

CHAPTER 6: QUADRATIC EQUATIONS

Specific Expectations Addressed in the Chapter

- Interpret real and non-real roots of quadratic equations, through investigation using graphing technology, and relate the roots to the x -intercepts of the corresponding relations. [6.1, 6.5]
- Express $y = ax^2 + bx + c$ in the form $y = a(x - h)^2 + k$ by completing the square in situations involving no fractions, using a variety of tools (e.g., concrete materials, diagrams, paper and pencil). [6.2, 6.3, 6.6, Chapter Task]
- Sketch or graph a quadratic relation whose equation is given in the form $y = ax^2 + bx + c$, using a variety of methods (e.g., sketching $y = x^2 - 2x - 8$ using intercepts and symmetry; sketching $y = 3x^2 - 12x + 1$ by completing the square and applying transformations; graphing $h = -4.9t^2 + 50t + 1.5$ using technology). [6.1, 6.3]
- Explore the algebraic development of the quadratic formula (e.g., given the algebraic development, connect the steps to a numerical example; follow a demonstration of the algebraic development [student reproduction of the development of the general case is not required]). [6.4]
- Solve quadratic equations that have real roots, using a variety of methods (i.e., factoring, using the quadratic formula, graphing). [6.1, 6.4, 6.6, Chapter Task]
- Determine the zeros and the maximum or minimum value of a quadratic relation from its graph (i.e., using graphing calculators or graphing software) or from its defining equation (i.e., by applying algebraic techniques). [6.1, 6.3, 6.6, Chapter Task]
- Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?). [6.1, 6.3, 6.4, 6.6, Chapter Task]

Prerequisite Skills Needed for the Chapter

- Graph a quadratic relation given in standard, factored, or vertex form.
- Expand and simplify an algebraic expression.
- Factor a quadratic expression.
- Solve a linear equation in one variable.
- Represent an expression with algebra tiles.

What “big ideas” should students develop in this chapter?

Students who have successfully completed the work of this chapter and who understand the essential concepts and procedures will know the following:

- A quadratic equation can be solved using different strategies: factoring the quadratic expression, setting the factors equal to zero, and then solving; and graphing with technology.
- In a perfect-square trinomial, the constant term is half the coefficient of the x term squared.
- By completing the square of a quadratic relation in standard form, the relation can be rewritten in vertex form.
- The quadratic formula is derived by completing the square of $ax^2 + bx + c$ and solving for x . The quadratic formula can be used to solve equations that cannot be factored.
- A quadratic equation has no real roots when the discriminant is less than zero, one real root when the discriminant is zero, and two real roots when the discriminant is greater than zero.

Chapter 6: Planning Chart			
Lesson Title	Lesson Goal	Pacing 11 days	Materials/Masters Needed
Getting Started , pp. 310–313	Use concepts and skills developed prior to this chapter.	2 days	grid paper; ruler; Diagnostic Test
Lesson 6.1: Solving Quadratic Equations , pp. 314–321	Use graphical and algebraic strategies to solve quadratic equations.	1 day	grid paper; ruler; graphing calculator; Lesson 6.1 Extra Practice
Lesson 6.2: Exploring the Creation of Perfect Squares , pp. 322–323	Recognize the relationship between the coefficients and constants of perfect-square trinomials.	1 day	algebra tiles
Lesson 6.3: Completing the Square , pp. 325–332	Write the equation of a parabola in vertex form by completing the square.	1 day	algebra tiles (optional); grid paper; ruler; Lesson 6.3 Extra Practice
Lesson 6.4: The Quadratic Formula , pp. 336–344	Understand the development of the quadratic formula, and use the quadratic formula to solve quadratic equations.	1 day	graphing calculator; Lesson 6.4 Extra Practice
Lesson 6.5: Interpreting Quadratic Equation Roots , pp. 345–351	Determine the number of roots of a quadratic equation, and relate these roots to the corresponding relation.	1 day	graphing calculator; grid paper; ruler; Lesson 6.5 Extra Practice
Lesson 6.6: Solving Problems Using Quadratic Models , pp. 352–359	Solve problems that can be modelled by quadratic relations using a variety of tools and strategies.	1 day	grid paper; ruler; graphing calculator; Lesson 6.6 Extra Practice
Mid-Chapter Review , pp. 333–335 Chapter Review , pp. 360–362 Chapter Self-Test , p. 363 Curious Math , p. 324 Chapter Task , p. 364		3 days	Mid-Chapter Review Extra Practice; Chapter Review Extra Practice; Chapter Test; Chapters 4–6 Cumulative Review Extra Practice

CHAPTER OPENER

Using the Chapter Opener

Introduce the chapter by initiating a discussion comparing the photographs on page 308 of the Student Book with the photographs on page 309.

Challenge students to think about why CO₂ emissions increased between 1995 and 2005, as shown in the table on page 308. Ask: What part of the scatter plot represents the data in the table?

Pose questions such as these:

- If CO₂ emissions continue to increase, what kind of model could be used to describe them? (linear)
- If CO₂ emissions level out, instead, and start to fall, what model could be used? (quadratic)

Ask students to explain, in mathematical terms, why they think the word “optimistic” is used in the title of the graph. (For example, a radical change of model, from linear to quadratic, would be required.) Ask students to describe the curve. Students might talk about the shape of the parabola, the vertex, and the direction of opening.

GETTING STARTED

Student Book Pages 310–313

Using the Words You Need to Know

After students complete question 1, ask them to describe the parabola in other ways. After they complete question 2, ask them to explain the strategies they used to match each form with the equation that represents it. For each equation, ask: What does the equation tell you about the graph?

Using the Skills and Concepts You Need

Work through each of the examples in the Student Book. After students have had a chance to ask questions about the examples, invite them to describe each strategy that was used in the first example in their own words. Then ask them which strategy in the second example they prefer.

Have students look over the Practice questions to see if there are any questions they do not know how to solve. Refer students to the Study Aid chart in the margin of the Student Book. Allow students to work on the Practice questions in class. Assign any unfinished questions for homework.

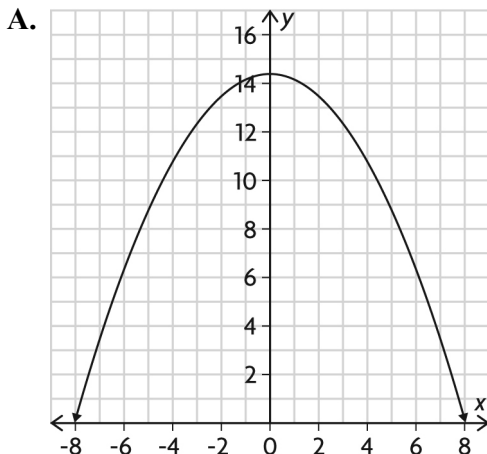
Using the Applying What You Know

Have students work in pairs on the activity. Have them read the whole activity before beginning their work. If students need help with parts B and C, discuss strategies. For part B, they can substitute for h and k in $y = a(x - h)^2 + k$ and then solve for a using one of the intercepts with the bottom of the grid. For part C, they can substitute for p and q in $y = a(x - p)(x - q)$ and then solve for a using the vertex coordinates.

After students have completed the activity, ask how they decided where to draw the axes in part A. Students may have placed the origin at or near the vertex of the parabola, directly below the vertex at the bottom of the grid, or at either of the intercepts of the parabola with the bottom of the grid. Ask: Does the location of the axes affect the answer? Why or why not?

Answers to Applying What You Know

Answers may vary, e.g.,



Preparation and Planning

Pacing

5–10 min	Words You Need to Know
60–65 min	Skills and Concepts You Need
25–35 min	Applying What You Know

Materials

- grid paper
- ruler

Nelson Website

<http://www.nelson.com/math>

B. $y = -0.225(x - 0)^2 + 14.4$

C. $y = -0.225(x - 8)(x + 8)$

D. The two models have the same coefficient for x^2 . The equation in part B shows the location of the vertex, while the equation in part C shows the location of the x -intercepts.

E. Vertex form:

$$y = -0.225(x - 0)^2 + 14.4$$

$$y = -0.225x^2 + 14.4$$

Factored form:

$$y = -0.225(x - 8)(x + 8)$$

$$y = -0.225(x^2 - 64)$$

$$y = -0.225x^2 + 14.4$$

All three models represent the same parabola. Expanding and simplifying the expressions on the right side of the vertex form and factored form resulted in the same equation in standard form.

F. Answers will vary, e.g.,

- I prefer the standard form because it is easiest to use to determine the height of the roof for each distance along the baseline.
- I prefer the vertex form because it shows the highest part of the building.
- I prefer the factored form because it gives the width of the building.

Initial Assessment	What You Will See Students Doing...
When students understand...	If students misunderstand...
Students write the vertex form and factored form correctly.	Students may not know the general vertex and factored forms. They may substitute values into the general forms incorrectly.
Students understand that different forms may represent the same parabola, and they explain the reason effectively.	Students may think that different forms of the same parabola represent different parabolas, or they may not know which parts of the equations must be the same. They may not be able to explain why different forms represent the same parabola, using appropriate mathematical terms.
Students provide analytical justification for their choice of the form of a quadratic relation.	Students may not correctly relate the form of a quadratic relation to the diagram in their explanation, or they may not justify their answer using correct vocabulary.

6.1 SOLVING QUADRATIC EQUATIONS

Lesson at a Glance

GOAL

Use graphical and algebraic strategies to solve quadratic equations.

Prerequisite Skills/Concepts

- Graph a quadratic relation given in standard or factored form.
- Expand and simplify an algebraic expression.
- Factor a quadratic expression.
- Solve a linear equation in one variable.

Specific Expectations

- Interpret real [and non-real] roots of quadratic equations, through investigation using graphing technology, and relate the roots to the x -intercepts of the corresponding relations.
- Sketch or graph a quadratic relation whose equation is given in the form $y = ax^2 + bx + c$, using a variety of methods (e.g., sketching $y = x^2 - 2x - 8$ using intercepts and symmetry; [sketching $y = 3x^2 - 12x + 1$ by completing the square and applying transformations;] graphing $h = -4.9t^2 + 50t + 1.5$ using technology).
- Solve quadratic equations that have real roots, using a variety of methods (i.e., factoring, [using the quadratic formula,] graphing).
- Determine the zeros [and the maximum or minimum value] of a quadratic relation from its graph (i.e., using graphing calculators or graphing software) or from its defining equation (i.e., by applying algebraic techniques).
- Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?).

Mathematical Process Focus

- Problem Solving
- Selecting Tools and Computational Strategies
- Connecting
- Representing

Student Book Pages 314–321

Preparation and Planning

Pacing

5 min	Introduction
25–30 min	Teaching and Learning
30–35 min	Consolidation

Materials

- grid paper
- ruler
- graphing calculator

Recommended Practice

Questions 5, 7, 9, 10, 11, 14, 16

Key Assessment Question

Question 11

Extra Practice

Lesson 6.1 Extra Practice

New Vocabulary/Symbols

quadratic equation
root

Nelson Website

<http://www.nelson.com/math>

MATH BACKGROUND | LESSON OVERVIEW

- Students should be familiar with graphing quadratic relations in different forms, with or without technology.
- Solving a quadratic equation involves rearranging it to make one side zero and then determining the x -intercepts or zeros of the relation.
- In this lesson, the two strategies that are used to solve quadratic equations are factoring algebraically and graphing using technology. The graphical strategy is often used to check an algebraic solution.

1

Introducing the Lesson

(5 min)

Ask students to think of a business they could run. Suggest that they choose a product or service to sell, such as school sweaters or tutoring services. Discuss the types of fixed costs that would be associated with their business. Ask the following questions:

- Why might their profit be negative? In other words, why might they lose money for low sales? (not enough sales to cover fixed costs)
- Why might their profit be negative for high sales? (Sales may exceed the capacity of the business.)

2

Teaching and Learning

(25 to 30 min)

Investigate the Math

Ask a few students to explain the situation in their own words. Then have students work in pairs to discuss their observations and share their work. For part C, students' graphs based on predictions may vary but should represent a relation with the intercepts 20 and 100. Discuss how the graphing calculator screen represents the relation exactly. If students need help with part F, ask:

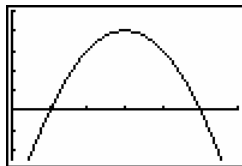
- Where on the graph would the profit be \$1200? (when $P = 1200$)
- How could you represent $P = 1200$? (with a line for $P = 1200$)
- What would the points of intersection for the parabola and the line represent? (the number of T-shirts sold for a profit of \$1200)

Answers to Investigate the Math

- A.** Susie's goal is to break even, which means that she would have no profit. If she had no profit, $P = 0$. Solving the quadratic equation involves determining the value(s) of x , the number of T-shirts, for which $P = 0$.
- B.** $-(x - 20)(x - 100) = 0$; $x = 20$ or $x = 100$; to achieve Susie's goal, 20 T-shirts or 100 T-shirts must be sold.
- C.** The graph crosses the x -axis at $x = 20$ and $x = 100$, and the graph is above the x -axis between $x = 20$ and $x = 100$.
- D.** My prediction for part C was correct.

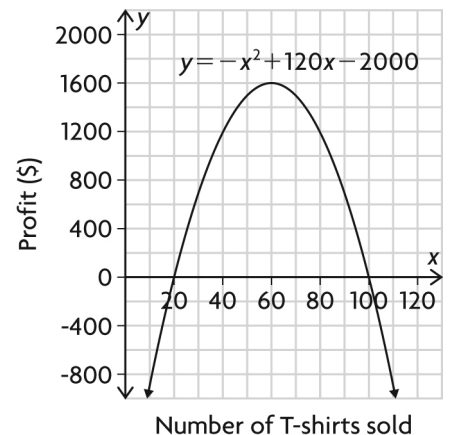
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WINDOW
Xmin=0
Xmax=120
Xscl=20
Ymin=-1000
Ymax=2000
Yscl=400
Xres=1
    
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- E.** For Andy's goal, $P = 1200$ for a profit of \$1200. So, the quadratic equation is $-x^2 + 120x - 2000 = 1200$.
- F.** Include the graph $Y_2 = 1200$, and determine the intersections with $Y_1 = -X^2 + 120X - 2000$.

Profit vs. Sales



G. To achieve Andy's goal, 40 T-shirts or 80 T-shirts must be sold.

Answers to Reflecting

- H. The break-even points are the values of x that make $P = 0$. Therefore, they are the points that make one of the factors equal to 0.
- I. Yes. These values make the expression $-x^2 + 120x - 2000$ equal to zero, so they are the solutions to the equation and the zeros or x -intercepts of the relation.
- J. When the right side is zero, you know that one of the factors of the left side must be zero. When the right side is not zero, the factors could be many values. The left side does not give the information about the factors you need to determine the number of T-shirts. You need to rearrange the equation so that the right side is zero, and then factor.
- K. The right side is zero, so factoring the left side is helpful. One of the factors must be zero.
- L. Isolating x^2 does not help you solve the problem because there is an unknown x on the other side of the equation.

3 Consolidation

(30 to 35 min)

Apply the Math

Using the Solved Examples

Example 1 presents a real-world problem that can be solved by factoring to determine the two values for which $t = 0$. Work through Amir's solution as a class, but have students complete the graphing-calculator check on their own or in pairs. Then have students work in pairs to reproduce Alex's solution, paying attention to the choice of window settings. Bring the class back together for a comparison of the two solutions. Discuss why other window settings would also display the graph.

Have students discuss *Example 2* in pairs, with one partner explaining the solution to the other. Then discuss each step of the solution as a class. Use a similar approach for *Example 3*, with partners trading roles. Ask: Why did Karl use a graphing calculator?

Example 4 introduces the important idea of a solution that is not realistic. Work through the solution with the class, emphasizing that a value is substituted for the variable. Discuss why time must be positive in this situation. Help students understand why the model only applies after the ball is thrown, which happens at $t = 0$. Guide students to solve the equation by graphing it, as in Jacqueline's solution. You might mention a few examples of real-world situations that could have negative solutions, such as situations involving distances below sea level or negative profits (that is, losses).

Technology-Based Alternative Lesson

If TI-*nspire* calculators are available, students can use them to work through *Example 1*. In a Graphs & Geometry application, have students enter the quadratic relation as $f1(x)$ and the value 60 as $f2(x)$. Then students can determine the points of intersection as shown in the screen at the right. You may want to go through the procedure with them, step by step.

To determine the height of the rocket after 3 s, students can place a point on the curve and then change the x -value to 3 by clicking on the coordinate. When they press Enter, the point will move to the appropriate location.

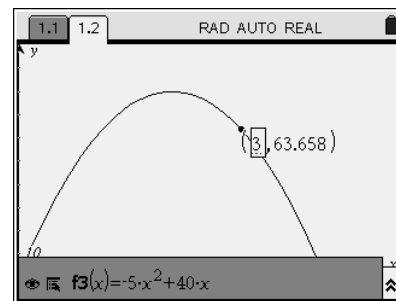
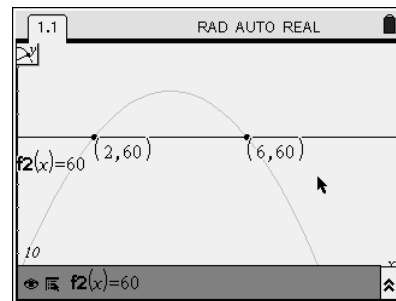
Answer to the Key Assessment Question

If students need help with question 11, encourage them to draw a diagram that represents the garden and walkway, and to mark the given information on their diagram. Students need to realize that they can substitute 900 for A since A represents the area, which is 900 m^2 , and then solve for x . After students complete the question, ask why the negative value of x (-45) is not used as the answer. Discuss that the width of the walkway must be positive.

11. The width of the walkway is 5 m.

Closing

Have students read question 16. For part a), students should think about why the factoring strategy requires rearranging the equation so that 0 is on one side. For part b), students might think about graphing.



Assessment and Differentiating Instruction

What You Will See Students Doing...

When students understand...

Students connect quadratic equations to real-world situations.

Students make reasonable decisions about when they should use factoring to determine the solutions to a quadratic equation.

Students correctly solve quadratic equations by using technology to graph the corresponding quadratic relation.

Students interpret the solution(s) to a quadratic equation in terms of the real-world situation that the equation models.

If students misunderstand...

Students do not connect quadratic equations to real-world situations; for example, they may add a profit expression to the desired profit instead of equating the expressions.

Students try to use factoring to solve quadratic equations that do not factor.

Students may try to solve a quadratic equation such as $x^2 + 3x = -5$ by graphing $x^2 + 3x$ and determining its zeros.

Students may not realize that some solutions are not realistic; for example, they may include negative-time solutions.

Key Assessment Question 11

Students substitute into the quadratic equation for the problem.

Students select a strategy (either factoring or graphing) and apply it correctly to solve the quadratic equation.

Students realize that the required width is the positive solution to the equation.

Students may not realize that they substitute for A or that the value they substitute is 900.

Students cannot identify a strategy to solve the quadratic equation, or they misapply their chosen strategy. They may not factor out 4, or they may not be able to determine the factors of the trinomial.

Students use the negative solution instead of the positive solution, or they incorrectly arrive at two positive solutions.

Differentiating Instruction | How You Can Respond

EXTRA SUPPORT

1. If students have difficulty arriving at the quadratic relation they need to factor or graph, suggest that they write out all the steps. For example, if a profit can be described by the equation $P = x^2 - 7x$ and the desired profit is 18, students can start by writing $P = 18$ and then substitute to get $x^2 - 7x = 18$. Emphasize that they need to have zero on one side of the equation, so they must subtract 18 from both sides. The equation they need to factor is $x^2 - 7x - 18 = 0$.
2. If students need help factoring a quadratic expression of the form $ax^2 + bx + c$, or deciding whether to factor the expression, remind them to list the factor pairs of the product ac to see if any pair adds to b . If not, then graphing is a better option.

EXTRA CHALLENGE

1. Have students write problems that involve quadratic models and then exchange their problems with a partner.
2. Challenge students to describe how they would use the strategies they learned in Lesson 6.1 to solve cubic equations of the form $ax^3 + bx^2 + cx = 0$, with zero constant.

6.2 EXPLORING THE CREATION OF PERFECT SQUARES

Lesson at a Glance

GOAL

Recognize the relationship between the coefficients and constants of perfect-square trinomials.

Prerequisite Skills/Concepts

- Expand and simplify an algebraic expression.
- Factor a quadratic expression.
- Represent an expression with algebra tiles.

Specific Expectation

- Express $y = ax^2 + bx + c$ in the form $y = a(x - h)^2 + k$ by completing the square in situations involving no fractions, using a variety of tools (e.g., concrete materials, diagrams, paper and pencil).

Mathematical Process Focus

- Reasoning and Proving
- Connecting
- Representing

Student Book Pages 322–324

Preparation and Planning

Pacing

5–10 min Introduction

35–40 min Teaching and Learning

15 min Consolidation

Materials

- algebra tiles

Recommended Practice

Questions 1, 2

Nelson Website

<http://www.nelson.com/math>

MATH BACKGROUND | LESSON OVERVIEW

- In this lesson, students recognize perfect-square trinomials and create perfect-square trinomials from general quadratic expressions of the form $x^2 + bx$ and $ax^2 + abx$.
- Students should be able to expand $(x + b)^2$ and use this skill to determine the constant term in a perfect-square trinomial.
- Students then move on to working with partial expressions of the form $ax^2 + abx$, adding $a\left(\frac{b}{2}\right)^2$ to $ax^2 + abx$ to get

the expression $a\left(x + \frac{b}{2}\right)^2$.

1 Introducing the Lesson

(5 to 10 min)

Have students work in pairs to arrange algebra tiles for $(x + 1)^2$ and then for $(x + 2)^2$. After students arrange the tiles for each expression, ask:

- What shape is it? (a square)
- What do you notice about the rows and columns? (The rows and columns are the same.)
- Why does this make sense? (When the width and length are equal, the figure is a square and represents a perfect-square trinomial.)

2 Teaching and Learning

(35 to 40 min)

Explore the Math

Ask students to explain how the tiles represent the quadratic expressions. Guide them to discuss how the tiles show whether the expression is a perfect-square trinomial.

Have students work in pairs to discuss their observations and share their work. If students need guidance to determine the value of c , suggest experimenting with tiles to get arrangements of tiles that represent the x^2 value and the x value. Students may find it helpful to start with the x^2 tiles in the corner and then build the tiles for x .

Answers to Explore the Math

A. $(x + 4)^2$, $(2x + 2)^2$; these expressions are called perfect-square trinomials because they have three terms when simplified and they can be written as the squares of expressions.

B. i) $x^2 + 2x + 1 = (x + 1)^2$

x^2	x
x	1

iii) $x^2 + 6x + 9 = (x + 3)^2$

x^2	x	x	x
x	1	1	1
x	1	1	1
x	1	1	1

v) $x^2 + 10x + 25 = (x + 5)^2$

x^2	x	x	x	x	x
x	1	1	1	1	1
x	1	1	1	1	1
x	1	1	1	1	1
x	1	1	1	1	1
x	1	1	1	1	1

ii) $x^2 + 4x + 4 = (x + 2)^2$

x^2	x	x
x	1	1
x	1	1

iv) $x^2 + 8x + 16 = (x + 4)^2$

x^2	x	x	x	x
x	1	1	1	1
x	1	1	1	1
x	1	1	1	1
x	1	1	1	1

vi) $x^2 + 12x + 36 = (x + 6)^2$

x^2	x	x	x	x	x	x
x	1	1	1	1	1	1
x	1	1	1	1	1	1
x	1	1	1	1	1	1
x	1	1	1	1	1	1
x	1	1	1	1	1	1
x	1	1	1	1	1	1

- C. The constant term is the square of half the coefficient of x .
 D. In part D, the coefficient of x is negative. In part B, the coefficient of x is positive.

E. i) $4; (x - 2)^2$

x^2	$-x$	$-x$
$-x$	1	1
$-x$	1	1

iii) $9; (x - 3)^2$

x^2	$-x$	$-x$	$-x$
$-x$	1	1	1
$-x$	1	1	1
$-x$	1	1	1

v) $1; (x - 1)^2$

x^2	$-x$
$-x$	1

ii) $16; (x - 4)^2$

x^2	$-x$	$-x$	$-x$	$-x$
$-x$	1	1	1	1
$-x$	1	1	1	1
$-x$	1	1	1	1
$-x$	1	1	1	1

iv) $36; (x - 6)^2$

x^2	$-x$	$-x$	$-x$	$-x$	$-x$	$-x$
$-x$	1	1	1	1	1	1
$-x$	1	1	1	1	1	1
$-x$	1	1	1	1	1	1
$-x$	1	1	1	1	1	1
$-x$	1	1	1	1	1	1
$-x$	1	1	1	1	1	1

vi) $25; (x - 5)^2$

x^2	$-x$	$-x$	$-x$	$-x$	$-x$
$-x$	1	1	1	1	1
$-x$	1	1	1	1	1
$-x$	1	1	1	1	1
$-x$	1	1	1	1	1
$-x$	1	1	1	1	1

- F. Yes. The value of c is the square of half the coefficient of x .

- G. i) $2(x^2 + 2x) + c = 2(x^2 + 2x + 1); 2; 2(x + 1)^2$
 ii) $3(x^2 - 4x) + c = 3(x^2 - 4x + 4); 12; 3(x - 2)^2$
 iii) $3(x^2 - 2x) + c = 3(x^2 - 2x + 1); 3; 3(x - 1)^2$
 iv) $-(x^2 - 4x) + c = -(x^2 - 4x + 4); -4; -(x - 2)^2$
 v) $5(x^2 + 5x) + c = 5(x^2 + 5x + 6.25); 31.25; 5(x + 2.5)^2$
 vi) $6(x^2 + 9x) + c = 6(x^2 + 9x + 20.25); 121.5; 6(x + 4.5)^2$

Answers to Reflecting

- H. Divide b by 2 and square the result.
 I. Divide b by 2, square the result, and multiply by a .

3 Consolidation

(15 min)

Students should understand how the algebra tiles have been used to represent perfect-square trinomials as a square. They could use algebra tiles to check any answer.

For question 2, you may need to explain how each expression is a multiple of a perfect-square trinomial, with a common factor for each term. If necessary, remind students about their work for part G.

Students should be able to answer the Further Your Understanding questions independently.

Curious Math

This Curious Math feature presents two related puzzles. The geometric version is a classic conundrum. If the two figures are drawn very accurately, using the same shapes with the same integer coordinates, a slight gap should be visible along the “diagonal” in the second figure.

Answers to Curious Math

1.	$a \times a = a \times b$	Multiply both sides by a .
	$a^2 = ab$	Simplify.
	$a^2 - b^2 = ab - b^2$	Subtract b^2 from both sides.
	$(a + b)(a - b) = b(a - b)$	Factor both sides.
	$a + b = b$	Divide both sides by $a - b$.
	$2 = 1$	Substitute for a and b .
	$2 + 63 = 1 + 63$	Add 63 to both sides.
	$65 = 64$	Simplify.

2. $a - b = 1 - 1 = 0$, and dividing by zero is undefined.

3. Answers may vary, e.g., it appears that each coloured section in the first figure is identical to the section with the same colour in the second figure.

4. 65 square units, 64 square units

5. Answers may vary, e.g., the figures appear to have identical sections. This would imply that their areas are equal and, therefore, that $65 = 64$.

6. Answers may vary, e.g., if it were true that $65 = 64$, it would also be true that $2 = 1$. So, every single object would be two identical objects.

7. Answers may vary, e.g., “prove” that $3 = \pi$.

$x = \frac{\pi + 3}{2}$	Define x as the mean of π and 3.
$2x = \pi + 3$	Multiply both sides by 2.
$2x(\pi - 3) = (\pi + 3)(\pi - 3)$	Multiply both sides by $(\pi - 3)$.
$2\pi x - 6x = \pi^2 - 9$	Expand and simplify.
$9 + 2\pi x - 6x = \pi^2$	Add 9 to both sides.
$9 - 6x = \pi^2 - 2\pi x$	Subtract $2\pi x$ from both sides.
$9 - 6x + x^2 = \pi^2 - 2\pi x + x^2$	Add x^2 to both sides.
$(3 - x)^2 = (\pi - x)^2$	Factor.
$3 - x = \pi - x$	Take the square root of both sides.
$3 = \pi$	Add x to both sides.

Explanation: In second-last line, the square root of $(3 - x)^2$ should be $x - 3$ not $3 - x$.

6.3 COMPLETING THE SQUARE

Lesson at a Glance

GOAL

Write the equation of a parabola in vertex form by completing the square.

Prerequisite Skills/Concepts

- Graph a quadratic relation given in vertex form.
- Expand and simplify an algebraic expression.
- Factor a quadratic expression.
- Represent an expression with algebra tiles.
- Create a perfect square for a quadratic relation.

Specific Expectations

- Express $y = ax^2 + bx + c$ in the form $y = a(x - h)^2 + k$ by completing the square in situations involving no fractions, using a variety of tools (e.g., concrete materials, diagrams, paper and pencil).
- Sketch or graph a quadratic relation whose equation is given in the form $y = ax^2 + bx + c$, using a variety of methods (e.g., sketching $y = x^2 - 2x - 8$ using intercepts and symmetry; sketching $y = 3x^2 - 12x + 1$ by completing the square and applying transformations; [graphing $h = -4.9t^2 + 50t + 1.5$ using technology]).
- Determine [the zeros and] the maximum or minimum value of a quadratic relation [from its graph (i.e., using graphing calculators or graphing software) or] from its defining equation (i.e., by applying algebraic techniques).
- Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, [with and] without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?).

Mathematical Process Focus

- Problem Solving
- Selecting Tools and Computational Strategies
- Connecting
- Representing

Student Book Pages 325–332

Preparation and Planning

Pacing

5 min	Introduction
25–30 min	Teaching and Learning
25–30 min	Consolidation

Materials

- algebra tiles (optional)
- grid paper
- ruler

Recommended Practice

Questions 5, 6, 8, 9, 11, 12, 13, 16

Key Assessment Question

Question 11

Extra Practice

Lesson 6.3 Extra Practice

New Vocabulary/Symbols

completing the square

Nelson Website

<http://www.nelson.com/math>

MATH BACKGROUND | LESSON OVERVIEW

- Students use what they learned in Lesson 6.2 to convert quadratic relations to vertex form by completing the square. (This will prepare them for the quadratic formula that is presented in Lesson 6.4.)
- Students connect the value of k in $y = a(x - h)^2 + k$ with the maximum or minimum value in a real-world situation. They also connect the value of h with a value in a real-world situation, such as the time at which the maximum or minimum occurs.

1 | Introducing the Lesson

(5 min)

Pose this question: What do you know about perfect-square trinomials and about representing them with algebra tiles? Have students take turns, stating one idea each. Make sure that the following ideas are mentioned:

- A perfect-square trinomial has three terms when it is simplified.
- The terms include an x^2 term, an x term, and a constant.
- The constant term is half the coefficient of the x term squared.
- Both factors are the same since the trinomial is a perfect square.
- A perfect square can be represented by algebra tiles arranged to form a square.
- Each side of the square represents one factor, so the sides are equivalent.

2 | Teaching and Learning

(25 to 30 min)

Learn About the Math

Ask for a volunteer to read the information about the automated hose. Focus students' attention on the photograph on the grid. Ask: What does the photograph show about the position of the water? Students' answers may include the following ideas:

- the coordinates of the point where the water spray starts
- the coordinates of the highest position of the water
- the shape of the water spray as a parabola
- the direction of opening of the parabola

Example 1 introduces completing the square by using a perfect-square trinomial to go from the standard form of a quadratic relation to the vertex form. Students may find it helpful to work in pairs with algebra tiles or to display algebra tiles on a board to represent each step in Joan's solution. Emphasize how Arianna's solution goes through the process of completing

$x^2 - 2x$ to get $x^2 - 2x + 1$, using $-\frac{2}{2}$.

Answers to Reflecting

- A. Joan did not have algebra tiles to represent $-2.25x^2$, so she factored out -2.25 to get 1 as the coefficient of x^2 . Arianna factored out -2.25 so that she could complete the square using 1 as the coefficient of x^2 .
- B. Answers may vary, e.g.,
- I prefer Joan's strategy because I can see the perfect square I'm completing. I can be sure that each algebraic expression I create is equivalent to the one I started with because I'm using the same algebra tiles and adding an equal number of positive and negative tiles.

- I prefer Arianna’s strategy because it’s more efficient—I’m just writing the solution. As well, I’m using what I know about perfect-square trinomials.
- C. Joan’s strategy introduces an extra positive unit tile, balanced by a negative unit tile, to complete the square with algebra tiles. Arianna’s solution completes the square by converting $x^2 - 2x$ into $(x^2 - 2x + 1) - 1 = (x - 1)^2 - 1$.

3 Consolidation

(25 to 30 min)

Apply the Math

Using the Solved Examples

Example 2 connects the algebra-tile strategy with the vertex form of a quadratic relation to graph the relation. *Example 3* introduces an area-diagram strategy, which is an extension of the algebra-tile strategy for quadratic relations with decimals. Direct students’ attention to the imperial unit—feet. *Example 4* illustrates the effectiveness of the algebraic strategy for non-integer coefficients.

You might want to have students work in small groups. Assign each group *Example 2, 3, or 4*, and ask students to brainstorm reasons why the strategy used to complete the square is the best strategy. Then bring the class together again to discuss the various strategies.

Answer to the Key Assessment Question

If students have difficulty identifying the mistakes in question 11, you could provide the correct solution. Then students could compare the correct solution with Neilles’s solution. Discuss why the mistakes result in incorrect values in the following steps, and why these incorrect values result in incorrect coordinates for the vertex.

$$11. y = -2x^2 + 16x - 7$$

$$y = -2(x^2 + 8x) - 7 \quad + 8x \text{ should be } - 8x.$$

$$y = -2(x^2 + 8x + 64 - 64) - 7 \quad + 64 - 64 \text{ should be } + 16 - 16.$$

$$y = -2(x + 8)^2 - 64 - 7 \quad 8 \text{ would be } 4, \text{ and } - 64 \text{ would be } - 64(-2).$$

$$y = -2(x + 8)^2 - 73$$

Therefore, the vertex is at $(73, -8)$. $(73, -8)$ would be $(-8, 73)$. (This “correct” vertex is incorrect, however, because of the mistakes in the solution.)

Correct solution:

$$y = -2x^2 + 16x - 7$$

$$y = -2(x^2 - 8x) - 7$$

$$y = -2(x^2 - 8x + 16 - 16) - 7$$

$$y = -2(x - 4)^2 - 16(-2) - 7$$

$$y = -2(x - 4)^2 + 25$$

Therefore, the vertex is at $(4, 25)$.

Closing

Have students read question 16 and respond on their own or in pairs. The strategies that students have encountered in this lesson involve algebra tiles, algebraic solutions, and area diagrams. The question “Which strategy do you prefer?” could be answered by “It depends.” Encourage students to use lists of “pros and cons” or decision trees to decide which strategy they prefer. Bring students back together to discuss and compare answers as a class.

Assessment and Differentiating Instruction	
What You Will See Students Doing...	
<p>When students understand...</p> <p>Students describe the connections among perfect-square trinomials, completing the square, and determining the vertex form of a quadratic relation.</p> <p>Students effectively interpret problems to identify the goal (for example, determining the maximum or minimum, locating the vertex, graphing the equation).</p> <p>Students select and apply strategies for completing the square in an informed and reasoned manner.</p>	<p>If students misunderstand...</p> <p>Students may have difficulty understanding different representations or the connections among representations. Communicating their understanding may be a challenge.</p> <p>Students have trouble identifying, or they misidentify, the goal of a problem. They may try to determine a maximum by locating the zeros of a quadratic relation.</p> <p>Students may not be able to choose a strategy, or they may be unable to justify adequately the selection of a strategy for completing the square. Students may attempt to complete the square in a relation such as $y = x^2 - 6x + 8$ by writing $y = (x - 6)^2 - 36 + 8$ or $y = (x - 3)^2 + 9 + 8$.</p>
<p>Key Assessment Question 11</p> <p>Students work accurately and efficiently through the steps in algebraic procedures.</p> <p>Students recognize multiple errors in algebraic procedures.</p> <p>Students correctly apply algebraic procedures to complete the square for the relation.</p>	<p>Students have difficulty working through the steps in an algebraic procedure. They may make errors, or they may fail to detect the errors in the given solution. They may not notice that $2(-64) = -128$, not -64.</p> <p>Students may miss some errors (such as a sign change), or they may incorrectly identify “errors” in a correct procedure.</p> <p>Students may fail to factor the same quantity from the x^2 and x terms, or they may forget to divide the (factored) coefficient of x before squaring it.</p>
Differentiating Instruction How You Can Respond	
<p>EXTRA SUPPORT</p> <p>1. Help students select an effective strategy for completing the square by looking at the coefficients of the quadratic relation. Remind students to factor out the coefficient of x^2 first, if necessary. Ask if they recognize any elements of a pattern, such as $x^2 + 14x$ or $x^2 - 9x$. If so, ask if they want to continue algebraically or if they want to use algebra tiles or an area diagram and then record their work. Invite students to explain the connections among representations. Pose prompting questions as needed.</p>	
<p>EXTRA CHALLENGE</p> <p>1. Have students create incorrect solutions for questions in the lesson and then trade solutions to determine the errors.</p> <p>2. Challenge students to work backwards from the vertex form by expanding the expression $a(x - h)^2 + k$ and matching the result to a given quadratic relation in standard form.</p>	

MID-CHAPTER REVIEW

Big Ideas Covered So Far

- A quadratic equation can be solved using different strategies: factoring the quadratic expression, setting the factors equal to zero, and then solving; and graphing with technology.
- In a perfect-square trinomial, the constant term is half the coefficient of the x term squared.
- By completing the square of a quadratic relation in standard form, the relation can be rewritten in vertex form.

Using the Frequently Asked Questions

Have students keep their Student Books closed. Display the Frequently Asked Questions on a board. Have students discuss the questions and use the discussion to draw out what the class thinks are good answers. Then have students compare the class answers with the answers on Student Book pages 333 and 334. Students can refer to the answers to the Frequently Asked Questions as they work through the Practice Questions.

Using the Mid-Chapter Review

Ask students if they have any questions about any of the topics covered so far in the chapter. Review any topics that students would benefit from considering again. Assign Practice Questions for class work and for homework.

To gain greater insight into students' understanding of the material covered so far in the chapter, you may want to ask questions such as the following:

- How would you solve a quadratic equation? What other strategy could you use? How would you choose the strategy to use? Do you think someone else would choose the same strategy? Why or why not?
- What is a perfect-square trinomial? Why do you think this trinomial is called a perfect square?
- If a quadratic relation is a perfect-square trinomial, what is true about the value of k when the relation is rewritten in vertex form?
- What different strategies could you use to complete the square of a quadratic equation?
- How does completing the square help you determine the maximum or minimum value of a quadratic relation? How does the equation show whether you are determining the maximum value or the minimum value?

6.4 THE QUADRATIC FORMULA

Lesson at a Glance

GOAL

Understand the development of the quadratic formula, and use the quadratic formula to solve quadratic equations.

Prerequisite Skills/Concepts

- Expand and simplify an algebraic expression.
- Complete the square for a quadratic equation.

Specific Expectations

- Explore the algebraic development of the quadratic formula (e.g., given the algebraic development, connect the steps to a numerical example; follow a demonstration of the algebraic development [student reproduction of the development of the general case is not required]).
- Solve quadratic equations that have real roots, using a variety of methods (i.e., factoring, using the quadratic formula, graphing).
- Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?).

Mathematical Process Focus

- Reasoning and Proving
- Selecting Tools and Computational Strategies
- Connecting

Student Book Pages 336–344

Preparation and Planning

Pacing

10 min	Introduction
20–25 min	Teaching and Learning
25–30 min	Consolidation

Materials

- graphing calculator

Recommended Practice

Questions 5, 6, 7, 9, 12, 14, 16, 17

Key Assessment Question

Question 16

Extra Practice

Lesson 6.4 Extra Practice

New Vocabulary/Symbols

quadratic formula

Nelson Website

<http://www.nelson.com/math>

MATH BACKGROUND | LESSON OVERVIEW

- This lesson begins by inviting students to think about quadratic relations that do not have integer solutions.
- In *Example 1*, students follow the development of the quadratic formula, which involves completing the square. Then they apply the formula in the rest of the examples.

1 | Introducing the Lesson

(10 min)

Ask students to work in pairs to solve the equation $x^2 - 9 = 0$. Then discuss students' solutions as a class, and elicit that the equation can be rearranged as $x^2 = 9$ and solved for $x = 3$ and -3 . Next ask the pairs to solve $x^2 - 7 = 0$. Discuss why this equation can be rearranged as $x^2 = 7$ and solved for $\sqrt{7}$ and $-\sqrt{7}$. Have students use a calculator to calculate the solutions as about 2.65 and about -2.65 .

Display these with a graphing calculator, or have students use graphing calculators to show the solutions for both equations.

2 | Teaching and Learning

(20 to 25 min)

Learn About the Math

Invite students to explain Devlin's steps for the graphing calculator screens. Ask how the screens show the solutions for $2x^2 + 4x - 10 = 0$.

Example 1 uses two strategies for solving a quadratic equation that has non-integer roots: using the vertex form and using the quadratic formula. Have students work through Kyle's solution in pairs, with one partner explaining the steps to the other. Then discuss both solutions as a class. Next, go through the steps in Liz's solution. Emphasize that she is using a general quadratic expression, rather than a quadratic expression with specific coefficients, to complete the square. Finally, have students work in pairs or small groups to brainstorm the advantages and disadvantages of each solution in *Example 1*.

Technology-Based Alternative Lesson

Students could use technology to confirm the results they obtain using the quadratic formula.

Answers to Reflecting

- A. A parabola may cross the x -axis in two places. Each place represents a solution to the quadratic equation.
- B. Both solutions involve completing the square. In Kyle's solution, the specific expression $2x^2 + 4x - 10$ is used. In Liz's solution, the general expression $ax^2 + bx - c$ is used and then the specific coefficients are substituted into the formula developed.
- C. When the values of a , b , or c are changed, the quadratic equation changes. The parabola that represents the equation also changes. So, the solutions to the equation, or the roots, change. Since the values of a , b , and c affect the parabola, all three values need to be part of the formula for determining the solution.

3 Consolidation

(25 to 30 min)

Apply the Math

Using the Solved Examples

Example 2 involves using the quadratic formula. Go through this example with the whole class, asking one or more student volunteers to explain each step.

Example 3 illustrates two situations, one in which the quadratic formula is not required and one in which it is required. Have students work in pairs, with each partner explaining one solution to the other. Then invite students to contribute to a class discussion about the solutions.

Example 4 is a real-world application of the quadratic formula. Discuss this example with the class, making sure that students understand how the equation $0 = -w^2 + 50w - 575$ is developed.

Answer to the Key Assessment Question

If students want to verify their answer for question 16, they can copy the diagram, mark the dimensions on it, and determine the two areas to check that these areas are equal. To help students visualize the equality of the areas, suggest that they divide the lawn and the walkway on their diagram to show equivalent parts.

16. The dimensions of the lawn, to the nearest tenth of a metre, are 9.7 m by 9.7 m.

Closing

Have students read question 17. It is important for students to understand that factoring may or may not work, while the quadratic formula can always be used. Ask students to think about which strategy they would try first if they were unsure whether a quadratic expression could be factored.

Assessment and Differentiating Instruction

What You Will See Students Doing...

When students understand...	If students misunderstand...
<p>Students correctly apply the quadratic formula to solve quadratic equations, simplifying first if necessary.</p> <p>Students identify appropriate strategies to solve a quadratic equation.</p> <p>Students correctly formulate quadratic equations in real-world applications and correctly interpret the solutions.</p>	<p>Students may not record the quadratic formula correctly, or they may not rearrange the equation first so that one side of the equation is zero. They may make errors substituting or calculating, or they may not use both the positive and negative square roots.</p> <p>Students may miss an equation with no x term and, as a result, not solve it by taking the square root. Students may try to factor a quadratic equation that has non-integer roots instead of using the quadratic formula.</p> <p>Students may be unable to create a quadratic equation to represent a situation, especially when the situation involves two variables, or they may not relate the solutions to the situation correctly.</p>
<p>Key Assessment Question 16</p> <p>Students formulate an equation that accurately relates the area of the walkway to the area of the lawn.</p> <p>Students correctly apply the quadratic formula to their equation, having first simplified it so that one side is zero.</p> <p>Students discard the negative solution of the quadratic formula and then state the solution correctly.</p>	<p>Students may confuse the area of the walkway with the area of the larger square. They may use two separate variables, x and y, for the dimensions of the two squares, without relating these variables.</p> <p>Students may neglect to simplify the equation, or they may make calculation errors when applying the quadratic formula.</p> <p>Students may fail to realize that the negative solution is not realistic for the situation. They may not round the dimensions correctly.</p>

Differentiating Instruction | How You Can Respond

EXTRA SUPPORT

1. If students tend to make calculation errors, encourage them to check the solutions they obtain using the quadratic formula. They could use a graphing calculator or substitute their solutions back into the original quadratic equation to verify that the result is zero.

EXTRA CHALLENGE

1. Have students work in pairs for this challenge. One partner should try to write a quadratic equation that cannot be solved by factoring, and the other partner should solve it. The first partner gets a point if the equation does not have integer roots. The second partner gets a point if his/her solution is correct and an extra point if he/she is able to factor the equation without using the quadratic formula. The partners can then trade roles.

6.5 INTERPRETING QUADRATIC EQUATION ROOTS

Lesson at a Glance

GOAL

Determine the number of roots of a quadratic equation, and relate these roots to the corresponding relation.

Prerequisite Skills/Concepts

- Graph a quadratic relation given in standard form.
- Expand and simplify an algebraic expression.
- Complete the square for a quadratic equation.
- Apply the quadratic formula.

Specific Expectation

- Interpret real and non-real roots of quadratic equations, through investigation using graphing technology, and relate the roots to the x -intercepts of the corresponding relations.

Mathematical Process Focus

- Reasoning and Proving
- Connecting
- Representing

Student Book Pages 345–351

Preparation and Planning

Pacing

5–10 min	Introduction
30–40 min	Teaching and Learning
15–20 min	Consolidation

Materials

- graphing calculator
- grid paper
- ruler

Recommended Practice

Questions 3, 5, 6, 8, 9, 13, 14, 15

Key Assessment Question

Question 8

Extra Practice

Lesson 6.5 Extra Practice

New Vocabulary/Symbols

real numbers
discriminant

Nelson Website

<http://www.nelson.com/math>

MATH BACKGROUND | LESSON OVERVIEW

- In this lesson, the significance of the discriminant is addressed. The graph of a quadratic relation meets the x -axis twice when the discriminant is positive and once when the discriminant is zero. The graph does not meet the x -axis when the discriminant is negative.
- Some students may be curious about the phrase “no real roots” (as opposed to “no roots”) when the discriminant is negative. Explain that they will learn about imaginary numbers, also known as complex numbers, in future math courses. Imaginary numbers involve the square roots of negative numbers.

1

Introducing the Lesson

(5 to 10 min)

Have students discuss, in small groups, what they have learned about using the quadratic formula to solve quadratic equations. If any groups need help getting started, suggest that they consider the following: the meaning of the formula, the reasons for using the formula, the connections between the results and a graph, and the connections between the results and the solutions obtained by factoring, when factoring is possible. Then ask a student from each group to explain one idea that the group discussed. Continue, as time allows, for other rounds with different students from each group reporting.

To conclude, ask the class to answer this question: What strategies can you suggest for remembering the formula? Encourage different ideas.

2

Teaching and Learning

(30 to 40 min)

Investigate the Math

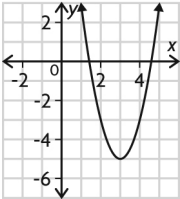
Ask students to explain how the explanations beside the parabolas on Student Book page 345 relate to the parabolas. Have students work in pairs to discuss their observations and share their work.

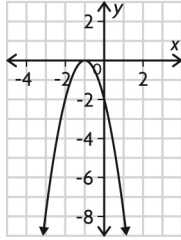
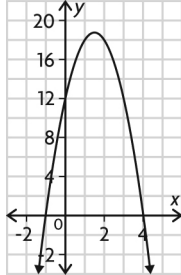
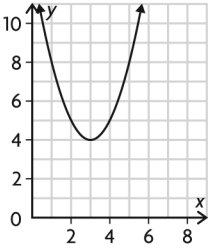
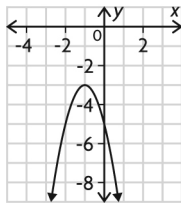
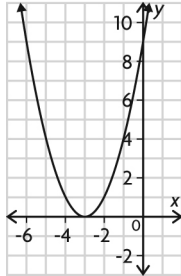
For part A, have students use the quadratic formula to solve all the relations in the table, even if the zeros are more easily determined with other strategies. It is important for students to see the effects of the discriminant.

Remind students that, although they are using a graphing calculator to complete the table, they need to sketch the graphs on grid paper or plain paper. Their sketches can be approximate.

Answers to Investigate the Math

A.

Quadratic Relation	Sketch of Graph	Quadratic Equation Used to Determine x-Intercepts	Roots of the Equation
$y = 2x^2 - 12x + 13$		$2x^2 - 12x + 13 = 0$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(2)(13)}}{2(2)}$ $x = \frac{12 \pm \sqrt{40}}{4}$ $x \doteq 1.42 \text{ or } x \doteq 4.58$

Quadratic Relation	Sketch of Graph	Quadratic Equation Used to Determine x-Intercepts	Roots of the Equation
$y = -2x^2 - 4x - 2$		$-2x^2 - 4x - 2 = 0$	$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-2)(-2)}}{2(-2)}$ $x = \frac{4 \pm \sqrt{0}}{-4}$ $x = -1$
$y = -3x^2 + 9x + 12$		$-3x^2 + 9x + 12 = 0$	$x = \frac{-9 \pm \sqrt{9^2 - 4(-3)(12)}}{2(-3)}$ $x = \frac{-9 \pm \sqrt{225}}{-6}$ $x = -1 \text{ or } x = 4$
$y = x^2 - 6x + 13$		$x^2 - 6x + 13 = 0$	$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)}$ $x = \frac{6 \pm \sqrt{-16}}{2}$ <p>no real solutions</p>
$y = -2x^2 - 4x - 5$		$-2x^2 - 4x - 5 = 0$	$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-2)(-5)}}{2(-2)}$ $x = \frac{4 \pm \sqrt{-24}}{-4}$ <p>no real solutions</p>
$y = x^2 + 6x + 9$		$x^2 + 6x + 9 = 0$	$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(9)}}{2(1)}$ $x = \frac{-6 \pm \sqrt{0}}{2}$ $x = -3$

B. If the discriminant is positive, there are two real number solutions. If the discriminant is zero, there is one real number solution. If the discriminant is negative, there are no real number solutions.

Answers to Reflecting

C. A negative number has no square root, so the quadratic formula does not result in a real number solution.

- D.** The square root of zero is zero. When the discriminant is zero, it is not possible to obtain a second solution by changing the sign in front of the square root.
- E.** In the quadratic formula, the two solutions are obtained by using different signs in front of the square root. When the discriminant is positive, one solution results from the positive square root and the other results from the negative square root.

3

Consolidation

(15 to 20 min)

Apply the Math

Using the Solved Examples

Example 1 connects the number of zeros of a quadratic relation, the value of the discriminant, the position of the vertex, and the direction of opening of the parabola. Discuss each part of the example with the class, prompting students to volunteer reasons for the conclusions about each graph.

Example 2 provides an opportunity for students to review and compare the strategies they have learned for solving quadratic equations. Have students work in pairs or small groups to brainstorm the advantages and disadvantages of each strategy.

Answer to the Key Assessment Question

After students complete question 8, ask them to share their strategies and reasoning. Some students may find sketching the graph or using a graphing calculator helpful for visualizing the graph.

- 8. a)** This relation will have one zero for $t \geq 0$ because the ball starts above the ground and eventually falls downward.
- b)** The zeros are about -0.10 and about 2.30 . The negative solution for the relation does not fit the context, since the time cannot be negative.
- c)** Answers may vary, e.g., the ball will pass through a height of 5 m twice, through a height of 7 m once, and through a height of 9 m zero times.
- d)** For 5 m, $D = 39.2$, which is greater than zero, so there are two zeros. For 7 m, $D = 0$, so there is one zero. For 9 m, $D = -39.2$, which is less than zero, so there are no zeros.

Closing

Have students read question 15 and answer it individually. Then have them share their work in groups. Some students may find it helpful to start by looking at equations in the lesson.

Assessment and Differentiating Instruction

What You Will See Students Doing...

When students understand...

Students connect the value of the discriminant with the position of the vertex and the direction of opening.

Students correctly predict the sign of the discriminant from the graph of a quadratic relation.

Students move confidently between quadratic equations in the general form $ax^2 + bx + c = 0$, quadratic equations that are not in this form, and the corresponding quadratic relations.

If students misunderstand...

Students may confuse the direction of opening with the sign of the discriminant. They may incorrectly think that upward means positive and downward means negative.

Students may not connect the number of zeros with the value of the discriminant.

Students may make errors, such as calculating the discriminant without first rearranging a quadratic equation so that one side is 0. They may use the value of $b^2 - 4ac$ as the discriminant, instead of using the square root of $b^2 - 4ac$.

Key Assessment Question 8

Students reason about the situation to make an accurate prediction about the number of zeros.

Students correctly apply the discriminant to confirm their prediction about the number of zeros.

Students do not understand how to make a prediction about the number of zeros, or they make a prediction without proper justification.

Students may misidentify the variables in the discriminant (for example, $c = 1.071$ in $-4.9t^2 + 10.78t + 1.071 = 7$), or they may incorrectly interpret the meaning of the discriminant.

Differentiating Instruction | How You Can Respond

EXTRA SUPPORT

- To help students make the connection between the number of zeros and the value of the discriminant, use dynamic geometry software. Set up parameters a , b , and c , plot $a \cdot x^2 + b \cdot x + c$, use the software to determine the discriminant, and observe the effect of adjusting or animating the parameters.

EXTRA CHALLENGE

- Challenge confident students to try this probability-based activity:
 - Roll a number cube three times to determine three positive integers between 1 and 6.
 - Toss a coin three times. For each tail, change the corresponding integer to be negative (for example, 3, 4, 6 and H, T, H \rightarrow 3, -4, 6).
 - Use the resulting integers as the values of a , b , and c in a quadratic relation (for example, 3, -4, 6 \rightarrow $y = 3x^2 - 4x + 6$).

Pose the following question: What is the probability of getting a quadratic relation with two real roots? one real root? no real roots? Students could use either a simulation or classical techniques (for example, determine the probability that the b -value gives a positive discriminant for each pair of values (a , c)).
- Have students try a simplified version of the activity above: Set $a = c$, and use just two rolls and tosses to determine the quadratic relation.

6.6 SOLVING PROBLEMS USING QUADRATIC MODELS

Lesson at a Glance

GOAL

Solve problems that can be modelled by quadratic relations using a variety of tools and strategies.

Prerequisite Skills/Concepts

- Graph a quadratic relation given in standard, factored, or vertex form.
- Expand and simplify an algebraic expression.
- Factor a quadratic expression.
- Complete the square for a quadratic relation.
- Apply the quadratic formula.

Specific Expectations

- Express $y = ax^2 + bx + c$ in the form $y = a(x - h)^2 + k$ by completing the square in situations involving no fractions, using a variety of tools (e.g., concrete materials, diagrams, paper and pencil).
- Solve quadratic equations that have real roots, using a variety of methods (i.e., factoring, using the quadratic formula, graphing).
- Determine the zeros and the maximum or minimum value of a quadratic relation from its graph (i.e., using graphing calculators or graphing software) or from its defining equation (i.e., by applying algebraic techniques).
- Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?).

Mathematical Process Focus

- Problem Solving
- Selecting Tools and Computational Strategies

Student Book Pages 352–359

Preparation and Planning

Pacing

5 min	Introduction
20–25 min	Teaching and Learning
30–35 min	Consolidation

Materials

- grid paper
- ruler
- graphing calculator

Recommended Practice

Questions 3, 5, 8, 10, 11, 12, 13, 15

Key Assessment Question

Question 3

Extra Practice

Lesson 6.6 Extra Practice

Nelson Website

<http://www.nelson.com/math>

MATH BACKGROUND | LESSON OVERVIEW

- Students have learned a variety of strategies for solving problems that involve quadratic relations. They can connect these strategies to the form of a quadratic relation and the requirements of a given problem.
- Specifically, these connections are
 - maximum/minimum problems: vertex form, completing the square
 - solving factorable quadratic equations: factored form, factoring
 - solving (possibly) non-factorable quadratic equations: standard form, the quadratic formula
- In this lesson, students connect real roots to realistic solutions for real-world problems.

1 | Introducing the Lesson

(5 min)

Ask students to suggest as many strategies as possible to determine

- the zero(s) of a quadratic relation
- the value of x that gives the maximum or minimum value
- the maximum or minimum value

Students could brainstorm in small groups before creating a class summary.

2 | Teaching and Learning

(20 to 25 min)

Learn About the Math

Briefly discuss the situation with the class, and ask students to explain how the relation models the profit.

Example 1 uses a break-even/maximum-profit problem as a basis for selecting and applying a strategy. Have students work in pairs, with each partner explaining one of the solutions to the other. Then bring the class together to discuss each step of Jack's solution and Dineke's solution.

Technology-Based Alternative Lesson

A variation on Jack's strategy for *Example 1* is to enter the model into a graphing calculator and use the Table application. Walk through the steps involved with the class:

- Enter $Y1 = -X^2 + 580X - 48000$.
- Use TBLSET to set TblStart=0, Δ Tbl=10.
- Select TABLE, and scroll down until a maximum of 36 100 appears to be reached at 290.
- Use this information to rewrite the profit relation as $P = -(x - 290)^2 + 36\,100$.

Answers to Reflecting

- A. Both solutions convert the relation in standard form to another form. Jack changed the relation to vertex form. Dineke changed it to factored form. Jack determined the vertex first and then located the zeros, or break-even points. Dineke determined the zeros first and then used their mean to locate the vertex.
- B. Jack's strategy will always work because every quadratic relation can be written in vertex form. Dineke's strategy will not always work because some quadratic relations cannot be factored.
- C. Answers may vary, e.g.,
- I would have used Jack's strategy because it will always work.
 - I would have used Dineke's strategy because the calculations are easier.

3

Consolidation

(30 to 35 min)

Apply the Math

Using the Solved Examples

Example 2 uses a strategy to create a quadratic model. Work through the example with the class. Ask students to identify two key ways in which the vertex form is used in the solution. Lead students to realize that the vertex form is used to create the model and solve the resulting equation.

In *Example 3*, a quadratic curve of good fit is used to answer questions about population.

In *Example 4*, the total fencing of a combined enclosure is used to determine possible dimensions.

As an alternative to completing the square, you might like to guide students through a graphing-calculator TABLE strategy. If you have already done this for *Example 1* (see the Technology-Based Alternative Lesson), have students work in pairs to apply the strategy themselves. (Use $Y1=30-5X$ and $Y2=X*Y1(X)$ with TBLSET settings such as $TblStart=0$, $\Delta Tbl=0.25$.)

Answer to the Key Assessment Question

For question 3, students could use technology to graph the equation, which would help them visualize the relation before completing the question.

Students could also use technology to confirm their answers. They could trace along the parabola or determine the values at various points to check their answers.

3. a) The maximum height that the water can reach is about 23.88 m.
- b) A firefighter could stand about 16.6 m or about 73.4 m back.

Closing

Have students read question 15. Then have students work in pairs to brainstorm ideas for each part. If students need help, you might need to hint that the problem for part a) should involve equating quadratic models either to zero or to some constant, and the problem for part b) should involve determining a maximum or minimum value.

Assessment and Differentiating Instruction

What You Will See Students Doing...

When students understand...

Students select effective strategies for problems that require them to determine a maximum or minimum value of a quadratic relation.

Students identify the correct quadratic equation they need to solve when they need to determine the x -values that correspond to a given y -value of a quadratic relation.

Students apply their chosen strategy correctly.

Students consider which solutions are realistic in the context of a problem situation.

If students misunderstand...

Students are not sure which strategy to apply. For example, when trying to determine a maximum or minimum value, they do not consider completing the square for the given relation.

Students may forget to simplify the expression first to get zero on one side of the equation.

Students make errors when applying their chosen strategy. For example, they fail to divide the coefficient of x by twice the coefficient of x^2 when completing the square.

Students may include unrealistic solutions, such as negative-time solutions or a ball below ground level.

Key Assessment Question 3

Students select and apply effective strategies to determine the maximum height of the water and the horizontal distance to reach the window.

Students identify the correct quadratic equation they need to solve in part b).

Students realize that there are two solutions, given the context of the question for the distance from the window.

Students may be unable to select effective strategies. For example, they may not consider completing the square for the given relation. Students may make errors when applying a chosen strategy. For example, they may fail to divide 0.99 by $(2)(-0.011)$ when completing the square.

Students may set the given relation equal to 0 rather than 15 .

Students may obtain only one solution or misidentify one solution as unrealistic.

Differentiating Instruction | How You Can Respond

EXTRA SUPPORT

1. Students may need more practice interpreting the given information. Encourage students to sketch a graph of the information before answering the question.
2. If students need help selecting a strategy, refer them to the suggestions given for the Key Idea on page 357 of the Student Book and to the examples. Students could create their own display of these suggestions, adding information that they find helpful.

EXTRA CHALLENGE

1. Give confident students this problem to solve: Tracey uses 60 cm of wire to make the framework of a square-based prism, which she then covers with tissue paper. What is the maximum surface area of the prism, and what are the dimensions that give this maximum?
2. Challenge students with good algebra skills to determine the relationship between the maximum or minimum value of a quadratic relation and the discriminant. (maximum/minimum value = $-\frac{D}{4a}$)

CHAPTER REVIEW

Big Ideas Covered So Far

- A quadratic equation can be solved using different strategies: factoring the quadratic expression, setting the factors equal to zero, and then solving; and graphing with technology.
- In a perfect-square trinomial, the constant term is half the coefficient of the x term squared.
- By completing the square of a quadratic relation in standard form, the relation can be rewritten in vertex form.
- The quadratic formula is derived by completing the square of $ax^2 + bx + c$ and solving for x . The quadratic formula can be used to solve equations that cannot be factored.
- A quadratic equation has no real roots when the discriminant is less than zero, one real root when the discriminant is zero, and two real roots when the discriminant is greater than zero.

Using the Frequently Asked Questions

Have students keep their Student Books closed. Display the Frequently Asked Questions on a board. Have students discuss the questions and use the discussion to draw out what the class thinks are good answers. Then have students compare the class answers with the answers on Student Book page 360. Students can refer to the answers to the Frequently Asked Questions as they work through the Practice Questions.

Using the Chapter Review

Ask students if they have any questions about any of the topics covered so far in the chapter. Review any topics that students would benefit from considering again. Assign Practice Questions for class work and for homework.

To gain greater insight into students' understanding of the material covered so far in the chapter, you may want to ask questions such as the following:

- When do you use the quadratic formula to solve an equation? What other strategies could you use to solve a quadratic equation?
- Suppose that the graph of a quadratic relation opens upward. Where is the vertex of the parabola located if the discriminant is greater than zero? If the discriminant is zero? If the discriminant is less than zero?
- What does the vertex form of a quadratic relation show? What does the standard form show? What does the factored form show?
- What are the advantages of using technology to solve equations? What are the disadvantages? Why might different people have different opinions about the advantages and disadvantages?

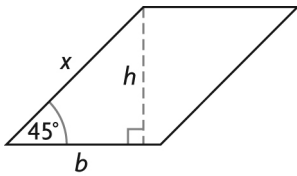
CHAPTER 6 TEST

For further assessment items, please use Nelson's Computerized Assessment Bank.

- Solve by factoring.
 - $x^2 + 10x - 11 = 0$
 - $3x^2 - 105 = 6x$
 - $6x(x - 2) = 5(1 - x)$
 - $4(x + 3)^2 = 36$
- Solve by any method. Round your answers to two decimal places, if necessary.
 - $x^2 - 6x - 4 = 0$
 - $4x^2 = 8x + 21$
 - $17 - 3(x - 7)^2 = 0$
 - $2x(2 - x) = 30(x + 1)$
- Determine the value of c that makes each expression a multiple of a perfect square.
 - $3x^2 + 6x + c$
 - $2x^2 - 108x + c$
 - $5x^2 - 30x + c$
 - $2x^2 + 6x + c$
- Determine the vertex of each quadratic relation by completing the square.
 - $y = x^2 - 6x + 22$
 - $y = -2x^2 - 20x - 65$
 - $y = 8x^2 - 104x + 338$
 - $y = -x^2 + 5x + 6$

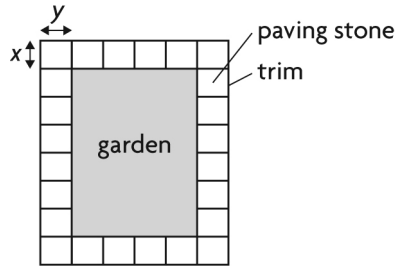
For questions 5 to 9, round your answers to two decimal places, if necessary.

- Solve each equation using the quadratic formula.
 - $2x^2 + 6x - 5 = 0$
 - $19x + 30 = 4x^2$
 - $x(2x + 15) = (x - 7)(x + 7)$
 - $-3x^2 - 7x + 8 = 0$
- A piece of wire, 70 cm long, is used to create a parallelogram, as shown in the diagram. Using the formula $A = bh$, determine the maximum area of the parallelogram. What do you notice about the parallelogram with the maximum area?



- Determine the number of roots for each equation.
 - $2x^2 + 3x + 1 = 0$
 - $3x^2 = 15x + 12$
 - $x(x + 14.1) = 4.7(x - 4.7)$
 - $\frac{1}{2}x^2 - \frac{1}{3}x + \frac{5}{6} = 0$
- A jet of water is spraying from the centre of a circular fountain. The height, h , in metres above the ground, of the jet of water is modelled by the relation $h = -0.5x^2 + 1.8x + 1.2$, where x represents the distance that the water travels horizontally, in metres.
 - What is the maximum height of the jet of water?
 - The rim of the fountain is 0.5 m high. What is the maximum possible radius of the fountain?

9. A landscape architect is planning a rectangular garden with a border of rectangular paving stones. The stones will be cut from a large piece of granite. The dimensions of the border will be six stones by eight stones. Each stone will be oriented the same way and will be surrounded by trim. The landscape architect has enough trim for 68 m.

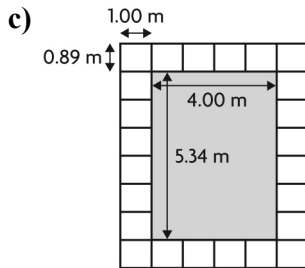


- Write an equation for the amount of trim in terms of x and y .
- What is the maximum area of the garden?
- Copy the plan. Label the dimensions of one of the paving stones, as well as the dimensions of the garden that give the maximum area.

CHAPTER 6 TEST ANSWERS

1. a) 1, -11 b) 7, -5 c) $\frac{5}{3}, -\frac{1}{2}$ d) 0, -6
2. a) -0.61, 6.61 b) 3.5, -1.5 c) 4.62, 9.38 d) -11.72, -1.28
3. a) 3 b) 1458 c) 45 d) 4.5
4. a) (3, 13) b) (-5, -15) c) (6.5, 0) d) (2.5, 12.25)
5. a) 0.68, -3.68 b) 6, -1.25 c) -4.81, -10.19 d) 0.84, -3.17
6. The maximum area is 216.55 cm². The parallelogram is a rhombus.
7. a) two b) two c) one d) zero
8. a) 2.82 m b) 3.95 m
9. a) $68 = 38x + 34y$

b) 21.36 m²



CHAPTER TASK

Up and Over

Specific Expectations

- Express $y = ax^2 + bx + c$ in the form $y = a(x - h)^2 + k$ by completing the square in situations involving no fractions, using a variety of tools (e.g., concrete materials, diagrams, paper and pencil).
- Solve quadratic equations that have real roots, using a variety of methods (i.e., factoring, using the quadratic formula, graphing).
- Determine the zeros and the maximum or minimum value of a quadratic relation from its graph (i.e., using graphing calculators or graphing software) or from its defining equation (i.e., by applying algebraic techniques).
- Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?).

Introducing the Chapter Task (Whole Class)

Have students look at the picture at the beginning of the Chapter Task, on Student Book page 364. Be sure to clarify that u is the initial upward velocity of the ball, not the horizontal velocity.

Have students read through the task, and then discuss the goal of the task. Lead students to understand that the goal is to investigate the connection between the initial upward velocity and the maximum height of the ball.

Using the Chapter Task

Make sure that students record the reasons for their conclusions in parts B, D, and E. For part E, lead students to notice that they have been squaring the initial velocity each time they completed the square, rather than formulating a precise relationship.

In part F, students need to produce hand-plotted graphs. In part G, they need to perform a quadratic regression of the data. They should compare the graphs they create by quadratic regression with the scatter plot they created in part F.

Assessing Students' Work

Use the Assessment of Learning chart as a guide for assessing students' work.

Student Book Page 364

Preparation and Planning

Pacing

10–15 min Introducing the Chapter Task

45–50 min Using the Chapter Task

Materials

- graphing calculator
- grid paper
- ruler

Nelson Website

<http://www.nelson.com/math>

Adapting the Task

You can adapt the task in the Student Book to suit the needs of your students. For example:

- If time is short, or if you wish to make the task co-operative, have students work in pairs, with one partner completing parts A, B, and E (4 m/s) and the other partner completing parts C, D, and E (6 m/s).
- If any students would benefit from a challenge, suggest that they use algebra to determine the relationship between the initial upward

velocity u and the maximum height H . ($h = -5\left(t - \frac{u}{10}\right)^2 + \frac{u^2}{20} + 6$,

so $H = \frac{u^2}{20} + 6$).

- Students could research gravity on the Moon and do calculations for a similar situation on the Moon.

Assessment of Learning—What to Look for in Student Work...

Assessment Strategy: Interview/Observation and Product Marking

Level of Performance	1	2	3	4
Knowledge and Understanding Knowledge of content	demonstrates limited knowledge of content	demonstrates some knowledge of content	demonstrates considerable knowledge of content	demonstrates thorough knowledge of content
Understanding of mathematical concepts	demonstrates limited understanding of concepts (e.g., does not communicate any understanding of the relationship between initial velocity and maximum height)	demonstrates some understanding of concepts (e.g., communicates a limited understanding of the relationship between initial velocity and maximum height)	demonstrates considerable understanding of concepts (e.g., communicates a good understanding of the relationship between initial velocity and maximum height)	demonstrates thorough understanding of concepts (e.g., communicates a thorough understanding of the relationship between initial velocity and maximum height)
Thinking Use of planning skills <ul style="list-style-type: none"> • understanding the problem • making a plan for solving the problem 	uses planning skills with limited effectiveness	uses planning skills with some effectiveness	uses planning skills with considerable effectiveness	uses planning skills with a high degree of effectiveness
Use of processing skills <ul style="list-style-type: none"> • carrying out a plan • looking back at the solution 	uses processing skills with limited effectiveness (e.g., demonstrates little ability to follow a plan and does not make adjustments if needed)	uses processing skills with some effectiveness (e.g., demonstrates some ability to follow a plan and attempts to make adjustments if needed)	uses processing skills with considerable effectiveness (e.g., demonstrates ability to follow a plan and makes some adjustments if needed)	uses processing skills with a high degree of effectiveness (e.g., demonstrates ability to follow a plan efficiently and makes reasonable adjustments if needed)
Use of critical/creative thinking processes	uses critical/creative thinking processes with limited effectiveness	uses critical/creative thinking processes with some effectiveness	uses critical/creative thinking processes with considerable effectiveness	uses critical/creative thinking processes with a high degree of effectiveness

Assessment of Learning—What to Look for in Student Work...

Assessment Strategy: Interview/Observation and Product Marking

Level of Performance	1	2	3	4
Communication Expression and organization of ideas and mathematical thinking, using oral, visual, and written forms	expresses and organizes mathematical thinking with limited effectiveness	expresses and organizes mathematical thinking with some effectiveness	expresses and organizes mathematical thinking with considerable effectiveness	expresses and organizes mathematical thinking with a high degree of effectiveness
Communication for different audiences and purposes in oral, visual, and written forms	communicates for different audiences and purposes with limited effectiveness	communicates for different audiences and purposes with some effectiveness	communicates for different audiences and purposes with considerable effectiveness	communicates for different audiences and purposes with a high degree of effectiveness
Use of conventions, vocabulary, and terminology of the discipline in oral, visual, and written forms	uses conventions, vocabulary, and terminology of the discipline with limited effectiveness	uses conventions, vocabulary, and terminology of the discipline with some effectiveness	uses conventions, vocabulary, and terminology of the discipline with considerable effectiveness	uses conventions, vocabulary, and terminology of the discipline with a high degree of effectiveness
Application Application of knowledge and skills in familiar contexts	applies knowledge and skills in familiar contexts with limited effectiveness (e.g., cannot complete the square to determine the maximum heights)	applies knowledge and skills in familiar contexts with some effectiveness (e.g., makes some errors when completing the square to determine the maximum heights)	applies knowledge and skills in familiar contexts with considerable effectiveness (e.g., almost always completes the square correctly to determine the maximum heights)	applies knowledge and skills in familiar contexts with a high degree of effectiveness (e.g., always completes the square correctly to determine the maximum heights)
Transfer of knowledge and skills to new contexts	transfers knowledge and skills to new contexts with limited effectiveness	transfers knowledge and skills to new contexts with some effectiveness	transfers knowledge and skills to new contexts with considerable effectiveness	transfers knowledge and skills to new contexts with a high degree of effectiveness
Making connections within and between various contexts	makes connections within and between various contexts with limited effectiveness	makes connections within and between various contexts with some effectiveness	makes connections within and between various contexts with considerable effectiveness	makes connections within and between various contexts with a high degree of effectiveness

CHAPTERS 4–6 CUMULATIVE REVIEW

1. D; students who answered incorrectly may need to review common factors in polynomials in Lesson 4.1.
2. B; students who answered incorrectly may need to review factoring quadratics of the form $ax^2 + bx + c$, where $a = 1$, in Lesson 4.3.
3. A; students who answered incorrectly may need to review factoring quadratics of the form $ax^2 + bx + c$, where $a \neq 1$, in Lesson 4.4.
4. C; students who answered incorrectly may need to review factoring perfect-square trinomials in Lesson 4.5.
5. A; students who answered incorrectly may need to review factoring differences of squares in Lesson 4.5.
6. D; students who answered incorrectly may need to review the strategies for factoring various types of polynomials in Lesson 4.6.
7. C; students who answered incorrectly may need to review the effect of the parameter a on the graph of the equation $y = ax^2$ in Lesson 5.1.
8. B; students who answered incorrectly may need to review the effects of h and k on the graph of the equation $y = a(x - h)^2 + k$ in Lesson 5.2.
9. D; students who answered incorrectly may need to review transformations applied to the graph of $y = ax^2$ in Lesson 5.3.
10. D; students who answered incorrectly may need to review determining the equation of a parabola in vertex form in Lesson 5.4.
11. C; students who answered incorrectly may need to review the equation of a parabola in vertex form in Lesson 5.4.
12. D; students who answered incorrectly may need to review solving problems using the vertex form of a quadratic relation in Lesson 5.5.
13. A; students who answered incorrectly may need to review the connection between a quadratic relation in partially factored form and a quadratic relation in vertex form in Lesson 5.6.
14. C; students who answered incorrectly may need to review using quadratic relations in partially factored form to solve problems in Lesson 5.6.
15. A; students who answered incorrectly may need to review connecting standard, factored, and vertex forms of quadratic relations in Lesson 5.6.
16. D; students who answered incorrectly may need to review solving quadratic equations in Lesson 6.1.
17. C; students who answered incorrectly may need to review creating perfect-square trinomials in Lesson 6.2.
18. A; students who answered incorrectly may need to review completing the square in Lesson 6.3.
19. C; students who answered incorrectly may need to review completing the square in Lesson 6.3.
20. A; students who answered incorrectly may need to review solving quadratic equations using the quadratic formula in Lesson 6.4.
21. C; students who answered incorrectly may need to review using the discriminant to determine the number of real roots to a quadratic equation in Lesson 6.5.
22. B; students who answered incorrectly may need to review using the discriminant to determine the number of real solutions to a quadratic equation in Lesson 6.5.
23. B; students who answered incorrectly may need to review using the discriminant to determine the number of real solutions to a quadratic equation in Lesson 6.5.

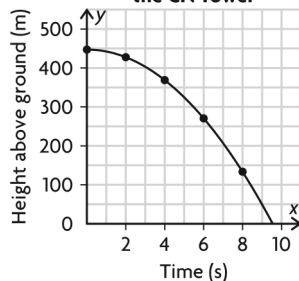
24. D; students who answered incorrectly may need to review solving problems using quadratic models in Lesson 6.6.
25. D; students who answered incorrectly may need to review connecting the standard, factored, and vertex forms of quadratic relations in Lesson 5.6 with the quadratic formula in Lesson 6.4.
26. Write each relation in factored form.

The relation for Sid is $P = -6(n - 4)(n - 8)$. The maximum profit occurs at (6, 24), which is the vertex of the graph of the relation. The maximum profit is \$24 000; it occurs when 6000 pairs of shoes are manufactured and sold. The break-even points are 4000 and 8000 pairs of shoes manufactured and sold.

The relation for Nancy is $P = -8(n - 1)(n - 4)$. The maximum profit occurs at (2.5, 18), which is the vertex of the graph of the relation. The maximum profit is \$18 000; it occurs when 2500 pairs of shoes are manufactured and sold. The break-even points are 1000 and 4000 pairs of shoes manufactured and sold.

Students who answered incorrectly may need to review solving problems using quadratic relations in Lesson 5.5.

27. a) **Dropping a Penny from the CN Tower**



b)

x	y	First Difference	Second Difference
0	447.0		
2	427.4	-19.6	
4	368.6	-58.8	-39.2
6	270.6	-98.0	-39.2
8	133.4	-137.2	-39.2

Yes. The second differences are constant, but not zero.

- c) Answers may vary, e.g., using (0, 447.0) as the vertex and (8, 133.4) as another point on the graph, $a = -4.9$ so the equation is $y = -4.9x^2 + 477$.
- d) $y = -4.9x^2 + 477$; answers may vary, e.g., comparing this equation with my equation for part c), the fit is perfect.
- e) The penny is about 298.8 m above the ground after it has fallen for 5.5 s.
Students who answered incorrectly may need to review quadratic models in Lesson 5.4.
28. a) Equation ①: maximum profit is \$1960 when $x = 6$; selling price is \$25.99.
Equation ②: maximum profit is \$1653.69 when $x = 2.25$; selling price is \$22.24.
- b) Equation ①: zeros occur at $x = -8$ and $x = 20$; break-even prices are \$11.99 and \$39.99.
Equation ②: zeros occur at $x = -10.01$ and $x = 14.51$; break-even prices are \$9.98 and \$34.50.
- c) Answers may vary, e.g., I would recommend the selling price of \$25.99, based on equation ①. This gives the greatest possible profit.
Students who answered incorrectly may need to review connecting standard, factored, and vertex forms of quadratic relations in Lesson 6.6.