CHAPTER 5:

Analytic Trigonometry

SECTION 5.1: FUNDAMENTAL TRIGONOMETRIC IDENTITIES

1)

Left Side	Right Side	Type of Identity (ID)
$\csc(x)$	$\dfrac{1}{\sin(x)}$	Reciprocal ID
$\tan(x)$	$\dfrac{1}{\cot(x)}$	Reciprocal ID
$\tan(x)$	$\dfrac{\sin(x)}{\cos(x)}$	Quotient ID
$\tan\left(\dfrac{\pi}{2}-x\right)$	$\cot(x)$	Cofunction ID
$\cos(x)$	$\sin\left(\dfrac{\pi}{2}-x\right)$	Cofunction ID
$\sin(-x)$	$-\sin(x)$	Even / Odd (Negative-Angle) ID
$\cos(-x)$	$\cos(x)$	Even / Odd (Negative-Angle) ID
$\tan(-x)$	$-\tan(x)$	Even / Odd (Negative-Angle) ID
$\sin^2(x)+\cos^2(x)$	1	Pythagorean ID
$\tan^2(x)+1$	$\sec^2(x)$	Pythagorean ID
$1+\cot^2(x)$	$\csc^2(x)$	Pythagorean ID

2) a) $-\sec(x)$; b) $\sec^2(\theta)$; c) 1; d) $\csc^4(x)$; e) $\sin(t)$; f) $\sin(\alpha)$

3) a) $4\cos(\theta)$; b) $6\sec(\theta)$; c) $3\tan(\theta)$

SECTION 5.2: VERIFYING TRIGONOMETRIC IDENTITIES

1) Solutions will vary.

SECTION 5.3: SOLVING TRIGONOMETRIC EQUATIONS

1)

a) $\left\{ x \in \mathbb{R} \,\middle|\, x = \frac{\pi}{3} + 2\pi n \text{ or } x = \frac{2\pi}{3} + 2\pi n \ (n \in \mathbb{Z}) \right\}$. In $[0, 2\pi)$: $\left\{ \frac{\pi}{3}, \frac{2\pi}{3} \right\}$.

b) $\left\{ \theta \in \mathbb{R} \,\middle|\, \theta = \pm \frac{3\pi}{4} + 2\pi n \ (n \in \mathbb{Z}) \right\}$, or, equivalently,

$\left\{ \theta \in \mathbb{R} \,\middle|\, \theta = \frac{3\pi}{4} + 2\pi n \text{ or } \theta = \frac{5\pi}{4} + 2\pi n \ (n \in \mathbb{Z}) \right\}$. In $[0, 2\pi)$: $\left\{ \frac{3\pi}{4}, \frac{5\pi}{4} \right\}$.

c) No real solutions; the solution set is $\varnothing$. No real solutions in $[0, 2\pi)$.

d) $\left\{ u \in \mathbb{R} \,\middle|\, u = \frac{3\pi}{2} + 2\pi n \ (n \in \mathbb{Z}) \right\}$. Solutions in $[0, 2\pi)$: $\left\{ \frac{3\pi}{2} \right\}$.

e) $\left\{ u \in \mathbb{R} \,\middle|\, u = \frac{\pi}{2} + \pi n \ (n \in \mathbb{Z}) \right\}$. Solutions in $[0, 2\pi)$: $\left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$.

f) $\left\{ u \in \mathbb{R} \,\middle|\, u = \frac{7\pi}{6} + 2\pi n \text{ or } u = \frac{11\pi}{6} + 2\pi n \ (n \in \mathbb{Z}) \right\}$, or, equivalently,

$\left\{ u \in \mathbb{R} \,\middle|\, u = \frac{7\pi}{6} + 2\pi n \text{ or } u = -\frac{\pi}{6} + 2\pi n \ (n \in \mathbb{Z}) \right\}$. In $[0, 2\pi)$: $\left\{ \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$.

g) $\left\{ x \in \mathbb{R} \,\middle|\, x = \pm \frac{\pi}{3} + 2\pi n \ (n \in \mathbb{Z}) \right\}$, or, equivalently,

$\left\{ x \in \mathbb{R} \,\middle|\, x = \frac{\pi}{3} + 2\pi n \text{ or } x = \frac{5\pi}{3} + 2\pi n \ (n \in \mathbb{Z}) \right\}$. In $[0, 2\pi)$: $\left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$.

h) No real solutions; the solution set is $\varnothing$. No real solutions in $[0, 2\pi)$.

i) $\left\{ x \in \mathbb{R} \,\middle|\, x = \frac{\pi}{6} + \pi n \ (n \in \mathbb{Z}) \right\}$. Solutions in $[0, 2\pi)$: $\left\{ \frac{\pi}{6}, \frac{7\pi}{6} \right\}$.

j) $\left\{ \theta \in \mathbb{R} \,\middle|\, \theta = \frac{\pi}{2} + \pi n \ (n \in \mathbb{Z}) \right\}$. Solutions in $[0, 2\pi)$: $\left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$.

k) $\left\{ \theta \in \mathbb{R} \,\middle|\, \theta = \pm \frac{\pi}{6} + \pi n \ (n \in \mathbb{Z}) \right\}$, or, equivalently,

$\left\{ \theta \in \mathbb{R} \,\middle|\, \theta = \frac{\pi}{6} + \pi n \text{ or } \theta = \frac{5\pi}{6} + \pi n \ (n \in \mathbb{Z}) \right\}$. In $[0,2\pi)$: $\left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$.

l) $\left\{ \theta \in \mathbb{R} \,\middle|\, \theta = \pi n \text{ or } \theta = \frac{3\pi}{4} + \pi n \ (n \in \mathbb{Z}) \right\}$, or, equivalently,

$\left\{ \theta \in \mathbb{R} \,\middle|\, \theta = \pi n \text{ or } \theta = -\frac{\pi}{4} + \pi n \ (n \in \mathbb{Z}) \right\}$. In $[0, 2\pi)$: $\left\{ 0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4} \right\}$.

m) $\left\{ x \in \mathbb{R} \,\middle|\, x = \frac{\pi}{6} + 2\pi n \text{ or } x = \frac{\pi}{2} + 2\pi n \text{ or } x = \frac{5\pi}{6} + 2\pi n \ (n \in \mathbb{Z}) \right\}$.

Solutions in $[0, 2\pi)$: $\left\{ \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6} \right\}$.

n) $\left\{ x \in \mathbb{R} \,\middle|\, x = \frac{\pi}{2} + \pi n \text{ or } x = \frac{2\pi n}{3} \ (n \in \mathbb{Z}) \right\}$, by rotational symmetry. Less efficiently: $\left\{ x \in \mathbb{R} \,\middle|\, x = \frac{\pi}{2} + \pi n \text{ or } x = 2\pi n \text{ or } x = \pm\frac{2\pi}{3} + 2\pi n \ (n \in \mathbb{Z}) \right\}$.

Solutions in $[0, 2\pi)$: $\left\{ 0, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2} \right\}$.

o) $\left\{ x \in \mathbb{R} \,\middle|\, x = \pm\frac{\pi}{12} + \frac{\pi n}{2} \ (n \in \mathbb{Z}) \right\}$. The following form may be more useful for later: $\left\{ x \in \mathbb{R} \,\middle|\, x = \frac{\pi}{12} + \frac{\pi n}{2} \text{ or } x = \frac{5\pi}{12} + \frac{\pi n}{2} \ (n \in \mathbb{Z}) \right\}$.

Solutions in $[0, 2\pi)$: $\left\{ \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12} \right\}$.

p) $\left\{ x \in \mathbb{R} \,\middle|\, x = \frac{\pi}{6} + \frac{2\pi n}{3} \ (n \in \mathbb{Z}) \right\}$. In $[0, 2\pi)$: $\left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2} \right\}$.

q) $\left\{ x \in \mathbb{R} \,\middle|\, x = \pm\frac{\pi}{9} + \frac{\pi n}{3} \ (n \in \mathbb{Z}) \right\}$. The following form may be more useful for later: $\left\{ x \in \mathbb{R} \,\middle|\, x = \frac{\pi}{9} + \frac{\pi n}{3} \text{ or } x = \frac{2\pi}{9} + \frac{\pi n}{3} \ (n \in \mathbb{Z}) \right\}$. Solutions in $[0, 2\pi)$:

$\left\{ \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{14\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9} \right\}$.

2) a) $\{\arctan 2, \ \pi + \arctan 2\}$; equivalently, $\{\tan^{-1} 2, \ \pi + \tan^{-1} 2\}$.

b) Approximately: $\{1.107, 4.249\}$. (Make sure your calculator is in radian mode.)

c) $\left\{ x \in \mathbb{R} \,\middle|\, x = \arctan 2 + \pi n \ (n \in \mathbb{Z}) \right\}$, or $\left\{ x \in \mathbb{R} \,\middle|\, x = \tan^{-1} 2 + \pi n \ (n \in \mathbb{Z}) \right\}$.

3)

a) Solutions in $[0, 2\pi)$: $\left\{ \arccos\left(-\frac{1}{5}\right),\ \pi + \arccos\left(\frac{1}{5}\right) \right\}$. Equivalent forms:

$\left\{ \cos^{-1}\left(-\frac{1}{5}\right),\ \pi + \cos^{-1}\left(\frac{1}{5}\right) \right\}$, $\left\{ \pi - \arccos\left(\frac{1}{5}\right),\ \pi + \arccos\left(\frac{1}{5}\right) \right\}$, and

$\left\{ \arccos\left(-\frac{1}{5}\right),\ 2\pi - \arccos\left(-\frac{1}{5}\right) \right\}$.

b) Approximately: $\{1.772, 4.511\}$. (Make sure your calculator is in radian mode.)

c) $\left\{ x \in \mathbb{R} \,\middle|\, x = \pm\arccos\left(-\frac{1}{5}\right) + 2\pi n\ \ (n \in \mathbb{Z}) \right\}$, or, equivalently,

$\left\{ x \in \mathbb{R} \,\middle|\, x = \pm\cos^{-1}\left(-\frac{1}{5}\right) + 2\pi n\ \ (n \in \mathbb{Z}) \right\}$, or, equivalently,

$\left\{ x \in \mathbb{R} \,\middle|\, x = \pm\arccos\left(\frac{1}{5}\right) + (2n+1)\pi\ \ (n \in \mathbb{Z}) \right\}$.

SECTIONS 5.4 and 5.5: MORE TRIGONOMETRIC IDENTITIES

1)

Left Side	Right Side	Type of Identity (ID)
$\sin(u+v)$	$\sin(u)\cos(v)+\cos(u)\sin(v)$	Sum ID
$\cos(u+v)$	$\cos(u)\cos(v)-\sin(u)\sin(v)$	Sum ID
$\tan(u+v)$	$\frac{\tan(u)+\tan(v)}{1-\tan(u)\tan(v)}$	Sum ID
$\sin(u-v)$	$\sin(u)\cos(v)-\cos(u)\sin(v)$	Difference ID
$\cos(u-v)$	$\cos(u)\cos(v)+\sin(u)\sin(v)$	Difference ID
$\tan(u-v)$	$\frac{\tan(u)-\tan(v)}{1+\tan(u)\tan(v)}$	Difference ID
$\sin(2u)$	$2\sin(u)\cos(u)$	Double-Angle ID

Left Side	**Right Side**	**Type of Identity (ID)**
$\cos(2u)$	$\cos^2(u)-\sin^2(u)$, $1-2\sin^2(u)$, and $2\cos^2(u)-1$	Double-Angle ID (write all three versions)
$\tan(2u)$	$\dfrac{2\tan(u)}{1-\tan^2(u)}$	Double-Angle ID
$\sin^2(u)$	$\dfrac{1-\cos(2u)}{2}$ or $\dfrac{1}{2}-\dfrac{1}{2}\cos(2u)$	Power-Reducing ID (PRI)
$\cos^2(u)$	$\dfrac{1+\cos(2u)}{2}$ or $\dfrac{1}{2}+\dfrac{1}{2}\cos(2u)$	Power-Reducing ID (PRI)
$\sin\left(\dfrac{\theta}{2}\right)$	$\pm\sqrt{\dfrac{1-\cos(\theta)}{2}}$ (Choose the sign appropriately.)	Half-Angle ID
$\cos\left(\dfrac{\theta}{2}\right)$	$\pm\sqrt{\dfrac{1+\cos(\theta)}{2}}$ (Choose the sign appropriately.)	Half-Angle ID
$\tan\left(\dfrac{\theta}{2}\right)$	$\pm\sqrt{\dfrac{1-\cos(\theta)}{1+\cos(\theta)}}=\dfrac{1-\cos(\theta)}{\sin(\theta)}=\dfrac{\sin(\theta)}{1+\cos(\theta)}$ (Choose the sign appropriately.)	Half-Angle ID (write all three versions)

2)
a) $\dfrac{\sqrt{2}+\sqrt{6}}{4}$; b) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$; c) $\sqrt{3}+2$ (rationalize the denominator in $\dfrac{\sqrt{3}+3}{3-\sqrt{3}}$).

3) $\dfrac{\sqrt{2-\sqrt{2}}}{2}$

4) $\dfrac{\sqrt{2+\sqrt{2}}}{2}$

5) a) $\dfrac{\sqrt{3}}{2}$; b) $\dfrac{1}{2}$; c) $\dfrac{\sqrt{2}}{2}$; d) $\dfrac{\sqrt{3}}{2}$

6) $\cos(2\theta)$

7) $\dfrac{\tan(4x)}{6}$

8)
a) Hint: Use a Sum Identity.
b) Hints: Use a Double-Angle Identity and a Pythagorean Identity.
c) Hints: Use the Sum Identities for sine and cosine, and then divide the numerator and the denominator by $\cos(u)\cos(v)$.

9)

a) All real solutions: $\left\{ x \in \mathbb{R} \,\middle|\, x = \frac{\pi}{12} + \pi n \text{ or } x = \frac{5\pi}{12} + \pi n \ (n \in \mathbb{Z}) \right\}$.

Solutions in $[0, 2\pi)$: $\left\{ \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12} \right\}$

b) All real solutions: $\left\{ x \in \mathbb{R} \,\middle|\, x = \pm\frac{2\pi}{3} + 2\pi n \text{ or } x = \pi + 2\pi n \ (n \in \mathbb{Z}) \right\}$, or,

equivalently, $\left\{ x \in \mathbb{R} \,\middle|\, x = \frac{2\pi}{3} + 2\pi n \text{ or } x = \frac{4\pi}{3} + 2\pi n \text{ or } x = \pi + 2\pi n \ (n \in \mathbb{Z}) \right\}$.

Solutions in $[0, 2\pi)$: $\left\{ \frac{2\pi}{3}, \pi, \frac{4\pi}{3} \right\}$

c) All real solutions: $\left\{ x \in \mathbb{R} \,\middle|\, x = \pi n \text{ or } x = \pm\frac{\pi}{3} + 2\pi n \ (n \in \mathbb{Z}) \right\}$, or,

equivalently, $\left\{ x \in \mathbb{R} \,\middle|\, x = \pi n \text{ or } x = \frac{\pi}{3} + 2\pi n \text{ or } x = \frac{5\pi}{3} + 2\pi n \ (n \in \mathbb{Z}) \right\}$,

or, equivalently, $\left\{ x \in \mathbb{R} \,\middle|\, x = 2\pi n \text{ or } x = \frac{\pi}{3} + \frac{2\pi n}{3} \ (n \in \mathbb{Z}) \right\}$.

Solutions in $[0, 2\pi)$: $\left\{ 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3} \right\}$

10) $2x\sqrt{1-x^2}$

11) $\cos^4(x) = \boxed{\frac{3}{8}} + \boxed{\frac{1}{2}}\cos(2x) + \boxed{\frac{1}{8}}\cos(4x)$

12)

a) $\frac{1}{2}\left[\cos(2\theta) + \cos(8\theta)\right]$, which is simplified from $\frac{1}{2}\left[\cos(-2\theta) + \cos(8\theta)\right]$;

b) $2\cos(4\alpha)\cos(\alpha)$; c) $2\sin(2x)\cos(x)$

d) $\frac{1}{2}\left[\sin(19\theta) - \sin(\theta)\right]$, which is simplified from $\frac{1}{2}\left[\sin(19\theta) + \sin(-\theta)\right]$;

e) $\frac{1}{2}\left[\cos(3x) - \cos(5x)\right]$; f) $-2\sin(4x)\sin(3x)$; g) $2\cos(5\alpha)\sin(3\alpha)$;

h) $\frac{1}{2}\left[\sin(9\alpha) - \sin(\alpha)\right]$

CHAPTER 6:

Additional Topics in Trigonometry

SECTION 6.1: THE LAW OF SINES

1) a) 35.0 m; b) 22.0 m; c) 372 m^2
2) a) 180.09 ft; b) 224.86 ft; c) 20,137 ft^2

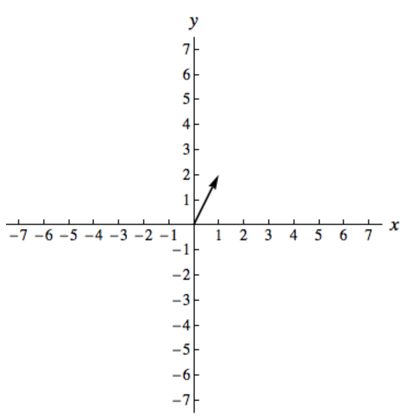
SECTION 6.2: THE LAW OF COSINES

1) a) 25.8°; b) 140.2°; c) No (that would violate the Triangle Inequality); d) 496 ft^2
2) 13.8 mi

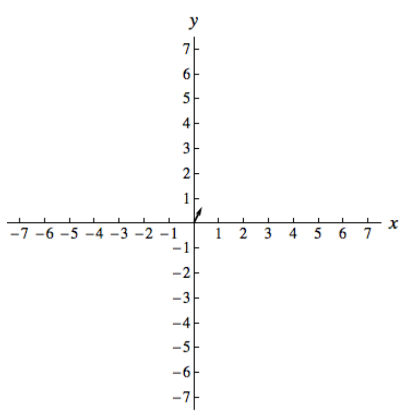
SECTION 6.3: VECTORS IN THE PLANE

1) a) $\langle 2, 3\rangle$ or $\langle 2\text{ m},\ 3\text{ m}\rangle$; b) $\sqrt{13}$ m; c) 56.3°
2) a) $\langle -5, -3\rangle$ or $\langle -5\text{ m},\ -3\text{ m}\rangle$; b) $\sqrt{34}$ m; c) 210.96°
3)

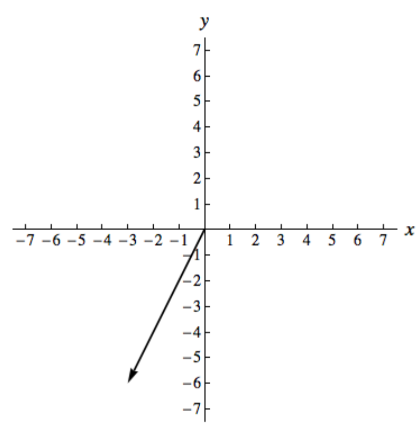
a) $\mathbf{v}$



$\frac{1}{3}\mathbf{v}$

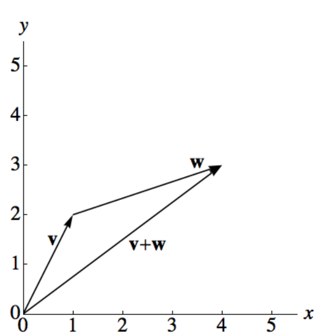


$-3\mathbf{v}$

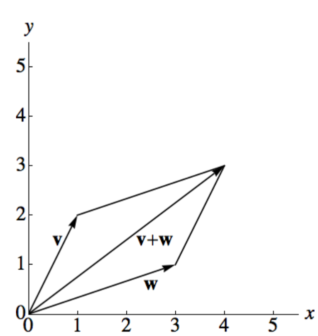


b) $\langle 4, 3\rangle$

c)



d)



e) $\langle 5, -5\rangle$

4) $\langle 8.0 \text{ ft}, \ 8.9 \text{ ft} \rangle$

5) a) $\left\langle -\frac{2\sqrt{29}}{29}, \frac{5\sqrt{29}}{29} \right\rangle$; b) 111.8°; c) $\left\langle -\frac{8\sqrt{29}}{29}, \frac{20\sqrt{29}}{29} \right\rangle$

6) a) 327.53°; b) $\left\langle \frac{22\sqrt{170}}{17}, -\frac{14\sqrt{170}}{17} \right\rangle$

7) Yes

8) No (they point in opposite directions)

9) a) 20.3 mph; b) 18.3 mph

SECTION 6.4: VECTORS AND DOT PRODUCTS

1) 14

2) a) scalar; b) vector; c) undefined; d) scalar; e) undefined; f) undefined

3) 10

4) Hint: $\| \mathbf{v} + \mathbf{w} \|^2 = (\mathbf{v} + \mathbf{w}) \bullet (\mathbf{v} + \mathbf{w})$.

5) $\| \mathbf{v} \|^2 + \| \mathbf{w} \|^2$

6) The Pythagorean Theorem

7) 19.7°; acute

8) 167.7°; obtuse

9) 47.7°. Hint: Find the angle between the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

10) a) 0°; b) 180°; c) 90°

11) Yes

12) No

13) 0 and 1

14) Hint: Use the formula: $\cos(\theta) = \frac{\mathbf{v} \bullet \mathbf{w}}{\|\mathbf{v}\| \|\mathbf{w}\|}$.

15) $\frac{14\sqrt{17}}{17}$