Chapter 7 Class 12

Integration **Formula Sheet**

by teachoo.com

Basic Formulae

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1.$ *Particularly*, $\int dx = x + c$
2. $\int \cos x\ dx = \sin x + C$
3. $\int \sin x\ dx = -\cos x + C$
4. $\int \sec^2 x\,dx = \tan x + c$
5. $\int \operatorname{cosec}^2 x\,dx = -\cot x + c$
6. $\int \sec x \tan x\,dx = \sec x + c$
7. $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + c$
8. $\int \frac{dx}{\sqrt{1 - x^2}} = \sin^{-1} x + c$
9. $\int \frac{dx}{\sqrt{1 - x^2}} = -\cos^{-1} x + c$
10. $\int \frac{dx}{1 + x^2} = \tan^{-1} x + c$

Questions in **Ex 7.2** and **Ex 7.3**

11. $\int \frac{dx}{1+x^2} = -\cot^{-1} x + c$

12. $\int e^x \, dx = e^x + c$

13. $\int a^x dx = \frac{a^x}{\log a} + c$

14. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c$

15. $\int \frac{dx}{x\sqrt{x^2-1}} = -\text{cosec}^{-1} x + c$

16. $\int \frac{1}{x} dx = \log|x| + c$

17. $\int \tan x \, dx = \log|\sec x| + c$

18. $\int \cot x \, dx = \log|\sin x| + c$

19. $\int \sec x \, dx = \log|\sec x + \tan x| + c$

20. $\int \text{cosec}\, x \, dx = \log|\text{cosec}\, x - \cot x| + c$

Integrals of some special functions

1. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left|\frac{x-a}{x+a}\right| + c$

2. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left|\frac{a+x}{a-x}\right| + c$

3. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

4. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left|x + \sqrt{x^2 - a^2}\right| + C$

5. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$

6. $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left|x + \sqrt{x^2 + a^2}\right| + C$

Questions in **Ex 7.4**

Integrals by partial fractions

1. $\frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$, $a \neq$ b

2. $\frac{px+q}{(x-a)^2} = \frac{A}{x-a} + \frac{B}{(x-a)^2}$

3. $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$

4. $\frac{px^2+qx+r}{(x-a)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$

5. $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)} = \frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$

Where x^2+ bx + c can not be factorised further.

Questions in **Ex 7.5**

Integration by parts

1. $\int f(x)\, g(x)\ dx = f(x) \int g(x)\, dx - \int \left(f'(x) \int g(x)\, dx \right) dx$

To decide first function. We use

I → Inverse *(Example $sin^{-1} x$)*

L → Log *(Example $\log x$)*

A → Algebra *(Example x^2, x^3)*

T → Trignometry *(Example $sin^2 x$)*

E → Exponential *(Example e^x)*

Questions in **Ex 7.6**

2. $\int e^x\ [f(x) + f'(x)]$ dx = $\int e^x$ f(x) dx + C

Other Special Integrals

1. $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$

2. $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + C$

3. $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{1}\frac{x}{a} + C$

Questions in
Ex 7.7

Area as a sum

$$\int f(x)\,dx$$

$$= (b-a)\lim_{n\to\infty}\frac{1}{n}\left(f(a)+f(a+h)+f(a+2h)\ldots+\right.$$

Questions in
Ex 7.8

Properties of definite integration

P₀ : $\int_a^b f(x)dx = \int_a^b f(t)dt =$

P₁ : $\int_a^b f(x)dx = -\int_b^a f(x)dx$. In particular, $\int_a^a f(x)dx = 0$

P₂ : $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

P₃ : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx.$

P₄ : $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

P₅ : $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

P₆ : $\int_0^{2a} f(x) = \begin{cases} 2\int_0^a f(x)dx, & if\ f(2a-x) = f(x) \\ 0, & if\ f(2a-x) = -f(x) \end{cases}$

P₆ : $\int_{-a}^{a} f(x) = \begin{cases} 2\int_0^a f(x)dx, & if\ f(-x) = f(x) \\ 0, & if\ f(-x) = -f(x) \end{cases}$

Questions in **Ex 7.11**