

# Chapter 5: Sequences and Series

## 1. Sequences



## 2. Arithmetic and Geometric Sequences



## 3. Summation Notation



## 4. Arithmetic Series



## 5. Geometric Series



## 6. Mortgage Payments



## LESSON 1

### SEQUENCES

In Common Core Algebra I, you studied **sequences**, which are ordered lists of numbers. Sequences are extremely important in mathematics, both theoretical and applied. A **sequence** is formally defined as **a function that has as its domain the set of positive integers**, i.e.  $\{1, 2, 3, \dots, n\}$ .

**Exercise #1:** A sequence is defined by the equation  $a(n) = 2n - 1$ .

- (a) Find the first three terms of this sequence, denoted by  $a_1$ ,  $a_2$ , and  $a_3$ .      (b) Which term has a value of 53?

- (c) Explain why there will not be a term that has a value of 70.

Recall that sequences can also be described by using **recursive definitions**. When a sequence is defined recursively, terms are found by operations on previous terms.

**Exercise #2:** A sequence is defined by the recursive formula:  $f(n) = f(n-1) + 5$  with  $f(1) = -2$ .

- (a) Generate the first five terms of this sequence. Label each term with proper **function** notation.      (b) Determine the value of  $f(20)$ . Hint - think about how many times you have added 5 to  $-2$ .

**Exercise #3:** Determine a recursive definition, in terms of  $f(n)$ , for the sequence shown below. Be sure to include a starting value.

5, 10, 20, 40, 80, 160, ...

**Exercise #4:** For the recursively defined sequence  $t_n = (t_{n-1})^2 + 2$  and  $t_1 = 2$ , the value of  $t_4$  is

(1) 18

(3) 456

(2) 38

(4) 1446

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**Exercise #5:** One of the most well-known sequences is the Fibonacci, which is defined recursively using two previous terms. Its definition is given below.

$$f(n) = f(n-1) + f(n-2) \text{ and } f(1) = 1 \text{ and } f(2) = 1$$

Generate values for  $f(3)$ ,  $f(4)$ ,  $f(5)$ , and  $f(6)$  (in other words, then next four terms of this sequence).

It is often possible to find algebraic formulas for simple sequence, and this skill should be practiced.

**Exercise #6:** Find an algebraic formula  $a(n)$ , similar to that in *Exercise #1*, for each of the following sequences. Recall that the domain that you map from will be the set  $\{1, 2, 3, \dots, n\}$ .

(a) 4, 5, 6, 7, ...

(b) 2, 4, 8, 16, ...

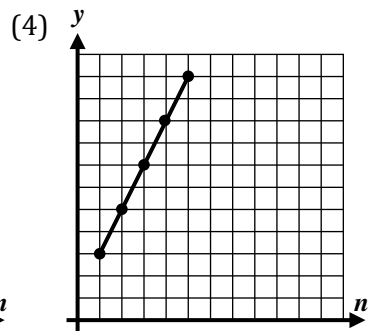
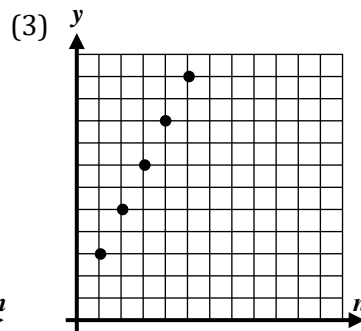
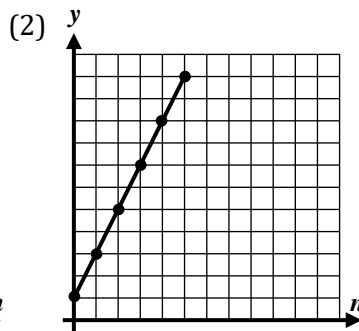
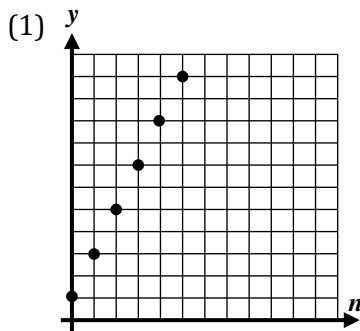
(c)  $5, \frac{5}{2}, \frac{5}{3}, \frac{5}{4}, \dots$

(d) -1, 1, -1, 1, ...

(e) 10, 15, 20, 25, ...

(f)  $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \dots$

**Exercise #7:** Which of the following would represent the graph of the sequence  $a_n = 2n + 1$ ? Explain your choice.



Explanation:

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**LESSON 1 HOMEWORK**  
**SEQUENCES**



**FLUENCY**

1. Given each of the following sequences defined by formulas, determine and label the first four terms. A variety of different notations is used below for practice purposes.

(a)  $f(n) = 7n + 2$

(b)  $a_n = n^2 - 5$

(c)  $t(n) = \left(\frac{2}{3}\right)^n$

(d)  $t_n = \frac{1}{n+1}$

2. Sequences below are defined recursively. Determine and label the **next** three terms of the sequence.

(a)  $f(1) = 4$  and  $f(n) = f(n-1) + 8$

(b)  $a(n) = a(n-1) \cdot \frac{1}{2}$  and  $a(1) = 24$

(c)  $b_n = b_{n-1} + 2n$  with  $b_1 = 5$

(d)  $f(n) = 2f(n-1) - n^2$  and  $f(1) = 4$



6. For each of the following sequences, determine an algebraic formula, similar to *Exercise #4* from the lesson, that defines the sequence. **Do not write it using the recursive formula.**

(a) 5, 10, 15, 20, ...

(b) 3, 9, 27, 81, ...

(c)  $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots$

7. For each of the following sequences, state a recursive definition. Be sure to include a starting value or values.

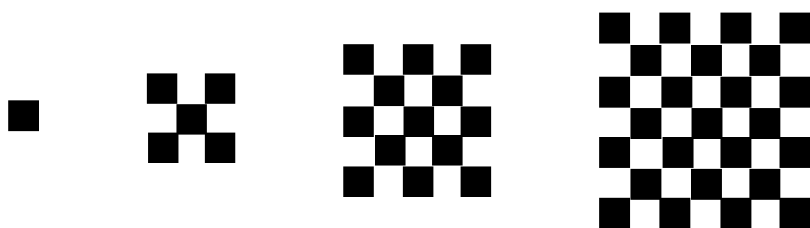
(a) 8, 6, 4, 2, ...

(b) 2, 6, 18, 54, ...

(c) 2, -2, 2, -2, ...

#### APPLICATIONS

8. A tiling pattern is created from a single square and then expanded as shown. If the number of squares in each pattern defines a sequence, then determine the number of squares in the seventh pattern. Explain how you arrived at your choice. Can you write a recursive definition for the pattern?



**REASONING**

9. Consider a sequence defined similarly to the Fibonacci, but with a slight twist:

$$f(n) = f(n-1) - f(n-2) \text{ with } f(1) = 2 \text{ and } f(2) = 5$$

Generate terms  $f(3)$ ,  $f(4)$ ,  $f(5)$ ,  $f(6)$ ,  $f(7)$ ,  $f(8)$ , and  $f(9)$ . Then, determine the value of  $f(25)$ .



## LESSON 2

### ARITHMETIC AND GEOMETRIC SEQUENCES

In Common Core Algebra I, you studied two particular sequences known as **arithmetic** (based on constant addition to get the next term) and **geometric** (based on constant multiplying to get the next term). In this lesson, we will review the basics of these two sequences.

#### ARITHMETIC SEQUENCE RECURSIVE DEFINITION

Given  $f(1)$ , then  $f(n) = f(n-1) + d$  or given  $a_1$  then  $a_n = a_{n-1} + d$

where  $d$  is called the **common difference** and can be positive or negative.

**Exercise #1:** Generate the next three terms of the given arithmetic sequences.

(a)  $f(n) = f(n-1) + 6$  with  $f(1) = 2$

(b)  $a_n = a_{n-1} + \frac{1}{2}$  and  $a_1 = \frac{3}{2}$

**Exercise #2:** For some number  $t$ , the first three terms of an arithmetic sequence are  $2t$ ,  $5t - 1$ , and  $6t + 2$ .

(a) What is the value of  $t$ ?

(b) What is the numerical value of the fourth term?

It is important to be able to determine a general term of an arithmetic sequence based on the value of the index variable (the subscript). The next exercise walks you through the thinking process involved.

**Exercise #3:** Consider  $a_n = a_{n-1} + 3$  with  $a_1 = 5$ .

(a) Determine the value of  $a_2$ ,  $a_3$ , and  $a_4$ .

(b) How many times was 3 added to 5 in order to produce  $a_4$ ?

(c) Use your result from part (b) to quickly find the value of  $a_{50}$ .

(d) Write a formula for the  $n^{\text{th}}$  term of an arithmetic sequence,  $a_n$ , based on the first term,  $a_1$ ,  $d$  and  $n$ .

**Exercise #4:** Given that  $a_1 = 6$  and  $a_4 = 18$  are members of an arithmetic sequence, determine the value of  $a_{20}$ .

**Exercise #5:** In an arithmetic sequence  $t_n = t_{n-1} - 5$ . If  $t_1 = 3$  determine the values of  $t_6$  and  $t_{25}$ . Show the calculations that lead to your answers.

**Exercise #6:** Find the number of terms in an arithmetic sequence whose first two terms are -3 and 4 and whose last term is 116.

**Geometric sequences** are defined very similarly to arithmetic, but with a multiplicative constant instead of an additive one.

**GEOMETRIC SEQUENCE RECURSIVE DEFINITION**

Given  $f(1)$  then  $f(n) = f(n-1) \cdot r$  or given  $a_1$ , then  $a_n = a_{n-1} \cdot r$

where  $r$  is called the **common ratio** and can be positive or negative and is often fractional.

**Exercise #7:** Generate the next three terms of the geometric sequences given below.

(a)  $a_1 = 4$  and  $r = 2$

(b)  $f(n) = f(n-1) \cdot \frac{1}{3}$  with  $f(1) = 9$

(c)  $t_n = t_{n-1} \cdot \sqrt{2}$  with  $t_1 = 3\sqrt{2}$

And, like arithmetic, we also need to be able to determine any given term of a geometric sequence based on the first value, the common ratio, and the index.

**Exercise #8:** Consider  $a_1 = 2$  and  $a_n = a_{n-1} \cdot 3$ .

(a) Generate the value of  $a_4$ .

(b) How many times did you need to multiply 2 by 3 in order to find  $a_4$ .

(c) Determine the value of  $a_{10}$ .

(d) Write a formula for the  $n^{\text{th}}$  term of a geometric sequence,  $a_n$ , based on the first term,  $a_1$ ,  $r$  and  $n$ .

**Exercise #9:** Given  $a_1 = -2$  and  $a_2 = 8$  are the first two terms of a geometric sequence, determine the values of  $a_4$  and  $a_8$ . Show calculations that lead to your answer.

**LESSON 2 HOMEWORK**  
**ARITHMETIC AND GEOMETRIC SEQUENCES**



**FLUENCY**

1. Generate the next **three** terms of each **arithmetic sequence** shown below.

(a)  $a_1 = -2$  and  $d = 4$

(b)  $f(n) = f(n-1) - 8$  with  $f(1) = 10$

(c)  $a_1 = 3, a_2 = 1$

2. In an arithmetic sequence  $t_n = t_{n-1} + 7$ . If  $t_1 = -5$  determine the values of  $t_4$  and  $t_{20}$ . Show the calculations that lead to your answers.

3. If  $x + 4, 2x + 5$ , and  $4x + 3$  represent the first three terms of an arithmetic sequence, then find the value of  $x$ . What is the fourth term?

4. If  $f(1) = 12$  and  $f(n) = f(n-1) - 4$  then which of the following represents the value of  $f(40)$ ?

(1)  $-148$

(3)  $-144$

(2)  $-140$

(4)  $-172$

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5. In an arithmetic sequence of numbers  $a_1 = -4$  and  $a_6 = 46$ . Which of the following is the value of  $a_{12}$ ?

(1) 120

(3) 92

(2) 146

(4) 106

6. The first term of an arithmetic sequence whose common difference is 7 and whose 22<sup>nd</sup> term is given by  $a_{22} = 143$  is which of the following? \_\_\_\_\_

(1) -25

(3) 7

(2) -4

(4) 28 \_\_\_\_\_

7. Generate the next **three** terms of each geometric sequence defined below.

(a)  $a_1 = -8$  with  $r = -1$

(b)  $a_n = a_{n-1} \cdot \frac{3}{2}$  and  $a_1 = 16$

(c)  $f(n) = f(n-1) \cdot -2$  and  $f(1) = 5$

8. Given that  $a_1 = 5$  and  $a_2 = 15$  are the first two terms of a geometric sequence, determine the values of  $a_3$  and  $a_{10}$ . Show the calculations that lead to your answers. \_\_\_\_\_

9. In a geometric sequence, it is known that  $a_1 = -1$  and  $a_4 = 64$ . The value of  $a_{10}$  is

(1)  $-65,536$

(3)  $512$

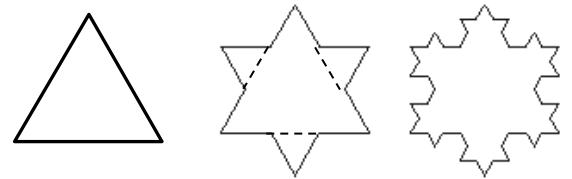
(2)  $262,144$

(4)  $-4096$

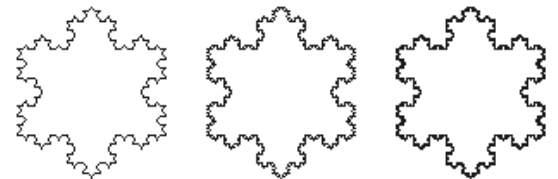
### APPLICATIONS

10. The Koch Snowflake is a mathematical shape known as a **fractal** that has many fascinating properties. It is created by repeatedly forming equilateral triangles off of the sides of other equilateral triangles. Its first six iterations are shown to the right. The perimeters of each of the figures form a geometric sequence.

(a) If the perimeter of the first snowflake (the equilateral triangle) is 3, what is the perimeter of the second snowflake? Note: the dashed lines in the second snowflake are not to be counted towards the perimeter. They are only there to show how the snowflake was constructed.



(b) Given that the perimeters form a geometric sequence, what is the perimeter of the sixth snowflake? Express your answer to the nearest tenth.



(c) If this process was allowed to continue forever, explain why the perimeter would become infinitely large.

### LESSON 3

## SUMMATION NOTATION

Much of our work in this unit will concern **adding the terms of a sequence**. In order to specify this addition or summarize it, we introduce a new notation, known as **summation or sigma notation** that will represent these sums. This notation will also be used later in the course when we want to write formulas used in statistics.

#### SUMMATION (SIGMA) NOTATION

$$\sum_{i=a}^n f(i) = f(a) + f(a+1) + f(a+2) + \cdots + f(n)$$

where  $i$  is called the **index variable**, which starts at a value of  $a$ , ends at a value of  $n$ , and moves by unit increments (increase by 1 each time).

**Exercise #1:** Evaluate each of the following sums.

(a)  $\sum_{i=3}^5 2i$

(b)  $\sum_{k=-1}^3 k^2$

(c)  $\sum_{j=-2}^2 2^j$

(d)  $\sum_{i=1}^5 (-1)^i$

(e)  $\sum_{k=0}^2 (2k+1)$

(f)  $\sum_{i=1}^3 i(i+1)$



**Exercise #2:** Which of the following represents the value of  $\sum_{i=1}^4 \frac{1}{i}$ ?

(1)  $\frac{1}{10}$

(3)  $\frac{25}{12}$

(2)  $\frac{9}{4}$

(4)  $\frac{31}{24}$

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**Exercise #3:** Consider the sequence defined recursively by  $a_n = a_{n-1} + 2a_{n-2}$  and  $a_1 = 0$  and  $a_2 = 1$ . Find the value of  $\sum_{i=4}^7 a_i$

It is also good to be able to place sums into sigma notation. These answers, though, will not be unique.

**Exercise #4:** Express each sum using sigma notation. Use  $i$  as your index variable. First, consider any patterns you notice amongst the terms involved in the sum. Then, work to put these patterns into a formula and sum.

(a)  $9+16+25+\dots+100$

(b)  $-6+-3+0+3+\dots+15$

(c)  $\frac{1}{25} + \frac{1}{5} + 1 + 5 + \dots + 625$

**Exercise #5:** Which of the following represents the sum  $3+6+12+24+48$ ?

(1)  $\sum_{i=1}^5 3^i$

(3)  $\sum_{i=0}^4 6^{i-1}$

(2)  $\sum_{i=0}^4 3(2)^i$

(4)  $\sum_{i=3}^{48} i$

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**Exercise #6:** Some sums are more interesting than others. Determine the value of  $\sum_{i=1}^{99} \left( \frac{1}{i} - \frac{1}{i+1} \right)$ . Show your reasoning. This is known as a **telescoping series (or sum)**.

**LESSON 3 HOMEWORK**  
**SUMMATION NOTATION**



**FLUENCY**

1. Evaluate each of the following. Place any non-integer answer in simplest rational form.

$$(a) \sum_{i=2}^5 4i$$

$$(b) \sum_{k=0}^3 (k^2 + 1)$$

$$(c) \sum_{j=-2}^0 (2j + 1)$$

$$(d) \sum_{i=1}^3 2^i$$

$$(e) \sum_{k=0}^2 (-1)^{2k+1}$$

$$(f) \sum_{i=1}^3 \log(10^i)$$

$$(g) \sum_{n=1}^4 \frac{n}{n+1}$$

$$(h) \frac{\sum_{i=2}^4 (i+1)^2}{\sum_{i=2}^4 (i^2 + 1)}$$

$$(i) \sum_{k=0}^3 256^{\frac{1}{2^k}}$$

2. Which of the following is the value of  $\sum_{k=0}^4 (4k + 1)$ ?

(1) 53

(3) 37

(2) 45

(4) 80

3. The sum  $\sum_{i=4}^7 2^{i-7}$  is equal to

(1)  $\frac{15}{8}$

(3)  $\frac{3}{4}$

(2)  $\frac{3}{2}$

(4)  $\frac{7}{8}$

4. Write each of the following sums using sigma notation. Use  $k$  as your index variable. Note, there are many correct ways to write each sum (and even more incorrect ways).

(a)  $-2 + 4 + -8 + \dots + 64 + -128$

(b)  $\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{81} + \frac{1}{100}$

(c)  $4 + 9 + 14 + \dots + 44 + 49$

5. Which of the following represents the sum  $2 + 5 + 10 + \dots + 82 + 101$ ?

(1)  $\sum_{j=1}^6 (4j - 3)$

(3)  $\sum_{j=1}^{10} (j^2 + 1)$

(2)  $\sum_{j=3}^{103} (j - 2)$

(4)  $\sum_{j=0}^{11} (4^j + 1)$

6. A sequence is defined recursively by the formula  $b_n = 4b_{n-1} - 2b_{n-2}$  with  $b_1 = 1$  and  $b_2 = 3$ . What is the value of  $\sum_{i=3}^5 b_i$ ? Show the work that leads to your answer.

### REASONING

7. A curious pattern occurs when we look at the behavior of the sum  $\sum_{k=1}^n (2k - 1)$ .

(a) Find the value of this sum for a variety of values of  $n$  below:

$$n = 2: \sum_{k=1}^2 (2k - 1) =$$

$$n = 4: \sum_{k=1}^4 (2k - 1) =$$

$$n = 3: \sum_{k=1}^3 (2k - 1) =$$

$$n = 5: \sum_{k=1}^5 (2k - 1) =$$

(b) What types of numbers are you summing?  
What types of numbers are the sums?

(c) Find the value of  $n$  such that  $\sum_{k=1}^n (2k - 1) = 196$ .

## LESSON 4 ARITHMETIC SERIES

A **series** is simply the **sum of the terms of a sequence**. The fundamental definition/notion of a series is below.

### THE DEFINITION OF A SERIES

If the set  $\{a_1, a_2, a_3, \dots\}$  represent the elements of a sequence then the series,  $S_n$ , is defined by:

$$S_n = \sum_{i=1}^n a_i$$

In truth, you have already worked extensively with series in previous lessons almost anytime you evaluated a summation problem.

**Exercise #1:** Given the arithmetic sequence defined by  $a_1 = -2$  and  $a_n = a_{n-1} + 5$ , then which of the following is the value of  $S_5 = \sum_{i=1}^5 a_i$ ?

(1) 32

(3) 25

(2) 40

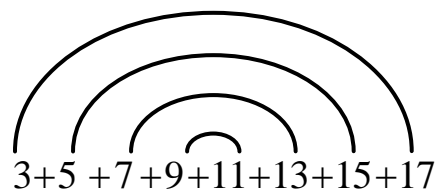
(4) 27

The sums associated with arithmetic sequences, known as **arithmetic series**, have interesting properties, many applications and values that can be predicted with what is commonly known as **rainbow addition**.

**Exercise #2:** Consider the arithmetic sequence defined by  $a_1 = 3$  and  $a_n = a_{n-1} + 2$ . The series, based on the first eight terms of this sequence, is shown below. Terms have been paired off as shown.

(a) What does each of the paired off sums equal?

(b) Why does it make sense that this sum is constant?



(c) How many of these pairs are there?

(d) Using your answers to (a) and (c) find the value of the sum using a multiplicative process.

(e) Generalize this now and create a formula for an arithmetic series sum based only on its first term,  $a_1$ , its last term,  $a_n$ , and the number of terms,  $n$ .

### SUM OF AN ARITHMETIC SERIES

Given an arithmetic series with  $n$  terms,  $\{a_1, a_2, \dots, a_n\}$ , then its sum is given by:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

**Exercise #3:** Which of the following is the sum of the first 100 natural numbers? Show the process that leads to your choice.

(1) 5,000

(3) 10,000

(2) 5,100

(4) 5,050

\_\_\_\_\_

**Exercise #4:** Find the sum of each arithmetic series described or shown below.

(a) The sum of the sixteen terms given by:

$$-10 + -6 + -2 + \dots + 46 + 50.$$

(b) The first term is  $-8$ , the common difference,  $d$ , is 6 and there are 20 terms

(c) The last term is  $a_{12} = -29$  and the common difference,  $d$ , is  $-3$ .

(d) The sum  $5 + 8 + 11 + \dots + 77$ .

**Exercise #5:** The first and last terms of an arithmetic series are 7 and -121, respectively, and the series has a sum of -1026. How many terms are in this series?

**Exercise #6:** Kirk has set up a college savings account for his son, Maxwell. If Kirk deposits \$100 per month in an account, increasing the amount he deposits by \$10 per month each month, then how much will be in the account after 10 years?



**LESSON 4 HOMEWORK**  
**ARITHMETIC SERIES**



**FLUENCY**

1. Which of the following represents the sum of  $3+10+\cdots+87+94$  if the arithmetic series has 14 terms?  
(1) 1,358                      (3) 679  
(2) 658                         (4) 1,276
  
2. The sum of the first 50 natural numbers is  
(1) 1,275                      (3) 1,250  
(2) 1,875                      (4) 950
  
3. If the first and last terms of an arithmetic series are 5 and 27, respectively, and the series has a sum 192, then the number of terms in the series is  
(1) 18                         (3) 14  
(2) 11                         (4) 12

4. Find the sum of each arithmetic series described or shown below.
- (a) The sum of the first 100 even, natural numbers.      (b) The sum of multiples of five from 10 to 75, inclusive.
- (c) A series whose first two terms are  $-12$  and  $-8$ , respectively, and whose last term is 124.      (d) A series of 20 terms whose last term is equal to 97 and whose common difference is five.
5. For an arithmetic series that sums to 1,485, it is known that the first term equals 6 and the last term equals 93. *Algebraically* determine the number of terms summed in this series.

## APPLICATIONS

6. Arlington High School recently installed a new black-box theatre for local productions. They only had room for 14 rows of seats, where the number of seats in each row constitutes an arithmetic sequence starting with eight seats and increasing by two seats per row thereafter. How many seats are in the new black-box theatre? Show the calculations that lead to your answer.
7. Simeon starts a retirement account where he will place \$50 into the account on the first month and increase his deposit by \$5 per month each month after. If he saves this way for the next 20 years, how much will the account contain in principal?

8. The distance an object falls per second while only under the influence of gravity forms an arithmetic sequence with it falling 16 feet in the first second, 48 feet in the second, 80 feet in the third, etcetera. What is the total distance an object will fall in 10 seconds? Show the work that leads to your answer.

9. A large grandfather clock strikes its bell once at 1:00, twice at 2:00, three times at 3:00, etcetera. What is the total number of times the bell will be struck in a day? Use an arithmetic series to help solve the problem and show how you arrived at your answer.

## LESSON 5 GEOMETRIC SERIES

Just as we can sum the terms of an arithmetic sequence to generate an arithmetic series, we can also sum the terms of a geometric sequence to generate a **geometric series**.

**Exercise #1:** Given a geometric series defined by the recursive formula  $a_1 = 3$  and  $a_n = a_{n-1} \cdot 2$ , which of the following is the value of  $S_5 = \sum_{i=1}^5 a_i$ ?

(1) 106

(3) 93

(2) 75

(4) 35

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### SUM OF A FINITE GEOMETRIC SERIES

For a geometric series defined by its first term,  $a_1$ , and its common ratio,  $r$ , the sum of  $n$  terms is given by:

$$S_n = \frac{a_1(1-r^n)}{1-r} \quad \text{or} \quad S_n = \frac{a_1 - a_1 r^n}{1-r}$$

**Exercise #2:** Which of the following represents the sum of a geometric series with 8 terms whose first term is 3 and whose common ratio is 4?

(1) 32,756

(3) 42,560

(2) 28,765

(4) 65,535

**Exercise #3:** Find the value of the geometric series shown below. Show the calculations that lead to your final answer.

$$6+12+24+\cdots+768$$

**Exercise #4:** Maria places \$500 at the beginning of each year into an account that earns 5% interest compounded annually. Maria would like to determine how much money is in her account after she has made her \$500 deposit at the end of 10 years.

(a) Determine a formula for the amount,  $A(t)$ , that a given \$500 has grown to  $t$ -years after it was placed into this account.

(b) At the end of 10 years, which will be worth more: the \$500 invested in the first year or the fourth year? Explain by showing how much each is worth at the beginning of the 11th year.

(c) Based on (b), write a geometric sum representing the amount of money in Maria's account after 10 years.

(d) Evaluate the sum in (c) using the formula above.

**Exercise #5:** A person places 1 penny in a piggy bank on the first day of the month, 2 pennies on the second day, 4 pennies on the third, and so on. Will this person be a millionaire at the end of a 31 day month? Show the calculations that lead to your answer.

**Exercise #6:** Find the sum of the first 10 terms of the sequence  $\frac{1}{5}, \frac{2}{5}, \frac{4}{5}, \frac{8}{5}, \dots$

**Exercise #7:** A geometric series has a first term of 8 and last term of  $\frac{1}{16}$ . Its common ratio is  $\frac{1}{2}$ . Find the value of this series.

**LESSON 5 HOMEWORK**  
**GEOMETRIC SERIES**



**FLUENCY**

1. Find the sums of geometric series with the following properties:

(a)  $a_1 = 6, r = 3$  and  $n = 8$

(b)  $a_1 = 20, r = \frac{1}{2}$ , and  $n = 6$

(c)  $a_1 = -5, r = -2$ , and  $n = 10$

2. If the geometric series  $54 + 36 + \dots + \frac{128}{27}$  has seven terms in its sum then the value of the sum is

(1)  $\frac{4118}{27}$

(3)  $\frac{1370}{9}$

(2)  $\frac{1274}{3}$

(4)  $\frac{8241}{54}$

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3. A geometric series has a first term of 32 and a final term of  $-\frac{1}{4}$  and a common ratio of  $-\frac{1}{2}$ . The value of this series is

(1) 19.75

(3) 22.5

(2) 16.25

(4) 21.25

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4. Which of the following represents the value of  $\sum_{i=0}^8 256 \left(\frac{3}{2}\right)^i$ ? Think carefully about how many terms this series has in it.

(1) 19,171

(3) 22,341

(2) 12,610

(4) 8,956

\_\_\_\_\_

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5. A geometric series whose first term is 3 and whose common ratio is 4 sums to 4095. The number of terms in this sum is

(1) 8

(3) 6

(2) 5

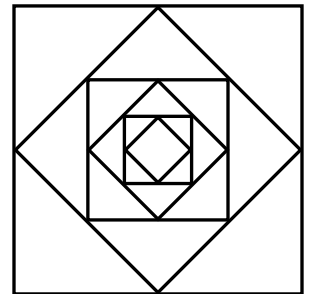
(4) 4

6. Find the sum of the geometric series shown below. Show the work that leads to your answer.

$$27 + 9 + 3 + \dots + \frac{1}{729}$$

#### APPLICATIONS

7. In the picture shown at the right, the outer most square has an area of 16 square inches. All other squares are constructed by connecting the midpoints of the sides of the square it is inscribed within. Find the sum of the areas of all of the squares shown. First, consider the how the area of each square relates to the larger square that surrounds (circumscribes) it.



8. A college savings account is constructed so that \$1000 is placed the account on January 1<sup>st</sup> of each year with a guaranteed 3% yearly return in interest, applied at the end of each year to the balance in the account. If this is repeatedly done, how much money is in the account after the \$1000 is deposited at the beginning of the 19<sup>th</sup> year? Show the sum that leads to your answer as well as relevant calculations.

9. A ball is dropped from 16 feet above a hard surface. After each time it hits the surface, it rebounds to a height that is  $\frac{3}{4}$  of its previous maximum height. What is the total vertical distance, to the nearest foot, the ball has traveled when it strikes the ground for the 10<sup>th</sup> time? Write out the first five terms of this sum to help visualize.

## LESSON 6

### MORTGAGE PAYMENTS

**Mortgages** are not just made on houses. They are large amounts of money borrowed from a bank on which interest is calculated. The interest is calculated on a regular basis (usually monthly). Regular payments are made on the amount of money owed so that over time the principal (original amount borrowed) is paid off as well as any interest on the amount owed.

This is a complex process that involved geometric series.

**Formula:**

$$m = \frac{P\left(\frac{r}{12}\right)}{1 - \left(1 + \frac{r}{12}\right)^{-n}}$$

$m$  = mortgage payment (monthly)  
 $p$  = principal (loan amount)  
 $r$  = interest rate (decimal)  
 $n$  = number of payments

**Examples:**

1.) Tom's house will cost \$300,000. He needs to pay a deposit of 10% and will pay the remaining 90% over 30 years at 8% interest rate per year. How much does Tom need per month to buy this house?

2.) Calculate the monthly payment needed to pay off a \$200,000 loan at 4% yearly interest rate over a 20 year period. Now calculate the pay off period to be 30 years. How much less is the monthly payment?

3.) You would like to buy a home priced at \$200,000. You plan to make a payment of 10% of the purchase price.

(a) Compute the total monthly payment for a 30-year mortgage at 4.8% annual interest.

(b) What is the total interest paid over the life of the loan?

(c) Compute the total monthly payment and the total interest paid over the life of a 20-year mortgage at 4.8% annual interest.

(d) Why would someone choose a 20-year mortgage over a 30-year mortgage? Why might another person choose a 30-year mortgage?

4.) Suppose you would like to buy a home priced at \$180,000. You qualify for a 30-year mortgage at 4.5% annual interest.

(a) Calculate the total monthly payment and the interest paid over the life of the loan if you make a 3% down payment.

(b) Calculate the total monthly payment and the interest paid over the life of the loan if you make a 10% down payment.

(c) Calculate the total monthly payment and the interest paid over the life of the loan if you make a 20% down payment.

5.) The following amortization table shows the amount of payments to principal and interest on a \$100,000 mortgage at the beginning and end of a 30-year loan.

| Month/ Year | Payment   | Principal Paid | Interest Paid | Total Interest | Balance      |
|-------------|-----------|----------------|---------------|----------------|--------------|
| Sept. 2014  | \$ 477.42 | \$ 144.08      | \$ 333.33     | \$ 333.33      | \$ 99,855.92 |
| Oct. 2014   | \$ 477.42 | \$ 144.56      | \$ 332.85     | \$ 666.19      | \$ 99,711.36 |
| Nov. 2014   | \$ 477.42 | \$ 145.04      | \$ 332.37     | \$ 998.56      | \$ 99,566.31 |
| Dec. 2014   | \$ 477.42 | \$ 145.53      | \$ 331.89     | \$ 1,330.45    | \$ 99,420.78 |
| Jan. 2015   | \$ 477.42 | \$ 146.01      | \$ 331.40     | \$ 1,661.85    | \$ 99,274.77 |
| Mar. 2044   | \$ 477.42 | \$ 467.98      | \$ 9.44       | \$ 71,845.82   | \$ 2,363.39  |
| April 2044  | \$ 477.42 | \$ 469.54      | \$ 7.88       | \$ 71,853.70   | \$ 1,893.85  |
| May 2044    | \$ 477.42 | \$ 471.10      | \$ 6.31       | \$ 71,860.01   | \$ 1,422.75  |
| June 2044   | \$ 477.42 | \$ 472.67      | \$ 4.74       | \$ 71,864.75   | \$ 950.08    |
| July 2044   | \$ 477.42 | \$ 474.25      | \$ 3.17       | \$ 71,867.92   | \$ 475.83    |
| Aug. 2044   | \$ 477.42 | \$ 475.83      | \$ 1.59       | \$ 71,869.51   | \$ 0.00      |

(a) Describe the changes in the amount of principal paid each month as the end of the loan approaches.

(b) Describe the changes in the amount of interest paid each month as the end of the loan approaches.

## LESSON 6 HOMEWORK MORTGAGE PAYMENTS

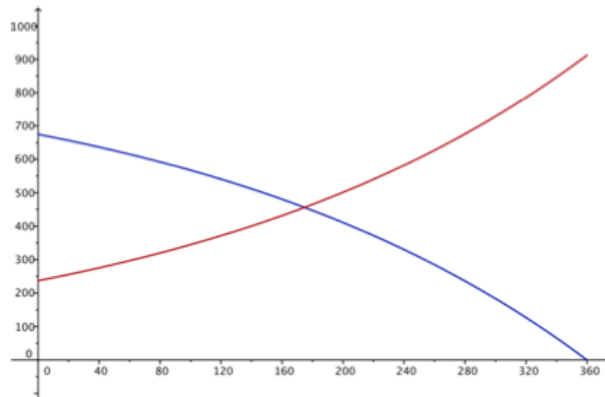


Complete the following questions. Show all work, including formulas.

1.) Christopher wants to buy a \$200,000 home with a 30-year mortgage at 4.5% annual interest paying 10% down.

(a) What is the monthly payment on the house?

(b) The graph below depicts the amount of your payment from part (b) that goes to the interest on the loan and the amount that goes to the principal on the loan. Explain how you can tell which graph is which.



2.) In the summer of 2014, the average listing price for homes for sale in the Hollywood Hills was **\$2,663,995**.

Suppose you want to buy a home at that price with a **30-year** mortgage at **5.25%** annual interest, paying **10%** as a down payment. What is your total monthly payment on this house?

3.) Suppose that you want to buy a house that costs \$175,000. You can make a 5% down payment, and

a. Find the monthly payment for a 30-year mortgage on this house at a 4.25% interest rate.

b. Find the monthly payment for a 15-year mortgage on this house at the same interest rate.

4.) Suppose that you would like to buy a home priced at **\$450,000**. You qualify for a **30-year** mortgage at **4.5%** annual interest

a. Calculate the total monthly payment and the total interest paid over the life of the loan if you make a **3%** down payment.

b. Calculate the total monthly payment and the total interest paid over the life of the loan if you make a **10%** down payment.

c. Calculate the total monthly payment and the total interest paid over the life of the loan if you make a **20%** down payment.