CHAPTER 6

# Chapter Summary

| *WHAT did you learn?* | *WHY did you learn it?* |
|---|---|
| Identify, name, and describe polygons. (6.1 ) | Lay the foundation for work with polygons. |
| Use the sum of the measures of the interior angles of a quadrilateral. (6.1) | Find an unknown measure of an angle of a quadrilateral. (p. 324) |
| Use properties of parallelograms. (6.2) | Solve problems in areas such as furniture design. (p. 333) |
| Prove that a quadrilateral is a parallelogram. (6.3) | Explore real-life tools, such as a bicycle derailleur. (p. 343) |
| Use coordinate geometry with parallelograms. (6.3) | Use coordinates to prove theorems. (p. 344) |
| Use properties of rhombuses, rectangles, and squares, including properties of diagonals. (6.4) | Simplify real-life tasks, such as building a rectangular frame. (p. 350) |
| Use properties of trapezoids and kites. (6.5) | Reach conclusions about geometric figures and real-life objects, such as a wedding cake. (p. 357) |
| Identify special types of quadrilaterals based on limited information. (6.6) | Describe real-world shapes, such as tents. (p. 367) |
| Prove that a quadrilateral is a special type of quadrilateral. (6.6) | Use alternate methods of proof. (p. 365) |
| Find the areas of rectangles, kites, parallelograms, squares, triangles, trapezoids, and rhombuses. (6.7) | Find areas of real-life surfaces, such as the roof of a covered bridge. (p. 378) |

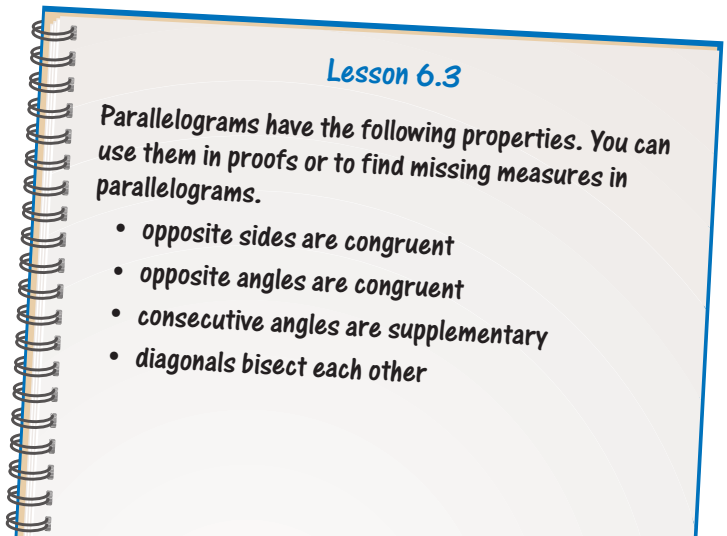
## *How does Chapter 6 fit into the BIGGER PICTURE of geometry?*

In this chapter, you studied properties of polygons, focusing on properties of quadrilaterals. You learned in Chapter 4 that a triangle is a rigid structure. Polygons with more than three sides do not form rigid structures. For instance, on page 336, you learned that a scissors lift can be raised and lowered because its beams form parallelograms, which are nonrigid figures. Quadrilaterals occur in many natural and manufactured structures. Understanding properties of special quadrilaterals will help you analyze real-life problems in areas such as architecture, design, and construction.

**STUDY STRATEGY**

## *How did your study group help you learn?*

The notes you made, following the **Study Strategy** on page 320, may resemble this one about order of operations.



CHAPTER 6

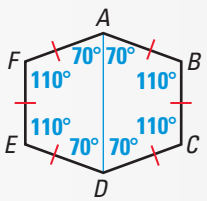
# Chapter Review

- polygon, p. 322
- sides of a polygon, p. 322
- vertex, vertices, p. 322
- convex, p. 323
- nonconvex, concave, p. 323
- equilateral polygon, p. 323
- equiangular polygon, p. 323
- regular polygon, p. 323
- diagonal of a polygon, p. 324
- parallelogram, p. 330
- rhombus, p. 347
- rectangle, p. 347
- square, p. 347
- trapezoid, p. 356
- bases of a trapezoid, p. 356
- base angles of a trapezoid, p. 356
- legs of a trapezoid, p. 356
- isosceles trapezoid, p. 356
- midsegment of a trapezoid, p. 357
- kite, p. 358

## 6.1 POLYGONS

Examples on pp. 322–324

**EXAMPLES** Hexagon $ABCDEF$ is convex and equilateral. It is not regular because it is not both equilateral and equiangular. $\overline{AD}$ is a diagonal of $ABCDEF$. The sum of the measures of the interior angles of quadrilateral $ABCD$ is 360°.



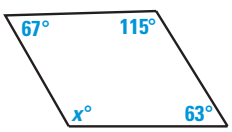

**Draw a figure that fits the description.**

**1.** a regular pentagon

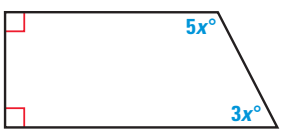
**2.** a concave octagon

**Find the value of *x*.**

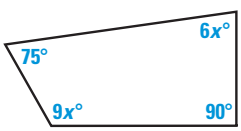
**3.**




**4.**



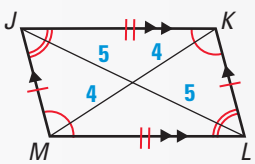

**5.**




## 6.2 PROPERTIES OF PARALLELOGRAMS

Examples on pp. 330–333

**EXAMPLES** Quadrilateral $JKLM$ is a parallelogram. Opposite sides are parallel and congruent. Opposite angles are congruent. Consecutive angles are supplementary. The diagonals bisect each other.

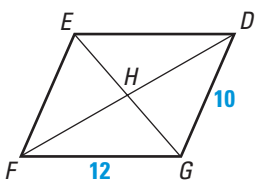



**Use parallelogram *DEFG* at the right.**

**6.** If $DH = 9.5$, find $FH$ and $DF$.

**7.** If $m\angle GDE = 65°$, find $m\angle EFG$ and $m\angle DEF$.

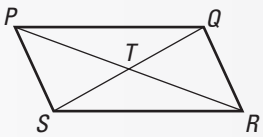
**8.** Find the perimeter of $\square DEFG$.



## 6.3 Proving Quadrilaterals are Parallelograms

Examples on pp. 338–341

**EXAMPLES** You are given that $\overline{PQ} \cong \overline{RS}$ and $\overline{PS} \cong \overline{RQ}$. Since both pairs of opposite sides are congruent, $PQRS$ must be a parallelogram.




**Is *PQRS* a parallelogram? Explain.**

**9.** $PQ = QR, RS = SP$

**10.** $\angle SPQ \cong \angle QRS, \angle PQR \cong \angle RSP$

**11.** $\overline{PS} \cong \overline{RQ}, \overline{PQ} \parallel \overline{RS}$

**12.** $m\angle PSR + m\angle SRQ = 180°, \angle PSR \cong \angle RQP$

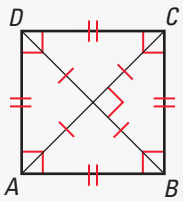
## 6.4 Rhombuses, Rectangles, and Squares

Examples on pp. 347–350

**EXAMPLES** $ABCD$ is a rhombus since it has 4 congruent sides. The diagonals of a rhombus are perpendicular and each one bisects a pair of opposite angles.

$ABCD$ is a rectangle since it has 4 right angles. The diagonals of a rectangle are congruent.

$ABCD$ is a square since it has 4 congruent sides and 4 right angles.



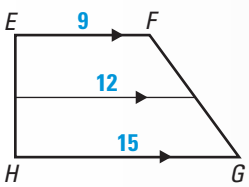

**List each special quadrilateral for which the statement is always true. Consider parallelograms, rectangles, rhombuses, and squares.**

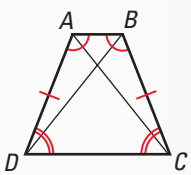
**13.** Diagonals are perpendicular. **14.** Opposite sides are parallel. **15.** It is equilateral.

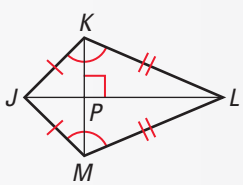
## 6.5 Trapezoids and Kites

Examples on pp. 356–358

**EXAMPLES** $EFGH$ is a trapezoid. $ABCD$ is an isosceles trapezoid. Its base angles and diagonals are congruent. $JKLM$ is a kite. Its diagonals are perpendicular, and one pair of opposite angles are congruent.




**Use the diagram of isosceles trapezoid *ABCD*.**

**16.** If $AB = 6$ and $CD = 16$, find the length of the midsegment.

**17.** If $m\angle DAB = 112°$, find the measures of the other angles of $ABCD$.

**18.** Explain how you could use congruent triangles to show that $\angle ACD \cong \angle BDC$.

## 6.6 SPECIAL QUADRILATERALS

Examples on pp. 364–366

**EXAMPLES** To prove that a quadrilateral is a rhombus, you can use any one of the following methods.

- Show that it has four congruent sides.
- Show that it is a parallelogram whose diagonals are perpendicular.
- Show that each diagonal bisects a pair of opposite angles.

**What special type of quadrilateral is *PQRS*? Give the most specific name, and justify your answer.**

**19.** $P(0, 3)$, $Q(5, 6)$, $R(2, 11)$, $S(-3, 8)$

**20.** $P(0, 0)$, $Q(6, 8)$, $R(8, 5)$, $S(4, -6)$

**21.** $P(2, -1)$, $Q(4, -5)$, $R(0, -3)$, $S(-2, 1)$

**22.** $P(-5, 0)$, $Q(-3, 6)$, $R(1, 6)$, $S(1, 2)$

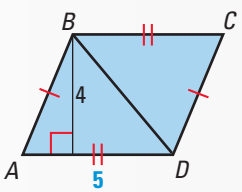
## 6.7 AREAS OF TRIANGLES AND QUADRILATERALS

Examples on pp. 372–375

**EXAMPLES**

Area of $\square ABCD = bh = 5 \cdot 4 = 20$

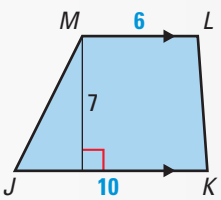
Area of $\triangle ABD = \frac{1}{2}bh = \frac{1}{2} \cdot 5 \cdot 4 = 10$




Area of trapezoid $JKLM = \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2} \cdot 7 \cdot (10 + 6)$

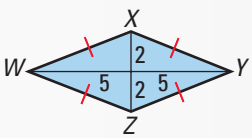
$= 56$




Area of rhombus $WXYZ = \frac{1}{2}d_1d_2$

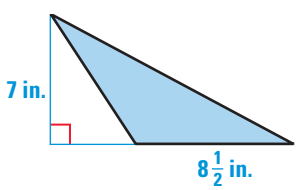
$= \frac{1}{2} \cdot 10 \cdot 4$

$= 20$

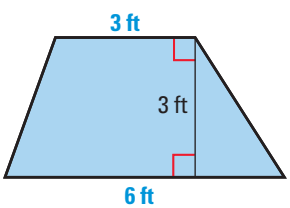



**Find the area of the triangle or quadrilateral.**

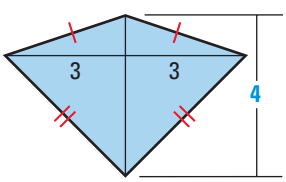
**23.**




**24.**




**25.**



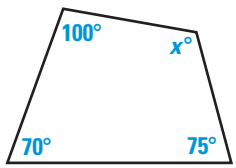
CHAPTER 6

# Chapter Test

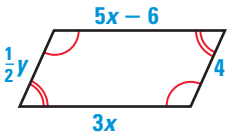
**1.** Sketch a concave pentagon.

**Find the value of each variable.**

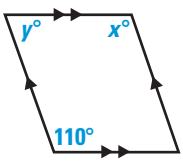
**2.**



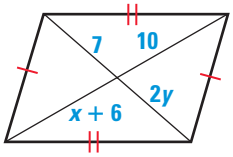

**3.**




**4.**




**5.**




**Decide if you are given enough information to prove that the quadrilateral is a parallelogram.**

**6.** Diagonals are congruent.

**7.** Consecutive angles are supplementary.

**8.** Two pairs of consecutive angles are congruent.

**9.** The diagonals have the same midpoint.

**Decide whether the statement is *always, sometimes,* or *never* true.**

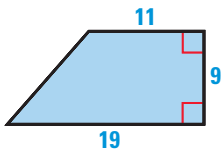
**10.** A rectangle is a square.

**11.** A parallelogram is a trapezoid.

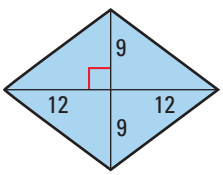
**12.** A rhombus is a parallelogram.

**What special type of quadrilateral is shown? Justify your answer.**

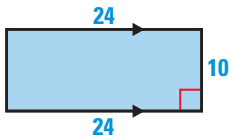
**13.**



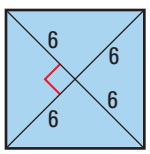

**14.**



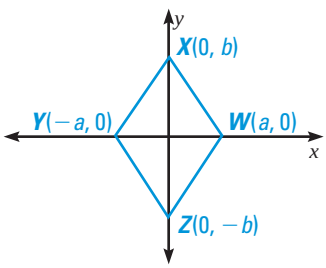

**15.**




**16.**

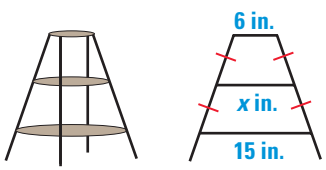



**17.** Refer to the coordinate diagram at the right. Use the Distance Formula to prove that *WXYZ* is a rhombus. Then explain how the diagram can be used to show that the diagonals of a rhombus bisect each other and are perpendicular.




**18.** Sketch a kite and label it *ABCD*. Mark all congruent sides and angles of the kite. State what you know about the diagonals $\overline{AC}$ and $\overline{BD}$ and justify your answer.

**19.** **PLANT STAND** You want to build a plant stand with three equally spaced circular shelves. You want the top shelf to have a diameter of 6 inches and the bottom shelf to have a diameter of 15 inches. The diagram at the right shows a vertical cross section of the plant stand. What is the diameter of the middle shelf?




**20.** **HIP ROOF** The sides of a *hip roof* form two trapezoids and two triangles, as shown. The two sides not shown are congruent to the corresponding sides that are shown. Find the total area of the sides of the roof.