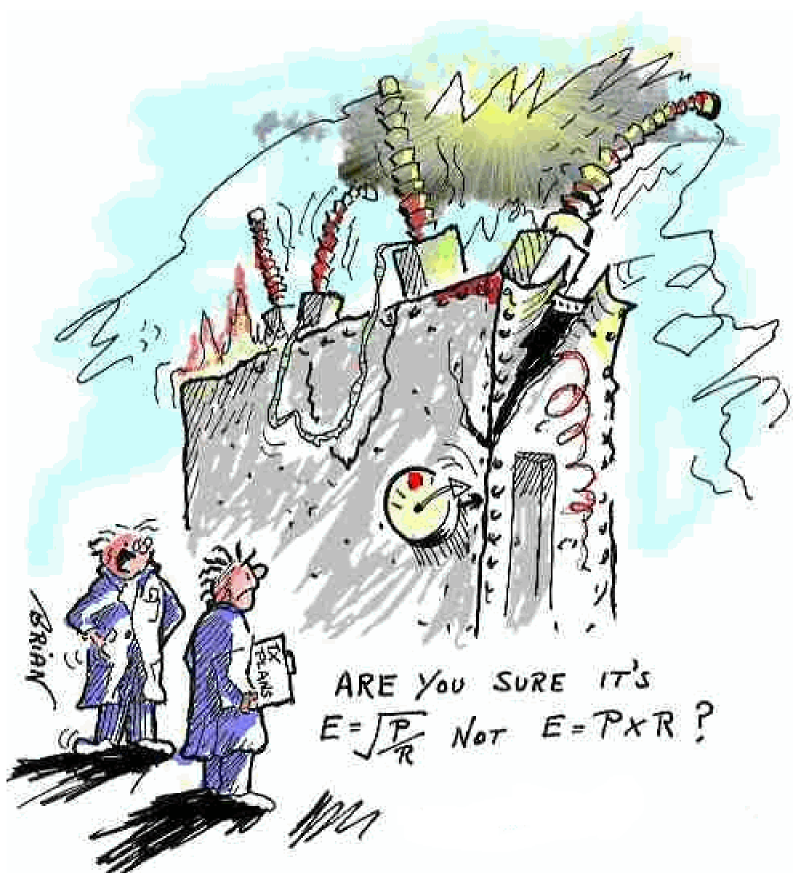
Chapter 4:

Solving Literal Equations



Day 1: The Basics of Literal Equations

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Warm-Up

The solution of the equation $5 - 2x = -4x - 7$ is

(1) 1	(3) -2
(2) 2	(4) -6

What is a literal equation?

A literal equation is an equation with two or more variables. Instead of solving for a numerical value, we solve for one variable in terms of another. A formula is one type of literal equation that has special applications in math or science.

Observe the similarities between the linear equation (left) and the **literal equation** (right):

One-Step Linear Equation:	*One-Step Literal Equation:*
1) $y + 10 = 55$	2) $y + x = 55$ *solve for y*
3) $s - 40 = 85$ *solve for s*	4) $s - x = 85$ *solve for s*

Two-Step Linear Equation:	*Two-Step Literal Equation:*
5) $2a - 13 = 87$ *solve for a*	6) $2a - b = c$ *solve for a*

*Solving for a variable using **division**:*

7) $3x = 45$	8) $3x = y$ *solve for x*

Quick Check for Understanding

9) $2x + y = 9$ *solve for y*	10) $-3b - c = d$ *solve for b*	11) $P = mv$ *solve for m*

Application

12) The formula $d = rt$ relates the distance an object travels, *d*, to its average rate of speed *r*, and amount of time *t* that it travels.

a) Solve the formula $d = rt$ for *t*.

b) How many hours would it take for a car to travel 150 miles at an average rate of 50 miles per hour?

Independent Practice   Solve for the variable indicated.

1) $d = rt \quad Solve\ for\ r$	2) $P = a + b \quad Solve\ for\ b$
3) $y = mx + b \quad Solve\ for\ x$	4) $T = M - N \quad Solve\ for\ N$
5) $Ax + B = C \quad Solve\ for\ x$	6) $Ax + By = C \quad Solve\ for\ y$
7) $I = prt \quad Solve\ for\ r$	8) $C = \pi d \quad Solve\ for\ d$

9) $s = r\theta$ $Solve\ for\ r$	10) $E = IR$ $Solve\ for\ R$
11) $E = mc^2$ $Solve\ for\ c^2$	12) $P = 2l + 2w$ $Solve\ for\ y$
13) $5m + n = 10$ $Solve\ for\ m$	14) $5 - b = 2t$ $Solve\ for\ t$
15) $PV = nRT$ $Solve\ for\ R$	16) $y = 3x - 1$ $Solve\ for\ x$

17) The volume of a prism is $V = lwh.$

a) Solve this formula for h.

b) If the volume of a prism is 64, its length 4, and its width 2, what is its height?

Summary

Literal Equations

Definition	**Facts**
Literal equations are equations with two or more variables.	Formulas are literal equations. Literal equations are solved the same way as equations, by using inverse operations.
Examples	**Non Examples**
$d = rt$ $A = \frac{1}{2}bh$ $m + n = 3p$	$-4x = 20$ $10 = \frac{1}{3}(y - 6)$ $n + 5 = 2n - 14$

<u>Homework</u>

Chapter 4- Day 1 -Textbook pp. 109-110 #2, 5, 8, 9, 10, 11, 14, 20, 23, 26, 36-37

Day 2: Solving Literal Equations with Proportions

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Warm-Up

If $3ax + b = c$, then x equals

1. $c - b + 3a$
2. $c + b - 3a$
3. $\frac{c-b}{3a}$
4. $\frac{b-c}{3a}$

Model Problems   Solving Proportions

Solve for x in each equation.

Linear Equations:	*Literal Equation:*
1) $\frac{x}{3} = 9$	2) $\frac{x}{3} = y$
3) $2x - 1 = \frac{3}{2}$	4) $\frac{x}{2} = m - 6$

5) $10 = \frac{2}{3}(x - 4)$	6) $D = \frac{11}{5}(x - 15)$ Reminder: Don't distribute a coefficient unless absolutely necessary!

Application

7) The formula to convert Celsius to Fahrenheit is given by $C = \frac{5}{9}(F - 32)$.

a) Solve this formula for *F*.

b) The boiling point of water is 100°F. What is the Fahrenheit equivalent of this temperature?

8) Check for Understanding Solve for the given variable.

a) $d = \frac{c}{n}$ solve for n	b) $A = \frac{a+b}{2}$ solve for b	c) $F = \frac{Gm_1m_2}{r^2}$ solve for m_1

d) The formula for the mean (average) A of two numbers y and z is one-half their sum, or $A = \frac{1}{2}(y + z)$.
If the average of two numbers is 7 and one of the numbers is 4, find the other number.

Cumulative Independent Practice Days 1-2 Solve for the value of the variable.

1) $\frac{m}{k} = x$ for k	2) $V = \frac{1}{3}Ah$ for A

3) $s = \frac{1}{2}gt^2 \; for \; g$	4) $s = \frac{w-10e}{m} \; for \; w$
5) $q + r = \frac{p}{5} \; for \; q$	6) $q + r = \frac{p}{5} \; for \; p$
7) $\frac{x}{7} - y = t \; for \; x$	8) $\frac{x-y}{7} = t \; for \; x$

9) $\frac{x-y}{7} = t \text{ for } y$	10) $P = R - C \text{ for } C$
11) $R = \frac{C-S}{t} \text{ for } C$	12) $2x + 7y = 14 \text{ for } y$
13) $m = \frac{y_2 - y_1}{x_2 - x_1} \text{ for } y_2$	14) $V = \frac{2}{3}(x + 2y) \text{ for } x$

15) The formula $V = \frac{1}{3}\pi r^2 h$ is the formula for the volume of a cylinder. To the nearest tenth, what is the height of a cylinder with volume 100 cm^3 and radius 2 cm?

HW Chapter 4-Day 2 Textbook pp. 109-110 #6, 7, 12, 13, 15, 18, 24, 30, 45

Day 3: Using the Distributive Property and Rational Equations

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Warm-Up

The formula $P = 2(L + W)$ is the formula for the perimeter of a rectangle. Solve this formula for L. What is the length of a rectangle whose perimeter is 48 and whose width is 6?

Distribution and Reverse distribution

1) When there is a common factor in all terms of an expression, we can use the distributive property in reverse to write it in factored form.

Simplest form	**Factored Form**
a) $2L + 2W$	$2(L + W)$
b) $3a - 3b$	
c) $2lw + 2l$	
d) $fb + fa$	
e) $2\pi rh + 2\pi r^2$	

2) Model Problem *Using the Distributive Property in Reverse*

Solve for *c* in terms of *a* and *b*: $ac + bc = ab$

3) Practice

a)	b)
If $a + ar = b + r$, the value of a in terms of b and r can be expressed as	If $k = am + 3mx$, the value of m in terms of a, k, and x can be expressed as

Using Rational Equations

4) Model Problem

The formula $\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$ relates an object's distance, a, and its image's distance, b, to the focal length of the lens, f. Solve this formula for f.

5) Practice

The total resistance in a circuit is given by the formula $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. Solve this formula for R_1.

Unit Summary So Far:

Look for STRUCTURE in equations:	
One- or Two-Step Equations $Ax + B = C$ $x + b = c$	*Proportions* $D = \frac{M}{V}$ $K = \frac{1}{2}mv^2$ $\bar{x} = \frac{x_1 + x_2}{2}$
Reverse Distribution (Common Factor) $S = 2\pi r^2 + 2\pi rh$	*Rational Equations (Sums and Differences)* $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

Cumulative Practice/Homework Chapter 4 – Day 3 Solve for the requested variable.

1) $r = pn \ for\ n$	2) $V = \frac{1}{3}Bh\ for\ B$
3) $s = \frac{2x+t}{r}\ for\ x$	4) $v = v_0 + at\ for\ v_0$

5) $J = mv_f - mv_i \text{ for } m$	6) $E = IR \text{ for } I$
7) $y - y_1 = m(x - x_1) \text{ for } x$	8) $U = mgh \text{ for } g$
9) $R = \frac{\rho l}{A} \text{ for } A$	10) $F + V - E = 2 \text{ for } V$

11) $U = \frac{1}{2}QV\ for\ V$	12) $z + y = x + xy^2\ for\ x$
13) $G = H - TS\ for\ H$	14) $F_C = \frac{mv^2}{r}$ for m
15) $P = P_0 + \rho gh$ for h	16) $e = \frac{T_H - T_C}{T_H}\ for\ T_H$

Multiple Choice Practice.

17)

The formula $V = \frac{Bh}{3}$ shows how to find the volume of a pyramid. Solve for B.

F $B = \frac{3V}{h}$

H $B = 3Vh$

G $B = 3V - h$

J $B = 3V + h$

18)

The cost of operating an electrical device is given by the formula $C = \frac{Wtc}{1000}$ where W is the power in watts, t is the time in hours, and c is the cost in cents per kilowatt-hour. Solve for W.

F $W = 1000C - tc$

G $W = \frac{Ctc}{1000}$

H $W = 1000C + tc$

J $W = \frac{1000C}{tc}$

Real-World Application.

19) The cost to mail a letter in the United States in 2008 was \$0.41 for the first ounce and \$0.26 for each additional ounce. Solve $C = 0.41 + 0.26(z - 1)$ for z.

If Patty paid 0.93 to mail her letter, how many ounces was it? (C = cost, z = ounces)

Day 4: Square Roots

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Warm-Up

The formula for the volume of a pyramid is $V = \frac{1}{3}Bh$. What is h expressed in terms of B and V?

1 $h = \frac{1}{3}VB$

2 $h = \frac{V}{3B}$

3 $h = \frac{3V}{B}$

4 $h = 3VB$

Mini-Lesson: *Using Square Roots*

To solve for a squared variable, take its square root.

Linear Equation: $64 = 16x^2$	*Literal Equation:* $A = \pi r^2$ *solve for r*

Check for Understanding Solve for the indicated variable.

1) The formula for kinetic energy is $K = \frac{1}{2}mv^2$. Write an expression for v in terms of K and m.	2) The gravitational force F that two planetary bodies exert on one another is given by $F = \frac{Gm_1m_2}{r^2}$. Solve this formula for r.

Cumulative Practice/Homework Solve for the value of the indicated variable.

1) $V = lwh$ *Solve for h*	2) $s = \frac{1}{2}at^2$ *solve for t*
3) $A = \frac{1}{2}h(b_1 + b_2)$ *Solve for h*	4) $Solve\ for\ r$: $\frac{p+r}{3} = m + 5$
5) Solve $R = \frac{l+3w}{2}$ *for w*	6) *Solve* $ax + by + c = 0\ \ for\ y$

7) $A = 2\pi rh + 2\pi r^2$ *Solve for π*	8) *Rewrite* $K = \frac{3}{2}kT$ *solved for T in terms of k and T.*
9) $q - 3r = 2 \quad Solve\ for\ r$	10) *In* $a + ax = b$, *what is a in terms of x and b?*
11) $\frac{5-c}{6} = d - 7$ *Solve for c*	12) $a_c = \frac{v^2}{r}$ *Solve for v*

Regents Practice.

13)

The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. The radius, r, of the cone may be expressed as

1 $\sqrt{\frac{3V}{\pi h}}$

2 $\sqrt{\frac{V}{3\pi h}}$

3 $3\sqrt{\frac{V}{\pi h}}$

4 $\frac{1}{3}\sqrt{\frac{V}{\pi h}}$

14)

The distance a free falling object has traveled can be modeled by the equation $d = \frac{1}{2}at^2$, where a is acceleration due to gravity and t is the amount of time the object has fallen. What is t in terms of a and d?

1 $t = \sqrt{\frac{da}{2}}$

2 $t = \sqrt{\frac{2d}{a}}$

3 $t = \left(\frac{da}{d}\right)^2$

4 $t = \left(\frac{2d}{a}\right)^2$

15)

The volume of a large can of tuna fish can be calculated using the formula $V = \pi r^2 h$. Write an equation to find the radius, r, in terms of V and h. Determine the diameter, to the *nearest inch*, of a large can of tuna fish that has a volume of 66 cubic inches and a height of 3.3 inches.

16)

The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$. Express b_1 in terms of A, h, and b_2. The area of a trapezoid is 60 square feet, its height is 6 ft, and one base is 12 ft. Find the number of feet in the other base.

Day 5: Review

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Look for STRUCTURE in equations:

One-Step Equations	
1) $I = prt$ *Solve for r.*	2) $T = M - N$ *Solve for M.*
Two-Step Equations	
3) $5t - 2r = 25$ *Solve for t.*	4) $vt - 16t^2$ *Solve for v.*

Proportions	
5) $F = \frac{lt}{d}$ *Solve for l.*	6) $P = \frac{144p}{y}$ *Solve for p.*
7) $A = \frac{1}{2}h(a + b)$ *Solve for a.*	8) $m = \frac{y_2 - y_1}{x_2 - x_1}$ *Solve for* y_2

Reverse Distribution	
9) $S = R - rR$ *Solve for R.*	10) $ax = bx + c$ *Solve for x.*
Rational Equations	
11) $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$ *Solve for* C_1	12) $\frac{x}{3} + \frac{x}{4} = d$ *Solve for x.*
Square Roots	
13) $K = \frac{1}{2}mv^2$ *Solve for v.*	14) $V = \frac{1}{3}\pi r^2 h$ *Solve for r.*

Applications. The surface area of a sphere is given by the formula $S = 4\pi r^2$. Solve this formula for r. What is the radius of a sphere whose surface area is 201 cm^2?