Algebra 2 and Trigonometry

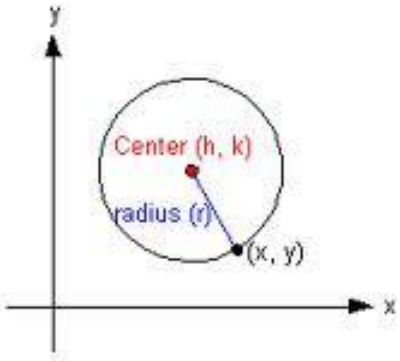
Chapter 5: Quadratic Equations/Circles

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Equation of a Circle (standard form)

$$(x-h)^2 + (y-k)^2 = r^2$$

Where r is the radius and (h, k) is the center.



Name:_____

Teacher:_____

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Day 1: Completing the Square

SWBAT: find the roots of a quadratic equation by completing the square, where a = 1.

Warm - Up:

1) Find the roots (solutions) of $x^2 - 3x - 10 = 0$

2) Find the roots of $x^2 = 9x - 18$.

Many quadratic equations contain expressions that cannot be easily factored. For equations containing these types of expressions, you can use square roots to find roots.

Square-Root Property

WORDS	NUMBERS	ALGEBRA
To solve a quadratic equation, you can take the square root of both sides. Be sure to consider the positive and negative square roots.	$x^2 = 15$ $\|x\| = \sqrt{15}$ $x = \pm\sqrt{15}$	If $x^2 = a$ and a is a nonnegative real number, then $x = \pm\sqrt{a}$.

Teacher Modeled	Student Try it!
Solve: $x^2 - 4 = 12$	**Solve:** $x^2 + 6 = 87$
Solve: $3x^2 - 4 = 68$	**Solve:** $4x^2 - 20 = 5$

You just practiced solving quadratic equations by using square roots. This only works if the quadratic expression is a perfect square. Remember that perfect square trinomials can be written as perfect squares.

$$x^2 + 8x + 16 = (x + 4)^2 \qquad x^2 - 10x + 25 = (x - 5)^2$$

If you have an equation of the form $x^2 + bx$, you can add the term $\left(\frac{b}{2}\right)^2$ to make a perfect square trinomial. This makes it possible to solve by using square roots.

Completing the Square

Steps to complete the square to form a perfect square trinomial. Example: $x^2 - 6x$

Step 1: Identify the "b" term.	
Step 2: Determine the number that will complete the perfect – square trinomial. You can do this simply by finding the value of $\left(\frac{b}{2}\right)^2$.	
Step 3: Add $\left(\frac{b}{2}\right)^2$.	
Step 4: Rewrite the perfect square trinomial as the square of a binomial.	

Practice:

Complete the square to form a perfect square trinomial by filling in the blanks. Then factor.

1. $x^2 - 14x$

$\left(\frac{b}{2}\right)^2 =$ _____

$x^2 - 14x +$ ☐

$(\square)^2$

2. $x^2 + 20x$

$\left(\frac{b}{2}\right)^2 =$ _____

$x^2 + 20x +$ ☐

$(\square)^2$

3. $x^2 + 6x$

$\left(\frac{b}{2}\right)^2 =$ _____

$x^2 + 6x +$ ☐

$(\square)^2$

Complete the square to form a perfect square trinomial. Then factor.

4. $x^2 + 18x$

_____

_____

5. $x^2 - 16x$

_____

_____

6. $x^2 + 5x$

_____

_____

Steps to solve a quadratic equation
by completing the square, follow these steps: Example: $x^2 - 6x - 7 = 0$

Step 1: Write the equation in the form $ax^2 + bx$ ____ $= c$ *Leave room to add a third term to this side.	
Step 2: Determine the number that will complete the perfect – square trinomial. You can do this simply by finding the value of $\left(\frac{b}{2}\right)^2$.	
Step 3: Add this number to each side of the equation.	
Step 4: Rewrite the perfect square trinomial as the square of a binomial.	
Step 5: Take the square root of each side of the equation. Remember to include $\pm$.	
Step 6: Solve for x.	

Solve each equation by completing the square.

Teacher Modeled	Student Try it!
Solve: $x^2 + 4x = 12$	**Solve:** $x^2 - 2x = 15$
Solve: $x^2 + 8x + 12 = 1$	**Solve:** $x^2 + 2x - 5 = -14$

Word Problems

A rectangular pool has an area of 880 ft². The length is 10 feet longer than the width. Find the dimensions of the pool. Solve by completing the square.
Round answers to the nearest tenth of a foot.

You Try it!

A gardener wants to create a rectangular vegetable garden in a backyard. She wants it to have a total area of 120 square feet, and it should be 12 feet longer than it is wide. What dimensions should she use for the vegetable garden? Round to the nearest hundredth of a foot.

A) 10.95 feet by 22.95 feet
B) 6.49 feet by 18.49 feet
C) 12.49 feet by 24.49 feet
D) 4.95 feet by 16.95 feet

CHALLENGE

Solve for x: $2x^2 - 8x + 3 = 0$

SUMMARY

Solving a Quadratic Equation by Completing the Square

Solve each equation by completing the square.

A $x^2 = 27 - 6x$

$x^2 + 6x = 27$ — *Collect variable terms on one side.*

$x^2 + 6x + \square = 27 + \square$ — *Set up to complete the square.*

$x^2 + 6x + \left(\frac{6}{2}\right)^2 = 27 + \left(\frac{6}{2}\right)^2$ — *Add $\left(\frac{b}{2}\right)^2$ to both sides.*

$x^2 + 6x + 9 = 27 + 9$ — *Simplify.*

$(x + 3)^2 = 36$ — *Factor.*

$x + 3 = \pm\sqrt{36}$ — *Take the square root of both sides.*

$x + 3 = \pm 6$ — *Simplify.*

$x + 3 = 6$ or $x + 3 = -6$ — *Solve for x.*

$x = 3$ or $x = -9$

B $2x^2 + 8x = 12$

$x^2 + 4x = 6$ — *Divide both sides by 2.*

$x^2 + 4x + \square = 6 + \square$ — *Set up to complete the square.*

$x^2 + 4x + \left(\frac{4}{2}\right)^2 = 6 + \left(\frac{4}{2}\right)^2$ — *Add $\left(\frac{b}{2}\right)^2$ to both sides.*

$x^2 + 4x + 4 = 6 + 4$ — *Simplify.*

$(x + 2)^2 = 10$ — *Factor.*

$x + 2 = \pm\sqrt{10}$ — *Take the square root of both sides.*

$x = -2 \pm \sqrt{10}$ — *Solve for x.*

Exit Ticket

If $x^2 + 2 = 6x$ is solved by completing the square, an intermediate step would be

(1) $(x + 3)^2 = 7$

(2) $(x - 3)^2 = 7$

(3) $(x - 3)^2 = 11$

(4) $(x - 6)^2 = 34$

More with Completing the Square – Day 2

SWBAT: find the roots of a quadratic equation by completing the square, where a ≠ 1.

Do Now:

Brian correctly used a method of completing the square to solve the equation $x^2 + 7x - 11 = 0$. Brian's first step was to rewrite the equation as $x^2 + 7x = 11$. He then added a number to both sides of the equation. Which number did he add?

(1) $\frac{7}{2}$

(2) $\frac{49}{4}$

(3) $\frac{49}{2}$

(4) 49

When using the procedure of completing the square, the leading coefficient should be equal to 1. If the coefficient of the quadratic term is not 1, divide both sides of the equation by the coefficient of the quadratic term. Then, follow the same steps we learned yesterday.

Steps to solve a quadratic equation
by completing the square, follow these steps: Example: $2x^2 + 4x + 1 = 0$

Step 1: Write the equation in the form $ax^2 + bx$ ____ = c *Leave room to add a third term to this side.	
Step 2: Divide both sides of the equation by "a"	
Step 3: Determine the number that will complete the perfect – square trinomial. You can do this simply by finding the value of $\left(\frac{b}{2}\right)^2$.	
Step 4: Add this number to each side of the equation.	
Step 5: Rewrite the perfect square trinomial as the square of a binomial.	
Step 6: Take the square root of each side of the equation. Remember to include ±.	
Step 7: Solve for x.	

Solve each equation by completing the square.

Teacher Modeled	Student Try it!
Solve: $2x^2 + 7x + 6 = 0$	**Solve:** $2x^2 - x - 6 = 0$
Solve: $3x^2 - 6x - 7 = -5$	**Solve:** $4x^2 + 4x - 1 = 2$

Word Problems

A rectangular garden has an area of 432 ft^2. The length is 2 more than 3 times the width. Find the dimensions of the garden. Solve by completing the square. Round your answer to the nearest tenth of a foot.

Word Problems

A small painting has an area of 400 cm^2. The length is 4 more than 2 times the width. Find the dimensions of the painting. Solve by completing the square. Round answers to the nearest tenth of a centimeter.

Challenge

Solve by completing the square.

$x^2 = (6\sqrt{2})x + 7$

SUMMARY

Given Equation:	$2x^2 + 3x - 2 = 0$
Divide through by coefficient of x^2: (in this case a 2)	$\frac{1}{2}\left(2x^2 + 3x - 2 = 0\right)$ $x^2 + \frac{3}{2}x - 1 = 0$
Move constant to other side:	$x^2 + \frac{3}{2}x = 1$
Add new constant term: (the square of half the coefficient of x, in this case 9/16):	$x^2 + \frac{3}{2}x + \frac{9}{16} = 1 + \frac{9}{16}$
Write as a binomial squared: (the constant in the binomial is half the coefficient of x)	$\left(x + \frac{3}{4}\right)^2 = \frac{25}{16}$
Square root both sides: (remember to use plus-or-minus)	$x + \frac{3}{4} = \pm\frac{5}{4}$
Solve for x:	$x = \frac{-3 \pm 5}{4}$

Thus $x = ½$ or $x = -2$

Exit Ticket

Solve by completing the square:

$2x^2 - 4x - 3 = 0$

[A] $\frac{2 \pm \sqrt{10}}{2}$ [B] $1 \pm \sqrt{10}$

[C] $\frac{-2 \pm \sqrt{10}}{2}$ [D] $-1 \pm \sqrt{10}$

Day 3: Equations of Circles in Center – Radius Form.

SWBAT: Write equations and graph circles in the coordinate plane.

Warm Up

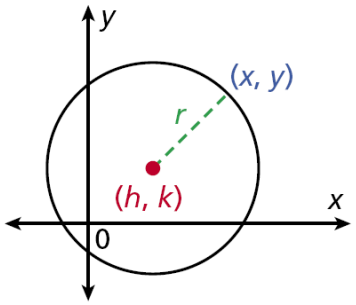
Solve by completing the square:

$4x^2 + 2x - 5 = 0$

[A] $\frac{1 \pm 2\sqrt{21}}{4}$ [B] $\frac{-1 \pm \sqrt{21}}{4}$

[C] $\frac{-1 \pm 2\sqrt{21}}{4}$ [D] $\frac{1 \pm \sqrt{21}}{4}$

In Geometry last year, you learned about the equation of a circle and its derivation from the Distance Formula and the fact that all points on a circle are equidistant from the center.



Center – Radius Form
For the Equation of a Circle

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$r = \sqrt{(x - h)^2 + (y - k)^2}$$

$$r^2 = (x - h)^2 + (y - k)^2$$

Part 1: Writing equations of circles given center and radius

Teacher Modeled	Student Try it!
Write the equation of a circle with ⊙***J* with center *J* (2, 2) and radius 4.**	Write the equation of a circle with ⊙***L* with center *L* (–5, –6) and radius 9**

Part 2: Writing equations of circles given center and point.

Teacher Modeled	Student Try it!
Write the equation of a circle with ⊙*K* that passes through *J*(6, 4) and has center *K*(1, –8) Step 1: Calculate radius Step 2: Plug in center and radius into formula.	**Write the equation of a circle with⊙*P* with center *P*(0, –3) and passes through point (6, 5).**

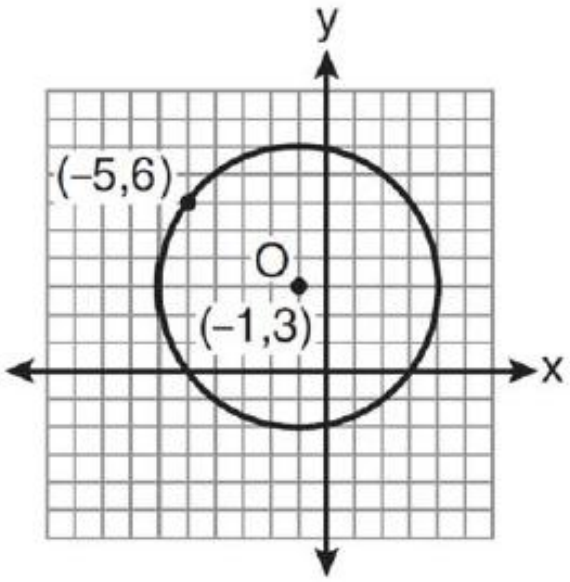
Identify the equation for the circle with center (1, 8) and containing the point (−2, 9).

- $(x+1)^2+(y+8)^2=\sqrt{2}$
- $(x+2)^2+(y-9)^2=\sqrt{10}$
- $(x-2)^2+(y-9)^2=2$
- $(x-1)^2+(y-8)^2=10$

Part 3: Write the equation of a circle given a graph.

Teacher Modeled

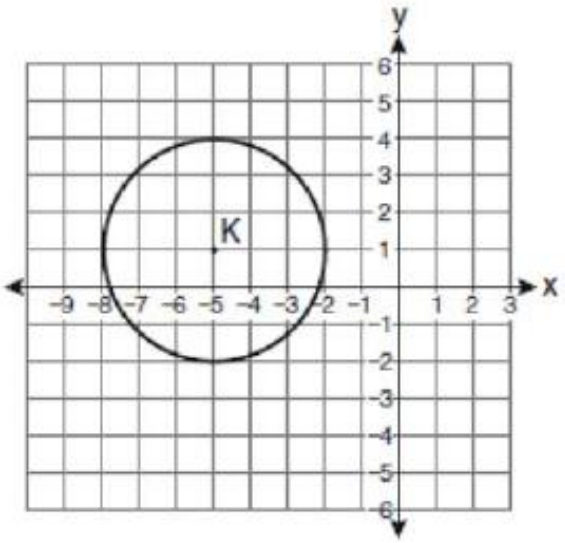
What is an equation of circle *O* shown in the graph below?



1) $(x+1)^2+(y-3)^2=25$
2) $(x-1)^2+(y+3)^2=25$
3) $(x-5)^2+(y+6)^2=25$
4) $(x+5)^2+(y-6)^2=25$

Student Try it!

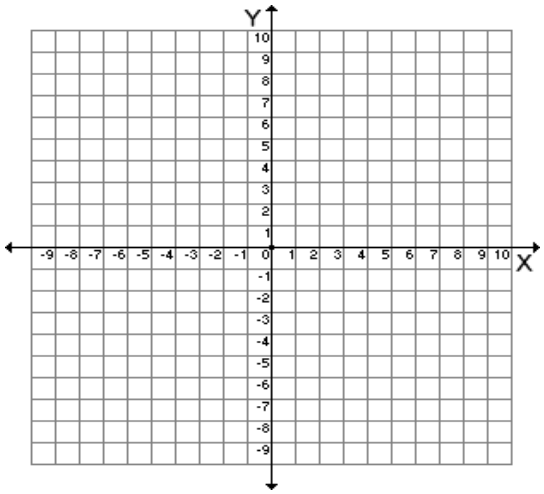
What is an equation of the circle shown in the graph below?



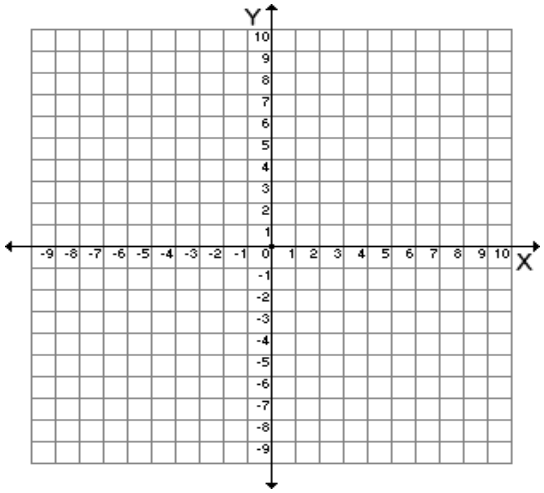
1) $(x+5)^2+(y-1)^2=3$
2) $(x+5)^2+(y-1)^2=9$
3) $(x-5)^2+(y+1)^2=3$
4) $(x-5)^2+(y+1)^2=9$

Part 4: Graphing Circles given equation in Center-Radius Form.

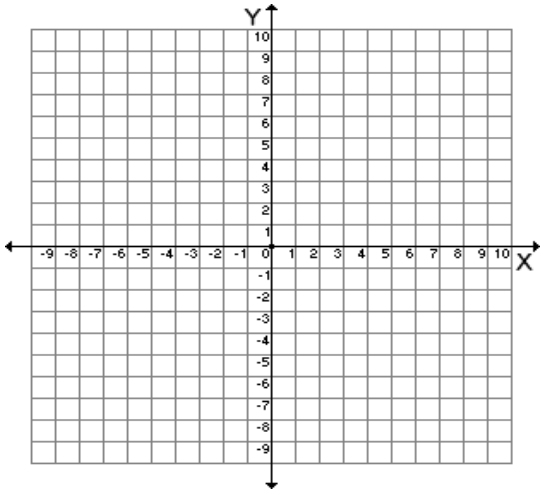
Graph $x^2 + y^2 = 16$.



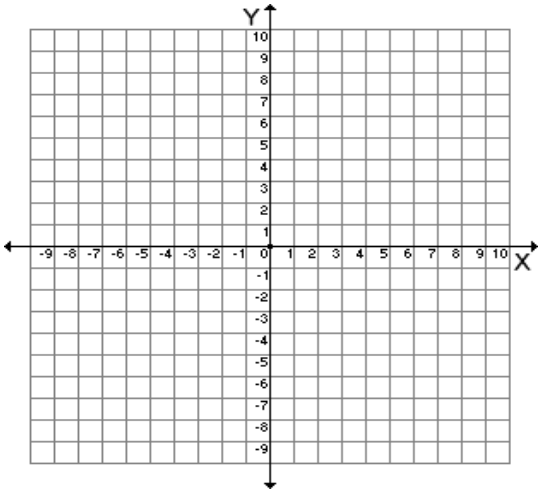
Graph $x^2 + y^2 = 36$.



Graph: $(x - 3)^2 + (y + 4)^2 = 9$.



Graph $(x + 5)^2 + (y - 2)^2 = 4$.



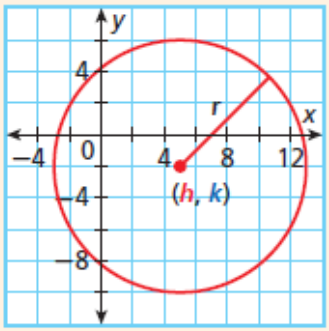
Challenge

Find the center, the radius, the diameter, the circumference, and the area of the circle represented by the equation $(x - 3)^2 + (y + 6)^2 = 100$.

SUMMARY

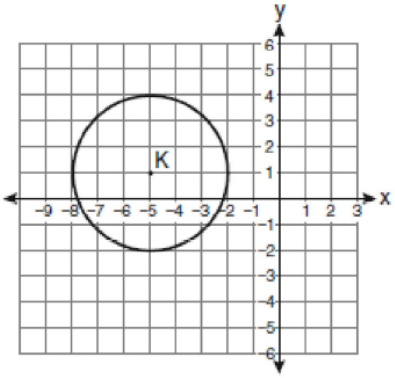
Know it! Note

Equation of a Circle

EQUATION	EXAMPLE	GRAPH
The equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.	The equation of the circle with center $(5, -2)$ and radius $r = 8$ is $(x - 5)^2 + (y - (-2))^2 = 8^2$ or $(x - 5)^2 + (y + 2)^2 = 64$.	

Exit Ticket

1. Which equation represents circle K shown in the graph below?



1) $(x + 5)^2 + (y - 1)^2 = 3$
2) $(x + 5)^2 + (y - 1)^2 = 9$
3) $(x - 5)^2 + (y + 1)^2 = 3$
4) $(x - 5)^2 + (y + 1)^2 = 9$

2. What is an equation of a circle with center $(7, -3)$ and radius 4?

1) $(x - 7)^2 + (y + 3)^2 = 4$
2) $(x + 7)^2 + (y - 3)^2 = 4$
3) $(x - 7)^2 + (y + 3)^2 = 16$
4) $(x + 7)^2 + (y - 3)^2 = 16$

Day 4: Equations of Circles in Standard Form.

SWBAT: write the equation of a circle from standard form to center-radius form.

Warm - Up:

1) What is the center and the radius of a circle whose equation is $(x + 3)^2 + (y - 5)^2 = 81$?

2) Write an equation of a circle with a radius of 7 and center at (-3,0).

Completing the Square of an Equation Containing Two Variables

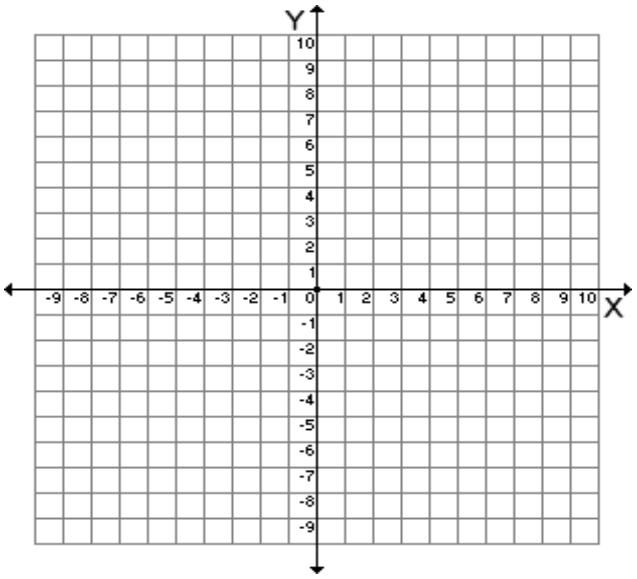
$$x^2 + 4x + y^2 - 6y + 1 = 0$$

If the coefficients of x^2 or $y^2 \neq 1$, divide everything in the equation by that value. Isolate the constant term on one side, and make sure all of the variables are grouped by letter (easiest if in this order: x^2, x, y^2, y.	$x^2 + 4x + y^2 - 6y + 1 = 0$
Isolate the constant term on one side.	$x^2 + 4x + y^2 - 6y + 1 = 0$ $\underline{\quad -1 = -1}$ $x^2 + 4x + y^2 - 6y = -1$
Determine the "magic constant" for the $x's$, and the "magic constant" for the $y's$ that compete each square: $c = \left(\frac{b}{2}\right)^2$. Add them to both sides, putting them in a place that makes sense. (MATH 1 ENTER on your calculator converts decimals to fractions)	$x^2 + 4x + \_\_ + y^2 - 6y + \_\_ = -1$ $c = \left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = 4 \qquad c = \left(\frac{b}{2}\right)^2 = \left(\frac{-6}{2}\right)^2 = 9$ $x^2 + 4x + \mathbf{4} + y^2 - 6y + \mathbf{9} = -1 + \mathbf{4} + \mathbf{9}$
Re-write the algebraic expressions as perfect squares. It will always be $(x + magic\#)^2$ and $(y + magic\#)^2$	$(x + 2)^2 + (y - 3)^2 = 12$
The equation of a circle is with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$ So the vertex is the (*opp, opp*) of what you see in the parentheses, and the radius is $\sqrt{\ }$ of the isolated constant	$(x + 2)^2 + (y - 3)^2 = 12$ Circle with radius (-2, 3) and radius $\sqrt{12} = 2\sqrt{3}$.

Write the following equations in a) standard form and b) center-radius form.

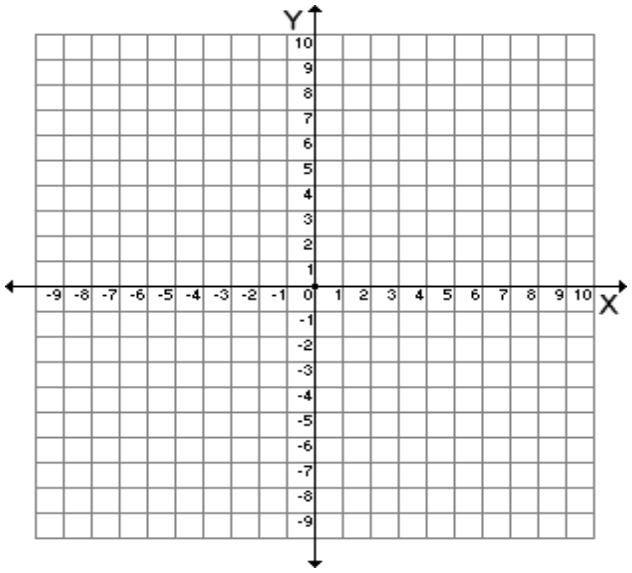
Teacher Modeled

$8x + x^2 - 2y = 64 - y^2$



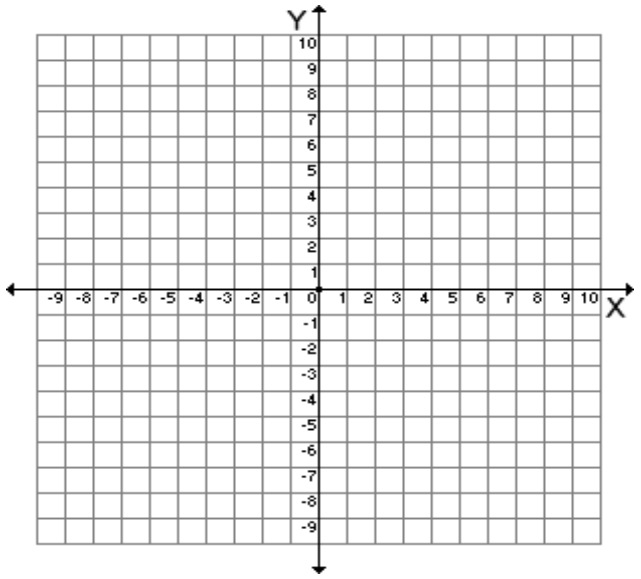
Student Try it!

$137 + 6y = -y^2 - x^2 - 24x$

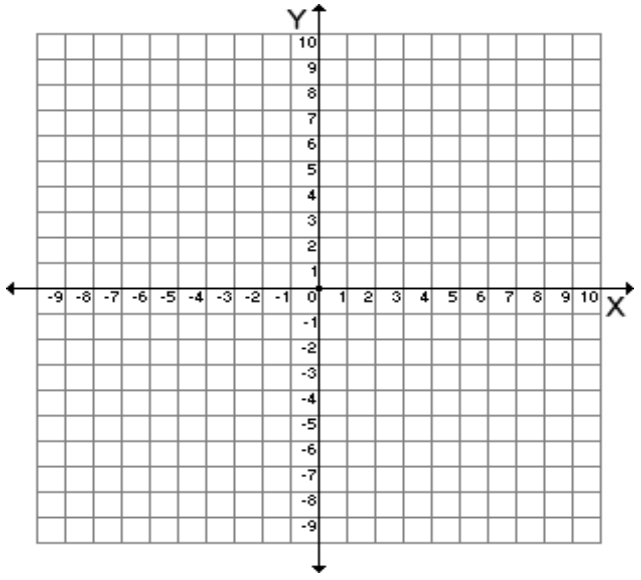


Practice:

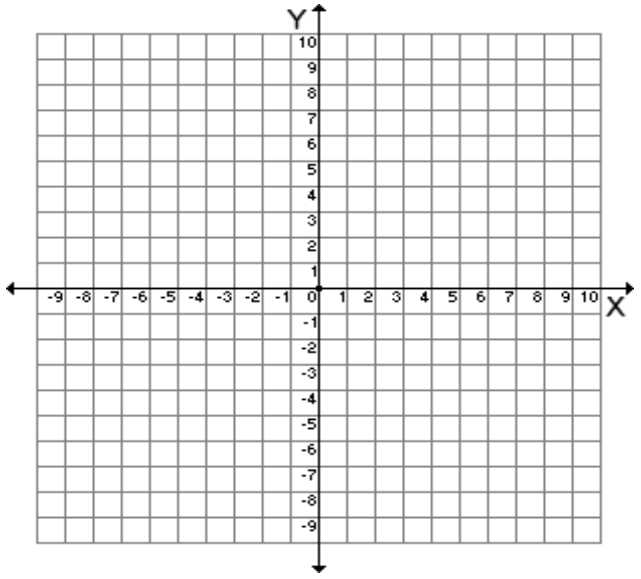
Example 1: Graph the equation: $x^2 + y^2 - 2x + 4y - 4 = 0$



Example 2: Graph: $x^2 + y^2 + 6x - 2y + 1 = 0$



Example 3: Graph: $x^2 + y^2 + x - y - \frac{1}{2} = 0$



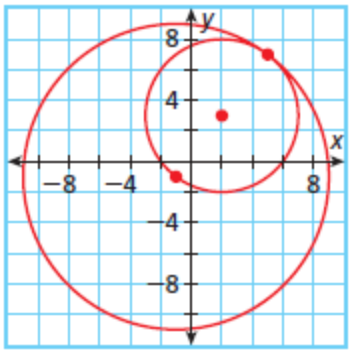
Challenge: (5 points)

Geometry The circle with center $(2, 3)$ and the circle with center $(-1, -1)$ are tangent at the point $(5, 7)$.

a. Find an equation for the small circle.

b. Find an equation for the large circle.

c. Find the equation of the line that is tangent to both circles.



Summary/Closure:

1. Convert $x^2 + y^2 - 4x - 6y + 8 = 0$ into center-radius form.

This conversion requires use of the technique of completing the square.

We will be creating two perfect square trinomials within the equation.

$x^2 + y^2 - 4x - 6y + 8 = 0$

$x^2 - 4x + y^2 - 6y = -8$

$x^2 - 4x + \square + y^2 - 6y + \square = -8 + \square + \square$

$x^2 - 4x + \boxed{4} + y^2 - 6y + \boxed{9} = -8 + \boxed{4} + \boxed{9}$

$(x-2)^2 + (y-3)^2 = 5$

- Start by grouping the x related terms together and the y related terms together. Move any numerical constants (plain numbers) to the other side.
- Get ready to insert the needed values for creating the perfect square trinomials. Remember to balance both sides of the equation.
- Find each missing value by taking half of the "middle term" and squaring. This value will always be positive as a result of the squaring process.
- Rewrite in factored form.

You can now read that the center of the circle is at (2, 3) and the radius is $\sqrt{5}$.

Exit Ticket:

The equation $x^2 + y^2 - 2x + 6y + 3 = 0$ is equivalent to

(1) $(x - 1)^2 + (y + 3)^2 = -3$

(2) $(x - 1)^2 + (y + 3)^2 = 7$

(3) $(x + 1)^2 + (y + 3)^2 = 7$

(4) $(x + 1)^2 + (y + 3)^2 = 10$

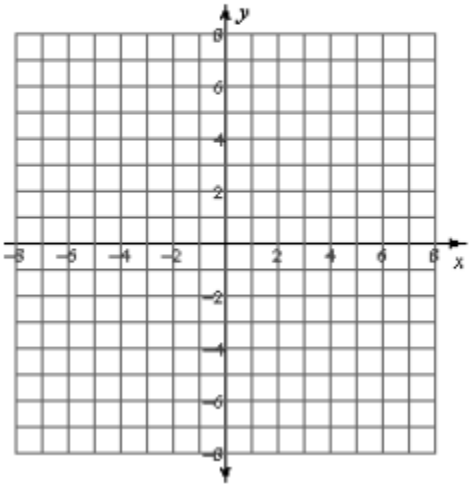
Day 5: Solving Quadratic Equations Using the Quadratic Formula

SWBAT: solve quadratic equations using the quadratic formula.

Warm - Up:

Identify the center and radius of each. Then sketch the graph.

$x^2 + y^2 + 8x - 4y + 14 = 0$



A quadratic equation is one whose highest power of *x* is ____. The standard form for a quadratic equation is:

The **roots** of a quadratic equation are where the graph of the equation hits the *x*-axis, or where *y* = ____.

We are used to solving quadratic equations that have ***rational roots*** by setting it equal to 0 and _____.

However, some quadratic equations aren't easily factorable because they have ***irrational roots***, meaning they contain a _____. For these situations the quadratic formula is employed:

The Quadratic Formula:

$x =$

The Quadratic Formula is the only method that can be used to solve any quadratic equation.

Example 1: Use the quadratic formula to find the roots of:
$2x^2 - 4x = 1$

Step 1: Is the quadratic written in standard form?	
Step 2: Determine the values of a, b, and c.	a = _____ b = _____ c = _____
Step 3: Substitute the values of a, b and c in the quadratic formula. Put parenthesis around your substitutions. Perform the computation. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
Step 4: Write in simplest radical form and simplify.	

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Steps:
(1) Put the quadratic equation in standard form $(ax^2 + bx + c = 0)$
(2) Determine the values for a, b, and c
(3) Plug the values into the formula
(4) Simplify as much as possible

Use the quadratic formula to find the roots of the following quadratic equations and check. Express irrational roots in *simplest radical form*.

1. $3x^2 + 12x = 3$

2. $x^2 - 6x = 12$

3. $5 - 4x = 7x^2 + 13$

4. $2x^2 + x = x^2 - 2x + 4$

Algebra2/Trig: Quadratics Word Problem Sampler:

Each of the following problems requires you to determine the roots, one of the coordinates of the vertex, or both. Determine by careful reading which the problem requires and find it. Show all work!!

Helpful reading hints:
"Hits the ground," "is empty," "is zero": ROOTS

"maximum," " minimum," "height":
VERTEX: $x_v = \frac{-b}{2a}$ y_v: $plug\ x_v\ into\ original\ equation$

EXAMPLE 5: A ball is thrown straight up at an initial velocity of 54 feet per second. The height of the ball t seconds after it is thrown is given by the formula $h(t) = 54t - 12t^2$. How many seconds after the ball is thrown will it return to the ground?

EXAMPLE 6: Barb pulled the plug in her bathtub and it started to drain. The amount of water in the bathtub as it drains is represented by the equation $L(t) = -5t^2 - 8t + 120$, where L represents the number of liters of water in the bathtub and t represents the amount of time, in minutes, since the plug was pulled. Determine the amount of time it takes for all the water in the bathtub to drain.

Example 7: A superhero is trying to leap over a tall building. The function $h(t) = -16t^2 + 200t$ gives the superhero's height in feet as a function of time. The building is 612 feet high. Will the superhero make it over the building? Show all work and give a sentence summary of why or why not.

Example 8: A model rocket is launched from ground level. Its height, *h* meters above the ground, is a function of time *t* seconds after launch and is given by the equation $h = -4.9t^2 + 68.6t$. What would be the maximum height, to the nearest meter, attained by the model?

SUMMARY

Find all roots of the equation $2x^2 - 3x - 2 = 0$.

SOLUTION

$$a = 2 \qquad b = -3 \qquad c = -2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-2)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{9 - (8)(-2)}}{4}$$

$$x = \frac{3 \pm \sqrt{25}}{4}$$

$$x = \frac{3 \pm 5}{4}$$

$$x_1 = \frac{3 + 5}{4} \qquad x_2 = \frac{3 - 5}{4}$$

$$x_1 = 2 \qquad x_2 = -\frac{1}{2}$$

Answer: The solutions to this equation are $\left\{-\frac{1}{2}, 2\right\}$.

Exit Ticket: Solve using the quadratic formula:

$5a^2 - 22 = 2a$

A) $\left\{\frac{1 + \sqrt{111}}{5}, \frac{1 - \sqrt{111}}{5}\right\}$

B) $\{1 + \sqrt{23}, 1 - \sqrt{23}\}$

C) $\{-1 + \sqrt{23}, -1 - \sqrt{23}\}$

D) $\left\{\frac{-1 + \sqrt{111}}{5}, \frac{-1 - \sqrt{111}}{5}\right\}$

Day 6: More with Solving Quadratic Equations Using the Quadratic Formula

SWBAT: solve quadratic equations using the quadratic formula.

Warm – Up:

Write an equation for the translation of $x^2 + y^2 = 64$ right 3 units and down 1 unit.

- $(x - 3)^2 + (y + 1)^2 = 72$
- $(x + 3)^2 + (y - 1)^2 = 80$
- $(x - 3)^2 + (y + 1)^2 = 64$
- $(x + 3)^2 + (y - 1)^2 = 64$

Identify the zeros of the function $f(x) = 4x^2 - 8x - 1$ using the Quadratic Formula.

- $1 \pm \frac{\sqrt{5}}{2}$
- $\frac{1}{2} \pm \sqrt{5}$
- $-1 \pm \frac{\sqrt{5}}{2}$
- $-\frac{1}{2} \pm \sqrt{5}$

An airplane pilot is fertilizing a field. The height y in feet of the fertilizer t seconds after it is dropped is modeled by $y(t) = -16t^2 - 3t + 300$. The horizontal distance x in feet between the fertilizer and its dropping point is modeled by $x(t) = 85t$. At approximately what horizontal distance from the field should the pilot start dropping the fertilizer?

- 531 ft
- 360 ft
- 300 ft
- 272 ft

Solve $5x^2 - 11x + 2 = 0$ using the Quadratic Formula.

- $-2, -\frac{1}{5}$
- $2, \frac{1}{5}$
- $-\frac{12}{5}, \frac{1}{5}$
- $\frac{12}{5}, -\frac{1}{5}$

Solve $x^2 + 8x - 2 = 0$ by completing the square.

- $2 \pm \sqrt{2}$
- ± 4
- $-4 \pm 3\sqrt{2}$
- $3\sqrt{2} \pm 4$

Solve $9x^2 + 6x + 1 = 64$.

- $-11, 5$
- $-\frac{65}{3}, 21$
- $\frac{7}{3}, 3$
- $-3, \frac{7}{3}$

Lane Manufacturing estimates that its weekly profit, P, in hundreds of dollars, can be approximated by the formula $P = -4x^2 + 14x + 3$ where x is the number of units produced per week, in thousands.
a. How many units should the company produce per week to earn the maximum profit?
b. Find the maximum weekly profit.

- **a.** 3000 units; **b.** $3900
- **a.** 3 units; **b.** $3900
- **a.** 2400 units; **b.** $3300
- **a.** 24 units; **b.** $3500

Day 6 - HW

1. What is the equation of a circle with center $(-3,1)$ and radius 7?

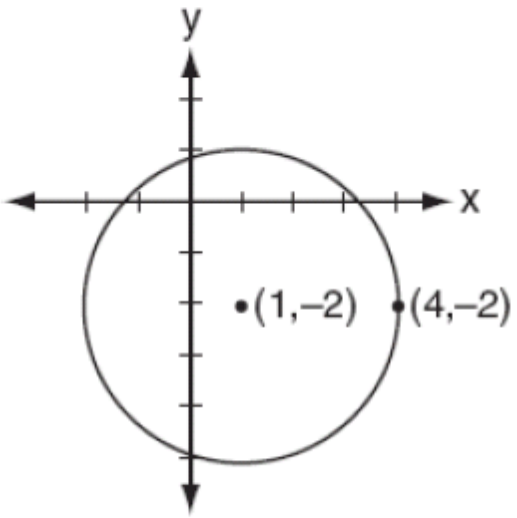
[A] $(x-3)^2+(y+1)^2=49$

[B] $(x-3)^2+(y+1)^2=7$

[C] $(x+3)^2+(y-1)^2=7$

[D] $(x+3)^2+(y-1)^2=49$

2. Which equation represents the circle shown in the accompanying graph?



[A] $(x+1)^2+(y-2)^2=9$

[B] $(x-1)^2-(y+2)^2=9$

[C] $(x+1)^2-(y-2)^2=9$

[D] $(x-1)^2+(y+2)^2=9$

3. The solutions of the equation $y^2-3y=9$ are

(1) $\frac{3\pm 3i\sqrt{3}}{2}$

(2) $\frac{3\pm 3i\sqrt{5}}{2}$

(3) $\frac{-3\pm 3\sqrt{5}}{2}$

(4) $\frac{3\pm 3\sqrt{5}}{2}$

4.

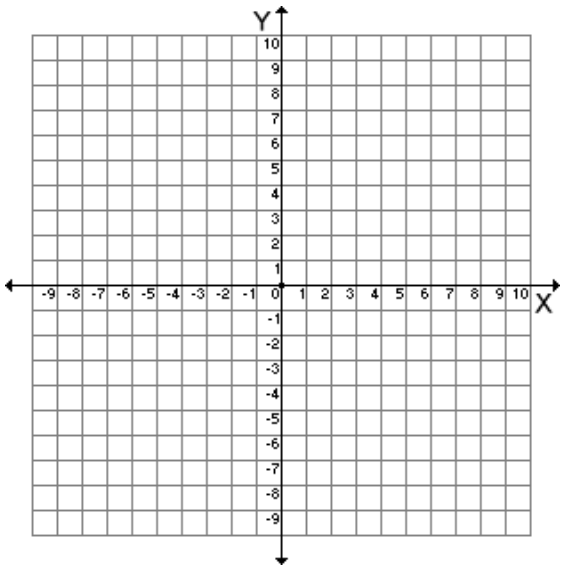
A circle has the equation $(x+1)^2+(y-3)^2=16$. What are the coordinates of its center and the length of its radius?

[A] (-1,3) and 16 [B] (1,-3) and 16

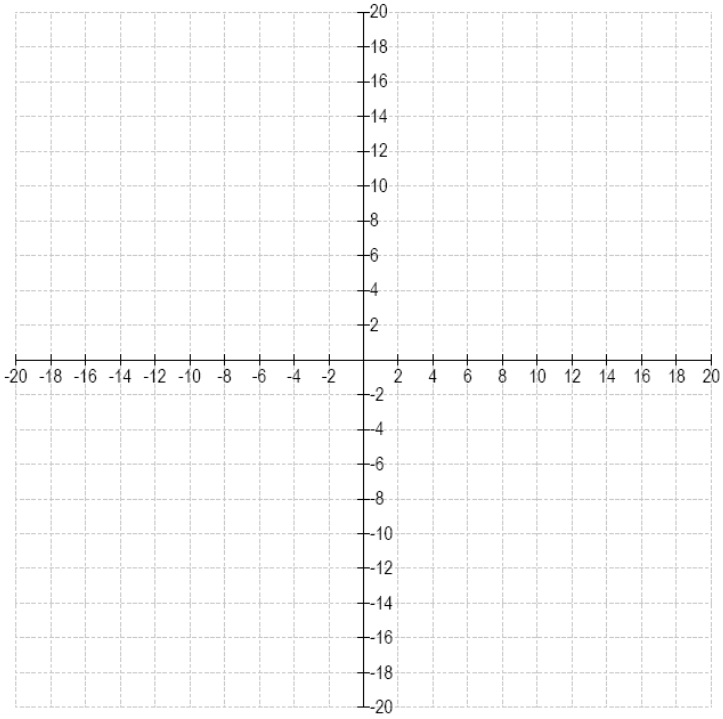
[C] (1,-3) and 4 [D] (-1,3) and 4

5. Find the roots of the equation $y = -x^2 - 4x + 2$ by completing the square.

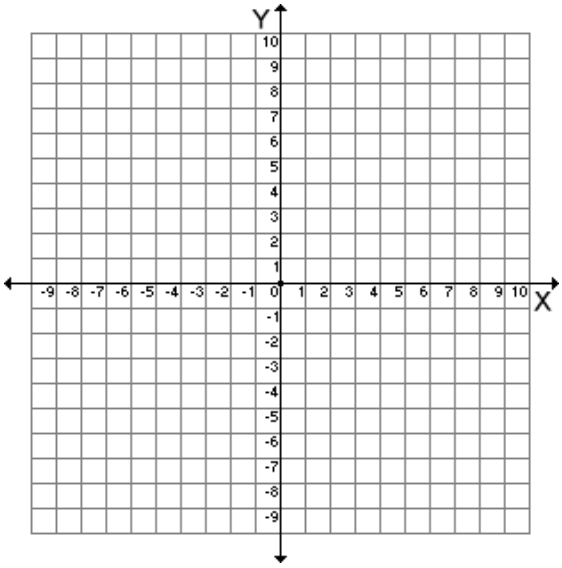
6. Write the equation of the circle in center-radius form: $x^2 + y^2 - 4x + 6y - 3 = 0$. Then graph.



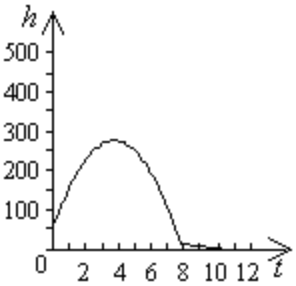
7. Write the equation of a circle given center (6, 5) and the point on the circle (0, -3). Then graph.



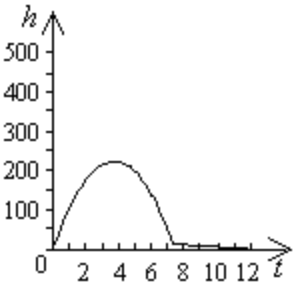
8. Graph the equation $y = x^2 - 10x + 18$. Find the exact value of the roots.



9. A rocket is launched from atop a 54-foot cliff with an initial velocity of 119 feet per second. The height of the rocket t seconds after launch is given by the equation $h = -16t^2 + 119t + 54$. Graph the equation to find out how long after the rocket is launched it will hit the ground. Estimate your answer to the nearest hundredth of a second.



○ 3.72 sec



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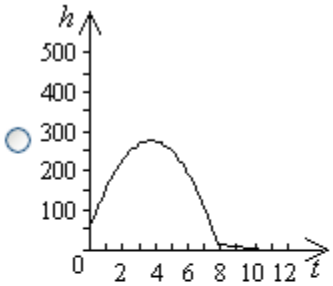

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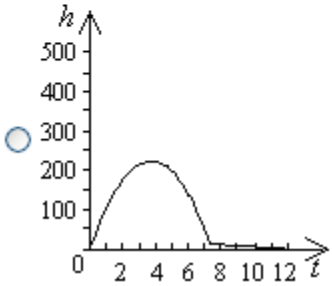


○ 7.44 sec

10. Randex Manufacturing estimates that its weekly profit, P, in hundreds of dollars, can be approximated by the formula $P = -2x^2 + 4x + 2$ where x is the number of units produced per week, in thousands.

a. How many units should the company produce per week to earn the maximum profit?

b. Find the maximum weekly profit.

○ **a.** 10 units; **b.** $1000

○ **a.** 1000 units; **b.** $400

○ **a.** 1 unit; **b.** $500

○ **a.** 100 units; **b.** $800

HW ANSWER KEYS

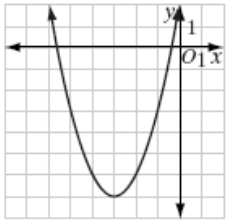
5-2 The Quadratic Formula *(pages 195–197)*

Writing About Mathematics

1. No. The denominator applies to all the terms in the numerator.
2. Yes. When $b^2 < 4ac$, the roots involve the square root of a negative number, which is not real.

Developing Skills

3. $-1, -4$
4. $-7, 1$
5. $\frac{3 \pm \sqrt{5}}{2}$
6. $\frac{1 \pm \sqrt{17}}{2}$
7. $\frac{-5 \pm \sqrt{33}}{2}$
8. $\pm 2\sqrt{2}$
9. $0, 3$
10. $-1 \pm \sqrt{5}$
11. $\frac{2}{3}, 1$
12. $\frac{1 \pm \sqrt{17}}{8}$
13. $\frac{5 \pm \sqrt{33}}{4}$
14. $\frac{1 \pm \sqrt{33}}{4}$
15. $3 \pm \sqrt{6}$
16. $\frac{1}{2} \pm \sqrt{3}$
17. $\frac{2 \pm \sqrt{10}}{3}$
18. **a.**



b. Answers will vary: $-0.4, -5.6$
c. $-3 \pm \sqrt{7}$
d. $-0.4, -5.6$

Applying Skills

19. $1 + \sqrt{6}, 7 + 2\sqrt{6}$ or $1 - \sqrt{6}, 7 - 2\sqrt{6}$
20. Width $= -1 + \sqrt{3}$ ft, length $= 1 + \sqrt{3}$ ft
21. Width $= -2 + \sqrt{46}$ cm, length $= 2 + \sqrt{46}$ cm
22. Altitude $= -3 + 3\sqrt{5}$ ft, base $= 3 + 3\sqrt{5}$ ft
23. Bases $= 8, 12$; height $= 4$
24. $DB = -2 + 2\sqrt{37}, AD = 2 + 2\sqrt{37}, AB = 4\sqrt{37}$

Chapter 5. Quadratic Functions and Complex Numbers

5-1 Real Roots of a Quadratic Equation *(pages 192–193)*

Writing About Mathematics

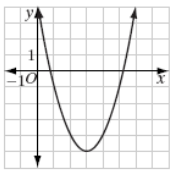
1. $0 = 2x^2 - x - 1$
   $0 = 16x^2 - 8x - 8$
   $0 = 8(x - 1)(2x + 1)$
   $x = 1, -\frac{1}{2}$
2. Yes. The resulting equation is equivalent to the original. The new equation can be solved by completing the square.

Developing Skills

3. $+9 = (x + 3)^2$
4. $+16 = (x - 4)^2$
5. $+1 = (x - 1)^2$
6. $+36 = (x - 6)^2$
7. $+2 = 2(x - 1)^2$
8. $+\frac{9}{4} = \left(x - \frac{3}{2}\right)^2$

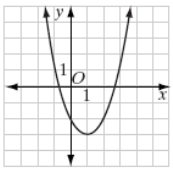
In 9–14, part **b**, answers will vary.

9. **a.**



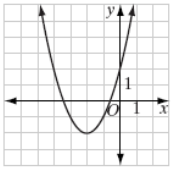
b. 0.8, 5.2
c. $3 \pm \sqrt{5}$

10. **a.**



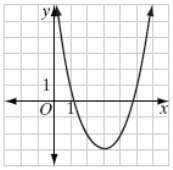
b. $-0.7, 2.7$
c. $1 \pm \sqrt{3}$

11. **a.**



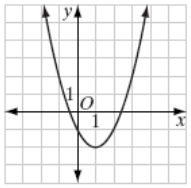
b. $-0.6, -3.4$
c. $-2 \pm \sqrt{2}$

12. **a.**



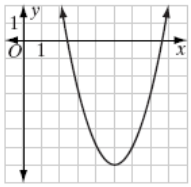
b. 1.3, 4.7
c. $3 \pm \sqrt{3}$

13. **a.**



b. $-0.4, 2.4$
c. $1 \pm \sqrt{2}$

14. **a.**



b. 2.4, 7.7
c. $5 \pm \sqrt{7}$

15. $1 \pm \sqrt{3}$
16. $-3 \pm \sqrt{5}$
17. $2 \pm \sqrt{3}$
18. $-1 \pm \sqrt{6}$
19. $3 \pm \sqrt{7}$
20. $4 \pm 2\sqrt{3}$
21. $-3 \pm \sqrt{\frac{15}{2}}$
22. $1 \pm \frac{2\sqrt{3}}{3}$
23. $\frac{3 \pm \sqrt{3}}{2}$
24. $\left\{\frac{1}{2}, \frac{9}{2}\right\}$
25. $-1 \pm \sqrt{7}$
26. $\frac{-3 \pm \sqrt{33}}{2}$

4-9 Circles *(pages 172–173)*

Writing About Mathematics

1. No. A circle does not pass the vertical line test.
2. In center-radius form, the constant term is the square of the radius, and this cannot be negative.

Developing Skills

3. **a.** $x^2 + y^2 = 4$
   b. $x^2 + y^2 - 4 = 0$
4. **a.** $x^2 + y^2 = 9$
   b. $x^2 + y^2 - 9 = 0$
5. **a.** $x^2 + y^2 = 16$
   b. $x^2 + y^2 - 16 = 0$
6. **a.** $(x - 4)^2 + (y - 2)^2 = 1$
   b. $x^2 + y^2 - 8x - 4y + 19 = 0$
7. **a.** $(x + 1)^2 + (y - 1)^2 = 16$
   b. $x^2 + y^2 + 2x - 2y - 14 = 0$
8. **a.** $(x - 6)^2 + (y - 5)^2 = 100$
   b. $x^2 + y^2 - 12x - 10y - 39 = 0$
9. **a.** $(x - 6)^2 + (y - 13)^2 = 169$
   b. $x^2 + y^2 - 12x - 26y + 36 = 0$
10. **a.** $x^2 + (y - 1)^2 = 17$
    b. $x^2 + y^2 - 2y - 16 = 0$
11. $x^2 + y^2 = 16$
12. $(x - 2)^2 + (y - 3)^2 = 1$
13. $(x - 1)^2 + (y + 1)^2 = 9$
14. $(x + 2)^2 + (y - 3)^2 = 4$
15. $(x - 1)^2 + (y + 1)^2 = 25$
16. $x^2 + (y + 1)^2 = 4$
17. $(x + 1)^2 + (y - 3)^2 = 9$
18. $(x - 1)^2 + (y - 1)^2 = 13$
19. $(x + 1)^2 + (y + 1)^2 = 13$
20. **a.** $x^2 + y^2 = 25$
    b. $(0, 0)$
    c. 5
21. **a.** $(x - 1)^2 + (y - 1)^2 = 9$
    b. $(1, 1)$
    c. 3
22. **a.** $(x + 1)^2 + (y - 2)^2 = 4$
    b. $(-1, 2)$
    c. 2
23. **a.** $(x - 3)^2 + (y + 1)^2 = 16$
    b. $(3, -1)$
    c. 4
24. **a.** $(x + 3)^2 + (y - 3)^2 = 12$
    b. $(-3, 3)$
    c. $2\sqrt{3}$
25. **a.** $x^2 + (y - 4)^2 = 16$
    b. $(0, 4)$
    c. 4
26. **a.** $(x + 5)^2 + (y - 2.5)^2 = 63.25$
    b. $(-5, 2.5)$
    c. $\sqrt{63.25} = \frac{\sqrt{253}}{2}$