

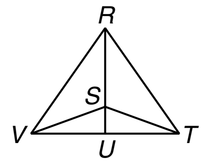
$\overline{DE} \cong \overline{GF}$

Prove: $\overline{CE} \cong \overline{CF}$

4-6 Practice

Isosceles Triangles

Refer to the figure.



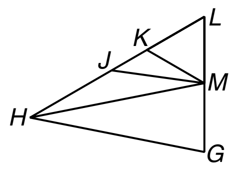
1. If $\overline{RV} \cong \overline{RT}$, name two congruent angles.

2. If $\overline{RS} \cong \overline{SV}$, name two congruent angles.

3. If $\angle SRT \cong \angle STR$, name two congruent segments.

4. If $\angle STV \cong \angle SVT$, name two congruent segments.

Triangles *GHM* and *HJM* are isosceles, with $\overline{GH} \cong \overline{MH}$ and $\overline{HJ} \cong \overline{MJ}$. Triangle *KLM* is equilateral, and $m\angle HMK = 50$. Find each measure.



5. $m\angle KML$ **6.** $m\angle HMG$ **7.** $m\angle GHM$

8. If $m\angle HJM = 145$, find $m\angle MHJ$.

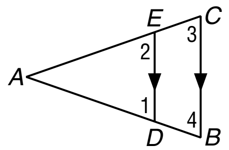
9. If $m\angle G = 67$, find $m\angle GHM$.

10. Write a two-column proof.

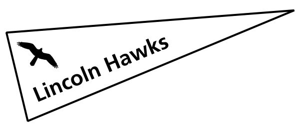
Given: $\overline{DE} \parallel \overline{BC}$

$\angle 1 \cong \angle 2$

Prove: $\overline{AB} \cong \overline{AC}$



11. SPORTS A pennant for the sports teams at Lincoln High School is in the shape of an isosceles triangle. If the measure of the vertex angle is 18, find the measure of each base angle.



NAME ______________________________ DATE ____________ PERIOD _____

4-6 Reading to Learn Mathematics

Isosceles Triangles

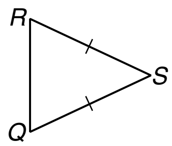
Pre-Activity **How are triangles used in art?**

Read the introduction to Lesson 4-6 at the top of page 216 in your textbook.

- Why do you think that isosceles and equilateral triangles are used more often than scalene triangles in art?
- Why might isosceles right triangles be used in art?

Reading the Lesson

1. Refer to the figure.



a. What kind of triangle is $\triangle QRS$?

b. Name the legs of $\triangle QRS$.

c. Name the base of $\triangle QRS$.

d. Name the vertex angle of $\triangle QRS$.

e. Name the base angles of $\triangle QRS$.

2. Determine whether each statement is *always*, *sometimes*, or *never* true.

a. If a triangle has three congruent sides, then it has three congruent angles.

b. If a triangle is isosceles, then it is equilateral.

c. If a right triangle is isosceles, then it is equilateral.

d. The largest angle of an isosceles triangle is obtuse.

e. If a right triangle has a 45° angle, then it is isosceles.

f. If an isosceles triangle has three acute angles, then it is equilateral.

g. The vertex angle of an isosceles triangle is the largest angle of the triangle.

3. Give the measures of the three angles of each triangle.

a. an equilateral triangle

b. an isosceles right triangle

c. an isosceles triangle in which the measure of the vertex angle is 70

d. an isosceles triangle in which the measure of a base angle is 70

e. an isosceles triangle in which the measure of the vertex angle is twice the measure of one of the base angles

Helping You Remember

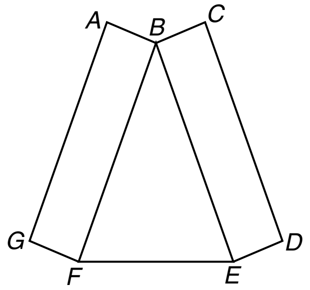
4. If a theorem and its converse are both true, you can often remember them most easily by combining them into an "if-and-only-if" statement. Write such a statement for the Isosceles Triangle Theorem and its converse.

4-6 Enrichment

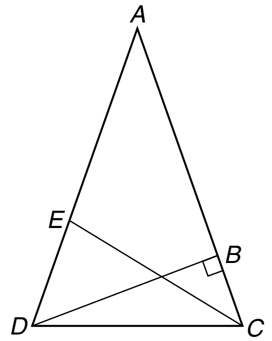
Triangle Challenges

Some problems include diagrams. If you are not sure how to solve the problem, begin by using the given information. Find the measures of as many angles as you can, writing each measure on the diagram. This may give you more clues to the solution.

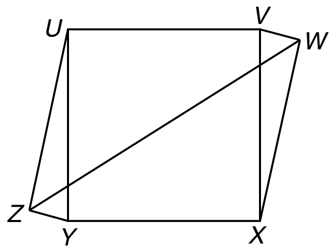
1. Given: $BE = BF$, $\angle BFG \cong \angle BEF \cong \angle BED$, $m\angle BFE = 82$ and $ABFG$ and $BCDE$ each have opposite sides parallel and congruent.
Find $m\angle ABC$.



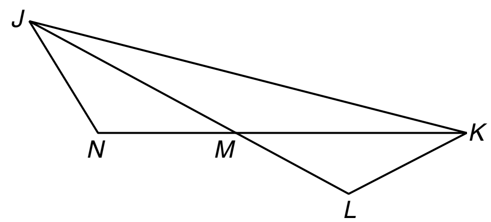
2. Given: $AC = AD$, and $\overline{AB} \perp \overline{BD}$, $m\angle DAC = 44$ and $\overline{CE}$ bisects $\angle ACD$.
Find $m\angle DEC$.



3. Given: $m\angle UZY = 90$, $m\angle ZWX = 45$, $\triangle YZU \cong \triangle VWX$, $UVXY$ is a square (all sides congruent, all angles right angles).
Find $m\angle WZY$.



4. Given: $m\angle N = 120$, $\overline{JN} \cong \overline{MN}$, $\triangle JNM \cong \triangle KLM$.
Find $m\angle JKM$.



4-7 Study Guide and Intervention

Triangles and Coordinate Proof

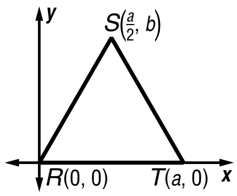
Position and Label Triangles A coordinate proof uses points, distances, and slopes to prove geometric properties. The first step in writing a coordinate proof is to place a figure on the coordinate plane and label the vertices. Use the following guidelines.

1. Use the origin as a vertex or center of the figure.
2. Place at least one side of the polygon on an axis.
3. Keep the figure in the first quadrant if possible.
4. Use coordinates that make the computations as simple as possible.

Example **Position an equilateral triangle on the coordinate plane so that its sides are *a* units long and one side is on the positive *x*-axis.**

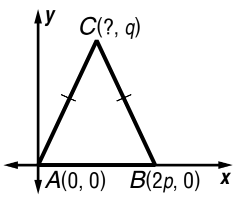
Start with $R(0, 0)$. If RT is a, then another vertex is $T(a, 0)$.

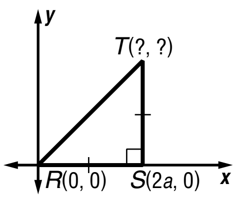
For vertex S, the x-coordinate is $\frac{a}{2}$. Use b for the y-coordinate, so the vertex is $S\left(\frac{a}{2}, b\right)$.

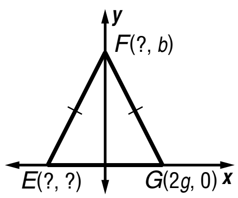


Exercises

Find the missing coordinates of each triangle.

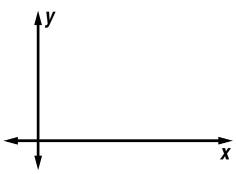
1.


2.


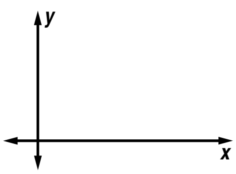
3.


Position and label each triangle on the coordinate plane.

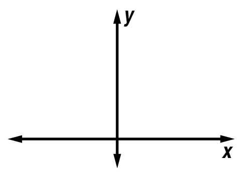
4. isosceles triangle $\triangle RST$ with base $\overline{RS}$ $4a$ units long



5. isosceles right $\triangle DEF$ with legs e units long



6. equilateral triangle $\triangle EQI$ with vertex $Q(0, a)$ and sides $2b$ units long



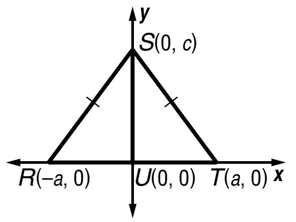
NAME ____ DATE ____________ PERIOD _____

4-7 Study Guide and Intervention *(continued)*

Triangles and Coordinate Proof

Write Coordinate Proofs Coordinate proofs can be used to prove theorems and to verify properties. Many coordinate proofs use the Distance Formula, Slope Formula, or Midpoint Theorem.

Example **Prove that a segment from the vertex angle of an isosceles triangle to the midpoint of the base is perpendicular to the base.**



First, position and label an isosceles triangle on the coordinate plane. One way is to use $T(a, 0)$, $R(-a, 0)$, and $S(0, c)$. Then $U(0, 0)$ is the midpoint of $\overline{RT}$.

Given: Isosceles $\triangle RST$; U is the midpoint of base $\overline{RT}$.
Prove: $\overline{SU} \perp \overline{RT}$

Proof:
U is the midpoint of $\overline{RT}$ so the coordinates of U are $\left(\frac{-a + a}{2}, \frac{0 + 0}{2}\right) = (0, 0)$. Thus $\overline{SU}$ lies on the y-axis, and $\triangle RST$ was placed so $\overline{RT}$ lies on the x-axis. The axes are perpendicular, so $\overline{SU} \perp \overline{RT}$.

Exercises

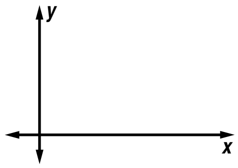
Prove that the segments joining the midpoints of the sides of a right triangle form a right triangle.

4-7 Skills Practice

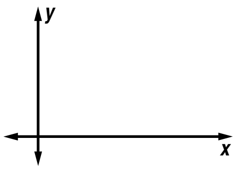
Triangles and Coordinate Proof

Position and label each triangle on the coordinate plane.

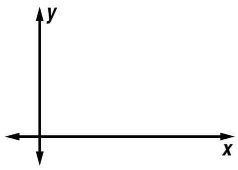
1. right $\triangle FGH$ with legs a units and b units



2. isosceles $\triangle KLP$ with base $\overline{KP}$ $6b$ units long

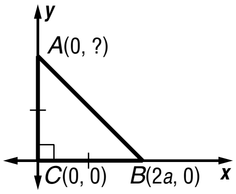


3. isosceles $\triangle AND$ with base $\overline{AD}$ $5a$ long

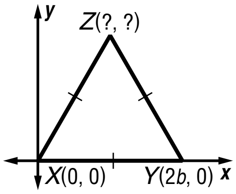


Find the missing coordinates of each triangle.

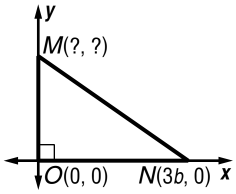
4.



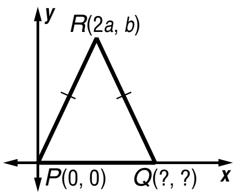
5.



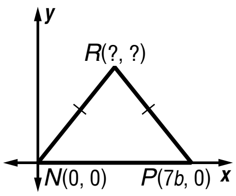
6.



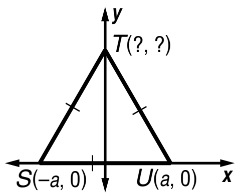
7.



8.



9.



10. Write a coordinate proof to prove that in an isosceles right triangle, the segment from the vertex of the right angle to the midpoint of the hypotenuse is perpendicular to the hypotenuse.

Given: isosceles right $\triangle ABC$ with $\angle ABC$ the right angle and M the midpoint of $\overline{AC}$

Prove: $\overline{BM} \perp \overline{AC}$

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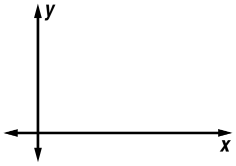
4-7 Practice

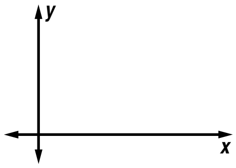
Triangles and Coordinate Proof

Position and label each triangle on the coordinate plane.

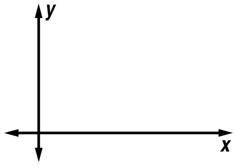
1. equilateral $\triangle SWY$ with sides $\frac{1}{4}a$ long



2. isosceles $\triangle BLP$ with base $\overline{BL}$ $3b$ units long

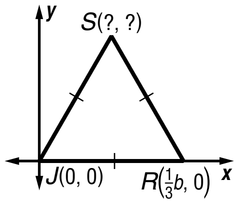


3. isosceles right $\triangle DGJ$ with hypotenuse $\overline{DJ}$ and legs $2a$ units long

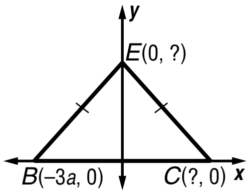


Find the missing coordinates of each triangle.

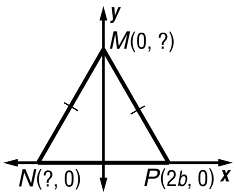
4.



5.



6.



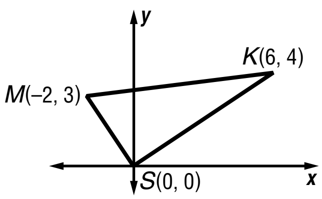
NEIGHBORHOODS For Exercises 7 and 8, use the following information.

Karina lives 6 miles east and 4 miles north of her high school. After school she works part time at the mall in a music store. The mall is 2 miles west and 3 miles north of the school.

7. Write a coordinate proof to prove that Karina's high school, her home, and the mall are at the vertices of a right triangle.

Given: $\triangle SKM$

Prove: $\triangle SKM$ is a right triangle.



8. Find the distance between the mall and Karina's home.

NAME ____ DATE ____________ PERIOD _____

4-7 Reading to Learn Mathematics

Triangles and Coordinate Proof

Pre-Activity **How can the coordinate plane be useful in proofs?**

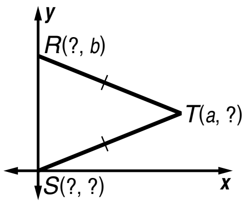
Read the introduction to Lesson 4-7 at the top of page 222 in your textbook.

From the coordinates of A, B, and C in the drawing in your textbook, what do you know about $\triangle ABC$?

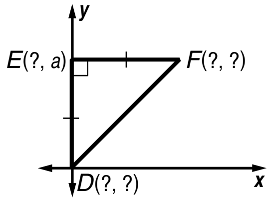
Reading the Lesson

1. Find the missing coordinates of each triangle.

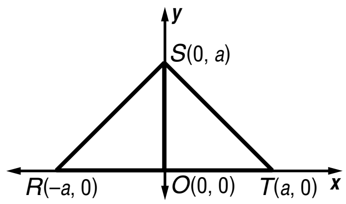
a.



b.



2. Refer to the figure.



a. Find the slope of $\overline{SR}$ and the slope of $\overline{ST}$.

b. Find the product of the slopes of $\overline{SR}$ and $\overline{ST}$. What does this tell you about $\overline{SR}$ and $\overline{ST}$?

c. What does your answer from part b tell you about $\triangle RST$?

d. Find SR and ST. What does this tell you about $\overline{SR}$ and $\overline{ST}$?

e. What does your answer from part d tell you about $\triangle RST$?

f. Combine your answers from parts c and e to describe $\triangle RST$ as completely as possible.

g. Find $m\angle SRT$ and $m\angle STR$.

h. Find $m\angle OSR$ and $m\angle OST$.

Helping You Remember

3. Many students find it easier to remember mathematical formulas if they can put them into words in a compact way. How can you use this approach to remember the slope and midpoint formulas easily?

Lesson 4-7

NAME ______________________________ DATE ____________ PERIOD _____

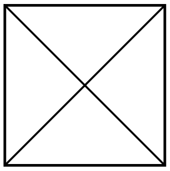
4-7 Enrichment

How Many Triangles?

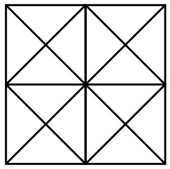
Each puzzle below contains many triangles. Count them carefully. Some triangles overlap other triangles.

How many triangles are there in each figure?

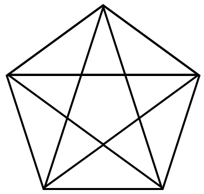
1.



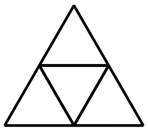
2.



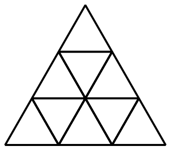
3.



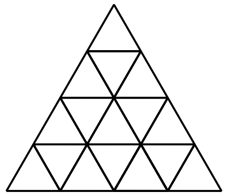
4.



5.

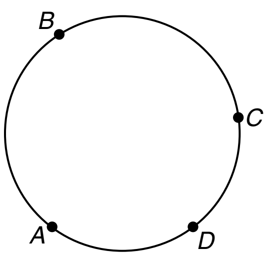


6.

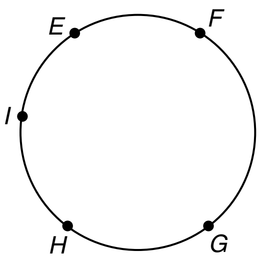


How many triangles can you form by joining points on each circle? List the vertices of each triangle.

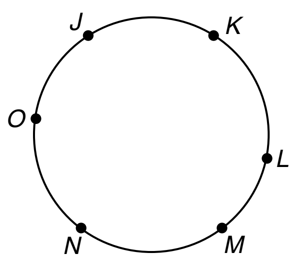
7.



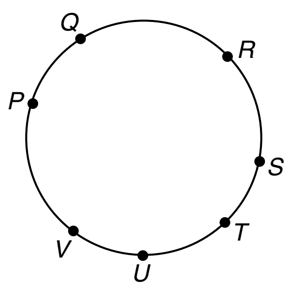
8.



8.



9.



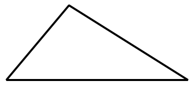
NAME ____________________ DATE __________ PERIOD ______

4 Chapter 4 Test, Form 1

SCORE ______

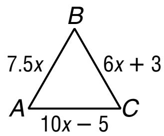
Write the letter for the correct answer in the blank at the right of each question.

1. How would this triangle be classified by angles? **1.** ______



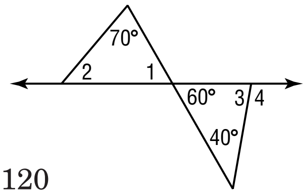
A. acute **B.** equiangular
C. obtuse **D.** right

2. What is the value of x if $\triangle ABC$ is equilateral? **2.** ______



A. -8 **B.** $-\frac{1}{8}$
C. $\frac{1}{2}$ **D.** 2

Use the figure for Questions 3–4 and write the letter for the correct answer in the blank at the right of each question.



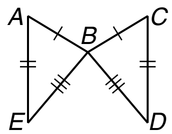
3. What is $m\angle 2$? **3.** ______

A. 50 **B.** 70 **C.** 110 **D.** 120

4. What is $m\angle 4$? **4.** ______

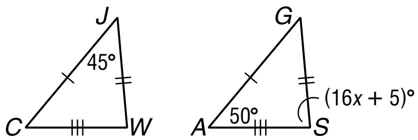
A. 10 **B.** 60 **C.** 100 **D.** 120

5. What are the congruent triangles in the diagram? **5.** ______



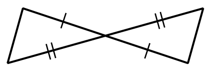
A. $\triangle ABC \cong \triangle EBD$ **B.** $\triangle ABE \cong \triangle CBD$
C. $\triangle AEB \cong \triangle CBD$ **D.** $\triangle ABE \cong \triangle CDB$

6. If $\triangle CJW \cong \triangle AGS$, $m\angle A = 50$, $m\angle J = 45$, and $m\angle S = 16x + 5$, what is x? **6.** ______



A. 17.5 **B.** 11.875
C. 6 **D.** 5

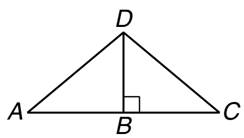
7. Which postulate can be used to prove the triangles congruent? **7.** ______



A. SSS **B.** SAS **C.** ASA **D.** AAS

8. What reason should be given for statement 5 in the proof? **8.** ______

Given: $\overline{DB}$ is the perpendicular bisector of $\overline{AC}$.
Prove: $\triangle ADB \cong \triangle CDB$



Statements	Reasons
1. DB is the perpendicular bisector of $\overline{AC}$.	1. Given
2. $\overline{AB} \cong \overline{CB}$	2. Midpoint Theorem
3. $\angle ABD \cong \angle CBD$	3. ⊥ line; all right ∠s are ≅.
4. $\overline{DB} \cong \overline{DB}$	4. Reflexive Property
5. $\triangle ADB \cong \triangle CDB$	5. __?__

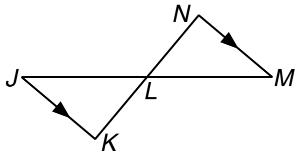
A. SSS **B.** AAS **C.** ASA **D.** SAS

Assessments

NAME ______________________ DATE __________ PERIOD ______

4 Chapter 4 Test, Form 1 *(continued)*

Use the proof for Questions 9–10 and write the letter for the correct answer in the blank at the right of each question.



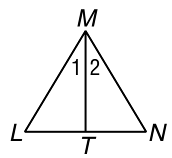
Given: L is the midpoint of $\overline{JM}$; $\overline{JK} \parallel \overline{NM}$.
Prove: $\triangle JKL \cong \triangle MNL$

Statements	Reasons
1. L is the midpoint of $\overline{JM}$.	1. Given
2. $\overline{JL} \cong \overline{ML}$	2. Definition of midpoint
3. $\overline{JK} \parallel \overline{MN}$	3. Given
4. $\angle JKL \cong \angle MNL$	4. Alt. int. ∡ are ≅.
5. $\angle JLK \cong \angle MLN$	5. (Question 9)
6. $\triangle JKL \cong \triangle MNL$	6. (Question 10)

9. What is the reason for $\angle JLK \cong \angle MLN$? **9.** ______
A. definition of midpoint
B. corresponding angles
C. vertical angles
D. alternate interior angles

10. What is the reason for $\triangle JKL \cong \triangle MNL$? **10.** ______
A. AAS **B.** ASA **C.** SAS **D.** SSS

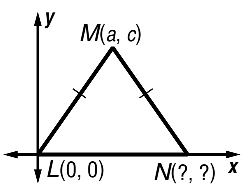
Use the figure for Questions 11–12 and write the letter for the correct answer in the blank at the right of each question.



11. If $\triangle LMN$ is isosceles and T is the midpoint of $\overline{LN}$, which postulate can be used to prove $\triangle MLT \cong \triangle MNT$? **11.** ______
A. AAA **B.** AAS **C.** SAS **D.** ABC

12. If $\triangle MLT \cong \triangle MNT$, what is used to prove $\angle 1 \cong \angle 2$? **12.** ______
A. CPCTC
B. definition of isosceles triangle
C. definition of perpendicular
D. definition of angle bisector

13. What are the missing coordinates of this triangle? **13.** ______
A. $(2a, 2c)$ **B.** $(2a, 0)$
C. $(0, 2a)$ **D.** $(a, 2c)$



Bonus What is the classification by sides of a triangle with coordinates $A(5, 0)$, $B(0, 5)$, and $C(-5, 0)$? **B:** ______________

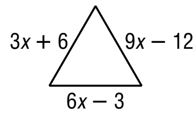
NAME ________________________ DATE ____________ PERIOD ______

4 Chapter 4 Test, Form 2A

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. What is the length of the sides of this equilateral triangle? **1.** ______

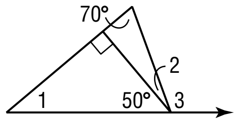


A. 42 **B.** 30
C. 15 **D.** 12

2. What is the classification of $\triangle ABC$ with vertices $A(4, 1)$, $B(2, -1)$, and $C(-2, -1)$ by its sides? **2.** ______

A. equilateral **B.** isosceles **C.** scalene **D.** right

Use the figure for Questions 3–4 and write the letter for the correct answer in the blank at the right of each question.



3. What is $m\angle 1$? **3.** ______

A. 40 **B.** 50 **C.** 70 **D.** 90

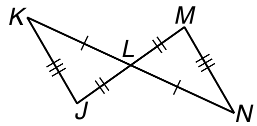
4. What is $m\angle 3$? **4.** ______

A. 40 **B.** 70 **C.** 90 **D.** 110

5. If $\triangle DJL \cong \triangle EGS$, which segment in $\triangle EGS$ corresponds to $\overline{DL}$? **5.** ______

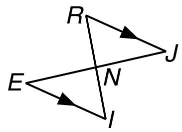
A. $\overline{EG}$ **B.** $\overline{ES}$ **C.** $\overline{GS}$ **D.** $\overline{GE}$

6. Which triangles are congruent in the figure? **6.** ______



A. $\triangle KLJ \cong \triangle MNL$ **B.** $\triangle JLK \cong \triangle NLM$
C. $\triangle JKL \cong \triangle LMN$ **D.** $\triangle JKL \cong \triangle MNL$

Use the proof for Questions 7–8 and write the letter for the correct answer in the blank at the right of each question.



Given: $\overline{RJ} \parallel \overline{EI}$; $\overline{RI}$ bisects $\overline{JE}$.
Prove: $\triangle RJN \cong \triangle IEN$

Statements	Reasons
1. $\overline{RJ} \parallel \overline{IE}$	1. Given
2. $\angle RJN \cong \angle IEN$	2. (Question 7)
3. $\overline{RI}$ bisects $\overline{JE}$.	3. Given
4. $\overline{JN} \cong \overline{EN}$	4. Definition of bisector
5. $\angle RNJ \cong \angle INE$	5. Vert. ∠s are ≅.
6. $\triangle RJN \cong \triangle IEN$	6. (Question 8)

7. What is the reason for statement 2 in the proof? **7.** ______

A. Isosceles Triangle Theorem **B.** same side interior angles
C. corresponding angles **D.** Alternate Interior Angle Theorem

8. What is the reason for statement 6? **8.** ______

A. ASA **B.** AAS **C.** SAS **D.** SSS

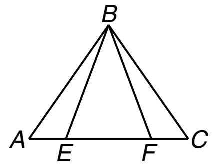
Assessments

4 Chapter 4 Test, Form 2A *(continued)*

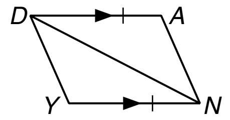
9. If $\triangle ABC$ is isosceles and $\overline{AE} \cong \overline{FC}$, which theorem or postulate can be used to prove $\triangle AEB \cong \triangle CFB$? **9.** ______

A. SSS **B.** SAS

C. ASA **D.** AAS



Use the proof for Questions 10–11 and write the letter for the correct answer in the blank at the right of each question.



Given: $\overline{DA} \parallel \overline{YN}$; $\overline{DA} \cong \overline{YN}$

Prove: $\angle NDY \cong \angle DNA$

Statements	Reasons
1. $\overline{DA} \parallel \overline{YN}$	1. Given
2. $\angle ADN \cong \angle YND$	2. Alt. int. $\angle$s are $\cong$.
3. $\overline{DA} \cong \overline{YN}$	3. Given
4. $\overline{DN} \cong \overline{DN}$	4. Reflexive Property
5. $\triangle NDY \cong \triangle DNA$	5. (Question 10)
6. $\angle NDY \cong \angle DNA$	6. (Question 11)

10. What is the reason for statement 5? **10.** ______

A. ASA **B.** AAS

C. SAS **D.** SSS

11. What is the reason for statement 6? **11.** ______

A. Alt. int. $\angle$s are $\cong$. **B.** CPCTC

C. Corr. angles are $\cong$. **D.** Isosceles Triangle Theorem

12. What is the classification of a triangle with vertices $A(3, 3)$, $B(6, -2)$, $C(0, -2)$ by its sides? **12.** ______

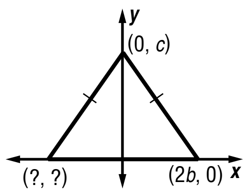
A. isosceles **B.** scalene

C. equilateral **D.** right

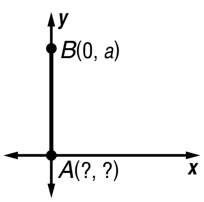
13. What are the missing coordinates of the triangle? **13.** ______

A. $(-2b, 0)$ **B.** $(0, 2b)$

C. $(-c, 0)$ **D.** $(0, -c)$



Bonus Name the coordinates of points A and C in isosceles right $\triangle ABC$ if point C is in the second quadrant. **B:** ____________________



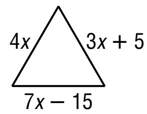
NAME ______________________ DATE ____________ PERIOD ______

4 Chapter 4 Test, Form 2B

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. What is the length of the sides of this equilateral triangle? **1.** ______

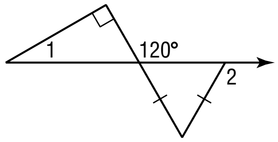


A. 2.5 **B.** 5
C. 15 **D.** 20

2. What is the classification of $\triangle ABC$ with vertices $A(0, 0)$, $B(4, 3)$, and $C(4, -3)$ by its sides? **2.** ______

A. equilateral **B.** isosceles **C.** scalene **D.** right

Use the figure for Questions 3–4 and write the letter for the correct answer in the blank at the right of each question.



3. What is $m\angle 1$? **3.** ______

A. 120 **B.** 90 **C.** 60 **D.** 30

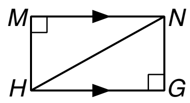
4. What is $m\angle 2$? **4.** ______

A. 120 **B.** 90 **C.** 60 **D.** 30

5. If $\triangle TGS \cong \triangle KEL$, which angle in $\triangle KEL$ corresponds to $\angle T$? **5.** ______

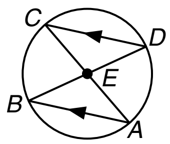
A. $\angle L$ **B.** $\angle E$ **C.** $\angle K$ **D.** $\angle A$

6. Which triangles are congruent in the figure? **6.** ______



A. $\triangle HMN \cong \triangle HGN$ **B.** $\triangle HMN \cong \triangle NGH$
C. $\triangle NMH \cong \triangle NGH$ **D.** $\triangle MNH \cong \triangle HGN$

Use the proof for Questions 7–8 and write the letter for the correct answer in the blank at the right of each question.



Given: $\overline{AB} \parallel \overline{CD}$; $\overline{AC}$ bisects $\overline{BD}$.
Prove: $\triangle ABE \cong \triangle CDE$

Statements	**Reasons**
1. $\overline{AC}$ bisects $\overline{BD}$.	1. Given
2. $\overline{BE} \cong \overline{DE}$	2. (Question 7)
3. $\overline{AB} \parallel \overline{CD}$	3. Given
4. $\angle ABE \cong \angle CDE$	4. Alt. int. ∠s are ≅.
5. (Question 8)	5.Vert. ∠s are ≅.
6. $\triangle ABE \cong \triangle CDE$	5. ASA

7. What is the reason for statement 2? **7.** ______

A. Definition of bisector **B.** Midpoint Theorem
C. Given **D.** Alternate Interior Angle Theorem

8. What is the statement for reason 5? **8.** ______

A. $\angle BEA \cong \angle DEC$ **B.** $\angle ABE \cong \angle CDE$
C. $\angle EAB \cong \angle ECD$ **D.** $\angle BEC \cong \angle DEA$

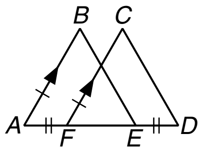
Assessments

NAME ______________________________ DATE ____________ PERIOD ______

4 Chapter 4 Test, Form 2B *(continued)*

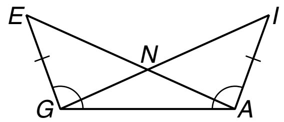
9. If $\overline{AF} \cong \overline{DE}$, $\overline{AB} \cong \overline{FC}$ and $\overline{AB} \parallel \overline{FC}$, which theorem or postulate can be used to prove $\triangle ABE \cong \triangle FCD$?

A. AAS **B.** ASA
C. SAS **D.** SSS



9. ______

Use the proof for Questions 10–11 and write the letter for the correct answer in the blank at the right of each question.



Given: $\overline{EG} \cong \overline{IA}$; $\angle EGA \cong \angle IAG$
Prove: $\angle GEN \cong \angle AIN$

Statements	**Reasons**
1. $\overline{EG} \cong \overline{IA}$	1. Given
2. $\angle EGA \cong \angle IAG$	2. Given
3. $\overline{GA} \cong \overline{GA}$	3. Reflexive Property
4. $\triangle EGA \cong \triangle IAG$	4. (Question 10)
5. $\angle GEN \cong \angle AIN$	5. (Question 11)

10. What is the reason for statement 4? **10.** ______

A. SSS **B.** ASA **C.** SAS **D.** AAS

11. What is the reason for statement 5? **11.** ______

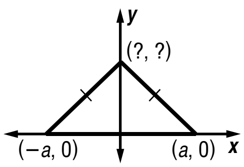
A. Alt. int. ∠s are ≅. **B.** Same Side Interior Angles
C. Corr. angles are ≅. **D.** CPCTC

12. What is the classification of a triangle with vertices $A(-3, -1)$, $B(-2, 2)$, $C(3, 1)$ by its sides? **12.** ______

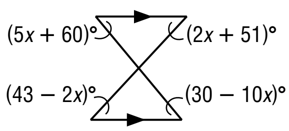
A. scalene **B.** isosceles
C. equilateral **D.** right

13. What are the missing coordinates of the triangle? **13.** ______

A. $(a, 0)$ **B.** $(b, 0)$
C. $(c, 0)$ **D.** $(0, c)$



Bonus Find x in the triangle.



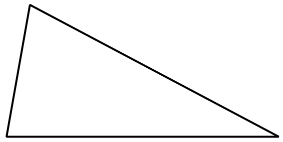
B: ____________________

NAME ______________________ DATE ____________ PERIOD ______

4 Chapter 4 Test, Form 2C

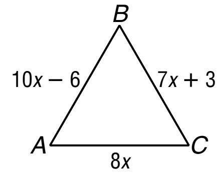
SCORE ______

1. Use a protractor and ruler to classify the triangle by its angles and sides.



1. ______________________

2. Find x, AB, BC, and AC if $\triangle ABC$ is equilateral.

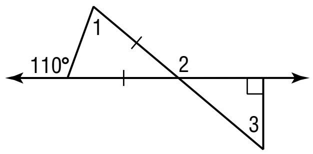


2. ______________________

3. Find the measure of the sides of the triangle if the vertices of $\triangle EFG$ are $E(-3, 3)$, $F(1, -1)$, and $G(-3, -5)$. Then classify the triangle by its sides.

3. ______________________

Find the measure of each angle.



4. $m\angle 1$

4. ______________________

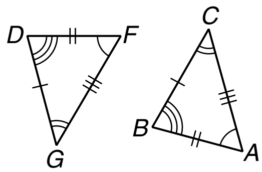
5. $m\angle 2$

5. ______________________

6. $m\angle 3$

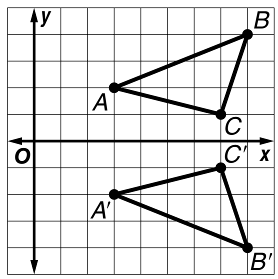
6. ______________________

7. Identify the congruent triangles and name their corresponding congruent angles.



7. ______________________

8. Verify that $\triangle ABC \cong \triangle A'B'C'$ preserves congruence, assuming that corresponding angles are congruent.

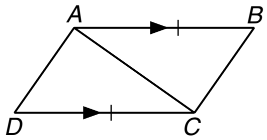


8. ______________________

NAME ____________________ DATE __________ PERIOD ______

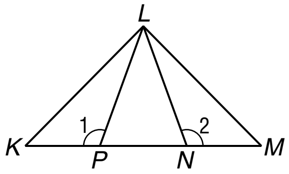
4 Chapter 4 Test, Form 2C *(continued)*

9. $ABCD$ is a quadrilateral with $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$. Name the postulate that could be used to prove $\triangle BAC \cong \triangle DCA$. Choose from SSS, SAS, ASA, and AAS.



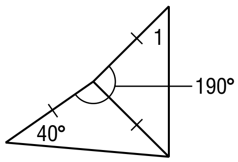
9. ____________________

10. $\triangle KLM$ is an isosceles triangle and $\angle 1 \cong \angle 2$. Name the theorem that could be used to determine $\angle LKP \cong \angle LMN$. Then name the postulate that could be used to prove $\triangle LKP \cong \triangle LMN$. Choose from SSS, SAS, ASA, and AAS.



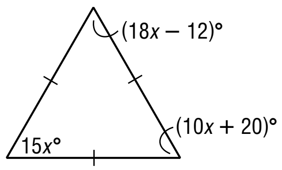
10. ____________________

11. Use the figure to find $m\angle 1$.



11. ____________________

12. Find x.

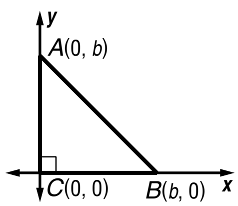


12. ____________________

13. Position and label isosceles $\triangle ABC$ with base $\overline{AB}$ b units long on the coordinate plane.

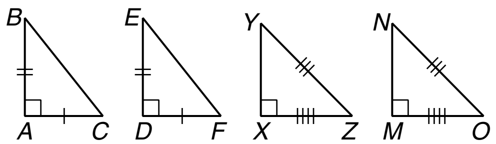
13. ____________________

14. $\overline{CP}$ joins point C in isosceles right $\triangle ABC$ to the midpoint P, of $\overline{AB}$. Name the coordinates of P. Then determine the relationship between $\overline{AB}$ and $\overline{CP}$.



14. ____________________

Bonus Without finding any other angles or sides congruent, which pair of triangles can be proved to be congruent by the HL Theorem?



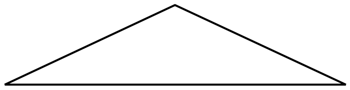
B: ____________________

NAME ______________________ DATE ____________ PERIOD ______

4 Chapter 4 Test, Form 2D

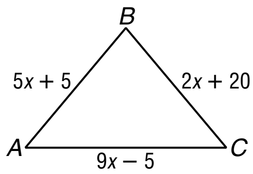
SCORE ______

1. Use a protractor and ruler to classify the triangle by its angles and sides.



1. ____________

2. Find x, AB, BC, AC if $\triangle ABC$ is isosceles.

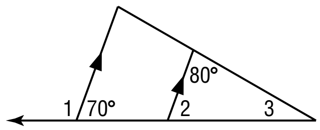


2. ____________

3. Find the measure of the sides of the triangle if the vertices of $\triangle EFG$ are $E(1, 4)$, $F(5, 1)$, and $G(2, -3)$. Then classify the triangle by its sides.

3. ____________

Find the measure of each angle.



4. $m\angle 1$

4. ____________

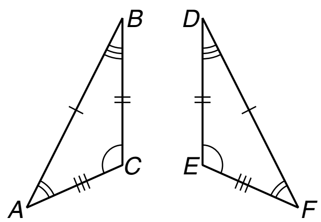
5. $m\angle 2$

5. ____________

6. $m\angle 3$

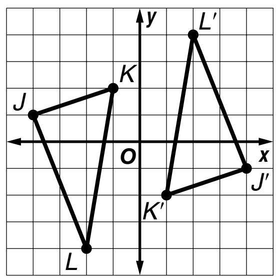
6. ____________

7. Identify the congruent triangles and name their corresponding congruent angles.



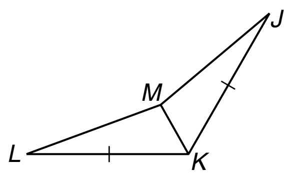
7. ____________

8. Verify that $\triangle JKL \cong \triangle J'K'L'$ preserves congruence, assuming that corresponding angles are congruent.



8. ____________

9. In quadrilateral $JKLM$, $\overline{JK} \cong \overline{LK}$ and $\overline{MK}$ bisects $\angle LKJ$. Name the postulate that could be used to prove $\triangle MKL \cong \triangle MKJ$. Choose from SSS, SAS, ASA, and AAS.



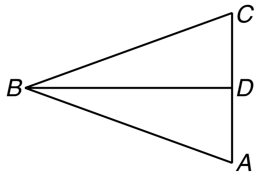
9. ____________

Assessments

NAME ______________________ DATE ____________ PERIOD ______

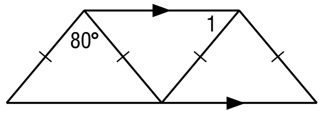
4 Chapter 4 Test, Form 2D *(continued)*

10. $\triangle ABC$ is an isosceles triangle with $\overline{BD} \perp \overline{AC}$. Name the theorem that could be used to determine $\angle A \cong \angle C$. Then name the postulate that could be used to prove $\triangle BDA \cong \triangle BDC$. Choose from SSS, SAS, ASA, and AAS.



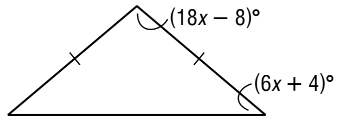
10. ______________

11. Use the figure to find $m\angle 1$.



11. ______________

12. Find x.

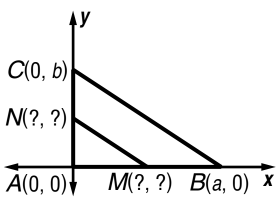


12. ______________

13. Position and label equilateral $\triangle KLM$ with side lengths $3a$ units long on the coordinate plane.

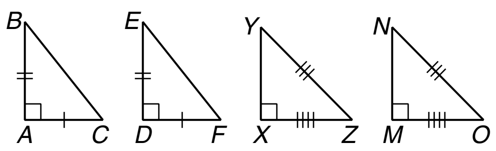
13. ______________

14. $\overline{MN}$ joins the midpoint of $\overline{AB}$ and the midpoint of $\overline{AC}$ in $\triangle ABC$. Find the coordinates of M and N, and the slopes of $\overline{MN}$ and $\overline{BC}$.



14. ______________

Bonus Without finding any other angles or sides congruent, which pair of triangles can be proved to be congruent by the LL Theorem?



B: ______________

NAME ________________ DATE ________ PERIOD ______

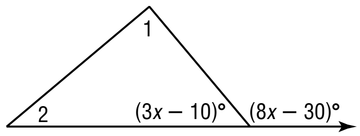
4 Chapter 4 Test, Form 3

SCORE ______

1. If $\triangle ABC$ is isosceles, $\angle B$ is the vertex angle, $AB = 20x - 2$, $BC = 12x + 30$, and $AC = 25x$, find x and the measure of each side of the triangle. **1.** ________

2. Given $A(0, 4)$, $B(5, 4)$, and $C(-3, -2)$, find the measure of the sides of the triangle. Then classify the triangle by its sides and angles. **2.** ________

Use the figure to answer Questions 3–5.



3. Find x. **3.** ________

4. $m\angle 1$, if $m\angle 1 = 4x + 10$. **4.** ________

5. $m\angle 2$ **5.** ________

6. Verify that the following preserves congruence, assuming that corresponding angles are congruent. $\triangle ABC$ is reflected over the x-axis as follows. **6.** ________

$A(-1, 1) \rightarrow A'(-1, -1)$

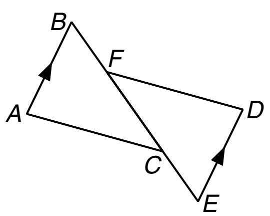
$B(4, 2) \rightarrow B'(4, -2)$

$C(1, 5) \rightarrow C'(1, -5)$

Verify $\triangle ABC \cong \triangle A'B'C'$.

7. Determine whether $\triangle GHI \cong \triangle JKL$, given $G(1, 2)$, $H(5, 4)$, $I(3, 6)$ and $J(-4, -5)$, $K(0, -3)$, $L(-2, -1)$. Explain. **7.** ________

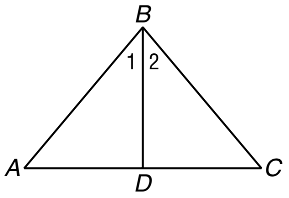
8. In the figure, $\overline{AC} \cong \overline{FD}$, $\overline{AB} \parallel \overline{DE}$, and $\overline{AC} \parallel \overline{FD}$. Name the postulate that could be used to prove $\triangle ABC \cong \triangle DEC$. Choose from SSS, SAS, ASA, and AAS. **8.** ________



Assessments

For Questions 9 and 10, complete this two-column proof.

Given: $\triangle ABC$ is an isosceles triangle with base $\overline{AC}$.
D is the midpoint of $\overline{AC}$.
Prove: $\overline{BD}$ bisects $\angle ABC$.

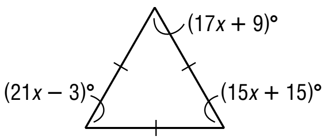


Statements	Reasons
1. $\triangle ABC$ is isosceles with base $\overline{AC}$.	1. Given
2. $\overline{AB} \cong \overline{CB}$	2. Def. of isosceles triangle.
3. $\angle A \cong \angle C$	3. (Question 9)
4. D is the midpoint of $\overline{AC}$.	4. Given
5. $\overline{AD} \cong \overline{CD}$	5. Midpoint Theorem
6. $\triangle ABD \cong \triangle CBD$	6. (Question 10)
7. $\angle 1 \cong \angle 2$	7. CPCTC
8. $\overline{BD}$ bisects $\angle ABC$.	8. Def. of angle bisector

9. ______________

10. ______________

11. Find x.

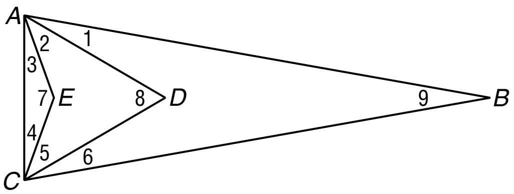


11. ______________

12. Position and label isosceles $\triangle ABC$ with base $\overline{AB}$ $(a + b)$ units long on a coordinate plane

12. ______________

Bonus In the figure, $\triangle ABC$ is isosceles, $\triangle ADC$ is equilateral, $\triangle AEC$ is isosceles, and the measures of $\angle 9$, $\angle 1$, and $\angle 3$ are all equal. Find the measures of the nine numbered angles.



B: ______________

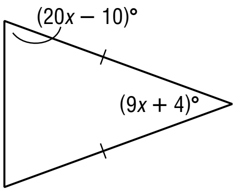
NAME ________________________ DATE ____________ PERIOD ______

4 Chapter 4 Open-Ended Assessment

SCORE ______

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1.



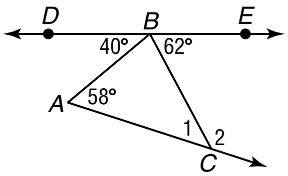
a. Classify the triangle by its angles and sides.

b. Show the steps needed to solve for x.

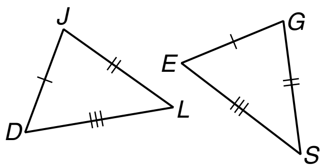
2. a. Describe how to determine whether a triangle with coordinates $A(1, 4)$, $B(1, -1)$, and $C(-4, 4)$ is an equilateral triangle.

b. Is the triangle equilateral? Explain.

3. Explain how to find $m\angle 1$ and $m\angle 2$ in the figure.



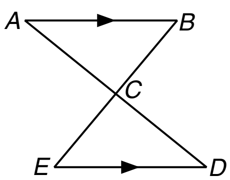
4.



a. State the theorem or postulate that can be used to prove that the triangles are congruent.

b. List their corresponding congruent angles and sides.

5.



Given: $\overline{AB} \parallel \overline{DE}$, $\overline{AD}$ bisects $\overline{BE}$.

Prove: $\triangle ABC \cong \triangle DEC$ by using the ASA postulate.

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4 Chapter 4 Vocabulary Test/Review

SCORE ______

acute triangle	coordinate proof	flow proof	remote interior angles
base angles	corollary	included angle	right triangle
congruence transformations	equiangular triangle	included side	scalene triangle
congruent triangles	equilateral triangle	isosceles triangle	vertex angle
	exterior angle	obtuse triangle	

Choose from the terms above to complete each sentence.

1. A triangle that is equilateral is also called a(n) ___?___. **1.** ____________

2. A(n) ___?___ has at least one obtuse angle. **2.** ____________

3. The sum of the ___?___ is equivalent to the exterior angle of a triangle. **3.** ____________

4. The ___?___ angles of an isosceles triangle are congruent. **4.** ____________

5. A triangle with different measures for each side is classified as a(n) ___?___. **5.** ____________

6. A ___?___ organizes a series of statements in logical order written in boxes and uses arrows to indicate the order of the statements. **6.** ____________

7. A triangle that is translated, reflected or rotated and preserves its shape, is said to be a(n) ___?___. **7.** ____________

8. The ASA postulate involves two corresponding angles and their corresponding ___?___. **8.** ____________

9. A ___?___ uses figures in the coordinate plane and algebra to prove geometric concepts. **9.** ____________

10. The ___?___ is formed by the congruent legs of an isosceles triangle. **10.** ____________

In your own words—

11. corollary **11.** ____________

12. congruent triangles **12.** ____________

13. acute triangle **13.** ____________

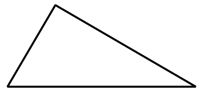
NAME ______________________ DATE __________ PERIOD ______

4 Chapter 4 Quiz

(Lessons 4–1 and 4–2)

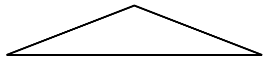
SCORE ______

1. Use a protractor to classify the triangle by its angles and sides.



1. ______________

2. **STANDARDIZED TEST PRACTICE** What is the best classification of this triangle by its angles and sides?



A. acute isosceles **B.** right isosceles
C. obtuse isosceles **D.** obtuse equilateral

2. ______________

3. If $\triangle ABC$ is an isosceles triangle, $\angle B$ is the vertex angle, $AB = 6x + 3$, $BC = 8x - 1$, and $AC = 10x - 10$, find x and the measures of each side of the triangle.

3. ______________

4. If $A(1, 5)$, $B(3, -2)$, and $C(-3, 0)$, find the measures of the sides of $\triangle ABC$. Then classify the triangle by its sides.

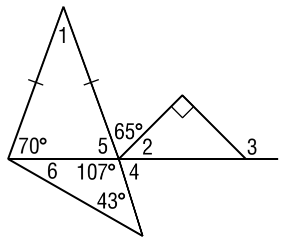
4. ______________

Find the measure of each angle in the figure.

5. $m\angle 1$ **6.** $m\angle 2$

7. $m\angle 3$ **8.** $m\angle 4$

9. $m\angle 5$ **10.** $m\angle 6$



5. ______________
6. ______________
7. ______________
8. ______________
9. ______________
10. ______________

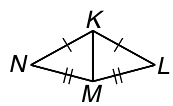
NAME ______________________ DATE __________ PERIOD ______

4 Chapter 4 Quiz

(Lessons 4–3 and 4–4)

SCORE ______

1. Identify the congruent triangles in the figure.



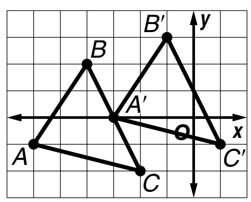
1. ______________

2. **STANDARDIZED TEST PRACTICE** If $\triangle JGO \cong \triangle RWI$, which angle corresponds to $\angle I$?

A. $\angle J$ **B.** $\angle R$ **C.** $\angle G$ **D.** $\angle O$

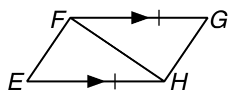
2. ______________

3. Verify that the following preserves congruence assuming that corresponding angles are congruent. $\triangle ABC \cong \triangle A'B'C'$



3. ______________

4. In quadrilateral $EFGH$, $\overline{FG} \cong \overline{HE}$, and $\overline{FG} \parallel \overline{HE}$. Name the postulate that could be used to prove $\triangle EHF \cong \triangle GFH$. Choose from SSS, SAS, ASA, and AAS.



4. ______________

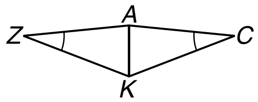
NAME ______________________ DATE ____________ PERIOD ______

4 Chapter 4 Quiz

SCORE ______

(Lessons 4–5 and 4–6)

For Questions 1 and 2, complete the two-column proof by supplying the missing information for each corresponding location.



Given: $\angle Z \cong \angle C$; $\overline{AK}$ bisects $\angle ZKC$.

Prove: $\triangle AKZ \cong \triangle AKC$

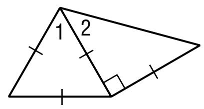
Statements	Reasons
1. $\angle Z \cong \angle C$; $\overline{AK}$ bisects $\angle ZKC$.	1. Given
2. $\angle ZKA \cong \angle CKA$	2. (Question 1)
3. $\overline{AK} \cong \overline{AK}$	3. Reflexive Property
4. $\triangle AKZ \cong \triangle AKC$	4. (Question 2)

1. ______________

2. ______________

Refer to the figure for Questions 3 and 4.

3. Find $m\angle 1$. **4.** Find $m\angle 2$.



3. ______________

4. ______________

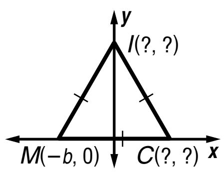
NAME ______________________ DATE ____________ PERIOD ______

4 Chapter 4 Quiz

SCORE ______

(Lesson 4–7)

1. Find the missing coordinates.



1. ______________

Position and label each triangle on a coordinate plane.

2. Right $\triangle DJL$ with hypotenuse $\overline{DJ}$; $LJ = \frac{1}{2}DL$ and $\overline{DL}$ is a units long.

2. ______________

3. isosceles $\triangle EGS$ with base $\overline{ES}$ $\frac{1}{2}b$ units long

3. ______________

For Questions 4 and 5, complete the coordinate proof by supplying the missing information for each corresponding location.

Given: $\triangle ABC$ with $A(-1, 1)$, $B(5, 1)$, and $C(2, 6)$.

Prove: $\triangle ABC$ is isosceles.

By the Distance Formula the lengths of the three sides are as follows: (Question 4). Since (Question 5), $\triangle ABC$ is isosceles.

4. ______________

5. ______________

NAME ________________ DATE ________ PERIOD ______

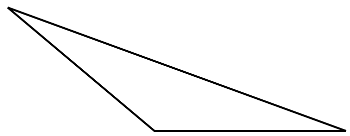
Chapter 4 Mid-Chapter Test

SCORE ______

(Lessons 4–1 through 4–3)

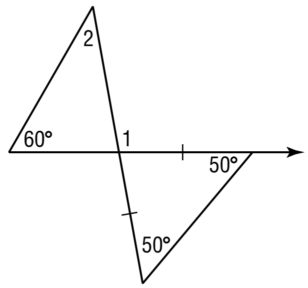
Part I ***Write the letter for the correct answer in the blank at the right of each question.***

1. What is the best classification for this triangle? **1.** ______



A. acute scalene
B. obtuse equilateral
C. acute isosceles
D. obtuse isosceles

Find the missing angle measures.



2. What is $m\angle 1$? **2.** ______

A. 50 **B.** 60
C. 100 **D.** 105

3. What is $m\angle 2$? **3.** ______

A. 40 **B.** 50
C. 60 **D.** 100

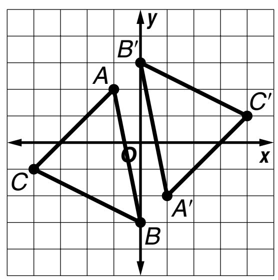
4. If $\triangle SJL \cong \triangle DMT$, which segment in $\triangle DMT$ corresponds to $\overline{LS}$ in $\triangle SJL$? **4.** ______

A. $\overline{DT}$ **B.** $\overline{TD}$
C. $\overline{MD}$ **D.** $\overline{MT}$

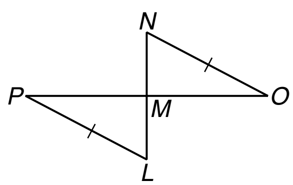
Part II

5. Find the measures of the sides of $\triangle ABC$ and classify it by its sides. $A(1, 3)$, $B(5, -2)$, and $C(0, -4)$ **5.** ________________

6. In $\triangle ABC$ and $\triangle A'B'C'$, $\angle A \cong \angle A'$, $\angle B \cong \angle B'$, and $\angle C \cong \angle C'$. Find the lengths needed to prove $\triangle ABC \cong \triangle A'B'C'$. **6.** ________________



7. What information would you need to know about $\overline{PO}$ and $\overline{LN}$ for $\triangle LMP$ to be congruent to $\triangle NMO$ by SSS? **7.** ________________



Assessments

NAME ______________________ DATE ____________ PERIOD ______

4 Chapter 4 Cumulative Review

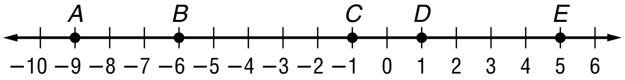
SCORE ______

(Chapters 1–4)

1. Name the geometric figure that is modeled by the second hand of a clock. (Lesson 1-1) **1.** ____________

2. Find the precision for a measurement of 36 inches. (Lesson 1-2) **2.** ____________

For Questions 3–5, use the number line.



3. Find BC. (Lesson 1-3) **3.** ____________

4. Find the coordinate of the midpoint of $\overline{AD}$. (Lesson 1-3) **4.** ____________

5. If B is the midpoint of a segment having one endpoint at E, what is the coordinate of its other endpoint? (Lesson 1-3) **5.** ____________

For Questions 6 and 7, determine whether each statement is *always*, *sometimes*, or *never* true. Explain your answer. (Lesson 2-5)

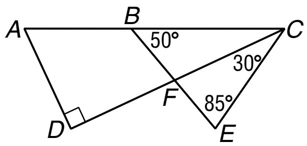
6. If $\overline{DE} \cong \overline{EF}$, then E is the midpoint of $\overline{DF}$. **6.** ____________

7. If points A and B lie in plane Q, then $\overleftrightarrow{AB}$ lies in Q. **7.** ____________

8. Find the slope of a line parallel to $x = 2$. (Lesson 3-3) **8.** ____________

9. Find the distance between $y = -9$ and $y = -5$. (Lesson 3-6) **9.** ____________

For Questions 10–12, use the figure.



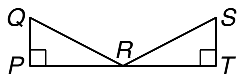
10. Name the segment that represents the distance from F to $\overleftrightarrow{AD}$. (Lesson 3-6) **10.** ____________

11. Classify $\triangle ADC$. (Lesson 4-1) **11.** ____________

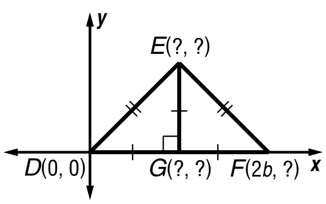
12. Find $m\angle ACD$. (Lesson 4-2) **12.** ____________

13. Name the corresponding congruent angles and sides for $\triangle PQR \cong \triangle HGB$. (Lesson 4-3) **13.** ____________

14. If $\angle QRP \cong \angle SRT$, and R is the midpoint of $\overline{PT}$, which theorem or postulate can be used to prove $\triangle QRP \cong \triangle SRT$? Choose from SSS, SAS, ASA, and AAS. (Lesson 4-5) **14.** ____________



15. Name the missing coordinates of $\triangle GEF$. (Lesson 4-7) **15.** ____________



NAME ______________________ DATE ____________ PERIOD ______

4 Standardized Test Practice

SCORE ______

(Chapters 1–4)

Part 1: Multiple Choice

Instructions: Fill in the appropriate oval for the best answer.

1. If $m\angle 1 = 5x - 4$, and $m\angle 2 = 52 - 9y$, which values for x and y would make $\angle 1$ and $\angle 2$ complementary? (Lesson 1-5)

A. $x = 2, y = 12$ **B.** $x = 12, y = 2$

C. $x = 27, y = \frac{1}{3}$ **D.** $x = \frac{1}{3}, y = 27$

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. Which is *not* a polygon? (Lesson 1-6)

E.  **F.**  **G.**  **H.** 

2. Ⓔ Ⓕ Ⓖ Ⓗ

3. Complete the statement so that its conditional *and* its converse are true.

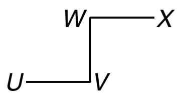
If $\angle 1 \cong \angle 2$, then $\angle 1$ and $\angle 2$ ___?___. (Lesson 2-3)

A. are supplementary. **B.** are complementary.

C. have the same measure. **D.** are alternate interior angles.

3. Ⓐ Ⓑ Ⓒ Ⓓ

4. Complete this proof. (Lesson 2-7)



Given: $\overline{UV} \cong \overline{VW}$

$\overline{VW} \cong \overline{WX}$

Prove: $UV = WX$

Proof:

Statements	**Reasons**
1. $\overline{UV} \cong \overline{VW}$; $\overline{VW} \cong \overline{WX}$	**1.** Given
2. $UV = VW$; $VW = WX$	**2.** ___?___
3. $UV = WX$	**3.** Transitive Property

E. Definition of congruent segments

F. Substitution Property

G. Segment Addition Postulate

H. Symmetric Property

4. Ⓔ Ⓕ Ⓖ Ⓗ

5. Which equation has a slope of $\frac{1}{3}$ and a y-intercept of -2? (Lesson 3-4)

A. $y = \frac{1}{3}x + 2$ **B.** $y = \frac{1}{3}x - 2$

C. $y = 2x - \frac{1}{3}$ **D.** $y = -2x + \frac{1}{3}$

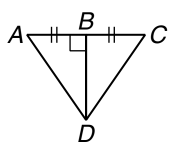
5. Ⓐ Ⓑ Ⓒ Ⓓ

6. Classify $\triangle DEF$ with vertices $D(2, 3)$, $E(5, 7)$ and $F(9, 4)$. (Lesson 4-1)

E. acute **F.** equiangular **G.** obtuse **H.** right

6. Ⓔ Ⓕ Ⓖ Ⓗ

7. Which postulate or theorem can be used to prove $\triangle ABD \cong \triangle CBD$? (Lesson 4-4)



A. SAS **B.** SSS

C. ASA **D.** AAS

7. Ⓐ Ⓑ Ⓒ Ⓓ

Assessments

NAME ______________________ DATE ____________ PERIOD ______

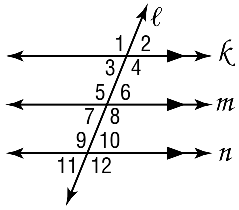
4 Standardized Test Practice *(continued)*

Part 2: Grid In

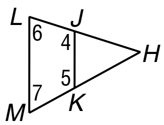
Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate oval that corresponds to that entry.

8. What is the y-coordinate of the midpoint of $A(12, 6)$ and $B(-15, -6)$? (Lesson 1-3)

9. If $m\angle 1 = 112$, find $m\angle 10$. (Lesson 3-2)

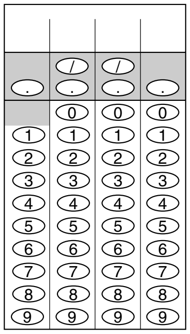


10. If $\overline{JK} \parallel \overline{LM}$, then $\angle 4$ must be supplementary to $\angle$ ___?___. (Lesson 3-5)

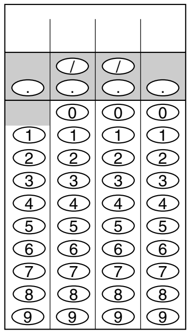


11. Find PR if $\triangle PQR$ is isosceles, $\angle Q$ is the vertex angle, $PQ = 4x - 8$, $QR = x + 7$, and $PR = 6x - 12$. (Lesson 4-1)

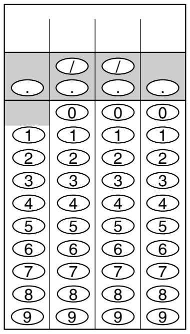
8.



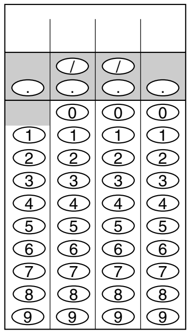
9.



10.



11.



Part 3: Short Response

Instructions: Show your work or explain in words how you found your answer.

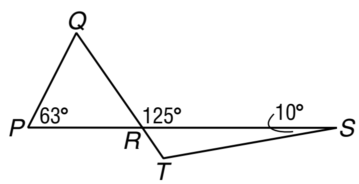
12. The perimeter of a regular pentagon is 14.5 feet. If each side length of the pentagon is doubled, what is the new perimeter? (Lesson 1-6)

12. ____________________

13. Make a conjecture about the next number in the sequence 5, 7, 11, 17, 25. (Lesson 2-1)

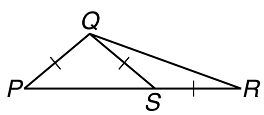
13. ____________________

14. Find $m\angle PQR$. (Lesson 4-2)



14. ____________________

15. If $PQ = QS$, $QS = SR$, and $m\angle R = 20$, find $m\angle PSQ$. (Lesson 4-6)



15. ____________________

NAME ________________ DATE ________ PERIOD ______

4

Standardized Test Practice

Student Record Sheet *(Use with pages 232–233 of the Student Edition.)*

Part 1 *Multiple Choice*

Select the best answer from the choices given and fill in the corresponding oval.

1 (A) (B) (C) (D)

2 (A) (B) (C) (D)

3 (A) (B) (C) (D)

4 (A) (B) (C) (D)

5 (A) (B) (C) (D)

6 (A) (B) (C) (D)

7 (A) (B) (C) (D)

8 (A) (B) (C) (D)

Part 2 *Short Response/Grid In*

Solve the problem and write your answer in the blank.

For Questions 12 and 14, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

9 ____________

10 ____________

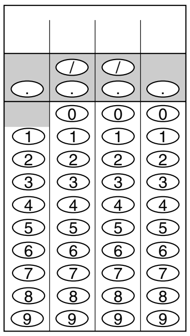
11 ____________

12 ____________ (grid in)

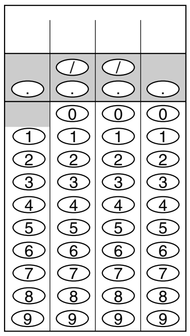
13 ____________

14 ____________ (grid in)

12



14



Part 3 *Open-Ended*

Record your answers for Questions 15–16 on the back of this paper.

Answers

NAME ______________________ DATE __________ PERIOD _____

4-1 Study Guide and Intervention

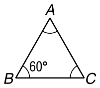
Classifying Triangles

Classify Triangles by Angles One way to classify a triangle is by the measures of its angles.

• If *one* of the angles of a triangle is an obtuse angle, then the triangle is an **obtuse triangle**.
• If *one* of the angles of a triangle is a right angle, then the triangle is a **right triangle**.
• If *all three* of the angles of a triangle are acute angles, then the triangle is an **acute triangle**.
• If all three angles of an acute triangle are congruent, then the triangle is an **equiangular triangle**.

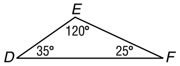
Example **Classify each triangle.**

a.



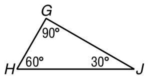
All three angles are congruent, so all three angles have measure 60°. The triangle is an equiangular triangle.

b.



The triangle has one angle that is obtuse. It is an obtuse triangle.

c.

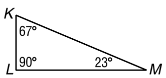


The triangle has one right angle. It is a right triangle.

Exercises

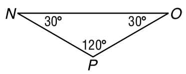
Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

1.



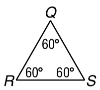
right

2.



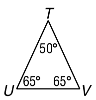
obtuse

3.



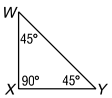
equiangular

4.



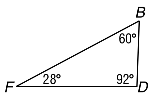
acute

5.



right

6.



obtuse

Lesson 4-1

NAME ______________________ DATE __________ PERIOD _____

4-1 Study Guide and Intervention *(continued)*

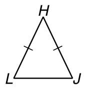
Classifying Triangles

Classify Triangles by Sides You can classify a triangle by the measures of its sides. Equal numbers of hash marks indicate congruent sides.

• If *all three* sides of a triangle are congruent, then the triangle is an **equilateral triangle**.
• If *at least two* sides of a triangle are congruent, then the triangle is an **isosceles triangle**.
• If *no two* sides of a triangle are congruent, then the triangle is a **scalene triangle**.

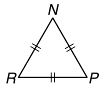
Example **Classify each triangle.**

a.



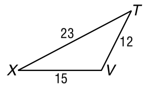
Two sides are congruent. The triangle is an isosceles triangle.

b.



All three sides are congruent. The triangle is an equilateral triangle.

c.

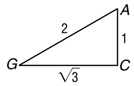


The triangle has no pair of congruent sides. It is a scalene triangle.

Exercises

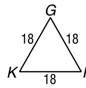
Classify each triangle as *equilateral*, *isosceles*, or *scalene*.

1.



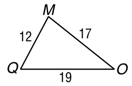
scalene

2.



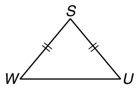
equilateral

3.



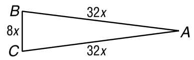
scalene

4.



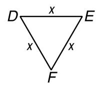
isosceles

5.



isosceles

6.



equilateral

7. Find the measure of each side of equilateral $\triangle RST$ with $RS = 2x + 2$, $ST = 3x$, and $TR = 5x - 4$. **2**

8. Find the measure of each side of isosceles $\triangle ABC$ with $AB = BC$ if $AB = 4y$, $BC = 3y + 2$, and $AC = 3y$.

***AB* = *BC* = 8, *AC* = 6**

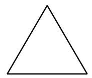
9. Find the measure of each side of $\triangle ABC$ with vertices $A(-1, 5)$, $B(6, 1)$, and $C(2, -6)$. Classify the triangle.

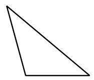
$AB = BC = \sqrt{65}$, $AC = \sqrt{130}$; $\triangle ABC$ is isosceles.

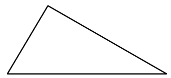
NAME ______________________ DATE __________ PERIOD _____

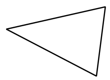
4-1 Skills Practice
Classifying Triangles

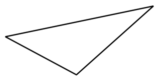
Use a protractor to classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

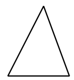
1. 
equiangular

2. 
obtuse

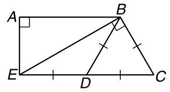
3. 
right

4. 
acute

5. 
obtuse

6. 
acute

Identify the indicated type of triangles.



7. right
△ABE, △BCE

8. isosceles
△BCD, △BDE

9. scalene
△ABE, △BCE

10. obtuse
△BDE

ALGEBRA Find x and the measure of each side of the triangle.

11. $\triangle ABC$ is equilateral with $AB = 3x - 2$, $BC = 2x + 4$, and $CA = x + 10$.
$x = 6$, $AB = 16$, $BC = 16$, $CA = 16$

12. $\triangle DEF$ is isosceles, $\angle D$ is the vertex angle, $DE = x + 7$, $DF = 3x - 1$, and $EF = 2x + 5$.
$x = 4$, $DE = 11$, $DF = 11$, $EF = 13$

Find the measures of the sides of $\triangle RST$ and classify each triangle by its sides.

13. $R(0, 2)$, $S(2, 5)$, $T(4, 2)$
$RS = \sqrt{13}$, $ST = \sqrt{13}$, $RT = 4$; isosceles

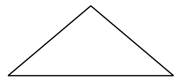
14. $R(1, 3)$, $S(4, 7)$, $T(5, 4)$
$RS = 5$, $ST = \sqrt{10}$, $RT = \sqrt{17}$; scalene

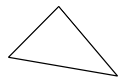
Lesson 4-1

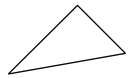
NAME ______________________ DATE __________ PERIOD _____

4-1 Practice (Average)
Classifying Triangles

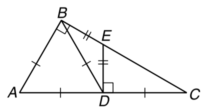
Use a protractor to classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

1. 
obtuse

2. 
acute

3. 
right

Identify the indicated type of triangles if $\overline{AB} \cong \overline{AD} \cong \overline{BD} \cong \overline{DC}$, $\overline{BE} \cong \overline{ED}$, $\overline{AB} \perp \overline{BC}$, and $\overline{ED} \perp \overline{DC}$.



4. right
△ABC, △CDE

5. obtuse
△BED, △BDC

6. scalene
△ABC, △CDE

7. isosceles
△ABD, △BED, △BDC

ALGEBRA Find x and the measure of each side of the triangle.

8. $\triangle FGH$ is equilateral with $FG = x + 5$, $GH = 3x - 9$, and $FH = 2x - 2$.
$x = 7$, $FG = 12$, $GH = 12$, $FH = 12$

9. $\triangle LMN$ is isosceles, $\angle L$ is the vertex angle, $LM = 3x - 2$, $LN = 2x + 1$, and $MN = 5x - 2$.
$x = 3$, $LM = 7$, $LN = 7$, $MN = 13$

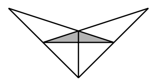
Find the measures of the sides of $\triangle KPL$ and classify each triangle by its sides.

10. $K(-3, 2)$ $P(2, 1)$, $L(-2, -3)$
$KP = \sqrt{26}$, $PL = 4\sqrt{2}$, $LK = \sqrt{26}$; isosceles

11. $K(5, -3)$, $P(3, 4)$, $L(-1, 1)$
$KP = \sqrt{53}$, $PL = 5$, $LK = 2\sqrt{13}$; scalene

12. $K(-2, -6)$, $P(-4, 0)$, $L(3, -1)$
$KP = 2\sqrt{10}$, $PL = 5\sqrt{2}$, $LK = 5\sqrt{2}$; isosceles

13. DESIGN Diana entered the design at the right in a logo contest sponsored by a wildlife environmental group. Use a protractor. How many right angles are there? **5**



Answers

NAME ______________ DATE ________ PERIOD _____

4-1 Reading to Learn Mathematics

Classifying Triangles

Pre-Activity **Why are triangles important in construction?**

Read the introduction to Lesson 4-1 at the top of page 178 in your textbook.

- Why are triangles used for braces in construction rather than other shapes? **Sample answer: Triangles lie in a plane and are rigid shapes.**
- Why do you think that isosceles triangles are used more often than scalene triangles in construction? **Sample answer: Isosceles triangles are symmetrical.**

Reading the Lesson

1. Supply the correct numbers to complete each sentence.

a. In an obtuse triangle, there are __2__ acute angle(s), __0__ right angle(s), and __1__ obtuse angle(s).

b. In an acute triangle, there are __3__ acute angle(s), __0__ right angle(s), and __0__ obtuse angle(s).

c. In a right triangle, there are __2__ acute angle(s), __1__ right angle(s), and __0__ obtuse angle(s).

2. Determine whether each statement is *always*, *sometimes*, or *never* true.

a. A right triangle is scalene. **sometimes**

b. An obtuse triangle is isosceles. **sometimes**

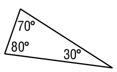
c. An equilateral triangle is a right triangle. **never**

d. An equilateral triangle is isosceles. **always**

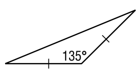
e. An acute triangle is isosceles. **sometimes**

f. A scalene triangle is obtuse. **sometimes**

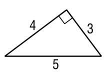
3. Describe each triangle by as many of the following words as apply: *acute*, *obtuse*, *right*, *scalene*, *isosceles*, or *equilateral*.

a. 

acute, scalene

b. 

obtuse, isosceles

c. 

right, scalene

Helping You Remember

4. A good way to remember a new mathematical term is to relate it to a nonmathematical definition of the same word. How is the use of the word *acute*, when used to describe *acute pain*, related to the use of the word *acute* when used to describe an *acute angle* or an *acute triangle*? **Sample answer: Both are related to the meaning of *acute* as *sharp*. An *acute pain* is a sharp pain, and an *acute angle* can be thought of as an angle with a sharp point. In an *acute triangle* all of the angles are acute.**

Lesson 4-1

NAME ______________ DATE ________ PERIOD _____

4-1 Enrichment

Reading Mathematics

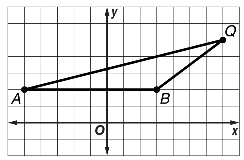
When you read geometry, you may need to draw a diagram to make the text easier to understand.

Example **Consider three points, *A*, *B*, and *C* on a coordinate grid. The *y*-coordinates of *A* and *B* are the same. The *x*-coordinate of *B* is greater than the *x*-coordinate of *A*. Both coordinates of *C* are greater than the corresponding coordinates of *B*. Is triangle *ABC* acute, right, or obtuse?**

To answer this question, first draw a sample triangle that fits the description.

Side *AB* must be a horizontal segment because the *y*-coordinates are the same. Point *C* must be located to the right and up from point *B*.

From the diagram you can see that triangle *ABC* must be obtuse.



Answer each question. Draw a simple triangle on the grid above to help you.

1. Consider three points, *R*, *S*, and *T* on a coordinate grid. The *x*-coordinates of *R* and *S* are the same. The *y*-coordinate of *T* is between the *y*-coordinates of *R* and *S*. The *x*-coordinate of *T* is less than the *x*-coordinate of *R*. Is angle *R* of triangle *RST* acute, right, or obtuse? **acute**

2. Consider three noncollinear points, *J*, *K*, and *L* on a coordinate grid. The *y*-coordinates of *J* and *K* are the same. The *x*-coordinates of *K* and *L* are the same. Is triangle *JKL* acute, right, or obtuse? **right**

3. Consider three noncollinear points, *D*, *E*, and *F* on a coordinate grid. The *x*-coordinates of *D* and *E* are opposites. The *y*-coordinates of *D* and *E* are the same. The *x*-coordinate of *F* is 0. What kind of triangle must $\triangle DEF$ be: scalene, isosceles, or equilateral? **isosceles**

4. Consider three points, *G*, *H*, and *I* on a coordinate grid. Points *G* and *H* are on the positive *y*-axis, and the *y*-coordinate of *G* is twice the *y*-coordinate of *H*. Point *I* is on the positive *x*-axis, and the *x*-coordinate of *I* is greater than the *y*-coordinate of *G*. Is triangle *GHI* scalene, isosceles, or equilateral? **scalene**

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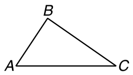
4-2 Study Guide and Intervention

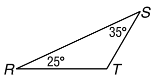
Angles of Triangles

Angle Sum Theorem If the measures of two angles of a triangle are known, the measure of the third angle can always be found.

Angle Sum Theorem	The sum of the measures of the angles of a triangle is 180. In the figure at the right, $m\angle A + m\angle B + m\angle C = 180$.

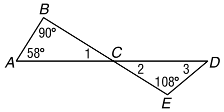


Example 1 **Find $m\angle T$.**



$m\angle R + m\angle S + m\angle T = 180$	Angle Sum Theorem
$25 + 35 + m\angle T = 180$	Substitution
$60 + m\angle T = 180$	Add.
$m\angle T = 120$	Subtract 60 from each side.

Example 2 **Find the missing angle measures.**

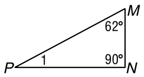


$m\angle 1 + m\angle A + m\angle B = 180$	Angle Sum Theorem
$m\angle 1 + 58 + 90 = 180$	Substitution
$m\angle 1 + 148 = 180$	Add.
$m\angle 1 = 32$	Subtract 148 from each side.
$m\angle 2 = 32$	Vertical angles are congruent.
$m\angle 3 + m\angle 2 + m\angle E = 180$	Angle Sum Theorem
$m\angle 3 + 32 + 108 = 180$	Substitution
$m\angle 3 + 140 = 180$	Add.
$m\angle 3 = 40$	Subtract 140 from each side.

Exercises

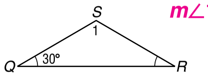
Find the measure of each numbered angle.

1.



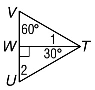
$m\angle 1 = 28$

2.



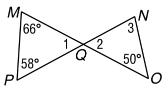
$m\angle 1 = 120$

3.



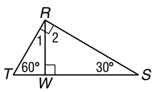
$m\angle 1 = 30$, $m\angle 2 = 60$

4.



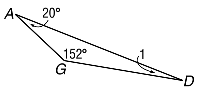
$m\angle 1 = 56$, $m\angle 2 = 56$, $m\angle 3 = 74$

5.



$m\angle 1 = 30$, $m\angle 2 = 60$

6.



$m\angle 1 = 8$

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Lesson 4-2

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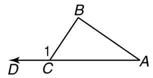
4-2 Study Guide and Intervention *(continued)*

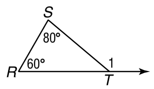
Angles of Triangles

Exterior Angle Theorem At each vertex of a triangle, the angle formed by one side and an extension of the other side is called an **exterior angle** of the triangle. For each exterior angle of a triangle, the **remote interior angles** are the interior angles that are not adjacent to that exterior angle. In the diagram below, $\angle B$ and $\angle A$ are the remote interior angles for exterior $\angle DCB$.

Exterior Angle Theorem	The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles. $m\angle 1 = m\angle A + m\angle B$

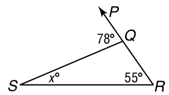


Example 1 **Find $m\angle 1$.**



$m\angle 1 = m\angle R + m\angle S$	Exterior Angle Theorem
$= 60 + 80$	Substitution
$= 140$	Add.

Example 2 **Find x.**

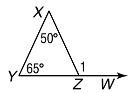


$m\angle PQS = m\angle R + m\angle S$	Exterior Angle Theorem
$78 = 55 + x$	Substitution
$23 = x$	Subtract 55 from each side.

Exercises

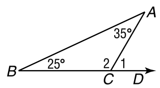
Find the measure of each numbered angle.

1.



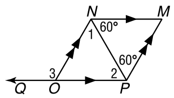
$m\angle 1 = 115$

2.



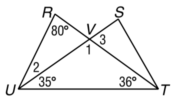
$m\angle 1 = 60$, $m\angle 2 = 120$

3.



$m\angle 1 = 60$, $m\angle 2 = 60$, $m\angle 3 = 120$

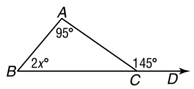
4.



$m\angle 1 = 109$, $m\angle 2 = 29$, $m\angle 3 = 71$

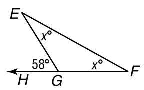
Find x.

5.



25

6.



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Answers

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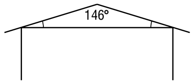
4-2 Skills Practice

Angles of Triangles

Find the missing angle measures.

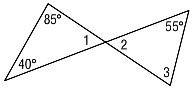
1. 

27

2. 

17, 17

Find the measure of each angle.

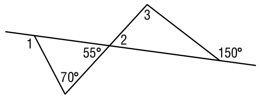


3. $m\angle 1$ 55

4. $m\angle 2$ 55

5. $m\angle 3$ 70

Find the measure of each angle.

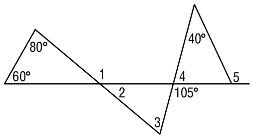


6. $m\angle 1$ 125

7. $m\angle 2$ 55

8. $m\angle 3$ 95

Find the measure of each angle.



9. $m\angle 1$ 140

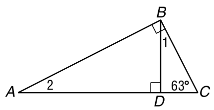
10. $m\angle 2$ 40

11. $m\angle 3$ 65

12. $m\angle 4$ 75

13. $m\angle 5$ 115

Find the measure of each angle.



14. $m\angle 1$ 27

15. $m\angle 2$ 27

Lesson 4-2

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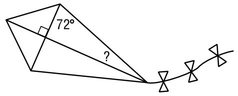
4-2 Practice (Average)

Angles of Triangles

Find the missing angle measures.

1. 

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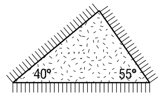
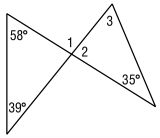
2. 

85

Find the measure of each angle.

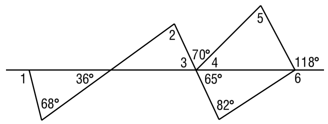


3. $m\angle 1$ 97

4. $m\angle 2$ 83

5. $m\angle 3$ 62

Find the measure of each angle.



6. $m\angle 1$ 104

7. $m\angle 4$ 45

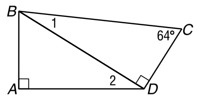
8. $m\angle 3$ 65

9. $m\angle 2$ 79

10. $m\angle 5$ 73

11. $m\angle 6$ 147

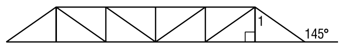
Find the measure of each angle if $\angle BAD$ and $\angle BDC$ are right angles and $m\angle ABC = 84$.



12. $m\angle 1$ 26

13. $m\angle 2$ 32

14. CONSTRUCTION The diagram shows an example of the Pratt Truss used in bridge construction. Use the diagram to find $m\angle 1$.
55



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4-2 Reading to Learn Mathematics

Angles of Triangles

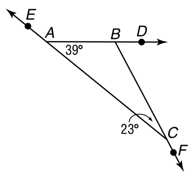
Pre-Activity How are the angles of triangles used to make kites?

Read the introduction to Lesson 4-2 at the top of page 185 in your textbook.

The frame of the simplest kind of kite divides the kite into four triangles. Describe these four triangles and how they are related to each other.
Sample answer: There are two pairs of right triangles that have the same size and shape.

Reading the Lesson

1. Refer to the figure.



   a. Name the three interior angles of the triangle. (Use three letters to name each angle.) **$\angle BAC$, $\angle ABC$, $\angle BCA$**
   b. Name three exterior angles of the triangle. (Use three letters to name each angle.) **$\angle EAB$, $\angle DBC$, $\angle FCA$**
   c. Name the remote interior angles of $\angle EAB$. **$\angle ABC$, $\angle BCA$**
   d. Find the measure of each angle without using a protractor.
      i. $\angle DBC$ **62** ii. $\angle ABC$ **118** iii. $\angle ACF$ **157** iv. $\angle EAB$ **141**

2. Indicate whether each statement is *true* or *false*. If the statement is false, replace the underlined word or number with a word or number that will make the statement true.
   a. The acute angles of a right triangle are <u>supplementary</u>. **false; complementary**
   b. The sum of the measures of the angles of any triangle is <u>100</u>. **false; 180**
   c. A triangle can have at most one right angle or <u>acute</u> angle. **false; obtuse**
   d. If two angles of one triangle are congruent to two angles of another triangle, then the third angles of the triangles are <u>congruent</u>. **true**
   e. The measure of an exterior angle of a triangle is equal to the <u>difference</u> of the measures of the two remote interior angles. **false; sum**
   f. If the measures of two angles of a triangle are 62 and 93, then the measure of the third angle is <u>35</u>. **false; 25**
   g. An <u>exterior</u> angle of a triangle forms a linear pair with an interior angle of the triangle. **true**

Helping You Remember

3. Many students remember mathematical ideas and facts more easily if they see them demonstrated visually rather than having them stated in words. Describe a visual way to demonstrate the Angle Sum Theorem.
Sample answer: Cut off the angles of a triangle and place them side-by-side on one side of a line so that their vertices meet at a common point. The result will show three angles whose measures add up to 180.

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4-2 Enrichment

Finding Angle Measures in Triangles

You can use algebra to solve problems involving triangles.

Example In triangle ABC, $m\angle A$, is twice $m\angle B$, and $m\angle C$ is 8 more than $m\angle B$. What is the measure of each angle?

Write and solve an equation. Let $x = m\angle B$.

$$m\angle A + m\angle B + m\angle C = 180$$
$$2x + x + (x + 8) = 180$$
$$4x + 8 = 180$$
$$4x = 172$$
$$x = 43$$

So, $m\angle A = 2(43)$ or 86, $m\angle B = 43$, and $m\angle C = 43 + 8$ or 51.

Solve each problem.

1. In triangle DEF, $m\angle E$ is three times $m\angle D$, and $m\angle F$ is 9 less than $m\angle E$. What is the measure of each angle?
$m\angle D = 27$, $m\angle E = 81$, $m\angle F = 72$

2. In triangle RST, $m\angle T$ is 5 more than $m\angle R$, and $m\angle S$ is 10 less than $m\angle T$. What is the measure of each angle?
$m\angle R = 60$, $m\angle S = 55$, $m\angle T = 65$

3. In triangle JKL, $m\angle K$ is four times $m\angle J$, and $m\angle L$ is five times $m\angle J$. What is the measure of each angle?
$m\angle J = 18$, $m\angle K = 72$, $m\angle L = 90$

4. In triangle XYZ, $m\angle Z$ is 2 more than twice $m\angle X$, and $m\angle Y$ is 7 less than twice $m\angle X$. What is the measure of each angle?
$m\angle X = 37$, $m\angle Y = 67$, $m\angle Z = 76$

5. In triangle GHI, $m\angle H$ is 20 more than $m\angle G$, and $m\angle G$ is 8 more than $m\angle I$. What is the measure of each angle?
$m\angle G = 56$, $m\angle H = 76$, $m\angle I = 48$

6. In triangle MNO, $m\angle M$ is equal to $m\angle N$, and $m\angle O$ is 5 more than three times $m\angle N$. What is the measure of each angle?
$m\angle M = m\angle N = 35$, $m\angle O = 110$

7. In triangle STU, $m\angle U$ is half $m\angle T$, and $m\angle S$ is 30 more than $m\angle T$. What is the measure of each angle?
$m\angle S = 90$, $m\angle T = 60$, $m\angle U = 30$

8. In triangle PQR, $m\angle P$ is equal to $m\angle Q$, and $m\angle R$ is 24 less than $m\angle P$. What is the measure of each angle?
$m\angle P = m\angle Q = 68$, $m\angle R = 44$

9. Write your own problems about measures of triangles.
See students' work.

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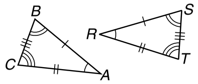
4-3 Study Guide and Intervention

Congruent Triangles

Corresponding Parts of Congruent Triangles

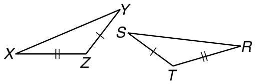
Triangles that have the same size and same shape are **congruent triangles**. Two triangles are congruent if and only if all three pairs of corresponding angles are congruent and all three pairs of corresponding sides are congruent. In the figure, $\triangle ABC \cong \triangle RST$.



Example **If $\triangle XYZ \cong \triangle RST$, name the pairs of congruent angles and congruent sides.**

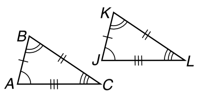
$\angle X \cong \angle R$, $\angle Y \cong \angle S$, $\angle Z \cong \angle T$

$\overline{XY} \cong \overline{RS}$, $\overline{XZ} \cong \overline{RT}$, $\overline{YZ} \cong \overline{ST}$

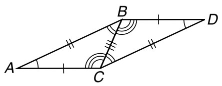


Exercises

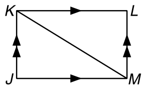
Identify the congruent triangles in each figure.

1. 

$\triangle ABC \cong \triangle JKL$

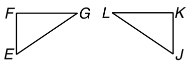
2. 

$\triangle ABC \cong \triangle DCB$

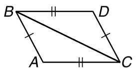
3. 

$\triangle JKM \cong \triangle LMK$

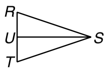
Name the corresponding congruent angles and sides for the congruent triangles.

4. 

$\angle E \cong \angle J$; $\angle F \cong \angle K$; $\angle G \cong \angle L$; $\overline{EF} \cong \overline{JK}$; $\overline{EG} \cong \overline{JL}$; $\overline{FG} \cong \overline{KL}$

5. 

$\angle A \cong \angle D$; $\angle ABC \cong \angle DCB$; $\angle ACB \cong \angle DBC$; $\overline{AB} \cong \overline{DC}$; $\overline{AC} \cong \overline{DB}$; $\overline{BC} \cong \overline{CB}$

6. 

$\angle R \cong \angle T$; $\angle RSU \cong \angle TSU$; $\angle RUS \cong \angle TUS$; $\overline{RU} \cong \overline{TU}$; $\overline{RS} \cong \overline{TS}$; $\overline{SU} \cong \overline{SU}$

NAME ______________________ DATE __________ PERIOD _____

4-3 Study Guide and Intervention *(continued)*

Congruent Triangles

Identify Congruence Transformations

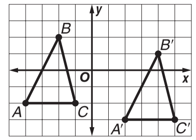
If two triangles are congruent, you can slide, flip, or turn one of the triangles and they will still be congruent. These are called **congruence transformations** because they do not change the size or shape of the figure. It is common to use prime symbols to distinguish between an original $\triangle ABC$ and a transformed $\triangle A'B'C'$.

Example **Name the congruence transformation that produces $\triangle A'B'C'$ from $\triangle ABC$.**

The congruence transformation is a slide.

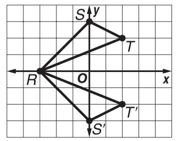
$\angle A \cong \angle A'$; $\angle B \cong \angle B'$; $\angle C \cong \angle C'$;

$\overline{AB} \cong \overline{A'B'}$; $\overline{AC} \cong \overline{A'C'}$; $\overline{BC} \cong \overline{B'C'}$

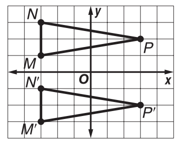


Exercises

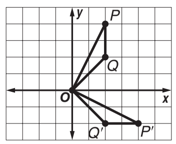
Describe the congruence transformation between the two triangles as a *slide*, a *flip*, or a *turn*. Then name the congruent triangles.

1. 

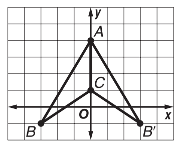
flip; $\triangle RST \cong \triangle RS'T'$

2. 

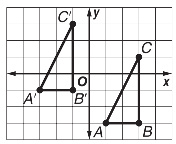
slide; $\triangle MNP \cong \triangle M'N'P'$

3. 

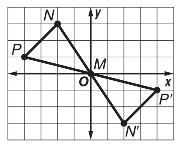
turn; $\triangle OPQ \cong \triangle OP'Q'$

4. 

flip; $\triangle ABC \cong \triangle AB'C$

5. 

slide; $\triangle ABC \cong \triangle A'B'C'$

6. 

turn; $\triangle MNP \cong \triangle MN'P'$

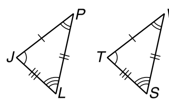
NAME ______________________ DATE ____________ PERIOD _____

4-3 Skills Practice

Congruent Triangles

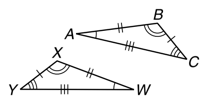
Identify the congruent triangles in each figure.

1.



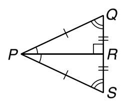
$\triangle JPL \cong \triangle TVS$

2.



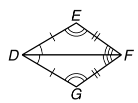
$\triangle ABC \cong \triangle WXY$

3.



$\triangle PQR \cong \triangle PSR$

4.



$\triangle DEF \cong \triangle DGF$

Name the congruent angles and sides for each pair of congruent triangles.

5. $\triangle ABC \cong \triangle FGH$

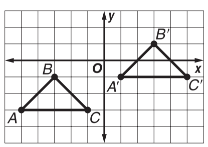
$\angle A \cong \angle F, \angle B \cong \angle G, \angle C \cong \angle H; \overline{AB} \cong \overline{FG}, \overline{BC} \cong \overline{GH}, \overline{AC} \cong \overline{FH}$

6. $\triangle PQR \cong \triangle STU$

$\angle P \cong \angle S, \angle Q \cong \angle T, \angle R \cong \angle U; \overline{PQ} \cong \overline{ST}, \overline{QR} \cong \overline{TU}, \overline{PR} \cong \overline{SU}$

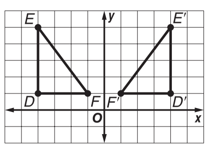
Verify that each of the following transformations preserves congruence, and name the congruence transformation.

7. $\triangle ABC \cong \triangle A'B'C'$



$AB = 2\sqrt{2}, A'B' = 2\sqrt{2},$
$BC = 2\sqrt{2}, B'C' = 2\sqrt{2},$
$AC = 4, A'C' = 4, \angle A \cong \angle A',$
$\angle B \cong \angle B', \angle C \cong \angle C'$; slide

8. $\triangle DEF \cong \triangle D'E'F'$



$DE = 4, D'E' = 4, EF = 5,$
$E'F' = 5, DF = 3, D'F' = 3,$
$\angle D \cong \angle D', \angle E \cong \angle E',$
$\angle F \cong \angle F'$; flip

Lesson 4-3

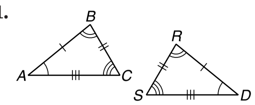
NAME ______________________ DATE ____________ PERIOD _____

4-3 Practice (Average)

Congruent Triangles

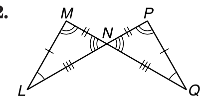
Identify the congruent triangles in each figure.

1.



$\triangle ABC \cong \triangle DRS$

2.



$\triangle LMN \cong \triangle QPN$

Name the congruent angles and sides for each pair of congruent triangles.

3. $\triangle GKP \cong \triangle LMN$

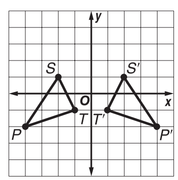
$\angle G \cong \angle L, \angle K \cong \angle M, \angle P \cong \angle N; \overline{GK} \cong \overline{LM}, \overline{KP} \cong \overline{MN}, \overline{GP} \cong \overline{LN}$

4. $\triangle ANC \cong \triangle RBV$

$\angle A \cong \angle R, \angle N \cong \angle B, \angle C \cong \angle V; \overline{AN} \cong \overline{RB}, \overline{NC} \cong \overline{BV}, \overline{AC} \cong \overline{RV}$

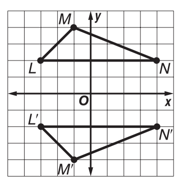
Verify that each of the following transformations preserves congruence, and name the congruence transformation.

5. $\triangle PST \cong \triangle P'S'T'$



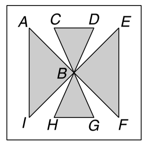
$PS = \sqrt{13}, P'S' = \sqrt{13},$
$ST = \sqrt{5}, S'T' = \sqrt{5}, PT = \sqrt{10},$
$P'T' = \sqrt{10}, \angle P \cong \angle P',$
$\angle S \cong \angle S', \angle T \cong \Delta T'$; flip

6. $\triangle LMN \cong \triangle L'M'N'$



$LM = 2\sqrt{2}, L'M' = 2\sqrt{2},$
$MN = \sqrt{29}, M'N' = \sqrt{29},$
$LN = 7, L'N' = 7, \angle L \cong \angle L',$
$\angle M \cong \angle M', \angle N \cong \angle N'$; flip

QUILTING For Exercises 7 and 8, refer to the quilt design.



7. Indicate the triangles that appear to be congruent.

$\triangle ABI \cong \triangle EBF, \triangle CBD \cong \triangle HBG$

8. Name the congruent angles and congruent sides of a pair of congruent triangles.

Sample answer: $\angle A \cong \angle E, \angle ABI \cong \angle EBF, \angle I \cong \angle F;$
$\overline{AB} \cong \overline{EB}, \overline{BI} \cong \overline{BF}, \overline{AI} \cong \overline{EF}$

Answers

NAME ______________________ DATE __________ PERIOD _____

4-3 Reading to Learn Mathematics

Congruent Triangles

Pre-Activity Why are triangles used in bridges?

Read the introduction to Lesson 4-3 at the top of page 192 in your textbook.

In the bridge shown in the photograph in your textbook, diagonal braces were used to divide squares into two isosceles right triangles. Why do you think these braces are used on the bridge? **Sample answer: The diagonal braces make the structure stronger and prevent it from being deformed when it has to withstand a heavy load.**

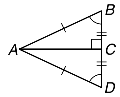
Reading the Lesson

1. If $\triangle RST \cong \triangle UWV$, complete each pair of congruent parts.

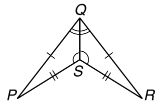
$\angle R \cong$ **$\angle U$**   **$\angle S$** $\cong \angle W$ $\angle T \cong$ **$\angle V$**

$\overline{RT} \cong$ **$\overline{UV}$**   **$\overline{RS}$** $\cong \overline{UW}$ **$\overline{ST}$** $\cong \overline{WV}$

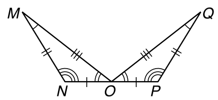
2. Identify the congruent triangles in each diagram.

a. 

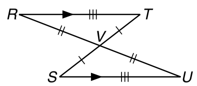
$\triangle ABC \cong \triangle ADC$

b. 

$\triangle PQS \cong \triangle RQS$

c. 

$\triangle MNO \cong \triangle QPO$

d. 

$\triangle RTV \cong \triangle USV$

3. Determine whether each statement says that congruence of triangles is *reflexive*, *symmetric*, or *transitive*.

a. If the first of two triangles is congruent to the second triangle, then the second triangle is congruent to the first. **symmetric**

b. If there are three triangles for which the first is congruent to the second and the second is congruent to the third, then the first triangle is congruent to the third. **transitive**

c. Every triangle is congruent to itself. **reflexive**

Helping You Remember

4. A good way to remember something is to explain it to someone else. Your classmate Ben is having trouble writing congruence statements for triangles because he thinks he has to match up three pairs of sides and three pairs of angles. How can you help him understand how to write correct congruence statements more easily? **Sample answer: Write the three vertices of one triangle in any order. Then write the corresponding vertices of the second triangle in the same order. If the angles are written in the correct correspondence, the sides will automatically be in the correct correspondence also.**

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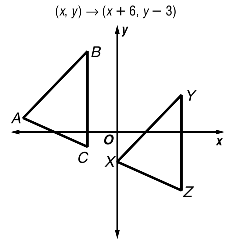
4-3 Enrichment

Transformations in The Coordinate Plane

The following statement tells one way to map preimage points to image points in the coordinate plane.

$(x, y) \rightarrow (x + 6, y - 3)$

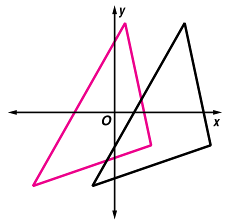
This can be read, "The point with coordinates (x, y) is mapped to the point with coordinates $(x + 6, y - 3)$." With this transformation, for example, (3, 5) is mapped to $(3 + 6, 5 - 3)$ or (9, 2). The figure shows how the triangle ABC is mapped to triangle XYZ.



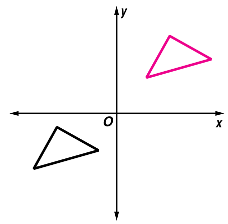
1. Does the transformation above appear to be a congruence transformation? Explain your answer. **Yes; the transformation slides the figure to the lower right without changing its size or shape.**

Draw the transformation image for each figure. Then tell whether the transformation is or is not a congruence transformation.

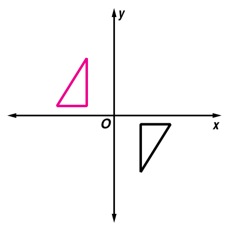
2. $(x, y) \rightarrow (x - 4, y)$ **yes**



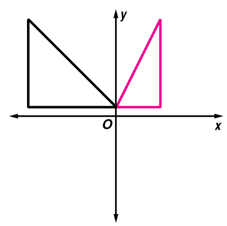
3. $(x, y) \rightarrow (x + 8, y + 7)$ **yes**



4. $(x, y) \rightarrow (-x, -y)$ **yes**



5. $(x, y) \rightarrow \left(-\frac{1}{2}x, y\right)$ **no**



NAME ______________________ DATE __________ PERIOD _____

4-4 Study Guide and Intervention

Proving Congruence—SSS, SAS

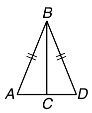
SSS Postulate You know that two triangles are congruent if corresponding sides are congruent and corresponding angles are congruent. The Side-Side-Side (SSS) Postulate lets you show that two triangles are congruent if you know only that the sides of one triangle are congruent to the sides of the second triangle.

SSS Postulate	If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent.

Example **Write a two-column proof.**

Given: $\overline{AB} \cong \overline{DB}$ and C is the midpoint of $\overline{AD}$.

Prove: $\triangle ABC \cong \triangle DBC$

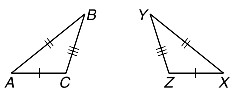


Statements	Reasons
1. $\overline{AB} \cong \overline{DB}$	**1.** Given
2. C is the midpoint of $\overline{AD}$.	**2.** Given
3. $\overline{AC} \cong \overline{DC}$	**3.** Definition of midpoint
4. $\overline{BC} \cong \overline{BC}$	**4.** Reflexive Property of ≅
5. $\triangle ABC \cong \triangle DBC$	**5.** SSS Postulate

Exercises

Write a two-column proof.

1.

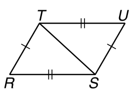


Given: $\overline{AB} \cong \overline{XY}$, $\overline{AC} \cong \overline{XZ}$, $\overline{BC} \cong \overline{YZ}$

Prove: $\triangle ABC \cong \triangle XYZ$

Statements	Reasons
1. $\overline{AB} \cong \overline{XY}$ $\overline{AC} \cong \overline{XZ}$ $\overline{BC} \cong \overline{YZ}$	**1.** Given
2. $\triangle ABC \cong \triangle XYZ$	**2.** SSS Post.

2.



Given: $\overline{RS} \cong \overline{UT}$, $\overline{RT} \cong \overline{US}$

Prove: $\triangle RST \cong \triangle UTS$

Statements	Reasons
1. $\overline{RS} \cong \overline{UT}$ $\overline{RT} \cong \overline{US}$	**1.** Given
2. $\overline{ST} \cong \overline{TS}$	**2.** Refl. Prop.
3. $\triangle RST \cong \triangle UTS$	**3.** SSS Post.

Lesson 4-4

NAME ______________________ DATE __________ PERIOD _____

4-4 Study Guide and Intervention *(continued)*

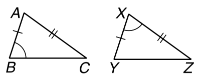
Proving Congruence—SSS, SAS

SAS Postulate Another way to show that two triangles are congruent is to use the Side-Angle-Side (SAS) Postulate.

SAS Postulate	If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

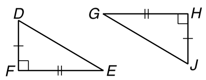
Example **For each diagram, determine which pairs of triangles can be proved congruent by the SAS Postulate.**

a.



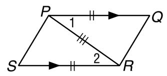
In $\triangle ABC$, the angle is not "included" by the sides $\overline{AB}$ and $\overline{AC}$. So the triangles cannot be proved congruent by the SAS Postulate.

b.



The right angles are congruent and they are the included angles for the congruent sides. $\triangle DEF \cong \triangle JGH$ by the SAS Postulate.

c.

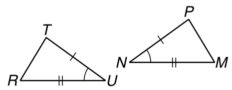


The included angles, $\angle 1$ and $\angle 2$, are congruent because they are alternate interior angles for two parallel lines. $\triangle PSR \cong \triangle RQP$ by the SAS Postulate.

Exercises

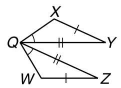
For each figure, determine which pairs of triangles can be proved congruent by the SAS Postulate.

1.



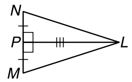
$\triangle TRU \cong \triangle PMN$ by the SAS Postulate.

2.



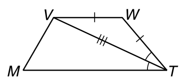
$\angle XQY$ and $\angle WQZ$ are not the included angles for the congruent segments. The triangles are not congruent by the SAS Postulate.

3.



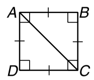
$\angle MPL \cong \angle NPL$ because both are right angles. $\triangle MPL \cong \triangle NPL$ by the SAS Postulate.

4.



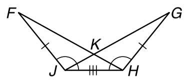
The triangles cannot be proved congruent by the SAS Postulate.

5.



$\angle D \cong \angle B$ because both are right angles. The two triangles are congruent by the SAS Postulate.

6.



The congruent angles are the included angles for the congruent sides. $\triangle FJH \cong \triangle GHJ$ by the SAS Postulate.

NAME ______________________ DATE ________ PERIOD ____

4-4 Skills Practice

Proving Congruence—SSS, SAS

Determine whether $\triangle ABC \cong \triangle KLM$ given the coordinates of the vertices. Explain.

1. $A(-3, 3)$, $B(-1, 3)$, $C(-3, 1)$, $K(1, 4)$, $L(3, 4)$, $M(1, 6)$

***AB* = 2, *KL* = 2, *BC* = $2\sqrt{2}$, *LM* = $2\sqrt{2}$, *AC* = 2, *KM* = 2.**

The corresponding sides have the same measure and are congruent, so $\triangle ABC \cong \triangle KLM$ by SSS.

2. $A(-4, -2)$, $B(-4, 1)$, $C(-1, -1)$, $K(0, -2)$, $L(0, 1)$, $M(4, 1)$

***AB* = 3, *KL* = 3, *BC* = $\sqrt{13}$, *LM* = 4, *AC* = $\sqrt{10}$, *KM* = 5.**

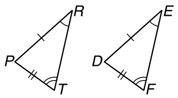
The corresponding sides are not congruent, so $\triangle ABC$ is not congruent to $\triangle KLM$.

3. Write a flow proof.

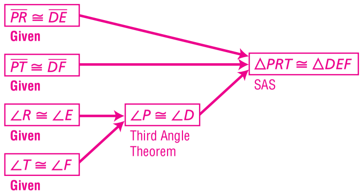
Given: $\overline{PR} \cong \overline{DE}$, $\overline{PT} \cong \overline{DF}$

$\angle R \cong \angle E$, $\angle T \cong \angle F$

Prove: $\triangle PRT \cong \triangle DEF$

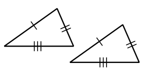


Proof:



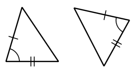
Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

4.



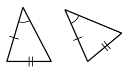
SSS

5.



SAS

6.



not possible

NAME ______________________ DATE ________ PERIOD ____

4-4 Practice (Average)

Proving Congruence—SSS, SAS

Determine whether $\triangle DEF \cong \triangle PQR$ given the coordinates of the vertices. Explain.

1. $D(-6, 1)$, $E(1, 2)$, $F(-1, -4)$, $P(0, 5)$, $Q(7, 6)$, $R(5, 0)$

***DE* = $5\sqrt{2}$, *PQ* = $5\sqrt{2}$, *EF* = $2\sqrt{10}$, *QR* = $2\sqrt{10}$, *DF* = $5\sqrt{2}$, *PR* = $5\sqrt{2}$.**
$\triangle DEF \cong \triangle PQR$ by SSS since corresponding sides have the same measure and are congruent.

2. $D(-7, -3)$, $E(-4, -1)$, $F(-2, -5)$, $P(2, -2)$, $Q(5, -4)$, $R(0, -5)$

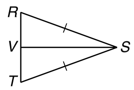
***DE* = $\sqrt{13}$, *PQ* = $\sqrt{13}$, *EF* = $2\sqrt{5}$, *QR* = $\sqrt{26}$, *DF* = $\sqrt{29}$, *PR* = $\sqrt{13}$.**
Corresponding sides are not congruent, so $\triangle DEF$ is not congruent to $\triangle PQR$.

3. Write a flow proof.

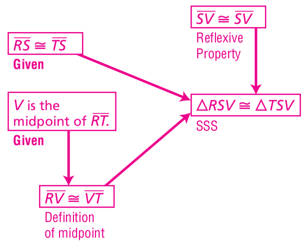
Given: $\overline{RS} \cong \overline{TS}$

V is the midpoint of $\overline{RT}$.

Prove: $\triangle RSV \cong \triangle TSV$

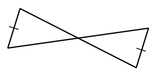


Proof:



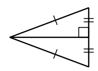
Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

4.



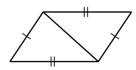
not possible

5.



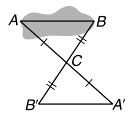
SAS or SSS

6.



SSS

7. INDIRECT MEASUREMENT To measure the width of a sinkhole on his property, Harmon marked off congruent triangles as shown in the diagram. How does he know that the lengths $A'B'$ and AB are equal?



Since $\angle ACB$ and $\angle A'CB'$ are vertical angles, they are congruent. In the figure, $\overline{AC} \cong \overline{A'C}$ and $\overline{BC} \cong \overline{B'C}$. So $\triangle ABC \cong \triangle A'B'C$ by SAS. By CPCTC, the lengths *A′B′* and *AB* are equal.

NAME ______________________ DATE __________ PERIOD _____

4-4 Reading to Learn Mathematics

Proving Congruence—SSS, SAS

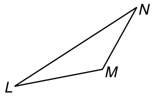
Pre-Activity **How do land surveyors use congruent triangles?**

Read the introduction to Lesson 4-4 at the top of page 200 in your textbook.

Why do you think that land surveyors would use congruent right triangles rather than other congruent triangles to establish property boundaries?
Sample answer: Land is usually divided into rectangular lots, so their boundaries meet at right angles.

Reading the Lesson

1. Refer to the figure.



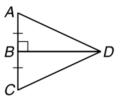
a. Name the sides of $\triangle LMN$ for which $\angle L$ is the included angle.
$\overline{LM}$, $\overline{LN}$

b. Name the sides of $\triangle LMN$ for which $\angle N$ is the included angle.
$\overline{NL}$, $\overline{NM}$

c. Name the sides of $\triangle LMN$ for which $\angle M$ is the included angle.
$\overline{ML}$, $\overline{MN}$

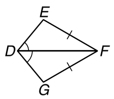
2. Determine whether you have enough information to prove that the two triangles in each figure are congruent. If so, write a congruence statement and name the congruence postulate that you would use. If not, write *not possible*.

a.



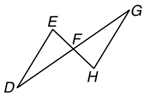
$\triangle ABD \cong \triangle CBD$; SAS

b.



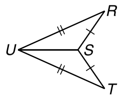
not possible

c. $\overline{EH}$ and $\overline{DG}$ bisect each other.



$\triangle DEF \cong \triangle GHF$; SAS

d.



$\triangle RSU \cong \triangle TSU$; SSS

Helping You Remember

3. Find three words that explain what it means to say that two triangles are congruent and that can help you recall the meaning of the SSS Postulate.
Sample answer: Congruent triangles are triangles that are the *same size* and *shape*, and the SSS Postulate ensures that two triangles with three corresponding sides congruent will be the same size and shape.

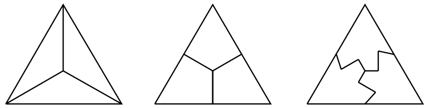
Lesson 4-4

NAME ______________________ DATE __________ PERIOD _____

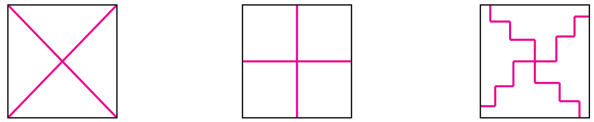
4-4 Enrichment

Congruent Parts of Regular Polygonal Regions

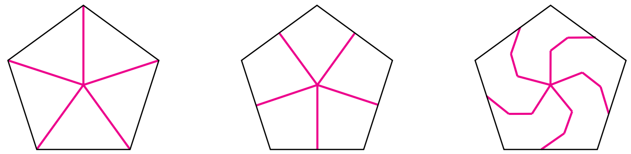
Congruent figures are figures that have exactly the same size and shape. There are many ways to divide regular polygonal regions into congruent parts. Three ways to divide an equilateral triangular region are shown. You can verify that the parts are congruent by tracing one part, then rotating, sliding, or reflecting that part on top of the other parts.



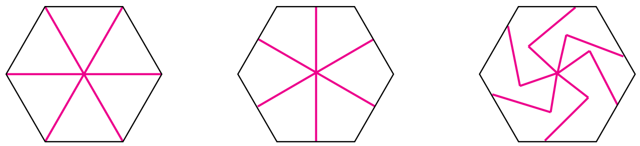
1. Divide each square into four congruent parts. Use three different ways. **Sample answers are shown.**



2. Divide each pentagon into five congruent parts. Use three different ways. **Sample answers are shown.**



3. Divide each hexagon into six congruent parts. Use three different ways. **Sample answers are shown.**



4. What hints might you give another student who is trying to divide figures like those into congruent parts? **See students' work.**

Answers

NAME ______________________ DATE __________ PERIOD _____

4-5 Study Guide and Intervention

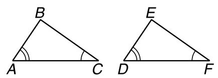
Proving Congruence—ASA, AAS

ASA Postulate The Angle-Side-Angle (ASA) Postulate lets you show that two triangles are congruent.

ASA Postulate	If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

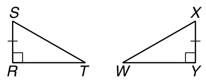
Example **Find the missing congruent parts so that the triangles can be proved congruent by the ASA Postulate. Then write the triangle congruence.**

a.



Two pairs of corresponding angles are congruent, $\angle A \cong \angle D$ and $\angle C \cong \angle F$. If the included sides $\overline{AC}$ and $\overline{DF}$ are congruent, then $\triangle ABC \cong \triangle DEF$ by the ASA Postulate.

b.

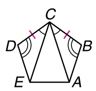


$\angle R \cong \angle Y$ and $\overline{SR} \cong \overline{XY}$. If $\angle S \cong \angle X$, then $\triangle RST \cong \triangle YXW$ by the ASA Postulate.

Exercises

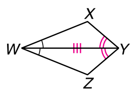
What corresponding parts must be congruent in order to prove that the triangles are congruent by the ASA Postulate? Write the triangle congruence statement.

1.



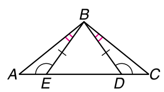
$\overline{DC} \cong \overline{BC}$;
$\triangle CDE \cong \triangle CBA$

2.



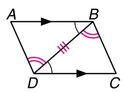
$\overline{WY} \cong \overline{WY}$;
$\angle XYW \cong \angle ZYW$;
$\triangle WXY \cong \triangle WZY$

3.



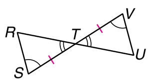
$\angle ABE \cong \angle CBD$;
$\triangle ABE \cong \triangle CBD$

4.



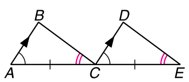
$\overline{BD} \cong \overline{DB}$;
$\angle ADB \cong \angle CBD$;
$\triangle ABD \cong \triangle CDB$

5.



$\overline{ST} \cong \overline{VT}$;
$\triangle RST \cong \triangle UVT$

6.



$\angle ACB \cong \angle E$;
$\triangle ABC \cong \triangle CDE$

NAME ______________________ DATE __________ PERIOD _____

4-5 Study Guide and Intervention *(continued)*

Proving Congruence—ASA, AAS

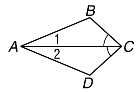
AAS Theorem Another way to show that two triangles are congruent is the Angle-Angle-Side (AAS) Theorem.

AAS Theorem	If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.

You now have five ways to show that two triangles are congruent.

- definition of triangle congruence
- SSS Postulate
- SAS Postulate
- ASA Postulate
- AAS Theorem

Example **In the diagram, $\angle BCA \cong \angle DCA$. Which sides are congruent? Which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the AAS Postulate?**

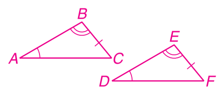


$\overline{AC} \cong \overline{AC}$ by the Reflexive Property of congruence. The congruent angles cannot be $\angle 1$ and $\angle 2$, because $\overline{AC}$ would be the included side. If $\angle B \cong \angle D$, then $\triangle ABC \cong \triangle ADC$ by the AAS Theorem.

Exercises

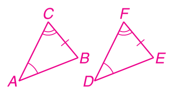
In Exercises 1 and 2, draw and label $\triangle ABC$ and $\triangle DEF$. Indicate which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the AAS Theorem.

1. $\angle A \cong \angle D$; $\angle B \cong \angle E$



If $\overline{BC} \cong \overline{EF}$ (or if $\overline{AC} \cong \overline{DF}$), then $\triangle ABC \cong \triangle DEF$ by the AAS Theorem.

2. $BC \cong EF$; $\angle A \cong \angle D$

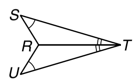


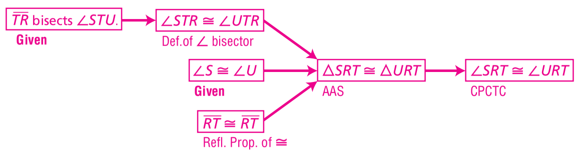
If $\angle C \cong \angle F$ (or if $\angle B \cong \angle E$), then $\triangle ABC \cong \triangle DEF$ by the AAS Theorem.

3. Write a flow proof.

Given: $\angle S \cong \angle U$; $\overline{TR}$ bisects $\angle STU$.

Prove: $\angle SRT \cong \angle URT$





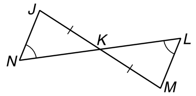
NAME ______________________ DATE __________ PERIOD _____

4-5 Skills Practice

Proving Congruence—ASA, AAS

Write a flow proof.

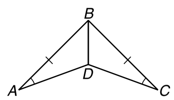
1. Given: $\angle N \cong \angle L$
$\overline{JK} \cong \overline{MK}$
Prove: $\triangle JKN \cong \triangle MKL$



Proof:

- $\angle N \cong \angle L$ (Given) → $\triangle JKN \cong \triangle MKL$
- $\overline{JK} \cong \overline{MK}$ (Given) → $\triangle JKN \cong \triangle MKL$
- $\angle JKN \cong \angle MKL$ (Vertical ∠s are ≅.) → $\triangle JKN \cong \triangle MKL$
- $\triangle JKN \cong \triangle MKL$ (AAS)

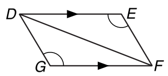
2. Given: $\overline{AB} \cong \overline{CB}$
$\angle A \cong \angle C$
$\overline{DB}$ bisects $\angle ABC$.
Prove: $\overline{AD} \cong \overline{CD}$



Proof:

- $\overline{AB} \cong \overline{CB}$ (Given) → $\triangle ABD \cong \triangle CBD$
- $\angle A \cong \angle C$ (Given) → $\triangle ABD \cong \triangle CBD$
- $\overline{DB}$ bisects $\angle ABC$. (Given) → $\angle ABD \cong \angle CBD$ (Def. of ∠ bisector) → $\triangle ABD \cong \triangle CBD$
- $\triangle ABD \cong \triangle CBD$ (ASA) → $\overline{AD} \cong \overline{CD}$ (CPCTC)

3. Write a paragraph proof.
Given: $\overline{DE} \parallel \overline{FG}$
$\angle E \cong \angle G$
Prove: $\triangle DFG \cong \triangle FDE$



Proof: Since it is given that $\overline{DE} \parallel \overline{FG}$, it follows that $\angle EDF \cong \angle GFD$, because alt. int. ∠s are ≅. It is given that $\angle E \cong \angle G$. By the Reflexive Property, $\overline{DF} \cong \overline{FD}$. So $\triangle DFG \cong \triangle FDE$ by AAS.

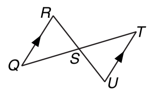
Lesson 4-5

NAME ______________________ DATE __________ PERIOD _____

4-5 Practice (Average)

Proving Congruence—ASA, AAS

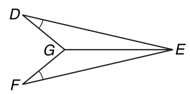
1. Write a flow proof.
Given: S is the midpoint of $\overline{QT}$.
$\overline{QR} \parallel \overline{TU}$
Prove: $\triangle QSR \cong \triangle TSU$



Sample proof:

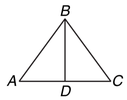
- S is the midpoint of $\overline{QT}$. (Given) → $\overline{QS} \cong \overline{TS}$ (Def. of midpoint) → $\triangle QSR \cong \triangle TSU$
- $\overline{QR} \parallel \overline{TU}$ (Given) → $\angle Q \cong \angle T$ (Alt. Int. ∠s are ≅.) → $\triangle QSR \cong \triangle TSU$
- $\angle QSR \cong \angle TSU$ (Vertical ∠s are ≅.) → $\triangle QSR \cong \triangle TSU$
- $\triangle QSR \cong \triangle TSU$ (ASA)

2. Write a paragraph proof.
Given: $\angle D \cong \angle F$
$\overline{GE}$ bisects $\angle DEF$.
Prove: $\overline{DG} \cong \overline{FG}$



Proof: Since it is given that $\overline{GE}$ bisects $\angle DEF$, $\angle DEG \cong \angle FEG$ by the definition of an angle bisector. It is given that $\angle D \cong \angle F$. By the Reflexive Property, $\overline{GE} \cong \overline{GE}$. So $\triangle DEG \cong \triangle FEG$ by AAS. Therefore $\overline{DG} \cong \overline{FG}$ by CPCTC.

ARCHITECTURE For Exercises 3 and 4, use the following information.
An architect used the window design in the diagram when remodeling an art studio. $\overline{AB}$ and $\overline{CB}$ each measure 3 feet.



3. Suppose D is the midpoint of $\overline{AC}$. Determine whether $\triangle ABD \cong \triangle CBD$. Justify your answer.
Since D is the midpoint of $\overline{AC}$, $\overline{AD} \cong \overline{CD}$ by the definition of midpoint. $\overline{AB} \cong \overline{CB}$ by the definition of congruent segments. By the Reflexive Property, $\overline{BD} \cong \overline{BD}$. So $\triangle ABD \cong \triangle CBD$ by SSS.

4. Suppose $\angle A \cong \angle C$. Determine whether $\triangle ABD \cong \triangle CBD$. Justify your answer.
We are given $\overline{AB} \cong \overline{CB}$ and $\angle A \cong \angle C$. $\overline{BD} \cong \overline{BD}$ by the Reflexive Property. Since SSA cannot be used to prove that triangles are congruent, we cannot say whether $\triangle ABD \cong \triangle CBD$.

Answers

NAME ______________________ DATE ________ PERIOD _____

4-5 Reading to Learn Mathematics

Proving Congruence—ASA, AAS

Pre-Activity **How are congruent triangles used in construction?**

Read the introduction to Lesson 4-5 at the top of page 207 in your textbook.

Which of the triangles in the photograph in your textbook appear to be congruent? **Sample answer: The four right triangles are congruent to each other. The two obtuse isosceles triangles are congruent to each other.**

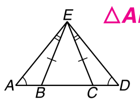
Reading the Lesson

1. Explain in your own words the difference between how the ASA Postulate and the AAS Theorem are used to prove that two triangles are congruent.
Sample answer: In ASA, you use two pairs of congruent angles and the *included* congruent sides. In AAS, you use two pairs of congruent angles and a pair of *nonincluded* congruent sides.

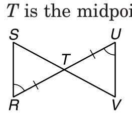
2. Which of the following conditions are sufficient to prove that two triangles are congruent? **B, D, E, G, H**

A. Two sides of one triangle are congruent to two sides of the other triangle.
B. The three sides of one triangles are congruent to the three sides of the other triangle.
C. The three angles of one triangle are congruent to the three angles of the other triangle.
D. All six corresponding parts of two triangles are congruent.
E. Two angles and the included side of one triangle are congruent to two sides and the included angle of the other triangle.
F. Two sides and a nonincluded angle of one triangle are congruent to two sides and a nonincluded angle of the other triangle.
G. Two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of the other triangle.
H. Two sides and the included angle of one triangle are congruent to two sides and the included angle of the other triangle.
I. Two angles and a nonincluded side of one triangle are congruent to two angles and a nonincluded side of the other triangle.

3. Determine whether you have enough information to prove that the two triangles in each figure are congruent. If so, write a congruence statement and name the congruence postulate or theorem that you would use. If not, write *not possible*.

a. 

$\triangle AEB \cong \triangle DEC$; AAS

b. T is the midpoint of $\overline{RU}$.


$\triangle RST \cong \triangle UVT$; ASA

Helping You Remember

4. A good way to remember mathematical ideas is to summarize them in a general statement. If you want to prove triangles congruent by using three pairs of corresponding parts, what is a good way to remember which combinations of parts will work?
Sample answer: At least one pair of corresponding parts must be sides. If you use two pairs of sides and one pair of angles, the angles must be the included angles. If you use two pairs of angles and one pair of sides, then the sides must both be included by the angles or must both be corresponding nonincluded sides.

NAME ______________________ DATE ________ PERIOD _____

4-5 Enrichment

Congruent Triangles in the Coordinate Plane

If you know the coordinates of the vertices of two triangles in the coordinate plane, you can often decide whether the two triangles are congruent. There may be more than one way to do this.

1. Consider $\triangle ABD$ and $\triangle CDB$ whose vertices have coordinates $A(0, 0)$, $B(2, 5)$, $C(9, 5)$, and $D(7, 0)$. Briefly describe how you can use what you know about congruent triangles and the coordinate plane to show that $\triangle ABD \cong \triangle CDB$. You may wish to make a sketch to help get you started.

Sample answer: Show that the slopes of $\overline{AB}$ and $\overline{CD}$ are equal and that the slopes of $\overline{AD}$ and $\overline{BC}$ are equal. Conclude that $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$. Use the angle relationships for parallel lines and a transversal and the fact that $\overline{BD}$ is a common side for the triangles to conclude that $\triangle ABD \cong \triangle CDB$ by ASA.

2. Consider $\triangle PQR$ and $\triangle KLM$ whose vertices are the following points.

$P(1, 2)$ $Q(3, 6)$ $R(6, 5)$
$K(-2, 1)$ $L(-6, 3)$ $M(-5, 6)$

Briefly describe how you can show that $\triangle PQR \cong \triangle KLM$.

Use the Distance Formula to find the lengths of the sides of both triangles. Conclude that $\triangle PQR \cong \triangle KLM$ by SSS.

3. If you know the coordinates of all the vertices of two triangles, is it *always* possible to tell whether the triangles are congruent? Explain.

Yes; you can use the Distance Formula and SSS.

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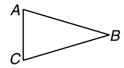
4-6 Study Guide and Intervention

Isosceles Triangles

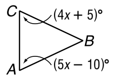
Properties of Isosceles Triangles An **isosceles triangle** has two congruent sides. The angle formed by these sides is called the **vertex angle**. The other two angles are called **base angles**. You can prove a theorem and its converse about isosceles triangles.

- If two sides of a triangle are congruent, then the angles opposite those sides are congruent. **(Isosceles Triangle Theorem)**
- If two angles of a triangle are congruent, then the sides opposite those angles are congruent.



If $\overline{AB} \cong \overline{CB}$, then $\angle A \cong \angle C$.
If $\angle A \cong \angle C$, then $\overline{AB} \cong \overline{CB}$.

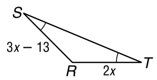
Example 1 **Find *x*.**



$BC = BA$, so

$m\angle A = m\angle C$.	Isos. Triangle Theorem
$5x - 10 = 4x + 5$	Substitution
$x - 10 = 5$	Subtract 4x from each side.
$x = 15$	Add 10 to each side.

Example 2 **Find *x*.**

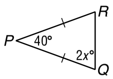


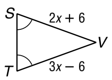
$m\angle S = m\angle T$, so

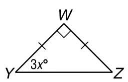
$SR = TR$.	Converse of Isos. △ Thm.
$3x - 13 = 2x$	Substitution
$3x = 2x + 13$	Add 13 to each side.
$x = 13$	Subtract 2x from each side.

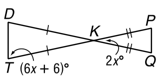
Exercises

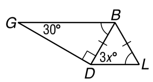
Find *x*.

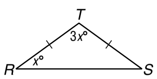
1.  **35**

2.  **12**

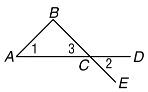
3.  **15**

4.  **12**

5.  **20**

6.  **36**

7. Write a two-column proof.
Given: $\angle 1 \cong \angle 2$
Prove: $\overline{AB} \cong \overline{CB}$



Statements	Reasons
1. $\angle 1 \cong \angle 2$	**1. Given**
2. $\angle 2 \cong \angle 3$	**2. Vertical angles are congruent.**
3. $\angle 1 \cong \angle 3$	**3. Transitive Property of ≅**
4. $\overline{AB} \cong \overline{CB}$	**4. If two angles of a triangle are ≅, then the sides opposite the angles are ≅.**

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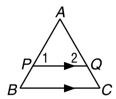
4-6 Study Guide and Intervention *(continued)*

Isosceles Triangles

Properties of Equilateral Triangles An **equilateral triangle** has three congruent sides. The Isosceles Triangle Theorem can be used to prove two properties of equilateral triangles.

1. A triangle is equilateral if and only if it is equiangular.
2. Each angle of an equilateral triangle measures 60°.

Example **Prove that if a line is parallel to one side of an equilateral triangle, then it forms another equilateral triangle.**

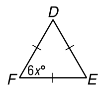


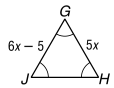
Proof:

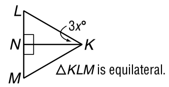
Statements	Reasons
1. $\triangle ABC$ is equilateral; $\overline{PQ} \parallel \overline{BC}$.	**1.** Given
2. $m\angle A = m\angle B = m\angle C = 60$	**2.** Each ∠ of an equilateral △ measures 60°.
3. $\angle 1 \cong \angle B$, $\angle 2 \cong \angle C$	**3.** If ∥ lines, then corres. ∠s are ≅.
4. $m\angle 1 = 60$, $m\angle 2 = 60$	**4.** Substitution
5. $\triangle APQ$ is equilateral.	**5.** If a △ is equiangular, then it is equilateral.

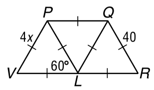
Exercises

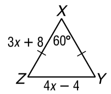
Find *x*.

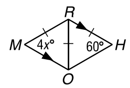
1.  **10**

2.  **5**

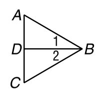
3.  **10**

4.  **10**

5.  **12**

6.  **15**

7. Write a two-column proof.
Given: $\triangle ABC$ is equilateral; $\angle 1 \cong \angle 2$.
Prove: $\angle ADB \cong \angle CDB$



Proof:

Statements	Reasons
1. $\triangle ABC$ is equilateral.	**1. Given**
2. $\overline{AB} \cong \overline{CB}$; $\angle A \cong \angle C$	**2. An equilateral △ has ≅ sides and ≅ angles.**
3. $\angle 1 \cong \angle 2$	**3. Given**
4. $\triangle ABD \cong \triangle CBD$	**4. ASA Postulate**
5. $\angle ADB \cong \angle CDB$	**5. CPCTC**

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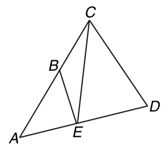
4-6 Skills Practice

Isosceles Triangles

Refer to the figure.



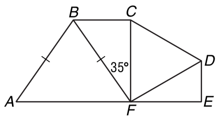
1. If $\overline{AC} \cong \overline{AD}$, name two congruent angles.
$\angle ACD \cong \angle CDA$

2. If $\overline{BE} \cong \overline{BC}$, name two congruent angles.
$\angle BEC \cong \angle BCE$

3. If $\angle EBA \cong \angle EAB$, name two congruent segments.
$\overline{EB} \cong \overline{EA}$

4. If $\angle CED \cong \angle CDE$, name two congruent segments.
$\overline{CE} \cong \overline{CD}$

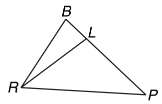
$\triangle ABF$ is isosceles, $\triangle CDF$ is equilateral, and $m\angle AFD = 150$. Find each measure.



5. $m\angle CFD$ **60** **6.** $m\angle AFB$ **55**

7. $m\angle ABF$ **70** **8.** $m\angle A$ **55**

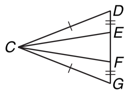
In the figure, $\overline{PL} \cong \overline{RL}$ and $\overline{LR} \cong \overline{BR}$.



9. If $m\angle RLP = 100$, find $m\angle BRL$. **20**

10. If $m\angle LPR = 34$, find $m\angle B$. **68**

11. Write a two-column proof.



Given: $\overline{CD} \cong \overline{CG}$
$\overline{DE} \cong \overline{GF}$
Prove: $\overline{CE} \cong \overline{CF}$

Proof:

Statements	Reasons
1. $\overline{CD} \cong \overline{CG}$	1. Given
2. $\angle D \cong \angle G$	2. If 2 sides of a △ are ≅, then the ∡ opposite those sides are ≅.
3. $\overline{DE} \cong \overline{GF}$	3. Given
4. $\triangle CDE \cong \triangle CGF$	4. SAS
5. $\overline{CE} \cong \overline{CF}$	5. CPCTC

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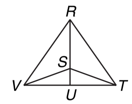
4-6 Practice (Average)

Isosceles Triangles

Refer to the figure.



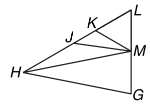
1. If $\overline{RV} \cong \overline{RT}$, name two congruent angles. **$\angle RTV \cong \angle RVT$**

2. If $\overline{RS} \cong \overline{SV}$, name two congruent angles. **$\angle SVR \cong \angle SRV$**

3. If $\angle SRT \cong \angle STR$, name two congruent segments. **$\overline{ST} \cong \overline{SR}$**

4. If $\angle STV \cong \angle SVT$, name two congruent segments. **$\overline{ST} \cong \overline{SV}$**

Triangles GHM and HJM are isosceles, with $\overline{GH} \cong \overline{MH}$ and $\overline{HJ} \cong \overline{MJ}$. Triangle KLM is equilateral, and $m\angle HMK = 50$. Find each measure.

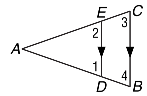


5. $m\angle KML$ **60** **6.** $m\angle HMG$ **70** **7.** $m\angle GHM$ **40**

8. If $m\angle HJM = 145$, find $m\angle MHJ$. **17.5**

9. If $m\angle G = 67$, find $m\angle GHM$. **46**

10. Write a two-column proof.

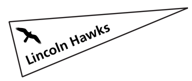


Given: $\overline{DE} \parallel \overline{BC}$
$\angle 1 \cong \angle 2$
Prove: $\overline{AB} \cong \overline{AC}$

Proof:

Statements	Reasons
1. $\overline{DE} \parallel \overline{BC}$	1. Given
2. $\angle 1 \cong \angle 4$ $\angle 2 \cong \angle 3$	2. Corr. ∡ are ≅.
3. $\angle 1 \cong \angle 2$	3. Given
4. $\angle 3 \cong \angle 4$	4. Congruence of ∡ is transitive.
5. $\overline{AB} \cong \overline{AC}$	5. If 2 ∡ of a △ are ≅, then the sides opposite those ∡ are ≅.

11. SPORTS A pennant for the sports teams at Lincoln High School is in the shape of an isosceles triangle. If the measure of the vertex angle is 18, find the measure of each base angle.
81, 81



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4-6 Reading to Learn Mathematics

Isosceles Triangles

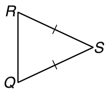
Pre-Activity **How are triangles used in art?**

Read the introduction to Lesson 4-6 at the top of page 216 in your textbook.

- Why do you think that isosceles and equilateral triangles are used more often than scalene triangles in art? **Sample answer: Their symmetry is pleasing to the eye.**
- Why might isosceles right triangles be used in art? **Sample answer: Two congruent isosceles right triangles can be placed together to form a square.**

Reading the Lesson

1. Refer to the figure.



a. What kind of triangle is $\triangle QRS$? **isosceles**

b. Name the legs of $\triangle QRS$. $\overline{QS}$, $\overline{RS}$

c. Name the base of $\triangle QRS$. $\overline{QR}$

d. Name the vertex angle of $\triangle QRS$. $\angle S$

e. Name the base angles of $\triangle QRS$. $\angle Q$, $\angle R$

2. Determine whether each statement is *always*, *sometimes*, or *never* true.

a. If a triangle has three congruent sides, then it has three congruent angles. **always**

b. If a triangle is isosceles, then it is equilateral. **sometimes**

c. If a right triangle is isosceles, then it is equilateral. **never**

d. The largest angle of an isosceles triangle is obtuse. **sometimes**

e. If a right triangle has a 45° angle, then it is isosceles. **always**

f. If an isosceles triangle has three acute angles, then it is equilateral. **sometimes**

g. The vertex angle of an isosceles triangle is the largest angle of the triangle. **sometimes**

3. Give the measures of the three angles of each triangle.

a. an equilateral triangle **60, 60, 60**

b. an isosceles right triangle **45, 45, 90**

c. an isosceles triangle in which the measure of the vertex angle is 70 **70, 55, 55**

d. an isosceles triangle in which the measure of a base angle is 70 **70, 70, 40**

e. an isosceles triangle in which the measure of the vertex angle is twice the measure of one of the base angles **90, 45, 45**

Helping You Remember

4. If a theorem and its converse are both true, you can often remember them most easily by combining them into an "if-and-only-if" statement. Write such a statement for the Isosceles Triangle Theorem and its converse. **Sample answer: Two sides of a triangle are congruent if and only if the angles opposite those sides are congruent.**

Lesson 4-6

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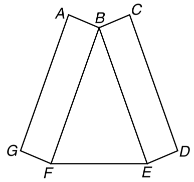
4-6 Enrichment

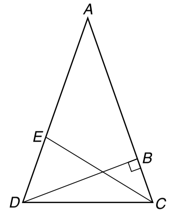
Triangle Challenges

Some problems include diagrams. If you are not sure how to solve the problem, begin by using the given information. Find the measures of as many angles as you can, writing each measure on the diagram. This may give you more clues to the solution.

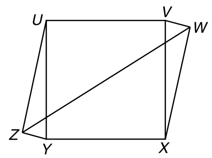
1. Given: $BE = BF$, $\angle BFG \cong \angle BEF \cong \angle BED$, $m\angle BFE = 82$ and $ABFG$ and $BCDE$ each have opposite sides parallel and congruent.
Find $m\angle ABC$. **148**



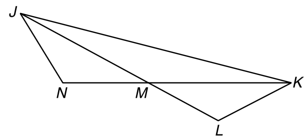
2. Given: $AC = AD$, and $\overline{AB} \perp \overline{BD}$, $m\angle DAC = 44$ and $\overline{CE}$ bisects $\angle ACD$.
Find $m\angle DEC$. **78**



3. Given: $m\angle UZY = 90$, $m\angle ZWX = 45$, $\triangle YZU \cong \triangle VWX$, $UVXY$ is a square (all sides congruent, all angles right angles).
Find $m\angle WZY$. **45**



4. Given: $m\angle N = 120$, $\overline{JN} \cong \overline{MN}$, $\triangle JNM \cong \triangle KLM$.
Find $m\angle JKM$. **15**



Answers

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4-7 Study Guide and Intervention

Triangles and Coordinate Proof

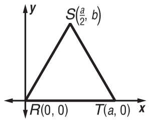
Position and Label Triangles A coordinate proof uses points, distances, and slopes to prove geometric properties. The first step in writing a coordinate proof is to place a figure on the coordinate plane and label the vertices. Use the following guidelines.

1. Use the origin as a vertex or center of the figure.
2. Place at least one side of the polygon on an axis.
3. Keep the figure in the first quadrant if possible.
4. Use coordinates that make the computations as simple as possible.

Example **Position an equilateral triangle on the coordinate plane so that its sides are a units long and one side is on the positive x-axis.**

Start with $R(0, 0)$. If RT is a, then another vertex is $T(a, 0)$.

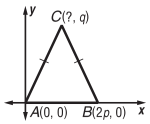
For vertex S, the x-coordinate is $\frac{a}{2}$. Use b for the y-coordinate, so the vertex is $S\left(\frac{a}{2}, b\right)$.



Exercises

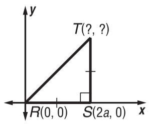
Find the missing coordinates of each triangle.

1.



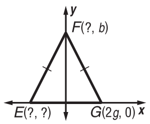
***C*(*p*, *q*)**

2.



***T*(2*a*, 2*a*)**

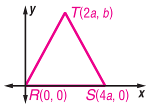
3.



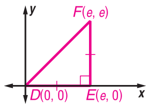
***E*(−2*g*, 0); *F*(0, *b*)**

Position and label each triangle on the coordinate plane. **Sample answers are given.**

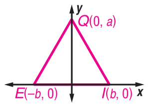
4. isosceles triangle $\triangle RST$ with base $\overline{RS}$ $4a$ units long



5. isosceles right $\triangle DEF$ with legs e units long



6. equilateral triangle $\triangle EQI$ with vertex $Q(0, a)$ and sides $2b$ units long



Lesson 4-7

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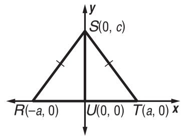
4-7 Study Guide and Intervention *(continued)*

Triangles and Coordinate Proof

Write Coordinate Proofs Coordinate proofs can be used to prove theorems and to verify properties. Many coordinate proofs use the Distance Formula, Slope Formula, or Midpoint Theorem.

Example **Prove that a segment from the vertex angle of an isosceles triangle to the midpoint of the base is perpendicular to the base.**

First, position and label an isosceles triangle on the coordinate plane. One way is to use $T(a, 0)$, $R(-a, 0)$, and $S(0, c)$. Then $U(0, 0)$ is the midpoint of $\overline{RT}$.



Given: Isosceles $\triangle RST$; U is the midpoint of base $\overline{RT}$.
Prove: $\overline{SU} \perp \overline{RT}$

Proof:
U is the midpoint of $\overline{RT}$ so the coordinates of U are $\left(\frac{-a + a}{2}, \frac{0 + 0}{2}\right) = (0, 0)$. Thus $\overline{SU}$ lies on the y-axis, and $\triangle RST$ was placed so $\overline{RT}$ lies on the x-axis. The axes are perpendicular, so $\overline{SU} \perp \overline{RT}$.

Exercises

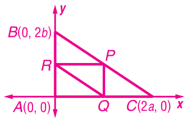
Prove that the segments joining the midpoints of the sides of a right triangle form a right triangle.

Sample answer: Position and label right $\triangle ABC$ with the coordinates $A(0, 0)$, $B(0, 2b)$, and $C(2a, 0)$.

The midpoint P of BC is $\left(\frac{0 + 2a}{2}, \frac{2b + 0}{2}\right) = (a, b)$.

The midpoint Q of AC is $\left(\frac{0 + 2a}{2}, \frac{0 + 0}{2}\right) = (a, 0)$.

The midpoint R of AB is $\left(\frac{0 + 0}{2}, \frac{0 + 2b}{2}\right) = (0, b)$.



The slope of $\overline{RP}$ is $\frac{b - b}{a - 0} = \frac{0}{a} = 0$, so the segment is horizontal.

The slope of $\overline{PQ}$ is $\frac{b - 0}{a - a} = \frac{b}{0}$, which is undefined, so the segment is vertical.

$\angle RPQ$ is a right angle because any horizontal line is perpendicular to any vertical line. $\triangle PRQ$ has a right angle, so $\triangle PRQ$ is a right triangle.

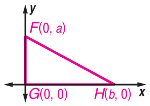
NAME ______________________ DATE __________ PERIOD _____

4-7 Skills Practice

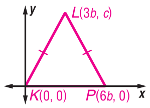
Triangles and Coordinate Proof

Position and label each triangle on the coordinate plane. **Sample answers are given.**

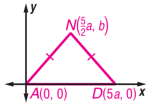
1. right $\triangle FGH$ with legs a units and b units



2. isosceles $\triangle KLP$ with base $\overline{KP}$ $6b$ units long

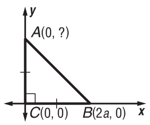


3. isosceles $\triangle AND$ with base $\overline{AD}$ $5a$ long



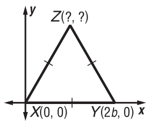
Find the missing coordinates of each triangle.

4.



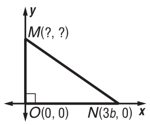
***A*(0, 2*a*)**

5.



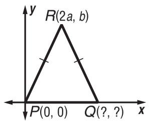
***Z*(*b*, *c*)**

6.



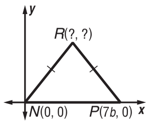
***M*(0, *c*)**

7.



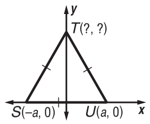
***Q*(4*a*, 0)**

8.



$R\left(\frac{7}{2}b, c\right)$

9.

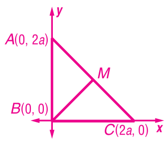


***T*(0, *b*)**

10. Write a coordinate proof to prove that in an isosceles right triangle, the segment from the vertex of the right angle to the midpoint of the hypotenuse is perpendicular to the hypotenuse.

Given: isosceles right $\triangle ABC$ with $\angle ABC$ the right angle and M the midpoint of $\overline{AC}$

Prove: $\overline{BM} \perp \overline{AC}$



Proof:

The Midpoint Formula shows that the coordinates of M are $\left(\frac{0 + 2a}{2}, \frac{2a + 0}{2}\right)$ or (a, a). The slope of $\overline{AC}$ is $\frac{2a - 0}{0 - 2a} = -1$. The slope of $\overline{BM}$ is $\frac{a - 0}{a - 0} = 1$. The product of the slopes is -1, so $\overline{BM} \perp \overline{AC}$.

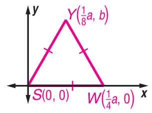
NAME ______________________ DATE __________ PERIOD _____

4-7 Practice (Average)

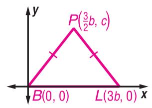
Triangles and Coordinate Proof

Position and label each triangle on the coordinate plane. **Sample answers are given.**

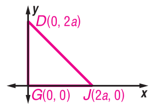
1. equilateral $\triangle SWY$ with sides $\frac{1}{4}a$ long



2. isosceles $\triangle BLP$ with base $\overline{BL}$ $3b$ units long

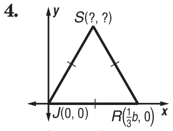


3. isosceles right $\triangle DGJ$ with hypotenuse $\overline{DJ}$ and legs $2a$ units long



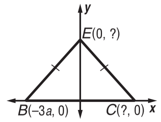
Find the missing coordinates of each triangle.

4.



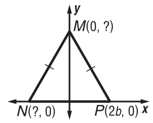
$S\left(\frac{1}{6}b, c\right)$

5.



***C*(3*a*, 0), *E*(0, *c*)**

6.



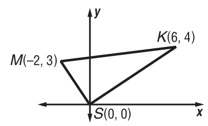
***M*(0, *c*), *N*(−2*b*, 0)**

NEIGHBORHOODS For Exercises 7 and 8, use the following information.
Karina lives 6 miles east and 4 miles north of her high school. After school she works part time at the mall in a music store. The mall is 2 miles west and 3 miles north of the school.

7. Write a coordinate proof to prove that Karina's high school, her home, and the mall are at the vertices of a right triangle.

Given: $\triangle SKM$

Prove: $\triangle SKM$ is a right triangle.



Proof:

Slope of $SK = \frac{4 - 0}{6 - 0}$ or $\frac{2}{3}$

Slope of $SM = \frac{3 - 0}{-2 - 0}$ or $-\frac{3}{2}$

Since the slope of $\overline{SM}$ is the negative reciprocal of the slope of $\overline{SK}$, $\overline{SM} \perp \overline{SK}$. Therefore, $\triangle SKM$ is right triangle.

8. Find the distance between the mall and Karina's home.

$KM = \sqrt{(-2 - 6)^2 + (3 - 4)^2} = \sqrt{64 + 1} = \sqrt{65}$ or ≈ 8.1 miles

NAME ______________________ DATE __________ PERIOD _____

4-7 Reading to Learn Mathematics

Triangles and Coordinate Proof

Pre-Activity **How can the coordinate plane be useful in proofs?**

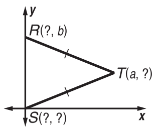
Read the introduction to Lesson 4-7 at the top of page 222 in your textbook.

From the coordinates of A, B, and C in the drawing in your textbook, what do you know about $\triangle ABC$? **Sample answer: $\triangle ABC$ is isosceles with $\angle C$ as the vertex angle.**

Reading the Lesson

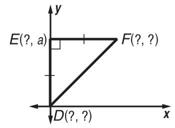
1. Find the missing coordinates of each triangle.

a.



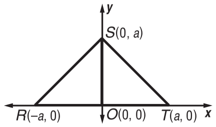
$R(0, b)$, $S(0, 0)$, $T\left(a, \frac{b}{2}\right)$

b.



$D(0, 0)$, $E(0, a)$, $F(a, a)$

2. Refer to the figure.



a. Find the slope of $\overline{SR}$ and the slope of $\overline{ST}$. **1; –1**

b. Find the product of the slopes of $\overline{SR}$ and $\overline{ST}$. What does this tell you about $\overline{SR}$ and $\overline{ST}$? **–1; $\overline{SR} \perp \overline{ST}$**

c. What does your answer from part b tell you about $\triangle RST$? **Sample answer: $\triangle RST$ is a right triangle with $\angle S$ as the right angle.**

d. Find SR and ST. What does this tell you about $\overline{SR}$ and $\overline{ST}$? **$SR = \sqrt{2a^2}$ or $a\sqrt{2}$; $ST = \sqrt{2a^2}$ or $a\sqrt{2}$; $\overline{SR} \cong \overline{ST}$**

e. What does your answer from part d tell you about $\triangle RST$? **Sample answer: $\triangle RST$ is isosceles with $\angle RST$ as the vertex angle.**

f. Combine your answers from parts c and e to describe $\triangle RST$ as completely as possible. **Sample answer: $\triangle RST$ is an isosceles right triangle. $\angle RST$ is the right angle and is also the vertex angle.**

g. Find $m\angle SRT$ and $m\angle STR$. **45; 45**

h. Find $m\angle OSR$ and $m\angle OST$. **45; 45**

Helping You Remember

3. Many students find it easier to remember mathematical formulas if they can put them into words in a compact way. How can you use this approach to remember the slope and midpoint formulas easily? **Sample answer: Slope Formula: change in y over change in x; Midpoint Formula: average of x-coordinates, average of y-coordinates**

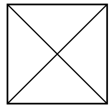
NAME ______________________ DATE __________ PERIOD _____

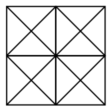
4-7 Enrichment

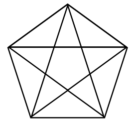
How Many Triangles?

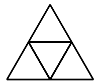
Each puzzle below contains many triangles. Count them carefully. Some triangles overlap other triangles.

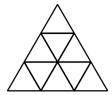
How many triangles are there in each figure?

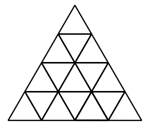
1.  **8**

2.  **40**

3.  **35**

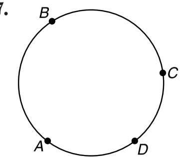
4.  **5**

5.  **13**

6.  **27**

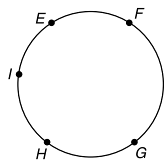
How many triangles can you form by joining points on each circle? List the vertices of each triangle.

7.



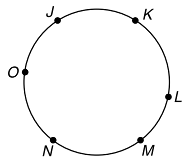
4; *ABC*, *ABD*, *ACD*, *BCD*

8.



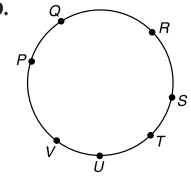
10; *EFG*, *EFH*, *EFI*, *EGH*, *EHI*, *FGH*, *FGI*, *FHI*, *EGI*, *GHI*

8.



20; *JKL*, *JKM*, *JKN*, *JKO*, *JLM*, *JLN*, *JLO*, *JMN*, *JMO*, *JNO*, *KLM*, *KLN*, *KLO*, *KMN*, *KMO*, *KNO*, *LMN*, *LMO*, *LNO*, *MNO*

9.



35; *PQR*, *PQS*, *PQT*, *PQU*, *PQV*, *PRS*, *PRT*, *PRU*, *PRV*, *PST*, *PSU*, *PSV*, *PTU*, *PTV*, *PUV*, *QRS*, *QRT*, *QRU*, *QRV*, *QST*, *QSU*, *QSV*, *QTU*, *QTV*, *QUV*, *RST*, *RSU*, *RSV*, *RTU*, *RTV*, *RUV*, *STU*, *STV*, *SUV*, *TUV*

Chapter 4 Assessment Answer Key

Form 1
Page 225

1. C
2. D
3. A
4. C
5. B
6. D
7. B
8. D

Page 226

9. C
10. A
11. C
12. A
13. B

B: isosceles

Form 2A
Page 227

1. C
2. C
3. A
4. D
5. B
6. D
7. D
8. A

(continued on the next page)

Answers

Chapter 4 Assessment Answer Key

Form 2A *(continued)*
Page 228

9. B

10. C

11. B

12. A

13. A

B: $A(0, 0)$, $C(-a, a)$

Form 2B
Page 229

1. D

2. B

3. D

4. A

5. C

6. B

7. A

8. A

Page 230

9. C

10. C

11. D

12. A

13. D

B: -2

Chapter 4 Assessment Answer Key

Form 2C

Page 231

1. acute scalene

2. $x = 3$, $AB = BC = AC = 24$

3. $EF = FG = 4\sqrt{2}$, $EG = 8$, isosceles

4. 70

5. 140

6. 50

7. $\triangle DFG \cong \triangle BAC$, $\angle D \cong \angle B$, $\angle F \cong \angle A$, $\angle G \cong \angle C$

8. $AB = A'B' = \sqrt{29}$, $BC = B'C' = \sqrt{10}$, $AC = A'C' = \sqrt{17}$

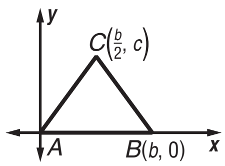
Page 232

9. SAS

10. Isosceles △ Theorem, AAS

11. 45

12. $x = 4$

13. 

14. $P\left(\frac{b}{2}, \frac{b}{2}\right)$, $\overline{CP} \perp \overline{AB}$

B: $\triangle XYZ \cong \triangle MNO$

Answers

Chapter 4 Assessment Answer Key

Form 2D

Page 233

1. obtuse isosceles

2. $x = 5$, $AB = BC = 30$, $AC = 40$

3. $EF = FG = 5$, $EG = 5\sqrt{2}$, isosceles

4. 110

5. 70

6. 30

7. $\triangle ABC \cong \triangle FDE$, $\angle A \cong \angle F$, $\angle B \cong \angle D$, $\angle C \cong \angle E$

8. $JK = J'K' = \sqrt{10}$, $JL = J'L' = \sqrt{29}$, $KL = K'L' = \sqrt{37}$

9. SAS

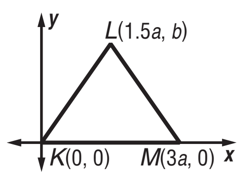
Page 234

10. Isosceles $\triangle$ Theorem, AAS

11. 50

12. $x = 6$

13.



14. $M\left(\frac{a}{2}, 0\right)$, $N\left(0, \frac{b}{2}\right)$, slopes: $\overline{MN} = -\frac{b}{a}$ and $\overline{BC} = -\frac{b}{a}$

B: $\triangle ABC \cong \triangle DEF$

Chapter 4 Assessment Answer Key

Form 3

Page 235

1. $x = 4$, $AB = 78$, $BC = 78$, $AC = 100$

2. $AB = 5$, $BC = 10$, $AC = 3\sqrt{5}$; scalene obtuse

3. 20

4. 90

5. 40

6. $AB = A'B' = \sqrt{26}$, $BC = B'C' = 3\sqrt{2}$, $AC = A'C' = 2\sqrt{5}$

7. $GH = JK = 2\sqrt{5}$, $IG = LJ = 2\sqrt{5}$, $IH = LK = 2\sqrt{2}$; $\triangle GHI \cong \triangle JKL$ by SSS.

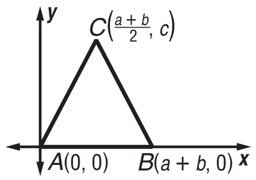
8. AAS

Page 236

9. Isosceles Triangle Theorem

10. SAS

11. $x = 3$

12. 

B: $m\angle 1$, $m\angle 3$, $m\angle 4$, $m\angle 6$, and $m\angle 9$ each equal 20, $m\angle 2 = 40$, $m\angle 5 = 40$, $m\angle 8 = 60$, and $m\angle 7 = 140$

Answers

Chapter 4 Assessment Answer Key

Page 237, Open-Ended Assessment Scoring Rubric

Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts of *using the Distance Formula to classify triangles and verify congruence*, *finding missing angles*, *solving algebraic equations in isosceles and equilateral triangles*, *proving triangles congruent*, *verifying congruence transformations*, *and writing coordinate proofs*. • Uses appropriate strategies to solve problems • Written explanations are exemplary. • Figures are accurate and appropriate. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows understanding of the concepts of *using the Distance Formula to classify triangles and verify congruence*, *finding missing angles*, *solving algebraic equations in isosceles and equilateral triangles*, *proving triangles congruent*, *verifying congruence transformations*, *and writing coordinate proofs*. • Uses appropriate strategies to solve problems • Computations are mostly correct. • Written explanations are effective. • Figures are mostly accurate and appropriate. • Satisfies all requirements of all problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows understanding of most of the concepts of *using the Distance Formula to classify triangles and verify congruence*, *finding missing angles*, *solving algebraic equations in isosceles and equilateral triangles*, *proving triangles congruent*, *verifying congruence transformations*, *and writing coordinate proofs*. • May not use appropriate strategies to solve problems • Computations are mostly correct. • Written explanations are satisfactory. • Figures are mostly accurate. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work is shown to substantiate the final computation. • Figures may be accurate but lack detail or explanation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts of *using the Distance Formula to classify triangles and verify congruence*, *finding missing angles*, *solving algebraic equations in isosceles and equilateral triangles*, *proving triangles congruent*, *verifying congruence transformations*, *and writing coordinate proofs*. • Does not use appropriate strategies to solve problems • Computations are incorrect. • Written explanations are unsatisfactory. • Figures are inaccurate or inappropriate. • Does not satisfy the requirements of the problems. • No answer may be given.

Chapter 4 Assessment Answer Key

Page 237, Open-Ended Assessment
Sample Answers

In addition to the scoring rubric found on page A28, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1. a. The figure is an acute isosceles triangle.

b. $9x + 4 + 2(20x - 10) = 180$
$9x + 4 + 40x - 20 = 180$
$49x - 16 = 180$
$49x = 196$
$x = 4$

2. a. To determine whether a triangle is equilateral, the coordinates should be graphed first to see if they form a triangle. Then list the segments which form the triangle and use the Distance Formula to find their lengths. A triangle is equilateral only if all three sides have equal measures.

b. This triangle is not equilateral since $AB = AC = 5$ and $BC = 5\sqrt{2}$, which makes this an isosceles triangle.

3. Since $\angle DBA$, $\angle ABC$, and $\angle EBC$ form a straight line, the sum of the angle measures is 180°. Therefore $m\angle ABC = 180 - 40 - 62$ or 78. Then since the sum of the measures of the angles of a triangle is 180°, $\angle 1 = 180 - 78 - 58$ or 44. Lastly, since $\angle 2$ is an exterior angle, its measure is equivalent to the sum of the measures of the two remote interior angles, $58 + 78$, which equals 136.

4. a. SSS postulate

b. $\angle J \cong \angle G$, $\angle D \cong \angle E$, $\angle L \cong \angle S$
$\overline{DJ} \cong \overline{EG}$, $\overline{DL} \cong \overline{ES}$, $\overline{JL} \cong \overline{GS}$

5.

Statements	**Reasons**
1. $\overline{AB} \parallel \overline{DE}$	1. Given
2. $\angle ABC \cong \angle DEC$	2. Alt. int. ∠s are ≅.
3. $\overline{AD}$ bisects $\overline{BE}$.	3. Given
4. $\overline{BC} \cong \overline{EC}$	4. Definition of segment bisector
5. $\angle ACB \cong \angle DCE$	5. Vert. ∠s are ≅.
6. $\triangle ABC \cong \triangle DEC$	6. ASA

Answers

Chapter 4 Assessment Answer Key

Vocabulary Test/Review
Page 238

1. equiangular triangle
2. obtuse triangle
3. remote interior angles
4. base
5. scalene triangle
6. flow proof
7. congruence transformation
8. included side
9. coordinate proof
10. vertex angle
11. a statement that can easily be proved using a theorem
12. two triangles in which all corresponding parts are congruent
13. a triangle in which all angle measures are between 0 and 90

Quiz 1
Page 239

1. right scalene
2. C
3. $x = 2$, $AB = 15$, $BC = 15$, $AC = 10$
4. $AB = \sqrt{53}$, $BC = 2\sqrt{10}$, $AC = \sqrt{41}$; scalene
5. 40
6. 45
7. 135
8. 73
9. 70
10. 30

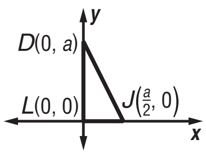
Quiz 2
Page 239

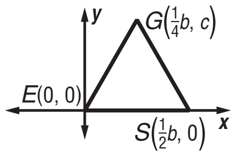
1. $\triangle KMN \cong \triangle KML$
2. D
3. $AB = A'B' = \sqrt{13}$, $BC = B'C' = 2\sqrt{5}$, $AC = A'C' = \sqrt{17}$
4. SAS

Quiz 3
Page 240

1. Def. of angle bisector
2. AAS
3. 60
4. 45

Quiz 4
Page 240

1. $I(0, c)$ and $C(b, 0)$
2. 

3. 

4. $AC = \sqrt{34}$, $AB = 6$, and $CB = \sqrt{34}$
5. $\overline{AC} \cong \overline{CB}$

Chapter 4 Assessment Answer Key

Mid-Chapter Test
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Part I

1. D

2. C

3. A

4. B

Part II

5. $AB = \sqrt{41}$, $BC = \sqrt{29}$, $AC = 5\sqrt{2}$; scalene

6. $AB = A'B' = \sqrt{26}$, $BC = B'C' = 2\sqrt{5}$, $AC = A'C' = 3\sqrt{2}$

7. $\overline{PO}$ and $\overline{LN}$ bisect each other.

Cumulative Review
Page 242

1. a ray

2. $35\frac{1}{2}$ in. to $36\frac{1}{2}$ in.

3. 5

4. −4

5. −17

6. Sometimes; *D*, *E*, and *F* can be noncollinear.

7. always

8. undefined

9. 4

10. $\overline{FD}$

11. right triangle

12. 15

13. $\angle P \cong \angle H$, $\angle Q \cong \angle G$, $\angle R \cong \angle B$, $\overline{PQ} \cong \overline{HG}$, $\overline{QR} \cong \overline{GB}$, $\overline{PR} \cong \overline{HB}$

14. ASA

15. $E(b, b)$; $F(2b, 0)$; $G(b, 0)$

Answers

Chapter 4 Assessment Answer Key

Standardized Test Practice

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1. Ⓐ **B** Ⓒ Ⓓ — B

2. Ⓔ Ⓕ **G** Ⓗ — G

3. Ⓐ Ⓑ **C** Ⓓ — C

4. **E** Ⓕ Ⓖ Ⓗ — E

5. Ⓐ **B** Ⓒ Ⓓ — B

6. Ⓔ Ⓕ Ⓖ **H** — H

7. **A** Ⓑ Ⓒ Ⓓ — A

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8. 0

9. 68

10. 6

11. 18

12. 29 ft

13. 35

14. 62

15. 40