CHAPTER 5
Analytic Trigonometry

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Section 5.1 Using Fundamental Identities

- You should know the fundamental trigonometric identities.

  (a) Reciprocal Identities

  $\sin u = \dfrac{1}{\csc u}$ $\qquad$ $\csc u = \dfrac{1}{\sin u}$

  $\cos u = \dfrac{1}{\sec u}$ $\qquad$ $\sec u = \dfrac{1}{\cos u}$

  $\tan u = \dfrac{1}{\cot u} = \dfrac{\sin u}{\cos u}$ $\qquad$ $\cot u = \dfrac{1}{\tan u} = \dfrac{\cos u}{\sin u}$

  (b) Pythagorean Identities

  $\sin^2 u + \cos^2 u = 1$

  $1 + \tan^2 u = \sec^2 u$

  $1 + \cot^2 u = \csc^2 u$

  (c) Cofunction Identities

  $\sin\left(\dfrac{\pi}{2} - u\right) = \cos u$ $\qquad$ $\cos\left(\dfrac{\pi}{2} - u\right) = \sin u$

  $\tan\left(\dfrac{\pi}{2} - u\right) = \cot u$ $\qquad$ $\cot\left(\dfrac{\pi}{2} - u\right) = \tan u$

  $\sec\left(\dfrac{\pi}{2} - u\right) = \csc u$ $\qquad$ $\csc\left(\dfrac{\pi}{2} - u\right) = \sec u$

  (d) Negative Angle Identities

  $\sin(-x) = -\sin x$ $\qquad$ $\csc(-x) = -\csc x$

  $\cos(-x) = \cos x$ $\qquad$ $\sec(-x) = \sec x$

  $\tan(-x) = -\tan x$ $\qquad$ $\cot(-x) = -\cot x$

- You should be able to use these fundamental identities to find function values.
- You should be able to convert trigonometric expressions to equivalent forms by using the fundamental identities.
- You should be able to check your answers with a graphing utility.

Vocabulary Check

1. $\sec u$ **2.** $\tan u$ **3.** $\cot u$ **4.** $\csc u$

5. $\tan^2 u$ **6.** $\csc^2 u$ **7.** $\sin u$ **8.** $\sec u$

9. $-\tan u$ **10.** $\cos u$

1. $\sin x = \frac{1}{2}, \cos x = \frac{\sqrt{3}}{2}$, x is in Quadrant I.

$\tan x = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\cot x = \sqrt{3}$

$\csc x = 2$

$\sec x = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

2. $\csc \theta = 2, \tan \theta = \frac{\sqrt{3}}{3}$, θ is in Quadrant I.

$\sin \theta = \frac{1}{2}$

$\cot \theta = \frac{3}{\sqrt{3}} = \sqrt{3}$

$\cos \theta = \cot \theta \sin \theta = \frac{\sqrt{3}}{2}$

$\sec \theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

3. $\sec \theta = \sqrt{2}, \sin \theta = -\frac{\sqrt{2}}{2} \implies \theta$ is in Quadrant IV.

$\cos \theta = \frac{1}{\sec \theta} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$

$\cot \theta = \frac{1}{\tan \theta} = -1$

$\csc \theta = -\sqrt{2}$

4. $\tan x = \frac{\sqrt{3}}{3}, \cos x = -\frac{\sqrt{3}}{2}$, x is in Quadrant III.

$\sin x = -\sqrt{1 - \left(-\frac{\sqrt{3}}{2}\right)^2} = -\sqrt{\frac{1}{4}} = -\frac{1}{2}$

$\csc x = \frac{1}{\sin x} = -2$

$\sec x = \frac{1}{\cos x} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

$\cot x = \frac{1}{\tan x} = \frac{3}{\sqrt{3}} = \sqrt{3}$

5. $\tan x = \frac{7}{24}, \sec x = \frac{-25}{24} \implies x$ is in Quadrant III.

$\cot x = \frac{24}{7}$

$\cos x = -\frac{24}{25}$

$\sin x = -\sqrt{1 - \cos^2 x} = -\frac{7}{25}$

$\csc x = \frac{1}{\sin x} = -\frac{25}{7}$

6. $\cot \phi = -5, \sin \phi = \frac{\sqrt{26}}{26}$, ϕ is in Quadrant II

$\cos \phi = \cot \phi \cdot \sin \phi = \frac{-5\sqrt{26}}{26}$

$\tan \phi = \frac{1}{\cot \phi} = -\frac{1}{5}$

$\csc \phi = \frac{1}{\sin \phi} = \frac{26}{\sqrt{26}} = \sqrt{26}$

$\sec \phi = \frac{1}{\cos \phi} = \frac{-26}{5\sqrt{26}} = \frac{-\sqrt{26}}{5}$

7. $\sec \phi = -\frac{17}{15}, \sin \phi = \frac{8}{17}$, ϕ is in Quadrant II.

$\cos \phi = -\frac{15}{17}$

$\csc \phi = \frac{17}{8}$

$\tan \phi = \frac{8/17}{-15/17} = -\frac{8}{15}$

$\cot \phi = -\frac{15}{8}$

8. $\cos\left(\frac{\pi}{2} - x\right) = \frac{3}{5}, \cos x = \frac{4}{5},$ x is in Quadrant I.

$$\sin x = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \frac{3}{5}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{3}{5} \cdot \frac{5}{4} = \frac{3}{4}$$

$$\csc x = \frac{1}{\sin x} = \frac{5}{3}$$

$$\sec x = \frac{1}{\cos x} = \frac{5}{4}$$

$$\cot x = \frac{1}{\tan x} = \frac{4}{3}$$

9. $\sin(-x) = -\sin x = -\frac{2}{3} \Rightarrow \sin x = \frac{2}{3}$

$$\sin x = \frac{2}{3},\ \tan x = -\frac{2\sqrt{5}}{5} \Rightarrow x \text{ is in Quadrant II.}$$

$$\cos x = -\sqrt{1 - \sin^2 x} = -\sqrt{1 - \frac{4}{9}} = -\frac{\sqrt{5}}{3}$$

$$\cot x = \frac{1}{\tan x} = -\frac{\sqrt{5}}{2}$$

$$\sec x = \frac{1}{\cos x} = -\frac{3\sqrt{5}}{5}$$

$$\csc x = \frac{1}{\sin x} = \frac{3}{2}$$

10. $\csc x = 5, \cos x > 0,$ x is in Quadrant I.

$$\sin x = \frac{1}{\csc x} = \frac{1}{5}$$

$$\cos x = \frac{2\sqrt{6}}{5}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{1}{5} \cdot \frac{5}{2\sqrt{6}} = \frac{\sqrt{6}}{12}$$

$$\sec x = \frac{1}{\cos x} = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$$

$$\cot x = \frac{1}{\tan x} = 2\sqrt{6}$$

11. $\tan\theta = 2, \sin\theta < 0 \Rightarrow \theta$ is in Quadrant III.

$$\sec\theta = -\sqrt{\tan^2\theta + 1} = -\sqrt{5}$$

$$\cos\theta = \frac{1}{-\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

$$\cot\theta = \frac{1}{2}$$

$$\sin\theta = -\sqrt{1 - \cos^2\theta}$$

$$= -\sqrt{1 - \frac{1}{5}} = -\sqrt{\frac{4}{5}} = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$$

$$\csc\theta = -\frac{\sqrt{5}}{2}$$

12. $\sec\theta = -3, \tan\theta < 0, \theta$ is in Quadrant II.

$$\cos\theta = -\frac{1}{3}$$

$$\tan^2\theta = \sec^2\theta - 1 = 9 - 1 = 8 \Rightarrow \tan\theta = -\sqrt{8} = -2\sqrt{2}$$

$$\cot\theta = \frac{1}{-2\sqrt{2}} = \frac{-\sqrt{2}}{4}$$

$$\sin\theta = \tan\theta\cos\theta = \left(-2\sqrt{2}\right)\left(-\frac{1}{3}\right) = \frac{2\sqrt{2}}{3}$$

$$\csc\theta = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

13. $\csc\theta$ is undefined and $\cos\theta < 0 \Rightarrow \theta = \pi$.

$\sin\theta = 0$

$\cos\theta = -1$

$\tan\theta = 0$

$\cot\theta$ is undefined.

$\sec\theta = -1$

14. $\tan\theta$ is undefined, $\sin\theta > 0$.

$\theta = \dfrac{\pi}{2}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ is undefined $\Rightarrow$ $\cos\theta = 0$.

$\sin\theta = \sqrt{1 - 0^2} = 1$

$\csc\theta = \dfrac{1}{\sin\theta} = 1$

$\sec\theta = \dfrac{1}{\cos\theta}$ is undefined.

$\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{0}{1} = 0$

15. $\sec x \cos x = \dfrac{1}{\cos x} \cdot \cos x = 1$

Matches (d).

16. $\tan x \csc x = \dfrac{\sin x}{\cos x}\dfrac{1}{\sin x} = \dfrac{1}{\cos x} = \sec x$

Matches (a).

17. $\cot^2 x - \csc^2 x = \cot^2 x - (1 + \cot^2 x) = -1$

Matches (b).

18. $(1 - \cos^2 x)\csc x = \sin^2 x\left(\dfrac{1}{\sin x}\right) = \sin x$

Matches (f).

19. $\dfrac{\sin(-x)}{\cos(-x)} = \dfrac{-\sin x}{\cos x} = -\tan x$

Matches (e).

20. $\dfrac{\sin[(\pi/2) - x]}{\cos[(\pi/2) - x]} = \dfrac{\cos x}{\sin x} = \cot x$

Matches (c).

21. $\sin x \sec x = \sin x\left(\dfrac{1}{\cos x}\right) = \tan x$

Matches (b).

22. $\cos^2 x(\sec^2 x - 1) = \cos^2 x \tan^2 x = \sin^2 x$

Matches (c).

23. $\sec^4 x - \tan^4 x = (\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x)$

$= (\sec^2 x + \tan^2 x)(1)$

$= \sec^2 x + \tan^2 x$

Matches (f).

24. $\cot x \sec x = \dfrac{\cos x}{\sin x} \cdot \dfrac{1}{\cos x} = \dfrac{1}{\sin x}$

$= \csc x$

Matches (a).

25. $\dfrac{\sec^2 x - 1}{\sin^2 x} = \dfrac{\tan^2 x}{\sin^2 x} = \dfrac{\sin^2 x}{\cos^2 x} \cdot \dfrac{1}{\sin^2 x} = \sec^2 x$

Matches (e).

26. $\dfrac{\cos^2[(\pi/2) - x]}{\cos x} = \dfrac{\sin^2 x}{\cos x} = \dfrac{\sin x}{\cos x}\sin x$

$= \tan x \sin x$

Matches (d).

27. $\cot x \sin x = \dfrac{\cos x}{\sin x}\sin x = \cos x$

28. $\cos\beta\tan\beta = \cos\beta\left(\dfrac{\sin\beta}{\cos\beta}\right) = \sin\beta$

29. $\sin\phi(\csc\phi - \sin\phi) = \sin\phi\csc\phi - \sin^2\phi$

$= \sin\phi \cdot \dfrac{1}{\sin\phi} - \sin^2\phi$

$= 1 - \sin^2\phi$

$= \cos^2\phi$

30. $\sec^2 x(1 - \sin^2 x) = \sec^2 x - \sec^2 x \sin^2 x$

$= \sec^2 x - \dfrac{1}{\cos^2 x} \cdot \sin^2 x$

$= \sec^2 x - \dfrac{\sin^2 x}{\cos^2 x}$

$= \sec^2 x - \tan^2 x = 1$

31. $\dfrac{\csc x}{\cot x} = \dfrac{1}{\sin x} \cdot \dfrac{\sin x}{\cos x} = \dfrac{1}{\cos x} = \sec x$

32. $\dfrac{\sec \theta}{\csc \theta} = \dfrac{1}{\cos \theta} \sin \theta = \tan \theta$

33. $\sec \alpha \dfrac{\sin \alpha}{\tan \alpha} = \dfrac{1}{\cos \alpha}(\sin \alpha) \cot \alpha$

$= \dfrac{1}{\cos \alpha}(\sin \alpha)\left(\dfrac{\cos \alpha}{\sin \alpha}\right) = 1$

34. $\dfrac{\tan^2 \theta}{\sec^2 \theta} = \dfrac{\sin^2 \theta}{\cos^2 \theta} \cdot \dfrac{1}{\sec^2 \theta}$

$= \dfrac{\sin^2 \theta}{\cos^2 \theta} \cdot \dfrac{1}{\dfrac{1}{\cos^2 \theta}} = \dfrac{\sin^2 \theta \cos^2 \theta}{\cos^2 \theta} = \sin^2 \theta$

35. $\sin\left(\dfrac{\pi}{2} - x\right) \csc x = \cos x \cdot \dfrac{1}{\sin x} = \cot x$

36. $\cot\left(\dfrac{\pi}{2} - x\right) \cos x = \tan x \cos x$

$= \dfrac{\sin x}{\cos x} \cdot \cos x = \sin x$

37. $\dfrac{\cos^2 y}{1 - \sin y} = \dfrac{1 - \sin^2 y}{1 - \sin y}$

$= \dfrac{(1 + \sin y)(1 - \sin y)}{1 - \sin y}$

$= 1 + \sin y$

38. $\dfrac{1}{\cot^2 x + 1} = \dfrac{1}{\csc^2 x} = \sin^2 x$

39. $\sin \theta + \cos \theta \cot \theta = \sin \theta + \cos \theta \dfrac{\cos \theta}{\sin \theta}$

$= \dfrac{\sin^2 \theta + \cos^2 \theta}{\sin \theta}$

$= \dfrac{1}{\sin \theta} = \csc \theta$

40. $(\sec \theta - \tan \theta)(\csc \theta + 1) = \dfrac{1}{\cos \theta}(1 - \sin \theta)\left(\dfrac{1}{\sin \theta} + 1\right)$

$= \dfrac{1}{\cos \theta}(1 - \sin \theta)(1 + \sin \theta)\dfrac{1}{\sin \theta}$

$= \dfrac{1}{\cos \theta \cdot \sin \theta}(1 - \sin^2 \theta)$

$= \dfrac{1}{\cos \theta \sin \theta} \cos^2 \theta = \cot \theta$

41. $\dfrac{\cos \theta}{1 - \sin \theta} = \dfrac{\cos \theta}{1 - \sin \theta} \cdot \dfrac{1 + \sin \theta}{1 + \sin \theta}$

$= \dfrac{\cos \theta(1 + \sin \theta)}{1 - \sin^2 \theta}$

$= \dfrac{\cos \theta(1 + \sin \theta)}{\cos^2 \theta}$

$= \dfrac{1 + \sin \theta}{\cos \theta} = \sec \theta + \tan \theta$

42. $\dfrac{1 + \csc \theta}{\cot \theta + \cos \theta} = \dfrac{1 + \csc \theta}{\cos \theta(\csc \theta + 1)}$

$= \dfrac{1}{\cos \theta} = \sec \theta$

43. $\dfrac{1 + \cos \theta}{\sin \theta} + \dfrac{\sin \theta}{1 + \cos \theta} = \dfrac{1 + 2 \cos \theta + \cos^2 \theta + \sin^2 \theta}{\sin \theta(1 + \cos \theta)}$

$$= \frac{2 + 2 \cos \theta}{\sin \theta(1 + \cos \theta)}$$

$$= \frac{2(1 + \cos \theta)}{\sin \theta(1 + \cos \theta)}$$

$$= \frac{2}{\sin \theta} = 2 \csc \theta$$

44. $\dfrac{\sin \theta + \cos \theta}{\sin \theta} - \dfrac{\cos \theta - \sin \theta}{\cos \theta} = 1 + \cot \theta - 1 + \tan \theta$

$$= \cot \theta + \tan \theta$$

$$= \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta}$$

$$= \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta}$$

$$= \frac{1}{\sin \theta \cos \theta} = \sec \theta \csc \theta$$

45. $\csc \theta \tan \theta = \dfrac{1}{\sin \theta} \cdot \dfrac{\sin \theta}{\cos \theta} = \dfrac{1}{\cos \theta} = \sec \theta$

46. $\sin \theta \csc \theta - \sin^2 \theta = 1 - \sin^2 \theta = \cos^2 \theta$

47. $1 - \dfrac{\sin^2 \theta}{1 - \cos \theta} = \dfrac{1 - \cos \theta - \sin^2 \theta}{1 - \cos \theta}$

$$= \frac{\cos^2 \theta - \cos \theta}{1 - \cos \theta}$$

$$= \frac{\cos \theta(\cos \theta - 1)}{1 - \cos \theta}$$

$$= -\cos \theta$$

48. $\dfrac{\tan \theta}{1 + \sec \theta} + \dfrac{1 + \sec \theta}{\tan \theta} = \dfrac{\tan^2 \theta + 1 + 2 \sec \theta + \sec^2 \theta}{(1 + \sec \theta) \tan \theta}$

$$= \frac{2 \sec^2 \theta + 2\sec \theta}{(1 + \sec \theta) \tan \theta}$$

$$= \frac{2 \sec \theta(\sec \theta + 1)}{(1 + \sec \theta) \tan \theta}$$

$$= \frac{2 \sec \theta}{\tan \theta} = 2 \csc \theta$$

49. $\dfrac{\cot(-\theta)}{\csc \theta} = \dfrac{\cos(-\theta)}{\sin(-\theta)} \sin \theta$

$$= \frac{\cos \theta}{-\sin \theta} \sin \theta = -\cos \theta$$

50. $\dfrac{\csc\left(\dfrac{\pi}{2} - \theta\right)}{\tan(-\theta)} = \dfrac{\sec \theta}{-\tan \theta} = -\csc \theta$

51. $\cot^2 x - \cot^2 x \cos^2 x = \cot^2 x(1 - \cos^2 x)$

$= \dfrac{\cos^2 x}{\sin^2 x}\sin^2 x = \cos^2 x$

52. $\sec^2 x \tan^2 x + \sec^2 x = \sec^2 x(\tan^2 x + 1)$

$= \sec^2 x(\sec^2 x) = \sec^4 x$

53. $\dfrac{\cos^2 x - 4}{\cos x - 2} = \dfrac{(\cos x + 2)(\cos x - 2)}{\cos x - 2} = \cos x + 2$

54. $\dfrac{\csc^2 x - 1}{\csc x - 1} = \dfrac{(\csc x - 1)(\csc x + 1)}{\csc x - 1} = \csc x + 1$

55. $\tan^4 x + 2\tan^2 x + 1 = (\tan^2 x + 1)^2$

$= (\sec^2 x)^2 = \sec^4 x$

56. $1 - 2\sin^2 x + \sin^4 x = (1 - \sin^2 x)^2$

$= (\cos^2 x)^2 = \cos^4 x$

57. $\sin^4 x - \cos^4 x = (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)$

$= (1)(\sin^2 x - \cos^2 x) = \sin^2 x - \cos^2 x$

58. $\sec^4 x - \tan^4 x = (\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x) = \sec^2 x + \tan^2 x$

59. $\csc^3 x - \csc^2 x - \csc x + 1 = \csc^2 x(\csc x - 1) - (\csc x - 1)$

$= (\csc^2 x - 1)(\csc x - 1)$

$= \cot^2 x(\csc x - 1)$

60. $\sec^3 x - \sec^2 x - \sec x + 1 = \sec^2 x(\sec x - 1) - (\sec x - 1)$

$= (\sec^2 x - 1)(\sec x - 1)$

$= \tan^2 x(\sec x - 1)$

61. $(\sin x + \cos x)^2 = \sin^2 x + 2\sin x\cos x + \cos^2 x$

$= (\sin^2 x + \cos^2 x) + 2\sin x\cos x$

$= 1 + 2\sin x\cos x$

62. $(\tan x + \sec x)(\tan x - \sec x) = \tan^2 x - \sec^2 x$

$= -1$

63. $(\csc x + 1)(\csc x - 1) = \csc^2 x - 1 = \cot^2 x$

64. $(5 - 5\sin x)(5 + 5\sin x) = 25 - 25\sin^2 x$

$= 25(1 - \sin^2 x)$

$= 25\cos^2 x$

65. $\dfrac{1}{1 + \cos x} + \dfrac{1}{1 - \cos x} = \dfrac{1 - \cos x + 1 + \cos x}{(1 + \cos x)(1 - \cos x)}$

$= \dfrac{2}{1 - \cos^2 x}$

$= \dfrac{2}{\sin^2 x}$

$= 2\csc^2 x$

66. $\dfrac{1}{\sec x + 1} - \dfrac{1}{\sec x - 1} = \dfrac{\sec x - 1 - (\sec x + 1)}{(\sec x + 1)(\sec x - 1)}$

$= \dfrac{\sec x - 1 - \sec x - 1}{\sec^2 x - 1}$

$= \dfrac{-2}{\tan^2 x}$

$= -2\left(\dfrac{1}{\tan^2 x}\right)$

$= -2\cot^2 x$

67. $\tan x - \dfrac{\sec^2 x}{\tan x} = \dfrac{\tan^2 x - \sec^2 x}{\tan x} = \dfrac{-1}{\tan x} = -\cot x$

68. $\dfrac{\cos x}{1 + \sin x} + \dfrac{1 + \sin x}{\cos x} = \dfrac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{\cos x(1 + \sin x)}$

$= \dfrac{2 + 2\sin x}{\cos x(1 + \sin x)}$

$= \dfrac{2}{\cos x} = 2\sec x$

69. $\dfrac{\sin^2 y}{1 - \cos y} = \dfrac{1 - \cos^2 y}{1 - \cos y}$

$= \dfrac{(1 + \cos y)(1 - \cos y)}{1 - \cos y}$

$= 1 + \cos y$

70. $\dfrac{5}{\tan x + \sec x} \cdot \dfrac{\tan x - \sec x}{\tan x - \sec x} = \dfrac{5(\tan x - \sec x)}{\tan^2 x - \sec^2 x}$

$= \dfrac{5(\tan x - \sec x)}{-1}$

$= 5(\sec x - \tan x)$

71. $\dfrac{3}{\sec x - \tan x} \cdot \dfrac{\sec x + \tan x}{\sec x + \tan x} = \dfrac{3(\sec x + \tan x)}{\sec^2 x - \tan^2 x}$

$= \dfrac{3(\sec x + \tan x)}{1}$

$= 3(\sec x + \tan x)$

72. $\dfrac{\tan^2 x}{\csc x + 1} \cdot \dfrac{\csc x - 1}{\csc x - 1} = \dfrac{\tan^2 x(\csc x - 1)}{\csc^2 x - 1}$

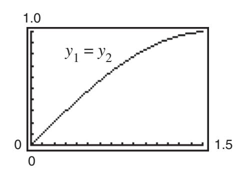
$= \dfrac{\tan^2 x(\csc x - 1)}{\cot^2 x}$

$= \tan^2 x(\csc x - 1)\tan^2 x$

$= \tan^4 x(\csc x - 1)$

73. $y_1 = \cos\left(\dfrac{\pi}{2} - x\right),\ y_2 = \sin x$

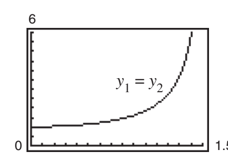
x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	0.1987	0.3894	0.5646	0.7174	0.8415	0.9320	0.9854
y_2	0.1987	0.3894	0.5646	0.7174	0.8415	0.9320	0.9854



Conjecture: $y_1 = y_2$

74. $y_1 = \cos x + \sin x \tan x, y_2 = \sec x$

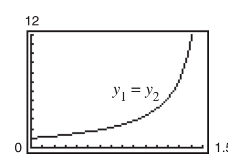
x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	1.0203	1.0857	1.2116	1.4353	1.8508	2.7597	5.8835
y_2	1.0203	1.0857	1.2116	1.4353	1.8508	2.7597	5.8835



Conjecture: $y_1 = y_2$

75. $y_1 = \dfrac{\cos x}{1 - \sin x},\ y_2 = \dfrac{1 + \sin x}{\cos x}$

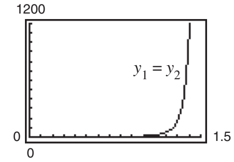
x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	1.2230	1.5085	1.8958	2.4650	3.4082	5.3319	11.6814
y_2	1.2230	1.5085	1.8958	2.4650	3.4082	5.3319	11.6814



Conjecture: $y_1 = y_2$

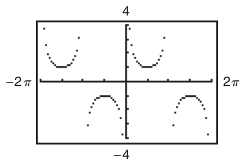
76. $y_1 = \sec^4 x - \sec^2 x, y_2 = \tan^2 x + \tan^4 x$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	0.0428	0.2107	0.6871	2.1841	8.3087	50.3869	1163.6143
y_2	0.0428	0.2107	0.6871	2.1841	8.3087	50.3869	1163.6143

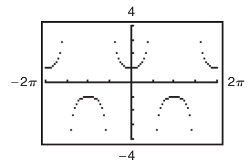


Conjecture: $y_1 = y_2$

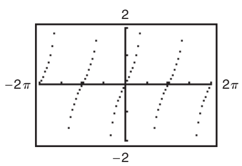
77. $y_1 = \cos x \cot x + \sin x = \csc x$



78. $\sin x(\cot x + \tan x) = \sec x$

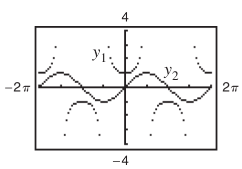


79. $y_1 = \sec x - \dfrac{\cos x}{1 + \sin x} = \tan x$

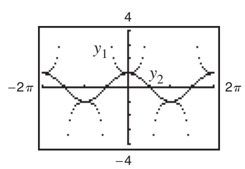


80. $y_1 = \dfrac{1}{2}\left(\dfrac{1 + \sin \theta}{\cos \theta} + \dfrac{\cos \theta}{1 + \sin \theta}\right)$

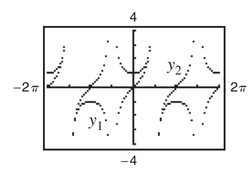
y_1 and $y_2 = \sin \theta$



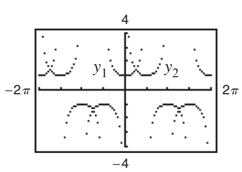
y_1 and $y_2 = \cos \theta$



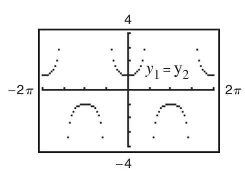
y_1 and $y_2 = \tan \theta$



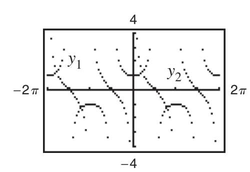
y_1 and $y_2 = \dfrac{1}{\sin \theta} = \csc \theta$



y_1 and $y_2 = \dfrac{1}{\cos \theta} = \sec \theta$



y_1 and $y_2 = \dfrac{1}{\tan \theta} = \cot \theta$



It appears that $\dfrac{1}{2}\left(\dfrac{1 + \sin \theta}{\cos \theta} + \dfrac{\cos \theta}{1 + \sin \theta}\right) = \sec \theta.$

81. $\sqrt{25 - x^2} = \sqrt{25 - (5 \sin \theta)^2}, \; x = 5 \sin \theta$

$= \sqrt{25 - 25 \sin^2 \theta}$

$= \sqrt{25(1 - \sin^2 \theta)}$

$= \sqrt{25 \cos^2 \theta}$

$= 5 \cos \theta$

82. Let $x = 2 \cos \theta$.

$\sqrt{64 - 16x^2} = \sqrt{64 - 16(4 \cos^2 \theta)}$

$= 8\sqrt{1 - \cos^2 \theta}$

$= 8 \sin \theta$

83. $\sqrt{x^2 - 9} = \sqrt{(3 \sec \theta)^2 - 9},\ x = 3 \sec \theta$

$= \sqrt{9 \sec^2 \theta - 9}$

$= \sqrt{9(\sec^2 \theta - 1)}$

$= \sqrt{9 \tan^2 \theta}$

$= 3 \tan \theta$

84. Let $x = 10 \tan \theta$.

$\sqrt{x^2 + 100} = \sqrt{(10 \tan \theta)^2 + 100}$

$= \sqrt{100(\tan^2 \theta + 1)}$

$= \sqrt{100 \sec^2 \theta}$

$= 10 \sec \theta$

85. $x = 3 \sin \theta, 0 < \theta < \dfrac{\pi}{2}$

$\sqrt{9 - x^2} = \sqrt{9 - 9 \sin^2 \theta}$

$= \sqrt{9 \cos^2 \theta} = 3 \cos \theta$

86. $x = 2 \cos \theta, 0 < \theta < \dfrac{\pi}{2}$

$\sqrt{4 - x^2} = \sqrt{4 - 4 \cos^2 \theta}$

$= \sqrt{4 \sin^2 \theta} = 2 \sin \theta$

87. $2x = 3 \tan \theta,\ 0 < \theta < \dfrac{\pi}{2}$

$\sqrt{4x^2 + 9} = \sqrt{9 \tan^2 \theta + 9}$

$= \sqrt{9 \sec^2 \theta} = 3 \sec \theta$

88. $3x = 2 \tan \theta,\ 0 < \theta < \dfrac{\pi}{2}$

$\sqrt{9x^2 + 4} = \sqrt{4 \tan^2 \theta + 4}$

$= \sqrt{4 \sec^2 \theta} = 2 \sec \theta$

89. $4x = 3 \sec \theta,\ 0 < \theta < \dfrac{\pi}{2}$

$\sqrt{16x^2 - 9} = \sqrt{9 \sec^2 \theta - 9}$

$= \sqrt{9 \tan^2 \theta} = 3 \tan \theta$

90. $3x = 5 \sec \theta,\ 0 < \theta < \dfrac{\pi}{2}$

$\sqrt{9x^2 - 25} = \sqrt{25 \sec^2 \theta - 25}$

$= \sqrt{25 \tan^2 \theta} = 5 \tan \theta$

91. $x = \sqrt{2} \sin \theta, 0 < \theta < \dfrac{\pi}{2}$

$\sqrt{2 - x^2} = \sqrt{2 - 2 \sin^2 \theta}$

$= \sqrt{2 \cos^2 \theta} = \sqrt{2} \cos \theta$

92. $x = \sqrt{5} \cos \theta,\ 0 < \theta < \dfrac{\pi}{2}$

$\sqrt{5 - x^2} = \sqrt{5 - 5 \cos^2 \theta}$

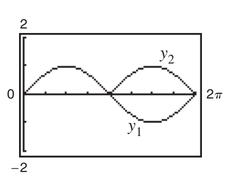
$= \sqrt{5 \sin^2 \theta} = \sqrt{5} \sin \theta$

93. $\sin \theta = \sqrt{1 - \cos^2 \theta}$

Let $y_1 = \sin x$ and $y_2 = \sqrt{1 - \cos^2 x}, 0 \le x < 2\pi$.

$y_1 = y_2$ for $0 \le x \le \pi$, so we have

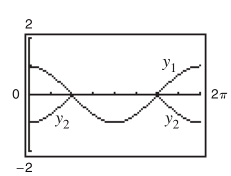
$\sin \theta = \sqrt{1 - \cos^2 \theta}$ for $0 \le \theta \le \pi$.



94. $\cos \theta = -\sqrt{1 - \sin^2 \theta}$

Let $y_1 = \cos \theta$ and $y_2 = -\sqrt{1 - \sin^2 \theta}$.

$y_1 = y_2$ for $\dfrac{\pi}{2} \le \theta \le \dfrac{3\pi}{2}$.

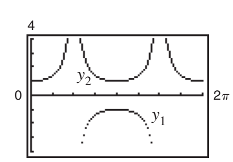


95. $\sec \theta = \sqrt{1 + \tan^2 \theta}$

Let $y_1 = \dfrac{1}{\cos x}$ and $y_2 = \sqrt{1 + \tan^2 x}, 0 \le x < 2\pi$.

$y_1 = y_2$ for $0 \le x < \dfrac{\pi}{2}$ and $\dfrac{3\pi}{2} < x < 2\pi$, so we

have $\sec \theta = \sqrt{1 + \tan^2 \theta}$ for $0 \le \theta < \dfrac{\pi}{2}$ and $\dfrac{3\pi}{2} < \theta < 2\pi$.



96. $\tan\theta = \sqrt{\sec^2\theta - 1}$

$0 \le \theta < \frac{\pi}{2},\ \pi \le \theta < \frac{3\pi}{2}$

97. $\ln|\cos\theta| - \ln|\sin\theta| = \ln\frac{|\cos\theta|}{|\sin\theta|} = \ln|\cot\theta|$

98. $\ln|\csc\theta| + \ln|\tan\theta| = \ln|\csc\theta \cdot \tan\theta|$

$= \ln|\sec\theta|$

99. $\ln(1 + \sin x) - \ln|\sec x| = \ln\left|\frac{1 + \sin x}{\sec x}\right|$

$= \ln|\cos x(1 + \sin x)|$

100. $\ln|\cot t| + \ln(1 + \tan^2 t) = \ln\left[|\cot t|(1 + \tan^2 t)\right]$

$= \ln\frac{(1 + \tan^2 t)}{|\tan t|}$

$= \ln\left|\frac{1}{\tan t} + \frac{\tan^2 t}{\tan t}\right|$

$= \ln|\cot t + \tan t|$

101. Let $\theta = \frac{7\pi}{6}$. Then

$\cos\theta = \cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2} \ne \sqrt{1 - \sin^2\theta} = \frac{\sqrt{3}}{2}.$

102. Let $\theta = \frac{2\pi}{3}$. Then

$\tan\theta = \tan\frac{2\pi}{3} = -\sqrt{3} \ne \sqrt{\sec^2\theta - 1} = \sqrt{3}.$

103. Let $\theta = \frac{5\pi}{3}$. Then

$\sin\theta = \sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2} \ne \sqrt{1 - \cos^2\theta} = \frac{\sqrt{3}}{2}.$

104. Let $\theta = \pi$. Then

$\sec\theta = \sec\pi = -1 \ne \sqrt{1 + \tan^2\theta} = 1.$

105. Let $\theta = \frac{7\pi}{4}$. Then

$\csc\theta = \csc\frac{7\pi}{4} = -\sqrt{2} \ne \sqrt{1 + \cot^2\theta} = \sqrt{2}.$

106. Let $\theta = \frac{3\pi}{4}$. Then

$\cot\theta = \cot\frac{3\pi}{4} = -1 \ne \sqrt{\csc^2\theta - 1} = 1.$

107. (a) $\csc^2 132° - \cot^2 132° \approx 1.8107 - 0.8107 = 1$

(b) $\csc^2\frac{2\pi}{7} - \cot^2\frac{2\pi}{7} \approx 1.6360 - 0.6360 = 1$

108. $\tan^2\theta + 1 = \sec^2\theta$

(a) $\theta = 346°$

$(\tan 346°)^2 + 1 \approx 1.0622$

$(\sec 346°)^2 = \left(\frac{1}{\cos 346°}\right)^2 \approx 1.0622$

(b) $\theta = 3.1$

$(\tan 3.1)^2 + 1 \approx 1.00173$

$(\sec 3.1)^2 = \left(\frac{1}{\cos 3.1}\right)^2 \approx 1.00173$

109. $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

(a) $\theta = 80°$

$\cos(90° - 80°) = \sin 80°$

$0.9848 = 0.9848$

(b) $\theta = 0.8$

$\cos\left(\frac{\pi}{2} - 0.8\right) = \sin 0.8$

$0.7174 = 0.7174$

110. $\sin(-\theta) = -\sin\theta$

(a) $\theta = 250°$

$\sin(-250°) \approx 0.9397$

$-(\sin 250°) \approx 0.9397$

(b) $\theta = \frac{1}{2}$

$\sin\left(-\frac{1}{2}\right) \approx -0.4794$

$-\left(\sin\frac{1}{2}\right) \approx -0.4794$

111. $\csc x \cot x - \cos x = \frac{1}{\sin x}\cdot\frac{\cos x}{\sin x} - \cos x$

$= \cos x(\csc^2 x - 1)$

$= \cos x \cdot \cot^2 x$

112. $\sec x \tan x - \sin x = \frac{1}{\cos x}\frac{\sin x}{\cos x} - \sin x$

$= \sin x[\sec^2 x - 1]$

$= \sin x \cdot \tan^2 x$

113. True for all $\theta \neq n\pi$

$\sin\theta \cdot \csc\theta = \sin\theta\left(\frac{1}{\sin\theta}\right) = 1$

114. False

$\cos 0 \sec\frac{\pi}{4} \neq 1$

115. As $x \to \frac{\pi}{2}^-$, $\sin x \to 1$ and $\csc x \to 1$.

116. As $x \longrightarrow 0^+$,

$\cos x \longrightarrow 1$ and $\sec x = \frac{1}{\cos x} \longrightarrow 1$.

117. As $x \to \frac{\pi}{2}^-$, $\tan x \to \infty$ and $\cot x \to 0$.

118. As $x \longrightarrow \pi^+$,

$\sin x \longrightarrow 0$ and $\csc x = \frac{1}{\sin x} \longrightarrow -\infty$.

119. $\sin\theta$

$\cos\theta = \pm\sqrt{1-\sin^2\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \pm\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$

$\csc\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{\pm 1}{\sqrt{1-\sin^2\theta}}$

$\cot\theta = \pm\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$

The sign + or − depends on the choice of θ.

120. $\cos\theta$

$\sin\theta = \pm\sqrt{1-\cos^2\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \pm\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$

$\csc\theta = \frac{1}{\sin\theta} = \pm\frac{1}{\sqrt{1-\cos^2\theta}}$

$\sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta} = \pm\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$

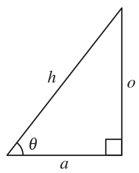
The sign + or − depends on the choice of θ.

121. $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}, \cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

From the Pythagorean Theorem,

$(\text{opp})^2 + (\text{adj})^2 = (\text{hyp})^2$

$\sin^2\theta + \cos^2\theta = 1.$



122. $\sin^2\theta + \cos^2\theta = 1$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$1 + \cot^2\theta = \csc^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

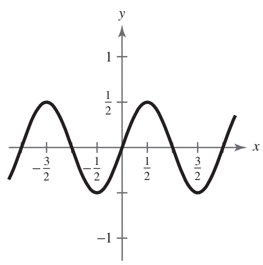
$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$\tan^2\theta + 1 = \sec^2\theta$

123. $f(x) = \dfrac{1}{2}\sin\pi x$

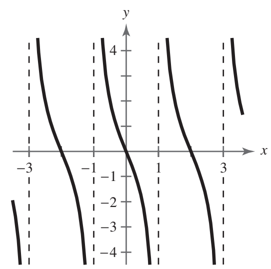
Period: $\dfrac{2\pi}{\pi} = 2$

Amplitude: $\dfrac{1}{2}$



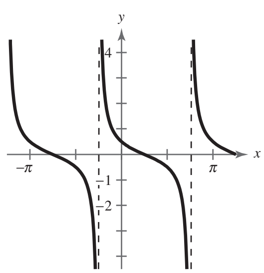
124. $f(x) = -2\tan\dfrac{\pi x}{2}$

Period: $\dfrac{\pi}{\pi/2} = 2$



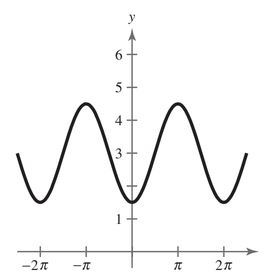
125. $f(x) = \dfrac{1}{2}\cot\left(x + \dfrac{\pi}{4}\right)$

Period: π



126. $f(x) = \dfrac{3}{2}\cos(x - \pi) + 3$

Amplitude: $\dfrac{3}{2}$



Section 5.2 Verifying Trigonometric Identities

- You should know the difference between an expression, a conditional equation, and an identity.
- You should be able to solve trigonometric identities, using the following techniques.
  - (a) Work with *one* side at a time. Do not "cross" the equal sign.
  - (b) Use algebraic techniques such as combining fractions, factoring expressions, rationalizing denominators, and squaring binomials.
  - (c) Use the fundamental identities.
  - (d) Convert all the terms into sines and cosines.

Vocabulary Check

1. conditional
2. identity
3. cot u
4. sin u
5. tan u
6. cos u
7. $\cos^2 u$
8. cot u
9. $-\sin u$
10. sec u

1. $\sin t \csc t = \sin t\left(\dfrac{1}{\sin t}\right) = 1$

2. $\sec y \cos y = \dfrac{1}{\cos y}\cos y = 1$

3. $\dfrac{\csc^2 x}{\cot x} = \dfrac{1}{\sin^2 x} \cdot \dfrac{\sin x}{\cos x} = \dfrac{1}{\sin x \cdot \cos x}$
$= \csc x \cdot \sec x$

4. $\dfrac{\sin^2 t}{\tan^2 t} = \dfrac{\sin^2 t}{\dfrac{\sin^2 t}{\cos^2 t}} = \cos^2 t$

5. $\cos^2 \beta - \sin^2 \beta = (1 - \sin^2 \beta) - \sin^2 \beta$
$= 1 - 2\sin^2 \beta$

6. $\cos^2 \beta - \sin^2 \beta = \cos^2 \beta - (1 - \cos^2 \beta)$
$= 2\cos^2 \beta - 1$

7. $\tan^2 \theta + 6 = (\tan^2 \theta + 1) + 5$
$= \sec^2 \theta + 5$

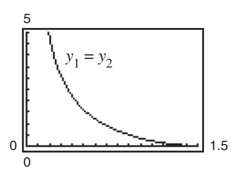
8. $2 - \csc^2 z = 2 - (\cot^2 z + 1) = 1 - \cot^2 z$

9. $(1 + \sin x)(1 - \sin x) = 1 - \sin^2 x = \cos^2 x$

10. $\tan^2 y(\csc^2 y - 1) = \tan^2 y \cot^2 y = 1$

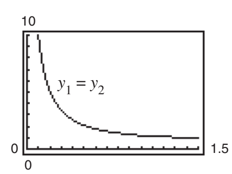
11. $\dfrac{1}{\sec x \tan x} = \cos x \cdot \dfrac{\cos x}{\sin x}$
$= \dfrac{\cos^2 x}{\sin x}$
$= \dfrac{1 - \sin^2 x}{\sin x}$
$= \dfrac{1}{\sin x} - \sin x$
$= \csc x - \sin x$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	4.8348	2.1785	1.2064	0.6767	0.3469	0.1409	0.0293
y_2	4.8348	2.1785	1.2064	0.6767	0.3469	0.1409	0.0293



12. $y_1 = \dfrac{\csc x - 1}{1 - \sin x} = \dfrac{\dfrac{1}{\sin x} - 1}{1 - \sin x}$
$= \dfrac{1 - \sin x}{\sin x}\dfrac{1}{1 - \sin x}$
$= \dfrac{1}{\sin x}$
$= \csc x = y_2$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	5.0335	2.5679	1.7710	1.3940	1.1884	1.0729	1.0148
y_2	5.0335	2.5679	1.7710	1.3940	1.1884	1.0729	1.0148



13. $\csc x - \sin x = \dfrac{1}{\sin x} - \sin x$

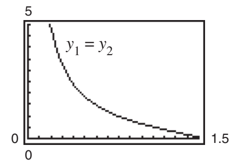
$$= \frac{1 - \sin^2 x}{\sin x}$$

$$= \frac{\cos^2 x}{\sin x}$$

$$= \cos x \cdot \frac{\cos x}{\sin x}$$

$$= \cos x \cdot \cot x$$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	4.8348	2.1785	1.2064	0.6767	0.3469	0.1409	0.0293
y_2	4.8348	2.1785	1.2064	0.6767	0.3469	0.1409	0.0293



14. $y_1 = \sec x - \cos x = \dfrac{1}{\cos x} - \cos x$

$$= \frac{1 - \cos^2 x}{\cos x}$$

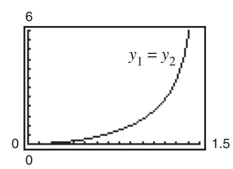
$$= \frac{\sin^2 x}{\cos x}$$

$$= \sin x\left(\frac{\sin x}{\cos x}\right)$$

$$= \sin x \tan x$$

$$= y_2$$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	0.0403	0.1646	0.3863	0.7386	1.3105	2.3973	5.7135
y_2	0.0403	0.1646	0.3863	0.7386	1.3105	2.3973	5.7135



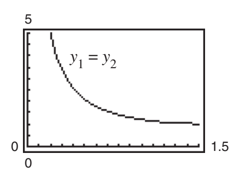
15. $\sin x + \cos x \cot x = \sin x + \cos x \dfrac{\cos x}{\sin x}$

$$= \frac{\sin^2 x + \cos^2 x}{\sin x}$$

$$= \frac{1}{\sin x}$$

$$= \csc x$$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	5.0335	2.5679	1.7710	1.3940	1.1884	1.0729	1.0148
y_2	5.0335	2.5679	1.7710	1.3940	1.1884	1.0729	1.0148



16. $y_1 = \cos x + \sin x \tan x$

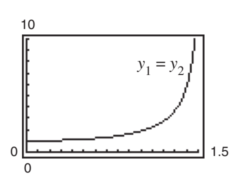
$$= \cos x + \frac{\sin^2 x}{\cos x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos x}$$

$$= \frac{1}{\cos x}$$

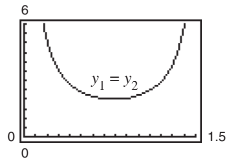
$$= \sec x = y_2$$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	1.0203	1.0857	1.2116	1.4353	1.8508	2.7597	5.8835
y_2	1.0203	1.0857	1.2116	1.4353	1.8508	2.7597	5.8835



17. $\dfrac{1}{\tan x} + \dfrac{1}{\cot x} = \dfrac{\cot x + \tan x}{\tan x \cdot \cot x}$

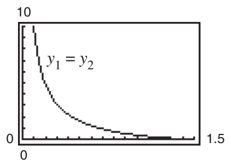
$= \cot x + \tan x$



x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	5.1359	2.7880	2.1458	2.0009	2.1995	2.9609	5.9704
y_2	5.1359	2.7880	2.1458	2.0009	2.1995	2.9609	5.9704

18. $y_1 = \dfrac{1}{\sin x} - \dfrac{1}{\csc x}$

$= \csc x - \sin x = y_2$



x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	4.8348	2.1785	1.2064	0.6767	0.3469	0.1409	0.0293
y_2	4.8348	2.1785	1.2064	0.6767	0.3469	0.1409	0.0293

19. The error is in line 1: $\cot(-x) \neq \cot x$.

20. There are two errors in line 1:

$\sec(-\theta) = \sec\theta$ and $\sin(-\theta) = -\sin\theta$.

21. $\sin^{1/2} x \cos x - \sin^{5/2} x \cos x = \sin^{1/2} x \cos x(1 - \sin^2 x) = \sin^{1/2} x \cos x \cdot \cos^2 x = \cos^3 x\sqrt{\sin x}$

22. $\sec^6 x(\sec x \tan x) - \sec^4 x(\sec x \tan x) = \sec^4 x(\sec x \tan x)(\sec^2 x - 1)$

$= \sec^4 x(\sec x \tan x)\tan^2 x$

$= \sec^5 x \tan^3 x$

23. $\cot\left(\dfrac{\pi}{2} - x\right)\csc x = \tan x \csc x$

$= \dfrac{\sin x}{\cos x} \cdot \dfrac{1}{\sin x}$

$= \dfrac{1}{\cos x} = \sec x$

24. $\dfrac{\sec(\pi/2 - x)}{\tan(\pi/2 - x)} = \dfrac{\csc x}{\cot x}$

$= \dfrac{1}{\sin x} \cdot \dfrac{\sin x}{\cos x}$

$= \dfrac{1}{\cos x} = \sec x$

25. $\dfrac{\csc(-x)}{\sec(-x)} = \dfrac{1/\sin(-x)}{1/\cos(-x)}$

$= \dfrac{\cos(-x)}{\sin(-x)}$

$= \dfrac{\cos x}{-\sin x}$

$= -\cot x$

26. $(1 + \sin y)[1 + \sin(-y)] = (1 + \sin y)(1 - \sin y)$

$= 1 - \sin^2 y$

$= \cos^2 y$

27. $\dfrac{\cos(-\theta)}{1 + \sin(-\theta)} = \dfrac{\cos\theta}{1 - \sin\theta} \cdot \dfrac{1 + \sin\theta}{1 + \sin\theta}$

$= \dfrac{\cos\theta(1 + \sin\theta)}{1 - \sin^2\theta}$

$= \dfrac{\cos\theta(1 + \sin\theta)}{\cos^2\theta}$

$= \dfrac{1 + \sin\theta}{\cos\theta}$

$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \sec\theta + \tan\theta$

28. $\dfrac{1 + \csc(-\theta)}{\cos(-\theta) + \cot(-\theta)} = \dfrac{1 - \csc\theta}{\cos\theta - \cot\theta}$

$= \dfrac{1 - \csc\theta}{\cos\theta\left(1 - \dfrac{1}{\sin\theta}\right)}$

$= \dfrac{1 - \csc\theta}{\cos\theta(1 - \csc\theta)}$

$= \dfrac{1}{\cos\theta}$

$= \sec\theta$

29. $\dfrac{\sin x\cos y + \cos x\sin y}{\cos x\cos y - \sin x\sin y} = \dfrac{\dfrac{\sin x\cos y}{\cos x\cos y} + \dfrac{\cos x\sin y}{\cos x\cos y}}{\dfrac{\cos x\cos y}{\cos x\cos y} - \dfrac{\sin x\sin y}{\cos x\cos y}} = \dfrac{\tan x + \tan y}{1 - \tan x\tan y}$

30. $\dfrac{\tan x + \tan y}{1 - \tan x\tan y} = \dfrac{\dfrac{1}{\cot x} + \dfrac{1}{\cot y}}{1 - \dfrac{1}{\cot x}\cdot\dfrac{1}{\cot y}} \cdot \dfrac{\cot x\cot y}{\cot x\cot y} = \dfrac{\cot y + \cot x}{\cot x\cot y - 1}$

31. $\dfrac{\cos x - \cos y}{\sin x + \sin y} + \dfrac{\sin x - \sin y}{\cos x + \cos y} = \dfrac{(\cos x + \cos y)(\cos x - \cos y) + (\sin x + \sin y)(\sin x - \sin y)}{(\sin x + \sin y)(\cos x + \cos y)}$

$= \dfrac{\cos^2 x - \cos^2 y + \sin^2 x - \sin^2 y}{(\sin x + \sin y)(\cos x + \cos y)}$

$= \dfrac{1 - 1}{(\sin x + \sin y)(\cos x + \cos y)}$

$= 0$

32. $\dfrac{\tan x + \cot y}{\tan x\cot y} = \dfrac{1}{\cot y} + \dfrac{1}{\tan x} = \tan y + \cot x$

33. $\sqrt{\dfrac{1 + \sin\theta}{1 - \sin\theta}} = \sqrt{\dfrac{1 + \sin\theta}{1 - \sin\theta}\cdot\dfrac{1 + \sin\theta}{1 + \sin\theta}}$

$= \sqrt{\dfrac{(1 + \sin\theta)^2}{1 - \sin^2\theta}}$

$= \sqrt{\dfrac{(1 + \sin\theta)^2}{\cos^2\theta}}$

$= \dfrac{1 + \sin\theta}{|\cos\theta|}$

Note: Check your answer with a graphing utility. What happens if you leave off the absolute value?

34. $\sqrt{\dfrac{1 - \cos\theta}{1 + \cos\theta}} = \sqrt{\dfrac{1 - \cos\theta}{1 + \cos\theta}\cdot\dfrac{1 - \cos\theta}{1 - \cos\theta}}$

$= \sqrt{\dfrac{(1 - \cos\theta)^2}{1 - \cos^2\theta}}$

$= \sqrt{\dfrac{(1 - \cos\theta)^2}{\sin^2\theta}}$

$= \dfrac{1 - \cos\theta}{|\sin\theta|}$

35. $\sin^2\left(\frac{\pi}{2} - x\right) + \sin^2 x = \cos^2 x + \sin^2 x = 1$

36. $\sec^2 y - \cot^2\left(\frac{\pi}{2} - y\right) = \sec^2 y - \tan^2 y = 1$

37. $\sin x \csc\left(\frac{\pi}{2} - x\right) = \sin x \sec x$
$$= \sin x\left(\frac{1}{\cos x}\right)$$
$$= \tan x$$

38. $\sec^2\left(\frac{\pi}{2} - x\right) - 1 = \csc^2 x - 1 = \cot^2 x$

39. $2\sec^2 x - 2\sec^2 x \sin^2 x - \sin^2 x - \cos^2 x = 2\sec^2 x(1 - \sin^2 x) - (\sin^2 x + \cos^2 x)$
$$= 2\sec^2 x(\cos^2 x) - 1$$
$$= 2 \cdot \frac{1}{\cos^2 x} \cdot \cos^2 x - 1$$
$$= 2 - 1 = 1$$

40. $\csc x(\csc x - \sin x) + \frac{\sin x - \cos x}{\sin x} + \cot x = \csc^2 x - \csc x \sin x + 1 - \frac{\cos x}{\sin x} + \cot x$
$$= \csc^2 x - 1 + 1 - \cot x + \cot x$$
$$= \csc^2 x$$

41. $\frac{\cot x \tan x}{\sin x} = \frac{1}{\sin x} = \csc x$

42. $\frac{1 + \csc\theta}{\sec\theta} - \cot\theta = \cos\theta\left(1 + \frac{1}{\sin\theta}\right) - \frac{\cos\theta}{\sin\theta}$
$$= \frac{\cos\theta}{\sin\theta}(\sin\theta + 1 - 1)$$
$$= \cos\theta$$

43. $\csc^4 x - 2\csc^2 x + 1 = (\csc^2 x - 1)^2$
$$= (\cot^2 x)^2 = \cot^4 x$$

44. $\sin x(1 - 2\cos^2 x + \cos^4 x) = \sin x(1 - \cos^2 x)^2$
$$= \sin x(\sin^2 x)^2$$
$$= \sin^5 x$$

45. $\sec^4\theta - \tan^4\theta = (\sec^2\theta + \tan^2\theta)(\sec^2\theta - \tan^2\theta)$
$$= (1 + \tan^2\theta + \tan^2\theta)(1)$$
$$= 1 + 2\tan^2\theta$$

46. $\csc^4\theta - \cot^4\theta = (\csc^2\theta - \cot^2\theta)(\csc^2\theta + \cot^2\theta)$
$$= \csc^2\theta + \cot^2\theta$$
$$= \csc^2\theta + (\csc^2\theta - 1)$$
$$= 2\csc^2\theta - 1$$

47. $\frac{\sin\beta}{1 - \cos\beta} \cdot \frac{1 + \cos\beta}{1 + \cos\beta} = \frac{\sin\beta(1 + \cos\beta)}{1 - \cos^2\beta}$
$$= \frac{\sin\beta(1 + \cos\beta)}{\sin^2\beta}$$
$$= \frac{1 + \cos\beta}{\sin\beta}$$

48. $\frac{\cot\alpha}{\csc\alpha - 1} \cdot \frac{\csc\alpha + 1}{\csc\alpha + 1} = \frac{\cot\alpha(\csc\alpha + 1)}{\csc^2\alpha - 1}$
$$= \frac{\cot\alpha(\csc\alpha + 1)}{\cot^2\alpha}$$
$$= \frac{\csc\alpha + 1}{\cot\alpha}$$

49. $\dfrac{\tan^3 \alpha - 1}{\tan \alpha - 1} = \dfrac{(\tan \alpha - 1)(\tan^2 \alpha + \tan \alpha + 1)}{\tan \alpha - 1} = \tan^2 \alpha + \tan \alpha + 1$

50. $\dfrac{\sin^3 \beta + \cos^3 \beta}{\sin \beta + \cos \beta} = \dfrac{(\sin \beta + \cos \beta)(\sin^2 \beta - \sin \beta \cos \beta + \cos^2 \beta)}{\sin \beta + \cos \beta}$

$= \sin^2 \beta + \cos^2 \beta - \sin \beta \cos \beta$

$= 1 - \sin \beta \cos \beta$

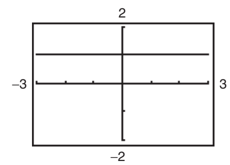
51. It appears that $y_1 = 1$. Analytically,

$\dfrac{1}{\cot x + 1} + \dfrac{1}{\tan x + 1} = \dfrac{\tan x + 1 + \cot x + 1}{(\cot x + 1)(\tan x + 1)}$

$= \dfrac{\tan x + \cot x + 2}{\cot x \tan x + \cot x + \tan x + 1}$

$= \dfrac{\tan x + \cot x + 2}{\tan x + \cot x + 2}$

$= 1.$



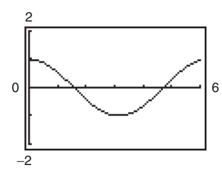
52. The function appears to be $y = \cos x$. Analytically,

$y = \dfrac{\cos x}{1 - \tan x} + \dfrac{\sin x \cdot \cos x}{\sin x - \cos x}$

$= \dfrac{\cos x}{1 - (\sin x/\cos x)} + \dfrac{\sin x \cos x}{\sin x - \cos x}$

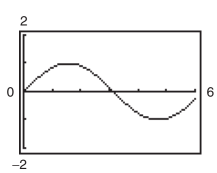
$= \dfrac{\cos^2 x}{\cos x - \sin x} - \dfrac{\sin x \cos x}{\cos x - \sin x}$

$= \dfrac{\cos x(\cos x - \sin x)}{\cos x - \sin x} = \cos x.$



53. It appears that $y_1 = \sin x$. Analytically,

$\dfrac{1}{\sin x} - \dfrac{\cos^2 x}{\sin x} = \dfrac{1 - \cos^2 x}{\sin x} = \dfrac{\sin^2 x}{\sin x} = \sin x.$

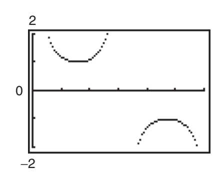


54. The function appears to be $y = \csc t$. Analytically,

$y = \sin t + \dfrac{\cot^2 t}{\csc t}$

$= \dfrac{1 + \cot^2 t}{\csc t}$

$= \dfrac{\csc^2 t}{\csc t} = \csc t.$



55. $\ln|\cot \theta| = \ln\left|\dfrac{\cos \theta}{\sin \theta}\right|$

$= \ln \dfrac{|\cos \theta|}{|\sin \theta|}$

$= \ln|\cos \theta| - \ln|\sin \theta|$

56. $\ln|\sec \theta| = \ln\left|\dfrac{1}{\cos \theta}\right|$

$= \ln|\cos \theta|^{-1}$

$= -\ln|\cos \theta|$

57. $-\ln(1+\cos\theta) = \ln(1+\cos\theta)^{-1}$

$$= \ln\left[\frac{1}{1+\cos\theta}\cdot\frac{1-\cos\theta}{1-\cos\theta}\right]$$

$$= \ln\frac{1-\cos\theta}{1-\cos^2\theta}$$

$$= \ln\frac{1-\cos\theta}{\sin^2\theta}$$

$$= \ln(1-\cos\theta) - \ln\sin^2\theta$$

$$= \ln(1-\cos\theta) - 2\ln|\sin\theta|$$

58. $-\ln|\csc\theta + \cot\theta| = -\ln\left|\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right|$

$$= \ln\left|\frac{1+\cos\theta}{\sin\theta}\right|^{-1}$$

$$= \ln\left|\frac{\sin\theta}{1+\cos\theta}\right|$$

$$= \ln\left|\frac{\sin\theta}{1+\cos\theta}\cdot\frac{1-\cos\theta}{1-\cos\theta}\right|$$

$$= \ln\left|\frac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta}\right|$$

$$= \ln\left|\frac{\sin\theta(1-\cos\theta)}{\sin^2\theta}\right|$$

$$= \ln\left|\frac{1-\cos\theta}{\sin\theta}\right|$$

$$= \ln|\csc\theta - \cot\theta|$$

59. $\sin^2 35° + \sin^2 55° = \cos^2(90° - 35°) + \sin^2 55°$

$$= \cos^2 55° + \sin^2 55° = 1$$

60. $\cos^2 14° + \cos^2 76° = \sin^2(90° - 14°) + \cos^2 76°$

$$= \sin^2 76° + \cos^2 76° = 1$$

61. $\cos^2 20° + \cos^2 52° + \cos^2 38° + \cos^2 70° = \cos^2 20° + \cos^2 52° + \sin^2(90° - 38°) + \sin^2(90° - 70°)$

$$= \cos^2 20° + \cos^2 52° + \sin^2 52° + \sin^2 20°$$

$$= (\cos^2 20° + \sin^2 20°) + (\cos^2 52° + \sin^2 52°)$$

$$= 1 + 1 = 2$$

62. $\sin^2 18° + \sin^2 40° + \sin^2 50° + \sin^2 72° = (\sin^2 18° + \sin^2 72°) + (\sin^2 40° + \sin^2 50°)$

$$= (\sin^2 18° + \cos^2 18°) + (\sin^2 40° + \cos^2 40°)$$

$$= 1 + 1 = 2$$

63. $\tan^5 x = \tan^3 x \cdot \tan^2 x$

$$= \tan^3 x(\sec^2 x - 1)$$

$$= \tan^3 x\sec^2 x - \tan^3 x$$

64. $\sec^4 x\tan^2 x = \sec^2 x(1+\tan^2 x)\tan^2 x$

$$= (\tan^2 x + \tan^4 x)\sec^2 x$$

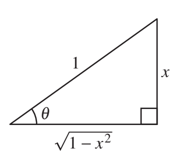
65. $(\sin^2 x - \sin^4 x)\cos x = \sin^2 x(1-\sin^2 x)\cos x = \sin^2 x\cdot\cos^2 x\cdot\cos x = \cos^3 x\sin^2 x$

66. $1 - 2\cos^2 x + 2\cos^4 x = [1 - 2\cos^2 x + \cos^4 x] + \cos^4 x = (1-\cos^2 x)^2 + \cos^4 x = \sin^4 x + \cos^4 x$

67. Let $\theta = \sin^{-1} x \implies \sin\theta = x = \dfrac{x}{1}$.

From the diagram,

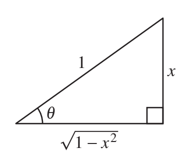
$\tan(\sin^{-1} x) = \tan\theta = \dfrac{x}{\sqrt{1-x^2}}$.



68. Let $\theta = \sin^{-1} x \implies \sin\theta = x = \dfrac{x}{1}$.

From the diagram,

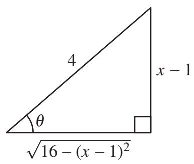
$\cos(\sin^{-1} x) = \cos\theta = \sqrt{1-x^2}$.



69. Let $\theta = \sin^{-1}\dfrac{x-1}{4} \Rightarrow \sin\theta = \dfrac{x-1}{4}$.

From the diagram,

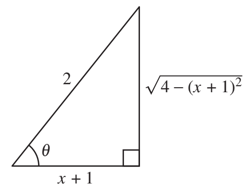
$$\tan\left(\sin^{-1}\frac{x-1}{4}\right) = \tan\theta = \frac{x-1}{\sqrt{16-(x-1)^2}}.$$



70. Let $\theta = \cos^{-1}\dfrac{x+1}{2} \Rightarrow \cos\theta = \dfrac{x+1}{2}$.

From the diagram,

$$\tan\left(\cos^{-1}\frac{x+1}{2}\right) = \tan\theta = \frac{\sqrt{4-(x+1)^2}}{x+1}.$$



71. $\mu W\cos\theta = W\sin\theta$

$$\mu = \frac{W\sin\theta}{W\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta,\ W \neq 0$$

72. $s = \dfrac{h\sin(90^\circ - \theta)}{\sin\theta} = h\dfrac{\cos\theta}{\sin\theta} = h\cot\theta$

73. True

74. True. Cosine and secant are even.

75. False. Just because the equation is true for one value of θ, you cannot conclude that the equation is an identity. For example,

$$\sin^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} = 1 \neq 1 + \tan^2\frac{\pi}{4}.$$

76. False. For example, $\sin(1^2) \neq \sin^2(1)$.

77. (a) $$\frac{\sin x}{1+\cos x} = \frac{\sin x}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x} = \frac{\sin x(1-\cos x)}{1-\cos^2 x} = \frac{\sin x(1-\cos x)}{\sin^2 x} = \frac{1-\cos x}{\sin x}$$

(b) Not true for $x = 0$ because $(1-\cos x)/\sin x$ is not defined for $x = 0$.

78. (a) $$\frac{\tan x}{\sec x - \cos x} = \frac{\tan x}{\dfrac{1}{\cos x} - \cos x} = \frac{\dfrac{\sin x}{\cos x}\cos x}{1-\cos^2 x} = \frac{\sin x}{\sin^2 x} = \frac{1}{\sin x}\cdot\frac{1/\cos x}{1/\cos x} = \frac{\sec x}{\tan x}$$

(b) Not true for $x = \pi$ because $\sec x/\tan x$ is not defined for $x = \pi$.

79. (a) $\dfrac{\sin x}{1+\cos x} = \dfrac{\sin x}{1+\cos x}\cdot\dfrac{1-\cos x}{1-\cos x}$

$= \dfrac{\sin x(1-\cos x)}{1-\cos^2 x}$

$= \dfrac{\sin x(1-\cos x)}{\sin^2 x}$

$= \dfrac{1-\cos x}{\sin x} = \csc x - \cot x$

(b) The identity is true for $x = \dfrac{\pi}{2}$:

$\dfrac{\sin(\pi/2)}{1+\cos(\pi/2)} = \dfrac{1}{1+0}$

$= 1 = \csc\dfrac{\pi}{2} - \cot\dfrac{\pi}{2}$

80. (a) $\dfrac{\cot x - 1}{\cot x + 1} = \dfrac{\cot x - 1}{\cot x + 1}\cdot\dfrac{\tan x}{\tan x}$

$= \dfrac{1-\tan x}{1+\tan x}$

(b) The identity is true for $x = \dfrac{\pi}{4}$:

$\dfrac{\cot(\pi/4)-1}{\cot(\pi/4)+1} = \dfrac{1-1}{1+1} = 0$

$\dfrac{1-\tan(\pi/4)}{1+\tan(\pi/4)} = \dfrac{1-1}{1+1} = 0$

81. $\sqrt{a^2-u^2} = \sqrt{a^2 - a^2\sin^2\theta}$

$= \sqrt{a^2(1-\sin^2\theta)}$

$= \sqrt{a^2\cos^2\theta}$

$= a\cos\theta$

82. $\sqrt{a^2-u^2} = \sqrt{a^2 - a^2\cos^2\theta}$

$= \sqrt{a^2(1-\cos^2\theta)}$

$= \sqrt{a^2\sin^2\theta}$

$= a\sin\theta$

83. $\sqrt{a^2+u^2} = \sqrt{a^2 + a^2\tan^2\theta}$

$= \sqrt{a^2(1+\tan^2\theta)}$

$= \sqrt{a^2\sec^2\theta}$

$= a\sec\theta$

84. $\sqrt{u^2-a^2} = \sqrt{a^2\sec^2\theta - a^2}$

$= \sqrt{a^2(\sec^2\theta - 1)}$

$= \sqrt{a^2\tan^2\theta}$

$= a\tan\theta$

85. $\sqrt{\tan^2 x} = |\tan x|$

Let $x = \dfrac{3\pi}{4}$. Then, $\sqrt{\tan^2 x} = \sqrt{(-1)^2} = 1 \neq \tan\left(\dfrac{3\pi}{4}\right) = -1$.

86. $\sin\theta = \sqrt{1-\cos^2\theta}$.

True identity is $\sin\theta = \pm\sqrt{1-\cos^2\theta}$.

For example, $\sin\theta \neq \sqrt{1-\cos^2\theta}$ for $\theta = \dfrac{3\pi}{2}$:

$\sin\left(\dfrac{3\pi}{2}\right) = -1 \neq \sqrt{1-0} = 1$

87. When n is even, $\cos\left[\dfrac{(2n+1)\pi}{2}\right] = \cos\dfrac{\pi}{2} = 0$.

When n is odd, $\cos\left[\dfrac{(2n+1)\pi}{2}\right] = \cos\dfrac{3\pi}{2} = 0$.

Thus, $\cos\left[\dfrac{(2n+1)\pi}{2}\right] = 0$ for all n.

88. $\sin\left[\dfrac{(12n+1)\pi}{6}\right] = \sin\left[\dfrac{1}{6}(12n\pi + \pi)\right]$

$= \sin\left(2n\pi + \dfrac{\pi}{6}\right) = \sin\dfrac{\pi}{6} = \dfrac{1}{2}$

Thus, $\sin\left[\dfrac{(12n+1)\pi}{6}\right] = \dfrac{1}{2}$ for all integers n.

89. $(x-1)(x-8i)(x+8i) = (x-1)(x^2+64)$

$= x^3 - x^2 + 64x - 64$

Answers will vary.

90. $(x - i)(x + i)(x - 4i)(x + 4i) = (x^2 + 1)(x^2 + 16) = x^4 + 17x^2 + 16$

91. $(x - 4)(x - 6 - i)(x - 6 + i) = (x - 4)((x - 6)^2 + 1)$

$= (x - 4)(x^2 - 12x + 37)$

$= x^3 - 16x^2 + 85x - 148$

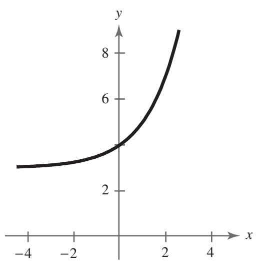
Answers will vary.

92. $x^2(x - 2)(x - (1 - i))(x - (1 + i)) = (x^3 - 2x^2)((x - 1)^2 + 1)$

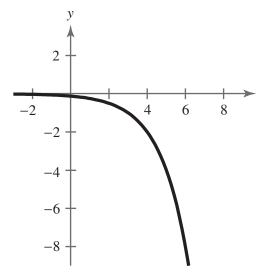
$= (x^3 - 2x^2)(x^2 - 2x + 2)$

$= x^5 - 4x^4 + 6x^3 - 4x^2$

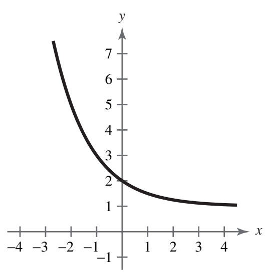
93. $f(x) = 2^x + 3$



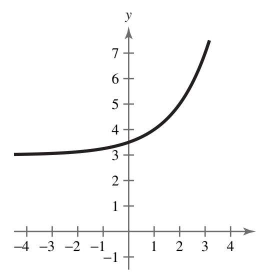
94. $f(x) = -2^{x-3}$



95. $f(x) = 2^{-x} + 1$



96. $f(x) = 2^{x-1} + 3$



97. $\csc \theta > 0$ and $\tan \theta < 0 \implies$ Quadrant II

98. Quadrant III

99. $\sec \theta > 0$ and $\sin \theta < 0 \implies$ Quadrant IV

100. Quadrant III

Section 5.3 Solving Trigonometric Equations

- You should be able to identify and solve trigonometric equations.
- A trigonometric equation is a conditional equation. It is true for a specific set of values.
- To solve trigonometric equations, use algebraic techniques such as collecting like terms, taking square roots, factoring, squaring, converting to quadratic form, using formulas, and using inverse functions. Study the examples in this section.
- Use your graphing utility to calculate solutions and verify results.

Vocabulary Check

1. general **2.** quadratic **3.** extraneous

1. $2\cos x - 1 = 0$

(a) $x = \frac{\pi}{3}$: $2\cos\frac{\pi}{3} - 1 = 2\left(\frac{1}{2}\right) - 1 = 0$

(b) $x = \frac{5\pi}{3}$: $2\cos\frac{5\pi}{3} - 1 = 2\left(\frac{1}{2}\right) - 1 = 0$

2. $\sec x - 2 = 0$

(a) $x = \frac{\pi}{3}$: $\sec\left(\frac{\pi}{3}\right) - 2 = \frac{1}{\cos(\pi/3)} - 2$

$= 2 - 2 = 0$

(b) $x = \frac{5\pi}{3}$: $\sec\left(\frac{5\pi}{3}\right) - 2 = 2 - 2 = 0$

3. $3\tan^2 2x - 1 = 0$

(a) $x = \frac{\pi}{12}$: $3\left[\tan\left(\frac{2\pi}{12}\right)\right]^2 - 1 = 3\tan^2\frac{\pi}{6} - 1 = 3\left(\frac{1}{\sqrt{3}}\right)^2 - 1 = 0$

(b) $x = \frac{5\pi}{12}$: $3\left[\tan\left(\frac{10\pi}{12}\right)\right]^2 - 1 = 3\tan^2\frac{5\pi}{6} - 1 = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 0$

4. $4\cos^2 2x - 2 = 0$

(a) $x = \frac{\pi}{8}$: $4\cos^2\left(2\cdot\frac{\pi}{8}\right) - 2 = 4\cos^2\left(\frac{\pi}{4}\right) - 2 = 4\left(\frac{\sqrt{2}}{2}\right)^2 - 2 = 0$

(b) $x = \frac{7\pi}{8}$: $4\cos^2\left(2\cdot\frac{7\pi}{8}\right) - 2 = 4\cos^2\left(\frac{7\pi}{4}\right) - 2 = 4\left(\frac{\sqrt{2}}{2}\right)^2 - 2 = 0$

5. $2\sin^2 x - \sin x - 1 = 0$

(a) $x = \frac{\pi}{2}$: $2\sin^2\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right) - 1 = 2 - 1 - 1 = 0$

(b) $x = \frac{7\pi}{6}$: $2\sin^2\left(\frac{7\pi}{6}\right) - \sin\left(\frac{7\pi}{6}\right) - 1 = 2\left(\frac{1}{4}\right) - \left(-\frac{1}{2}\right) - 1 = 0$

6. $\sec^4 x - 3\sec^2 x - 4 = 0$

(a) $x = \frac{2\pi}{3}$: $\sec\frac{2\pi}{3} = -2$ and $\sec^4 x - 3\sec^2 x - 4 = (-2)^4 - 3(-2)^2 - 4 = 0$

(b) $x = \frac{5\pi}{3}$: $\sec\frac{5\pi}{3} = 2$ and $\sec^4 x - 3\sec^2 x - 4 = 2^4 - 3(2)^2 - 4 = 0$

7. $\sin x = \frac{1}{2}$

$x = 30°, 150°$

8. $\cos x = \frac{\sqrt{3}}{2}$

$x = 30°, 330°$

9. $\cos x = -\frac{1}{2}$

$x = 120°, 240°$

10. $\sin x = -\frac{\sqrt{2}}{2}$

$x = 225°, 315°$

11. $\tan x = 1$

$x = 45°, 225°$

12. $\tan x = -\sqrt{3}$

$x = 120°, 300°$

13. $\cos x = -\frac{\sqrt{3}}{2}$

$x = \frac{5\pi}{6}, \frac{7\pi}{6}$

14. $\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

15. $\cot x = -1$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

16. $\sin x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$

17. $\tan x = -\frac{\sqrt{3}}{3}$

$x = \frac{5\pi}{6}, \frac{11\pi}{6}$

18. $\cos x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{7\pi}{4}$

19. $\csc x = -2 \implies \sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

20. $\sec x = \sqrt{2} \implies \cos x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{7\pi}{4}$

21. $\cot x = \sqrt{3} \implies \tan x = \frac{\sqrt{3}}{3}$

$x = \frac{\pi}{6}, \frac{7\pi}{6}$

22. $\sec x = 2 \implies \cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

23. $\tan x = -1$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

24. $\csc x = -\sqrt{2} \implies \sin x = -\frac{\sqrt{2}}{2}$

$x = \frac{5\pi}{4}, \frac{7\pi}{4}$

25. $2\cos x + 1 = 0$

$$2\cos x = -1$$
$$\cos x = -\frac{1}{2}$$
$$x = \frac{2\pi}{3} + 2n\pi$$
$$\text{or } x = \frac{4\pi}{3} + 2n\pi$$

26. $\sqrt{2}\sin x + 1 = 0$

$$\sin x = -\frac{1}{\sqrt{2}}$$
$$x = \frac{5\pi}{4} + 2n\pi$$
$$x = \frac{7\pi}{4} + 2n\pi$$

27. $\sqrt{3}\sec x - 2 = 0$

$$\sqrt{3}\sec x = 2$$
$$\sec x = \frac{2}{\sqrt{3}}$$
$$\cos x = \frac{\sqrt{3}}{2}$$
$$x = \frac{\pi}{6} + 2n\pi$$
$$\text{or } x = \frac{11\pi}{6} + 2n\pi$$

28. $\cot x + 1 = 0$

$$\cot x = -1$$
$$x = \frac{3\pi}{4} + n\pi$$

29. $3\csc^2 x - 4 = 0$

$$\csc^2 x = \frac{4}{3}$$
$$\csc x = \pm\frac{2}{\sqrt{3}}$$
$$\sin x = \pm\frac{\sqrt{3}}{2}$$
$$x = \frac{\pi}{3} + n\pi \text{ or } x = \frac{2\pi}{3} + n\pi$$

30. $3\cot^2 x - 1 = 0$

$$\cot^2 x = \frac{1}{3}$$
$$\cot x = \pm\frac{\sqrt{3}}{3}$$
$$x = \frac{\pi}{3} + n\pi$$
$$x = \frac{2\pi}{3} + n\pi$$

31. $4\cos^2 x - 1 = 0$

$$\cos^2 x = \frac{1}{4}$$
$$\cos x = \pm\frac{1}{2}$$
$$x = \frac{\pi}{3} + n\pi \text{ or } x = \frac{2\pi}{3} + n\pi$$

32. $\cos x(\cos x - 1) = 0$

$\cos x = 0 \quad \text{or} \quad \cos x = 1$

$x = \frac{\pi}{2} + n\pi \quad \text{or} \quad x = 2n\pi$

33. $\sin^2 x = 3\cos^2 x$

$\sin^2 x - 3(1 - \sin^2 x) = 0$

$4\sin^2 x = 3$

$\sin x = \pm\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3} + n\pi$

$\text{or } x = \frac{2\pi}{3} + n\pi$

34. $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$

$\tan^2 x = \frac{1}{3} \quad \text{or} \quad \tan^2 x = 3$

$\tan x = \pm\frac{1}{\sqrt{3}} \quad \text{or} \quad \tan x = \pm\sqrt{3}$

$x = \frac{\pi}{6} + n\pi \qquad x = \frac{\pi}{3} + n\pi$

$x = \frac{5\pi}{6} + n\pi \qquad x = \frac{2\pi}{3} + n\pi$

35. $\tan x + \sqrt{3} = 0$

$\tan x = -\sqrt{3}$

$x = \frac{2\pi}{3}, \frac{5\pi}{3}$

36. $2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

37. $\csc^2 x - 2 = 0$

$\csc^2 x = 2$

$\csc x = \pm\sqrt{2}$

$\sin x = \pm\frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

38. $\tan^2 x - 1 = 0$

$\tan^2 x = 1$

$\tan x = \pm 1$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

39. $3\tan^3 x - \tan x = 0$

$\tan x(3\tan^2 x - 1) = 0$

$\tan x = 0 \quad \text{or} \quad 3\tan^2 x - 1 = 0$

$x = 0, \pi \qquad \tan x = \pm\frac{\sqrt{3}}{3}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

40. $2\sin^2 x = 2 + \cos x$

$2 - 2\cos^2 x = 2 + \cos x$

$2\cos^2 x + \cos x = 0$

$\cos x(2\cos x + 1) = 0$

$\cos x = 0 \quad \text{or} \quad 2\cos x + 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad 2\cos x = -1$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

41. $\sec^2 x - \sec x - 2 = 0$

$(\sec x - 2)(\sec x + 1) = 0$

$\sec x - 2 = 0 \quad \text{or} \quad \sec x + 1 = 0$

$\sec x = 2 \qquad \sec x = -1$

$x = \frac{\pi}{3}, \frac{5\pi}{3} \qquad x = \pi$

42. $\sec x \csc x = 2 \csc x$

$\sec x \csc x - 2 \csc x = 0$

$\csc x(\sec x - 2) = 0$

$\csc x = 0$ or $\sec x - 2 = 0$

No solution $\quad \sec x = 2$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

43. $2 \sin x + \csc x = 0$

$2 \sin x + \frac{1}{\sin x} = 0$

$2 \sin^2 x + 1 = 0$

Since $2 \sin^2 x + 1 > 0$, there are no solutions.

44. $\sec x + \tan x = 1$

$(\sec x + \tan x)(\sec x - \tan x) = \sec x - \tan x$

$\sec^2 x - \tan^2 x = \sec x - \tan x$

$1 = \sec x - \tan x$

Hence, $\sec x + \tan x = \sec x - \tan x \implies \tan x = 0$.

$\sec x = 1, \tan x = 0 \implies x = 0$

45. $\cos x + \sin x \tan x = 2$

$\cos x + \frac{\sin^2 x}{\cos x} = 2$

$\frac{\cos^2 x + \sin^2 x}{\cos x} = 2$

$\frac{1}{\cos x} = 2$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

46. $\sin^2 x + \cos x + 1 = 0$

$(1 - \cos^2 x) + \cos x + 1 = 0$

$\cos^2 x - \cos x - 2 = 0$

$(\cos x - 2)(\cos x + 1) = 0$

$\cos x = 2$, Impossible

$\cos x + 1 = 0 \implies x = \pi$

47. $\sec^2 x + \tan x = 3$

$(1 + \tan^2 x) + \tan x = 3$

$\tan^2 x + \tan x - 2 = 0$

$(\tan x + 2)(\tan x - 1) = 0$

$\tan x = -2$ or $\tan x = 1$

$x \approx 2.0344, 5.1760 \qquad x = \frac{\pi}{4}, \frac{5\pi}{4}$

48. $2 \cos^2 x + \cos x - 1 = (2 \cos x - 1)(\cos x + 1) = 0$

$2 \cos x = 1$ or $\cos x = -1$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3} \qquad x = \pi$

49. $2 \sin^2 x + 3 \sin x + 1 = 0$

$y = 2 \sin^2 x + 3 \sin x + 1$

$x \approx 3.6652, 5.7596, 4.7124$

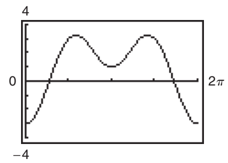
50. $2 \sec^2 x + \tan^2 x - 3 = 0$

$y = \frac{2}{\cos^2 x} + \tan^2 x - 3$

$x \approx 0.5236, 2.6180, 3.6652, 5.7596$

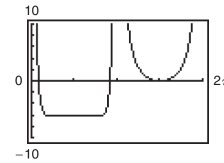
51. $y = 4\sin^2 x - 2\cos x - 1$

$x \approx 0.8614, 5.4218$



52. $y = \dfrac{1}{\sin^2 x} - \dfrac{3}{\sin x} - 4$

$x \approx 0.2527, 2.8889, 4.7124$

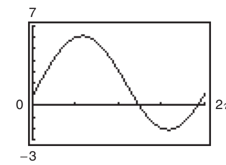


53. $y = \csc x + \cot x - 1 = \dfrac{1}{\sin x} + \dfrac{\cos x}{\sin x} - 1$

$x \approx 1.5708, \quad \left(\dfrac{\pi}{2}\right)$

54. $y = 4\sin x - \cos x + 2$

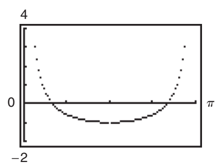
$x \approx 3.8930, 6.0217$



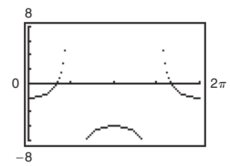
55. $\dfrac{\cos x \cot x}{1 - \sin x} = 3$

Graph $y = \dfrac{\cos x}{(1 - \sin x)\tan x} - 3.$

The solutions are approximately $x \approx 0.5236, x \approx 2.6180.$

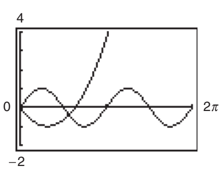


56. $\dfrac{1 + \sin x}{\cos x} + \dfrac{\cos x}{1 + \sin x} - 4 = 0$



$x \approx 1.0472, 5.2360$

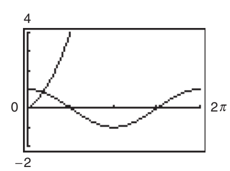
57. (a)



(b) $\sin 2x = x^2 - 2x$

(c) Points of intersection: $(0, 0), (1.7757, -0.3984)$

58. (a)



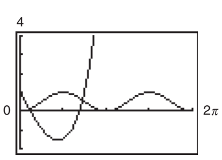
(b) $\cos x = x + x^2$

(c) Points of intersection:

$(-1.2512, 0.3142)$ (outside interval),

$(0.5500, 0.8525)$

59. (a)

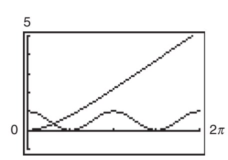


(b) $\sin^2 x = e^x - 4x$

(c) Points of intersection:

$(0.3194, 0.0986), (2.2680, 0.5878)$

60. (a)



(b) $\cos^2 x = e^{-x} + x - 1$

(c) Points of intersection:

$(0.9510, 0.3374)$, and

$(-0.8266, -0.4589)$ (outside interval)

61. $\cos\frac{x}{4} = 0$

$\frac{x}{4} = \frac{\pi}{2} + 2n\pi$ or $\frac{x}{4} = \frac{3\pi}{2} + 2n\pi$

$x = 2\pi + 8n\pi$ or $x = 6\pi + 8n\pi$

Combining, $x = 2\pi + 4n\pi$.

62. $\sin\frac{x}{2} = 0$

$\frac{x}{2} = n\pi$

$x = 2n\pi$

63. $\sin 4x = 1$

$4x = \frac{\pi}{2} + 2n\pi$

$x = \frac{\pi}{8} + \frac{n\pi}{2}$

64. $\cos 2x = -1$

$2x = \pi + 2n\pi$

$x = \frac{\pi}{2} + n\pi$

65. $\sin 2x = -\frac{\sqrt{3}}{2}$

$2x = \frac{4\pi}{3} + 2n\pi$ or $2x = \frac{5\pi}{3} + 2n\pi$

$x = \frac{2\pi}{3} + n\pi$ $\quad x = \frac{5\pi}{6} + n\pi$

66. $\sec 4x = 2$

$4x = \frac{\pi}{3} + 2n\pi$ or $4x = \frac{5\pi}{3} + 2n\pi$

$x = \frac{\pi}{12} + \frac{n\pi}{2}$ $\quad x = \frac{5\pi}{12} + \frac{n\pi}{2}$

67. $2\sin^2 2x = 1$

$\sin^2 2x = \frac{1}{2}$

$\sin 2x = \pm\frac{\sqrt{2}}{2}$

$2x = \frac{\pi}{4} + \frac{n\pi}{2}$

$x = \frac{\pi}{8} + \frac{n\pi}{4}$

68. $\tan^2 3x = 3$

$\tan 3x = \pm\sqrt{3}$

$3x = \frac{\pi}{3} + n\pi$ or $3x = \frac{2\pi}{3} + n\pi$

$x = \frac{\pi}{9} + \frac{n\pi}{3}$ or $x = \frac{2\pi}{9} + \frac{n\pi}{3}$

69. $\tan 3x(\tan x - 1) = 0$

$\tan 3x = 0$ or $\tan x - 1 = 0$

$3x = n\pi$ or $x = \frac{\pi}{4} + n\pi$

$x = \frac{n\pi}{3}$ $\quad x = \frac{\pi}{4} + n\pi$

70. $\cos 2x(2\cos x + 1) = 0$

$\cos 2x = 0$ or $2\cos x + 1 = 0$

$2x = \frac{\pi}{2} + n\pi$ $\quad \cos x = -\frac{1}{2}$

$x = \frac{\pi}{4} + \frac{n\pi}{2}$ $\quad x = \frac{2\pi}{3} + 2n\pi$

$x = \frac{4\pi}{3} + 2n\pi$

71. $\cos\frac{x}{2} = \frac{\sqrt{2}}{2}$

$\frac{x}{2} = \frac{\pi}{4} + 2n\pi$ or $\frac{x}{2} = \frac{7\pi}{4} + 2n\pi$

$x = \frac{\pi}{2} + 4n\pi$ $\qquad x = \frac{7\pi}{2} + 4n\pi$

72. $\tan\frac{x}{3} = 1$

$\frac{x}{3} = \frac{\pi}{4} + n\pi$

$x = \frac{3\pi}{4} + 3n\pi$

73. $y = \sin\frac{\pi x}{2} + 1$

From the graph in the textbook we see that the curve has x-intercepts at $x = -1$ and at $x = 3$.

74. $y = \sin \pi x + \cos \pi x$

From the graph in the textbook, we see that the curve has x-intercepts at $x = -0.25, 0.75, 1.75,$ and 2.75.

75. $y = \tan^2\left(\frac{\pi x}{6}\right) - 3$

From the graph in the textbook, we see that the curve has x-intercepts at $x = \pm 2$.

76. $y = \sec^4\left(\frac{\pi x}{8}\right) - 4$

From the graph in the textbook, we see that the curve has x-intercepts at $x = -2, 2$.

77. $2\cos x - \sin x = 0$

Graph $y_1 = 2\cos x - \sin x$ and estimate the zeros.

$x \approx 1.1071, 4.2487$

78. $y = 2\sin x + \cos x$

$x \approx 2.6779, 5.8195$

79. $x \tan x - 1 = 0$

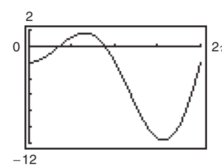
Graph $y_1 = x \tan x - 1$ and estimate the zeros.

$x \approx 0.8603, 3.4256$

80. $2x \sin x - 2 = 0$

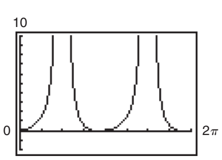
$y = 2x \sin x - 2$

$x \approx 1.1142, 2.7726$



81. $\sec^2 x + 0.5 \tan x - 1 = 0$

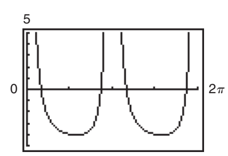
Graph $y_1 = \frac{1}{(\cos x)^2} + 0.5 \tan x - 1.$



$x = 0, x \approx 2.6779, 3.1416, 5.8195$

82. $\csc^2 x + 0.5 \cot x - 5 = 0$

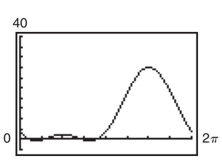
$y_1 = \left(\frac{1}{\sin x}\right)^2 + \frac{1}{2\tan x} - 5$



$x \approx 0.5153, 2.7259, 3.6569, 5.8675$

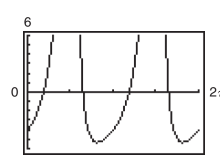
83. $12\sin^2 x - 13\sin x + 3 = 0$

Graph $y_1 = 12\sin^2 x - 13\sin x + 3.$



$x \approx 0.3398, 0.8481, 2.2935, 2.8018$

84. $3\tan^2 x + 4\tan x - 4 = 0$



$x \approx 0.5880, 2.0344, 3.7296, 5.1760$

85. $3\tan^2 x + 5\tan x - 4 = 0, \quad \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$x \approx -1.154, 0.534$

86. $y = \cos^2 x - 2\cos x - 1 = 0, \ [0, \pi]$

$x \approx 1.998$

87. $4\cos^2 x - 2\sin x + 1 = 0, \quad \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

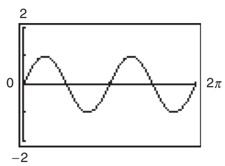
$x \approx 1.110$

88. $y = 2\sec^2 x + \tan x - 6 = 0, \quad \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$x \approx -1.035, 0.870$

89. $f(x) = \sin 2x$

(a)



Maxima: (0.7854, 1), (3.9270, 1)

Minima: (2.3562, −1), (5.4978, −1)

(b) $2\cos 2x = 0$

$\cos 2x = 0$

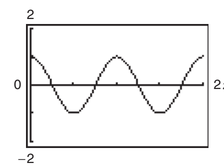
$2x = \frac{\pi}{2} + n\pi$

$x = \frac{\pi}{4} + \frac{n\pi}{2}$

The zeros are 0.7854, 2.3562, 3.9270, and 5.4978.

90. $f(x) = \cos 2x$

(a)



Maxima: (0, 1), (3.1416, 1), (6.2832, 1)

Minima: (1.5708, −1), (4.7124, −1)

(b) $-2\sin 2x = 0$

$\sin 2x = 0$

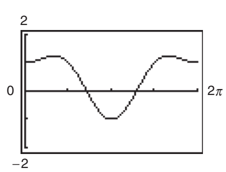
$2x = n\pi$

$x = \frac{n\pi}{2}$

The zeros are 0, 1.5708, 3.1416, 4.7124, and 6.2832.

91. $f(x) = \sin^2 x + \cos x$

(a)



Maxima: (1.0472, 1.25), (5.2360, 1.25)

Minima: (3.1416, −1)

(b) $2\sin x\cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

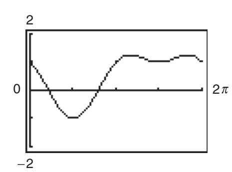
$\sin x = 0 \implies x = n\pi$

$\cos x = \frac{1}{2} \implies x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$

The zeros are 1.0472, 3.1416, and 5.2360.

92. $f(x) = \cos^2 x - \sin x$

(a)



Maxima: (3.6652, 1.25), (5.7596, 1.25)

Minima: (1.5708, −1)

(b) $-2\sin x\cos x - \cos x = 0$

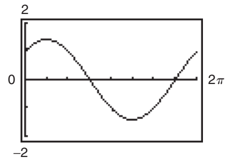
$\cos x(2\sin x + 1) = 0$

$\cos x = 0 \implies x = \frac{\pi}{2} + n\pi$

$\sin x = -\frac{1}{2} \implies x = \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi$

The zeros are 1.5708, 3.6652, 5.7596, and 4.7124.

93. (a) $f(x) = \sin x + \cos x$



Maximum: (0.7854, 1.4142)

Minimum: (3.9270, −1.4142)

(b) $\cos x - \sin x = 0$

$$\cos x = \sin x$$

$$1 = \frac{\sin x}{\cos x}$$

$$\tan x = 1$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$$

$$f\left(\frac{5\pi}{4}\right) = \sin\frac{5\pi}{4} + \cos\frac{5\pi}{4}$$

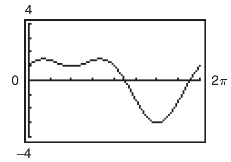
$$= -\sin\frac{\pi}{4} + \left(-\cos\frac{\pi}{4}\right)$$

$$= -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$$

Therefore, the maximum point in the interval $[0, 2\pi)$ is $(\pi/4, \sqrt{2})$ and the minimum point is $(5\pi/4, -\sqrt{2})$.

94. $y = 2\sin x + \cos 2x$

(a)



Maximum: (0.5236, 1.5), (2.6180, 1.5)

Minimum: (4.7124, −3.0)

(b) $2\cos x - 4\sin x\cos x = 0$

$$2\cos x(1 - 2\sin x) = 0$$

$$\cos x = 0 \implies x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$1 - 2\sin x = 0 \implies \sin x = \frac{1}{2} \implies x = \frac{\pi}{6}, \frac{5\pi}{6}$$

The zeros are 0.5236, 2.618, 4.712 and 1.571. The first three correspond to the values in (a).

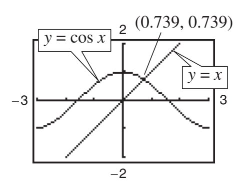
95. $f(x) = \tan\dfrac{\pi x}{4}$

$\tan 0 = 0$, but 0 is not positive. By graphing

$$y = \tan\frac{\pi x}{4} - x,$$

you see that the smallest positive fixed point is $x = 1$.

96. Graph $y = \cos x$ and $y = x$ on the same set of axes. Their point of intersection gives the value of c such that $f(c) = c \implies \cos c = c$.



$c \approx 0.739$

97. $f(x) = \cos\dfrac{1}{x}$

(a) The domain of $f(x)$ is all real numbers except 0.

(b) The graph has y-axis symmetry and a horizontal asymptote at $y = 1$.

(c) As $x \to 0$, $f(x)$ oscillates between −1 and 1.

(d) There are an infinite number of solutions in the interval $[-1, 1]$.

$$\frac{1}{x} = \frac{\pi}{2} + n\pi = \frac{\pi + 2n\pi}{2} \implies x = \frac{2}{\pi(2n + 1)}$$

(e) The greatest solution appears to occur at $x \approx 0.6366$.

98. $f(x) = \dfrac{\sin x}{x}$

(a) Domain: all real numbers except $x = 0$.

(b) The graph has y-axis symmetry.

Horizontal asymptote: $y = 0$

(c) As $x \longrightarrow 0, f(x) \longrightarrow 1$.

(d) $\sin x/x = 0$ has four solutions in the interval $[-8, 8]$.

$$(\sin x)\left(\frac{1}{x}\right) = 0$$

$$\sin x = 0$$

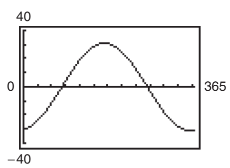
$$x = -2\pi, -\pi, \pi, 2\pi$$

99. $S = 74.50 - 43.75 \cos \dfrac{\pi t}{6}$

t	1	2	3	4	5	6	7	8	9	10	11	12
S	36.6	52.6	74.5	96.4	112.4	118.3	112.4	96.4	74.5	52.6	36.6	30.8

$S > 100$ for $t = 5, 6, 7$ (May, June, July)

100. $D = 31 \sin\left(\dfrac{2\pi}{365}t - 1.4\right)$



$D > 20°$ for $123 \le t \le 223$ days

101. $$y = \frac{1}{12}(\cos 8t - 3 \sin 8t)$$

$$\frac{1}{12}(\cos 8t - 3 \sin 8t) = 0$$

$$\cos 8t = 3 \sin 8t$$

$$\frac{1}{3} = \tan 8t$$

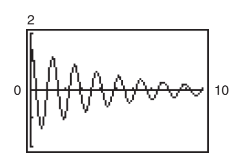
$$8t = 0.32175 + n\pi$$

$$t = 0.04 + \frac{n\pi}{8}$$

In the interval $0 \le t \le 1$, $t = 0.04, 0.43$, and 0.83 second.

102. $y_1 = 1.56e^{-0.22t} \cos 4.9t$ intersects $y_2 = -1$ at $t \approx 1.96$ (and other points).

The displacement does not exceed one foot from equilibrium after $t = 1.96$ seconds.



103. $$r = \frac{1}{32}v_0^2 \sin 2\theta$$

$$300 = \frac{1}{32}(100)^2 \sin 2\theta$$

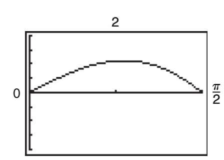
$\sin 2\theta = 0.96$

$2\theta \approx 1.287$ or $2\theta \approx \pi - 1.287 \approx 1.855$

$\theta \approx 0.6435 \approx 37°$ or $\theta \approx 0.9275 \approx 53°$

104. $A = 2x \cos x,\ 0 \le x \le \dfrac{\pi}{2}$

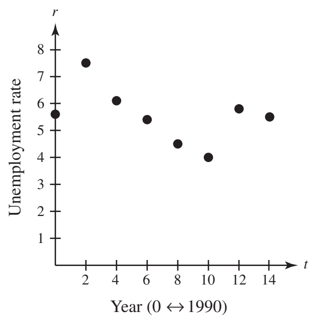
(a)



The maximum area of $A \approx 1.12$ occurs when $x \approx 0.86$.

(b) $A \ge 1$ for $0.6 < x < 1.1$

105. (a)



(b) Models 1 and 2 are both good fits, but model 1 seems better.

(c) The constant term 5.45 gives the average unemployment rate, 5.45%.

(d) The length is approximately one period

$\frac{2\pi}{0.47} \approx 13.37$ years.

(e) $r = 1.24 \sin(0.47t + 0.40) + 5.45 = 5.0$

Using a graphing utility, $t \approx 19.99 \approx 20$, or 2010.

106. $f(x) = 3 \sin(0.6x - 2)$

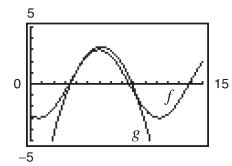
(a) Zero: $\sin(0.6x - 2) = 0$

$$0.6x - 2 = 0$$

$$0.6x = 2$$

$$x = \frac{2}{0.6} = \frac{10}{3}$$

(b) $g(x) = -0.45x^2 + 5.52x - 13.70$



For $3.5 \le x \le 6$ the approximation appears to be good. Answers will vary.

(c) $-0.45x^2 + 5.52x - 13.70 = 0$

$$x = \frac{-5.52 \pm \sqrt{(5.52)^2 - 4(-0.45)(-13.70)}}{2(-0.45)}$$

$x \approx 3.46, 8.81$

The zero of g on $[0, 6]$ is 3.46. The zero is close to the zero $\frac{10}{3} \approx 3.33$ of f.

107. False. $\sin x - x = 0$ has one solution, $x = 0$.

108. False. There might not be periodicity, as in the equation $\sin(x^2) = 0$.

109. False. The equation has no solution because $-1 \le \sin x \le 1$.

110. Answers will vary.

111. $124° = 124°\left(\frac{\pi}{180°}\right) \approx 2.164$ radians

112. $486° = 486°\left(\frac{\pi}{180°}\right) \approx 8.482$ radians

113. $-0.41° = -0.41°\left(\frac{\pi}{180°}\right) \approx -0.007$ radian

114. $-210.55° = -210.55°\left(\frac{\pi}{180°}\right) \approx -3.675$ radians

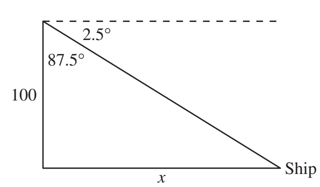
115. $\tan 30° = \frac{14}{x} \implies x = \frac{14}{\tan 30°} = \frac{14}{\sqrt{3}/3} \approx 24.249$

116. $\sin 70° = \frac{x}{10} \implies x = 10 \cdot \sin 70° \approx 9.397 \approx 9.4$

117. $\tan 87.5° = \frac{x}{100}$

$$x = 100 \tan 87.5°$$

$$\approx 2290.4 \text{ feet} \approx 0.43 \text{ mile}$$



118. Answers will vary.

Section 5.4 Sum and Difference Formulas

- You should memorize the sum and difference formulas.

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

- You should be able to use these formulas to find the values of the trigonometric functions of angles whose sums or differences are special angles.
- You should be able to use these formulas to solve trigonometric equations.

Vocabulary Check

1. $\sin u \cos v - \cos u \sin v$
2. $\cos u \cos v - \sin u \sin v$
3. $\dfrac{\tan u + \tan v}{1 - \tan u \tan v}$
4. $\sin u \cos v + \cos u \sin v$
5. $\cos u \cos v + \sin u \sin v$
6. $\dfrac{\tan u - \tan v}{1 + \tan u \tan v}$

1. (a) $\cos(240° - 0°) = \cos(240°) = -\frac{1}{2}$

(b) $\cos(240°) - \cos 0° = -\frac{1}{2} - 1 = -\frac{3}{2}$

2. (a) $\sin(405° + 120°) = \sin 405° \cos 120° + \cos 405° \sin 120°$

$$= \frac{\sqrt{2}}{2}\left(-\frac{1}{2}\right) + \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{6} - \sqrt{2}}{4}$$

(b) $\sin 405° + \sin 120° = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{2} + \sqrt{3}}{2}$

3. (a) $\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$$= \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

(b) $\cos\frac{\pi}{4} + \cos\frac{\pi}{3} = \dfrac{\sqrt{2}}{2} + \dfrac{1}{2} = \dfrac{\sqrt{2} + 1}{2}$

4. (a) $\sin\left(\frac{2\pi}{3} + \frac{5\pi}{6}\right) = \sin\left(\frac{9\pi}{6}\right) = -1$

(b) $\sin\frac{2\pi}{3} + \sin\frac{5\pi}{6} = \dfrac{\sqrt{3}}{2} + \dfrac{1}{2} = \dfrac{\sqrt{3} + 1}{2}$

5. (a) $\sin(315° - 60°) = \sin 315° \cos 60° - \cos 315° \sin 60° = -\dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2} - \dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} = \dfrac{-\sqrt{2} - \sqrt{6}}{4}$

(b) $\sin 315° - \sin 60° = -\dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{3}}{2} = -\dfrac{\sqrt{2} + \sqrt{3}}{2}$

6. (a) $\sin\left(\frac{7\pi}{6} - \frac{\pi}{3}\right) = \sin\left(\frac{5\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$

(b) $\sin\frac{7\pi}{6} - \sin\frac{\pi}{3} = -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2} = \dfrac{-1 - \sqrt{3}}{2}$

7. $\sin 105° = \sin(60° + 45°)$

$= \sin 60° \cos 45° + \sin 45° \cos 60°$

$= \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2}$

$= \dfrac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$

$\cos 105° = \cos(60° + 45°)$

$= \cos 60° \cos 45° - \sin 60° \sin 45°$

$= \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2}$

$= \dfrac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$

$\tan 105° = \tan(60° + 45°)$

$= \dfrac{\tan 60° + \tan 45°}{1 - \tan 60° \tan 45°}$

$= \dfrac{\sqrt{3} + 1}{1 - \sqrt{3}} = \dfrac{\sqrt{3} + 1}{1 - \sqrt{3}} \cdot \dfrac{1 + \sqrt{3}}{1 + \sqrt{3}}$

$= \dfrac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3}$

8. $165° = 135° + 30°$

$\sin 165° = \sin(135° + 30°)$

$= \sin 135° \cos 30° + \sin 30° \cos 135°$

$= \sin 45° \cos 30° - \sin 30° \cos 45°$

$= \dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} - \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$

$\cos 165° = \cos(135° + 30°)$

$= \cos 135° \cos 30° - \sin 135° \sin 30°$

$= -\cos 45° \cos 30° - \sin 45° \sin 30°$

$= -\dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2}$

$= -\dfrac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$

$\tan 165° = \tan(135° + 30°)$

$= \dfrac{\tan 135° + \tan 30°}{1 - \tan 135° \tan 30°}$

$= \dfrac{-\tan 45° + \tan 30°}{1 + \tan 45° \tan 30°}$

$= \dfrac{-1 + \left(\sqrt{3}/3\right)}{1 + \left(\sqrt{3}/3\right)} = -2 + \sqrt{3}$

9. $\sin 195° = \sin(225° - 30°)$

$= \sin 225° \cos 30° - \sin 30° \cos 225°$

$= -\sin 45° \cos 30° + \sin 30° \cos 45°$

$= -\dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} + \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$

$\cos 195° = \cos(225° - 30°)$

$= \cos 225° \cos 30° + \sin 225° \sin 30°$

$= -\cos 45° \cos 30° - \sin 45° \sin 30°$

$= -\dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2} = -\dfrac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$

$\tan 195° = \tan(225° - 30°)$

$= \dfrac{\tan 225° - \tan 30°}{1 + \tan 225° \tan 30°}$

$= \dfrac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$

$= \dfrac{1 - \left(\sqrt{3}/3\right)}{1 + \left(\sqrt{3}/3\right)} = \dfrac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \dfrac{3 - \sqrt{3}}{3 - \sqrt{3}}$

$= \dfrac{12 - 6\sqrt{3}}{6} = 2 - \sqrt{3}$

10. $255° = 300° - 45°$

$\sin 255° = \sin(300° - 45°)$

$= \sin 300° \cos 45° - \cos 300° \cos 45°$

$= \left(-\dfrac{\sqrt{3}}{2}\right)\dfrac{\sqrt{2}}{2} - \dfrac{1}{2}\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{-\sqrt{6} - \sqrt{2}}{4}$

$\cos 255° = \cos(300° - 45°)$

$= \cos 300° \cos 45° + \sin 300° \sin 45°$

$= \dfrac{1}{2}\left(\dfrac{\sqrt{2}}{2}\right) + \left(-\dfrac{\sqrt{3}}{2}\right)\dfrac{\sqrt{2}}{2} = \dfrac{-\sqrt{6} + \sqrt{2}}{4}$

$\tan 255° = \tan(300° - 45°)$

$= \dfrac{\tan 300° - \tan 45°}{1 + \tan 300° \tan 45°}$

$= \dfrac{-\sqrt{3} - 1}{1 + \left(-\sqrt{3}\right)(1)}$

$= \dfrac{1 + \sqrt{3}}{\sqrt{3} - 1}$

$= \dfrac{1 + \sqrt{3}}{\sqrt{3} - 1} \cdot \dfrac{\sqrt{3} + 1}{\sqrt{3} + 1}$

$= \dfrac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3}$

11. $\sin\dfrac{11\pi}{12} = \sin\left(\dfrac{3\pi}{4} + \dfrac{\pi}{6}\right)$

$= \sin\dfrac{3\pi}{4}\cos\dfrac{\pi}{6} + \sin\dfrac{\pi}{6}\cos\dfrac{3\pi}{4}$

$= \dfrac{\sqrt{2}}{2}\cdot\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}\left(-\dfrac{\sqrt{2}}{2}\right) = \dfrac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$

$\cos\dfrac{11\pi}{12} = \cos\left(\dfrac{3\pi}{4} + \dfrac{\pi}{6}\right)$

$= \cos\dfrac{3\pi}{4}\cos\dfrac{\pi}{6} - \sin\dfrac{3\pi}{4}\sin\dfrac{\pi}{6}$

$= -\dfrac{\sqrt{2}}{2}\cdot\dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{2}}{2}\cdot\dfrac{1}{2}$

$= -\dfrac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$

$\tan\dfrac{11\pi}{12} = \tan\left(\dfrac{3\pi}{4} + \dfrac{\pi}{6}\right)$

$= \dfrac{\tan(3\pi/4) + \tan(\pi/6)}{1 - \tan(3\pi/4)\tan(\pi/6)}$

$= \dfrac{-1 + \left(\sqrt{3}/3\right)}{1 - (-1)\left(\sqrt{3}/3\right)}$

$= \dfrac{-3 + \sqrt{3}}{3 + \sqrt{3}}\cdot\dfrac{3 - \sqrt{3}}{3 - \sqrt{3}}$

$= \dfrac{-12 + 6\sqrt{3}}{6} = -2 + \sqrt{3}$

12. $\dfrac{17\pi}{12} = \dfrac{7\pi}{6} + \dfrac{\pi}{4}$

$\sin\dfrac{17\pi}{12} = \sin\left(\dfrac{7\pi}{6} + \dfrac{\pi}{4}\right)$

$= \sin\dfrac{7\pi}{6}\cos\dfrac{\pi}{4} + \cos\dfrac{7\pi}{6}\sin\dfrac{\pi}{4}$

$= \left(-\dfrac{1}{2}\right)\left(\dfrac{\sqrt{2}}{2}\right) + \left(-\dfrac{\sqrt{3}}{2}\right)\dfrac{\sqrt{2}}{2}$

$= \dfrac{-\sqrt{6} - \sqrt{2}}{4}$

$\cos\dfrac{17\pi}{12} = \cos\left(\dfrac{7\pi}{6} + \dfrac{\pi}{4}\right)$

$= \cos\dfrac{7\pi}{6}\cos\dfrac{\pi}{4} - \sin\dfrac{7\pi}{6}\sin\dfrac{\pi}{4}$

$= \left(-\dfrac{\sqrt{3}}{2}\right)\dfrac{\sqrt{2}}{2} - \left(-\dfrac{1}{2}\right)\dfrac{\sqrt{2}}{2}$

$= \dfrac{\sqrt{2} - \sqrt{6}}{4}$

$\tan\dfrac{17\pi}{12} = \tan\left(\dfrac{7\pi}{6} + \dfrac{\pi}{4}\right)$

$= \dfrac{\tan\dfrac{7\pi}{6} + \tan\dfrac{\pi}{4}}{1 - \tan\dfrac{7\pi}{6}\tan\dfrac{\pi}{4}} = \dfrac{\dfrac{\sqrt{3}}{3} + 1}{1 - \dfrac{\sqrt{3}}{3}(1)}$

$= \dfrac{\sqrt{3} + 3}{3 - \sqrt{3}} = 2 + \sqrt{3}$

13. $-\dfrac{\pi}{12} = \dfrac{\pi}{6} - \dfrac{\pi}{4}$

$\sin\left(-\dfrac{\pi}{12}\right) = \sin\left(\dfrac{\pi}{6} - \dfrac{\pi}{4}\right)$

$= \sin\dfrac{\pi}{6}\cos\dfrac{\pi}{4} - \sin\dfrac{\pi}{4}\cos\dfrac{\pi}{6}$

$= \dfrac{1}{2}\cdot\dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2}\cdot\dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$

$\cos\left(-\dfrac{\pi}{12}\right) = \cos\left(\dfrac{\pi}{6} - \dfrac{\pi}{4}\right)$

$= \cos\dfrac{\pi}{6}\cos\dfrac{\pi}{4} + \sin\dfrac{\pi}{6}\sin\dfrac{\pi}{4}$

$= \dfrac{\sqrt{3}}{2}\cdot\dfrac{\sqrt{2}}{2} + \dfrac{1}{2}\cdot\dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$

$\tan\left(-\dfrac{\pi}{12}\right) = \tan\left(\dfrac{\pi}{6} - \dfrac{\pi}{4}\right)$

$= \dfrac{\tan(\pi/6) - \tan(\pi/4)}{1 + \tan(\pi/6)\tan(\pi/4)}$

$= \dfrac{\left(\sqrt{3}/3\right) - 1}{1 + \left(\sqrt{3}/3\right)} = \dfrac{\sqrt{3} - 3}{\sqrt{3} + 3}\cdot\dfrac{\sqrt{3} - 3}{\sqrt{3} - 3}$

$= \dfrac{12 - 6\sqrt{3}}{-6} = -2 + \sqrt{3}$

14. $-\frac{19\pi}{12} = \frac{2\pi}{3} - \frac{9\pi}{4}$

$$\sin\left(-\frac{19\pi}{12}\right) = \sin\left(\frac{2\pi}{3} - \frac{9\pi}{4}\right)$$
$$= \sin\frac{2\pi}{3}\cos\frac{9\pi}{4} - \cos\frac{2\pi}{3}\sin\frac{9\pi}{4}$$
$$= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \left(-\frac{1}{2}\right)\frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$
$$\cos\left(-\frac{19\pi}{12}\right) = \cos\left(\frac{2\pi}{3} - \frac{9\pi}{4}\right)$$
$$= \cos\frac{2\pi}{3}\cos\frac{9\pi}{4} + \sin\frac{2\pi}{3}\sin\frac{9\pi}{4}$$
$$= \left(-\frac{1}{2}\right)\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$$
$$\tan\left(-\frac{19\pi}{12}\right) = \tan\left(\frac{2\pi}{3} - \frac{9\pi}{4}\right) = \frac{\tan\frac{2\pi}{3} - \tan\frac{9\pi}{4}}{1 + \tan\frac{2\pi}{3}\tan\frac{9\pi}{4}}$$
$$= \frac{-\sqrt{3}-1}{1+\left(-\sqrt{3}\right)(1)}$$
$$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$$
$$= 2+\sqrt{3}$$

15. $\sin 75° = \sin(30° + 45°)$

$$= \sin 30°\cos 45° + \sin 45°\cos 30°$$
$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{4}\left(1+\sqrt{3}\right)$$
$$\cos 75° = \cos(30° + 45°)$$
$$= \cos 30°\cos 45° - \sin 30°\sin 45°$$
$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}\left(\sqrt{3}-1\right)$$
$$\tan 75° = \tan(30° + 45°)$$
$$= \frac{\tan 30° + \tan 45°}{1 - \tan 30°\tan 45°}$$
$$= \frac{\left(\sqrt{3}/3\right)+1}{1-\left(\sqrt{3}/3\right)} = \frac{\sqrt{3}+3}{3-\sqrt{3}}\cdot\frac{3+\sqrt{3}}{3+\sqrt{3}}$$
$$= \frac{6\sqrt{3}+12}{6} = \sqrt{3}+2$$

16. $15° = 45° - 30°$

$$\sin 15° = \sin(45° - 30°) = \sin 45°\cos 30° - \cos 45°\sin 30°$$
$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{2}\left(\sqrt{3}-1\right)}{4} = \frac{\sqrt{2}}{4}\left(\sqrt{3}-1\right)$$
$$\cos 15° = \cos(45° - 30°) = \cos 45°\cos 30° + \sin 45°\sin 30°$$
$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{2}\left(\sqrt{3}+1\right)}{4} = \frac{\sqrt{2}}{4}\left(\sqrt{3}+1\right)$$
$$\tan 15° = \tan(45° - 30°) = \frac{\tan 45° - \tan 30°}{1 + \tan 45°\tan 30°}$$
$$= \frac{1-\frac{\sqrt{3}}{3}}{1+(1)\left(\frac{\sqrt{3}}{3}\right)} = \frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}} = \frac{3-\sqrt{3}}{3+\sqrt{3}}\cdot\frac{3-\sqrt{3}}{3-\sqrt{3}}$$
$$= \frac{12-6\sqrt{3}}{6} = 2-\sqrt{3}$$

17. $-225°$ is coterminal with $135°$, and lies in Quadrant II.

$$\sin(-225°) = \frac{\sqrt{2}}{2}$$

$$\cos(-225°) = -\frac{\sqrt{2}}{2}$$

$$\tan(-225°) = -1$$

18. $-165° = -135° - 30°$

(a) $$\sin(-165°) = \sin(-135° - 30°)$$
$$= \sin(-135°)\cos 30° - \cos(-135°)\sin 30°$$
$$= \left(-\frac{\sqrt{2}}{2}\right)\frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\frac{1}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$$

(b) $$\cos(-165°) = \cos(-135° - 30°)$$
$$= \cos(-135°)\cos 30° + \sin(-135°)\sin 30°$$
$$= \left(-\frac{\sqrt{2}}{2}\right)\frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{2}}{2}\right)\frac{1}{2} = \frac{-\sqrt{6} - \sqrt{2}}{4}$$

(c) $$\tan(-165°) = \tan(-135° - 30°)$$
$$= \frac{\tan(-135°) - \tan 30°}{1 + \tan(-135°)\tan 30°}$$
$$= \frac{1 - \sqrt{3}/3}{1 + 1\left(\sqrt{3}/3\right)}$$
$$= \frac{3 - \sqrt{3}}{3 + \sqrt{3}} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}}$$
$$= \frac{12 - 6\sqrt{3}}{6} = 2 - \sqrt{3}$$

19. $\dfrac{13\pi}{12} = \dfrac{3\pi}{4} + \dfrac{\pi}{3}$

$$\sin\frac{13\pi}{12} = \sin\left(\frac{3\pi}{4} + \frac{\pi}{3}\right) = \sin\frac{3\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos\frac{3\pi}{4}$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2}\left(-\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2} - \sqrt{6}}{4}$$

$$\cos\frac{13\pi}{12} = \cos\left(\frac{3\pi}{4} + \frac{\pi}{3}\right) = \cos\frac{3\pi}{4}\cos\frac{\pi}{3} - \sin\frac{3\pi}{4}\sin\frac{\pi}{3}$$
$$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\tan\frac{13\pi}{12} = \tan\left(\frac{3\pi}{4} + \frac{\pi}{3}\right) = \frac{\tan(3\pi/4) + \tan(\pi/3)}{1 - \tan(3\pi/4)\tan(\pi/3)}$$
$$= \frac{(-1) + \sqrt{3}}{1 - (-1)\sqrt{3}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = 2 - \sqrt{3}$$

20. $\frac{5\pi}{12} = \frac{\pi}{4} + \frac{\pi}{6}$

$$\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{6}\cos\frac{\pi}{4}$$

$$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{1}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$$

$$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\tan\left(\frac{5\pi}{12}\right) = \frac{\sin(5\pi/12)}{\cos(5\pi/12)} = \frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}} = 2 + \sqrt{3}$$

21. $-\frac{7\pi}{12} = \frac{\pi}{6} - \frac{3\pi}{4}$

$$\sin\left(-\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{6} - \frac{3\pi}{4}\right) = \sin\frac{\pi}{6}\cos\frac{3\pi}{4} - \sin\frac{3\pi}{4}\cos\frac{\pi}{6}$$

$$= \frac{1}{2}\left(-\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2} + \sqrt{6}}{4}$$

$$\cos\left(-\frac{7\pi}{12}\right) = \cos\left(\frac{\pi}{6} - \frac{3\pi}{4}\right) = \cos\frac{\pi}{6}\cos\frac{3\pi}{4} + \sin\frac{\pi}{6}\sin\frac{3\pi}{4}$$

$$= \frac{\sqrt{3}}{2}\left(-\frac{\sqrt{2}}{2}\right) + \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$$

$$\tan\left(-\frac{7\pi}{12}\right) = \tan\left(\frac{\pi}{6} - \frac{3\pi}{4}\right) = \frac{\tan(\pi/6) - \tan(3\pi/4)}{1 + \tan(\pi/6)\tan(3\pi/4)}$$

$$= \frac{(\sqrt{3}/3) - (-1)}{1 + (\sqrt{3}/3)(-1)} = \frac{3 + \sqrt{3}}{3 - \sqrt{3}} = 2 + \sqrt{3}$$

22. $\frac{-13\pi}{12} = \frac{-3\pi}{4} - \frac{\pi}{3}$

$$\sin\left(\frac{-13\pi}{12}\right) = \sin\left(\frac{-3\pi}{4} - \frac{\pi}{3}\right) = \sin\left(\frac{-3\pi}{4}\right)\cos\frac{\pi}{3} - \cos\left(\frac{-3\pi}{4}\right)\sin\frac{\pi}{3}$$

$$= \frac{-\sqrt{2}}{2}\left(\frac{1}{2}\right) - \left(\frac{-\sqrt{2}}{2}\right)\frac{\sqrt{3}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\cos\left(\frac{-13\pi}{12}\right) = \cos\left(\frac{-3\pi}{4} - \frac{\pi}{3}\right) = \cos\left(\frac{-3\pi}{4}\right)\cos\frac{\pi}{3} + \sin\left(\frac{-3\pi}{4}\right)\sin\frac{\pi}{3}$$

$$= \frac{-\sqrt{2}}{2}\left(\frac{1}{2}\right) + \left(\frac{-\sqrt{2}}{2}\right)\frac{\sqrt{3}}{2} = -\frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\tan\left(\frac{-13\pi}{12}\right) = \frac{\sin\left(\frac{-13\pi}{12}\right)}{\cos\left(\frac{-13\pi}{12}\right)} = \frac{\sqrt{2} - \sqrt{6}}{\sqrt{2} + \sqrt{6}} = \sqrt{3} - 2$$

23. $\cos 60^\circ \cos 20^\circ - \sin 60^\circ \sin 20^\circ = \cos(60^\circ + 20^\circ) = \cos 80^\circ$

24. $\sin 110° \cos 80° + \cos 110° \sin 80° = \sin(110° + 80°) = \sin(190°)$

25. $\dfrac{\tan 325° - \tan 86°}{1 + \tan 325° \tan 86°} = \tan(325° - 86°) = \tan 239°$

26. $\dfrac{\tan 154° - \tan 49°}{1 + \tan 154° \tan 49°} = \tan(154° - 49°) = \tan 105°$

27. $\sin 3.5 \cos 1.2 - \cos 3.5 \sin 1.2 = \sin(3.5 - 1.2) = \sin 2.3$

28. $\cos 0.96 \cos 0.42 + \sin 0.96 \sin 0.42 = \cos(0.96 - 0.42) = \cos(0.54)$

29. $\cos \dfrac{\pi}{9} \cos \dfrac{\pi}{7} - \sin \dfrac{\pi}{9} \sin \dfrac{\pi}{7} = \cos\left(\dfrac{\pi}{9} + \dfrac{\pi}{7}\right) = \cos\left(\dfrac{16\pi}{63}\right)$

30. $\sin \dfrac{4\pi}{9} \cos \dfrac{\pi}{8} + \cos \dfrac{4\pi}{9} \sin \dfrac{\pi}{8} = \sin\left(\dfrac{4\pi}{9} + \dfrac{\pi}{8}\right) = \sin\left(\dfrac{41\pi}{72}\right)$

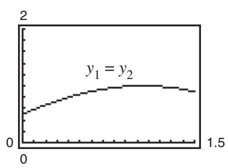
31. $y_1 = \sin\left(\dfrac{\pi}{6} + x\right)$

$= \sin \dfrac{\pi}{6} \cos x + \sin x \cdot \cos \dfrac{\pi}{6}$

$= \dfrac{1}{2} \cos x + \dfrac{\sqrt{3}}{2} \sin x$

$= \dfrac{1}{2}\left(\cos x + \sqrt{3} \sin x\right) = y_2$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	0.6621	0.7978	0.9017	0.9696	0.9989	0.9883	0.9384
y_2	0.6621	0.7978	0.9017	0.9696	0.9989	0.9883	0.9384



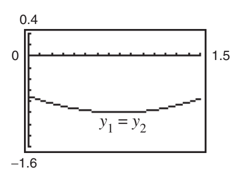
32.

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	−0.8335	−0.9266	−0.9829	−0.9999	−0.9771	−0.9153	−0.8170
y_2	−0.8335	−0.9266	−0.9829	−0.9999	−0.9771	−0.9153	−0.8170

$y_1 = \cos\left(\dfrac{5\pi}{4} - x\right) = \cos \dfrac{5\pi}{4} \cos x + \sin \dfrac{5\pi}{4} \sin x$

$= -\dfrac{\sqrt{2}}{2} \cos x - \dfrac{\sqrt{2}}{2} \sin x$

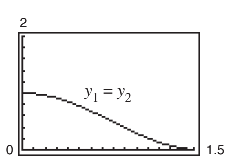
$= -\dfrac{\sqrt{2}}{2}(\cos x + \sin x) = y_2$



33. $y_1 = \cos(x + \pi) \cos(x - \pi)$

$= (\cos x \cdot \cos \pi - \sin x \cdot \sin \pi)[\cos x \cos \pi + \sin x \sin \pi]$

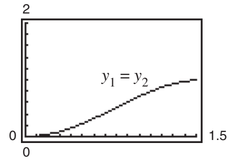
$= [-\cos x][-\cos x] = \cos^2 x = y_2$



x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	0.9605	0.8484	0.6812	0.4854	0.2919	0.1313	0.0289
y_2	0.9605	0.8484	0.6812	0.4854	0.2919	0.1313	0.0289

34. $y_1 = \sin(x + \pi)\sin(x - \pi)$

$= [\sin x \cos \pi + \sin \pi \cos \pi][\sin x \cos \pi - \sin \pi \cos x] = [-\sin x][-\sin x] = \sin^2 x = y_2$



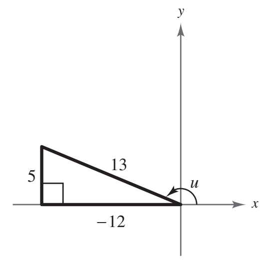
x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	0.0395	0.1516	0.3188	0.5146	0.7081	0.8687	0.9711
y_2	0.0395	0.1516	0.3188	0.5146	0.7081	0.8687	0.9711

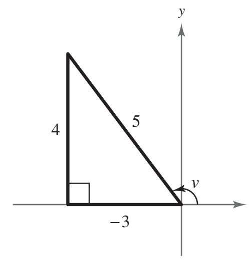
For Exercises 35–38,

$\sin u = \frac{5}{13}$ and u in Quadrant II $\Rightarrow$ $\cos u = -\frac{12}{13}$

$\cos v = -\frac{3}{5}$ and v in Quadrant II $\Rightarrow$ $\sin v = \frac{4}{5}$

$\tan u = -\frac{5}{12}$ and $\tan v = -\frac{4}{3}$.





35. $\sin(u + v) = \sin u \cos v + \sin v \cos u$

$$= \frac{5}{13}\left(\frac{-3}{5}\right) + \frac{4}{5}\left(\frac{-12}{13}\right) = -\frac{63}{65}$$

36. $\cos(v - u) = \cos v \cos u + \sin v \sin u$

$$= \left(\frac{-3}{5}\right)\left(\frac{-12}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) = \frac{56}{65}$$

37. $\tan(u + v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v}$

$$= \frac{(-5/12) - (4/3)}{1 - (-5/12)(-4/3)}$$

$$= \frac{-63/36}{16/36} = -\frac{63}{16}$$

38. $\sin(u - v) = \sin u \cos v - \sin v \cos u$

$$= \frac{5}{13}\left(\frac{-3}{5}\right) - \left(\frac{4}{5}\right)\left(\frac{-12}{13}\right)$$

$$= \frac{33}{65}$$

For Exercises 39–42, $\sin u = -\frac{8}{17}$, $\cos u = -\frac{15}{17}$, $\sin v = -\frac{3}{5}$, $\cos v = -\frac{4}{5}$, $\tan u = \frac{8}{15}$, $\tan v = \frac{3}{4}$.

39. $\cos(u + v) = \cos u \cos v - \sin u \sin v$

$$= \left(-\frac{15}{17}\right)\left(-\frac{4}{5}\right) - \left(-\frac{8}{17}\right)\left(-\frac{3}{5}\right) = \frac{36}{85}$$

40. $\tan(u + v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v}$

$$= \frac{(8/15) + (3/4)}{1 - (8/15)(3/4)} = \frac{32 + 45}{60 - 24} = \frac{77}{36}$$

41. $\sin(v - u) = \sin v \cos u - \cos v \sin u$

$$= \left(-\tfrac{3}{5}\right)\left(-\tfrac{15}{17}\right) - \left(-\tfrac{4}{5}\right)\left(-\tfrac{8}{17}\right) = \tfrac{13}{85}$$

42. $\cos(u - v) = \cos u \cos v + \sin u \sin v$

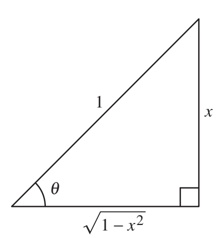
$$= \left(-\tfrac{15}{17}\right)\left(-\tfrac{4}{5}\right) + \left(-\tfrac{8}{17}\right)\left(-\tfrac{3}{5}\right) = \tfrac{84}{85}$$

43. $\sin(\arcsin x + \arccos x) = \sin(\arcsin x)\cos(\arccos x) + \sin(\arccos x)\cos(\arcsin x)$

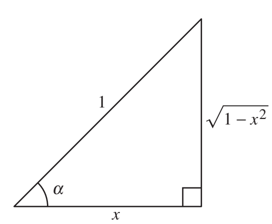
$$= x \cdot x + \sqrt{1 - x^2} \cdot \sqrt{1 - x^2}$$

$$= x^2 + 1 - x^2$$

$$= 1$$



$\theta = \arcsin x$

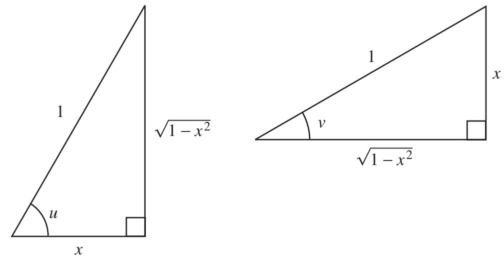


$\alpha = \arccos x$

44. Let: $u = \arccos x$ and $v = \arcsin x$

$\cos u = x$ $\quad\quad$ $\sin v = x$

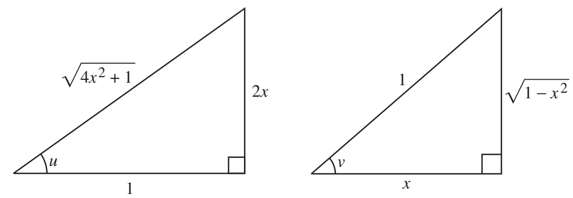
$$\begin{aligned}\cos(\arccos x - \arcsin x) &= \cos(\arccos x)\cos(\arcsin x) + \sin(\arccos x)\sin(\arcsin x)\\ &= x\sqrt{1 - x^2} + \sqrt{1 - x^2}\,x\\ &= 2x\sqrt{1 - x^2}\end{aligned}$$



45. Let: $u = \arctan 2x$ and $v = \arccos x$

$\tan u = 2x$ $\quad\quad$ $\cos v = x$

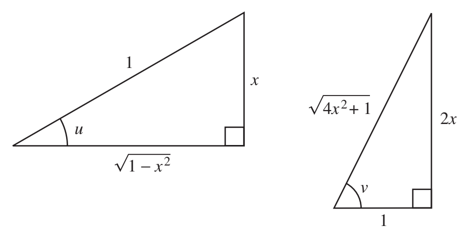
$$\begin{aligned}\sin(\arctan 2x - \arccos x) &= \sin(u - v)\\ &= \sin u \cos v - \cos u \sin v\\ &= \frac{2x}{\sqrt{4x^2 + 1}}(x) - \frac{1}{\sqrt{4x^2 + 1}}\left(\sqrt{1 - x^2}\right)\\ &= \frac{2x^2 - \sqrt{1 - x^2}}{\sqrt{4x^2 + 1}}\end{aligned}$$



46. Let: $u = \arcsin x$ and $v = \arctan 2x$

$\sin u = x$ $\quad\quad$ $\tan v = 2x$

$$\begin{aligned}\cos(\arcsin x - \arctan 2x) &= \cos(\arcsin x)\cos(\arctan 2x) + \sin(\arcsin x)\sin(\arctan 2x)\\ &= \sqrt{1 - x^2}\,\frac{1}{\sqrt{4x^2 + 1}} + x\,\frac{2x}{\sqrt{4x^2 + 1}}\\ &= \frac{2x^2 + \sqrt{1 - x^2}}{\sqrt{4x^2 + 1}}\end{aligned}$$



47. $\sin^{-1} 1 = \dfrac{\pi}{2}$ because $\sin \dfrac{\pi}{2} = 1$.

$\cos^{-1} 1 = 0$ because $\cos 0 = 1$.

$$\sin(\sin^{-1} 1 + \cos^{-1} 1) = \sin\left(\frac{\pi}{2} + 0\right) = 1$$

48. $\sin^{-1}(-1) = -\dfrac{\pi}{2}$ and $\cos^{-1} 0 = \dfrac{\pi}{2}$

$$\begin{aligned}\cos(\sin^{-1}(-1) + \cos^{-1} 0) &= \cos\left(-\frac{\pi}{2} + \frac{\pi}{2}\right)\\ &= \cos 0 = 1\end{aligned}$$

49. $\sin^{-1} 1 = \dfrac{\pi}{2}$ and $\cos^{-1}(-1) = \pi$

$$\begin{aligned}\sin(\sin^{-1} 1 - \cos^{-1}(-1)) &= \sin\left(\frac{\pi}{2} - \pi\right)\\ &= \sin\left(-\frac{\pi}{2}\right) = -1\end{aligned}$$

50. $\cos^{-1}(-1) = \pi$ and $\cos^{-1} 1 = 0$

$$\cos(\cos^{-1}(-1) - \cos^{-1} 1) = \cos(\pi - 0) = -1$$

51. $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$ and $\cos^{-1}\frac{1}{2} = \frac{\pi}{3}$

$$\sin\left(\sin^{-1}\frac{1}{2} - \cos^{-1}\frac{1}{2}\right) = \sin\left(\frac{\pi}{6} - \frac{\pi}{3}\right)$$
$$= \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$$

52. $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$ and $\sin^{-1} 1 = \frac{\pi}{2}$

$$\cos\left(\cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1} 1\right) = \cos\left(\frac{2\pi}{3} + \frac{\pi}{2}\right)$$
$$= \cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$$

53. $\sin^{-1} 0 = 0$ and $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

$$\tan\left(\sin^{-1} 0 + \sin^{-1}\frac{1}{2}\right) = \tan\left(0 + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$$

54. $\cos^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$ and $\sin^{-1} 0 = 0$

$$\tan\left(\cos^{-1}\frac{\sqrt{2}}{2} - \sin^{-1} 0\right) = \tan\left(\frac{\pi}{4} - 0\right) = 1$$

55. $\sin^{-1}(-1) = -\frac{\pi}{2}$

$$\sin\left(\frac{\pi}{2} + \sin^{-1}(-1)\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{2}\right) = \sin 0 = 0$$

56. $\cos^{-1}(-1) = \pi$

$$\sin(\cos^{-1}(-1) + \pi) = \sin(\pi + \pi) = \sin 2\pi = 0$$

57. $\sin^{-1} 1 = \frac{\pi}{2}$

$$\cos(\pi + \sin^{-1} 1) = \cos\left(\pi + \frac{\pi}{2}\right) = \cos\frac{3\pi}{2} = 0$$

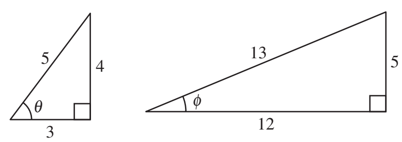
58. $\cos^{-1}(-1) = \pi$

$$\cos(\pi - \cos^{-1}(-1)) = \cos(\pi - \pi) = \cos 0 = 1$$

59. Let $\theta = \cos^{-1}\frac{3}{5} \implies \cos\theta = \frac{3}{5}$.

Let $\phi = \sin^{-1}\frac{5}{13} \implies \sin\phi = \frac{5}{13}$.

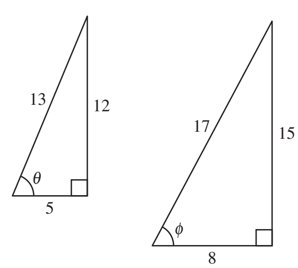
$$\sin\left(\cos^{-1}\tfrac{3}{5} - \sin^{-1}\tfrac{5}{13}\right) = \sin(\theta - \phi)$$
$$= \sin\theta\cos\phi - \cos\theta\sin\phi$$
$$= \left(\tfrac{4}{5}\right)\left(\tfrac{12}{13}\right) - \left(\tfrac{3}{5}\right)\left(\tfrac{5}{13}\right) = \tfrac{33}{65}$$



60. Let $\theta = \sin^{-1}\frac{12}{13} \implies \sin\theta = \frac{12}{13}$.

Let $\phi = \cos^{-1}\frac{8}{17} \implies \cos\phi = \frac{8}{17}$.

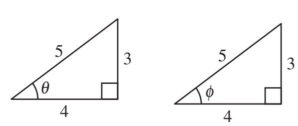
$$\cos\left(\sin^{-1}\tfrac{12}{13} + \cos^{-1}\tfrac{8}{17}\right) = \cos(\theta + \phi)$$
$$= \cos\theta\cos\phi - \sin\theta\sin\phi$$
$$= \left(\tfrac{5}{13}\right)\left(\tfrac{8}{17}\right) - \left(\tfrac{12}{13}\right)\left(\tfrac{15}{17}\right) = -\tfrac{140}{221}$$



61. Let $\theta = \tan^{-1}\frac{3}{4} \implies \tan\theta = \frac{3}{4}$.

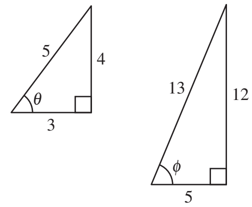
Let $\phi = \sin^{-1}\frac{3}{5} \implies \sin\phi = \frac{3}{5}$.

$$\sin\left(\tan^{-1}\tfrac{3}{4} + \sin^{-1}\tfrac{3}{5}\right) = \sin(\theta + \phi)$$
$$= \sin\theta\cos\phi + \sin\phi\cos\theta$$
$$= \tfrac{3}{5}\left(\tfrac{4}{5}\right) + \tfrac{3}{5}\left(\tfrac{4}{5}\right) = \tfrac{24}{25}$$



Note: $\theta = \phi$

62. Let $\theta = \sin^{-1}\frac{4}{5} \Rightarrow \sin\theta = \frac{4}{5}$.



Let $\phi = \cos^{-1}\frac{5}{13} \Rightarrow \cos\phi = \frac{5}{13}$.

$$\tan\left(\sin^{-1}\frac{4}{5} - \cos^{-1}\frac{5}{13}\right) = \tan(\theta - \phi)$$
$$= \frac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi}$$
$$= \frac{(4/3) - (12/5)}{1 + (4/3)\cdot(12/5)} = \frac{-16}{63}$$

63. $\sin\left(\frac{\pi}{2} + x\right) = \sin\frac{\pi}{2}\cos x + \sin x\cos\frac{\pi}{2}$
$= (1)\cos x + 0 = \cos x$

64. $\sin(3\pi - x) = \sin 3\pi\cos x - \cos 3\pi\sin x$
$= (0)\cos x - (-1)\sin x = \sin x$

65. $$\tan(x + \pi) - \tan(\pi - x) = \frac{\tan x + \tan\pi}{1 - \tan x\cdot\tan\pi} - \frac{\tan\pi - \tan x}{1 + \tan\pi\tan x}$$
$$= \frac{\tan x}{1} - \left(-\frac{\tan x}{1}\right) = 2\tan x$$

66. $$\tan\left(\frac{\pi}{4} - \theta\right) = \frac{\tan(\pi/4) - \tan\theta}{1 + \tan(\pi/4)\tan\theta} = \frac{1 - \tan\theta}{1 + \tan\theta}$$

67. $\sin(x + y) + \sin(x - y) = \sin x\cos y + \sin y\cos x + \sin x\cos y - \sin y\cos x = 2\sin x\cos y$

68. $\cos(x + y) + \cos(x - y) = \cos x\cos y - \sin x\sin y + \cos x\cos y + \sin x\sin y = 2\cos x\cos y$

69. $$\cos(x + y)\cos(x - y) = [\cos x\cos y - \sin x\sin y][\cos x\cos y + \sin x\sin y]$$
$$= \cos^2 x\cos^2 y - \sin^2 x\sin^2 y = \cos^2 x(1 - \sin^2 y) - \sin^2 x\sin^2 y$$
$$= \cos^2 x - \sin^2 y(\cos^2 x + \sin^2 x) = \cos^2 x - \sin^2 y$$

70. $$\sin(x + y)\sin(x - y) = [\sin x\cos y + \cos x\sin y][\sin x\cos y - \cos x\sin y]$$
$$= \sin^2 x\cos^2 y - \cos^2 x\sin^2 y = \sin^2 x(1 - \sin^2 y) - \cos^2 x\sin^2 y$$
$$= \sin^2 x - \sin^2 y(\sin^2 x + \cos^2 x) = \sin^2 x - \sin^2 y$$

71. $$\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 1$$
$$\sin x\cos\frac{\pi}{3} + \cos x\sin\frac{\pi}{3} + \sin x\cos\frac{\pi}{3} - \cos x\sin\frac{\pi}{3} = 1$$
$$2\sin x(0.5) = 1$$
$$\sin x = 1$$
$$x = \frac{\pi}{2}$$

72.

$$\cos\left(x + \frac{\pi}{6}\right) - \cos\left(x - \frac{\pi}{6}\right) = 1$$

$$\left(\cos x \cos \frac{\pi}{6} - \sin x \sin \frac{\pi}{6}\right) - \left(\cos x \cos \frac{\pi}{6} + \sin x \sin \frac{\pi}{6}\right) = 1$$

$$-2 \sin x \sin \frac{\pi}{6} = 1$$

$$-2 \sin x\left(\frac{1}{2}\right) = 1$$

$$\sin x = -1$$

$$x = \frac{3\pi}{2}$$

73.

$$\tan(x + \pi) + 2 \sin(x + \pi) = 0$$

$$\frac{\tan x + \tan \pi}{1 - \tan x \tan \pi} + 2(\sin x \cos \pi + \cos x \sin \pi) = 0$$

$$\frac{\tan x + 0}{1 - \tan x(0)} + 2[\sin x(-1) + \cos x(0)] = 0$$

$$\frac{\tan x}{1} - 2 \sin x = 0$$

$$\frac{\sin x}{\cos x} = 2 \sin x$$

$$\sin x = 2 \sin x \cos x$$

$$\sin x(1 - 2 \cos x) = 0$$

$$\sin x = 0 \quad \text{or} \quad \cos x = \frac{1}{2}$$

$$x = 0, \pi \qquad x = \frac{\pi}{3}, \frac{5\pi}{3}$$

74.

$$2 \sin\left(x + \frac{\pi}{2}\right) + 3 \tan(\pi - x) = 0$$

$$2\left[\sin x \cos \frac{\pi}{2} + \cos x \sin \frac{\pi}{2}\right] + 3 \tan(-x) = 0$$

$$2 \cos x - 3 \frac{\sin x}{\cos x} = 0$$

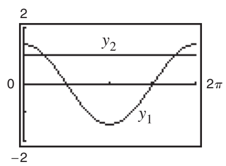
$$2 \cos^2 x - 3 \sin x = 0$$

$$2(1 - \sin^2 x) - 3 \sin x = 0$$

$$2 \sin^2 x + 3 \sin x - 2 = 0$$

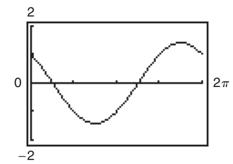
$$(2 \sin x - 1)(\sin x + 2) \implies \sin x = \frac{1}{2} \implies x = \frac{\pi}{6}, \frac{5\pi}{6}$$

75. Graph $y_1 = \cos\left(x + \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right)$ and $y_2 = 1$.



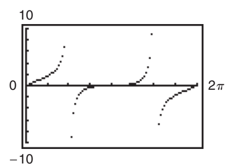
$x \approx 0.7854, 5.4978$

76. $\sin\left(x + \frac{\pi}{2}\right) - \cos\left(x + \frac{3\pi}{2}\right) = 0$



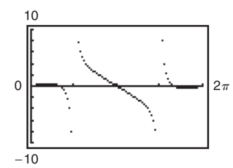
$x \approx 0.7854, 3.9270$

77. $\tan(x + \pi) - \cos\left(x + \frac{\pi}{2}\right) = 0$



Answers: 0.0, 3.1416 $(x = 0, \pi)$

78. $\tan(\pi - x) + 2\cos\left(x + \frac{3\pi}{2}\right) = 0$

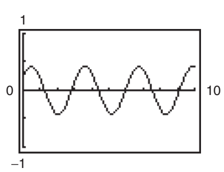


$x = 0, 1.0472, 3.1416, 5.2360$

79.
$$\begin{aligned} y_1 + y_2 &= A\cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + A\cos 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right) \\ &= A\left[\cos\left(\frac{2\pi t}{T}\right)\cos\left(\frac{2\pi x}{\lambda}\right) + \sin\left(\frac{2\pi t}{T}\right)\sin\left(\frac{2\pi x}{\lambda}\right)\right] + A\left[\cos\left(\frac{2\pi t}{T}\right)\cos\left(\frac{2\pi x}{\lambda}\right) - \sin\left(\frac{2\pi t}{T}\right)\sin\left(\frac{2\pi x}{\lambda}\right)\right] \\ &= 2A\cos\left(\frac{2\pi t}{T}\right)\cos\left(\frac{2\pi x}{\lambda}\right) \end{aligned}$$

80. $y = \frac{1}{3}\sin 2t + \frac{1}{4}\cos 2t$

(a)



(b) $a = \frac{1}{3},\ b = \frac{1}{4},\ B = 2$

$C = \arctan\frac{b}{a} = \arctan\frac{3}{4} \approx 0.6435$

$y \approx \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{4}\right)^2}\sin(2t + 0.6435) = \frac{5}{12}\sin(2t + 0.6435)$

(c) Amplitude: $\frac{5}{12}$

(d) Frequency: $\frac{1}{\text{period}} = \frac{B}{2\pi} = \frac{2}{2\pi} = \frac{1}{\pi}$

81. False. See page 380.

82. True. $\sin\left(x - \frac{11\pi}{2}\right) = \sin x\cos\frac{11\pi}{2} - \cos x\sin\frac{11\pi}{2} = 0 - \cos x(-1) = \cos x$

83.
$$\begin{aligned} \cos(n\pi + \theta) &= \cos n\pi\cos\theta - \sin n\pi\sin\theta \\ &= (-1)^n(\cos\theta) - (0)(\sin\theta) \\ &= (-1)^n(\cos\theta), \text{ where } n \text{ is an integer.} \end{aligned}$$

84.
$$\begin{aligned} \sin(n\pi + \theta) &= \sin n\pi\cos\theta + \sin\theta\cos n\pi \\ &= (0)(\cos\theta) + (\sin\theta)(-1)^n \\ &= (-1)^n(\sin\theta), \text{ where } n \text{ is an integer.} \end{aligned}$$

85. $C = \arctan \frac{b}{a} \implies \tan C = \frac{b}{a} \implies \sin C = \frac{b}{\sqrt{a^2 + b^2}}, \cos C = \frac{a}{\sqrt{a^2 + b^2}}$

$$\sqrt{a^2 + b^2} \sin(B\theta + C) = \sqrt{a^2 + b^2}\left(\sin B\theta \cdot \frac{a}{\sqrt{a^2 + b^2}} + \frac{b}{\sqrt{a^2 + b^2}} \cdot \cos B\theta\right) = a \sin B\theta + b \cos B\theta$$

86. $C = \arctan \frac{a}{b} \implies \sin C = \frac{a}{\sqrt{a^2 + b^2}}, \cos C = \frac{b}{\sqrt{a^2 + b^2}}$

$$\begin{aligned}\sqrt{a^2 + b^2} \cos(B\theta - C) &= \sqrt{a^2 + b^2}\left(\cos B\theta \cdot \frac{b}{\sqrt{a^2 + b^2}} + \sin B\theta \cdot \frac{a}{\sqrt{a^2 + b^2}}\right)\\ &= b \cos B\theta + a \sin B\theta\\ &= a \sin B\theta + b \cos B\theta\end{aligned}$$

87. $\sin \theta + \cos \theta$

$a = 1,\ b = 1,\ B = 1$

(a) $C = \arctan \frac{b}{a} = \arctan 1 = \frac{\pi}{4}$

$$\begin{aligned}\sin \theta + \cos \theta &= \sqrt{a^2 + b^2} \sin(B\theta + C)\\ &= \sqrt{2} \sin\left(\theta + \frac{\pi}{4}\right)\end{aligned}$$

(b) $C = \arctan \frac{a}{b} = \arctan 1 = \frac{\pi}{4}$

$$\begin{aligned}\sin \theta + \cos \theta &= \sqrt{a^2 + b^2} \cos(B\theta - C)\\ &= \sqrt{2} \cos\left(\theta - \frac{\pi}{4}\right)\end{aligned}$$

88. $3 \sin 2\theta + 4 \cos 2\theta; a = 3, b = 4, B = 2$

(a) $C = \arctan \frac{b}{a} = \arctan \frac{4}{3} \approx 0.9273$

$$\begin{aligned}3 \sin 2\theta + 4 \cos 2\theta &= \sqrt{a^2 + b^2} \sin(B\theta + C)\\ &\approx 5 \sin(2\theta + 0.9273)\end{aligned}$$

(b) $C = \arctan \frac{a}{b} = \arctan \frac{3}{4} \approx 0.6435$

$$\begin{aligned}3 \sin 2\theta + 4 \cos 2\theta &= \sqrt{a^2 + b^2} \cos(B\theta - C)\\ &\approx 5 \cos(2\theta - 0.6435)\end{aligned}$$

89. $12 \sin 3\theta + 5 \cos 3\theta; a = 12,\ b = 5,\ B = 3$

(a) $C = \arctan \frac{b}{a} = \arctan \frac{5}{12} \approx 0.3948$

$$\begin{aligned}12 \sin 3\theta + 5 \cos 3\theta &= \sqrt{a^2 + b^2} \sin(B\theta + C)\\ &\approx 13 \sin(3\theta + 0.3948)\end{aligned}$$

(b) $C = \arctan \frac{a}{b} = \arctan \frac{12}{5} \approx 1.1760$

$$\begin{aligned}12 \sin 3\theta + 5 \cos 3\theta &= \sqrt{a^2 + b^2} \cos(B\theta - C)\\ &\approx 13 \cos(3\theta - 1.1760)\end{aligned}$$

90. $\sin 2\theta - \cos 2\theta; a = 1, b = -1, B = 2$

(a) $C = \arctan \frac{b}{a} = \arctan(-1) = -\frac{\pi}{4}$

$$\begin{aligned}\sin 2\theta - \cos 2\theta &= \sqrt{a^2 + b^2} \sin(B\theta + C)\\ &= \sqrt{2} \sin\left(2\theta - \frac{\pi}{4}\right)\end{aligned}$$

(b) Because $b > 0$ in the formula, we write the given expression as

$-(-\sin 2\theta + \cos 2\theta); a = -1, b = 1, B = 2,$

$C = \arctan\left(\frac{a}{b}\right) = \arctan(-1) = -\frac{\pi}{4}.$

Hence,

$$\begin{aligned}-(-\sin 2\theta + \cos 2\theta) &= -\sqrt{a^2 + b^2} \cos(B\theta - C)\\ &= -\sqrt{2} \cos\left(2\theta + \frac{\pi}{4}\right).\end{aligned}$$

91. $C = \arctan \frac{b}{a} = \frac{\pi}{2} \implies a = 0$

$\sqrt{a^2 + b^2} = 2 \implies b = 2$

$B = 1$

$2 \sin\left(\theta + \frac{\pi}{2}\right) = (0)(\sin \theta) + (2)(\cos \theta) = 2 \cos \theta$

92. $C = -\frac{\pi}{4} = \arctan\left(\frac{a}{b}\right) \implies \frac{a}{b} = -1$

$\implies a = -1, b = 1$

$\sqrt{a^2 + b^2} = \sqrt{2}$

Hence, $B = 1$ and

$$5 \cos\left(\theta + \frac{\pi}{4}\right) = \frac{5}{\sqrt{2}}\sqrt{2} \cos\left[\theta - \left(-\frac{\pi}{4}\right)\right]$$

$$= \frac{5}{\sqrt{2}}[-\sin \theta + \cos \theta]$$

$$= -\frac{5}{\sqrt{2}} \sin \theta + \frac{5}{\sqrt{2}} \cos \theta$$

$$= -\frac{5\sqrt{2}}{2} \sin \theta + \frac{5\sqrt{2}}{2} \cos \theta.$$

93. $$\frac{\sin(x + h) - \sin x}{h} = \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \frac{\sin x(\cos h - 1) + \cos x \sin h}{h}$$

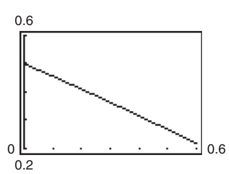
$$= \frac{\cos x \sin h}{h} - \frac{\sin x(1 - \cos h)}{h}$$

94. (a) Domains of f and g are all real numbers, $h \neq 0$.

(b)

h	0.01	0.02	0.05	0.1	0.2	0.5
$f(h)$	0.4957	0.4913	0.4781	0.4559	0.4104	0.2674
$g(h)$	0.4957	0.4913	0.4781	0.4559	0.4104	0.2674

(c)



(d) As $h \to 0, f \to \frac{1}{2}$ and $g \to \frac{1}{2}$. In fact, $f = g$.

95. From the figure, it appears that $u + v = w$. Assume that u, v, and w are all in Quadrant I. From the figure:

$\tan u = \frac{s}{3s} = \frac{1}{3}$

$\tan v = \frac{s}{2s} = \frac{1}{2}$

$\tan w = \frac{s}{s} = 1$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} = \frac{(1/3) + (1/2)}{1 - (1/3)(1/2)} = \frac{5/6}{1 - (1/6)} = 1 = \tan w.$$

Thus, $\tan(u + v) = \tan w$. Because u, v, and w are all in Quadrant I, we have

$\arctan[\tan(u + v)] = \arctan[\tan w]$

$u + v = w.$

96. (a) $\sin(u + v + w) = \sin u \cos(v + w) + \cos u \sin(v + w)$

$= \sin u[\cos v \cos w - \sin v \sin w] + \cos u[\sin v \cos w + \sin w \cos v]$

$= \sin u \cos v \cos w - \sin u \sin v \sin w + \cos u \sin v \cos w + \cos u \sin w \cos v$

(b) $$\tan(u + v + w) = \frac{\tan(u + v) + \tan w}{1 - \tan(u + v)\tan w} = \frac{\left[\frac{\tan u + \tan v}{1 - \tan u \tan v}\right] + \tan w}{1 - \left[\frac{\tan u + \tan v}{1 - \tan u \tan v}\right]\tan w}$$

$$= \frac{\tan u + \tan v + \tan w(1 - \text{an } u \tan v)}{(1 - \tan u \tan v) - (\tan u + \tan v)\tan w} = \frac{\tan u + \tan v + \tan w - \tan u \tan v \tan w}{1 - \tan u \tan v - \tan u \tan w - \tan v \tan w}$$

97. $x = 0$: $y = -\frac{1}{2}(0 - 10) + 14 = 5 + 14 = 19$

y-intercept: $(0, 19)$

$y = 0$: $0 = -\frac{1}{2}(x - 10) + 14$

$= -\frac{1}{2}x + 19 \implies x = 38$

x-intercept: $(38, 0)$

98. $y = 0$: $x^2 - 3x - 40 = (x - 8)(x + 5) = 0$

x-intercepts: $(8, 0), (-5, 0)$

$x = 0 \implies y = -40$

y-intercept: $(0, -40)$

99. $x = 0$: $|2(0) - 9| - 5 = 9 - 5 = 4$

y-intercept: $(0, 4)$

$y = 0$: $|2x - 9| = 5 \implies x = 7, 2$

x-intercepts: $(2, 0), (7, 0)$

100. $y = 0$: $2x\sqrt{x + 7} = 0 \implies x = 0, -7$

x-intercepts: $(0, 0), (-7, 0)$

$x = 0 \implies y = 0$

y-intercept: $(0, 0)$

101. $\arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$ because $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$.

102. $\arctan(-\sqrt{3}) = -\frac{\pi}{3}$ because $\tan\left(-\frac{\pi}{3}\right) = -\sqrt{3}$.

103. $\arcsin 1 = \frac{\pi}{2}$ because $\sin\frac{\pi}{2} = 1$.

104. $\arctan 0 = 0$

Section 5.5 Multiple-Angle and Product-to-Sum Formulas

- You should know the following double-angle formulas.

  (a) $\sin 2u = 2\sin u \cos u$ (b) $\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$ (c) $\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$

- You should be able to reduce the power of a trigonometric function.

  (a) $\sin^2 u = \frac{1 - \cos 2u}{2}$ (b) $\cos^2 u = \frac{1 + \cos 2u}{2}$ (c) $\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$

- You should be able to use the half-angle formulas.

  (a) $\sin\frac{u}{2} = \pm\sqrt{\frac{1 - \cos u}{2}}$ (b) $\cos\frac{u}{2} = \pm\sqrt{\frac{1 + \cos u}{2}}$ (c) $\tan\frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$

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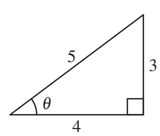
—CONTINUED—

- You should be able to use the product-sum formulas.
  (a) $\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$
  (b) $\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$
  (c) $\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$
  (d) $\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$
- You should be able to use the sum-product formulas.
  (a) $\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$
  (b) $\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$
  (c) $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$
  (d) $\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

Vocabulary Check

1. $2\sin u \cos u$
2. $\frac{1 + \cos 2u}{2}$
3. $\cos 2u$
4. $\tan \frac{u}{2}$
5. $\frac{2\tan u}{1 - \tan^2 u}$
6. $\frac{1}{2}[\cos(u - v) + \cos(u + v)]$
7. $\sin^2 u$
8. $\cos \frac{u}{2}$
9. $\frac{1}{2}[\sin(u + v) + \sin(u - v)]$
10. $2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$

1. (a) $\sin\theta = \frac{3}{5}$



(b) $\cos\theta = \frac{4}{5}$

(c) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

(d) $\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$

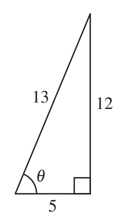
(e) $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{24}{7}$

(f) $\sec 2\theta = \frac{1}{\cos 2\theta} = \frac{25}{7}$

(g) $\csc 2\theta = \frac{1}{\sin 2\theta} = \frac{25}{24}$

(h) $\cot 2\theta = \frac{1}{\tan 2\theta} = \frac{7}{24}$

2. (a) $\sin\theta = \frac{12}{13}$



(b) $\cos\theta = \frac{5}{13}$

(c) $\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right) = \frac{120}{169}$

(d) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \frac{25}{169} - \frac{144}{169} = \frac{-119}{169}$

(e) $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{-120}{119}$

(f) $\cot 2\theta = \frac{1}{\tan 2\theta} = \frac{-119}{120}$

(g) $\sec 2\theta = \frac{1}{\cos 2\theta} = \frac{-169}{119}$

(h) $\csc 2\theta = \frac{1}{\sin 2\theta} = \frac{169}{120}$

3. $\sin 2x - \sin x = 0$

Solutions: 0, 1.047, 3.142, 5.236

Analytically:

$$\sin 2x - \sin x = 0$$
$$2 \sin x \cos x - \sin x = 0$$
$$\sin x(2 \cos x - 1) = 0$$

$\sin x = 0$ or $2 \cos x - 1 = 0$

$x = 0, \pi$ $\qquad \cos x = \frac{1}{2}$

$x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ $\qquad x = \frac{\pi}{3}, \frac{5\pi}{3}$

4. $\sin 2x + \cos x = 0$

Solutions: 1.5708, 3.6652, 4.7124, 5.7596

Analytically:

$$\sin 2x + \cos x = 0$$
$$2 \sin x \cos x + \cos x = 0$$
$$\cos x(2 \sin x + 1) = 0$$

$\cos x = 0$ $\qquad \sin x = -\frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad x = \frac{7\pi}{6}, \frac{11\pi}{6}$

5. $4 \sin x \cos x = 1$

$x \approx 0.2618, 1.3090, 3.4034, 4.4506$

Analytically:

$$4 \sin x \cos x = 1$$
$$2 \sin(2x) = 1$$
$$\sin(2x) = \frac{1}{2}$$
$$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$
$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$$

6. $\sin 2x \sin x = \cos x$

$x \approx 0.7854, 1.5708, 2.3562, 3.9270, 4.7124, 5.4978$

Analytically:

$$2 \sin x \cos x \sin x - \cos x = 0$$
$$\cos x(2 \sin^2 x - 1) = 0$$

$\cos x = 0$ or $2 \sin^2 x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad \sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

7. $\cos 2x - \cos x = 0$

$x \approx 0, 2.094, 4.189$, (6.283 not in interval)

Analytically:

$$\cos 2x - \cos x = 0$$
$$2 \cos^2 x - 1 - \cos x = 0$$
$$(2 \cos x + 1)(\cos x - 1) = 0$$

$\cos x = -\frac{1}{2}$, $\qquad \cos x = 1$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}, 0$, ($2\pi$ not in interval)

8. $\tan 2x - \cot x = 0$

$x \approx 0.5236, 1.5708, 2.6180, 3.6652, 4.7124, 5.7596$

Analytically:

$$\frac{2\tan x}{1-\tan^2 x} = \cot x$$
$$2\tan x = \cot x(1-\tan^2 x)$$
$$2\tan x = \cot x - \cot x\tan^2 x$$
$$2\tan x = \cot x - \tan x$$
$$3\tan x = \cot x$$
$$3\tan x - \cot x = 0$$
$$3\tan x - \frac{1}{\tan x} = 0$$
$$\frac{3\tan^2 x - 1}{\tan x} = 0$$
$$\frac{1}{\tan x}(3\tan^2 x - 1) = 0$$
$$\cot x(3\tan^2 x - 1) = 0$$
$$\cot x = 0 \quad \text{or} \quad 3\tan^2 x - 1 = 0$$
$$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \tan^2 x = \frac{1}{3}$$
$$\tan x = \pm\frac{\sqrt{3}}{3}$$
$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$
$$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$$

9. Solutions: 0, 1.571, 3.142, 4.712

$$\sin 4x = -2\sin 2x$$
$$\sin 4x + 2\sin 2x = 0$$
$$2\sin 2x\cos 2x + 2\sin 2x = 0$$
$$2\sin 2x(\cos 2x + 1) = 0$$
$$2\sin 2x = 0 \quad \text{or} \quad \cos 2x + 1 = 0$$
$$\sin 2x = 0 \qquad \cos 2x = -1$$
$$2x = n\pi \qquad 2x = \pi + 2n\pi$$
$$x = \frac{n}{2}\pi \qquad x = \frac{\pi}{2} + n\pi$$
$$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \qquad x = \frac{\pi}{2}, \frac{3\pi}{2}$$

10. $(\sin 2x + \cos 2x)^2 = 1$

$x \approx 0.0, 0.7854, 1.5708, 2.3562, 3.1416, 3.9270,$
$4.7124, 5.4978$

Analytically:

$$\sin^2 2x + 2\sin 2x\cos 2x + \cos^2 2x = 1$$
$$2\sin 2x\cos 2x = 0$$
$$\sin 4x = 0$$
$$4x = n\pi$$
$$x = \frac{n\pi}{4}$$
$$x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4},$$
$$\pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$$

11. $\cos 2x + \sin x = 0$

Using a graphing utility, $x \approx 1.5708, 3.6652, 5.7596$.

Algebraically:

$$\cos 2x + \sin x = 0$$
$$1 - 2\sin^2 x + \sin x = 0$$
$$2\sin^2 x - \sin x - 1 = 0$$
$$(2\sin x + 1)(\sin x - 1) = 0$$
$$\sin x = -\frac{1}{2} \implies x = \frac{7\pi}{6}, \frac{11\pi}{6}$$
$$\sin x = 1 \implies x = \frac{\pi}{2}$$

12. $\tan 2x - 2\cos x = 0$

Using a graphing utility,

$x \approx 0.5236, 1.5708, 2.6180, 4.7124.$

Algebraically:

$$\tan 2x = 2\cos x$$

$$\frac{2\tan x}{1-\tan^2 x} = 2\cos x$$

$$\frac{\sin x}{\cos x} = \cos x\left(1 - \frac{\sin^2 x}{\cos^2 x}\right)$$

$\left(\textbf{Note: } \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}\right)$

$$\sin x = \cos^2 x - \sin^2 x$$

$$\sin x = 1 - 2\sin^2 x$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$\sin x = \frac{1}{2} \implies x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\sin x = -1 \implies x = \frac{3\pi}{2}$$

$$\cos x = 0 \implies x = \frac{\pi}{2}$$

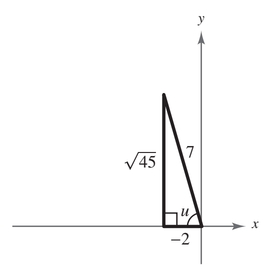
13. $\sin u = \frac{3}{5},\ 0 < u < \frac{\pi}{2} \implies \cos u = \frac{4}{5}$

$$\sin 2u = 2\sin u\cos u = 2\cdot\frac{3}{5}\cdot\frac{4}{5} = \frac{24}{25}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u} = \frac{2(3/4)}{1-(9/16)} = \frac{24}{7}$$

14. $\cos u = -\frac{2}{7}, \frac{\pi}{2} < u < \pi,$ Quadrant II



$$\sin 2u = 2\sin u\cos u = 2\left(\frac{\sqrt{45}}{7}\right)\left(-\frac{2}{7}\right) = -\frac{12\sqrt{5}}{49}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{4}{49} - \frac{45}{49} = -\frac{41}{49}$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u} = \frac{2(-\sqrt{45}/2)}{1-(45/4)} = \frac{-\sqrt{45}}{(4-45)/4} = \frac{12\sqrt{5}}{41}$$

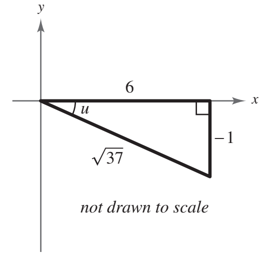
15. $\tan u = \frac{1}{2},\ \pi < u < \frac{3\pi}{2} \implies \sin u = -\frac{1}{\sqrt{5}}$ and $\cos u = -\frac{2}{\sqrt{5}}$

$$\sin 2u = 2\sin u\cos u = 2\left(-\frac{1}{\sqrt{5}}\right)\left(-\frac{2}{\sqrt{5}}\right) = \frac{4}{5}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \left(-\frac{2}{\sqrt{5}}\right)^2 - \left(-\frac{1}{\sqrt{5}}\right)^2 = \frac{3}{5}$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u} = \frac{2(1/2)}{1-(1/4)} = \frac{4}{3}$$

16. $\cot u = -6, \frac{3\pi}{2} < u < 2\pi,$ Quadrant IV



$$\sin 2u = 2\sin u\cos u = 2\left(-\frac{1}{\sqrt{37}}\right)\left(\frac{6}{\sqrt{37}}\right) = -\frac{12}{37}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{36}{37} - \frac{1}{37} = \frac{35}{37}$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u} = \frac{2(-1/6)}{1-(-1/6)^2} = \frac{-2/6}{35/36} = -\frac{12}{35}$$

17. $\sec u = -\frac{5}{2}, \frac{\pi}{2} < u < \pi$

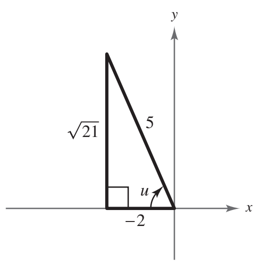
$$\cos u = -\frac{2}{5} \Rightarrow \sin u = \frac{\sqrt{21}}{5}$$

$$\sin 2u = 2\sin u\cos u = 2\left(\frac{\sqrt{21}}{5}\right)\left(-\frac{2}{5}\right) = \frac{-4\sqrt{21}}{25}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{4}{25} - \frac{21}{25} = -\frac{17}{25}$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u}$$

$$= \frac{2(\sqrt{21}/-2)}{1-(21/4)} = \frac{-\sqrt{21}}{-17/4} = \frac{4\sqrt{21}}{17}$$



18. $\csc u = 3, \frac{\pi}{2} < u < \pi$

$$\sin u = \frac{1}{3}, \cos u = -\frac{2\sqrt{2}}{3}, \tan u = \frac{-1}{2\sqrt{2}} = \frac{-\sqrt{2}}{4}$$

$$\sin 2u = 2\sin u\cos u = 2\left(\frac{1}{3}\right)\left(-\frac{2\sqrt{2}}{3}\right) = \frac{-4\sqrt{2}}{9}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{8}{9} - \frac{1}{9} = \frac{7}{9}$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u} = \frac{(-2\sqrt{2})/4}{1-(1/8)} = \frac{-4\sqrt{2}}{7}$$

19. $8\sin x\cos x = 4(2\sin x\cos x) = 4\sin 2x$

20. $4\sin x\cos x + 1 = 2\sin 2x + 1$

21. $6 - 12\sin^2 x = 6(1 - 2\sin^2 x) = 6\cos 2x$

22. $(\cos x + \sin x)(\cos x - \sin x) = \cos^2 x - \sin^2 x$
$= \cos 2x$

23. $\cos^4 x = (\cos^2 x)(\cos^2 x) = \left(\frac{1+\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) = \frac{1+2\cos 2x+\cos^2 2x}{4}$

$$= \frac{1+2\cos 2x+(1+\cos 4x)/2}{4} = \frac{2+4\cos 2x+1+\cos 4x}{8}$$

$$= \frac{3+4\cos 2x+\cos 4x}{8} = \frac{1}{8}(3+4\cos 2x+\cos 4x)$$

24. $$\begin{aligned}\sin^4 x &= (\sin^2 x)(\sin^2 x)\\ &= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)\\ &= \frac{1-2\cos 2x+\cos^2 2x}{4}\\ &= \frac{1-2\cos 2x+\left(\frac{1+\cos 4x}{2}\right)}{4}\\ &= \frac{2-4\cos 2x+1+\cos 4x}{8}\\ &= \frac{1}{8}(3-4\cos 2x+\cos 4x)\end{aligned}$$

25. $$\begin{aligned}(\sin^2 x)(\cos^2 x) &= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)\\ &= \frac{1-\cos^2 2x}{4}\\ &= \frac{1}{4}\left(1-\frac{1+\cos 4x}{2}\right)\\ &= \frac{1}{8}(2-1-\cos 4x)\\ &= \frac{1}{8}(1-\cos 4x)\end{aligned}$$

26. $$\begin{aligned}\cos^6 x = (\cos^2 x)^3 &= \left(\frac{1+\cos 2x}{2}\right)^3\\ &= \frac{1}{8}[1+3\cos 2x+3\cos^2 2x+\cos^3 2x]\\ &= \frac{1}{8}\left[1+3\cos 2x+3\cdot\frac{1+\cos 4x}{2}+\cos 2x\left(\frac{1+\cos 4x}{2}\right)\right]\\ &= \frac{1}{8}\left[\frac{5}{2}+3\cos 2x+\frac{3}{2}\cos 4x+\frac{1}{2}\cos 2x+\frac{1}{2}\cos 2x\cdot\cos 4x\right]\\ &= \frac{1}{8}\left[\frac{5}{2}+\frac{7}{2}\cos 2x+\frac{3}{2}\cos 4x+\frac{1}{2}\frac{1}{2}(\cos 2x+\cos 6x)\right]\\ &= \frac{1}{32}[10+15\cos 2x+6\cos 4x+\cos 6x]\end{aligned}$$

27. $$\begin{aligned}\sin^2 x\cos^4 x = \sin^2 x\cos^2 x\cos^2 x &= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)\\ &= \frac{1}{8}(1-\cos 2x)(1+\cos 2x)(1+\cos 2x)\\ &= \frac{1}{8}(1-\cos^2 2x)(1+\cos 2x)\\ &= \frac{1}{8}(1+\cos 2x-\cos^2 2x-\cos^3 2x)\\ &= \frac{1}{8}\left[1+\cos 2x-\left(\frac{1+\cos 4x}{2}\right)-\cos 2x\left(\frac{1+\cos 4x}{2}\right)\right]\\ &= \frac{1}{16}[2+2\cos 2x-1-\cos 4x-\cos 2x-\cos 2x\cos 4x]\\ &= \frac{1}{16}\left[1+\cos 2x-\cos 4x-\left(\frac{1}{2}\cos 2x+\frac{1}{2}\cos 6x\right)\right]\\ &= \frac{1}{32}(2+2\cos 2x-2\cos 4x-\cos 2x-\cos 6x)\\ &= \frac{1}{32}(2+\cos 2x-2\cos 4x-\cos 6x)\end{aligned}$$

28. $\sin^4 x \cos^2 x = \sin^2 x \sin^2 x \cos^2 x$

$$= \left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 + \cos 2x}{2}\right)$$

$$= \frac{1}{8}(1 - \cos 2x)(1 - \cos^2 2x)$$

$$= \frac{1}{8}(1 - \cos 2x - \cos^2 2x + \cos^3 2x)$$

$$= \frac{1}{8}\left[1 - \cos 2x - \left(\frac{1 + \cos 4x}{2}\right) + \cos 2x\left(\frac{1 + \cos 4x}{2}\right)\right]$$

$$= \frac{1}{16}[2 - 2\cos 2x - 1 - \cos 4x + \cos 2x + \cos 2x \cos 4x]$$

$$= \frac{1}{16}\left[1 - \cos 2x - \cos 4x + \frac{1}{2}\cos 2x + \frac{1}{2}\cos 6x\right]$$

$$= \frac{1}{32}[2 - 2\cos 2x - 2\cos 4x + \cos 2x + \cos 6x]$$

$$= \frac{1}{32}[2 - \cos 2x - 2\cos 4x + \cos 6x]$$

29. $\sin^2 2x = \dfrac{1 - \cos 4x}{2}$

$$= \frac{1}{2} - \frac{1}{2}\cos 4x$$

$$= \frac{1}{2}(1 - \cos 4x)$$

30. $\cos^2 2x = \dfrac{1 + \cos 4x}{2}$

$$= \frac{1}{2} + \frac{1}{2}\cos 4x$$

$$= \frac{1}{2}(1 + \cos 4x)$$

31. $\cos^2 \dfrac{x}{2} = \dfrac{1 + \cos x}{2}$

$$= \frac{1}{2} + \frac{1}{2}\cos x$$

$$= \frac{1}{2}(1 + \cos x)$$

32. $\sin^2 \dfrac{x}{2} = \dfrac{1 - \cos x}{2}$

$$= \frac{1}{2} - \frac{1}{2}\cos x$$

$$= \frac{1}{2}(1 - \cos x)$$

33. $\sin^2 2x \cos^2 2x = \left(\dfrac{1 - \cos 4x}{2}\right)\left(\dfrac{1 + \cos 4x}{2}\right)$

$$= \frac{1}{4}(1 - \cos 4x)(1 + \cos 4x)$$

$$= \frac{1}{4}(1 - \cos^2 4x)$$

$$= \frac{1}{4}\left(1 - \frac{1 + \cos 8x}{2}\right)$$

$$= \frac{1 - \cos 8x}{8}$$

34. $\sin^2 \dfrac{x}{2} \cos^2 \dfrac{x}{2} = \left(\dfrac{1 - \cos x}{2}\right)\left(\dfrac{1 + \cos x}{2}\right)$

$$= \frac{1}{4}(1 - \cos x)(1 + \cos x)$$

$$= \frac{1}{4}(1 - \cos^2 x)$$

$$= \frac{1}{4}\left(1 - \frac{1 + \cos 2x}{2}\right)$$

$$= \frac{1}{8}(1 - \cos 2x)$$

35. $\sin^4 \frac{x}{2} = \left(\sin^2 \frac{x}{2}\right)\left(\sin^2 \frac{x}{2}\right)$

$= \left(\frac{1 - \cos x}{2}\right)\left(\frac{1 - \cos x}{2}\right)$

$= \frac{1}{4}[1 - 2\cos x + \cos^2 x]$

$= \frac{1}{4}\left[1 - 2\cos x + \frac{1 + \cos 2x}{2}\right]$

$= \frac{1}{8}[2 - 4\cos x + 1 + \cos 2x]$

$= \frac{1}{8}[3 - 4\cos x + \cos 2x]$

36. $\cos^4 \frac{x}{2} = \left(\cos^2 \frac{x}{2}\right)\left(\cos^2 \frac{x}{2}\right)$

$= \left(\frac{1 + \cos x}{2}\right)\left(\frac{1 + \cos x}{2}\right)$

$= \frac{1}{4}[1 + 2\cos x + \cos^2 x]$

$= \frac{1}{4}\left[1 + 2\cos x + \frac{1 + \cos 2x}{2}\right]$

$= \frac{1}{8}[2 + 4\cos x + 1 + \cos 2x]$

$= \frac{1}{8}[3 + 4\cos x + \cos 2x]$

37. (a) $\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos\theta}{2}} = \sqrt{\frac{1 + (15/17)}{2}} = \sqrt{\frac{16}{17}} = \frac{4}{\sqrt{17}} = \frac{4\sqrt{17}}{17}$

(b) $\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos\theta}{2}} = \sqrt{\frac{1 - (15/17)}{2}} = \frac{\sqrt{17}}{17}$

(c) $\tan \frac{\theta}{2} = \frac{\sin\theta}{1 + \cos\theta} = \frac{8/17}{1 + (15/17)} = \frac{8}{32} = \frac{1}{4}$

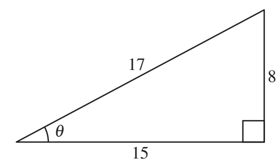
(d) $\sec \frac{\theta}{2} = \frac{1}{\cos(\theta/2)} = \frac{1}{\sqrt{(1 + \cos\theta)/2}} = \frac{\sqrt{2}}{\sqrt{1 + (15/17)}} = \frac{\sqrt{17}}{4}$

(e) $\csc \frac{\theta}{2} = \frac{1}{\sin(\theta/2)} = \frac{1}{\sqrt{(1 - \cos\theta)/2}} = \frac{1}{\sqrt{[1 - (15/17)]/2}} = \frac{1}{1/\sqrt{17}} = \sqrt{17}$

(f) $\cot \frac{\theta}{2} = \frac{1 + \cos\theta}{\sin\theta} = \frac{1 + (15/17)}{8/17} = 4$

(g) $2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2\left(\frac{1}{\sqrt{17}}\right)\left(\frac{4\sqrt{17}}{17}\right) = \frac{8}{17}$, $\quad (= \sin\theta)$

(h) $2\cos\frac{\theta}{2}\tan\frac{\theta}{2} = 2\sin\frac{\theta}{2} = \frac{2\sqrt{17}}{17}$

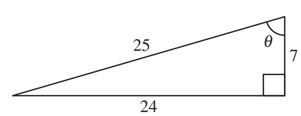


$\sin\theta = \frac{8}{17}$

$\cos\theta = \frac{15}{17}$

38. (a) $\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos\theta}{2}} = \sqrt{\frac{1 - (7/25)}{2}} = \sqrt{\frac{18}{50}} = \frac{3}{5}$

(b) $\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos\theta}{2}} = \sqrt{\frac{1 + (7/25)}{2}} = \frac{4}{5}$



(c) $\tan \frac{\theta}{2} = \frac{\sin(\theta/2)}{\cos(\theta/2)} = \frac{3}{4}$

(d) $\cot \frac{\theta}{2} = \frac{1}{\tan(\theta/2)} = \frac{4}{3}$

(e) $\sec \frac{\theta}{2} = \frac{1}{\cos(\theta/2)} = \frac{5}{4}$

(f) $\csc \frac{\theta}{2} = \frac{1}{\sin(\theta/2)} = \frac{5}{3}$

(g) $2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \sin\theta = \frac{24}{25}$

(h) $\cos 2\theta = 2\cos^2\theta - 1 = 2\left(\frac{7}{25}\right)^2 - 1 = -\frac{527}{625}$

39. $\sin 15° = \sin\left(\frac{1}{2} \cdot 30°\right) = \sqrt{\frac{1 - \cos 30°}{2}} = \sqrt{\frac{1 - \left(\sqrt{3}/2\right)}{2}} = \frac{1}{2}\sqrt{2 - \sqrt{3}}$

$\cos 15° = \cos\left(\frac{1}{2} \cdot 30°\right) = \sqrt{\frac{1 + \cos 30°}{2}} = \sqrt{\frac{1 + \left(\sqrt{3}/2\right)}{2}} = \frac{1}{2}\sqrt{2 + \sqrt{3}}$

$\tan 15° = \tan\left(\frac{1}{2} \cdot 30°\right) = \frac{\sin 30°}{1 + \cos 30°} = \frac{1/2}{1 + \left(\sqrt{3}/2\right)} = \frac{1}{2 + \sqrt{3}} = 2 - \sqrt{3}$

40. $\sin 165° = \sin\left(\frac{1}{2} \cdot 330°\right) = \sqrt{\frac{1 - \cos 330°}{2}} = \sqrt{\frac{1 - \left(\sqrt{3}/2\right)}{2}} = \frac{1}{2}\sqrt{2 - \sqrt{3}}$

$\cos 165° = \cos\left(\frac{1}{2} \cdot 330°\right) = -\sqrt{\frac{1 + \cos 330°}{2}} = -\sqrt{\frac{1 + \left(\sqrt{3}/2\right)}{2}} = -\frac{1}{2}\sqrt{2 + \sqrt{3}}$

$\tan 165° = \tan\left(\frac{1}{2} \cdot 330°\right) = \frac{\sin 330°}{1 + \cos 330°} = \frac{-1/2}{1 + \left(\sqrt{3}/2\right)} = \frac{-1}{2 + \sqrt{3}} = \sqrt{3} - 2$

41. $\sin 112° 30' = \sin\left(\frac{1}{2} \cdot 225°\right) = \sqrt{\frac{1 - \cos 225°}{2}} = \sqrt{\frac{1 + \left(\sqrt{2}/2\right)}{2}} = \frac{1}{2}\sqrt{2 + \sqrt{2}}$

$\cos 112° 30' = \cos\left(\frac{1}{2} \cdot 225°\right) = -\sqrt{\frac{1 + \cos 225°}{2}} = -\sqrt{\frac{1 - \left(\sqrt{2}/2\right)}{2}} = -\frac{1}{2}\sqrt{2 - \sqrt{2}}$

$\tan 112° 30' = \tan\left(\frac{1}{2} \cdot 225°\right) = \frac{\sin 225°}{1 + \cos 225°} = \frac{-\sqrt{2}/2}{1 - \left(\sqrt{2}/2\right)} = -1 - \sqrt{2}$

42. $157° 30' = 157.5° = \frac{1}{2}(315°)$, Quadrant II

$\sin(157° 30') = \sin\left(\frac{1}{2} \cdot 315°\right) = \sqrt{\frac{1 - \cos 315°}{2}} = \sqrt{\frac{1 - \sqrt{2}/2}{2}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$

$\cos(157° 30') = \cos\left(\frac{1}{2} \cdot 315°\right) = -\sqrt{\frac{1 + \cos 315°}{2}} = -\sqrt{\frac{1 + \sqrt{2}/2}{2}} = -\frac{\sqrt{2 + \sqrt{2}}}{2}$

$\tan(157° 30') = \tan\left(\frac{1}{2} \cdot 315°\right) = \frac{\sin 315°}{1 + \cos 315°} = \frac{-\sqrt{2}/2}{1 + \sqrt{2}/2} = \frac{-\sqrt{2}}{2 + \sqrt{2}} = 1 - \sqrt{2}$

43. $\sin \frac{\pi}{8} = \sin\left[\frac{1}{2}\left(\frac{\pi}{4}\right)\right] = \sqrt{\frac{1 - \cos(\pi/4)}{2}} = \frac{1}{2}\sqrt{2 - \sqrt{2}}$

$\cos \frac{\pi}{8} = \cos\left[\frac{1}{2}\left(\frac{\pi}{4}\right)\right] = \sqrt{\frac{1 + \cos(\pi/4)}{2}} = \frac{1}{2}\sqrt{2 + \sqrt{2}}$

$\tan \frac{\pi}{8} = \tan\left[\frac{1}{2}\left(\frac{\pi}{4}\right)\right] = \frac{\sin(\pi/4)}{1 + \cos(\pi/4)} = \frac{\sqrt{2}/2}{1 + \left(\sqrt{2}/2\right)} = \sqrt{2} - 1$

44. $\sin \frac{\pi}{12} = \sin\left[\frac{1}{2}\left(\frac{\pi}{6}\right)\right] = \sqrt{\frac{1 - \cos(\pi/6)}{2}} = \sqrt{\frac{1 - \left(\sqrt{3}/2\right)}{2}} = \frac{1}{2}\sqrt{2 - \sqrt{3}}$

$\cos \frac{\pi}{12} = \cos\left[\frac{1}{2}\left(\frac{\pi}{6}\right)\right] = \sqrt{\frac{1 + \cos(\pi/6)}{2}} = \frac{1}{2}\sqrt{2 + \sqrt{3}}$

$\tan \frac{\pi}{12} = \tan\left[\frac{1}{2}\left(\frac{\pi}{6}\right)\right] = \frac{\sin(\pi/6)}{1 + \cos(\pi/6)} = \frac{1/2}{1 + \left(\sqrt{3}/2\right)} = 2 - \sqrt{3}$

45. $\sin\frac{3\pi}{8} = \sin\left(\frac{1}{2}\cdot\frac{3\pi}{4}\right) = \sqrt{\frac{1-\cos(3\pi/4)}{2}} = \sqrt{\frac{1+(\sqrt{2}/2)}{2}} = \frac{1}{2}\sqrt{2+\sqrt{2}}$

$\cos\frac{3\pi}{8} = \cos\left(\frac{1}{2}\cdot\frac{3\pi}{4}\right) = \sqrt{\frac{1+\cos(3\pi/4)}{2}} = \sqrt{\frac{1-(\sqrt{2}/2)}{2}} = \frac{1}{2}\sqrt{2-\sqrt{2}}$

$\tan\frac{3\pi}{8} = \tan\left(\frac{1}{2}\cdot\frac{3\pi}{4}\right) = \frac{\sin(3\pi/4)}{1+\cos(3\pi/4)} = \frac{\sqrt{2}/2}{1-(\sqrt{2}/2)} = \frac{\sqrt{2}}{2-\sqrt{2}} = \sqrt{2}+1$

46. $\frac{7\pi}{12} = \frac{1}{2}\left(\frac{7\pi}{6}\right)$, Quadrant II

$\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{1}{2}\cdot\frac{7\pi}{6}\right) = \sqrt{\frac{1-\cos(7\pi/6)}{2}} = \sqrt{\frac{1+(\sqrt{3}/2)}{2}} = \frac{\sqrt{2+\sqrt{3}}}{2}$

$\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{1}{2}\cdot\frac{7\pi}{6}\right) = -\sqrt{\frac{1+\cos(7\pi/6)}{2}} = -\sqrt{\frac{1-(\sqrt{3}/2)}{2}} = -\frac{\sqrt{2-\sqrt{3}}}{2}$

$\tan\left(\frac{7\pi}{12}\right) = \tan\left(\frac{1}{2}\cdot\frac{7\pi}{6}\right) = \frac{\sin(7\pi/6)}{1+\cos(7\pi/6)} = \frac{-(1/2)}{1-(\sqrt{3}/2)} = \frac{1}{\sqrt{3}-2} = -2-\sqrt{3}$

47. $\sin u = \frac{5}{13}, \frac{\pi}{2} < u < \pi \implies \cos u = -\frac{12}{13}$

$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1-\cos u}{2}}$

$= \sqrt{\frac{1+(12/13)}{2}} = \frac{5\sqrt{26}}{26}$

$\cos\left(\frac{u}{2}\right) = \sqrt{\frac{1+\cos u}{2}}$

$= \sqrt{\frac{1-(12/13)}{2}} = \frac{\sqrt{26}}{26}$

$\tan\left(\frac{u}{2}\right) = \frac{\sin u}{1+\cos u} = \frac{5/13}{1-(12/13)} = \frac{5}{1} = 5$

48. $\cos u = \frac{7}{25}, 0 < u < \frac{\pi}{2}$, Quadrant I

$\sin u = \sqrt{1-\cos^2 u} = \sqrt{1-\frac{49}{625}} = \frac{24}{25}$

$\sin\frac{u}{2} = \sqrt{\frac{1-\cos u}{2}}$

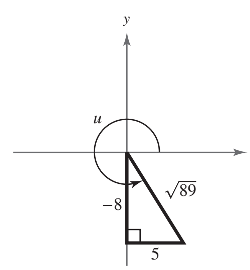
$= \sqrt{\frac{1-(7/25)}{2}} = \sqrt{\frac{9}{25}} = 0.6 = \frac{3}{5}$

$\cos\frac{u}{2} = \sqrt{\frac{1+\cos u}{2}}$

$= \sqrt{\frac{1+(7/25)}{2}} = \sqrt{\frac{16}{25}} = 0.8 = \frac{4}{5}$

$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{1-(7/25)}{24/25} = \frac{18}{24} = \frac{3}{4}$

49. $\tan u = -\frac{8}{5}, \frac{3\pi}{2} < u < 2\pi$, Quadrant IV



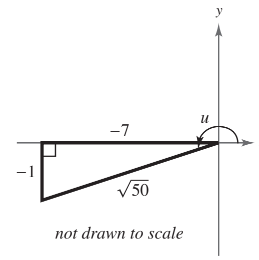
$\sin u = -\frac{8}{\sqrt{89}}, \cos u = \frac{5}{\sqrt{89}}$

$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1-(5/\sqrt{89})}{2}} = \sqrt{\frac{\sqrt{89}-5}{2\sqrt{89}}} = \sqrt{\frac{89-5\sqrt{89}}{178}}$

$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1+\cos u}{2}} = -\sqrt{\frac{1+(5/\sqrt{89})}{2}} = -\sqrt{\frac{\sqrt{89}+5}{2\sqrt{89}}} = -\sqrt{\frac{89+5\sqrt{89}}{178}}$

$\tan\left(\frac{u}{2}\right) = \frac{1-\cos u}{\sin u} = \frac{1-(5/\sqrt{89})}{-8/\sqrt{89}} = \frac{5-\sqrt{89}}{8}$

50. $\cot u = 7,\ \pi < u < \dfrac{3\pi}{2}$, Quadrant III

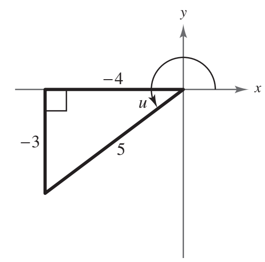


$$\sin\frac{u}{2} = \sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1+(7/\sqrt{50})}{2}} = \sqrt{\frac{\sqrt{50}+7}{2\sqrt{50}}} = \frac{\sqrt{50+7\sqrt{50}}}{10}$$

$$\cos\frac{u}{2} = -\sqrt{\frac{1+\cos u}{2}} = -\sqrt{\frac{1-7/\sqrt{50}}{2}} = -\sqrt{\frac{\sqrt{50}-7}{2\sqrt{50}}} = -\frac{\sqrt{50-7\sqrt{50}}}{10}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{1+7/\sqrt{50}}{-1/\sqrt{50}} = -\left(\sqrt{50}+7\right)$$

51. $\csc u = -\dfrac{5}{3},\ \pi < u < \dfrac{3\pi}{2}$, Quadrant III



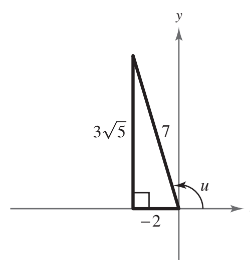
$$\sin u = -\frac{3}{5},\ \cos u = -\frac{4}{5}$$

$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1+(4/5)}{2}} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1+\cos u}{2}} = -\sqrt{\frac{1-(4/5)}{2}} = \frac{-1}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1-\cos u}{\sin u} = \frac{1+(4/5)}{-3/5} = -3$$

52. $\sec u = \dfrac{-7}{2},\ \dfrac{\pi}{2} < u < \pi$



$$\cos u = \frac{-2}{7},\ \sin u = \frac{3\sqrt{5}}{7}$$

$$\sin\frac{u}{2} = \sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1+(2/7)}{2}} = \sqrt{\frac{9}{14}} = \frac{3\sqrt{14}}{14}$$

$$\cos\frac{u}{2} = \sqrt{\frac{1+\cos u}{2}} = \sqrt{\frac{1-(2/7)}{2}} = \sqrt{\frac{5}{14}} = \frac{\sqrt{70}}{14}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{1+(2/7)}{3\left(\sqrt{5}/7\right)} = \frac{9}{3\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

53. $\sqrt{\dfrac{1-\cos 6x}{2}} = |\sin 3x|$

54. $\sqrt{\dfrac{1+\cos 4x}{2}} = \left|\cos\dfrac{4x}{2}\right| = |\cos 2x|$

55. $-\sqrt{\dfrac{1-\cos 8x}{1+\cos 8x}} = -\dfrac{\sqrt{(1-\cos 8x)/2}}{\sqrt{(1+\cos 8x)/2}}$

$$= -\left|\frac{\sin 4x}{\cos 4x}\right| = -|\tan 4x|$$

56. $-\sqrt{\dfrac{1-\cos(x-1)}{2}} = -\left|\sin\left(\dfrac{x-1}{2}\right)\right|$

57. $\sin\dfrac{x}{2} - \cos x = 0$

$$\pm\sqrt{\frac{1-\cos x}{2}} = \cos x$$

$$\frac{1-\cos x}{2} = \cos^2 x$$

$$0 = 2\cos^2 x + \cos x - 1$$

$$= (2\cos x - 1)(\cos x + 1)$$

$$\cos x = \frac{1}{2} \quad \text{or} \quad \cos x = -1$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3} \qquad x = \pi$$

By checking these values in the original equations, we see that $x = \pi/3$ and $x = 5\pi/3$ are the only solutions. $x = \pi$ is extraneous.

58. $h(x) = \sin\dfrac{x}{2} + \cos x - 1$

$$\sin\frac{x}{2} + \cos x - 1 = 0$$

$$\pm\sqrt{\frac{1-\cos x}{2}} = 1 - \cos x$$

$$\frac{1-\cos x}{2} = 1 - 2\cos x + \cos^2 x$$

$$1 - \cos x = 2 - 4\cos x + 2\cos^2 x$$

$$2\cos^2 x - 3\cos x + 1 = 0$$

$$(2\cos x - 1)(\cos x - 1) = 0$$

$$2\cos x - 1 = 0 \quad \text{or} \quad \cos x - 1 = 0$$

$$\cos x = \frac{1}{2} \qquad \cos x = 1$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3} \qquad x = 0$$

59. $\cos\dfrac{x}{2} - \sin x = 0$

$$\pm\sqrt{\frac{1+\cos x}{2}} = \sin x$$

$$\frac{1+\cos x}{2} = \sin^2 x$$

$$1 + \cos x = 2\sin^2 x$$

$$1 + \cos x = 2 - 2\cos^2 x$$

$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$$2\cos x - 1 = 0 \quad \text{or} \quad \cos x + 1 = 0$$

$$\cos x = \frac{1}{2} \qquad \cos x = -1$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3} \qquad x = \pi$$

$$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

$\pi/3$, π, and $5\pi/3$ are all solutions to the equation.

60. $g(x) = \tan\dfrac{x}{2} - \sin x$

$$\tan\frac{x}{2} - \sin x = 0$$

$$\frac{1-\cos x}{\sin x} = \sin x$$

$$1 - \cos x = \sin^2 x$$

$$1 - \cos x = 1 - \cos^2 x$$

$$\cos^2 x - \cos x = 0$$

$$\cos x(\cos x - 1) = 0$$

$$\cos x = 0 \quad \text{or} \quad \cos x - 1 = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \cos x = 1$$

$$x = 0$$

0, $\pi/2$, and $3\pi/2$ are all solutions to the equation.

61. $6\sin\dfrac{\pi}{3}\cos\dfrac{\pi}{3} = 6\cdot\dfrac{1}{2}\left[\sin\left(\dfrac{\pi}{3}+\dfrac{\pi}{3}\right) + \sin\left(\dfrac{\pi}{3}-\dfrac{\pi}{3}\right)\right] = 3\left[\sin\dfrac{2\pi}{3} + \sin 0\right] = 3\sin\dfrac{2\pi}{3}$

62. $4\sin\dfrac{\pi}{3}\cos\dfrac{5\pi}{6} = 4\cdot\dfrac{1}{2}\left[\sin\left(\dfrac{\pi}{3}+\dfrac{5\pi}{6}\right) + \sin\left(\dfrac{\pi}{3}-\dfrac{5\pi}{6}\right)\right] = 2\left[\sin\dfrac{7\pi}{6} + \sin\left(-\dfrac{\pi}{2}\right)\right] = 2\left(\sin\dfrac{7\pi}{6} - \sin\dfrac{\pi}{2}\right)$

63. $\sin 5\theta \cos 3\theta = \frac{1}{2}[\sin(5\theta + 3\theta) + \sin(5\theta - 3\theta)] = \frac{1}{2}(\sin 8\theta + \sin 2\theta)$

64. $5 \sin 3\alpha \sin 4\alpha = 5 \cdot \frac{1}{2}[\cos(3\alpha - 4\alpha) - \cos(3\alpha + 4\alpha)] = \frac{5}{2}[\cos(-\alpha) - \cos(7\alpha)] = \frac{5}{2}[\cos \alpha - \cos 7\alpha]$

65. $10 \cos 75^\circ \cos 15^\circ = 5[\cos(75^\circ - 15^\circ) + \cos(75^\circ + 15^\circ)] = 5[\cos 60^\circ + \cos 90^\circ]$

66. $6 \sin 45^\circ \cos 15^\circ = 3[\sin(45^\circ + 15^\circ) + \sin(45^\circ - 15^\circ) = 3[\sin 60^\circ + \sin 30^\circ]$

67. $5 \cos(-5\beta) \cos 3\beta = 5 \cdot \frac{1}{2}[\cos(-5\beta - 3\beta) + \cos(-5\beta + 3\beta)]$

$= \frac{5}{2}[\cos(-8\beta) + \cos(-2\beta)] = \frac{5}{2}(\cos 8\beta + \cos 2\beta)$

68. $\cos 2\theta \cos 4\theta = \frac{1}{2}[\cos(2\theta - 4\theta) + \cos(2\theta + 4\theta)] = \frac{1}{2}[\cos(-2\theta) + \cos 6\theta] = \frac{1}{2}(\cos 2\theta + \cos 6\theta)$

69. $\sin(x + y) \sin(x - y) = \frac{1}{2}[\cos((x + y) - (x - y)) - \cos((x + y) + (x - y))] = \frac{1}{2}[\cos 2y - \cos 2x]$

70. $\sin(x + y) \cos(x - y) = \frac{1}{2}[\sin((x + y) + (x - y)) + \sin((x + y) - (x - y))] = \frac{1}{2}[\sin 2x + \sin 2y]$

71. $\cos(\theta - \pi) \sin(\theta + \pi) = \frac{1}{2}[\sin((\theta - \pi) + (\theta + \pi)) - \sin((\theta - \pi) - (\theta + \pi))]$

$= \frac{1}{2}[\sin 2\theta - \sin(-2\pi)] = \frac{1}{2}[\sin 2\theta + \sin 2\pi]$

72. $\sin(\theta + \pi) \sin(\theta - \pi) = \frac{1}{2}[\cos((\theta + \pi) - (\theta - \pi)) - \cos((\theta + \pi) + (\theta - \pi))] = \frac{1}{2}[\cos 2\pi - \cos 2\theta]$

73. $\sin 5\theta - \sin \theta = 2 \cos\left(\frac{5\theta + \theta}{2}\right) \sin\left(\frac{5\theta - \theta}{2}\right)$

$= 2 \cos 3\theta \cdot \sin 2\theta$

74. $\sin 3\theta + \sin \theta = 2 \sin\left(\frac{3\theta + \theta}{2}\right) \cos\left(\frac{3\theta - \theta}{2}\right)$

$= 2 \sin 2\theta \cos \theta$

75. $\cos 6x + \cos 2x = 2 \cos\left(\frac{6x + 2x}{2}\right) \cos\left(\frac{6x - 2x}{2}\right)$

$= 2 \cos 4x \cos 2x$

76. $\sin x + \sin 7x = 2 \sin\left(\frac{x + 7x}{2}\right) \cos\left(\frac{x - 7x}{2}\right)$

$= 2 \sin 4x \cos(-3x)$

$= 2 \sin 4x \cos 3x$

77. $\sin(\alpha + \beta) - \sin(\alpha - \beta) = 2 \cos\left(\frac{\alpha + \beta + \alpha - \beta}{2}\right) \sin\left(\frac{\alpha + \beta - \alpha + \beta}{2}\right) = 2 \cos \alpha \sin \beta$

78. $\cos(\phi + 2\pi) + \cos \phi = 2 \cos\left(\frac{\phi + 2\pi + \phi}{2}\right) \cos\left(\frac{\phi + 2\pi - \phi}{2}\right) = 2 \cos(\phi + \pi) \cos \pi = -2 \cos(\phi + \pi)$

79. $\cos\left(\theta + \frac{\pi}{2}\right) - \cos\left(\theta - \frac{\pi}{2}\right) = -2 \sin\left(\frac{\theta + (\pi/2) + \theta - (\pi/2)}{2}\right) \sin\left(\frac{\theta + (\pi/2) - \theta + (\pi/2)}{2}\right)$

$= -2 \sin \theta \sin \frac{\pi}{2} = -2 \sin \theta$

80. $\sin\left(x + \frac{\pi}{2}\right) + \sin\left(x - \frac{\pi}{2}\right) = 2 \sin\left(\frac{x + (\pi/2) + x - (\pi/2)}{2}\right) \cos\left(\frac{x + (\pi/2) - x + (\pi/2)}{2}\right) = 2 \sin x \cos \frac{\pi}{2} = 0$

81. $\sin 195° + \sin 105° = 2\sin\left(\frac{195° + 105°}{2}\right)\cos\left(\frac{195° - 105°}{2}\right) = 2\sin(150°)\cos(45°) = 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}$

82. $\cos 165° - \cos 75° = -2\sin\left(\frac{165° + 75°}{2}\right)\sin\left(\frac{165° - 75°}{2}\right) = -2\sin(120°)\sin(45°)$

$$= -2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{6}}{2}$$

83. $\cos\frac{5\pi}{12} + \cos\frac{\pi}{12} = 2\cos\left(\frac{(5\pi/12) + (\pi/12)}{2}\right)\cos\left(\frac{(5\pi/12) - (\pi/12)}{2}\right)$

$$= 2\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) = 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{2\sqrt{6}}{4} = \frac{\sqrt{6}}{2}$$

84. $\sin\frac{11\pi}{2} - \sin\frac{7\pi}{12} = 2\cos\left(\frac{(11\pi/12) + (7\pi/12)}{2}\right)\sin\left(\frac{(11\pi/12) - (7\pi/12)}{2}\right)$

$$= 2\cos\left(\frac{3\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) = 2\left(\frac{-\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{-\sqrt{2}}{2}$$

85.

$$\sin 6x + \sin 2x = 0$$

$$2\sin\left(\frac{6x + 2x}{2}\right)\cos\left(\frac{6x - 2x}{2}\right) = 0$$

$$\sin 4x \cos 2x = 0$$

$\sin 4x = 0$ or $\cos 2x = 0$

$4x = n\pi$ $\quad 2x = \frac{\pi}{2} + n\pi$

$x = \frac{n\pi}{4}$ $\quad x = \frac{\pi}{4} + \frac{n\pi}{2}$

In the interval we have $x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$.

86.

$$h(x) = \cos 2x - \cos 6x$$

$$\cos 2x - \cos 6x = 0$$

$$-2\sin 4x \sin(-2x) = 0$$

$$2\sin 4x \sin 2x = 0$$

$\sin 4x = 0$ OR $\sin 2x = 0$

$4x = n\pi$ $\quad 2x = n\pi$

$x = \frac{n\pi}{4}$ $\quad x = \frac{n\pi}{2}$

$x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$

87. $\dfrac{\cos 2x}{\sin 3x - \sin x} - 1 = 0$

$$\frac{\cos 2x}{\sin 3x - \sin x} = 1$$

$$\frac{\cos 2x}{2\cos 2x \sin x} = 1$$

$$2\sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

88. $f(x) = \sin^2 3x - \sin^2 x$

$$\sin^2 3x - \sin^2 x = 0$$

$$(\sin 3x + \sin x)(\sin 3x - \sin x) = 0$$

$$(2\sin 2x \cos x)(2\cos 2x \sin x) = 0$$

$$\sin 2x = 0 \implies x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \quad \text{or}$$

$$\cos x = 0 \implies x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \text{or}$$

$$\cos 2x = 0 \implies x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \quad \text{or}$$

$$\sin x = 0 \implies x = 0, \pi$$

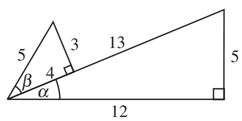


Figure for Exercises 89–92

89. $\sin^2 \alpha = \left(\frac{5}{13}\right)^2 = \frac{25}{169}$

$\sin^2 \alpha = 1 - \cos^2 \alpha = 1 - \left(\frac{12}{13}\right)^2 = 1 - \frac{144}{169} = \frac{25}{169}$

90. $\cos^2 \alpha = (\cos \alpha)^2 = \left(\frac{12}{13}\right)^2 = \frac{144}{169}$

$\cos^2 \alpha = 1 - \sin^2 \alpha = 1 - \left(\frac{5}{13}\right)^2 = 1 - \frac{25}{169} = \frac{144}{169}$

91. $\sin \alpha \cos \beta = \left(\dfrac{5}{13}\right)\left(\dfrac{4}{5}\right) = \dfrac{4}{13}$

$$\sin \alpha \cos \beta = \cos\left(\frac{\pi}{2} - \alpha\right) \sin\left(\frac{\pi}{2} - \beta\right)$$

$$= \left(\frac{5}{13}\right)\left(\frac{4}{5}\right) = \frac{4}{13}$$

92. $\cos \alpha \sin \beta = \left(\dfrac{12}{13}\right)\left(\dfrac{3}{5}\right) = \dfrac{36}{65}$

$$\cos \alpha \sin \beta = \sin\left(\frac{\pi}{2} - \alpha\right) \cos\left(\frac{\pi}{2} - \beta\right)$$

$$= \left(\frac{12}{13}\right)\left(\frac{3}{5}\right) = \frac{36}{65}$$

93. $\csc 2\theta = \dfrac{1}{\sin 2\theta}$

$$= \frac{1}{2\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta} \cdot \frac{1}{2\cos\theta}$$

$$= \frac{\csc\theta}{2\cos\theta}$$

94. $\sec 2\theta = \dfrac{1}{\cos 2\theta} = \dfrac{1}{\cos^2\theta - \sin^2\theta}$

$$= \frac{1/\cos^2\theta}{1 - (\sin^2\theta/\cos^2\theta)}$$

$$= \frac{\sec^2\theta}{1 - \tan^2\theta}$$

$$= \frac{\sec^2\theta}{1 - (\sec^2\theta - 1)}$$

$$= \frac{\sec^2\theta}{2 - \sec^2\theta}$$

95. $\cos^2 2\alpha - \sin^2 2\alpha = \cos[2(2\alpha)]$

$$= \cos 4\alpha$$

96. $\cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$

$$= (\cos 2x)(1) = \cos 2x$$

97. $(\sin x + \cos x)^2 = \sin^2 x + 2\sin x\cos x + \cos^2 x$

$$= (\sin^2 x + \cos^2 x) + 2\sin x \cos x = 1 + \sin 2x$$

98. $\sin\frac{\alpha}{3}\cos\frac{\alpha}{3} = \frac{1}{2}\left[\sin\left(\frac{\alpha}{3}+\frac{\alpha}{3}\right) + \sin\left(\frac{\alpha}{3}-\frac{\alpha}{3}\right)\right]$

$= \frac{1}{2}\sin\frac{2\alpha}{3}$

99. $1 + \cos 10y = 1 + \cos^2 5y - \sin^2 5y$

$= 1 + \cos^2 5y - (1 - \cos^2 5y)$

$= 2\cos^2 5y$

100. $\frac{\cos 3\beta}{\cos\beta} = \frac{\cos(2\beta + \beta)}{\cos\beta}$

$= \frac{\cos 2\beta\cos\beta - \sin 2\beta\sin\beta}{\cos\beta}$

$= \frac{(1 - 2\sin^2\beta)\cos\beta - 2\sin\beta\cos\beta\sin\beta}{\cos\beta}$

$= (1 - 2\sin^2\beta) - 2\sin^2\beta$

$= 1 - 4\sin^2\beta$

101. $\sec\frac{u}{2} = \frac{1}{\cos(u/2)}$

$= \pm\sqrt{\frac{2}{1 + \cos u}}$

$= \pm\sqrt{\frac{2\sin u}{\sin u(1 + \cos u)}}$

$= \pm\sqrt{\frac{2\sin u}{\sin u + \sin u\cos u}}$

$= \pm\sqrt{\frac{(2\sin u)/(\cos u)}{(\sin u)/(\cos u) + (\sin u\cos u)/(\cos u)}}$

$= \pm\sqrt{\frac{2\tan u}{\tan u + \sin u}}$

102. $\tan\frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{1}{\sin u} - \frac{\cos u}{\sin u} = \csc u - \cot u$

103. $\cos 3\beta = \cos(2\beta + \beta)$

$= \cos 2\beta\cos\beta - \sin 2\beta\sin\beta = (\cos^2\beta - \sin^2\beta)\cos\beta - 2\sin\beta\cos\beta\sin\beta$

$= \cos^3\beta - \sin^2\beta\cos\beta - 2\sin^2\beta\cos\beta = \cos^3\beta - 3\sin^2\beta\cos\beta$

104. $\sin 4\beta = 2\sin 2\beta\cos 2\beta$

$= 2[2\sin\beta\cos\beta(\cos^2\beta - \sin^2\beta)]$

$= 2[2\sin\beta\cos\beta(1 - \sin^2\beta - \sin^2\beta)]$

$= 4\sin\beta\cos\beta(1 - 2\sin^2\beta)$

105. $\frac{\cos 4x - \cos 2x}{2\sin 3x} = \frac{-2\sin\left(\frac{4x + 2x}{2}\right)\sin\left(\frac{4x - 2x}{2}\right)}{2\sin 3x}$

$= \frac{-2\sin 3x\sin x}{2\sin 3x}$

$= -\sin x$

106. $\frac{\cos 3x - \cos x}{\sin 3x - \sin x} = \frac{-2\sin\left(\frac{3x + x}{2}\right)\sin\left(\frac{3x - x}{2}\right)}{2\cos\left(\frac{3x + x}{2}\right)\sin\left(\frac{3x - x}{2}\right)}$

$= \frac{-2\sin 2x\sin x}{2\cos 2x\sin x} = -\tan 2x$

107. $\frac{\cos 4x + \cos 2x}{\sin 4x + \sin 2x} = \frac{2\cos(3x)\cos x}{2\sin(3x)\cos x}$

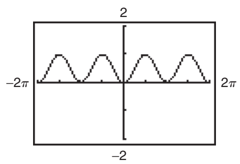
$= \cot 3x$

108. $\frac{\cos t + \cos 3t}{\sin 3t - \sin t} = \frac{2\cos 2t\cos(-t)}{2\cos 2t\sin t} = \frac{\cos t}{\sin t} = \cot t$

109. $\sin\left(\frac{\pi}{6} + x\right) + \sin\left(\frac{\pi}{6} - x\right) = 2\sin\left(\frac{\frac{\pi}{6} + x + \frac{\pi}{6} - x}{2}\right)\cos\left(\frac{\frac{\pi}{6} + x - \frac{\pi}{6} + x}{2}\right) = 2\sin\frac{\pi}{6}\cos x = \cos x$

110. $\cos\left(\frac{\pi}{3}+x\right)+\cos\left(\frac{\pi}{3}-x\right)=2\cos\left(\frac{\frac{\pi}{3}+x+\frac{\pi}{3}-x}{2}\right)\cos\left(\frac{\frac{\pi}{3}+x-\frac{\pi}{3}+x}{2}\right)=2\cos\frac{\pi}{3}\cos x=\cos x$

111. $\sin^2 x=\frac{1-\cos 2x}{2}=\frac{1}{2}-\frac{\cos 2x}{2}$

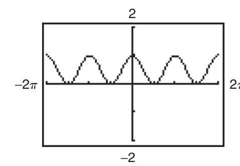


112. $f(x)=\cos^2 x=\frac{1+\cos 2x}{2}=\frac{1}{2}+\frac{\cos 2x}{2}$

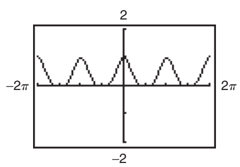
Shifted upward by $\frac{1}{2}$ unit.

Amplitude: $|a|=\frac{1}{2}$

Period: $\frac{2\pi}{2}=\pi$

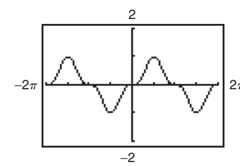


113. $f(x)=\cos^4 x=\frac{1}{8}(3+4\cos 2x+\cos 4x)$



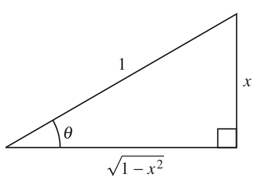
114. $f(x)=\sin^3 x$

$=\sin x\left(\frac{1-\cos 2x}{2}\right)$



115. $\sin(2\arcsin x)=2\sin(\arcsin x)\cos(\arcsin x)$

$=2x\sqrt{1-x^2}$

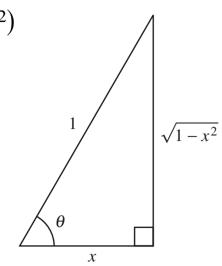


116. Let $\theta=\arccos x$.

$\cos(2\arccos x)=\cos^2(\arccos x)-\sin^2(\arccos x)$

$=x^2-(1-x^2)$

$=2x^2-1$



117. $\cos(2\arcsin x)=1-2\sin^2(\arcsin x)$

$=1-2x^2$

118. Let $u=\arccos x$.

$\sin(2\arccos x)=2\sin(\arccos x)\cos(\arccos x)$

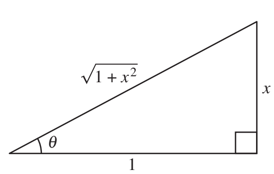
$=2\sqrt{1-x^2}(x)=2x\sqrt{1-x^2}$

119. $\cos(2\arctan x)=1-2\sin^2(\arctan x)$

$=1-2\left(\frac{x}{\sqrt{1+x^2}}\right)^2$

$=1-\frac{2x^2}{1+x^2}$

$=\frac{1-x^2}{1+x^2}$

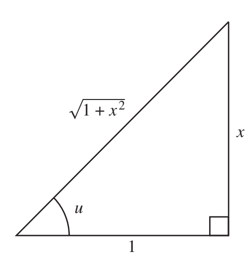


120. Let $u=\arctan x$.

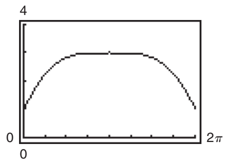
$\sin(2\arctan x)=2\sin(\arctan x)\cos(\arctan x)$

$=2\frac{x}{\sqrt{1+x^2}}\cdot\frac{1}{\sqrt{1+x^2}}$

$=\frac{2x}{1+x^2}$



121. (a) $y = 4\sin\frac{x}{2} + \cos x$

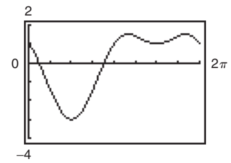


Maximum: $(\pi, 3)$

(b)
$$2\cos\frac{x}{2} - \sin x = 0$$
$$2\left(\pm\sqrt{\frac{1+\cos x}{2}}\right) = \sin x$$
$$4\left(\frac{1+\cos x}{2}\right) = \sin^2 x$$
$$2(1+\cos x) = 1 - \cos^2 x$$
$$\cos^2 x + 2\cos x + 1 = 0$$
$$(\cos x + 1)^2 = 0$$
$$\cos x = -1$$
$$x = \pi$$

122. $f(x) = \cos 2x - 2\sin x$

(a)



Maximum points: $(3.6652, 1.5)$, $(5.7596, 1.5)$

Minimum point: $(1.5708, -3)$

(b) $-2\cos x(2\sin x + 1) = 0$

$-2\cos x = 0$ or $2\sin x + 1 = 0$

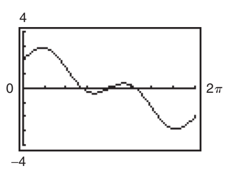
$\cos x = 0$ $\qquad$ $\sin x = -\frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad$ $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\frac{\pi}{2} \approx 1.5708$ $\qquad$ $\frac{7\pi}{6} \approx 3.6652$

$\frac{3\pi}{2} \approx 4.7124$ $\qquad$ $\frac{11\pi}{6} \approx 5.7596$

123. (a) $y = 2\cos\frac{x}{2} + \sin 2x$



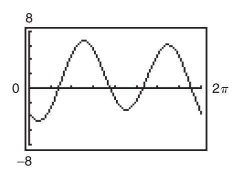
Maximum: $(0.699, 2.864)$

Minimum: $(5.584, -2.864)$

(b) $2\cos 2x - \sin(x/2) = 0$ has four zeros on $[0, 2\pi)$. Two of the zeros are $x \approx 0.699$ and $x \approx 5.584$. (The other two are 2.608, 3.675.)

124. $f(x) = 2\sin\frac{x}{2} - 5\cos\left(2x - \frac{\pi}{4}\right)$

(a)



Maximum point: $(1.9907, 6.6705)$

Minimum point: $(0.3434, -4.6340)$

(b) $10\sin\left(2x - \frac{\pi}{4}\right) + \cos\frac{x}{2} = 0$

$x \approx 0.343, 1.991, 3.544, 5.064$

The first and second solutions correspond to the maximum and minimum points in part (a).

125. (a) $f(x) = \sin 2x - \sin x = 0$

$2\sin x\cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0 \implies x = 0, \pi, 2\pi$

$\cos x = \frac{1}{2} \implies x = \frac{\pi}{3}, \frac{5\pi}{3}$

(b) $2\cos 2x - \cos x = 0$

$2(2\cos^2 x - 1) - \cos x = 0$

$4\cos^2 x - \cos x - 2 = 0$

$\cos x = \frac{1 \pm \sqrt{1 + 32}}{8} = \frac{1 \pm \sqrt{33}}{8}$

$x = \arccos\left(\frac{1 \pm \sqrt{33}}{8}\right)$

$x = 2\pi - \arccos\left(\frac{1 \pm \sqrt{33}}{8}\right)$

$x \approx 0.5678, 2.2057, 4.0775, 5.7154$

126. (a) $f(x) = \cos 2x + \sin x = 0$

$1 - 2\sin^2 x + \sin x = 0$

$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -\frac{1}{2} \implies x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\sin x = 1 \implies x = \frac{\pi}{2}$

(b) $-2\sin 2x + \cos x = 0$

$-4\sin x\cos x + \cos x = 0$

$\cos x(1 - 4\sin x) = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \arcsin\left(\frac{1}{4}\right), \pi - \arcsin\left(\frac{1}{4}\right)$

$x \approx 1.5708, 4.7124, 0.2527, 2.8889$

127. (a) $r = \frac{1}{32}v_0^2 \sin 2\theta$

$= \frac{1}{32}v_0^2(2\sin\theta\cos\theta)$

$= \frac{1}{16}v_0^2 \sin\theta\cos\theta$

(b) $r = \frac{1}{16}v_0^2 \sin\theta\cos\theta$

$= \frac{1}{16}(80)^2 \sin 42° \cos 42° \approx 198.90$ feet

(c) $r = \frac{1}{16}v_0^2 \sin\theta\cos\theta$

$300 = \frac{1}{16}v_0^2 \sin 40° \cos 40°$

$v_0^2 = \frac{300(16)}{\sin 40° \cos 40°} \approx 9748.0955$

$v_0 \approx 98.73$ feet per second

(d) $\sin 2\theta$ is greatest when $\theta = 45°$.

128. (a) $\sin\left(\frac{\theta}{2}\right) = \frac{b/2}{10} \implies b = 20\sin\frac{\theta}{2}$

$\cos\left(\frac{\theta}{2}\right) = \frac{h}{10} \implies h = 10\cos\frac{\theta}{2}$

$A = \frac{1}{2}bh = \frac{1}{2}\left(20\sin\frac{\theta}{2}\right)\left(10\cos\frac{\theta}{2}\right)$

$= 100\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

(b) $A = 50\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) = 50\sin\theta$

The area is maximum when $\theta = \frac{\pi}{2}, A = 50.$

129. $\frac{x}{2} = 2r\sin^2\frac{\theta}{2},\ x = 4r\left[\sin\frac{\theta}{2}\right]^2 = 4r\,\frac{1 - \cos\theta}{2} = 2r(1 - \cos\theta)$

130. $\sin\frac{\theta}{2} = \frac{1}{M}$

(a) $\sin\frac{\theta}{2} = \frac{1}{1} = 1 \Rightarrow \frac{\theta}{2} = \frac{\pi}{2} \Rightarrow \theta = \pi = 180°$

(b) $\sin\frac{\theta}{2} = \frac{1}{4.5} = \frac{2}{9}$

$\frac{\theta}{2} = \arcsin\left(\frac{2}{9}\right) \approx 0.2241$

$\theta \approx 0.4482 \approx 25.7°$

(c) $M = 1 \Rightarrow$ Speed $= 760$ mph

$M = 4.5 \Rightarrow \frac{\text{Speed}}{760} = 4.5 \Rightarrow$ Speed $= 3420$ mph

(d) $\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}} = \frac{1}{M}$

$\frac{1-\cos\theta}{2} = \frac{1}{M^2}$

$1 - \cos\theta = \frac{2}{M^2}$

$\cos\theta = 1 - \frac{2}{M^2}$

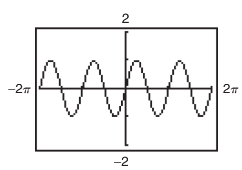
131. False. If $x = \pi$, $\sin\frac{x}{2} = \sin\frac{\pi}{2} = 1$, whereas

$-\sqrt{\frac{1-\cos\pi}{2}} = -1.$

132. True. $\sin(\pi) = 0$ and $y = 4 - 8\sin^2\pi = 4$, a maximum.

133. $f(x) = 2\sin x\left[2\cos^2\left(\frac{x}{2}\right) - 1\right]$

(a)



(b) The graph appears to be that of $y = \sin 2x$.

(c) $2\sin x\left[2\cos^2\left(\frac{x}{2}\right) - 1\right] = 2\sin x\left[2\,\frac{1+\cos x}{2} - 1\right]$

$= 2\sin x[\cos x] = \sin 2x$

134. (a) $f(x) = \sin^4 x + \cos^4 x$. From Example 5 and Exercise 23,

$f(x) = \frac{1}{8}(3 - 4\cos 2x + \cos 4x) + \frac{1}{8}(3 + 4\cos 2x + \cos 4x) = \frac{3}{4} + \frac{1}{4}\cos 4x$

(b) Sample answer:

$f(x) = \sin^2 x \cdot \sin^2 x + \cos^2 x \cdot \cos^2 x = \left(\frac{1-\cos 2x}{2}\right)^2 + \left(\frac{1+\cos 2x}{2}\right)^2$

$= \frac{1}{4}[2 + 2\cos^2 2x] = \frac{1}{2}(1 + \cos^2 2x)$

(c) $f(x) = \sin^4 x + 2\sin^2 x\cos^2 x + \cos^4 x - 2\sin^2 x\cos^2 x$

$= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x = 1 - 2\sin^2 x\cos^2 x$

(d) $f(x) = 1 - \frac{1}{2}\sin^2 2x$

(e) Answers will vary.

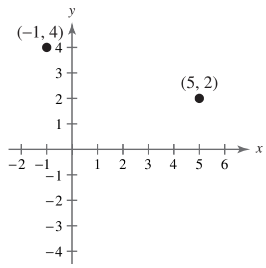
135. Answers will vary.

136. (a) Sample answer: $\cos(3\theta) = \cos(2\theta + \theta) = \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$

$= (\cos^2\theta - \sin^2\theta)\cos\theta - 2\sin\theta\cos\theta\sin\theta = \cos^3\theta - 3\sin^2\theta\cos\theta$

(b) Sample answer: $\cos 4\theta = 2\cos^2 2\theta - 1 = 2(2\cos^2\theta - 1)^2 - 1 = 8\cos^4\theta - 8\cos^2\theta + 1$

137. (a)

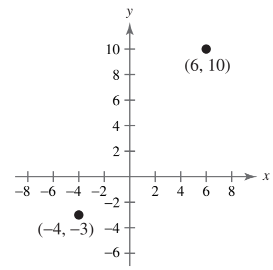


(b) Distance:

$\sqrt{(5+1)^2 + (2-4)^2} = \sqrt{40} = 2\sqrt{10}$

(c) Midpoint: $\left(\frac{-1+5}{2}, \frac{4+2}{2}\right) = (2, 3)$

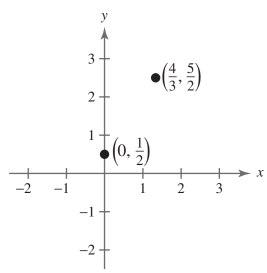
138. (a)



(b) Distance: $\sqrt{(6+4)^2 + (10+3)^2} = \sqrt{269}$

(c) Midpoint: $\left(\frac{6-4}{2}, \frac{10-3}{2}\right) = \left(1, \frac{7}{2}\right)$

139. (a)



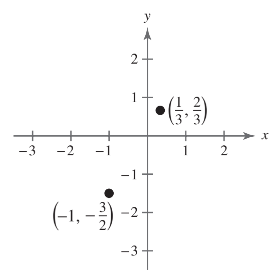
(b) Distance:

$\sqrt{\left(\frac{4}{3}\right)^2 + \left(\frac{5}{2} - \frac{1}{2}\right)^2} = \sqrt{\frac{16}{9} + 4}$

$= \frac{\sqrt{52}}{3} = \frac{2\sqrt{13}}{3}$

(c) Midpoint: $\left(\frac{0 + (4/3)}{2}, \frac{(1/2) + (5/2)}{2}\right) = \left(\frac{2}{3}, \frac{3}{2}\right)$

140. (a)



(b) Distance: $\sqrt{\left(\frac{1}{3} + 1\right)^2 + \left(\frac{2}{3} + \frac{3}{2}\right)^2} = \frac{\sqrt{233}}{6}$

(c) Midpoint:

$\left(\frac{(1/3) - 1}{2}, \frac{(2/3) - (3/2)}{2}\right) = \left(-\frac{1}{3}, -\frac{5}{12}\right)$

141. (a) Complement: $90° - 55° = 35°$

Supplement: $180° - 55° = 125°$

(b) Complement: None

Supplement: $180° - 162° = 18°$

142. (a) Complement: None

Supplement: $180° - 109° = 71°$

(b) Complement: $90° - 78° = 12°$

Supplement: $180° - 78° = 102°$

143. (a) Complement: $\frac{\pi}{2} - \frac{\pi}{18} = \frac{8\pi}{18} = \frac{4\pi}{9}$

Supplement: $\pi - \frac{\pi}{18} = \frac{17\pi}{18}$

(b) Complement: $\frac{\pi}{2} - \frac{9\pi}{20} = \frac{\pi}{20}$

Supplement: $\pi - \frac{9\pi}{20} = \frac{11\pi}{20}$

144. (a) Complement: $\frac{\pi}{2} - 0.95 \approx 0.6208$

Supplement: $\pi - 0.95 \approx 2.1916$

(b) Complement: None

Supplement: $\pi - 2.76 \approx 0.3816$

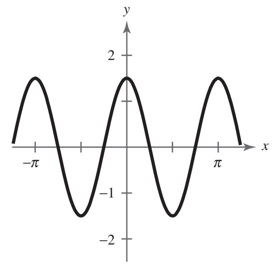
145. $s = r\theta \implies \theta = \dfrac{s}{r} = \dfrac{7}{15} \approx 0.467$ rad

146. $s = r\,\theta = 21(35°)\left(\dfrac{\pi}{180°}\right) \approx 12.8282$ cm

147. $f(x) = \dfrac{3}{2}\cos(2x)$

Period: $\dfrac{2\pi}{2} = \pi$

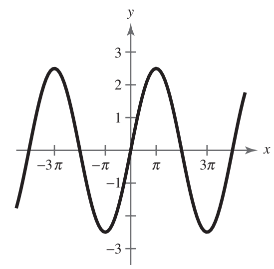
Amplitude: $\dfrac{3}{2}$



148. $f(x) = \dfrac{5}{2}\sin\dfrac{x}{2}$

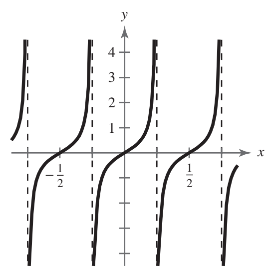
Period: 4π

Amplitude: $\dfrac{5}{2}$



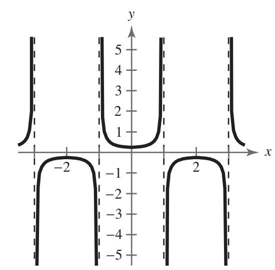
149. $f(x) = \dfrac{1}{2}\tan(2\pi x)$

Period: $\dfrac{\pi}{2\pi} = \dfrac{1}{2}$



150. $f(x) = \dfrac{1}{4}\sec\dfrac{\pi x}{2}$

Period: $\dfrac{2\pi}{\pi/2} = 4$



Review Exercises for Chapter 5

1. $\dfrac{1}{\cos x} = \sec x$

2. $\dfrac{1}{\sin x} = \csc x$

3. $\dfrac{1}{\sec x} = \cos x$

4. $\dfrac{1}{\tan x} = \cot x$

5. $\sqrt{1 - \cos^2 x} = |\sin x|$

6. $\sqrt{1 + \tan^2 x} = |\sec x|$

7. $\csc\left(\dfrac{\pi}{2} - x\right) = \sec x$

8. $\cot\left(\dfrac{\pi}{2} - x\right) = \tan x$

9. $\sec(-x) = \sec x$

10. $\tan(-x) = -\tan x$

11. $\sin x = \dfrac{4}{5}, \cos x = \dfrac{3}{5}$, Quadrant I

$\tan x = \dfrac{\sin x}{\cos x} = \dfrac{4}{3}$

$\cot x = \dfrac{3}{4}$

$\sec x = \dfrac{5}{3}$

$\csc x = \dfrac{5}{4}$

12. $\tan\theta = \dfrac{2}{3}, \sec\theta = \dfrac{\sqrt{13}}{3}$, Quadrant I

$\cos\theta = \dfrac{3\sqrt{13}}{13}$

$\sin\theta = \tan\theta\cos\theta = \dfrac{2}{3}\left(\dfrac{3\sqrt{13}}{13}\right) = \dfrac{2\sqrt{13}}{13}$

$\csc\theta = \dfrac{\sqrt{13}}{2}$

$\cot\theta = \dfrac{3}{2}$

13. $\sin\left(\frac{\pi}{2} - x\right) = \cos x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2},$

$\sin x = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2},$ Quadrant IV

$\tan x = -1$

$\cot x = -1$

$\sec x = \sqrt{2}$

$\csc x = -\sqrt{2}$

14. $\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta = 3, \sin\theta = \frac{2\sqrt{2}}{3},$ Quadrant I

$\cos\theta = \frac{1}{3}$

$\tan\theta = 2\sqrt{2}$

$\cot\theta = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

$\csc\theta = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$

15. $\frac{1}{\tan^2 x + 1} = \frac{1}{\sec^2 x} = \cos^2 x$

16. $\frac{\sec^2 x - 1}{\sec x - 1} = \frac{(\sec x - 1)(\sec x + 1)}{\sec x - 1} = \sec x + 1$

17. $\frac{\sin^2\alpha - \cos^2\alpha}{\sin^2\alpha - \sin\alpha\cos\alpha} = \frac{(\sin\alpha + \cos\alpha)(\sin\alpha - \cos\alpha)}{\sin\alpha(\sin\alpha - \cos\alpha)} = \frac{\sin\alpha + \cos\alpha}{\sin\alpha} = 1 + \cot\alpha$

18. $\frac{\sin^3\beta + \cos^3\beta}{\sin\beta + \cos\beta} = \frac{(\sin\beta + \cos\beta)(\sin^2\beta - \sin\beta\cos\beta + \cos^2\beta)}{\sin\beta + \cos\beta} = 1 - \sin\beta\cos\beta$

19. $\tan^2\theta(\csc^2\theta - 1) = \tan^2\theta(\cot^2\theta)$

$= \tan^2\theta\left(\frac{1}{\tan^2\theta}\right) = 1$

20. $\csc^2 x(1 - \cos^2 x) = \csc^2 x(\sin^2 x) = 1$

21. $\tan\left(\frac{\pi}{2} - x\right)\sec x = \cot x \sec x$

$= \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} = \frac{1}{\sin x} = \csc x$

22. $\frac{\sin(-x)\cot x}{\sin\left(\frac{\pi}{2} - x\right)} = \frac{(-\sin x)\left(\frac{\cos x}{\sin x}\right)}{\cos x} = -1$

23. $\sin^{-1/2} x \cos x = \frac{\cos x}{\sin^{1/2} x}$

$= \frac{\cos x}{\sqrt{\sin x}} \cdot \frac{\sqrt{\sin x}}{\sqrt{\sin x}}$

$= \frac{\cos x}{\sin x}\sqrt{\sin x} = \cot x\sqrt{\sin x}$

24. $\csc^2 x - \csc x \cot x = \frac{1}{\sin^2 x} - \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}$

$= \frac{1 - \cos x}{\sin^2 x}$

25. $\cos x(\tan^2 x + 1) = \cos x \sec^2 x$

$= \frac{1}{\sec x}\sec^2 x = \sec x$

26. $\sec^2 x \cot x - \cot x = \cot x(\sec^2 x - 1)$

$= \cot x \tan^2 x$

$= \frac{1}{\tan x}\tan^2 x = \tan x$

27. $\sin^3\theta + \sin\theta\cos^2\theta = \sin\theta(\sin^2\theta + \cos^2\theta)$

$= \sin\theta$

28. $\cot^2 x - \cos^2 x = \frac{\cos^2 x}{\sin^2 x} - \cos^2 x$

$= \cos^2 x[\csc^2 x - 1]$

$= \cos^2 x \cdot \cot^2 x$

29. $\sin^5 x\cos^2 x = \sin^4 x\cos^2 x\sin x$

$= (1 - \cos^2 x)^2\cos^2 x\sin x$

$= (1 - 2\cos^2 x + \cos^4 x)\cos^2 x\sin x$

$= (\cos^2 x - 2\cos^4 x + \cos^6 x)\sin x$

30. $\cos^3 x\sin^2 x = \cos x(\cos^2 x)\sin^2 x$

$= \cos x(1 - \sin^2 x)\sin^2 x$

$= (\sin^2 x - \sin^4 x)\cos x$

31. $\sqrt{\dfrac{1 - \sin\theta}{1 + \sin\theta}} = \sqrt{\dfrac{1 - \sin\theta}{1 + \sin\theta}\cdot\dfrac{1 - \sin\theta}{1 - \sin\theta}} = \sqrt{\dfrac{(1 - \sin\theta)^2}{1 - \sin^2\theta}} = \sqrt{\dfrac{(1 - \sin\theta)^2}{\cos^2\theta}} = \dfrac{|1 - \sin\theta|}{|\cos\theta|} = \dfrac{1 - \sin\theta}{|\cos\theta|}$

Note: We can drop the absolute value on $1 - \sin\theta$ since it is always nonnegative.

32. $\sqrt{1 - \cos x} = \sqrt{(1 - \cos x)\dfrac{1 + \cos x}{1 + \cos x}}$

$= \sqrt{\dfrac{\sin^2 x}{1 + \cos x}} = \dfrac{|\sin x|}{\sqrt{1 + \cos x}}$

33. $\dfrac{\csc(-x)}{\sec(-x)} = -\dfrac{\csc x}{\sec x} = -\dfrac{\cos x}{\sin x} = -\cot x$

34. $\dfrac{1 + \sec(-x)}{\sin(-x) + \tan(-x)} = \dfrac{1 + \sec x}{-\sin x - \tan x}$

$= \dfrac{1 + \sec x}{-\sin x(1 + \sec x)}$

$= -\dfrac{1}{\sin x} = -\csc x$

35. $\csc^2\left(\dfrac{\pi}{2} - x\right) - 1 = \sec^2 x - 1 = \tan^2 x$

36. $\tan\left(\dfrac{\pi}{2} - x\right)\sec x = \cot x\sec x$

$= \dfrac{\cos x}{\sin x}\cdot\dfrac{1}{\cos x}$

$= \dfrac{1}{\sin x} = \csc x$

37. $2\sin x - 1 = 0$

$\sin x = \dfrac{1}{2}$

$x = \dfrac{\pi}{6} + 2n\pi$

$x = \dfrac{5\pi}{6} + 2n\pi$

38. $\tan x + 1 = 0$

$\tan x = -1$

$x = \dfrac{3\pi}{4} + n\pi$

39. $\sin x = \sqrt{3} - \sin x$

$2\sin x = \sqrt{3}$

$\sin x = \dfrac{\sqrt{3}}{2}$

$x = \dfrac{\pi}{3} + 2n\pi$

$x = \dfrac{2\pi}{3} + 2n\pi$

40. $4\cos x = 1 + 2\cos x$

$2\cos x = 1$

$\cos x = \dfrac{1}{2}$

$x = \dfrac{\pi}{3} + 2n\pi$

$x = \dfrac{5\pi}{3} + 2n\pi$

41. $3\sqrt{3}\tan x = 3$

$\tan x = \dfrac{1}{\sqrt{3}}$

$x = \dfrac{\pi}{6} + n\pi$

42. $\frac{1}{2}\sec x - 1 = 0$

$$\sec x = 2$$
$$\cos x = \frac{1}{2}$$
$$x = \frac{\pi}{3} + 2n\pi$$
$$x = \frac{5\pi}{3} + 2n\pi$$

43. $3\csc^2 x = 4$

$$\csc^2 x = \frac{4}{3}$$
$$\sin^2 x = \frac{3}{4}$$
$$\sin x = \pm\frac{\sqrt{3}}{2}$$
$$x = \frac{\pi}{3} + n\pi$$
$$x = \frac{2\pi}{3} + n\pi$$

44. $4\tan^2 x - 1 = \tan^2 x$

$$3\tan^2 x = 1$$
$$\tan^2 x = \frac{1}{3}$$
$$\tan x = \pm\frac{1}{\sqrt{3}}$$
$$x = \frac{\pi}{6} + n\pi$$
$$x = \frac{5\pi}{6} + n\pi$$

45. $4\cos^2 x - 3 = 0$

$$\cos^2 x = \frac{3}{4}$$
$$\cos x = \pm\frac{\sqrt{3}}{2}$$
$$x = \frac{\pi}{6} + n\pi$$
$$x = \frac{5\pi}{6} + n\pi$$

46. $\sin x(\sin x + 1) = 0$

$\sin x = 0$ or $\sin x = -1$

$x = n\pi$ or $x = \frac{3\pi}{2} + 2n\pi$

47. $\sin x - \tan x = 0$

$$\sin x - \frac{\sin x}{\cos x} = 0$$
$$\sin x\cos x - \sin x = 0$$
$$\sin x(\cos x - 1) = 0$$

$\sin x = 0$ or $\cos x - 1 = 0$

$x = n\pi$ $\quad\cos x = 1$

48. $\csc x - 2\cot x = 0$

$$\frac{1}{\sin x}(1 - 2\cos x) = 0$$
$$\cos x = \frac{1}{2}$$
$$x = \frac{\pi}{3} + 2n\pi$$
$$x = \frac{5\pi}{3} + 2n\pi$$

49. $2\cos^2 x - \cos x - 1 = 0$

$$(2\cos x + 1)(\cos x - 1) = 0$$

$2\cos x + 1 = 0$ or $\cos x - 1 = 0$

$\cos x = -\frac{1}{2}$ $\quad\cos x = 1$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\quad x = 0$

50. $2\sin^2 x - 3\sin x + 1 = 0$

$$(2\sin x - 1)(\sin x - 1) = 0$$

$\sin x = \frac{1}{2}$ or $\sin x = 1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ or $x = \frac{\pi}{2}$

51.
$$\cos^2 x + \sin x = 1$$
$$1 - \sin^2 x + \sin x = 1$$
$$\sin x(\sin x - 1) = 0$$
$$\sin x = 0 \quad \text{or} \quad \sin x = 1$$
$$x = 0, \pi \qquad x = \frac{\pi}{2}$$

52.
$$\sin^2 x + 2\cos x = 2$$
$$(1 - \cos^2 x) + 2\cos x - 2 = 0$$
$$\cos^2 x - 2\cos x + 1 = 0$$
$$(\cos x - 1)^2 = 0$$
$$\cos x = 1$$
$$x = 0$$

53. $2\sin 2x = \sqrt{2}$
$$\sin 2x = \frac{\sqrt{2}}{2}$$
$$2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$$
$$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$$

54. $\sqrt{3}\tan 3x = 0$
$$\tan 3x = 0$$
$$3x = k\pi$$
$$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$$

55. $\cos 4x(\cos x - 1) = 0$

$\cos 4x = 0$ or $\cos x - 1 = 0$
$$4x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}, \frac{13\pi}{2}, \frac{15\pi}{2}$$
or $\cos x = 1$
$$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}, 0$$

56. $3\csc^2 5x = -4$
$$\csc^2 5x = -\frac{4}{3}$$
No solutions

57.
$$\cos 4x - 7\cos 2x = 8$$
$$2\cos^2 2x - 1 - 7\cos 2x = 8$$
$$2\cos^2 2x - 7\cos 2x - 9 = 0$$
$$(2\cos 2x - 9)(\cos 2x + 1) = 0$$
$$2\cos 2x - 9 = 0 \quad \text{or} \quad \cos 2x + 1 = 0$$
$$\cos 2x = \frac{9}{2} \qquad \cos 2x = -1$$
No solution
$$2x = \pi + 2n\pi$$
$$x = \frac{\pi}{2} + n\pi$$
$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

58. $\sin 4x - \sin 2x = 0$
$$2\cos 3x \sin x = 0$$
$$\cos 3x = 0 \quad \text{or} \quad \sin x = 0$$
$$3x = \frac{\pi}{2} + n\pi \qquad x = 0, \pi$$
$$x = \frac{\pi}{6} + \frac{n\pi}{3}$$
$$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$$

59. $2\sin 2x - 1 = 0$

$$\sin 2x = \frac{1}{2}$$

$$2x = \frac{\pi}{6} + 2n\pi \quad \text{or} \quad 2x = \frac{5\pi}{6} + 2n\pi$$

$$x = \frac{\pi}{12} + n\pi \quad \text{or} \quad x = \frac{5\pi}{12} + n\pi$$

60. $2\cos 4x + \sqrt{3} = 0$

$$\cos 4x = \frac{-\sqrt{3}}{2}$$

$$4x = \frac{5\pi}{6} + 2n\pi \quad \text{or} \quad 4x = \frac{7\pi}{6} + 2n\pi$$

$$x = \frac{5\pi}{24} + \frac{n\pi}{2} \quad \text{or} \quad x = \frac{7\pi}{24} + \frac{n\pi}{2}$$

61. $2\sin^2 3x - 1 = 0$

$$\sin^2 3x = \frac{1}{2}$$

$$\sin 3x = \pm\frac{\sqrt{2}}{2}$$

$$3x = \frac{\pi}{4} + \frac{n\pi}{2}$$

$$x = \frac{\pi}{12} + \frac{n\pi}{6}$$

62. $4\cos^2 2x - 3 = 0$

$$\cos^2 2x = \frac{3}{4}$$

$$\cos 2x = \pm\frac{\sqrt{3}}{2}$$

$$2x = \frac{\pi}{6} + n\pi \quad \text{or} \quad 2x = \frac{5\pi}{6} + n\pi$$

$$x = \frac{\pi}{12} + \frac{n\pi}{2} \quad \text{or} \quad x = \frac{5\pi}{12} + \frac{n\pi}{2}$$

63. $\sin^2 x - 2\sin x = 0$

$\sin x(\sin x - 2) = 0$

$\sin x = 0$ or $\sin x = 2$ (impossible)

$x = 0, \pi$

64. $3\cos^2 x + 5\cos x = 0$

$\cos x(3\cos x + 5) = 0$

$\cos x = 0 \implies x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos x = -\frac{5}{3}$ (impossible)

65. $\tan^2\theta + 3\tan\theta - 10 = 0$

$(\tan\theta + 5)(\tan\theta - 2) = 0$

$\tan\theta = -5 \implies$

$\theta = \arctan(-5) + \pi, \arctan(-5) + 2\pi$

$\tan\theta = 2 \implies \theta = \arctan(2), \arctan(2) + \pi$

$\theta \approx 1.1071, 1.7682, 4.2487, 4.9098$

66. $\sec^2 x + 6\tan x + 4 = 0$

$(1 + \tan^2 x) + 6\tan x + 4 = 0$

$\tan^2 x + 6\tan x + 5 = 0$

$(\tan x + 1)(\tan x + 5) = 0$

$\tan x = -1$ or $\tan x = -5$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$ or $x \approx 1.7682, 4.9098$

67. $\sin 285° = \sin(315° - 30°)$

$$= \sin 315°\cos 30° - \cos 315°\sin 30°$$

$$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = -\frac{\sqrt{6} + \sqrt{2}}{4}$$

$\cos 285° = \cos(315° - 30°) = \cos 315°\cos 30° + \sin 315°\sin 30°$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\tan 285° = -\frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}} = -2 - \sqrt{3}$$

68. $\sin 345° = \sin(300° + 45°) = \sin 300° \cos 45° + \cos 300° \sin 45° = -\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$

$\cos 345° = \cos(300° + 45°) = \cos 300° \cos 45° - \sin 300° \sin 45° = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{2} + \sqrt{6}}{4}$

$\tan 345° = \frac{\sin 345°}{\cos 345°} = \frac{\sqrt{2} - \sqrt{6}}{\sqrt{2} + \sqrt{6}} = \sqrt{3} - 2$

69. $\sin \frac{31\pi}{12} = \sin\left(\frac{11\pi}{6} + \frac{3\pi}{4}\right) = \sin \frac{11\pi}{6} \cos \frac{3\pi}{4} + \sin \frac{3\pi}{4} \cos \frac{11\pi}{6}$

$= \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2} + \sqrt{6}}{4}$

$\cos \frac{31\pi}{12} = \cos\left(\frac{11\pi}{6} + \frac{3\pi}{4}\right) = \cos \frac{11\pi}{6} \cos \frac{3\pi}{4} - \sin \frac{11\pi}{6} \sin \frac{3\pi}{4}$

$= \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2} - \sqrt{6}}{4}$

$\tan \frac{31\pi}{12} = \frac{\sin(31\pi/12)}{\cos(31\pi/12)} = \frac{\sqrt{2} + \sqrt{6}}{\sqrt{2} - \sqrt{6}} = -2 - \sqrt{3}$

70. $\sin\left(\frac{13\pi}{12}\right) = \sin\left(\frac{11\pi}{6} - \frac{3\pi}{4}\right) = \sin \frac{11\pi}{6} \cos \frac{3\pi}{4} - \sin \frac{3\pi}{4} \cos \frac{11\pi}{6}$

$= \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2} - \sqrt{6}}{4}$

$\cos\left(\frac{13\pi}{12}\right) = \cos\left(\frac{11\pi}{6} - \frac{3\pi}{4}\right) = \cos \frac{11\pi}{6} \cos \frac{3\pi}{4} + \sin \frac{11\pi}{6} \sin \frac{3\pi}{4}$

$= \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{6} - \sqrt{2}}{4}$

$\tan\left(\frac{13\pi}{12}\right) = \frac{\sin(13\pi/12)}{\cos(13\pi/12)} = \frac{\sqrt{2} - \sqrt{6}}{-\sqrt{6} - \sqrt{2}} = 2 - \sqrt{3}$

71. $\sin 130° \cos 50° + \cos 130° \sin 50° = \sin(130° + 50°) = \sin 180° = 0$

72. $\cos 45° \cos 120° - \sin 45° \sin 120° = \cos(45° + 120°) = \cos(165°)$

73. $\frac{\tan 25° + \tan 10°}{1 - \tan 25° \tan 10°} = \tan(25° + 10°) = \tan 35°$

74. $\frac{\tan 63° - \tan 118°}{1 + \tan 63° \tan 118°} = \tan(63° - 118°) = \tan(-55°) = -\tan(55°)$

For Exercises 75–80, $\sin u = \frac{3}{5}$, $\cos v = -\frac{7}{25}$, $\cos u = -\frac{4}{5}$, $\sin v = \frac{24}{25}$.

75. $\sin(u + v) = \sin u \cos v + \sin v \cos u$

$= \frac{3}{5}\left(-\frac{7}{25}\right) + \frac{24}{25}\left(-\frac{4}{5}\right) = \frac{-117}{125}$

76. $\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$

$= \frac{(-3/4) + (-24/7)}{1 - (-3/4)(-24/7)} = \frac{117}{44}$

77. $\tan(u - v) = \dfrac{\tan u - \tan v}{1 + \tan u \tan v}$

$= \dfrac{(-3/4) - (-24/7)}{1 + (-3/4)(-24/7)} = \dfrac{3}{4}$

78. $\sin(u - v) = \sin u \cos v - \cos u \sin v$

$= \dfrac{3}{5}\left(-\dfrac{7}{25}\right) - \left(-\dfrac{4}{5}\right)\left(\dfrac{24}{25}\right) = \dfrac{3}{5}$

79. $\cos(u + v) = \cos u \cos v - \sin u \sin v$

$= \left(-\frac{4}{5}\right)\left(-\frac{7}{25}\right) - \left(\frac{3}{5}\right)\left(\frac{24}{25}\right) = -\frac{44}{125}$

80. $\cos(u - v) = \cos u \cos v + \sin u \sin v$

$= \left(-\frac{4}{5}\right)\left(-\frac{7}{25}\right) + \left(\frac{3}{5}\right)\left(\frac{24}{25}\right) = \frac{4}{5}$

81. $\sin^{-1} 0 = 0$ and $\cos^{-1}(-1) = \pi$

$\sin(\sin^{-1} 0 + \cos^{-1}(-1)) = \sin(0 + \pi) = 0$

82. $\cos^{-1} 1 = 0$ and $\sin^{-1} 0 = 0$

$\cos(\cos^{-1} 1 + \sin^{-1} 0) = \cos(0 + 0) = 1$

83. $\cos^{-1} 1 = 0$ and $\sin^{-1}(-1) = -\dfrac{\pi}{2}$

$\cos(\cos^{-1} 1 - \sin^{-1}(-1)) = \cos\left(0 + \dfrac{\pi}{2}\right) = 0$

84. $\cos^{-1}(-1) = \pi$ and $\cos^{-1} 1 = 0$

$\tan(\cos^{-1}(-1) + \cos^{-1} 1) = \tan(\pi + 0) = 0$

85. $\cos\left(x + \dfrac{\pi}{2}\right) = \cos x \cos \dfrac{\pi}{2} - \sin x \sin \dfrac{\pi}{2}$

$= (\cos x)(0) - (\sin x)(1) = -\sin x$

86. $\sin\left(x - \dfrac{3\pi}{2}\right) = \sin x \cos \dfrac{3\pi}{2} - \sin \dfrac{3\pi}{2} \cos x$

$= (\sin x)(0) - (-1)(\cos x) = \cos x$

87. $\cot\left(\dfrac{\pi}{2} - x\right) = \dfrac{\cos[(\pi/2) - x]}{\sin[(\pi/2) - x]}$

$= \dfrac{\cos(\pi/2) \cos x + \sin(\pi/2) \sin x}{\sin(\pi/2) \cos x - \sin x \cos(\pi/2)}$

$= \dfrac{\sin x}{\cos x} = \tan x$

88. $\sin(\pi - x) = \sin \pi \cos x - \sin x \cos \pi$

$= (0)(\cos x) - (\sin x)(-1)$

$= \sin x$

89. $\cos 3x = \cos(2x + x)$

$= \cos 2x \cos x - \sin 2x \sin x$

$= (\cos^2 x - \sin^2 x) \cos x - 2 \sin x \cos x \sin x$

$= \cos^3 x - 3 \sin^2 x \cos x$

$= \cos^3 x - 3 \cos x(1 - \cos^2 x)$

$= \cos^3 x - 3 \cos x + 3 \cos^3 x$

$= 4 \cos^3 x - 3 \cos x$

90. $\dfrac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta} = \dfrac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta}$

$= \dfrac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \dfrac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}$

$= \tan \alpha + \tan \beta$

91. $\sin\left(x + \dfrac{\pi}{2}\right) - \sin\left(x - \dfrac{\pi}{2}\right) = \sqrt{2}$

$2 \cos x \sin \dfrac{\pi}{2} = \sqrt{2}$ (Sum-to-Product)

$\cos x = \dfrac{\sqrt{2}}{2}$

$x = \dfrac{\pi}{4}, \dfrac{7\pi}{4}$

92. $\cos\left(x + \dfrac{\pi}{4}\right) - \cos\left(x - \dfrac{\pi}{4}\right) = 1$

$-2 \sin x \sin \dfrac{\pi}{4} = 1$ (Sum-to-Product)

$\sin x = -\dfrac{\sqrt{2}}{2}$

$x = \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$

93. $\sin u = -\frac{5}{7}, \pi < u < \frac{3\pi}{2},$ Quadrant III

$\cos^2 u = 1 - \left(-\frac{5}{7}\right)^2 = \frac{24}{49} \implies \cos u = -\frac{2\sqrt{6}}{7}$

$\sin 2u = 2 \sin u \cos u = 2\left(-\frac{5}{7}\right)\left(-\frac{2\sqrt{6}}{7}\right) = \frac{20\sqrt{6}}{49}$

$\cos 2u = 1 - 2\sin^2 u$

$= 1 - 2\left(-\frac{5}{7}\right)^2 = 1 - \frac{50}{49} = -\frac{1}{49}$

$\tan 2u = \frac{\sin 2u}{\cos 2u} = \frac{20\sqrt{6}}{-1} = -20\sqrt{6}$

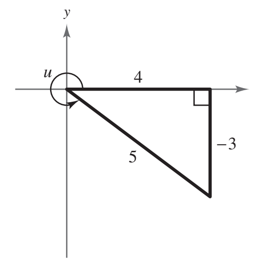
94. $\cos u = \frac{4}{5}, \frac{3\pi}{2} < u < 2\pi,$ Quadrant IV

$\sin u = \frac{-3}{5}, \tan u = \frac{-3}{4}$

$\sin 2u = 2 \sin u \cos u = 2\left(\frac{-3}{5}\right)\left(\frac{4}{5}\right) = \frac{-24}{25}$

$\cos 2u = \cos^2 u - \sin^2 u = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

$\tan 2u = \frac{\sin 2u}{\cos 2u} = \frac{-24}{7}$



95. $\tan u = -\frac{2}{9}, \frac{\pi}{2} < u < \pi,$ Quadrant II

$\sec^2 u = \tan^2 u + 1 = \frac{4}{81} + 1 = \frac{85}{81} \implies$

$\sec u = -\frac{\sqrt{85}}{9}$

$\cos u = \frac{-9\sqrt{85}}{85}, \sin u = (\tan u)(\cos u) = \frac{2\sqrt{85}}{85}$

$\sin 2u = 2 \sin u \cos u$

$= 2\left(\frac{2\sqrt{85}}{85}\right)\left(\frac{-9\sqrt{85}}{85}\right) = -\frac{36}{85}$

$\cos 2u = 1 - 2\sin^2 u = 1 - 2\left(\frac{4}{85}\right) = \frac{77}{85}$

$\tan 2u = \frac{\sin 2u}{\cos 2u} = -\frac{36}{77}$

96. $\cos u = -\frac{2}{\sqrt{5}}, \frac{\pi}{2} < u < \pi,$ Quadrant II

$\sin^2 u = 1 - \cos^2 u = 1 - \frac{4}{5} = \frac{1}{5} \implies \sin u = \frac{1}{\sqrt{5}}$

$\sin 2u = 2 \sin u \cos u = 2\left(\frac{1}{\sqrt{5}}\right)\left(-\frac{2}{\sqrt{5}}\right) = -\frac{4}{5}$

$\cos 2u = \cos^2 u - \sin^2 u = \frac{4}{5} - \frac{1}{5} = \frac{3}{5}$

$\tan 2u = \frac{\sin 2u}{\cos 2u} = -\frac{4}{3}$

97. $6 \sin x \cos x = 3[2 \sin x \cos x] = 3 \sin 2x$

98. $4 \sin x \cos x + 2 = 2(2 \sin x \cos x) + 2$

$= 2 \sin 2x + 2$

99. $1 - 4\sin^2 x \cos^2 x = 1 - (2 \sin x \cos x)^2$

$= 1 - \sin^2 2x = \cos^2 2x$

100. $\sin 4x = 2 \sin 2x \cos 2x$

$= 2[2 \sin x \cos x(\cos^2 x - \sin^2 x)]$

$= 4 \sin x \cos x(2\cos^2 x - 1)$

$= 8\cos^3 x \sin x - 4 \cos x \sin x$

101. $r = \frac{1}{32}v_0^2 \sin 2\theta$

$100 = \frac{1}{32}(80)^2 \sin 2\theta$

$\sin 2\theta = 0.5$

$2\theta = 30°$ or $2\theta = 180° - 30° = 150°$

$\theta = 15°$ $\qquad \theta = 75°$

102. $r = \frac{1}{32}v_0^2 \sin 2\theta$

$77 = \frac{1}{32}(50)^2 \sin 2\theta$

$\sin 2\theta = 0.9856$

$2\theta = 80.2649°$ or $99.7351°$

$\theta = 40.13°$ or $49.87°$

103. $\sin^6 x = \left(\frac{1 - \cos 2x}{2}\right)^3 = \frac{1}{8}(1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x)$

$= \frac{1}{8}\left[1 - 3\cos 2x + 3\left(\frac{1 + \cos 4x}{2}\right) - \cos 2x\left(\frac{1 + \cos 4x}{2}\right)\right]$

$= \frac{1}{8}\left(1 - 3\cos 2x + \frac{3}{2} + \frac{3}{2}\cos 4x - \frac{1}{2}\cos 2x - \frac{1}{2}\cos 2x \cos 4x\right)$

$= \frac{1}{16}\left(5 - 7\cos 2x + 3\cos 4x - \frac{1}{2}[\cos 2x + \cos 6x]\right)$

$= \frac{1}{32}(10 - 15\cos 2x + 6\cos 4x - \cos 6x)$

104. $\cos^4 x \sin^4 x = \left(\frac{1 + \cos 2x}{2}\right)^2\left(\frac{1 - \cos 2x}{2}\right)^2$

$= \frac{(1 + 2\cos 2x + \cos^2 2x)(1 - 2\cos 2x + \cos^2 2x)}{16}$

$= \frac{\left(1 + 2\cos 2x + \frac{1 + \cos 4x}{2}\right)\left(1 - 2\cos 2x + \frac{1 + \cos 4x}{2}\right)}{16}$

$= \frac{(3 + 4\cos 2x + \cos 4x)(3 - 4\cos 2x + \cos 4x)}{64}$

$= \frac{1}{64}[(3 + \cos 4x) + 4\cos 2x][(3 + \cos 4x) - 4\cos 2x]$

$= \frac{1}{64}[(3 + \cos 4x)^2 - 16\cos^2 2x]$

$= \frac{1}{64}[9 + 6\cos 4x + \cos^2 4x - 16\cos^2 2x]$

$= \frac{1}{64}\left[9 + 6\cos 4x + \frac{1 + \cos 8x}{2} - 16 \cdot \frac{1 + \cos 4x}{2}\right]$

$= \frac{1}{64}\left[\frac{3}{2} + \frac{1}{2}\cos 8x - 2\cos 4x\right]$

$= \frac{1}{128}(\cos 8x - 4\cos 4x + 3)$

105. $\cos^4 2x = \left(\frac{1 + \cos 4x}{2}\right)^2$

$= \frac{1}{4}(1 + 2\cos 4x + \cos^2 4x)$

$= \frac{1}{4}\left(1 + 2\cos 4x + \frac{1 + \cos 8x}{2}\right)$

$= \frac{1}{8}(2 + 4\cos 4x + 1 + \cos 8x)$

$= \frac{1}{8}(3 + 4\cos 4x + \cos 8x)$

106. $\sin^4 2x = \left(\frac{1 - \cos 4x}{2}\right)^2$

$= \frac{1}{4}[1 - 2\cos 4x + \cos^2 4x]$

$= \frac{1}{4}\left[1 - 2\cos 4x + \frac{1 + \cos 8x}{2}\right]$

$= \frac{1}{4}\left[\frac{3}{2} - 2\cos 4x + \frac{1}{2}\cos 8x\right]$

$= \frac{1}{8}(3 + \cos 8x - 4\cos 4x)$

107. $\sin 105° = \sin\left(\frac{1}{2} \cdot 210°\right) = \sqrt{\frac{1 - \cos 210°}{2}} = \sqrt{\frac{1 + (\sqrt{3}/2)}{2}} = \frac{\sqrt{2 + \sqrt{3}}}{2}$

$\cos 105° = \cos\left(\frac{1}{2} \cdot 210°\right) = -\sqrt{\frac{1 + \cos 210°}{2}} = -\sqrt{\frac{1 - (\sqrt{3}/2)}{2}} = \frac{-\sqrt{2 - \sqrt{3}}}{2}$

$\tan 105° = \tan\left(\frac{1}{2} \cdot 210°\right) = \frac{\sin 210°}{1 + \cos 210°} = \frac{-1/2}{1 - (\sqrt{3}/2)} = \frac{1}{\sqrt{3} - 2} = -2 - \sqrt{3}$

108. $112° \, 30' = 112.5°$

$\sin(112.5°) = \sin\left(\frac{1}{2} \cdot 225°\right) = \sqrt{\frac{1 - \cos 225°}{2}} = \sqrt{\frac{1 + (\sqrt{2}/2)}{2}} = \frac{\sqrt{2 + \sqrt{2}}}{2}$

$\cos(112.5°) = \cos\left(\frac{1}{2} \cdot 225°\right) = -\sqrt{\frac{1 + \cos 225°}{2}} = -\sqrt{\frac{1 - (\sqrt{2}/2)}{2}} = \frac{-\sqrt{2 - \sqrt{2}}}{2}$

$\tan(112.5°) = \tan\left(\frac{1}{2} \cdot 225°\right) = \frac{\sin 225°}{1 + \cos 225°} = \frac{-\sqrt{2}/2}{1 - (\sqrt{2}/2)} = \frac{-\sqrt{2}}{2 - \sqrt{2}} \cdot \frac{2 + \sqrt{2}}{2 + \sqrt{2}}$

$= \frac{-2 - 2\sqrt{2}}{2} = -1 - \sqrt{2}$

109. $\sin\left(\frac{7\pi}{8}\right) = \sin\left(\frac{1}{2} \cdot \frac{7\pi}{4}\right) = \sqrt{\frac{1 - \cos(7\pi/4)}{2}} = \sqrt{\frac{1 - (\sqrt{2}/2)}{2}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$

$\cos\left(\frac{7\pi}{8}\right) = \cos\left(\frac{1}{2} \cdot \frac{7\pi}{4}\right) = -\sqrt{\frac{1 + \cos(7\pi/4)}{2}} = -\sqrt{\frac{1 + (\sqrt{2}/2)}{2}} = \frac{-\sqrt{2 + \sqrt{2}}}{2}$

$\tan\left(\frac{7\pi}{8}\right) = \tan\left(\frac{1}{2} \cdot \frac{7\pi}{4}\right) = \frac{\sin(7\pi/4)}{1 + \cos(7\pi/4)} = \frac{-\sqrt{2}/2}{1 + (\sqrt{2}/2)} = \frac{-\sqrt{2}}{2 + \sqrt{2}} = 1 - \sqrt{2}$

110. $\frac{11\pi}{12} = \frac{1}{2}\left(\frac{11\pi}{6}\right)$, Quadrant II

$\sin\left(\frac{11\pi}{12}\right) = \sin\left(\frac{1}{2} \cdot \frac{11\pi}{6}\right) = \sqrt{\frac{1 - \cos(11\pi/6)}{2}} = \sqrt{\frac{1 - (\sqrt{3}/2)}{2}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$

$\cos\left(\frac{11\pi}{12}\right) = \cos\left(\frac{1}{2} \cdot \frac{11\pi}{6}\right) = -\sqrt{\frac{1 + \cos(11\pi/6)}{2}} = -\sqrt{\frac{1 + (\sqrt{3}/2)}{2}} = -\frac{\sqrt{2 + \sqrt{3}}}{2}$

$\tan\left(\frac{11\pi}{12}\right) = \tan\left(\frac{1}{2} \cdot \frac{11\pi}{6}\right) = \frac{\sin(11\pi/6)}{1 + \cos(11\pi/6)} = \frac{-1/2}{1 + (\sqrt{3}/2)} = \frac{-1}{2 + \sqrt{3}} = -2 + \sqrt{3}$

111. $\sin u = \frac{3}{5}, 0 < u < \frac{\pi}{2} \Rightarrow \cos u = \frac{4}{5}$

$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1 - \cos u}{2}}$$

$$= \sqrt{\frac{1 - (4/5)}{2}} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\cos\left(\frac{u}{2}\right) = \sqrt{\frac{1 + \cos u}{2}}$$

$$= \sqrt{\frac{1 + (4/5)}{2}} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1 - \cos u}{\sin u} = \frac{1 - (4/5)}{3/5} = \frac{1}{3}$$

112. $\tan u = \frac{21}{20}, \pi < u < \frac{3\pi}{2}$, Quadrant III

$$\sec^2 u = \tan^2 u + 1 = \frac{441}{400} + 1 = \frac{841}{400} \Rightarrow$$

$$\sec u = \frac{-29}{20}$$

$$\cos u = \frac{-20}{29}$$

$$\sin u = \cos u \tan u = \frac{-20}{29} \cdot \frac{21}{20} = \frac{-21}{29}$$

$$\sin \frac{u}{2} = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 + (20/29)}{2}} = \frac{7\sqrt{58}}{58}$$

$$\cos \frac{u}{2} = -\sqrt{\frac{1 + \cos u}{2}} = -\sqrt{\frac{1 - (20/29)}{2}}$$

$$= \frac{-3\sqrt{58}}{58}$$

$$\tan \frac{u}{2} = \frac{\sin(u/2)}{\cos(u/2)} = -\frac{7}{3}$$

113. $\cos u = -\frac{2}{7}, \frac{\pi}{2} < u < \pi \Rightarrow \sin u = \sqrt{1 - \frac{4}{49}} = \frac{\sqrt{45}}{7}$

$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 + (2/7)}{2}} = \sqrt{\frac{9}{14}} = \frac{3\sqrt{14}}{14}$$

$$\cos\left(\frac{u}{2}\right) = \sqrt{\frac{1 + \cos u}{2}} = \sqrt{\frac{1 - (2/7)}{2}} = \sqrt{\frac{5}{14}} = \frac{\sqrt{70}}{14}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1 - \cos u}{\sin u} = \frac{1 + (2/7)}{\sqrt{45}/7} = \frac{9}{\sqrt{45}} = \frac{9\sqrt{45}}{45} = \frac{\sqrt{45}}{5} = \frac{3\sqrt{5}}{5}$$

114. $\sec u = -6, \frac{\pi}{2} < u < \pi,$ Quadrant II

$$\cos u = \frac{-1}{6}, \sin u = \frac{\sqrt{35}}{6}, \tan u = -\sqrt{35}$$

$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 + (1/6)}{2}} = \sqrt{\frac{7}{12}} = \frac{\sqrt{21}}{6}$$

$$\cos\left(\frac{u}{2}\right) = \sqrt{\frac{1 + \cos u}{2}} = \sqrt{\frac{1 - (1/6)}{2}} = \sqrt{\frac{5}{12}} = \frac{\sqrt{15}}{6}$$

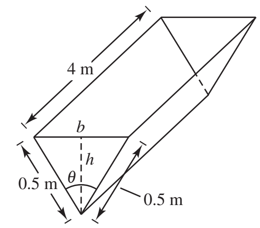
$$\tan\left(\frac{u}{2}\right) = \frac{1 - \cos u}{\sin u} = \frac{1 + (1/6)}{\sqrt{35}/6} = \frac{7}{\sqrt{35}} = \frac{\sqrt{35}}{5}$$

Note: $u \approx 99.6°$

115. $-\sqrt{\frac{1 + \cos 8x}{2}} = -|\cos 4x|$

116. $\frac{\sin 10x}{1 + \cos 10x} = \tan 5x$

117. Volume V of the trough will be the area A of the isosceles triangle times the length l of the trough.



$V = A \cdot l$

$A = \frac{1}{2}bh$

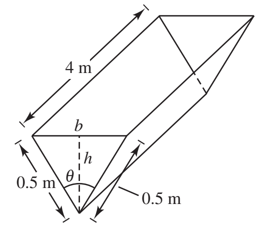
$\cos\frac{\theta}{2} = \frac{h}{0.5} \implies h = 0.5\cos\frac{\theta}{2}$

$\sin\frac{\theta}{2} = \frac{b/2}{0.5} \implies \frac{b}{2} = 0.5\sin\frac{\theta}{2}$

$A = 0.5\sin\frac{\theta}{2}\,0.5\cos\frac{\theta}{2} = (0.5)^2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 0.25\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ square meters

$V = (0.25)(4)\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ cubic meters $= \sin\frac{\theta}{2}\cos\frac{\theta}{2}$ cubic meters

118. Volume V of the trough will be the area A of the isosceles triangle times the length l of the trough.



$V = A \cdot l$

$A = \frac{1}{2}bh$

$\cos\frac{\theta}{2} = \frac{h}{0.5} \implies h = 0.5\cos\frac{\theta}{2}$

$\sin\frac{\theta}{2} = \frac{b/2}{0.5} \implies \frac{b}{2} = 0.5\sin\frac{\theta}{2}$

$A = 0.5\sin\frac{\theta}{2}\,0.5\cos\frac{\theta}{2} = (0.5)^2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 0.25\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ square meters

$V = (0.25)(4)\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ cubic meters $= \sin\frac{\theta}{2}\cos\frac{\theta}{2}$ cubic meters

$V = \sin\frac{\theta}{2}\cos\frac{\theta}{2} = \frac{1}{2}\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) = \frac{1}{2}\sin\theta$ cubic meters

Volume is maximum when $\theta = \pi/2$.

119. $6\sin\frac{\pi}{4}\cos\frac{\pi}{4} = 6\left[\frac{1}{2}\sin\left(\frac{\pi}{4} + \frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4} - \frac{\pi}{4}\right)\right] = 3\left(\sin\frac{\pi}{2} + \sin 0\right) = 3$

120. $4\sin 15^\circ \sin 45^\circ = 4 \cdot \frac{1}{2}[\cos(15^\circ - 45^\circ) - \cos(15^\circ + 45^\circ) = 2[\cos(-30^\circ) - \cos(60^\circ)]$

$= 2[\cos(30^\circ) - \cos(60^\circ)]$

121. $\sin 5\alpha \sin 4\alpha = \frac{1}{2}[\cos(5\alpha - 4\alpha) - \cos(5\alpha + 4\alpha)] = \frac{1}{2}[\cos\alpha - \cos 9\alpha]$

122. $\cos 6\theta \sin 8\theta = \frac{1}{2}[\sin(6\theta + 8\theta) - \sin(6\theta - 8\theta)] = \frac{1}{2}[\sin 14\theta - \sin(-2\theta)] = \frac{1}{2}[\sin 14\theta + \sin 2\theta]$

123. $\cos 5\theta + \cos 4\theta = 2\cos\left(\frac{9\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$

124. $\sin 3\theta + \sin 2\theta = 2\sin\left(\frac{5\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$

125. $\sin\left(x + \frac{\pi}{4}\right) - \sin\left(x - \frac{\pi}{4}\right) = 2 \cos x \sin \frac{\pi}{4} = \sqrt{2} \cos x$

126. $\cos\left(x + \frac{\pi}{6}\right) - \cos\left(x - \frac{\pi}{6}\right) = -2 \sin\left(\frac{x + \frac{\pi}{6} + x - \frac{\pi}{6}}{2}\right) \sin\left(\frac{x + \frac{\pi}{6} - x + \frac{\pi}{6}}{2}\right) = -2 \sin x \sin \frac{\pi}{6}$

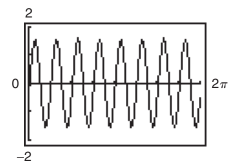
127. $y = 1.5 \sin 8t - 0.5 \cos 8t$

$a = \frac{3}{2},\ b = -\frac{1}{2},\ B = 8,\ C = \arctan\left(-\frac{1/2}{3/2}\right)$

$y = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \sin\left(8t + \arctan\left(-\frac{1}{3}\right)\right)$

$y = \frac{1}{2}\sqrt{10} \sin\left(8t - \arctan \frac{1}{3}\right)$

128. $y = \frac{1}{2}\sqrt{10} \sin\left(8t - \arctan \frac{1}{3}\right)$



129. The amplitude is $\frac{\sqrt{10}}{2}$.

130. Frequency $= \frac{1}{\text{period}} = \frac{4}{\pi}$

131. If $\frac{\pi}{2} < \theta < \pi$, then $\cos \frac{\theta}{2} < 0$. False, if

$\frac{\pi}{2} < \theta < \pi \implies \frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{2}$,

which is in Quadrant I $\implies \cos\left(\frac{\theta}{2}\right) > 0$.

132. $\sin(x + y) = \sin x + \sin y$. False.

$\sin(x + y) = \sin x \cos y + \cos x \sin y$

133. $4 \sin(-x) \cos(-x) = -2 \sin 2x$. True.

$4 \sin(-x) \cos(-x) = 4(-\sin x)(\cos x) = -4 \sin x \cos x = -2(2 \sin x \cos x) = -2 \sin 2x$

134. $4 \sin 45° \cos 15° = 1 + \sqrt{3}$. True.

$4 \sin 45° \cos 15° = 4\left(\frac{1}{2}[\sin(45° + 15°) + \sin(45° - 15°)]\right) = 2[\sin 60° + \sin 30°]$

$= 2\left[\frac{\sqrt{3}}{2} + \frac{1}{2}\right] = 2\left(\frac{\sqrt{3} + 1}{2}\right) = 1 + \sqrt{3}$

135. Answers will vary. See page 352.

136. No. $\cos \theta = \pm\sqrt{1 - \sin^2 \theta}$

137. $y_1 = \sec^2\left(\frac{\pi}{2} - x\right) = \csc^2 x$

$y_2 = \cot^2 x$

$\csc^2 x = \cot^2 x + 1$

Let $y_3 = y_2 + 1 = \cot^2 x + 1 = y_1$.

138. $y_1 = \frac{\cos 3x}{\cos x}$

$y_2 = (2 \sin x)^2$

From the graphs, $y_3 = -y_2 + 1 = y_1$.

Chapter 5 Practice Test

1. Find the value of the other five trigonometric functions, given $\tan x = \frac{4}{11}$, $\sec x < 0$.

2. Simplify $\dfrac{\sec^2 x + \csc^2 x}{\csc^2 x(1 + \tan^2 x)}$.

3. Rewrite as a single logarithm and simplify $\ln|\tan \theta| - \ln|\cot \theta|$.

4. True or false: $\cos\left(\dfrac{\pi}{2} - x\right) = \dfrac{1}{\csc x}$

5. Factor and simplify: $\sin^4 x + (\sin^2 x) \cos^2 x$

6. Multiply and simplify: $(\csc x + 1)(\csc x - 1)$

7. Rationalize the denominator and simplify:

$$\frac{\cos^2 x}{1 - \sin x}$$

8. Verify:

$$\frac{1 + \cos \theta}{\sin \theta} + \frac{\sin \theta}{1 + \cos \theta} = 2 \csc \theta$$

9. Verify:

$$\tan^4 x + 2 \tan^2 x + 1 = \sec^4 x$$

10. Use the sum or difference formulas to determine:

(a) $\sin 105°$ (b) $\tan 15°$

11. Simplify: $(\sin 42°) \cos 38° - (\cos 42°) \sin 38°$

12. Verify: $\tan\left(\theta + \dfrac{\pi}{4}\right) = \dfrac{1 + \tan \theta}{1 - \tan \theta}$

13. Write $\sin(\arcsin x - \arccos x)$ as an algebraic expression in x.

14. Use the double-angle formulas to determine:

(a) $\cos 120°$ (b) $\tan 300°$

15. Use the half-angle formulas to determine:

(a) $\sin 22.5°$ (b) $\tan \dfrac{\pi}{12}$

16. Given $\sin \theta = 4/5$, θ lies in Quadrant II, find $\cos \theta/2$.

17. Use the power-reducing identities to write $(\sin^2 x) \cos^2 x$ in terms of the first power of cosine.

18. Rewrite as a sum: $6(\sin 5\theta) \cos 2\theta$

19. Rewrite as a product: $\sin(x + \pi) + \sin(x - \pi)$

20. Verify: $\dfrac{\sin 9x + \sin 5x}{\cos 9x - \cos 5x} = -\cot 2x$

21. Verify: $(\cos u) \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$

22. Find all solutions in the interval $[0, 2\pi)$:

$$4 \sin^2 x = 1$$

23. Find all solutions in the interval $[0, 2\pi)$:

$$\tan^2 \theta + \left(\sqrt{3} - 1\right) \tan \theta - \sqrt{3} = 0$$

24. Find all solutions in the interval $[0, 2\pi)$:

$$\sin 2x = \cos x$$

25. Use the Quadratic Formula to find all solutions in the interval $[0, 2\pi)$:

$$\tan^2 x - 6 \tan x + 4 = 0$$