

Chapter 4: Transformations

Addressed or Prepped VA SOL:

- G.3 The student will solve problems involving symmetry and transformation. This will include
- c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
 - d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.

SOL Progression

Middle School:

- Draw polygons in the coordinate plane given the vertices, and find the lengths of sides
- Identify congruent figures and similar figures
- Verify the properties of rotations, reflections and translations

Algebra I:

- Translate, reflect, stretch and shrink graphs of functions
- Combine transformations of graphs of functions
- Use slope to solve real-life problems

Geometry:

- Perform translations, reflections, rotations, dilations and compositions of transformations
- Solve real-life problems involving transformations
- Identify lines of symmetry and rotational symmetry
- Describe and perform congruence and similarity transformations



Chapter 4: Transformations

Section 4-1: Translations

SOL: G.3.d

Objectives:

- Perform translations
- Perform compositions
- Solve real-life problems involving compositions

Vocabulary:

Component form – combines horizontal and vertical components; $\langle x, y \rangle$

Composition of transformations – when two or more transformations are combined to form a single transformation

Horizontal component – vector travel in the “x” direction

Image – figure after the transformation

Initial point – starting point of a vector; initial position in motion problems

Magnitude – the length of a vector; found by using Pythagorean Theorem on its components

Preimage – figure before the transformation

Rigid motion – a transformation that preserves length and angle measure; congruent transformation

Terminal point – the ending point of the vector

Transformation – a function that moves or changes a figure in some way to produce a new figure (called the image)

Translation – moves every point of a figure the same distance in the same direction

Vector – a quantity that has both direction and magnitude

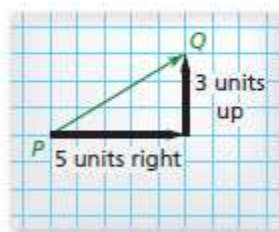
Vertical component – vector travel in the “y” direction

Core Concepts:

Core Concept

Vectors

The diagram shows a vector. The **initial point**, or starting point, of the vector is P , and the **terminal point**, or ending point, is Q . The vector is named \overline{PQ} , which is read as “vector PQ .” The **horizontal component** of \overline{PQ} is 5, and the **vertical component** is 3. The **component form** of a vector combines the horizontal and vertical components. So, the component form of \overline{PQ} is $\langle 5, 3 \rangle$.



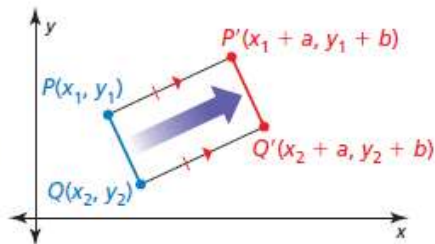
Chapter 4: Transformations

Core Concept

Translations

A **translation** moves every point of a figure the same distance in the same direction. More specifically, a translation *maps*, or moves, the points P and Q of a plane figure along a vector $\langle a, b \rangle$ to the points P' and Q' , so that one of the following statements is true.

- $PP' = QQ'$ and $\overline{PP'} \parallel \overline{QQ'}$, or
- $PP' = QQ'$ and $\overline{PP'}$ and $\overline{QQ'}$ are collinear.



STUDY TIP

You can use *prime notation* to name an image. For example, if the preimage is point P , then its image is point P' , read as "point P prime."

Postulate

Postulate 4.1 Translation Postulate

A translation is a rigid motion.

Theorem

Theorem 4.1 Composition Theorem

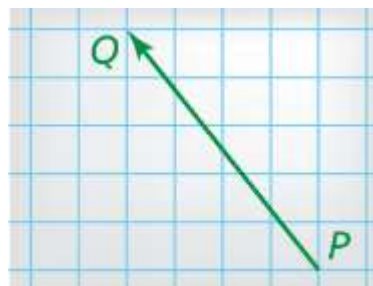
The composition of two (or more) rigid motions is a rigid motion.

Proof Ex. 35, p. 162

Examples:

Example 1

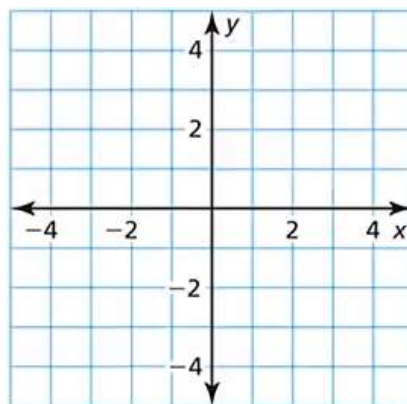
Name the vector and write its component form.



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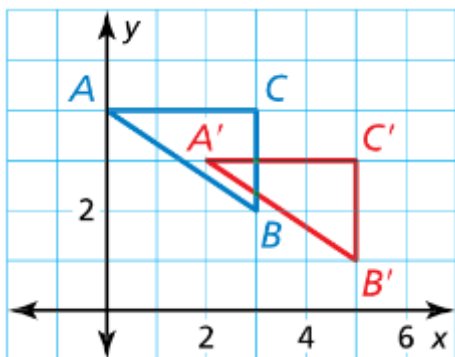
Example 2

The vertices of $\triangle ABC$ are $A(0, 3)$, $B(2, 4)$, and $C(1, 0)$. Translate $\triangle ABC$ using the vector $\langle -1, -2 \rangle$.



Example 3

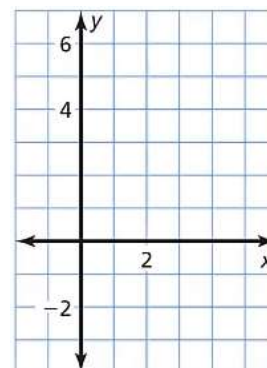
Write a rule for the translation of $\triangle ABC$ to $\triangle A'B'C'$.



$(x, y) \rightarrow (x \quad , y \quad)$

Example 4

Graph quadrilateral $ABCD$ with vertices $A(1, -2)$, $B(2, 1)$, $C(4, 1)$, and $D(4, -2)$ and its image after the translation $(x, y) \rightarrow (x - 1, y + 4)$.



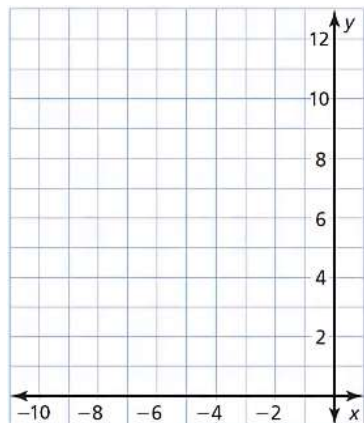
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Example 5

Graph \overline{RS} with endpoints $R(-8, 5)$ and $S(-6, 8)$ and its image after the composition.

Translation: $(x, y) \rightarrow (x - 1, y + 4)$

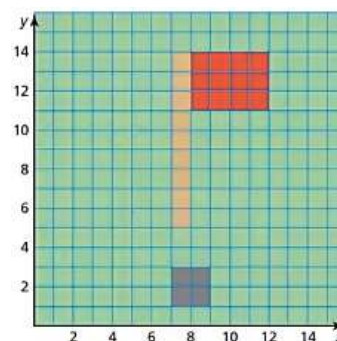
Translation: $(x, y) \rightarrow (x + 4, y - 6)$



Example 6

A graphic artist is designing a favicon for a golf website.

In an image-editing program, she moves the red rectangle 3 units right and 1 unit down. Then she moves the red rectangle 1 unit left and 4 units up. Rewrite the composition as a single transformation.



Concept Summary:

A translation maintains length and angles (rigid motion)

A translation moves all parts of the figure the same distance and direction

Khan Academy Videos:

1. [Rigid transformations](#) introduction
2. [Translating points](#)
3. [Determining translations](#)
4. [Translating shapes](#)

Homework: [Translation worksheet](#)

Reading Assignment: student notes section 4-2

Chapter 4: Transformations

Section 4-2: Reflections

SOL: G.3.c and .d

Objectives:

- Perform reflections
- Perform glide reflections
- Identify lines of symmetry
- Solve real-life problems involving reflections

Vocabulary:

- Glide reflection* – a transformation involving a translation followed by a reflection
- Line of reflection* – the mirror line in the reflection
- Line of symmetry* – the line of reflection that generates line symmetry
- Line symmetry* – when a figure can be mapped onto itself by a reflection in that line
- Reflection* – a transformation that use a line like a mirror to reflect a figure

Core Concepts:

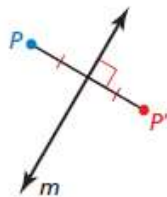
Core Concept

Reflections

A **reflection** is a transformation that uses a line like a mirror to reflect a figure. The mirror line is called the **line of reflection**.

A reflection in a line m maps every point P in the plane to a point P' , so that for each point one of the following properties is true.

- If P is not on m , then m is the perpendicular bisector of $\overline{PP'}$, or
- If P is on m , then $P = P'$.



point P not on m



point P on m

Core Concept

Coordinate Rules for Reflections

- If (a, b) is reflected in the x -axis, then its image is the point $(a, -b)$.
- If (a, b) is reflected in the y -axis, then its image is the point $(-a, b)$.
- If (a, b) is reflected in the line $y = x$, then its image is the point (b, a) .
- If (a, b) is reflected in the line $y = -x$, then its image is the point $(-b, -a)$.

Reflection over the origin is a reflection of both axes: $(a, b) \rightarrow (-a, -b)$

Chapter 4: Transformations

Postulate

Postulate 4.2 Reflection Postulate

A reflection is a rigid motion.

STUDY TIP

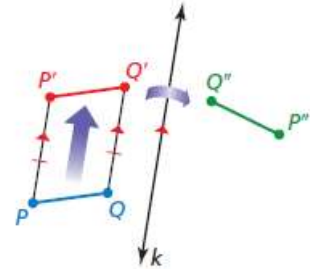
The line of reflection must be parallel to the direction of the translation to be a glide reflection.



A **glide reflection** is a transformation involving a translation followed by a reflection in which every point P is mapped to a point P'' by the following steps.

Step 1 First, a translation maps P to P' .

Step 2 Then, a reflection in a line k parallel to the direction of the translation maps P' to P'' .

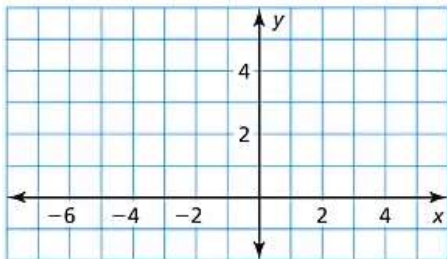


Examples:

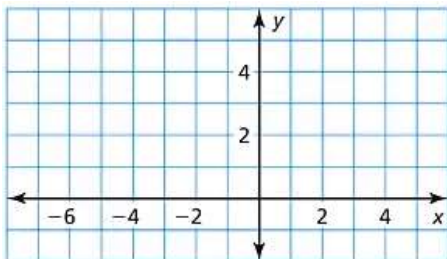
Example 1

Graph $\triangle ABC$ with vertices $A(1, 3)$, $B(5, 2)$, and $C(2, 1)$ and its image after the reflection described.

a. In the line $n: x = -1$



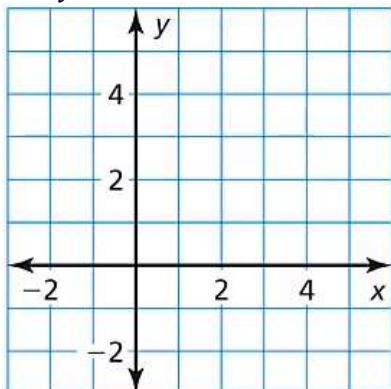
b. In the line $m: y = 3$



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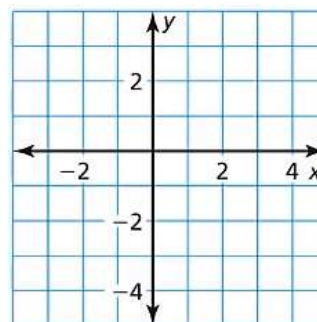
Example 2

Graph \overline{AB} with endpoints $A(3, -1)$ and $B(3, 2)$ and its image after the reflection in the line $y = x$.



Example 3

Graph \overline{AB} with endpoints $A(3, -1)$ and $B(3, 2)$ and its image after the reflection in the line $y = -x$.

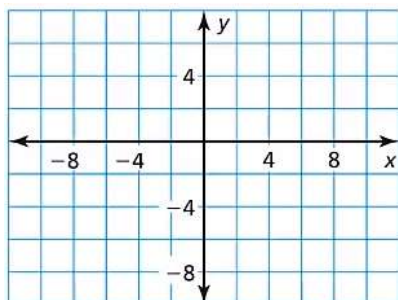


Example 4

Graph $\triangle ABC$ with vertices $A(3, 2)$, $B(6, 3)$, and $C(7, 1)$ and its image after the glide reflection.

Translation: $(x, y) \rightarrow (x, y - 6)$

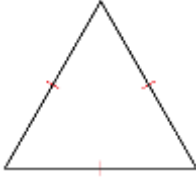
Reflection: in the y -axis



Chapter 4: Transformations

Example 5

How many lines of symmetry does the triangle have?



Example 6

You are going to the music store. Your friend is going to buy craft supplies. Where should you park to minimize the distance you will both walk?



Concept Summary:

A reflection maintains length and angles (rigid motion)

A reflection maintains the distance from the line or point of reflection

A reflection can be done across

x-axis,

y-axis,

origin (both x-axis and y-axis),

line $y = x$,

line $y = -x$,

or any other horizontal or vertical line

Khan Academy Videos:

1. [Reflecting points](#)
2. Determining reflections ([basic](#) and [advanced](#))
3. [Reflecting shapes](#)

Homework: [Reflections worksheet](#)

Reading Assignment: student notes section 4.3

Chapter 4: Transformations

Section 4-3: Rotations

SOL: G.3.c and .d

Objectives:

- Perform rotations
- Perform compositions with rotations
- Identify rotational symmetry and point symmetry

Vocabulary:

- Angle of rotation* – determined by the rays drawn from the center of rotation to a point in the preimage and its point in the image
- Center of rotation* – the point about which a figure is turned
- Center of symmetry* – the center of the figure
- Point symmetry* – a figure can be mapped onto itself by a rotation of 180°
- Rotation* – a transformation in which a figure is turned about a fixed point (center of rotation)
- Rotational symmetry* – when a figure can be mapped onto itself by a rotation of 180° or less
- Rotational symmetry magnitude* – the angle turn of rotation; $360/\text{order}$
- Rotational symmetry order* – how many times a figure maps back onto itself; for regular polygons it is the number of sides

Core Concepts:

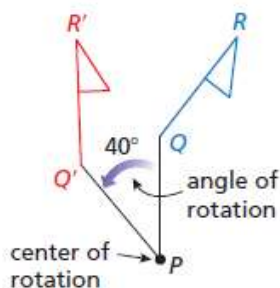
Core Concept

Rotations

A **rotation** is a transformation in which a figure is turned about a fixed point called the **center of rotation**. Rays drawn from the center of rotation to a point and its image form the **angle of rotation**.

A rotation about a point P through an angle of x° maps every point Q in the plane to a point Q' so that one of the following properties is true.

- If Q is not the center of rotation P , then $QP = Q'P$ and $m\angle QPQ' = x^\circ$, or
- If Q is the center of rotation P , then $Q = Q'$.

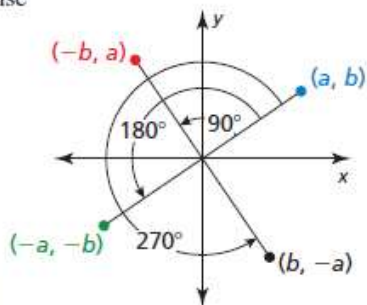


Core Concept

Coordinate Rules for Rotations about the Origin

When a point (a, b) is rotated counterclockwise about the origin, the following are true.

- For a rotation of 90° ,
 $(a, b) \rightarrow (-b, a)$.
- For a rotation of 180° ,
 $(a, b) \rightarrow (-a, -b)$.
- For a rotation of 270° ,
 $(a, b) \rightarrow (b, -a)$.



Postulate

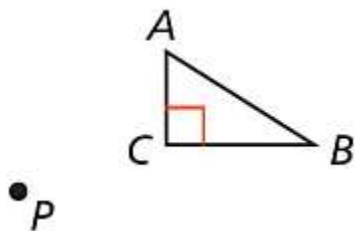
Postulate 4.3 Rotation Postulate

A rotation is a rigid motion.

Examples:

Example 1

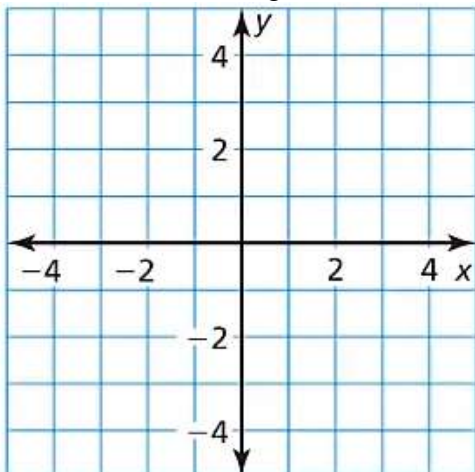
Draw a 60° rotation of $\triangle ABC$ about point P .



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Example 2

Graph $\triangle ABC$ with vertices $A(3, 1)$, $B(3, 4)$, and $C(1, 1)$ and its image after a 180° rotation about the origin.

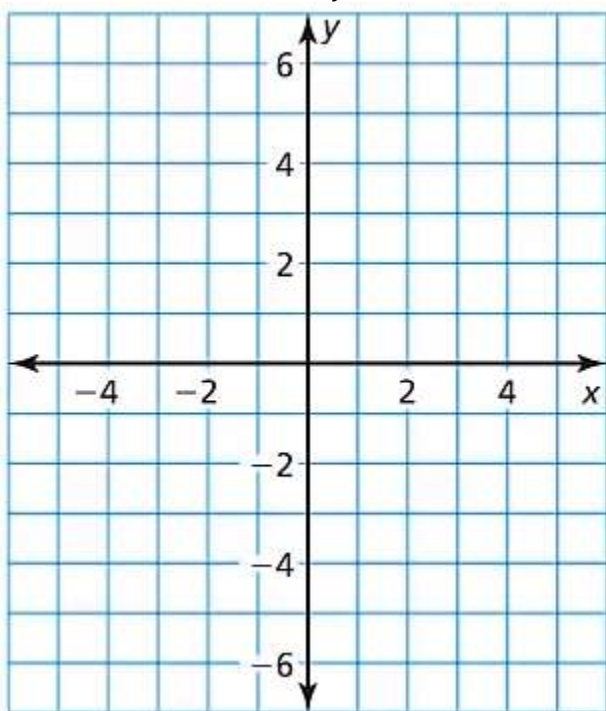


Example 3

Graph \overline{RS} with endpoints $R(1, -3)$ and $S(2, -6)$ and its image after the composition.

Rotation: 180° about the origin

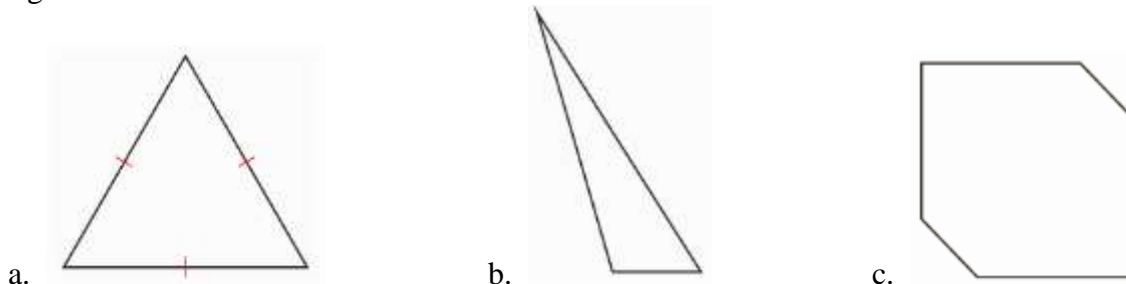
Reflection: in the y -axis



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Example 4

Does the figure have rotational symmetry? If so, describe any rotations that map the figure onto itself.



Concept Summary:

- A rotation turns each point in a figure through the same angle about a fixed point
- Rules for 90° rotations, clockwise vs counterclockwise

Rotation	Counterclockwise Rule	Clockwise	Reminders
90°	$(a, b) \rightarrow (-b, a)$	$(a, b) \rightarrow (b, -a)$	CW 270 CCW 90
180°	$(a, b) \rightarrow (-a, -b)$	$(a, b) \rightarrow (-a, -b)$	same
270°	$(a, b) \rightarrow (b, -a)$	$(a, b) \rightarrow (-b, a)$	CW 90 CCW 270

- An object has rotational symmetry when you can rotate it less than 360° and the pre-image and the image are indistinguishable (can't tell them apart)
- Rotational Symmetry for regular figures:
 - Order is the number of sides
 - Magnitude = $360 / \text{order}$ (later we learn it's the exterior angle)

Khan Academy Videos:

1. [Rotating Points](#)
2. [Determining Rotations](#)
3. [Rotating Shapes](#)

Homework: [Rotation worksheet](#)

Reading Assignment: student notes Section 4.4

Chapter 4: Transformations

Section 4-4: Congruence and Transformations

SOL: G.3.d

Objectives:

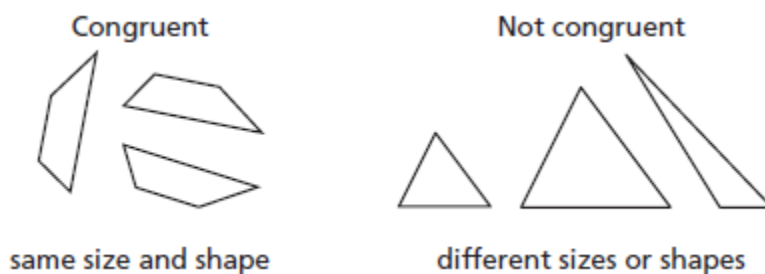
- Identify congruent figures
- Describe congruence transformations
- Use theorems about congruence transformations

Vocabulary:

Congruent figures – figures are congruent, if and only if, there is a rigid motion or composition of rigid motions that maps one of the figures onto the other; congruent figures have the same size and shape

Congruence transformation – preimage and the image are congruent; terms “rigid motion” and “congruence transformation” are interchangeable

Core Concepts:



Theorem

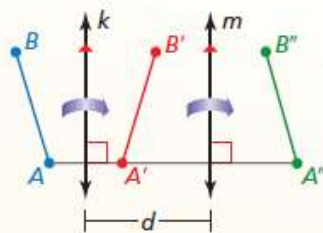
Theorem 4.2 Reflections in Parallel Lines Theorem

If lines k and m are parallel, then a reflection in line k followed by a reflection in line m is the same as a translation.

If A'' is the image of A , then

- $\overline{AA''}$ is perpendicular to k and m , and
- $AA'' = 2d$, where d is the distance between k and m .

Proof Ex. 31, p. 186

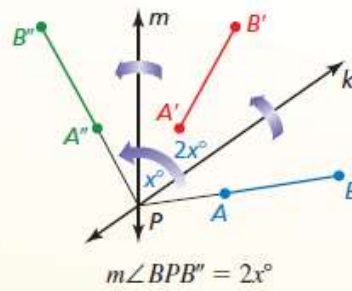


Theorem

Theorem 4.3 Reflections in Intersecting Lines Theorem

If lines k and m intersect at point P , then a reflection in line k followed by a reflection in line m is the same as a rotation about point P .

The angle of rotation is $2x^\circ$, where x° is the measure of the acute or right angle formed by lines k and m .

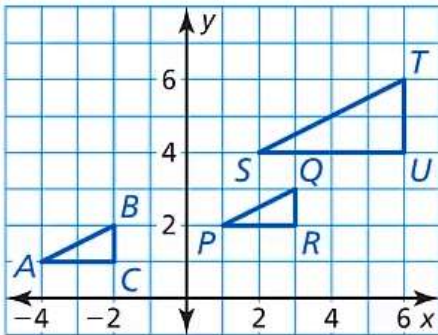


Proof Ex. 31, p. 226

Examples:

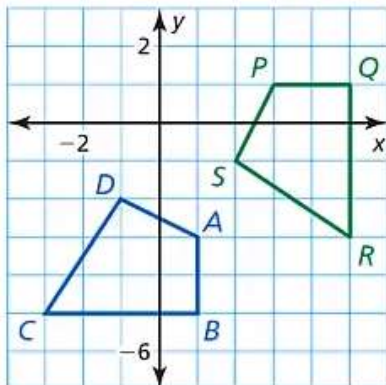
Example 1

Identify the congruent figures in the coordinate plane. Explain.



Example 2

Describe a congruence transformation that maps quadrilateral $ABCD$ to quadrilateral $PQRS$.

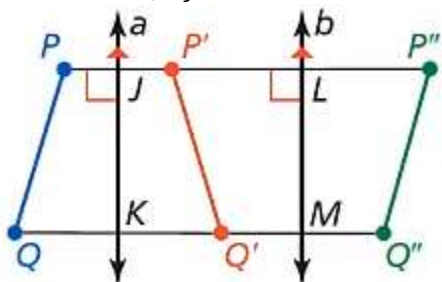


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Example 3

In the diagram, a reflection in line a maps \overline{PQ} to $\overline{P'Q'}$. A reflection in line b maps $\overline{P'Q'}$ to $\overline{P''Q''}$.

Also, $PJ = 3$ and $LP'' = 8$.



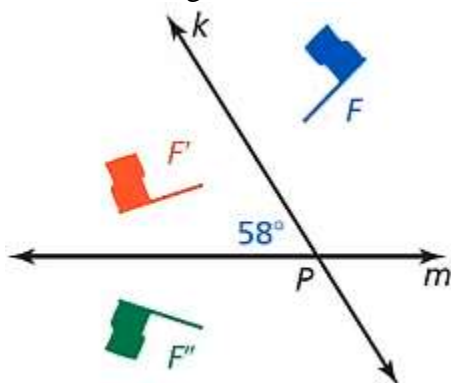
a. Name any segments congruent to each segment: \overline{PQ} , \overline{PJ} , and \overline{QK} .

b. Does $JK = LM$? Explain.

c. What is the length of $\overline{PP''}$?

Example 4

In the diagram, the preimage is reflected in line k . The image is then reflected in line m . Describe a single transformation that maps F to F'' .



Chapter 4: Transformations

Important other pieces:

Tessellations:

A tessellation is when a figure (or combinations of figures) covers a plane with no gaps or overlaps. This only happens if the sum of the interior angles of the figures at the point of intersection sum to 360° . If the sum of the angles is less than 360° , then we have a gap in the pattern. If the sum of the angles is more than 360° , then we have an overlap.

Regular polygons that tessellate: Triangles, Squares and Hexagons

Symmetry:

A *line of symmetry* cuts a figure into two equal halves. You can fold a figure over a line of symmetry, like a napkin, to match up the two halves. Regular polygons have the same number of lines of symmetry as they have sides. A triangle has 3, a square has 4 and so on.

A *point of symmetry* has many definitions. The easiest for us is that if we turn the figure 180° , does the figure look like what we started with. For example with common letters: X and W. Turn X 180° and it still looks like an X. Turn W 180° and it looks like an M, not a W. So X would have point symmetry, but M would not. We can do the same thing with figures. In regular polygons, only even numbered sided figures (squares, hexagons, octagons, etc) have point symmetry; odd numbered sided figures (triangles, pentagons, heptagons, etc) do not have point symmetry

Concept Summary:

- A composition of several reflections can act like a single translation
- A composition of several reflections can act like a single rotation.

Khan Academy Videos:

1. Introduction to [reflective](#) symmetry
2. Introduction to [rotational](#) symmetry
3. Finding a [quadrilateral](#) from its symmetries

Homework: [Alphabet Symmetry WS](#)

Reading Assignment: student notes section 4.5

Chapter 4: Transformations

Section 4-5: Dilations

SOL: G.3.d

Objectives:

Identify and perform dilations

Solve real-life problems involving scale factors and dilations

Vocabulary:

Center of dilation – fixed point around which the dilation occurs

Congruence transformation – when the absolute value of the scaling factor is equal to one

Dilation – a transformation in which a figure is enlarged or reduced with respect to a fixed point C (center of dilation) and a scale factor k

Enlargement – the absolute value of the scaling factor is greater than one

Reduction – the absolute value of the scaling factor is less than one

Scale factor – the ratio of the lengths of corresponding sides of the image and the preimage

Core Concepts:

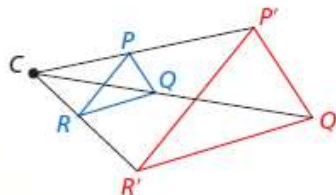
Core Concept

Dilations

A **dilation** is a transformation in which a figure is enlarged or reduced with respect to a fixed point C called the **center of dilation** and a **scale factor** k , which is the ratio of the lengths of the corresponding sides of the image and the preimage.

A dilation with center of dilation C and scale factor k maps every point P in a figure to a point P' so that the following are true.

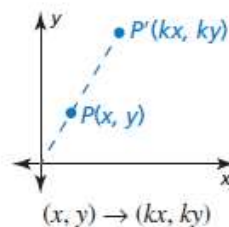
- If P is the center point C , then $P = P'$.
- If P is not the center point C , then the image point P' lies on \overline{CP} . The scale factor k is a positive number such that $k = \frac{CP'}{CP}$.
- Angle measures are preserved.



Core Concept

Coordinate Rule for Dilations

If $P(x, y)$ is the preimage of a point, then its image after a dilation centered at the origin $(0, 0)$ with scale factor k is the point $P'(kx, ky)$.

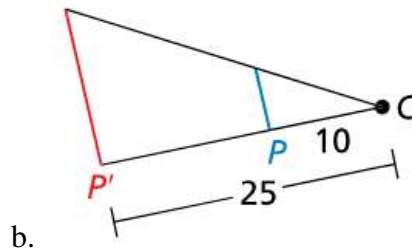
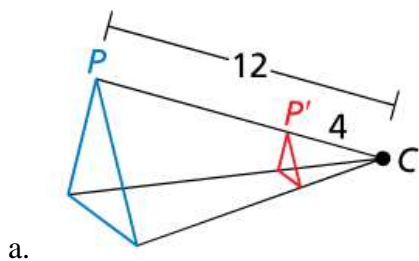


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Examples:

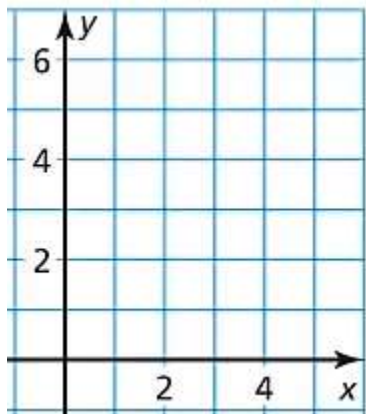
Example 1

Find the scale factor of the dilation. Then tell whether the dilation is a *reduction* or an *enlargement*.



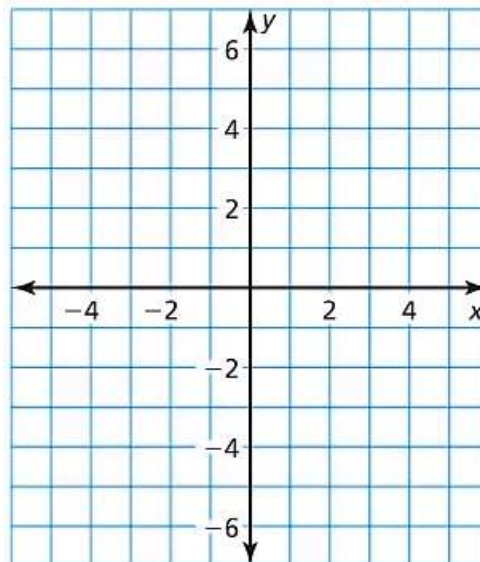
Example 2

Graph $\triangle PQR$ with vertices $P(0, 2)$, $Q(1, 0)$, and $R(2, 2)$ and its image after a dilation with scale factor 3.



Example 3

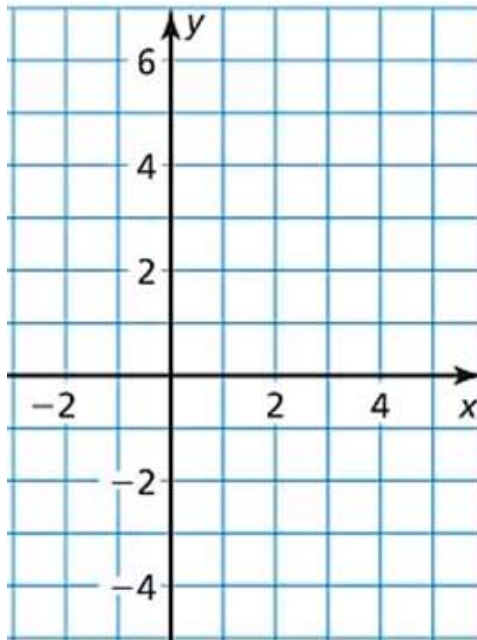
Graph $\triangle PQR$ with vertices $P(4, 6)$, $Q(-4, 2)$, and $R(2, -6)$ and its image after a dilation with scale factor 0.5.



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Example 4

Graph $\triangle FGH$ with vertices $F(3, 6)$, $G(3, -3)$, and $H(6, 6)$ and its image after a dilation with a scale factor of $-\frac{1}{3}$.



Example 5

You are using word processing software to type the online school newsletter. You change the size of the text in one headline from 0.5 inch tall to 1.25 inches tall. What is the scale factor of this dilation?

Example 6

You are using a magnifying glass that shows the image of an object that is six times the object's actual size. The image of a spider seen through the magnifying glass is 13.5 centimeters. Find the actual size of the spider.

Concept Summary:

Dilations: If the $|k|$, the scaling factor
is equal to one, then it's a congruence transformation
is less than one, then it's a reduction
is more than one, then it's an enlargement
Dilations are similar transformations (not congruent).

Khan Academy Videos:

1. [Dilating points](#)
2. Dilations: [scale factor](#) and [center](#)
3. Dilations shapes ([expanding](#) and [shrinking](#))
4. [Dilations and properties](#)

Homework: Dilations worksheet

Reading Assignment: student notes section 4.6

Chapter 4: Transformations

Section 4-6: Similarity and Transformations

SOL: G.3.d

Objectives:

- Perform similarity transformations
- Describe similarity transformations
- Prove that figures are similar

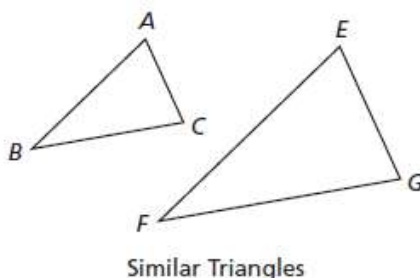
Vocabulary:

Similar figures – a similarity transformation maps one of the figures onto the other
Similarity transformation – dilation or a composition of rigid motions and dilations

Core Concept:

Similar figures have all corresponding sides with the same ratio, called the scaling factor, k , and have all corresponding angles congruent.

Two figures are *similar figures* when they have the same shape but not necessarily the same size.



In the picture above, corresponding angles A and E are congruent; corresponding angles C and G are congruent; corresponding angles B and F are congruent. The sides have the same ratio:

$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{AC}{EG} = k, \text{ the scaling factor}$$

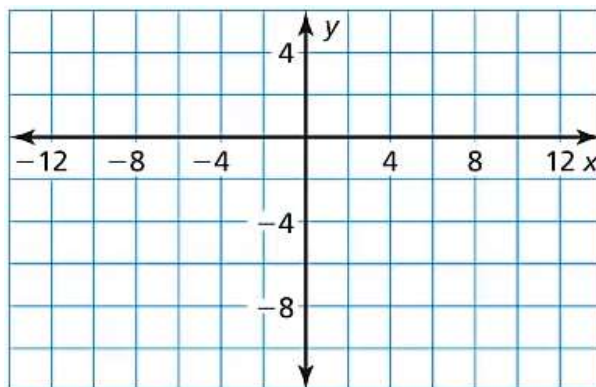
Examples:

Example 1

Graph \overline{AB} with endpoints $A(12, -6)$ and $B(0, -3)$ and its image after the similarity transformation.

Reflection: in the y -axis

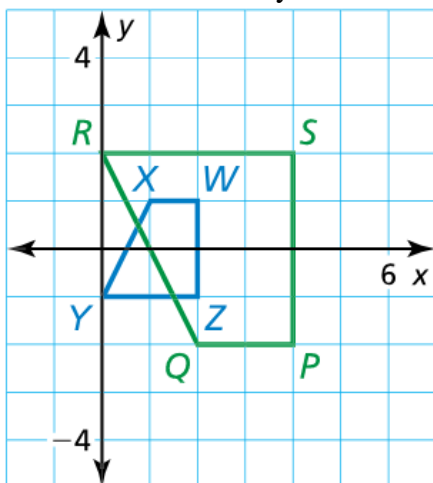
Dilation: $(x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)$



Chapter 4: Transformations

Example 2

Describe the similarity transformation that maps trapezoid $WXYZ$ to trapezoid $PQRS$.

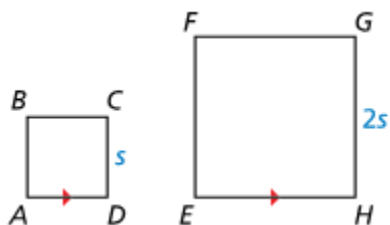


Example 3

Prove that square $ABCD$ is similar to square $EFGH$.

Given Square $ABCD$ with side length s , square $EFGH$ with side length $2s$, $\overline{AD} \parallel \overline{EH}$

Prove Square $ABCD$ is similar to square $EFGH$.



Concept Summary:

In a congruence transformation, the position of the image may differ from the preimage, but the two figures remain congruent.

In a similar transformation, the position and size of the image may differ from the preimage, but the two figures keep the shape (angles congruent and sides scaled).

Flips, turns, and slides are congruence transformations

Dilations are similar transformations

Khan Academy Videos: None related

Homework: TBD

Reading Assignment: chapter 4 review

Chapter 4: Transformations

Section 4-R: Chapter Review

SOL: G.3.d

Objectives:

Review material from chapter 4

Vocabulary:

None new

Core Concept:



Examples: none

Concept Summary:

In a congruence transformation, the position of the image may differ from the preimage, but the two figures remain congruent.

Flips, turns, and slides are congruence transformations

Dilations are similar transformations

Khan Academy Videos:

Homework: [Quiz Review WS](#)

Reading Assignment: none

Chapter 4: Transformations

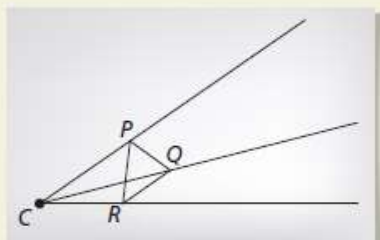
Constructions:

CONSTRUCTION Constructing a Dilation

Use a compass and straightedge to construct a dilation of $\triangle PQR$ with a scale factor of 2. Use a point C outside the triangle as the center of dilation.

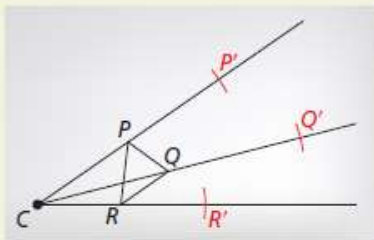
SOLUTION

Step 1



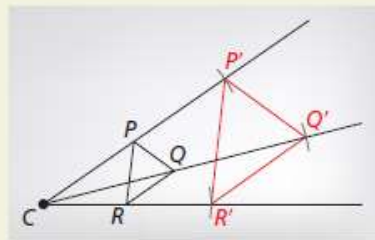
Draw a triangle Draw $\triangle PQR$ and choose the center of the dilation C outside the triangle. Draw rays from C through the vertices of the triangle.

Step 2



Use a compass Use a compass to locate P' on \overrightarrow{CP} so that $CP' = 2(CP)$. Locate Q' and R' using the same method.

Step 3



Connect points Connect points P' , Q' , and R' to form $\triangle P'Q'R'$.

Tesselations

A figure, or series of figures, tessellates a plane by covering it without gaps or overlaps (like tile on a floor)

Gaps – interior angles add up to less than 360

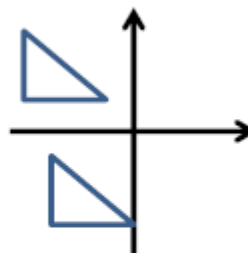
Overlaps – interior angles add up to more than 360

Only 3 regular figures tessellate: Triangle, Square, Hexagon

Transformations

Translation – “Slides”

- Up – adds to y
- Down – subtracts from y
- Right – adds to x
- Left – subtracts from x



Translation function: $(x, y) \rightarrow (x \pm h, y \pm k)$

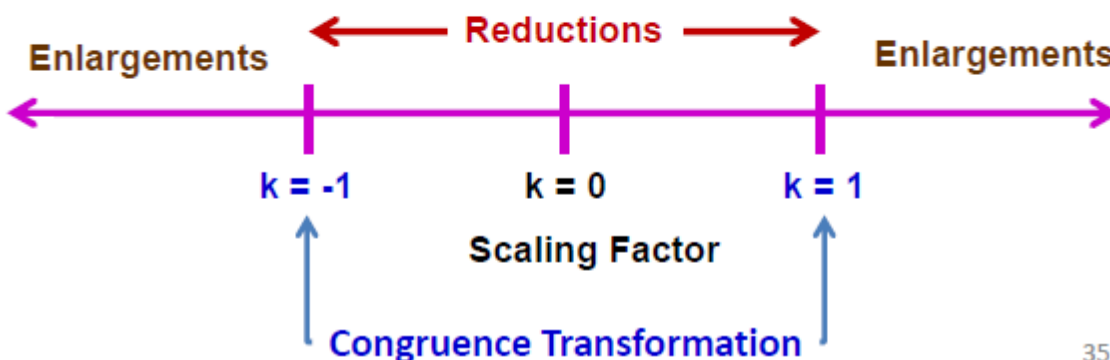
In picture above: Down 7 and right 3: $(x, y) \rightarrow (x + 3, y - 7)$

Dilations – “Shrinks or Blow-ups”

Similar figures (from before)

- Scaling factor r : if $|k| > 1$ then enlargement
- if $|k| < 1$ then reduction
- if $|k| = 1$ then congruence transformation

Negative numbers flip figure over dilation center point



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Chapter 4: Transformations

Rotations – “Turns” (usually around the origin)

Clockwise:



Counterclockwise:



90° increment turns can be done without using trig

180° rotation (in either direction) is same
as reflection across the origin

Clockwise	Counterclockwise	Same places turning in different directions
90°	270°	
180°	180°	
270°	90°	

Symmetry

Line symmetry – 1) folding the figure in half
2) regular figures have lines of symmetry
equal to the number of sides

Point symmetry – 1) where two lines of symmetry intersect
2) midpoint of every point and its reflection
3) even-sided regular figures have it

Rotational symmetry – number of times a figure can be turned

Order – in regular figures equal to the number of sides

Magnitude – equal to $360/\text{order}$ (degrees per turn)
(equal to the exterior angle of figure)

Chapter 4: Transformations

Transformations

Mirror Image across a line or point

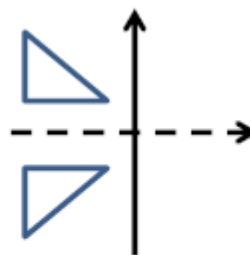
Reflections – “Flips”

Equal distance from reflection point/line

Over x-axis (or horizontal lines)

Equation: $(x, y) \rightarrow (x, -y)$

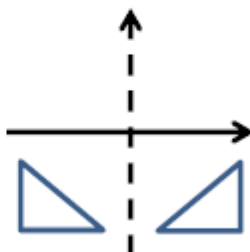
Up/down distance



Over y-axis (or vertical lines)

Equation: $(x, y) \rightarrow (-x, y)$

Left/right distance

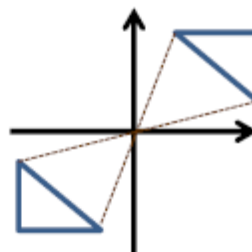


Over origin

Equation: $(x, y) \rightarrow (-x, -y)$

Origin is midpoint of before and after points

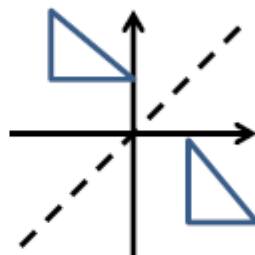
Both distances



Over (diagonal) line $y = x$

Equation: $(x, y) \rightarrow (y, x)$

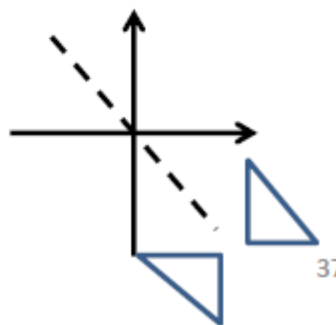
Diagonal distance



Over (diagonal) line $y = -x$

Equation: $(x, y) \rightarrow (-y, -x)$

Diagonal distance



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Chapter 4: Transformations

Transformations:

transformation is when a figure is moved in some manner; congruence transformations include **reflections, rotations and translations**. Key concept is equal distance!

Reflections across:

Reflections can be done over any horizontal line (like x-axis), any vertical line (like y-axis) or any diagonal line (like lines $y = x$ and $y = -x$) – before/after, equal distance from the line of reflection

Reflection	x-axis	y-axis	origin	$y = x$	$y = -x$
Pre-image to image	$(a, b) \rightarrow (a, -b)$	$(a, b) \rightarrow (-a, b)$	$(a, b) \rightarrow (-a, -b)$	$(a, b) \rightarrow (b, a)$	$(a, b) \rightarrow (-b, -a)$
Find coordinates	Multiply y coordinate by -1	Multiply x coordinate by -1	Multiply both coordinates by -1	Interchange x and y coordinates	Interchange x and y coordinates and Multiply both coordinates by -1

“Negate y” “Negate x” “Negate both” “Flip x and y” “Flip and Negate”

90° Rotations:

SSM: Use your graph paper plot the original points and then rotate the paper clockwise or counterclockwise the proper amount and read off the new points and then plot them

Translations:

Moves the figure the same distance in the x and y directions. Orientation does not change.
Finish – start = change

Dilations:

Can change a figures' size (like similar triangles) if the scaling factor, k, is not equal to 1 or -1. If equal to 1 or -1, then it is a congruence transformation.

Reductions are when $|k| < 1$

Enlargements are when $|k| > 1$

Symmetry:

Point – flip figure 180 degrees, is it the same figure?

Line – divides the figure into *foldable* halves