# Chapter 7

# Decimals: Rational Numbers and Percent

"Students understand decimal notation as an extension of the base-ten system of writing whole numbers that is useful for representing more numbers, including numbers between 0 and 1, between 1 and 2, and so on. Students relate their understanding of fractions to reading and writing decimals that are greater than or less than 1, identifying equivalent decimals, comparing and ordering decimals, and estimating decimal or fractional amounts in problem solving."

*—Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*

"Percents, which can be thought about in ways that combine aspects of both fractions and decimals, offer students another useful form of rational numbers. Percents are particularly useful when comparing fractional parts of sets or numbers of unequal size, and they are also frequently encountered in problem-solving situations that arise in everyday life. As with fractions and decimals, conceptual difficulties need to be carefully addressed in instruction."

*—Principles and Standards for School Mathematics*

Instruction related to decimals and to computation with decimals will have its greatest impact when it is based on the same models and understanding as fractions and whole numbers. The activities in this chapter extend and reinforce the models that have been used previously with whole numbers and fractions. Decimal numbers will be modeled with base-ten blocks in a variety of ways.

Example:  *If a rod = 1, then the small cube = 0.1.*
*If a flat = 1, then a rod = 0.1, and the small cube = 0.01*

These models reinforce the concept of a decimal number being part of a whole.

Number and operation sense and estimation skills with whole numbers developed in Chapters 2 and 3 will be reviewed and applied in the development of algorithms for multiplication of decimals.

The concept of percent will be developed by extending rational number concepts and applying models that have been used previously with whole numbers, fractions, and decimals. The word percent means part of 100. The idea of part of a whole relates to the concept of a fraction. When the whole is 100, there is also a direct connection to decimals.

## Activity 1:  What's My Name?

| | |
|---|---|
| **PURPOSE** | Develop the relationship between fractions and decimals. |
| **MATERIALS** | Pouch: Base Ten Blocks |
| **GROUPING** | Work individually. |
| **GETTING STARTED** | Let 1 rod = 1, and 1 cube = 0.1. |

Write the fraction and the decimal for the shaded part of each figure. If necessary, use the cubes to determine the decimal part.

**Fraction   Decimal**

1. _____  _____

2. _____  _____

3. _____  _____

4. _____  _____

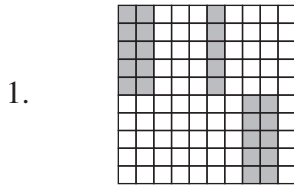5. _____  _____

6. _____  _____

Match the numbers of the problems that have equal decimal answers.
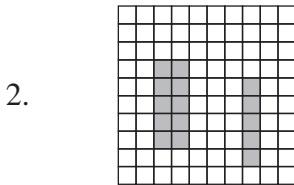
1.  _____  =  _____

2.  _____  =  _____

3.  _____  =  _____

In the following problems, a flat = 1, a rod = 0.1, and a small cube = 0.01. Write the fraction and the decimal for the shaded part of each figure.

|  | **Fraction** | **Decimal** |
|---|---|---|
| 1. | _____ | _____ |
| 2. | _____ | _____ |
| 3. | _____ | _____ |
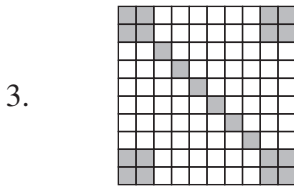
The 100 grid represents a flat. Shade in a part to show the correct fraction or decimal and fill in the missing number in each problem.

|  | **Fraction** | **Decimal** |
|---|---|---|
| 1. | $\dfrac{63}{100}$ | _____ |
| 2. | _____ | 0.07 |
| 3. | $\dfrac{14}{100}$ | _____ |

# Activity 2:  Deci-Order

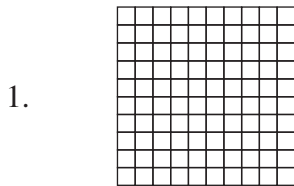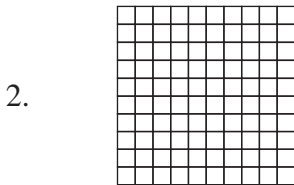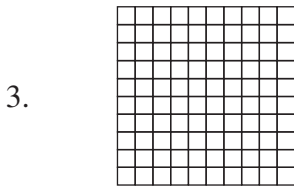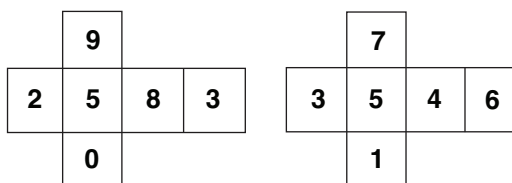| | |
|---|---|
| **PURPOSE** | Reinforce ordering of decimals. |
| **MATERIALS** | Online: Deci-Order Grids |
| | Other: Two polyhedral dice (8 or 10 faces) or cubes marked as shown |



| | |
|---|---|
| **GROUPING** | Work in pairs. |
| **GETTING STARTED** | Follow the rules below to play *Deci-Order*. |

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### RULES FOR DECI-ORDER

1. Player 1 rolls the dice and forms a decimal number with the numbers showing on the dice. All numbers will be hundredths. For example, if 3 and 5 are rolled, 0.35 or 0.53 can be formed. Player 1 enters the number formed in any square on the grid.

2. Players then alternate turns rolling the dice, forming a decimal number, and placing it on the grid. Players must try to enter their numbers so that the entries in each row, column, or diagonal are in ascending or descending order.

3. A player scores three points by completing a row, column, or diagonal in which each entry is in the correct order. One point is lost if a player enters a number in a square that interrupts the order of that row, column, or diagonal. No points are lost for playing a number in a square where the order of the row, column, or diagonal is already interrupted.

   **Examples:**

   **Deci-Order**

   | | | |
   |---|---|---|
   | .39 | .56 | .21 |
   | .95 | | .39 |
   | | .51 | |
   | | .62 | .71 | .85 |

   A. If a player rolls 7 and 5, two points would be scored by playing 0.75 in row 4, column 1 of the grid at the left, since three points would be scored for completing the diagonal and one point would be lost for interrupting the order of the row.

   B. If 0.57 were placed in that position instead of 0.75, six points would be scored for completing both the row and the diagonal.

   C. No points would be scored by placing 0.57 or 0.75 in row 1, column 1, since the order of row 1 was previously interrupted.

4. Players keep a running score after each turn; play continues until the grid is filled.

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## Activity 3:  Race for the Flat

**PURPOSE**    Model addition of decimals using Base Ten Blocks.

**MATERIALS**   Pouch: Base Ten Blocks
Other: Number cubes labeled as shown

| | .7 | | | | | .09 | | |
|---|---|---|---|---|---|---|---|---|
| .1 | .3 | .5 | .2 | | .04 | .02 | .03 | .05 |
| | .8 | | | | | .07 | | |
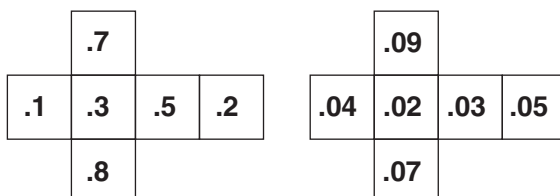
**GROUPING**    Work in pairs.

**GETTING STARTED**    In this game, the flat = 1, the rod = 0.1, and the cube = 0.01. Players alternate turns rolling the dice. After each roll, a player collects the correct number of rods and cubes corresponding to the numbers on the dice and places them on his or her flat. On each roll after the first, a player adds the new blocks to those on the flat, trading when necessary. Each addition must be recorded and validated by the blocks on the flat as shown below. The winner is the first player to exactly cover the flat. On any turn, a player may choose to roll only one die.

**Example:**

Turn 1

.2  .07  = 0.27

Turn 2

.1  .05  = 0.15

**0.27**                    **0.27 + 0.15          =          0.42**
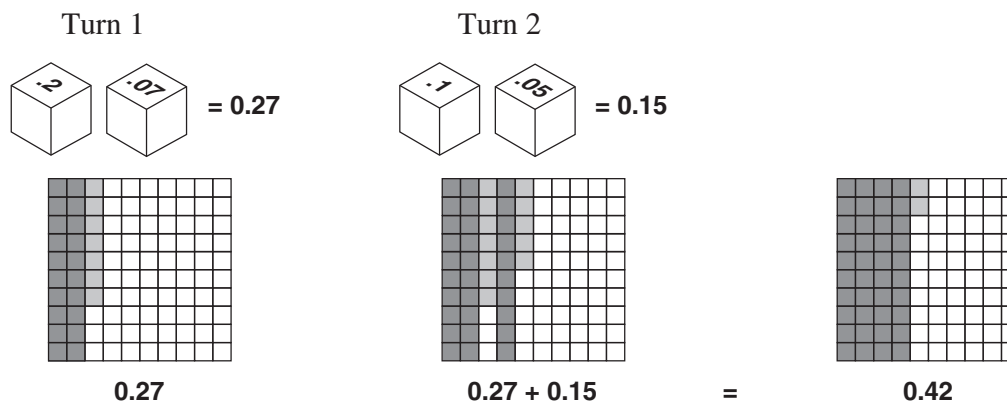
Play another game in which a player does not have to fill the flat exactly. Choose a goal for the game from one of the following:

A.   The winner is the person whose number is closest to 1.
B.   The winner is the person whose number is the greatest.

Players roll the dice and add blocks to the flat as above. For Goal A, a player's strategy will determine when to stop rolling the dice. For Goal B, players stop adding blocks on the roll that covers the flat, with or without some blocks left over.

At the conclusion of a game, players compare the decimal numbers represented by all of their blocks. The winner is the person whose number meets the goal of the game.
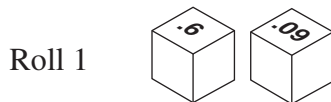
# Activity 4: Empty the Board

**PURPOSE** Model subtraction of decimals using Base Ten Blocks.

**MATERIALS** Pouch: Base Ten Blocks
Online: Place-Value Operations Board
Other: Tenths and hundredths decimal dice (see Activity 3) and
a colored chip

**GROUPING** Work in pairs.

**GETTING STARTED** In this activity, a flat = 1. Place the chip on the place-value operations board as shown to represent the decimal point. Each player arranges a stack of five flats on his or her board. Players alternate turns rolling the dice. After each roll, the player who rolled the dice removes blocks equivalent to the sum of the numbers on the dice, making trades when necessary. Then the player must record the subtraction problem that was modeled by removing the blocks. The first player to empty the entire board is the winner.
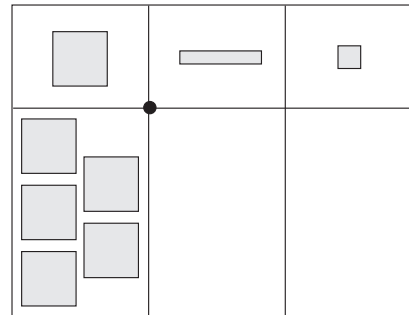
**Example:**

**PLACE-VALUE OPERATIONS BOARD**

Roll 1

$$\begin{array}{r} 5.00 \\ -\phantom{0}.69 \\ \hline 4.31 \end{array}$$

Roll 2
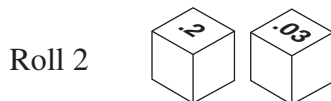
$$\begin{array}{r} 4.31 \\ -\phantom{0}.23 \\ \hline 4.08 \end{array}$$



**EXTENSION** How would you adapt this game to reinforce addition of decimals, and how would you adapt the game in Activity 3 for subtraction?

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**DECIMAL PUZZLE**

In each of the given numbers, the decimal point may be placed in front of the first digit, behind the last digit, or between any two digits. Place the decimal point in each number so that the sum of the resulting numbers is 100.

$$\begin{array}{r} 6\,8 \\ 9\,5\,9 \\ 4\,8\,1 \\ 1\,7\,3\,4 \\ 2\,2\,7\,9 \\ +\ 4\,0\,1\,1 \\ \hline \end{array}$$   **Answer is 100**

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## Activity 5:  Decimal Arrays

**PURPOSE**    Develop an algorithm for multiplication of decimals.

**MATERIALS**    Pouch: Base Ten Blocks

**GROUPING**    Work individually.

**GETTING STARTED**    Recall from Activity 8 in Chapter 6 on multiplication of fractions that 0.2 × 0.3 means 0.2 of 0.3. In this activity, a flat = 1, a rod = 0.1, and the small cube = 0.01.

**Example:**    To determine 0.2 × 0.3, place a flat on your paper and label the factors as shown.
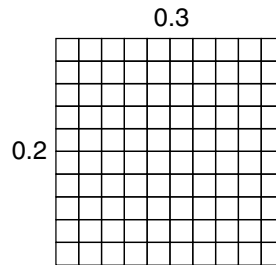


Place three rods vertically on the flat as shown to model 0.3.
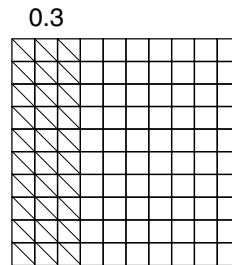


Place two rods horizontally as shown to model 0.2.
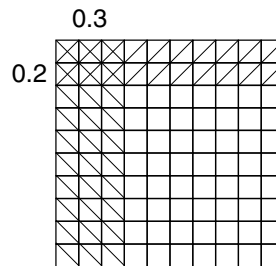


The product of 0.2 × 0.3 is determined by the rectangular array formed by the overlapping parts of the rods. The six squares in the array represent *6 parts of 100*.

So, 0.2 of 0.3 or 0.2 × 0.3 = 0.06.

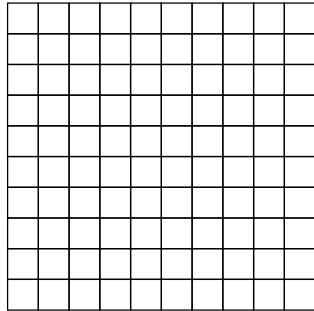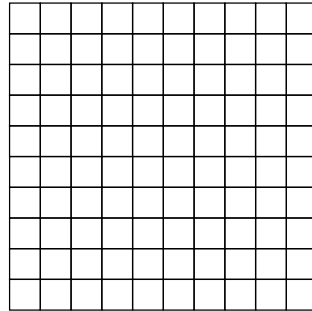Use base ten blocks to solve these multiplication problems. Record your results on the 100 grids by labeling the dimensions (factors) and shading in the correct number of rods. Find the number of squares in the overlapping area to determine each product.

$0.4 \times 0.7 =$ _____        $0.5 \times 0.2 =$ _____        $0.8 \times 0.9 =$ _____
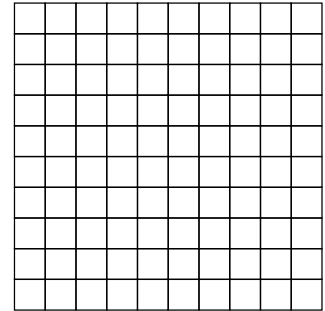
From what you have observed in this activity, write a rule for multiplying decimals.
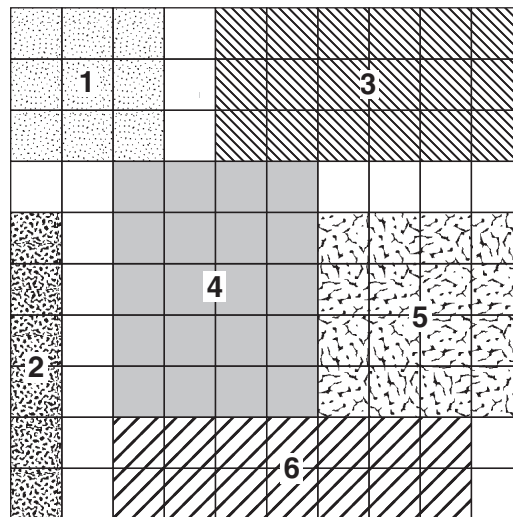
The 100 grid above represents a flat that is equal to 1. Write the dimensions for each shaded rectangle and determine its area.

| Rectangle | Dimensions | Area | Rectangle | Dimensions | Area |
|-----------|-----------|------|-----------|-----------|------|
| 1 | ____ × ____ | ____ | 2 | ____ × ____ | ____ |
| 3 | ____ × ____ | ____ | 4 | ____ × ____ | ____ |
| 5 | ____ × ____ | ____ | 6 | ____ × ____ | ____ |

How can you use the dimensions of the rectangle to determine its area?

## Activity 6:  Decimal Multiplication

**PURPOSE**    Use Base Ten Blocks to extend multiplication of decimals.

**MATERIALS**    Pouch: Base Ten Blocks
Online: Multiplication and Division Frame

**GROUPING**    Work individually.

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For these problems, a flat = 1. Model the multiplication by using base ten blocks to construct a rectangle in the multiplication and division frame. Then determine the product of the two factors. Record your work as shown in the example.

**Example: 2.4 $\times$ 1.3 =**

$$
\begin{array}{rrr}
 & 2 & + & .4 \\
\times & 1 & + & .3 \\
\hline
 & 2 & + & .4 \\
 & & .6 & + & .12 \\
\hline
 & 2 & + & 1.0 & + & .12 & = & 3.12
\end{array}
$$
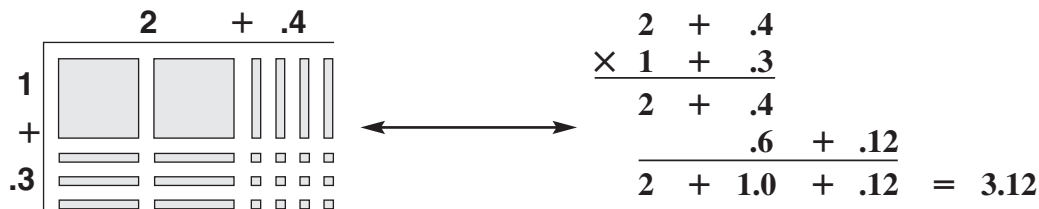
1.  3 $\times$ 2.3 =  _____

2.  1.4 $\times$ 4.2 =  _____

3.  3.4 $\times$ 1.2 =  _____

4.  2.2 $\times$ 3.5 =  _____

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**EXTENSION**    Construct some rectangular arrays to represent the product of two decimals. Give the arrays to another student, identifying the correct decimal value for each base ten block. Instruct the student to determine the factors of your number.

# Activity 7:  Dice and Decimals

|   |   |
|---|---|
| **PURPOSE** | Reinforce operations with decimal numbers. |
| **MATERIALS** | Other: Dice and Decimals game board (on next page), a pair of dice, and chips for markers |
| **GROUPING** | Work in groups of two to four players. |
| **GETTING STARTED** | Follow the rules below to play *Dice and Decimals.* |

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**RULES FOR DICE AND DECIMALS**

1.  The dots on each face of the dice represent a decimal number in tenths: 0.1, 0.2, 0.3, 0.4, 0.5, and 0.6. To begin play, place a chip on each **FREE** square on the game board.

2.  Each player rolls the dice and finds the sum of the numbers showing on them. The player with the LEAST sum begins; play progresses to the next player on the right.

3.  On his or her turn, the player rolls the dice and performs any operation with the two numbers showing on them. The player then places a marker on the resulting number on the game board. A player may not place a marker on a number that is already covered. When using division, a remainder is not allowed.

    **Example:** $0.6 \div 0.5 = 1.2$ can be played, but $0.5 \div 0.6 = 0.8\overline{3}$ is not allowed.

4.  To score points, a player must place a marker on a number on the game board that is adjacent vertically, horizontally, or diagonally to a previously covered square. One point is scored for each adjacent covered square. No points are scored by placing a marker on a square that touches the **NO COUNT** square.

5.  If a player is unable to produce an uncovered number, he or she must pass the dice to the next player. If another player knows a play that can be made with the numbers on the dice, that player may call attention to the mistake and tell the other players the operation that will result in an uncovered number. The player citing the mistake may then place a marker on that number and earn points. This does not affect the turn of the player citing the mistake. If more than one player calls attention to a mistake, the first player to do so makes the play.

6.  Players keep a running total of their scores. A player who cannot produce an uncovered number in three successive turns is eliminated from the game. When the game board is filled, or if all players have failed to play in three successive turns, the game ends. The player with the highest score is the winner.

# Dice and Decimals
## Game Board

| | | | | | | |
|---|---|---|---|---|---|---|
| FREE | 0 | .01 | .02 | .03 | .04 | FREE |
| .05 | .06 | .08 | .09 | .1 | .12 | .15 |
| .16 | .18 | .2 | .24 | .25 | .3 | .36 |
| .4 | .5 | .6 | NO COUNT | .7 | .75 | .8 |
| .9 | 1 | 1.1 | 1.2 | 1.25 | 1.5 | 2 |
| FREE | 2.5 | 3 | 4 | 5 | 6 | FREE |

# Activity 8: Target Number Revisited

|  |  |
|---|---|
| **PURPOSE** | Develop number and operation sense, estimation skills, and an increased understanding of multiplication and division of decimals. |
| **MATERIALS** | Other: A calculator for each pair of students |
| **GROUPING** | Work in pairs. |
| **GETTING STARTED** | Recall the Target Number game in Activity 10 in Chapter 3. All computations will be done on the calculator, but determining the guesses to enter must be done mentally. |
| **Game 1** | Player 1 chooses the target product and Player 2 chooses a factor. Then follow the steps in the flow chart. |
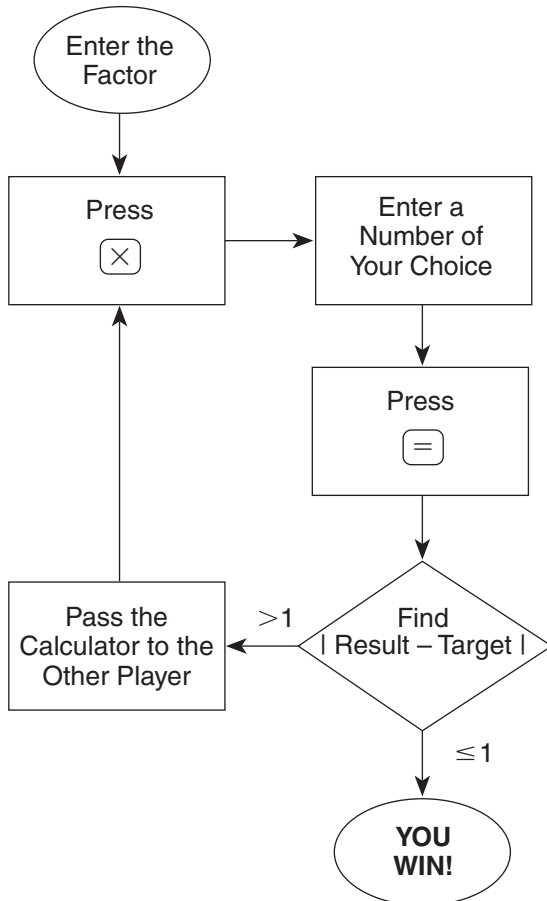
**SAMPLE GAME**

**Target = 238        Factor = 9**

| Player | Keys Pressed | | | | | Display |
|---|---|---|---|---|---|---|
| 1 | 9 | $\times$ | 15 | $=$ | | 135 |
| 2 | | $\times$ | 1.4 | $=$ | | 189 |
| 1 | | $\times$ | 1.2 | $=$ | | 226.8 |
| 2 | | $\times$ | 1.05 | $=$ | | 238.14 |

Since $|238.14 - 238| \leq 1$, player 2 is the winner.

Try these games for practice.

| Game | Target | Factor |
|---|---|---|
| 1 | 585 | 15 |
| 2 | 1107 | 27 |
| 3 | 393 | 7 |
| 4 | 2139 | 13 |

Flow chart:

Enter the Factor → Press $\times$ → Enter a Number of Your Choice → Press $=$ → Find |Result − Target|

- $>1$: Pass the Calculator to the Other Player (back to Press $\times$)
- $\leq 1$: YOU WIN!

|  |  |
|---|---|
| **EXTENSION** | **Game 2**   Player 1 chooses a target quotient, and Player 2 chooses a product. Follow the steps in the flow chart, but enter the product and use the $\div$ key instead of the $\times$ key. *The winner is the player who gets within 0.5 of the target quotient.* |

## Activity 9:  Repeating Decimals

**PURPOSE**   Apply the patterns problem-solving strategy to discover the decimal equivalent for proper fractions whose denominators contain only 9s or a combination of 9s followed by 0s.

**MATERIALS**   Other: Calculator

**GROUPING**   Work individually.

1.  Convert each of the following fractions to its decimal equivalent. If your calculator rounds the last digit, ignore the rounded digit.

   a. $\dfrac{1}{9} =$ _____

   b. $\dfrac{7}{9} =$ _____

   c. $\dfrac{5}{9} =$ _____

   d. $\dfrac{3}{9} =$ _____

2.  a. How many digits are there in the repeating block of the decimal equivalent of each fraction? _____

   b. How many 9s are there in the denominator of each fraction? _____

   c. What is the relationship between the numerator of each fraction and its decimal equivalent?

3.  Convert each of the following fractions to its decimal equivalent.

   a. $\dfrac{13}{99} =$ _____

   b. $\dfrac{47}{99} =$ _____

   c. $\dfrac{5}{99} =$ _____

   d. $\dfrac{214}{999} =$ _____

   e. $\dfrac{75}{999} =$ _____

   f. $\dfrac{235}{9999} =$ _____

   g. $\dfrac{8457}{9999} =$ _____

   h. $\dfrac{53071}{99999} =$ _____

4.  What is the relationship between the number of digits in the repeating block and the number of 9s in the denominator?

5. What is the relationship between the numerator of each fraction and its decimal equivalent?

6. Predict the decimal equivalent for each of the following fractions. Then use your calculator to check your predictions.

   a. $\dfrac{615}{999} = $ _____   b. $\dfrac{8}{99} = $ _____

   c. $\dfrac{137}{9999} = $ _____   d. $\dfrac{7421}{99999} = $ _____

7. Write a rule for finding the decimal equivalent of any proper fraction whose denominator contains only 9s.

8. Explain how the rule that you wrote above can be extended to include the decimal equivalent for an improper fraction such as $\dfrac{25}{9}$ or $\dfrac{137}{99}$.

9. a. Convert $\dfrac{1}{3}$ and $\dfrac{2}{3}$ to decimals. $\dfrac{1}{3} = $ _____ $\dfrac{2}{3} = $ _____.

   b. What is the sum of the decimal numbers? _____

   c. $\dfrac{1}{3} + \dfrac{2}{3} = $ _____

   d. What can you conclude from your answers to Parts b and c?

10. Use the fact that $\dfrac{25}{90} = \dfrac{25}{9} \times \dfrac{1}{10} = \left(2 + \dfrac{7}{9}\right) \times \dfrac{1}{10}$, to determine the decimal equivalent of $\dfrac{25}{90}$.

    $\dfrac{25}{90} = $ _____

11. Use the procedure from Exercise 10 to predict the decimal equivalent for each of the following fractions. Then use your calculator to check your predictions.

    a. $\dfrac{32}{90} = $ _____   b. $\dfrac{76}{90} = $ _____   c. $\dfrac{583}{900} = $ _____

    d. $\dfrac{347}{900} = $ _____   e. $\dfrac{28}{990} = $ _____   f. $\dfrac{2743}{9900} = $ _____

**EXTENSION**  1.  Complete the following table.

| Fraction | Decimal Equivalent | Repeating or Terminating | Prime Factorization of Denominator | Number of Digits in the Repeating Block | Number of Non-Repeating Digits after the Decimal |
|---|---|---|---|---|---|
| $\dfrac{2}{3}$ | $0.\overline{6}$ | Repeats | $3^1$ | 1 | 0 |
| $\dfrac{29}{90}$ | $0.3\overline{2}$ | Repeats | $2^1 \cdot 3^2 \cdot 5^1$ | 1 | 1 |
| $\dfrac{17}{99}$ | $0.\overline{17}$ | | | | |
| $\dfrac{3}{125}$ | | Terminates | | | 3 |
| $\dfrac{1}{8}$ | | | | | |
| $\dfrac{17}{400}$ | | Terminates | | | |
| $\dfrac{119}{5000}$ | | | | | |
| $\dfrac{4179}{4950}$ | | | | | |
| $\dfrac{5}{24}$ | | | | 1 | |
| $\dfrac{121}{600}$ | | | | | |

2. a. What are the prime numbers in the prime factorization of the denominators of those fractions whose decimal equivalents terminate? _____

   b. Does the prime factorization of the denominator of any fraction whose decimal equivalent repeats contain only the prime factors in Part a? If so, which fractions?

3. When a decimal is converted to a fraction, the denominator will be 10 or _____ or _____ or _____ or _____, etc.

4. Write the prime factorization of each power of 10.

   a. 10        b. 100        c. 1000        d. 10,000

5. Based on your answers to Exercises 2 through 4, how can you predict whether the decimal equivalent of a fraction in lowest terms will terminate or repeat?

6. For fractions in lowest terms whose decimal equivalents terminate, how is the number of digits in the decimal equivalent related to the exponents in the prime factorization of the denominator?

7. For fractions in lowest terms whose decimal equivalents repeat, how can you predict the number of digits between the decimal point and the repeating block in the decimal equivalent?

# Activity 10:  What Is Percent?

**PURPOSE**   Develop the concept of percent as a part of 100 and the relationship among fractions, decimals, and percents.

**MATERIALS**   Online: A transparent copy of a 100 Grid
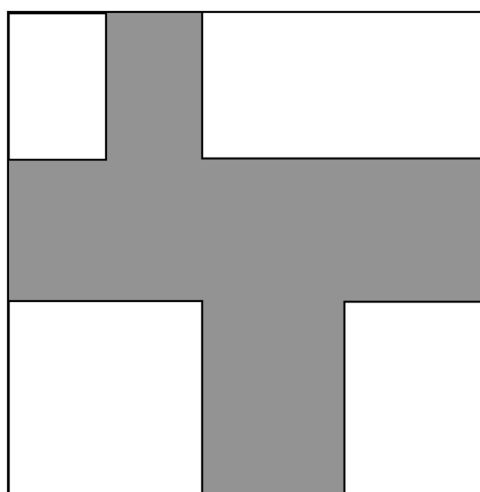
**GROUPING**   Work individually.

**GETTING STARTED**   Percent means *part of* 100. 40% means 40 equal parts out of 100. 80 correct out of 100 problems $= \frac{80}{100} = 80\%$.

1.  Place a copy of the 100 grid over each square. Count the shaded squares in the grid to determine what part of 100 and the percent of each figure that is shaded.
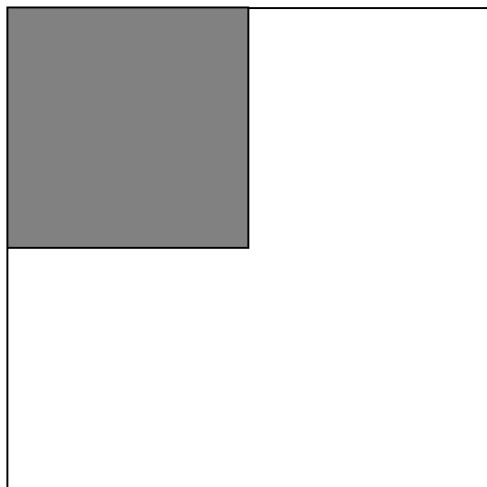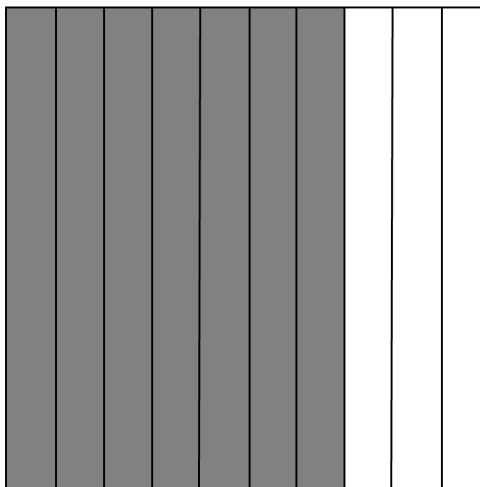
$= \dfrac{\quad}{100}$

$= \underline{\quad}\ \%$

$= \dfrac{\quad}{100}$

$= \underline{\quad}\ \%$
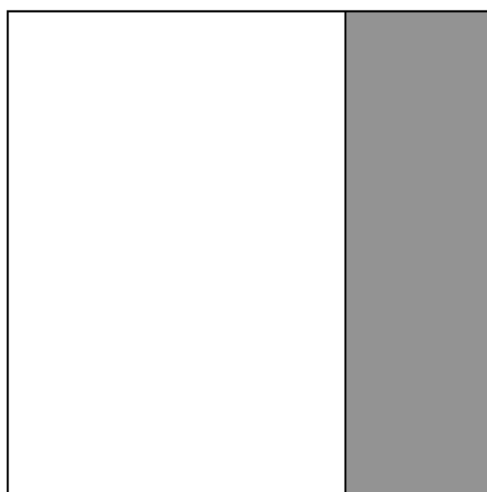
2.  Name the fractional part of each figure that is shaded. Use your 100 grid to determine the equivalent decimal and percent.

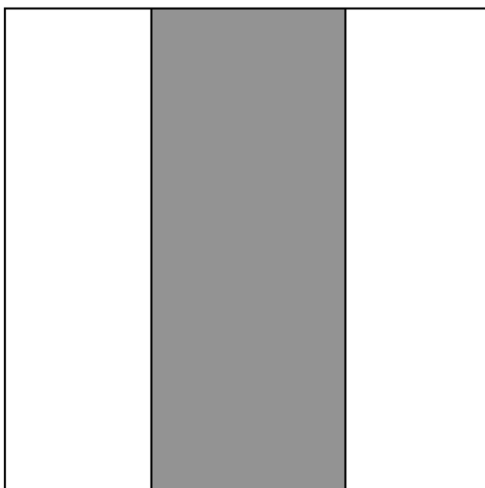$\dfrac{}{\text{Fraction}} = \dfrac{}{\text{Decimal}} = \dfrac{}{\text{Percent}}$

$\dfrac{}{\text{Fraction}} = \dfrac{}{\text{Decimal}} = \dfrac{}{\text{Percent}}$

Complete the following.

1.

15% means _____ out of 100,
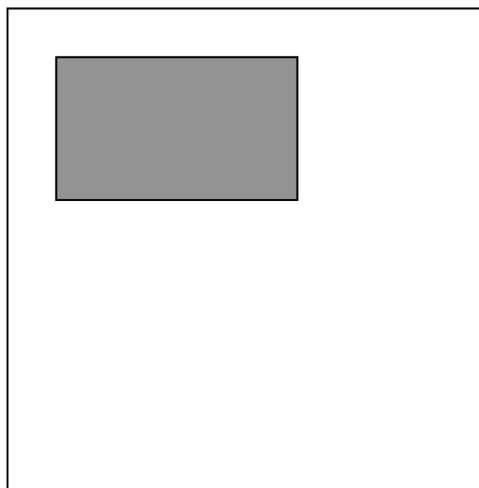
40% means _____ out of 100,
or 20 out of 50,
or _____ out of 25,
or 4 out of _____,
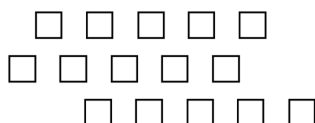or 6 out of _____,
or 60 out of _____.

2.

15% means _____ out of 100,
or _____ out of 20,
or _____ out of $5.00,
or 12 out of _____,
or 24 out of _____,
or _____ out of $25.00.

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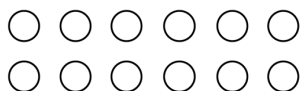1. Shade 20% of the squares.
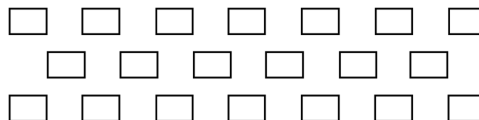$$\frac{20}{100} = \frac{}{15}$$

2. Shade 75% of the triangles.
$$\frac{75}{100} = \frac{}{8}$$

3. Shade 25% of the circles.
   Circle 50% of the circles.

4. Put an X in 35% of the rectangles.
   Put a Y in 25% of the rectangles.
   Put a Z in 15% of the rectangles.

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**EXTENSION**

1. List five real-world applications of percent between 28% and 57%.

2. List three real-world applications of 200%.

3. A headline on the newspaper business page reads, *ABC stock price increases 200% in the past six months from $7\frac{1}{2}$ to 15.* Do you believe the headline? Explain your answer.

# Chapter Summary

The activities in this chapter stressed the conceptual development of decimals. The activities continued the constructivist approach to fractions in Chapter 6 and progressed from the concrete to the representational (pictorial) to the abstract or symbolic level.

Activity 1 used models to demonstrate the relationship between fractions and decimals. The game in Activity 2 provided an opportunity to practice ordering of decimal numbers. The activity also involved problem solving in order to determine the maximum score in each play.

The games in Activities 3 and 4 provided opportunities to physically place rods and cubes on a flat during the addition and subtraction process and then to make the necessary trades in order to represent the answer with the least number of blocks. This physical modeling of the operations and the trading and regrouping process are all essential for understanding the computational algorithms.

Activities 5 and 6 used the rectangular array model for multiplication to extend the processes developed with whole numbers and fractions to decimals. The array model will be revisited in the geometry section. Using this model in a variety of settings provides a firm foundation for understanding the concept of area.

A game format was also used in Activity 7 to provide practice in all operations with decimal numbers. In order to maximize the points earned on a play, problem solving strategies were employed to determine all of the possible outcomes using the two numbers on the dice and then deciding which result produced the greatest score.

Activity 8 revisited the Target Number game from Chapter 3. The guess and check strategy was used with the calculator to reinforce number and operation sense, and estimation skills. This activity also developed an increased understanding of multiplication and division of decimal numbers.

In Activity 9, you explored patterns in the decimal equivalents of fractions whose denominators contain only 9s or a combination of 9s followed by 0s. In addition, you learned how to determine whether the decimal equivalent of a fraction terminates or repeats.

The use of the 100 Grid in Activity 10 established the relationship between percent and 100 and also helped to develop the connections among fractions, decimals, ratios, and percents. These connections were applied in determining the percent of a number by finding a fractional part or multiplying by a decimal.