Chapter 4: Systems

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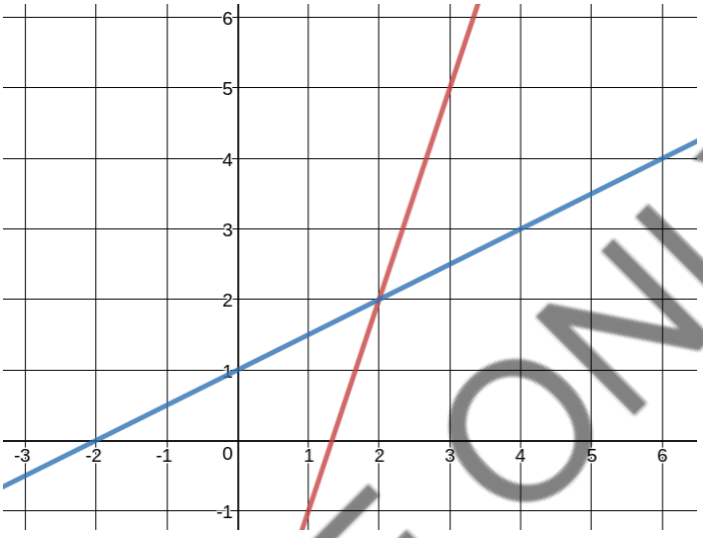
SAMPLE ONLY

Lesson 1: Graphing Systems of Linear Equations

In Chapter 3, we graphed linear equations in slope-intercept form. In this chapter we will be looking at systems of equations, or two or more equations graphed on the same coordinate plane.

Exercise #1: Take a look at the *system of linear equations* below.



a) Where do the two lines intersect?

b) What are the equations of these linear functions? You may need to use either slope-intercept form, the point-slope formula, or both. Label these functions with their appropriate equations.

c) Now that you have found the equations of these lines, substitute the intersection point from part "a" into each equation. What do you notice? Why do you think this is the case?

The point of intersection is the point at which the first equation is equal to the second equation. This will be helpful to know when solving systems of linear equations.

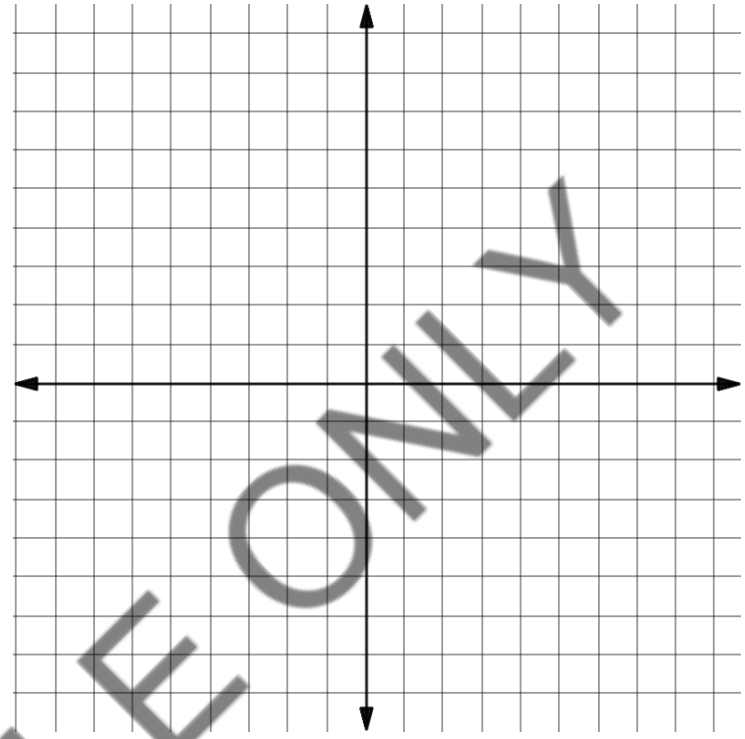
SAMPLE ONLY

Now that we recognize how to find a solution to a system of equations, let's try to solve a system of equations by graphing.

Exercise #2: Graph the system of equations on the graph provided. Then, state the solution.

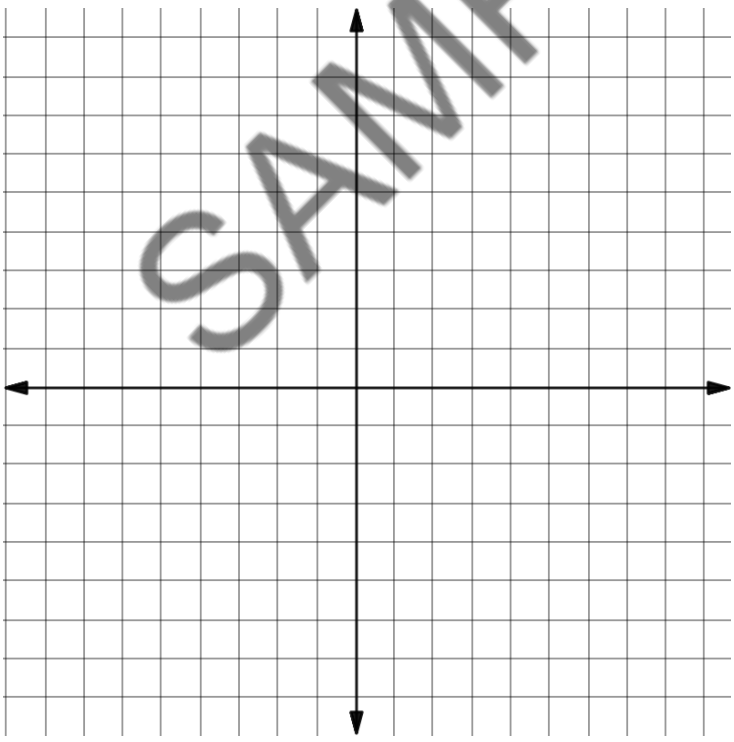
$y = \frac{-3}{4}x + 2$

$y = \frac{1}{2}x - 3$



SAMPLE ONLY

Exercise #3: Graph the system of equations and state the solution.



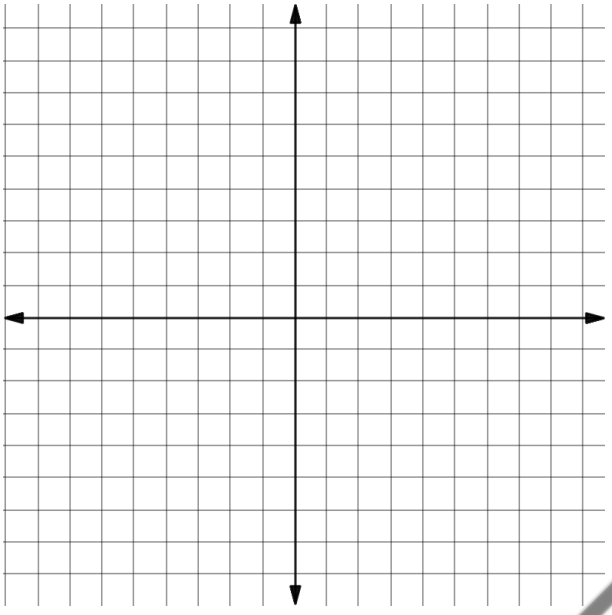
$4x + 2y = 8$

$6y = 3x + 9$

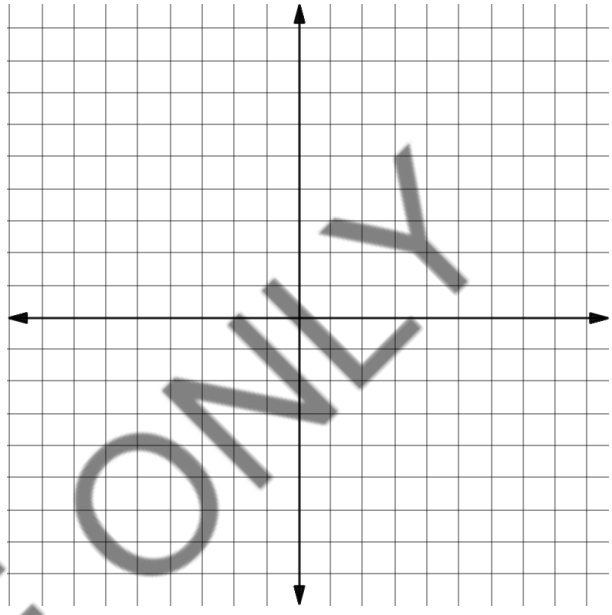
Lesson 1 Extra Practice

EP1. For each of the following, graph the system of equations, and state the solution. Be sure to label your functions.

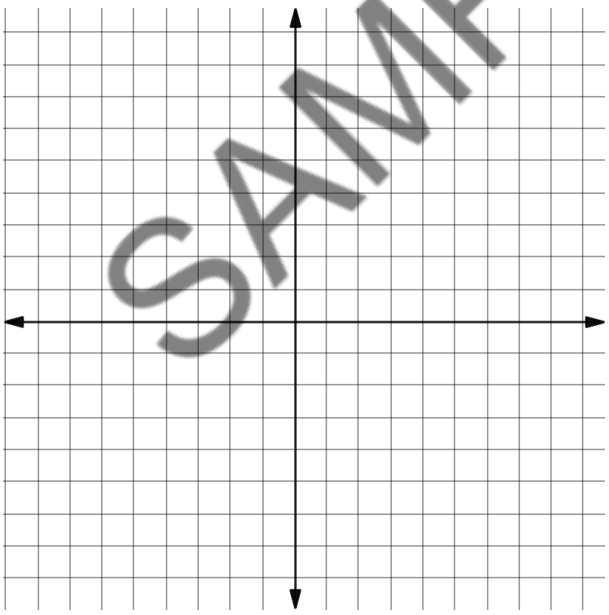
a) $y = -2x$ and $y = -\frac{1}{2}x + 3$



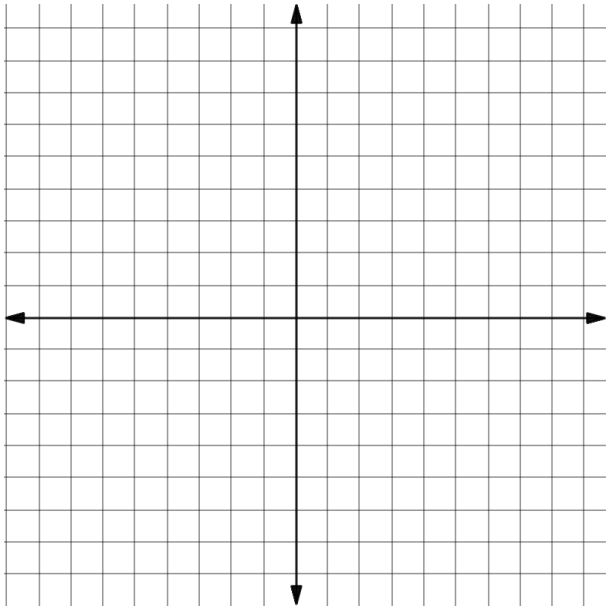
b) $y = \frac{3}{2}x - 2$ and $y = -\frac{5}{2}x + 6$



c) $f(x) = -\frac{1}{3}x - 1$ and $g(x) = \frac{1}{6}x - 4$

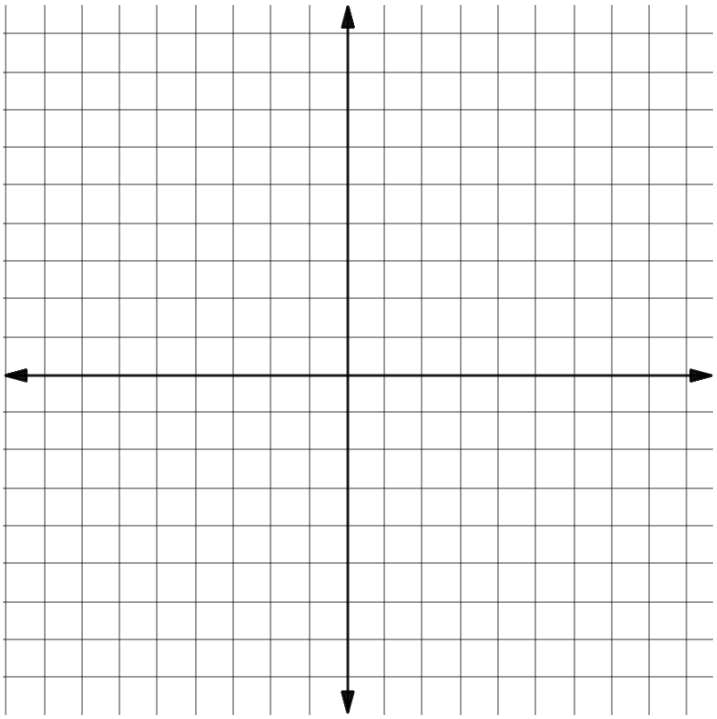


d) $j(x) = -4$ and $k(x) = -4x$



SAMPLE ONLY

EP2. Graph the system of equations on the graph below.



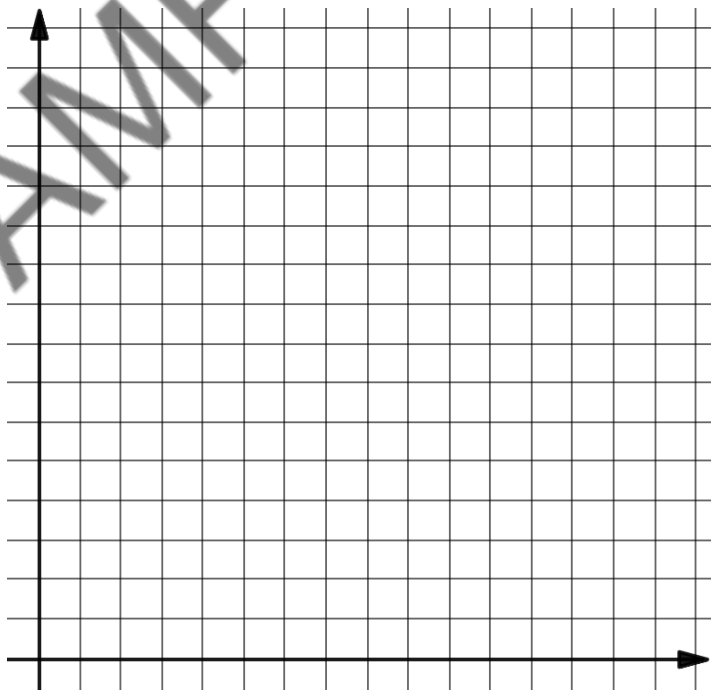
$y = -3$

$x = -2$

$y = -\frac{1}{2}x$

After graphing the system of equations above, find the area of the shape enclosed by the linear functions.

EP3. Patrick is debating between two different gym memberships. Planetary Gym offers a flat rate of $50 that includes all specialty classes. Long Island Sports Club charges $25 for the membership plus an additional $5 per specialty class. At what number of specialty classes will it be cheaper for Patrick to choose Planetary Gym? Support your answer with the graph.

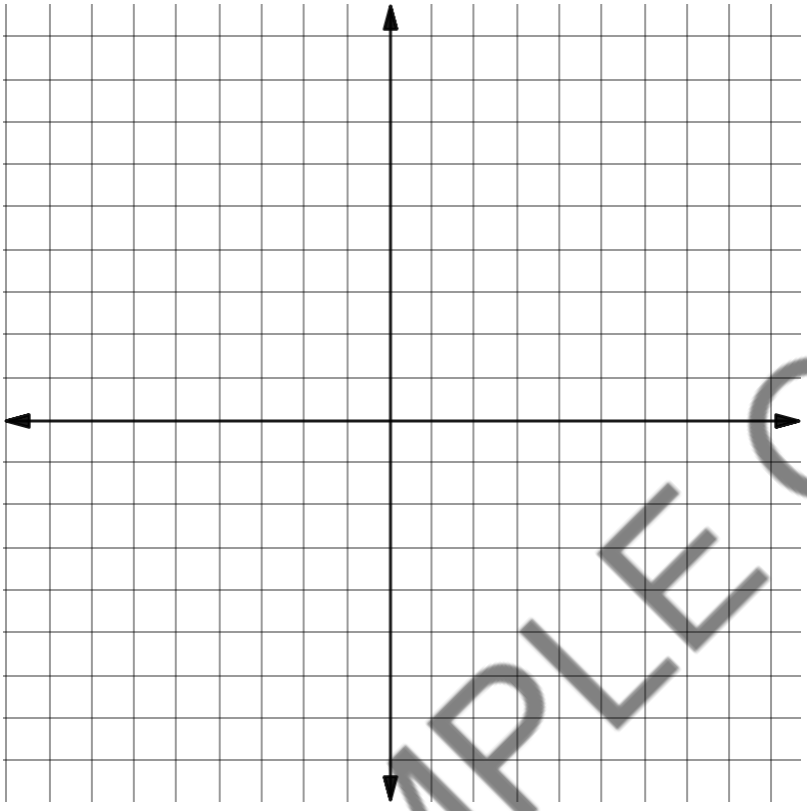


SAMPLE ONLY

Lesson 2: Solving Systems of Equations by Substitution

In Lesson 1, you were able to graph a system of equations and state the solution by finding the intersection point. In Algebra, the solution won't always be an integer so we will need to know how to solve systems using other methods as well. Today, we will learn how to solve systems of equations by substitution.

Exercise #1: Let's begin with the following system of linear equations.



$y = -2x + 5$

$y = \frac{3}{2}x$

a) What do you notice about the intersection point?

Now, let's try to solve the system using the substitution method.

b) Since y is equal to $-2x + 5$, substitute $-2x + 5$ in for y in the second equation. Then, solve for the *exact* value of x.

c) Use the exact value you found for x to substitute into either equation. Now, find the *exact* value of y.

SAMPLE ONLY

Exercise #2: Solve the following systems of equations using the substitution method.

a) $y = 2x + 6$
$3x + y = -9$

b) $y = -3$
$2x + 2y = 0$

c) $x - y = 4$
$y = -4x + 6$

d) $y = 0.5x - 1.5$
$0.25x + 0.75y = 4.25$

SAMPLE ONLY

Exercise #3: Each of the examples in Exercise #2 were presented where one variable was already isolated. This will not always be the case. How would you go about solving the system of equations here?

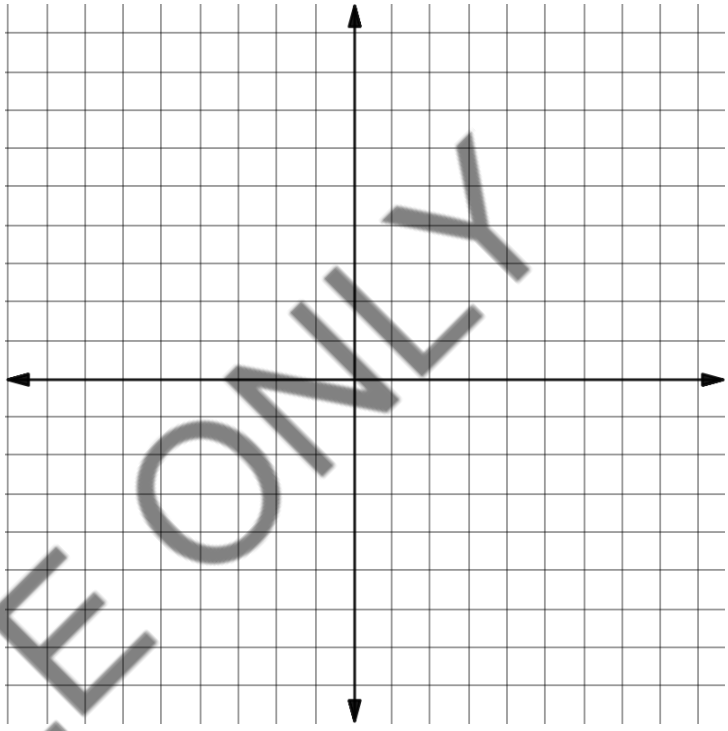
$8x - 2y = 4$
$2y + 3x = 7$

In Lesson 3, you will learn how to solve a system of equations by elimination, for examples that are in the form of the one above.

In each of the examples in this lesson so far, you have successfully solved a system of equations using the substitution method, and obtained a rational solution. Now, we are going to look at various types of solutions you may encounter during your study of Algebra 1.

Exercise #4:
Solve the following system of equations using the substitution method.

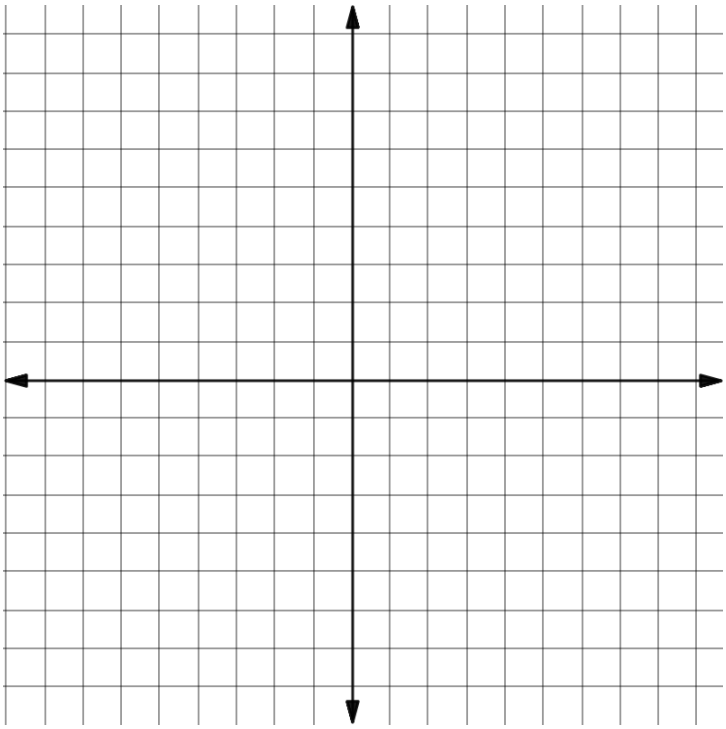
$$y = x + 3$$
$$-3x + 3y = -3$$



Now, graph the two linear equations. What do you notice? Does this make sense with your solution in part "a"?

Exercise #5:
Solve the following system of equations using the substitution method.

$$y = 4 - 2x$$
$$-4y = 8x - 16$$



Now, graph the two linear equations. What do you notice? Does this make sense with your solution in part "a"?

SAMPLE ONLY

Lesson 2 Extra Practice

EP1. Solve the following systems of equations using the substitution method.

a) $2x - 3y = -1$
$y = x - 1$

b) $y = -3x + 5$
$5x - 4y = -3$

c) $y = 5x - 7$
$-3x - 2y = -12$

d) $-4x + y = 6$
$-5x - y = 21$

e) $x = 3y$
$2x - 6y = 0$

f) $y = 2x$
$-6x + 3y = 16$

SAMPLE ONLY

Lesson 3: Solving Systems of Equations by Elimination

At this point in Chapter 4, you have learned how to solve a system of equations using two different methods. The final method you will become familiar with is the elimination method.

"If equals be added to equals, the wholes are equal."
- Euclid, c. 300 BCE

Exercise #1: Consider the equation 4 + 5 = 9. Is it true?

Now, multiply the equation by 3. Is the resulting equation true?

The resulting equation is true because the properties of equality always hold true for algebra. Any real number that you choose to multiply the equation by, the equation will always remain true.

Exercise #2: Create two equations that are true. Use the blank spaces below to choose numbers you would like to use.

_____ + _____ = _____

_____ + _____ = _____

Add the equations together. Is the resulting equation true?

The resulting equation is true because, once again, the properties of equality in algebra are preserved. Any two equations that are true, when added together, will result in a true equation.

SAMPLE ONLY

Exercise #3: Let's take a look at a system of linear equations.

$$2x + 3y = 12$$
$$3x - 3y = 3$$

Since we have two variables in each equation, we are unable to solve for an exact value of either variable. In order to solve for one variable, the other variable must be eliminated.

As you saw in Exercise #2, we are able to add equations together and the resulting equation remains true.

Add the given system of equations together. What do you notice about the resulting equation?

Now, solve the equation for *x*.

Use the value you found for *x*, and substitute it into either original equation to find the value of *y*.

Exercise #4: Solve each system of equations using the elimination method.

a) $2x + 7y = 4$
$-2x - y = 8$

b) $5x - 3y = 12$
$2x + 3y = -33$

SAMPLE ONLY

Exercise #5: In this exercise, you will need to eliminate a variable by multiplying one (or both) of the equations by a constant. Remember from Exercise #1, when an equation is multiplied by a constant the resulting equation will always remain true.

Solve the following systems of equations using the elimination method.

a) $3x + 6y = 24$
$4x - 3y = 10$

b) $-7x + 2y = 54$
$5x - 8y = -32$

c) $9x - 4y = 52$
$3x - 4y = 28$

d) $2x + 3y = 6$
$3x + 2y = 4$

SAMPLE ONLY

Lesson 3 Extra Practice

EP1. Solve the following systems of equations using the elimination method.

a) $-6x + 5y = 1$
$6x + 4y = -10$

b) $x - y = 11$
$2x + y = 19$

c) $-2x - 9y = -25$
$-4x - 9y = -23$

d) $5x + y = 9$
$10x - 7y = -18$

e) $8x + 14y = 4$
$-6x - 7y = -10$

f) $3x - 2y = 2$
$5x - 5y = 10$

SAMPLE ONLY

Lesson 4: Modeling Systems of Equations

In this lesson you will be combining all of the information you have learned on systems of equations, and applying it to various word problems.

Exercise #1: Two different high schools are planning a field trip to Philadelphia. Long Beach High School has 3 filled buses, and 4 filled vans with a total of 186 students and teachers. Wantagh High School has 4 filled buses and 2 filled vans with a total of 218 students and teachers. How many people does a full bus hold? How many people does a full van hold?

Exercise #2: The sum of two integers is 93. Their difference is 35. What are the two numbers?

Exercise #3: Kylie has a bank consisting of only nickels and dimes. The bank has exactly 350 coins in it that total $23.10. How many of each type of coin does Kylie have in her bank?

SAMPLE ONLY

Exercise #4: Natalie is sixteen years younger than three times Arjun's age. Cecelia is five years older than twice Arjun's age. If Natalie and Cecelia are twins, how old is Arjun?

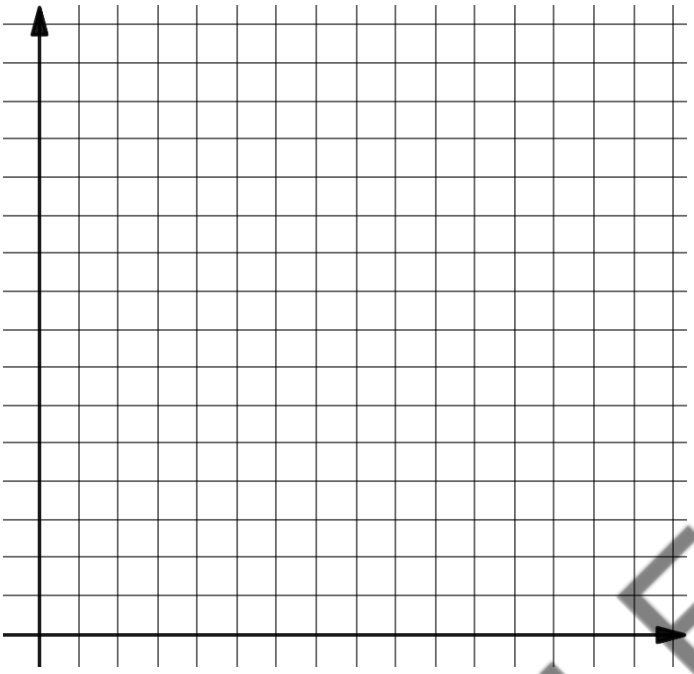
Exercise #5: Sage is going to the movies with 5 of her friends. The movie theater sells tickets for $12 each, popcorn for $6.25, and soda for $5.50. The group of friends spent $124.50 on tickets and 9 additional concession stand items, consisting of popcorn and soda. How many popcorns and how many sodas did Sage and her friends purchase?

Exercise #6: Oceanside High School had 14 students on the track team in 2015. Over the next few years, the track team saw an increase of 6 members per year. That same school had 46 students on the swim team and saw a decrease of two members per year. What year will both teams have the same number of members?

SAMPLE ONLY

Lesson 4 Extra Practice

EP1. Zachary and his friends are going to a baseball game. They buy x order of fries for $4.00 each, and y hamburgers for $6.00 each. If they bought 12 total items and spent $60.00, how many of each item did they buy? Set up a system of equations, and graph your solution on the grid below.



EP2. Greta and Stefan are at the local bakery buying muffins and brownies. Greta buys 2 muffins and 4 brownies for $11. Stefan buys 3 muffins and 2 brownies for $8.70. What is the price of one muffin and one brownie?

SAMPLE ONLY

EP3. The profit of Company A can be modeled by the function $A(x) = 25x + 50$, where A(x) represents the total profit, in dollars, and x represents the number of months since December 2018. The profit of Company B can be modeled by the function $B(x) = -10x + 400$, where B(x) represents the total profit, in dollars, and x represents the number of months since December 2018. . Determine how many months it will take for both companies to make the same amount of profit.

EP4. A picture has a length that is 6 more inches than twice its width. If the perimeter of the picture is 66 inches, what are its dimensions?

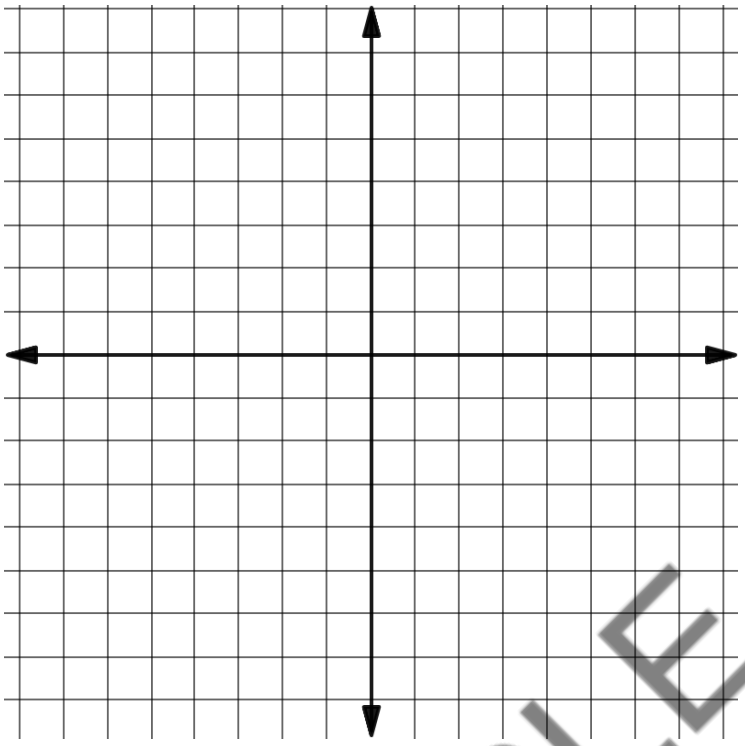
EP5. Isabella and Cole are preparing for junior class picnic. Isabella goes to SaveMart and buys 125 bottles of water and 50 bottles of lemonade for a total of $160. Cole buys 50 bottles of water and 80 bottles of lemonade and spends $136. What is the cost of one bottle of water? What is the cost of one bottle of lemonade?

SAMPLE ONLY

Lesson 5: Graphing Systems of Inequalities

In Chapter 3, you learned how to solve and graph a linear inequality. You will need to apply that knowledge in order to graph a system of linear inequalities.

Exercise #1: Graph the system of inequalities on the grid below.

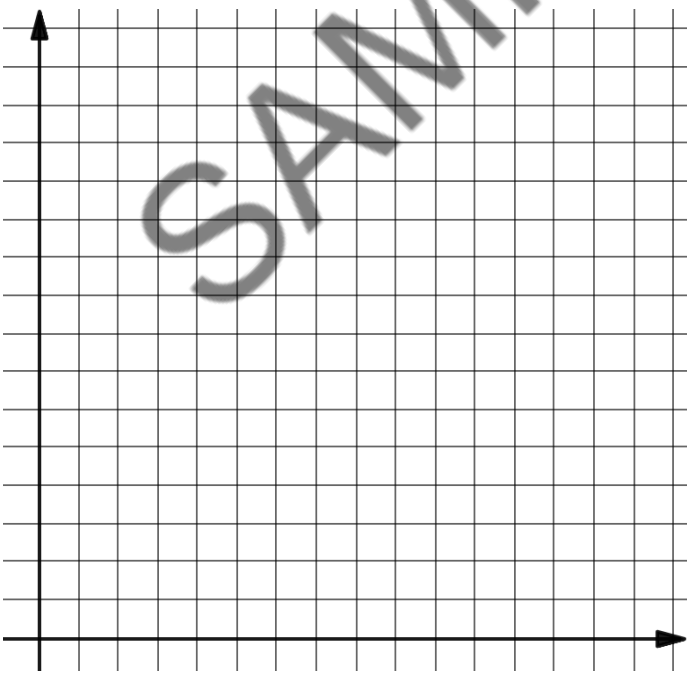


$$y < \frac{1}{5}x + 2$$

$$y \geq 2x - 7$$

a) Explain why (-5, -3) is a point in the solution but (5, 3) is not.

Exercise #2: Graph the system of inequalities on the grid below.



$$20x + 50y > 200$$

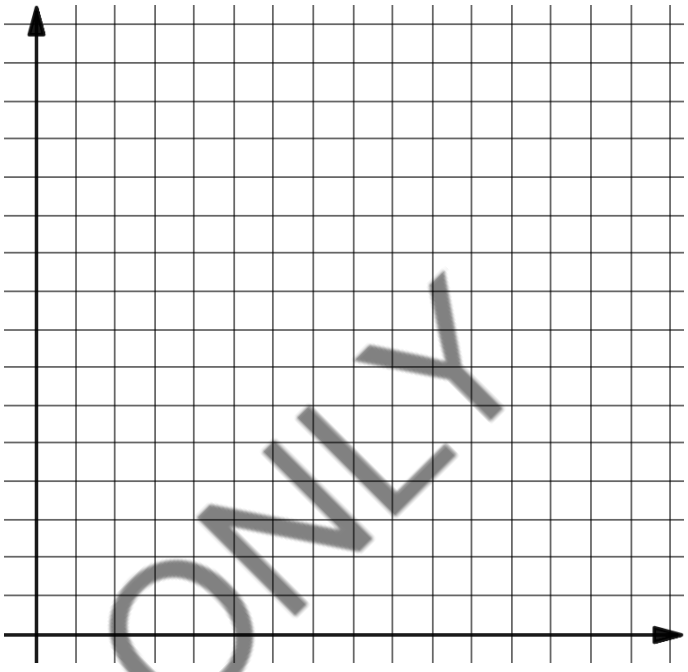
$$50x - 250y \geq -500$$

SAMPLE ONLY

Exercise #3: Graph the following system of inequalities.

$$y \geq 40x - 20$$

$$y \geq -20x + 280$$



a) State whether each of the following points is in the solution set.

(10, 140)

(7, 300)

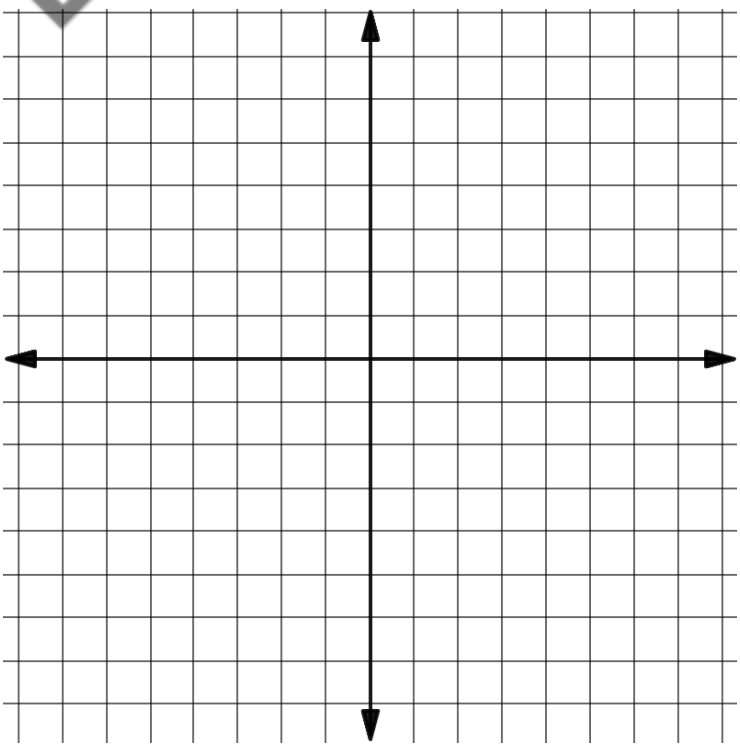
(5, 180)

Exercise #4: Graph the system of inequalities on the grid provided.

$$y \geq -5$$

$$x \leq 5$$

$$y \leq 2x - 1$$



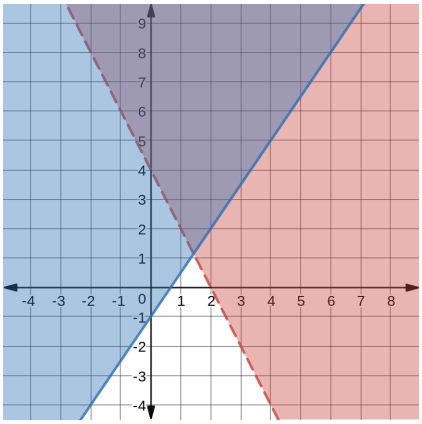
a) What is the area of the figure enclosed by solution set?

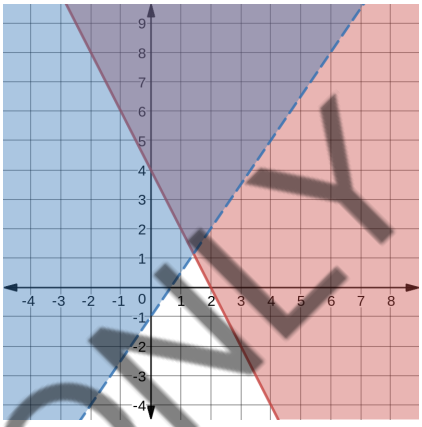
SAMPLE ONLY

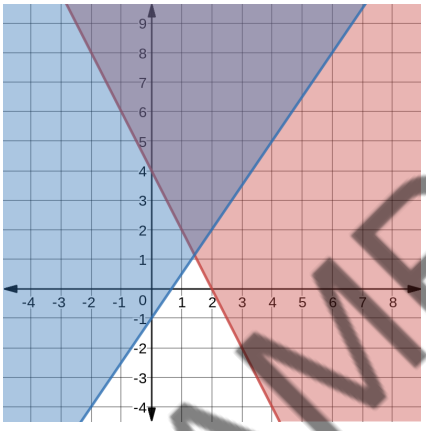
Lesson 5 Extra Practice

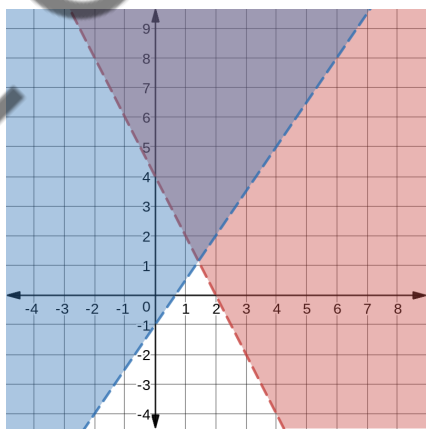
EP1. Which of the following is the graph of the following system:

$$2y + 4x > 8$$
$$3x - 2y < 2$$







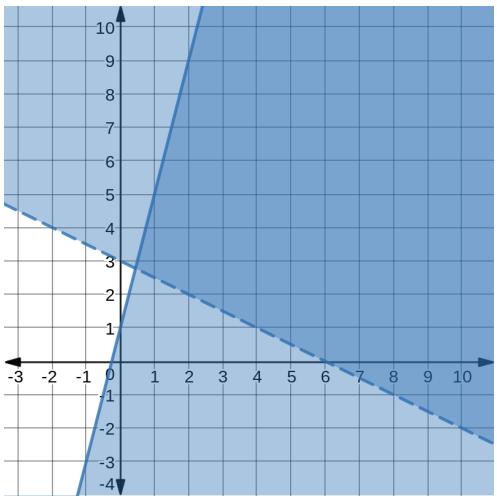


SAMPLE ONLY

EP2. Andre graphed the system of inequalities on the graph below. What error, if any, did Andre make?

$$y \leq 4x + 1$$

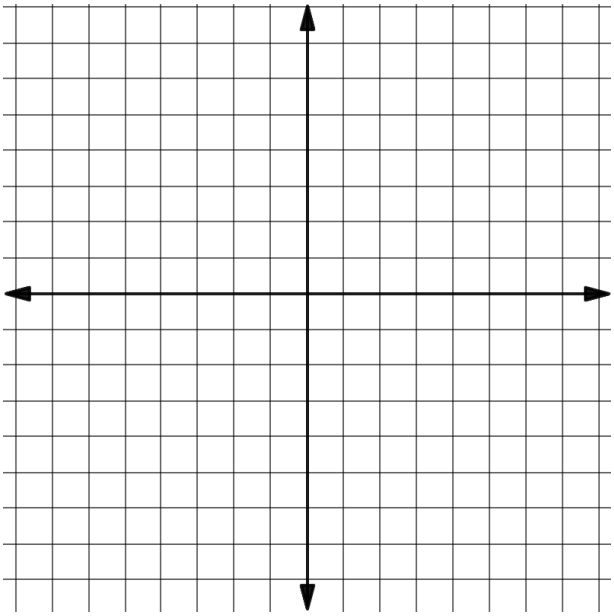
$$-2y > x - 6$$



EP3. On the set of axes below, graph the systems of inequalities.

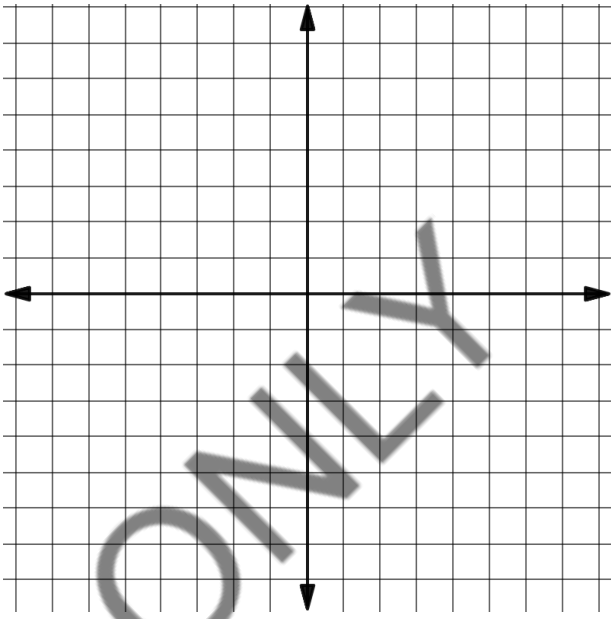
$y \leq 3x - 2$

$-4y < x - 8$



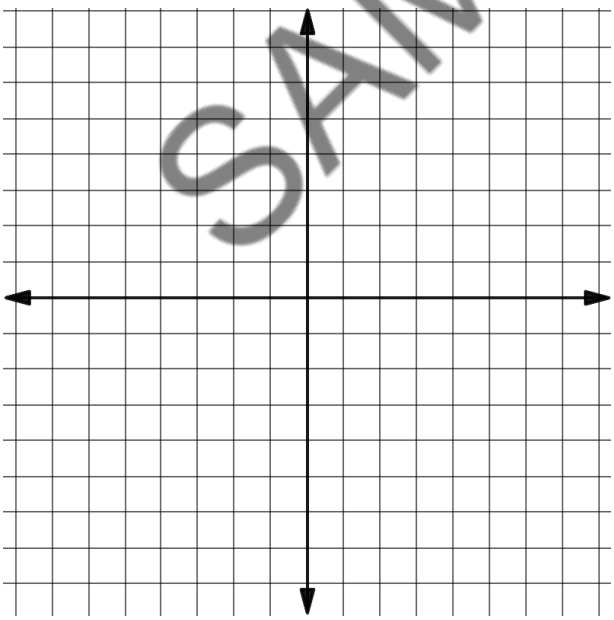
$y < \frac{1}{3}x + 7$

$y \geq 5x - 7$



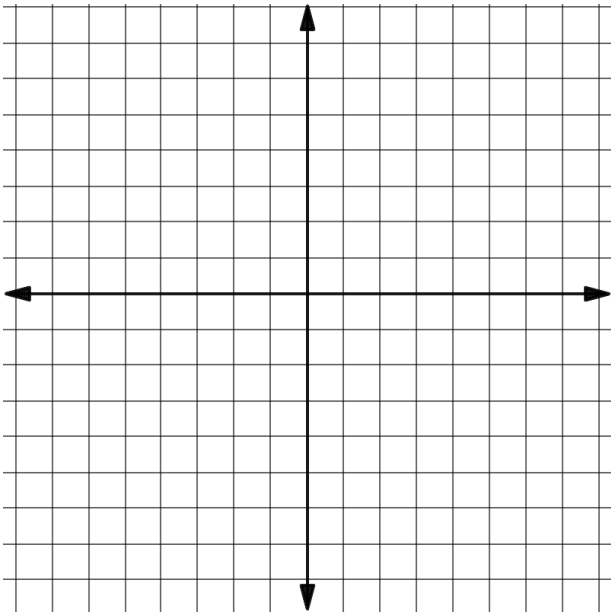
$2y \geq -3x - 10$

$-5y \leq -15x + 25$



$4x + 2y < 14$

$6x + 8y \geq -16$



SAMPLE ONLY

Lesson 6: Modeling Systems of Inequalities

As you saw in Lesson 5, systems of linear inequalities have infinitely many solutions, compared to systems of linear equations that oftentimes only have one solution.

A major component of Algebra I is being able to model a system, whether it be equations or inequalities. Today, we will examine real-life scenarios where systems of inequalities would be useful.

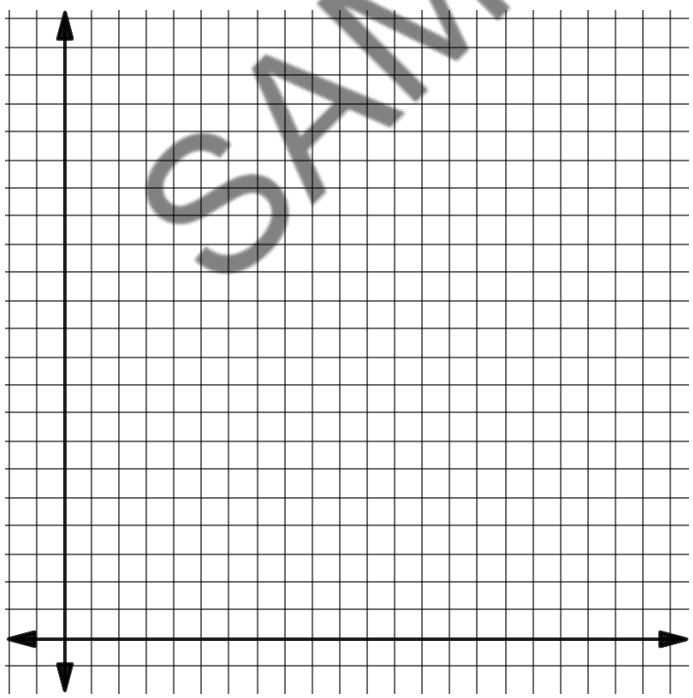
Exercise #1: The swim team is fundraising for a new diving board. A local bagel store agrees to donate bagels and juice. The swim team will sell the bagels for $1.50 each and the juice for $2.00 each. The bagel store will donate a maximum of 500 items, and the team must raise at least $800.

a) Write a system of linear inequalities to represent this situation.

b) The swim team sells 350 cartons of juice. What is the least number of bagels that must be sold to be able to raise enough money to purchase the diving board?

SAMPLE ONLY

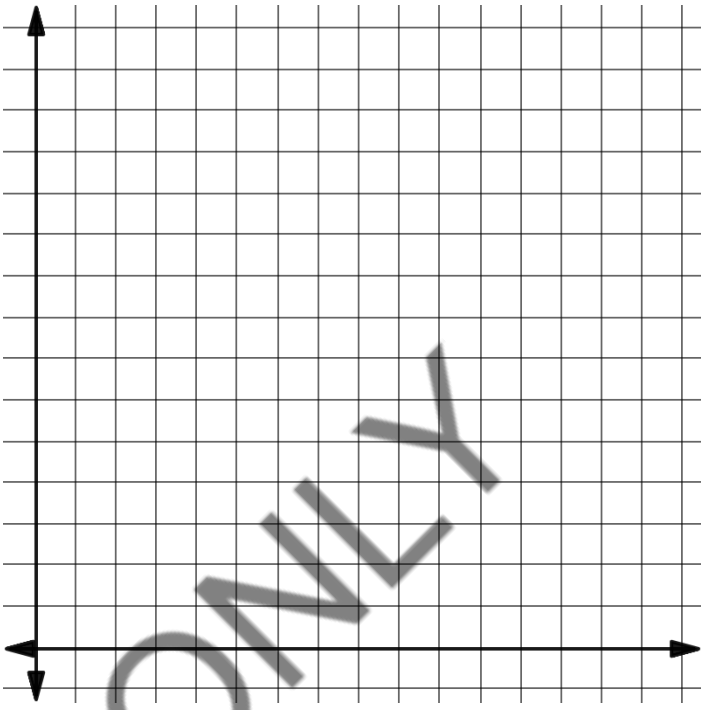
Exercise #2: The sum of two integers, x and y, is more than 20. If you take the sum of x and four times y, the result is less than 50.



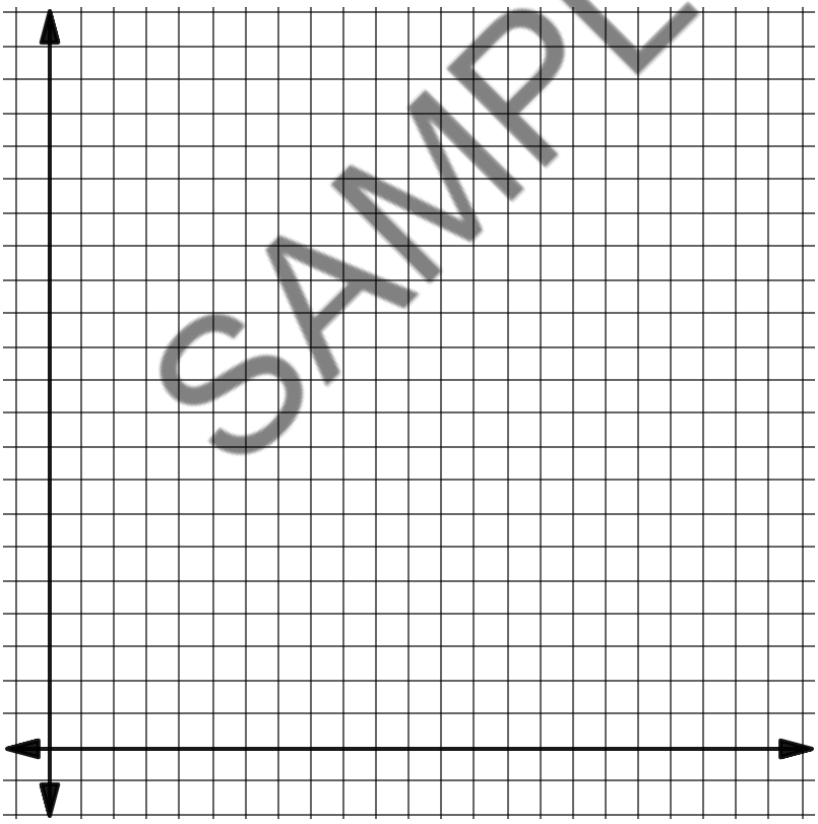
a) Set up a system of inequalities and graph the system on the grid.

b) State one possible combination of integers that satisfy the constraints of the problem.

Exercise #3: In football, field goals are three points each, and touchdowns with an extra point are seven points each. The Massapequa High School football team scored more than 49 points in their game last Saturday. The team had at most 11 scoring plays that were a combination of only field goals, x, and touchdowns with the extra point, y. Graph a system of inequalities and state one possible combination of field goals and touchdowns that the Massapequa High School football team could have scored.



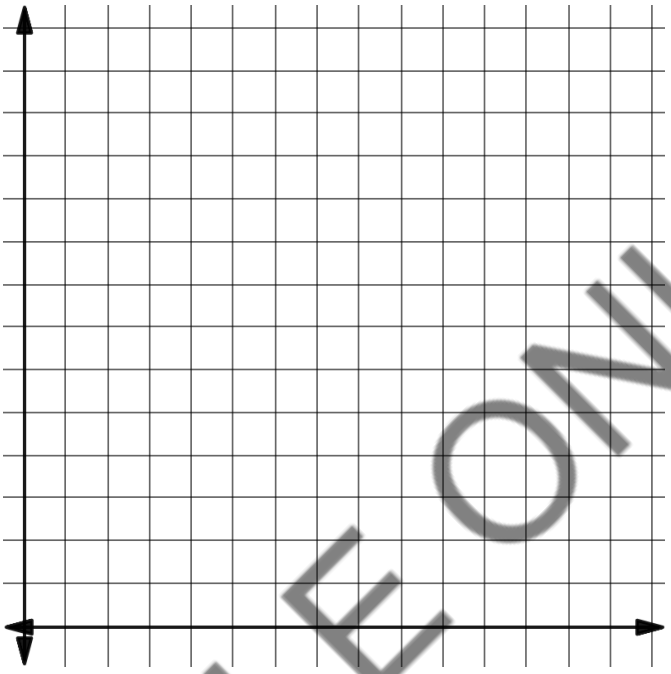
Exercise #4: A drama production is giving away tickets as part of a promotion. Orchestra tickets are $20 each, and Balcony tickets are $10 each. The production wants to give away no more than 20 tickets that have a value of at most $300. Set up a system of inequalities and graph the inequalities on the grid provided.



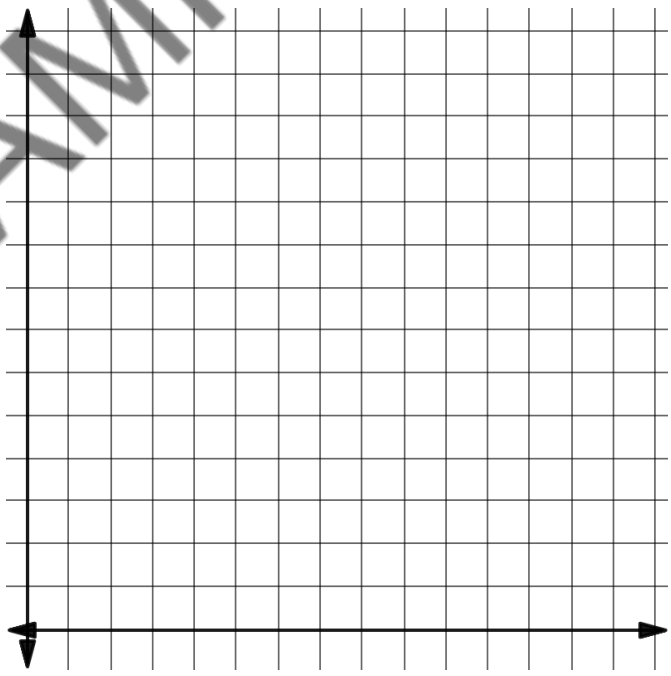
SAMPLE ONLY

Lesson 6 Extra Practice

EP1. Ariel has two jobs. She works as a babysitter and a camp counselor. She makes $10 per hour at her babysitting job and $8 per hour at the camp. She is trying to save at least $200 dollars per week, but can only work a maximum of 24 hours per week. Graph a system of inequalities to show how many hours she is able to work at each job, while still making at least $200 for her savings.



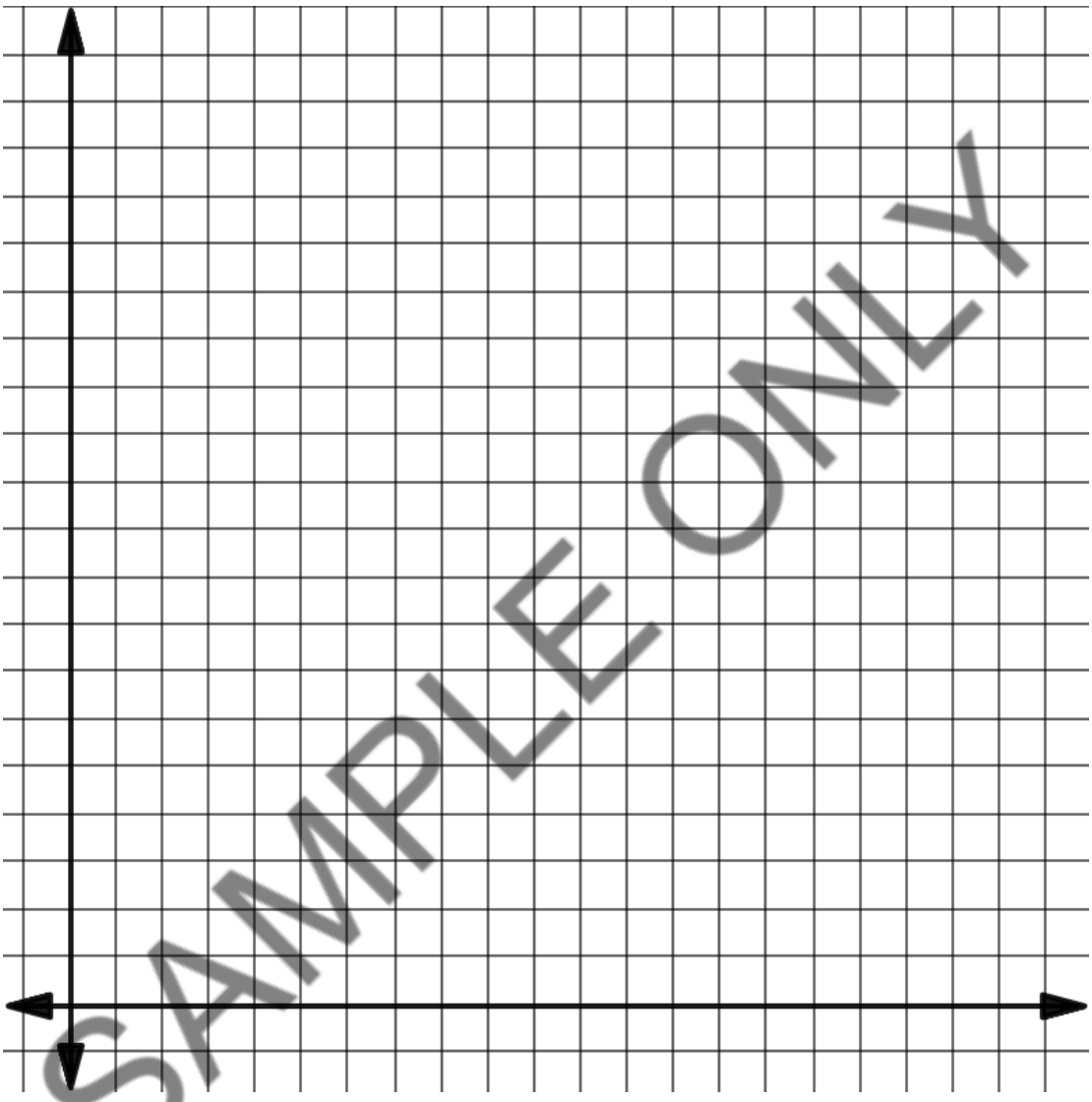
EP2. Bora is packing his books into boxes. The small boxes hold 10 books and the large boxes hold 20 books. If he must pack at least 200 books into a maximum of 20 boxes, how many small boxes and large boxes must he pack with books?



SAMPLE ONLY

EP3. Orange Computer Company must sell at least $10,000 worth of desktops and laptops each day. Each desktop, x, costs $200 and each laptop, y, costs $1000. The company can only ship a maximum of 14 items per day.

On the set of axes below, graph a system of inequalities to model this situation.



Determine one possible combination of desktops and laptops that the company could sell to fulfill all of the requirements.

Chapter Review

Part I Questions: **For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question.**

CR1. Which system of equations will yield the same result as the system below?

$$x + y = 4$$
$$2x + 4y = 10$$

1) $-2x - 2y = 4$
$2x + 4y = 10$

2) $-2x - 2y = -4$
$2x + 4y = 10$

3) $-2x - 2y = 8$
$2x + 4y = 10$

4) $-2x - 2y = -8$
$2x + 4y = 10$

CR2. A system of equations is shown below.

Equation A: $5x + 2y = 4$
Equation B: $6x - 6y = 30$

Which method eliminates one of the variables?

1) Multiply Equation A by -3 and add the result to Equation B.

2) Multiply Equation B by $-\frac{1}{3}$ and add the result to Equation A.

3) Multiply Equation A by 6 and Equation B by 2 and add the results together.

4) Multiply Equation B by 5 and Equation A by 6 and add the results together.

CR3. The equations $8x + 11y = 250$ and $4x + 8y = 160$ represent the money collected in advance from selling student and non-student tickets to the school's drama production. If x represents the cost for each student ticket and y represents the cost for each non-student ticket, what is the cost for each student ticket?

1) \$30 2) \$18 3) \$14 4) \$12

CR4. Ja'niah has 36 coins that total \$7.35. All of her coins are dimes, D, and quarters, Q. Which system of equations models this situation?

1) $D + Q = 7.35$
$.10D + .25Q = 36$

2) $D + Q = 36$
$.10D + .25Q = 7.35$

3) $D + Q = 36$
$.25D + .10Q = 7.35$

4) $D + Q = 7.35$
$.25D + .10Q = 36$

SAMPLE ONLY

CR5. During the 2018 season, baseball player Greinke's earnings, g, were 0.08 million dollars less than those of Trout's earnings, t. The two players earned a total of 68 million dollars. Which system of equations could be used to determine the amount each player earned, in millions of dollars?

1) $g + t = 68$
$g + 0.08 = t$

2) $g - 68 = t$
$t + 0.08 = g$

3) $t - 68 = g$
$g + 0.08 = t$

4) $g + t = 68$
$t + 0.08 = g$

CR6. Last week, a used book store received $180.70 for selling 24 books. Paperback books sell for $5.85 and hardcover books sell for $8.95. How many hardcover books did the store sell?

1) 9 2) 11 3) 13 4) 15

SAMPLE ONLY

Open Response Questions: **For each question, clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.**

CR7. Marco and Vincent work at a used car dealership. Marco is paid $500 per week plus 2.5% of his total sales in dollars, x, which can be represented by $m(x) = 500 + 0.025x$. Vincent is paid $460 per week plus 3.5% of his total sales in dollars, x, which can be represented by $f(x) = 460 + 0.035x$.

Determine the value of x, in dollars, that will make their weekly pay the same.

CR8. In attempting to solve the system of equations $y = 4x - 5$ and $12x - 3y = 15$, Deanna graphed the two equations on her graphing calculator. Because she saw only one line, Deanna wrote that there are no solutions to this system. Is she correct? Explain your answer.

CR9. At Lawrence High School, the sophomore class has 50 more students than the freshman class. The junior class has 30 fewer students than three times the students in the freshman class. The senior class is twice as large as the freshman class. If there are a total of 1,406 students at Lawrence High School, how many students are in the freshman class?

CR10. Three friends went to a pizzeria and ordered one plain pizza and three sodas. Their bill totaled $19.50. Later that day, six friends went to the same pizzeria. They ordered two plain pizzas and each person had one soda. Their bill totaled $39.00.

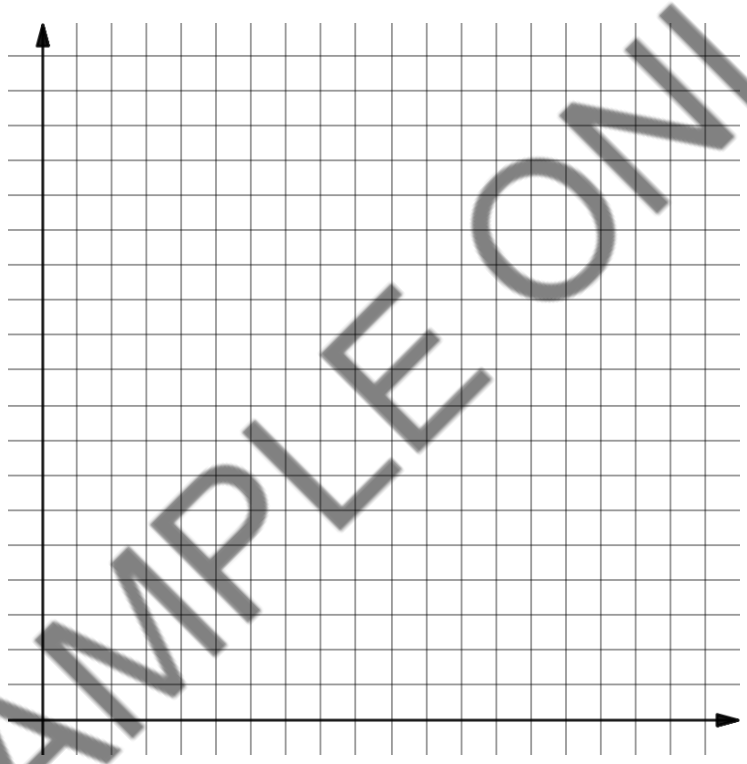
Write and solve a system of equations to determine the price of one plain pizza.

SAMPLE ONLY

CR11. Bellmore High School had twenty members in their honor society in 2014. Over the next several years, the honor society increased by an average of 8 members per year. The same school had 40 members in their band in 2014. The band saw an increase of 4 members per year.

Write a system of equations to model this situation, where x represents the number of years since 2014.

Graph this system of equations on the set of axes below.



SAMPLE ONLY

Explain in detail what each coordinate of the point of intersection of these equations means in the context of this problem.

SAMPLE ONLY