Chapter 7 — Exponential and logarithmic functions

Exercise 7A — Index laws

1 a $x^2 \times x^5 \times x^3 = x^{2+5+3}$
$= x^{10}$

b $m^3 \times m^2 p \times p^4$
$= m^3 \times m^2 \times p^1 \times p^4$
$= m^5 p^5$

c $5^2 \times 5^7 \times (5^3)^3 = 5^9 \times 5^9$
$= 5^{18}$

d $4y^3 \times 2y \times y^7$
$= 4 \times 2 \times y^3 \times y^1 \times y^7$
$= 8y^{11}$

e $(xy)^3 \times x^4 y^5 = x^3 y^3 \times x^4 y^5$
$= x^3 \times x^4 \times y^3 \times y^5$
$= x^7 y^8$

f $(2x^4)^2 \times (4x^2)^5 = 4x^8 \times 4^5 x^{10}$
$= 4 \times 4^5 \times x^8 \times x^{10}$
$= 4^6 x^{18}$

g $3m^2 p^5 \times (mp^2)^3 \times 2m^4 p^6$
$= 3m^2 p^5 \times m^3 p^6 \times 2m^4 p^6$
$= 3 \times 2 \times m^2 \times m^3 \times m^4 \times p^5 \times p^6$
$= 6m^9 p^{17}$

h $5x^2 y^3 \times (5xy^2)^4 \times (5x^2 y)^2$
$= 5x^2 y^3 \times 5^4 x^4 y^8 \times 5^2 x^4 y^2$
$= 5 \times 5^4 \times 5^2 \times x^2 \times x^4 \times x^4 \times y^3 \times y^8 \times y^2$
$= 5^7 x^{10} y^{13}$

2 a $a^7 b^8 \div a^2 b^5 = a^{7-2} b^{8-5}$
$= a^5 b^3$

b $2a^{12} b^9 \div (2a)^3 b^4$
$= 2^1 a^{12} b^9 \div 2^3 a^3 b^4$
$= 2^{1-3} a^{12-3} b^{9-4}$
$= 2^{-2} a^9 b^5$
$= \frac{a^9 b^5}{4}$

c $(3x^5) y^{11} \div 6x^2 y^2 = 3x^5 y^{11} \div 6x^2 y^2$
$= \frac{3x^5 y^{11}}{6x^2 y^2}$
$= \frac{x^3 y^9}{2}$

d $p^{13} q^{10} \div (pq^4)^2 = \frac{p^{13} q^{10}}{p^2 q^8}$
$= p^{11} q^2$

e $(4mn^4)^2 \div 14n^3 = \frac{16m^2 n^8}{14n^3}$
$= \frac{8m^2 n^5}{7}$

f $\frac{a^3 b^4}{ab^2} = a^{3-1} b^{4-2}$
$= a^2 b^2$

g $25r^{15} s^{10} t^4 \div r^5 (s^5)^2 (5t)^3$
$= \frac{5^2 r^{15} s^{10} t^4}{r^5 s^{10} 5^3 t^3}$
$= 5^{2-3} r^{15-5} s^{10-10} t^{4-3}$
$= 5^{-1} r^{10} s^0 t^1$
$= \frac{r^{10} t}{5}$

h $\frac{15a^6 b^7}{3a^3 b^4} = 5a^{6-3} b^{7-4}$
$= 5a^3 b^3$

i $\frac{24x^4 y^3}{20x^2 y^3} = \frac{6x^{4-2} y^{3-3}}{5}$
$= \frac{6x^2 y^0}{5}$
$= \frac{6x^2}{5}$

3 a $\frac{6p^8 m^4 \times 2p^7 m^6}{9p^5 m^2}$
$= \frac{6 \times 2 \times p^8 \times p^7 \times m^4 \times m^6}{9p^5 m^2}$
$= \frac{4p^{8+7-5} m^{4+6-2}}{3}$
$= \frac{4p^{10} m^8}{3}$

b $\frac{(3x)^2 y^2 \times 5x^6 y^3}{10x^7 y}$
$= \frac{3^2 x^2 y^2 \times 5x^6 y^3}{10x^7 y}$
$= \frac{9 \times 5 \times x^2 \times x^6 \times y^2 \times y^3}{10x^7 y^1}$
$= \frac{9x^{2+6-7} y^{2+3-1}}{2}$
$= \frac{9xy^4}{2}$

c $\frac{14u^{11} v^9 \times (3u^2)^3 v}{21u^6 v^5}$
$= \frac{14u^{11} v^9 \times 3^3 u^6 v}{21u^6 v^5}$
$= \frac{14 \times 27 \times u^{11} \times u^6 \times v^9 \times v^1}{21u^6 v^5}$
$= 18u^{11+6-6} v^{9+1-5}$
$= 18u^{11} v^5$

d $\frac{(5e^3)^2 f^4 \times 8e^4 f^3}{20ef^5}$
$= \frac{5^2 e^6 f^4 \times 8e^4 f^3}{20ef^5}$
$= \frac{25 \times 8 \times e^6 \times e^4 \times f^4 \times f^3}{20e^1 f^5}$
$= 10e^{6+4-1} f^{4+3-5}$
$= 10e^9 f^2$

e $\frac{6w^2 t^7 \times 9w^4 t^{12}}{(3w)^5 t^{13}}$
$= \frac{6w^2 t^7 \times 9w^4 t^{12}}{3^5 w^5 t^{13}}$
$= \frac{6 \times 9 \times w^2 \times w^4 \times t^7 \times t^{12}}{243 w^5 t^{13}}$
$= \frac{2w^{2+4-5} t^{7+12-13}}{9}$
$= \frac{2wt^6}{9}$

f $\frac{(2x)^4 y \times (3x^7 y)^2}{18x^5 (2y)^3}$
$= \frac{2^4 x^4 y \times 3^2 x^{14} y^2}{18x^5 2^3 y^3}$
$= \frac{16 \times 9x^4 \times x^{14} \times y^1 \times y^2}{8 \times 18x^5 y^3}$
$= x^{4+14-5} y^{1+2-3}$
$= x^{13} y^0$
$= x^{13}$

g $\frac{(-3x^3 y^2)^3}{2x^3 y^6} \times \frac{6x^7 y^5}{(x^2 y)^2}$
$= \frac{(-3)^3 \times x^{3\times 3} \times y^{2\times 3} \times 6x^7 y^5}{2x^3 y^6 \times x^{2\times 2} \times y^{1\times 2}}$
$= \frac{-27 \times 6 \times x^9 \times x^7 \times y^6 \times y^5}{2x^3 \times x^4 \times y^6 \times y^2}$
$= -81x^{9+7-3-4} y^{6+5-6-2}$
$= -81x^9 y^3$

h $\frac{(-3mp)^2 \times 4m^4 p}{12(mp)^2}$
$= \frac{(-3)^2 m^2 p^2 \times 4m^2 p}{12m^2 p^2}$
$= \frac{9 \times 4 \times m^2 \times m^4 \times p^2 \times p^1}{12m^2 p^2}$
$= 3 \times m^{2+4-2} \times p^{2+1-2}$
$= 3m^4 p$

i $\frac{m^3 p^4 \times (mp^3)^2}{(-mp^2)^4}$
$= \frac{m^3 p^4 \times m^2 p^6}{m^4 p^8}$
$= \frac{m^3 \times m^2 \times p^4 \times p^6}{m^4 p^8}$
$= m^{3+2-4} p^{4+6-8}$
$= mp^2$

j $\frac{4(u^7 v^6)^3}{(-2u^3 v^2)^2 \times u^4 (3v^5)^2}$
$= \frac{4u^{7\times 3} v^{6\times 3}}{(-2)^2 \times u^{3\times 2} \times v^{2\times 2} \times u^4 \times 3^2 \times v^{5\times 2}}$
$= \frac{4u^{21} v^{18}}{4 \times 9 \times u^6 \times u^4 \times v^4 \times v^{10}}$
$= \frac{u^{21-6-4} v^{18-4-10}}{9}$
$= \frac{u^{11} v^4}{9}$

4 a $\dfrac{15a^8b^3}{9a^4b^5} \div \left(\dfrac{2a^3b}{3ab^2}\right)^2$

$= \dfrac{15a^8b^3}{9a^4b^5} \times \left(\dfrac{3ab^2}{2a^3b}\right)^2$

$= \dfrac{15a^8b^3}{9a^4b^5} \times \dfrac{3^2a^2b^4}{2^2a^6b^2}$

$= \dfrac{15 \times 9 \times a^{8+2-4-6}b^{3+4-5-2}}{9 \times 4}$

$= \dfrac{15a^0b^0}{4}$

$= \frac{15}{4}$

b $\dfrac{5k^{12}d}{(2k^3)^2} \div \dfrac{6kd^4}{25(k^2d^3)^3}$

$= \dfrac{5k^{12}d}{(2k^3)^2} \times \dfrac{25(k^2d^3)^3}{6kd^4}$

$= \dfrac{5k^{12}d^1}{2^2k^6} \times \dfrac{25k^6d^9}{6k^1d^4}$

$= \dfrac{5 \times 25k^{12+6-6-1}d^{1+9-4}}{24}$

$= \dfrac{125k^{11}d^6}{24}$

c $\dfrac{4g^4(2p^{11})^2}{g^3p^7} \div \dfrac{8g^4p}{(2gp)^3}$

$= \dfrac{4g^4(2p^{11})^2}{g^3p^7} \times \dfrac{(2gp)^3}{8g^4p}$

$= \dfrac{4g^4 2^2 p^{22}}{g^3p^7} \times \dfrac{2^3g^3p^3}{8g^4p^1}$

$= \dfrac{4 \times 4 \times 8g^{4+3-3-4}p^{22+3-7-1}}{8}$

$= 16g^0p^{17}$

$= 16p^{17}$

d $\left(\dfrac{3jn^2}{n^5}\right)^3 \div \dfrac{(4j^2n)^2}{n^{13}(2j)^4}$

$= \left(\dfrac{3jn^2}{n^5}\right)^3 \times \dfrac{n^{13}(2j)^4}{(4j^2n)^2}$

$= \dfrac{3^3j^3n^6}{n^{15}} \times \dfrac{n^{13}2^4j^4}{4^2j^4n^2}$

$= \dfrac{27 \times 16j^{3+4-4}n^{6+13-15-2}}{16}$

$= 27j^3n^2$

e $\dfrac{x^4y^7}{x^3y^2} \div \dfrac{x^3y^2}{x^5y}$

$= \dfrac{x^4y^7}{x^3y^2} \times \dfrac{x^5y^1}{x^3y^2}$

$= x^{4+5-3-3}y^{7+1-2-2}$

$= x^3y^4$

f $\dfrac{6x^3y^8}{(x^2y^3)^3} \div \dfrac{(2xy^3)^2}{8x^5y^7}$

$= \dfrac{6x^3y^8}{(x^2y^3)^3} \times \dfrac{8x^5y^7}{(2xy^3)^2}$

$= \dfrac{6x^3y^8}{x^6y^9} \times \dfrac{8x^5y^7}{2^2x^2y^6}$

$= \dfrac{6 \times 8x^{3+5-6-2}y^{8+7-9-6}}{4}$

$= 12x^0y^0$

$= 12$

5 a $\dfrac{3p^3m^4}{p^1m^2} = 3p^{3-1}m^{4-2}$

$= 3p^2m^2$

The answer is **A**

b $\dfrac{6x^6y^5}{x^5y^3} \times \dfrac{x^4}{(2y)^2}$

$= \dfrac{6x^6y^5}{x^5y^3} \times \dfrac{x^4}{2^2y^2}$

$= \dfrac{6x^{6+4-5}y^{5-3-2}}{4}$

$= \dfrac{3x^5y^0}{2}$

$= \dfrac{3x^5}{2}$

The answer is **E**

c $\dfrac{3ab^3}{-ab} \div \left(\dfrac{a^2b}{a^5}\right)^2$

$= \dfrac{3ab^3}{-ab} \times \left(\dfrac{a^5}{a^2b}\right)^2$

$= \dfrac{3a^1b^3}{-a^1b^1} \times \dfrac{a^{10}}{a^4b^2}$

$= \dfrac{3a^{1+10-1-4}b^{3-1-2}}{-1}$

$= -3a^6b^0$

$= -3a^6$

The answer is **B**

6 a $\dfrac{x^{n+1} \times y^5 \times z^{4-n}}{x^{n-2} \times y^{4-n} \times z^{3-n}}$

$= x^{n+1-(n-2)}y^{5-(4-n)}z^{4-n-(3-n)}$

$= x^{n+1-n+2}y^{5-4+n}z^{4-n-3+n}$

$= x^3y^{1+n}z$

b $\dfrac{(x^ny^{m+3})}{x^{n+2}y^{3-m}} \times \dfrac{x^2y}{x^{n-5} \times y^{5-3m}}$

$= \dfrac{x^{2n}y^{2m+6}}{x^{n+2}y^{3-m}} \times \dfrac{x^2y}{x^{n-5}y^{5-3m}}$

$= x^{2n+2-(n+2)-(n-5)}y^{2m+6+1-(3-m)-(5-3m)}$

$= x^{2n+2-n-2-n+5}y^{2m+6+1-3+m-5+3m}$

$= x^5y^{6m-1}$

7 a $2^4 \times 4^2 \times 8$

$= 2^4 \times (2 \times 2)^2 \times (2 \times 2 \times 2)$

$= 2^4 \times (2^2)^2 \times 2^3$

$= 2^4 \times 2^4 \times 2^3$

$= 2^{11}$

b $3^7 \times 9^2 \times 27^3 \times 81$

$= 3^7 \times (3 \times 3)^2 \times (3 \times 3 \times 3)^3 \times (3 \times 3 \times 3 \times 3)$

$= 3^7 \times (3^2)^2 \times (3^3)^3 \times 3^4$

$= 3^7 \times 3^4 \times 3^9 \times 3^4$

$= 3^{24}$

c $5^3 \times 15^2 \times 3^2 = 5^3 \times (5 \times 3)^2 \times 3^2$
$= 5^3 \times 5^2 \times 3^2 \times 3^2$
$= 5^5 \times 3^4$

d $20^5 \times 8^4 \times 125$
$= (2 \times 2 \times 5)^5 \times (2 \times 2 \times 2)^4 \times (5 \times 5 \times 5)$
$= (2^2 \times 5)^5 \times (2^3)^4 \times 5^3$
$= 2^{10} \times 5^5 \times 2^{12} \times 5^3$
$= 2^{22} \times 5^8$

e $\dfrac{3^4 \times 27^2}{6^4 \times 3^5} = \dfrac{3^4 \times (3 \times 3 \times 3)^2}{(2 \times 3)^4 \times 3^5}$
$= \dfrac{3^4 \times (3^3)^2}{2^4 \times 3^4 \times 3^5}$
$= \dfrac{3^4 \times 3^6}{2^4 \times 3^4 \times 3^5}$
$= \dfrac{3^{4+6-4-5}}{2^4}$
$= \dfrac{3}{2^4}$

f $\dfrac{8 \times 5^2}{2^3 \times 10} = \dfrac{(2 \times 2 \times 2) \times 5^2}{2^3 \times (5 \times 2)}$
$= \dfrac{2^3 \times 5^2}{2^3 \times 5^1 \times 2^1}$
$= 2^{3-3-1} \times 5^{2-1}$
$= 2^{-1} \times 5$
$= \frac{5}{2}$

8 a $\dfrac{4^5}{2^7} = \dfrac{(2 \times 2)^5}{2^7}$
$= \dfrac{(2^2)^5}{2^7}$
$= \dfrac{2^{10}}{2^7}$
$= 2^3$
$= 8$

b $9^4 \times 3^5 \div 27 = \dfrac{9^4 \times 3^5}{27}$
$= \dfrac{(3 \times 3)^4 \times 3^5}{(3 \times 3 \times 3)}$
$= \dfrac{(3^2)^4 \times 3^5}{3^3}$
$= \dfrac{3^8 \times 3^5}{3^3}$
$= 3^{10}$
$= 59\,049$

c $\dfrac{(16^2)^3}{(2^5)^4} = \dfrac{16^6}{2^{20}}$
$= \dfrac{(2 \times 2 \times 2 \times 2)^6}{2^{20}}$
$= \dfrac{(2^4)^6}{2^{20}}$
$= \dfrac{2^{24}}{2^{20}}$
$= 2^4$
$= 16$

d $\dfrac{27^2}{(3^2)^3} = \dfrac{(3 \times 3 \times 3)^2}{(3^2)^3}$
$= \dfrac{(3^3)^2}{(3^2)^3}$
$= \dfrac{3^6}{3^6}$
$= 3^0$
$= 1$

e $\dfrac{(625)^4}{(5^3)^5} = \dfrac{(5 \times 5 \times 5 \times 5)^4}{(5^3)^5}$
$= \dfrac{(5^4)^4}{(5^3)^5}$
$= \dfrac{5^{16}}{5^{15}}$
$= 5$

f $\dfrac{(25)^4}{(125)^3} = \dfrac{(5 \times 5)^4}{(5 \times 5 \times 5)^3}$
$= \dfrac{(5^2)^4}{(5^3)^3}$
$= \dfrac{5^8}{5^9}$
$= 5^{-1}$
$= \frac{1}{5}$

g $\dfrac{4^{11} \div 8^2}{16^3} = \dfrac{(2 \times 2)^{11} \div (2 \times 2 \times 2)^2}{(2 \times 2 \times 2 \times 2)^3}$
$= \dfrac{(2^2)^{11} \div (2^3)^2}{(2^4)^3}$
$= \dfrac{2^{22} \div 2^6}{2^{12}}$
$= 2^{22-6-12}$
$= 2^4$
$= 16$

h $\dfrac{27^2 \times 81}{9^3 \times 3^5}$
$= \dfrac{(3 \times 3 \times 3)^2 \times (3 \times 3 \times 3 \times 3)}{(3 \times 3)^3 \times 3^5}$
$= \dfrac{(3^3)^2 \times 3^4}{(3^2)^3 \times 3^5}$
$= \dfrac{3^6 \times 3^4}{3^6 \times 3^5}$
$= 3^{6+4-6-5}$
$= 3^{-1}$
$= \frac{1}{3}$

9 a $\dfrac{2^n \times 9^{2n+1}}{6^{n-2}} = \dfrac{2^n \times (3 \times 3)^{2n+1}}{(2 \times 3)^{n-2}}$
$= \dfrac{2^n \times (3^2)^{2n+1}}{(2 \times 3)^{n-2}}$
$= \dfrac{2^n \times 3^{4n+2}}{2^{n-2} \times 3^{n-2}}$
$= 2^{n-(n-2)} \times 3^{4n+2-(n-2)}$
$= 2^{n-n+2} \times 3^{4n+2-n+2}$
$= 2^2 \times 3^{3n+4}$

b $\dfrac{25^{3n}\times 5^{n-3}}{5^{4n+3}}$

$= \dfrac{(5\times 5)^{3n}\times 5^{n-3}}{5^{4n+3}}$

$= \dfrac{(5^2)^{3n}\times 5^{n-3}}{5^{4n+3}}$

$= \dfrac{5^{6n}\times 5^{n-3}}{5^{4n+3}}$

$= 5^{6n+n-3-(4n+3)}$

$= 5^{6n+n-3-4n-3}$

$= 5^{3n-6}$

c $\dfrac{12^{x-2}\times 4^{x}}{6^{x-2}}$

$= \dfrac{(2\times 2\times 3)^{x-2}\times (2\times 2)^{x}}{(2\times 3)^{x-2}}$

$= \dfrac{(2^2\times 3)^{x-2}\times (2^2)^{x}}{(2\times 3)^{x-2}}$

$= \dfrac{2^{2x-4}\times 3^{x-2}\times 2^{2x}}{2^{x-2}\times 3^{x-2}}$

$= 2^{2x-4+2x-(x-2)}\times 3^{x-2-(x-2)}$

$= 2^{2x-4+2x-x+2}\times 3^{x-2-x+2}$

$= 2^{3x-2}\times 3^{0}$

$= 2^{3x-2}$

d $\dfrac{12^{n-3}\times 27^{1-n}}{9^{2n}\times 8^{n-1}\times 16^{n}}$

$= \dfrac{(2^2\times 3)^{n-3}\times (3^3)^{1-n}}{(3^2)^{2n}\times (2^3)^{n-1}\times (2^4)^{n}}$

$= \dfrac{2^{2n-6}\times 3^{n-3}\times 3^{3-3n}}{3^{4n}\times 2^{3n-3}\times 2^{4n}}$

$= 2^{2n-6-(3n-3)-4n}$
$\times 3^{n-3+3-3n-4n}$

$= 2^{2n-6-3n+3-4n}$
$\times 3^{n-3+3-3n-4n}$

$= 2^{-5n-3}\times 3^{-6n}$

e $\dfrac{4^{n}\times 7^{n-3}\times 49^{3n+1}}{14^{n+2}}$

$= \dfrac{(2\times 2)^{n}\times 7^{n-3}\times (7\times 7)^{3n+1}}{(2\times 7)^{n+2}}$

$= \dfrac{(2^2)^{n}\times 7^{n-3}\times (7^2)^{3n+1}}{(2\times 7)^{n+2}}$

$= \dfrac{2^{2n}\times 7^{n-3}\times 7^{6n+2}}{2^{n+2}\times 7^{n+2}}$

$= 2^{2n-(n+2)}\times 7^{n-3+6n+2-(n+2)}$

$= 2^{2n-n-2}\times 7^{n-3+6n+2-n-2}$

$= 2^{n-2}\times 7^{6n-3}$

f $\dfrac{35^2\times 5^5\times 7^6}{25^4\times 49^3}$

$= \dfrac{(5\times 7)^2\times 5^5\times 7^6}{(5\times 5)^4\times (7\times 7)^3}$

$= \dfrac{5^2\times 7^2\times 5^5\times 7^6}{(5^2)^4\times (7^2)^3}$

$= \dfrac{5^2\times 7^2\times 5^5\times 7^6}{5^8\times 7^6}$

$= 5^{2+5-8}\times 7^{2+6-6}$

$= 5^{-1}\times 7^{2}$

$= \dfrac{7^2}{5}$

$= \frac{49}{5}$

g $\dfrac{3^{5n-4}\times 16^{n}\times 9^{3}}{4^{n+1}\times 18^{1-n}\times 6^{3-2n}}$

$= \dfrac{3^{5n-4}\times (2^4)^{n}\times (3^2)^{3}}{(2^2)^{n+1}\times (2\times 3^2)^{1-n}\times (2\times 3)^{3-2n}}$

$= \dfrac{3^{5n-4}\times 2^{4n}\times 3^{6}}{2^{2n+2}\times 2^{1-n}\times 3^{2-2n}\times 2^{3-2n}\times 3^{3-2n}}$

$= 3^{5n-4+6-(2-2n)-(3-2n)}$
$\times 2^{4n-(2n+2)-(1-n)-(3-2n)}$

$= 3^{5n-4+6-2+2n-3+2n}$
$\times 2^{4n-2n-2-1+n-3+2n}$

$= 3^{9n-3}\times 2^{5n-6}$

h $\dfrac{3^{n}+3^{n+1}}{3^{n}+3^{n-1}} = \dfrac{3^{n}(1+3)}{3^{n}(1+3^{-1})}$

$= \dfrac{1+3}{1+3^{-1}}$

$= \dfrac{4}{1\frac{1}{3}}$

$= 3$

i $\dfrac{5^{n}-5^{n+1}}{5^{n+1}+5^{n}} = \dfrac{5^{n}(1-5)}{5^{n}(5+1)}$

$= \frac{-4}{6}$

$= -\frac{2}{3}$

10 $\dfrac{36^{2n}\times 6^{n+3}}{216^{n-2}} = \dfrac{(6^2)^{2n}\times 6^{n+3}}{(6^3)^{n-2}}$

$= \dfrac{6^{4n}\times 6^{n+3}}{6^{3n-6}}$

$= 6^{4n+n+3-(3n-6)}$

$= 6^{4n+n+3-3n+6}$

$= 6^{2n+9}$

The answer is **E**

Exercise 7B — Negative and rational powers

1 a $6^{-3} = \dfrac{1}{6^3}$

b $5^{-4} = \dfrac{1}{5^4}$

c $\left(\dfrac{3}{5}\right)^{-2} = \dfrac{3^{-2}}{5^{-2}}$

$= \dfrac{5^2}{3^2}$

d $\left(\dfrac{7}{4}\right)^{-5} = \dfrac{7^{-5}}{4^{-5}}$

$= \dfrac{4^5}{7^5}$

e $\left(\frac{1}{9}\right)^{-2} = \frac{1^{-2}}{9^{-2}}$
$= \frac{9^2}{1^2}$
$= 9^2$

f $(64^{-2})^3 = 64^{-6}$
$= \frac{1}{64^6}$

g $(-3)^{-1} = \frac{1}{-3}$

h $\left(\frac{3^4}{2^3}\right)^{-4} = \frac{3^{-16}}{2^{-12}}$
$= \frac{2^{12}}{3^{16}}$

2 a $\frac{(-2)^3 \times 2^{-4}}{2^{-3}} = \frac{(-1\times 2)^3 \times 2^{-4}}{2^{-3}}$
$= \frac{(-1)^3 \times 2^3 \times 2^{-4}}{2^{-3}}$
$= -1 \times 2^{3-4--3}$
$= -1 \times 2^{3-4+3}$
$= -1 \times 2^2$
$= -2^2$

b $\frac{(x^{-2})^3 \times (y^4)^{-2}}{x^{-5} \times (y^{-2})^3} = \frac{x^{-6} \times y^{-8}}{x^{-5} \times y^{-6}}$
$= x^{-6--5} \times y^{-8--6}$
$= x^{-6+5} \times y^{-8+6}$
$= x^{-1} y^{-2}$
$= \frac{1}{xy^2}$

c $\frac{(-m)^2 \times m^{-3}}{(p^{-2})^{-1} \times p^{-4}} = \frac{m^2 \times m^{-3}}{p^2 \times p^{-4}}$
$= \frac{m^{-1}}{p^{-2}}$
$= \frac{p^2}{m}$

d $\frac{x^5}{x^{-3}} \div \frac{(x^4)^{-2}}{(x^2)^{-3}} = \frac{x^5}{x^{-3}} \times \frac{(x^2)^{-3}}{(x^4)^{-2}}$
$= \frac{x^5}{x^{-3}} \times \frac{x^{-6}}{x^{-8}}$
$= \frac{x^{-1}}{x^{-11}}$
$= x^{-1--11}$
$= x^{10}$

e $\frac{(3^{-2})^2 \times (2^{-5})^{-1}}{(2^4)^{-2} \times (3^4)^{-3}} = \frac{3^{-4} \times 2^5}{2^{-8} \times 3^{-12}}$
$= 3^{-4--12} \times 2^{5--8}$
$= 3^8 \times 2^{13}$

f $\frac{x^3 y^{-2} \times (xy^2)^{-3}}{(2x^3)^2 \times (y^{-3})^2} = \frac{x^3 y^{-2} \times x^{-3} y^{-6}}{2^2 x^6 \times y^{-6}}$
$= \frac{x^0 y^{-8}}{4x^6 y^{-6}}$
$= \frac{x^{0-6} y^{-8--6}}{4}$
$= \frac{x^{-6} y^{-2}}{4}$
$= \frac{1}{4x^6 y^2}$

3 a $9^{\frac{1}{2}} = (3^2)^{\frac{1}{2}}$
$= 3$

b $27^{\frac{1}{3}} = (3^3)^{\frac{1}{3}}$
$= 3$

c $625^{\frac{1}{4}} = (5^4)^{\frac{1}{4}}$
$= 5$

d $256^{\frac{1}{8}} = (2^8)^{\frac{1}{8}}$
$= 2$

e $8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}}$
$= 2^2$
$= 4$

f $81^{\frac{3}{4}} = (3^4)^{\frac{3}{4}}$
$= 3^3$
$= 27$

g $125^{\frac{4}{3}} = (5^3)^{\frac{4}{3}}$
$= 5^4$
$= 625$

h $\left(\frac{8}{125}\right)^{\frac{1}{3}} = \left(\frac{2^3}{5^3}\right)^{\frac{1}{3}}$
$= \frac{2}{5}$

i $\left(\frac{16}{81}\right)^{\frac{1}{4}} = \left(\frac{2^4}{3^4}\right)^{\frac{1}{4}}$
$= \frac{2}{3}$

j $\left(\frac{25}{16}\right)^{\frac{3}{2}} = \left(\frac{5^2}{4^2}\right)^{\frac{3}{2}}$
$= \frac{5^3}{4^3}$
$= \frac{125}{64}$

k $\left(\frac{27}{64}\right)^{\frac{2}{3}} = \left(\frac{3^3}{4^3}\right)^{\frac{2}{3}}$
$= \frac{3^2}{4^2}$
$= \frac{9}{16}$

l $32^{-\frac{2}{5}} = (2^5)^{-\frac{2}{5}}$
$= 2^{-2}$
$= \frac{1}{2^2}$
$= \frac{1}{4}$

m $81^{-\frac{3}{4}} = (3^4)^{-\frac{3}{4}}$
$= 3^{-3}$
$= \frac{1}{3^3}$
$= \frac{1}{27}$

n $\left(\frac{8}{27}\right)^{-\frac{2}{3}} = \left(\frac{2^3}{3^3}\right)^{-\frac{2}{3}}$
$= \frac{2^{-2}}{3^{-2}}$
$= \frac{3^2}{2^2}$
$= \frac{9}{4}$

o $\left(\frac{16}{121}\right)^{-\frac{1}{2}} = \left(\frac{4^2}{11^2}\right)^{-\frac{1}{2}}$
$= \frac{4^{-1}}{11^{-1}}$
$= \frac{11}{4}$

p $\left(\frac{125}{216}\right)^{-\frac{1}{3}} = \left(\frac{5^3}{6^3}\right)^{-\frac{1}{3}}$
$= \frac{5^{-1}}{6^{-1}}$
$= \frac{6}{5}$

4 a $6^{-2} = \frac{1}{6^2}$
$= \frac{1}{36}$
The answer is **D**

b $\left(\frac{27}{8}\right)^{-\frac{1}{3}} = \left(\frac{3^3}{2^3}\right)^{-\frac{1}{3}}$
$= \frac{3^{-1}}{2^{-1}}$
$= \frac{2}{3}$
The answer is **C**

c $\sqrt[3]{25} \times \sqrt{125} = 25^{\frac{1}{3}} \times 125^{\frac{1}{2}}$
$= (5^2)^{\frac{1}{3}} \times (5^3)^{\frac{1}{2}}$
$= 5^{\frac{2}{3}} \times 5^{\frac{3}{2}}$
$= 5^{\frac{2}{3}+\frac{3}{2}}$
$= 5^{\frac{13}{6}}$
The answer is **E**

5 a $\sqrt{9} \times \sqrt[3]{81} = 9^{\frac{1}{2}} \times 81^{\frac{1}{3}}$
$= (3^2)^{\frac{1}{2}} \times (3^4)^{\frac{1}{3}}$
$= 3^1 \times 3^{\frac{4}{3}}$
$= 3^{1+\frac{4}{3}}$
$= 3^{\frac{7}{3}}$

b $x^{\frac{2}{3}} \times x^{\frac{1}{6}} = x^{\frac{2}{3}+\frac{1}{6}}$
$= x^{\frac{5}{6}}$

c $x^{-\frac{3}{4}} \times x^{\frac{9}{8}} = x^{-\frac{3}{4}+\frac{9}{8}}$
$= x^{\frac{3}{8}}$

d $x^{\frac{5}{2}} \div (x^{\frac{1}{3}})^4 = x^{\frac{5}{2}} \div x^{\frac{4}{3}}$
$= x^{\frac{5}{2}-\frac{4}{3}}$
$= x^{\frac{7}{6}}$

e $\sqrt[3]{(xy^3)} \div \sqrt{(x^2y)}$
$= (xy^3)^{\frac{1}{3}} \div (x^2y)^{\frac{1}{2}}$
$= x^{\frac{1}{3}}y^1 \div x^1y^{\frac{1}{2}}$
$= x^{\frac{1}{3}-1}y^{1-\frac{1}{2}}$
$= x^{-\frac{2}{3}}y^{\frac{1}{2}}$
$= \frac{y^{\frac{1}{2}}}{x^{\frac{2}{3}}}$

f $\sqrt[5]{32} \times \sqrt[4]{8} = 32^{\frac{1}{5}} \times 8^{\frac{1}{4}}$
$= (2^5)^{\frac{1}{5}} \times (2^3)^{\frac{1}{4}}$
$= 2^1 \times 2^{\frac{3}{4}}$
$= 2^{1+\frac{3}{4}}$
$= 2^{\frac{7}{4}}$

g $2^{\frac{5}{4}} \times 4^{-\frac{1}{2}} \times 8^{-\frac{2}{3}}$
$= 2^{\frac{5}{4}} \times (2^2)^{-\frac{1}{2}} \times (2^3)^{-\frac{2}{3}}$
$= 2^{\frac{5}{4}} \times 2^{-1} \times 2^{-2}$
$= 2^{\frac{5}{4}-1-2}$
$= 2^{-\frac{7}{4}}$
$= \frac{1}{2^{\frac{7}{4}}}$

h $27^{-\frac{1}{4}} \times 9^{\frac{2}{3}} \times 3^{-\frac{5}{4}}$
$= (3^3)^{-\frac{1}{4}} \times (3^2)^{\frac{2}{3}} \times 3^{-\frac{5}{4}}$
$= 3^{-\frac{3}{4}} \times 3^{\frac{4}{3}} \times 3^{-\frac{5}{4}}$
$= 3^{-\frac{3}{4}+\frac{4}{3}-\frac{5}{4}}$
$= 3^{-\frac{8}{12}}$
$= 3^{-\frac{2}{3}} = \frac{1}{3^{\frac{2}{3}}}$

i $\frac{18^{\frac{1}{2}}}{9^{\frac{4}{3}} \times 4^{\frac{3}{4}}} = \frac{(2\times 3^2)^{\frac{1}{2}}}{(3^2)^{\frac{4}{3}} \times (2^2)^{\frac{3}{2}}}$
$= \frac{2^{\frac{1}{2}} \times 3^1}{3^{\frac{8}{3}} \times 2^{\frac{3}{2}}}$
$= 2^{\frac{1}{2}-\frac{3}{2}} \times 3^{1-\frac{8}{3}}$
$= 2^{-1} \times 3^{-\frac{5}{3}}$
$= \frac{1}{2 \times 3^{\frac{5}{3}}}$

j $(\sqrt[4]{x^3})^{\frac{2}{3}} \times (\sqrt[3]{x^4})^{\frac{3}{8}} = (x^{\frac{3}{4}})^{\frac{2}{3}} \times (x^{\frac{4}{3}})^{\frac{3}{8}}$
$= x^{\frac{1}{2}} \times x^{\frac{1}{2}}$
$= x^{\frac{1}{2}+\frac{1}{2}}$
$= x^1$
$= x$

k $\frac{(64m^6)^{\frac{4}{3}}}{4m^{-2}} = \frac{(4^3m^6)^{\frac{4}{3}}}{4m^{-2}}$
$= \frac{4^4m^8}{4^1m^{-2}}$
$= 4^{(4-1)}m^{8--2}$
$= 4^3m^{10}$
$= 64m^{10}$

l $\frac{\sqrt{x^3}}{\sqrt{x}} = \frac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}}$
$= x^{\frac{3}{2}-\frac{1}{2}}$
$= x^1$
$= x$

m $\frac{1}{\sqrt{x^{-4}}} = \frac{1}{x^{-\frac{4}{2}}}$
$= \frac{1}{x^{-2}}$
$= x^2$

n $\frac{(x+1)^2}{\sqrt{x+1}} = \frac{(x+1)^2}{(x+1)^{\frac{1}{2}}}$
$= (x+1)^{2-\frac{1}{2}}$
$= (x+1)^{1\frac{1}{2}}$

o $\sqrt{x} - \frac{1}{\sqrt{x}} = x^{\frac{1}{2}} - \frac{1}{x^{\frac{1}{2}}}$
$= \frac{x-1}{x^{\frac{1}{2}}}$

p $\sqrt{x+2} + \frac{x}{\sqrt{x+2}}$
$= (x+2)^{\frac{1}{2}} + \frac{x}{(x+2)^{\frac{1}{2}}}$
$= \frac{x+2+x}{(x+2)^{\frac{1}{2}}}$
$= \frac{2x+2}{(x+2)^{\frac{1}{2}}}$

q $(y-4)\sqrt{y-4}$
$= (y-4)^1 \times (y-4)^{\frac{1}{2}}$
$= (y-4)^{1\frac{1}{2}}$

r $(p+3)(p+3)^{-\frac{2}{5}} = (p+3)^{1-\frac{2}{5}}$
$= (p+3)^{\frac{3}{5}}$

Exercise 7C — Indicial equations

1 a $2^x = 32$
$2^x = 2^5$
$x = 5$

b $5^x = 625$
$5^x = 5^4$
$x = 4$

c $3^x = 243$
$3^x = 3^5$
$x = 5$

d $10^{-x} = \frac{1}{100}$
$10^{-x} = \frac{1}{10^2}$
$10^{-x} = 10^{-2}$
$-x = -2$
$x = 2$

e $4^{-x} = 16$
$4^{-x} = 4^2$
$-x = 2$
$x = -2$

f $6^x = \frac{1}{216}$
$6^x = \frac{1}{6^3}$
$6^x = 6^{-3}$
$x = -3$

g $3^{-x} = \frac{1}{81}$
$3^{-x} = \frac{1}{3^4}$
$3^{-x} = 3^{-4}$
$-x = -4$
$x = 4$

h $2^{-x} = 1$
$2^{-x} = 2^0$
$-x = 0$
$x = 0$

i $8^x = 2^6$
$(2^3)^x = 2^6$
$2^{3x} = 2^6$
$3x = 6$
$x = 2$

j $25^x = 5$
$(5^2)^x = 5^1$
$5^{2x} = 5^1$
$2x = 1$
$x = \frac{1}{2}$

k $81^x = 3^3$
$(3^4)^x = 3^3$
$3^{4x} = 3^3$
$4x = 3$
$x = \frac{3}{4}$

l $125^x = 5^4$
$(5^3)^x = 5^4$
$5^{3x} = 5^4$
$3x = 4$
$x = \frac{4}{3}$

2 a $2^{3n+1} = 64$
$2^{3n+1} = 2^6$
$3n + 1 = 6$
$3n = 5$
$n = \frac{5}{3}$

b $5^{2n+3} = 25$
$5^{2n+3} = 5^2$
$2n + 3 = 2$
$2n = -1$
$n = -\frac{1}{2}$

c $3^{2-n} = 27$
$3^{2-n} = 3^3$
$2 - n = 3$
$-n = 1$
$n = -1$

d $16^{n+3} = 2^3$
$(2^4)^{n+3} = 2^3$
$2^{4n+12} = 2^3$
$4n + 12 = 3$
$4n = -9$
$n = -\frac{9}{4}$

e $49^{5-3n} = \frac{1}{7}$
$(7^2)^{5-3n} = 7^{-1}$
$7^{10-6n} = 7^{-1}$
$10 - 6n = -1$
$-6n = -11$
$n = \frac{11}{6}$

f $36^{4n-3} = 216$
$(6^2)^{4n-3} = 6^3$
$6^{8n-6} = 6^3$
$8n - 6 = 3$
$8n = 9$
$n = \frac{9}{8}$

3 a $4^{2x} = 8^{x-1}$
$(2^2)^{2x} = (2^3)^{x-1}$
$2^{4x} = 2^{3x-3}$
$4x = 3x - 3$
$x = -3$

b $27^{4-x} = 9^{2x+1}$
$(3^3)^{4-x} = (3^2)^{2x+1}$
$3^{12-3x} = 3^{4x+2}$
$12 - 3x = 4x + 2$
$-7x = -10$
$x = \frac{10}{7}$

c $16^{3x+1} = 128^{x-2}$
$(2^4)^{3x+1} = (2^7)^{x-2}$
$2^{12x+4} = 2^{7x-14}$
$12x + 4 = 7x - 14$
$5x = -18$
$x = -\frac{18}{5}$

d $25^{2x-3} = \frac{1}{125}$
$(5^2)^{2x-3} = 5^{-3}$
$5^{4x-6} = 5^{-3}$
$4x - 6 = -3$
$4x = 3$
$x = \frac{3}{4}$

e $32^{5-x} = 4^{3x+2}$
$(2^5)^{5-x} = (2^2)^{3x+2}$
$2^{25-5x} = 2^{6x+4}$
$25 - 5x = 6x + 4$
$-11x = -21$
$x = \frac{21}{11}$

f $64^{2-3x} = 16^{x+1}$
$(2^6)^{2-3x} = (2^4)^{x+1}$
$2^{12-18x} = 2^{4x+4}$
$12 - 18x = 4x + 4$
$-22x = -8$
$x = \frac{8}{22}$
$x = \frac{4}{11}$

g $9^{3x+5} = \frac{1}{243}$
$(3^2)^{3x+5} = \frac{1}{3^5}$
$3^{6x+10} = 3^{-5}$
$6x + 10 = -5$
$6x = -15$
$x = -\frac{15}{6}$
$= -\frac{5}{2}$

h $16^{4-3x} = \frac{1}{8^{x+3}}$
$16^{4-3x} = 8^{-x-3}$
$(2^4)^{4-3x} = (2^3)^{-x-3}$
$2^{16-12x} = 2^{-3x-9}$
$16 - 12x = -3x - 9$
$-9x = -25$
$x = \frac{25}{9}$

4 a $2^x \times 8^{3x-1} = 64$
$2^x \times (2^3)^{3x-1} = 2^6$
$2^x \times 2^{9x-3} = 2^6$
$2^{10x-3} = 2^6$
$10x - 3 = 6$
$10x = 9$
$x = \frac{9}{10}$

b $5^{2x} \times 125^{3-x} = 25$
$5^{2x} \times (5^3)^{3-x} = 5^2$
$5^{2x} \times 5^{9-3x} = 5^2$
$5^{-x+9} = 5^2$
$-x + 9 = 2$
$-x = -7$
$x = 7$

c $3^{4x} \times 27^{x+3} = 81$
$3^{4x} \times (3^3)^{x+3} = 81$
$3^{4x} \times 3^{3x+9} = 3^4$
$3^{7x+9} = 3^4$
$7x + 9 = 4$
$7x = -5$
$x = \frac{-5}{7}$

d $16^{x+4} \times 2^{3+2x} = 4^{5x}$
$(2^4)^{x+4} \times 2^{3+2x} = (2^2)^{5x}$
$2^{4x+16} \times 2^{3+2x} = 2^{10x}$
$2^{6x+19} = 2^{10x}$
$6x + 19 = 10x$
$4x = 19$
$x = \frac{19}{4}$

e $3125 \times 25^{2x+1} = 5^{3x+4}$
$5^5 \times (5^2)^{2x+1} = 5^{3x+4}$
$5^5 \times 5^{4x+2} = 5^{3x+4}$
$5^{4x+7} = 5^{3x+4}$
$4x + 7 = 3x + 4$
$x = -3$

f $\frac{81^{2-x}}{27^{x+3}} = 9^{2x}$
$\frac{(3^4)^{2-x}}{(3^3)^{x+3}} = (3^2)^{2x}$
$\frac{3^{8-4x}}{3^{3x+9}} = 3^{4x}$
$3^{-1-7x} = 3^{4x}$
$-1 - 7x = 4x$
$11x = -1$
$x = \frac{-1}{11}$

g $49^{3-4x} \times 7^{2x+3} = 343^{2-x}$
$(7^2)^{3-4x} \times 7^{2x+3} = (7^3)^{2-x}$
$7^{6-8x} \times 7^{2x+3} = 7^{6-3x}$
$7^{9-6x} = 7^{6-3x}$
$9 - 6x = 6 - 3x$
$3x = 3$
$x = 1$

h $5^{7x+5} \div 625^{2x+1} = 25^{3-x}$
$5^{7x+5} \div (5^4)^{2x+1} = (5^2)^{3-x}$
$5^{7x+5} \div 5^{8x+4} = 5^{6-2x}$
$5^{-x+1} = 5^{6-2x}$
$-x + 1 = 6 - 2x$
$x = 5$

i $256^{2-x} \times 4^{3x+1} = 64^{x-1}$
$(2^8)^{2-x} \times (2^2)^{3x+1} = (2^6)^{x-1}$
$2^{16-8x} \times 2^{6x+2} = 2^{6x-6}$
$2^{18-2x} = 2^{6x-6}$
$18 - 2x = 6x - 6$
$8x = 24$
$x = 3$

j $\frac{36^{2x+1}}{216^{x-2}} = \frac{1}{6}$
$\frac{(6^2)^{2x+1}}{(6^3)^{x-2}} = \frac{1}{6}$
$\frac{6^{4x+2}}{6^{3x-6}} = \frac{1}{6^1}$
$6^{x+8} = 6^{-1}$
$x + 8 = -1$
$x = -9$

5 a $3^{2x} - 4(3^x) + 3 = 0$
$(3^x)^2 - 4(3^x) + 3 = 0$
and now let $y = 3^x$
$y^2 - 4y + 3 = 0$
$(y - 3)(y - 1) = 0$
$y = 3$ or $y = 1$
$3^x = 3^1$ or $3^x = 1$ or 3^0
$x = 1$ or $x = 0$

b $2^{2x} - 6(2^x) + 8 = 0$
$(2^x)^2 - 6(2^x) + 8 = 0$
and let $y = 2^x$
$y^2 - 6y + 8 = 0$
$(y - 4)(y - 2) = 0$
$y = 4$ or $y = 2$
$2^x = 2^2$ or $2^x = 2^1$
$x = 2$ or $x = 1$

c $3(2^{2x}) - 36(2^x) + 96 = 0$
$2^{2x} - 12(2^x) + 32 = 0$
$(2^x)^2 - 12(2^x) + 32 = 0$
and let $y = 2^x$
$y^2 - 12y + 32 = 0$
$(y - 8)(y - 4) = 0$
$y = 8$ or $y = 4$
$2^x = 8$ or $2^x = 4$
$2^x = 2^3$ or $2^x = 2^2$
$x = 3$ or $x = 2$

d $2(5^{2x}) - 12(5^x) + 10 = 0$
$5^{2x} - 6(5^x) + 5 = 0$
$(5^x)^2 - 6(5^x) + 5 = 0$
and let $y = 5^x$
$y^2 - 6y + 5 = 0$
$(y - 5)(y - 1) = 0$
$y = 5$ or $y = 1$
$5^x = 5$ or $5^x = 1$
$5^x = 5^1$ or $5^x = 5^0$
$x = 1$ or $x = 0$

e $3(4^{2x}) = 15(4^x) - 12$
$3(4^{2x}) - 15(4^x) + 12 = 0$
$4^{2x} - 5(4^x) + 4 = 0$
$(4^x)^2 - 5(4^x) + 4 = 0$
and let $y = 4^x$
$y^2 - 5y + 4 = 0$
$(y - 4)(y - 1) = 0$
$y = 4$ or $y = 1$
$4^x = 4$ or $4^x = 1$
$4^x = 4^1$ or $4^x = 4^0$
$x = 1$ or $x = 0$

f $25^x - 30(5^x) + 125 = 0$
$5^{2x} - 30(5^x) + 125 = 0$
$(5^x)^2 - 30(5^x) + 125 = 0$
and let $y = 5^x$

$y^2 - 30y + 125 = 0$
$(y - 25)(y - 5) = 0$
$y = 25$ or $y = 5$
$5^x = 25$ or $5^x = 5$
$5^x = 5^2$ or $5^x = 5^1$
$x = 2$ or $x = 1$

g $4^x - 16(2^x) = -64$
$4^x - 16(2^x) + 64 = 0$
$2^{2x} - 16(2^x) + 64 = 0$
$(2^x)^2 - 16(2^x) + 64 = 0$
and let $y = 2^x$
$y^2 - 16y + 64 = 0$
$(y - 8)(y - 8) = 0$
$y = 8$
$2^x = 8$
$2^x = 2^3$
$x = 3$

h $2(25^x) + 10 = 12(5^x)$
$2(25^x) - 12(5^x) + 10 = 0$
$25^x - 6(5^x) + 5 = 0$
$5^{2x} - 6(5^x) + 5 = 0$
$(5^x)^2 - 6(5^x) + 5 = 0$
and let $y = 5^x$
$y^2 - 6y + 5 = 0$
$(y - 1)(y - 5) = 0$
$y = 1$ or $y = 5$
$5^x = 1$ or $5^x = 5$
$5^x = 5^0$ or $5^x = 5^1$
$x = 0$ or $x = 1$

6 $3^{2x} - 12(3^x) + 27 = 0$
$(3^x)^2 - 12(3^x) + 27 = 0$
Let $y = 3^x$
The answer is **A**

7 $y^2 - 12y + 27 = 0$
Continued from question **6**
The answer is **D**

8 Continuing from question **7**
$(y - 9)(y - 3) = 0$
$y = 9$ or $y = 3$
$3^x = 9$ or $3^x = 3$
$3^x = 3^2$ or $3^x = 3^1$
$x = 2$ or $x = 1$
The answer is **B**

9 a $2^x = 3$
Since $2^{1.5} = 2.828$ and
$2^{1.6} = 3.031$, then x is between 1.5 and 1.6
$2^{1.58} = 2.990$ too small
$2^{1.59} = 3.010$ too big
$2^{1.584} = 2.999$ too small
$2^{1.585} = 3.000$ required value
$x = 1.58$ correct to 2 decimal places.

b $2^x = 12$
Since $2^{3.5} = 11.314$ and
$2^{3.6} = 12.126$, then x is between 3.5 and 3.6
$2^{3.58} = 11.959$ too small
$2^{3.59} = 12.042$ too big
$2^{3.584} = 11.992$
$2^{3.585} = 12.000$
$x = 3.58$ correct to 2 decimal places.

c $10^x = 45$
Since $10^1 = 10$ and $10^2 = 100$, then x is between 1 and 2.
$10^{1.6} = 39.811$
$10^{1.7} = 50.119$
$10^{1.65} = 44.668$
$10^{1.55} = 45.709$
$10^{1.655} = 45.186$
$x = 1.65$ correct to 2 decimal places.

d $3^x = 8$
Since $3^1 =$ and $3^2 = 9$, then x is between 1 and 2.
$3^{1.8} = 7.225$ too small
$3^{1.9} = 8.064$ too big
$3^{1.89} = 7.976$
$3^{1.895} = 8.019$
$x = 1.89$ correct to 2 decimal places.

e $10^x = 19$
Since $10^1 = 10$ and $10^2 = 100$, then x is between 1 and 2.
$10^{1.2} = 15.849$
$10^{1.3} = 19.953$
$10^{1.27} = 18.621$
$10^{1.28} = 19.055$
$10^{1.275} = 18.836$
$x = 1.28$ correct to 2 decimal places.

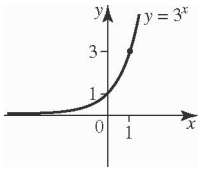
f $4^x = 10$
Since $4^1 = 4$ and $4^2 = 16$, then x is between 1 and 2.
$4^{1.6} = 9.190$
$4^{1.7} = 10.556$
$4^{1.66} = 9.987$
$4^{1.67} = 10.126$
$4^{1.665} = 10.056$
$x = 1.66$ correct to 2 decimal places.

10 $3^x = 10$
Since $3^2 = 9$ and $3^3 = 27$, then x is between 2 and 3.
$3^{2.0} = 9$
$3^{2.1} = 10.045$
$3^{2.09} = 9.935$
$3^{2.095} = 9.990$
$x = 2.10$ correct to 2 decimal places.
The answer is **E**

Exercise 7D — Graphs of exponential functions

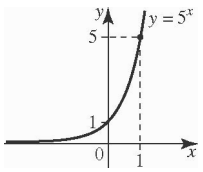
1 a $y = 3^x$

x	–3	–2	–1	0	1	2	3
y	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27



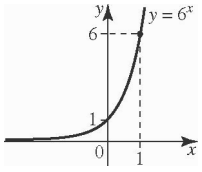
b $y = 5^x$

x	–3	–2	–1	0	1	2	3
y	$\frac{1}{125}$	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25	125



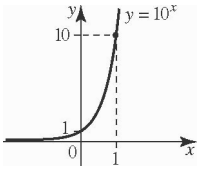
c $y = 6^x$

x	–3	–2	–1	0	1	2	3
y	$\frac{1}{216}$	$\frac{1}{36}$	$\frac{1}{6}$	1	6	36	216



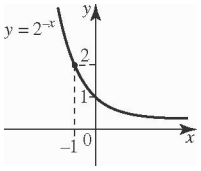
d $y = 10^x$

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{1000}$	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100	1000



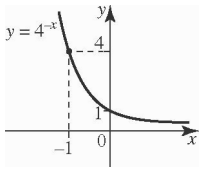
e $y = 2^{-x}$

x	-3	-2	-1	0	1	2	3
y	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$



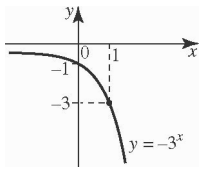
f $y = 4^{-x}$

x	-3	-2	-1	0	1	2	3
y	64	16	4	1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$



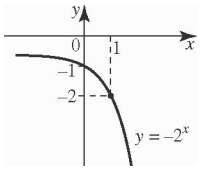
g $y = -3^x$

x	-3	-2	-1	0	1	2	3
y	$-\frac{1}{27}$	$-\frac{1}{9}$	$-\frac{1}{3}$	-1	-3	-9	-27



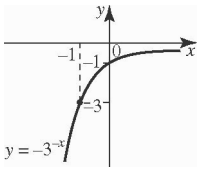
h $y = -2^x$

x	-3	-2	-1	0	1	2	3
y	$-\frac{1}{8}$	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	-2	-4	-8



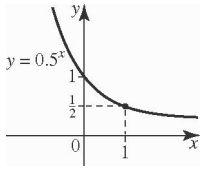
i $y = -3^{-x}$

x	-3	-2	-1	0	1	2	3
y	-27	-9	-3	-1	$-\frac{1}{3}$	$-\frac{1}{9}$	$-\frac{1}{27}$



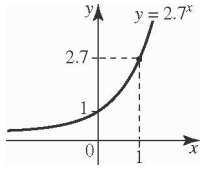
j $y = 0.5^x$

x	-3	-2	-1	0	1	2	3
y	8	4	2	1	0.5	0.25	0.125



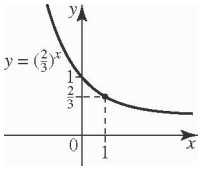
k $y = 2.7^x$

x	-3	-2	-1	0	1	2	3
y	0.05	0.14	0.37	1	2.7	7.29	19.6



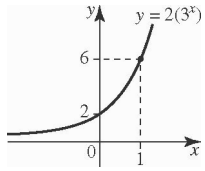
l $y = \left(\frac{2}{3}\right)^x$

x	-3	-2	-1	0	1	2	3
y	3.375	2.25	1.5	1	0.67	0.44	0.30



2 a $y = 2(3^x)$

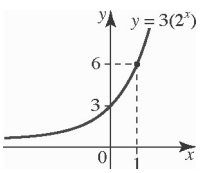
x	-2	-1	0	1	2
y	0.22	0.67	2	6	18



Asymptote $y = 0$,
y-intercept $= 2$

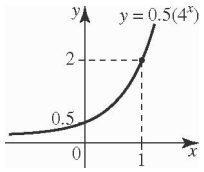
b $y = 3(2^x)$

x	-2	-1	0	1	2
y	0.75	1.5	3	6	12



c $y = 0.5(4^x)$

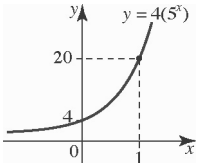
x	–2	–1	0	1	2
y	0.03	0.125	0.5	2	8



Asymptote $y = 0$,
y-intercept = 0.5

d $y = 4(5^x)$

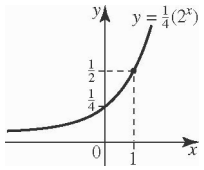
x	–2	–1	0	1	2
y	0.16	0.8	4	20	100



Asymptote $y = 0$,
y-intercept = 4

e $y = \frac{1}{4}(2^x)$

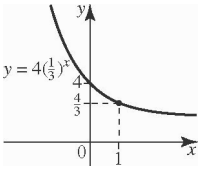
x	–2	–1	0	1	2
y	0.06	0.125	0.25	0.5	2



Asymptote $y = 0$,
y-intercept = 0.25

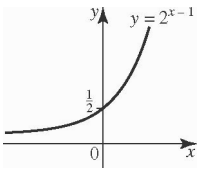
f $y = 4\left(\frac{1}{3}\right)^x$

x	–2	–1	0	1	2
y	36	12	4	1.33	0.44



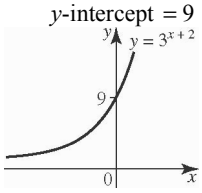
Asymptote $y = 0$,
y-intercept = 4

3 a $y = 2^{x-1}$
Asymptote $y = 0$
When $x = 0$, $y = 2^{0-1}$
$= 2^{-1}$
$= \frac{1}{2}$
y-intercept $= \frac{1}{2}$



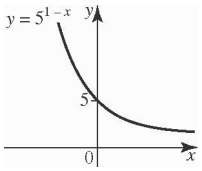
Domain R
Range $(0, \infty)$

b $y = 3^{x+2}$
Asymptote $y = 0$
When $x = 0$, $y = 3^{0+2}$
$= 3^2$
$= 9$
y-intercept $= 9$



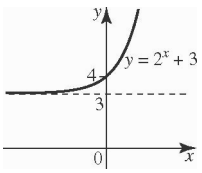
Domain R
Range $(0, \infty)$

c $y = 5^{1-x}$
Asymptote $y = 0$
When $x = 0$, $y = 5^{1-0}$
$= 5$
y-intercept = 5



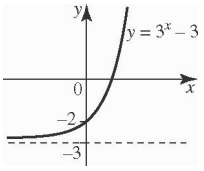
Domain R
Range $(0, \infty)$

d $y = 2^x + 3$
Asymptote $y = 3$
When $x = 0$, $y = 2^0 + 3$
$= 1 + 3$
$= 4$
y-intercept = 4



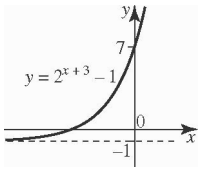
Domain R
Range $(3, \infty)$

e $y = 3^x - 3$
Asymptote $y = -3$
When $x = 0$, $y = 3^0 - 3$
$= 1 - 3$
$= -2$
y-intercept = –2



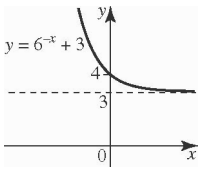
Domain R
Range $(-3, \infty)$

f $y = 2^{x+3} - 1$
Asymptote $y = -1$
When $x = 0$, $y = 2^3 - 1$
$= 7$
y-intercept = 7



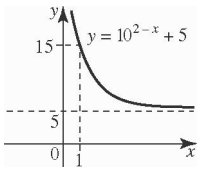
Domain R
Range $(-1, \infty)$

g $y = 6^{-x} + 3$
Asymptote $y = 3$
When $x = 0$, $y = 6^0 + 3$
$= 1 + 3$
$= 4$
y-intercept = 4



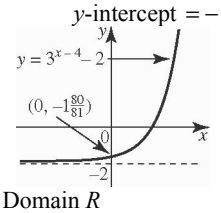
Domain R
Range $(3, \infty)$

h $y = 10^{2-x} + 5$
Asymptote $y = 5$
When $x = 0$, $y = 10^2 + 5$
$= 105$
y-intercept = 105



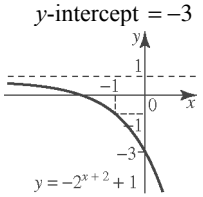
Domain R
Range $(5, \infty)$

i $y = 3^{x-4} - 2$
Asymptote $y = -2$
When $x = 0$, $y = 3^{-4} - 2$
$= -1.99$
y-intercept = –1.99



Domain R
Range $(-2, \infty)$

j $y = -2^{x+2} + 1$
Asymptote $y = 1$
When $x = 0$, $y = -2^2 + 1$
$= -3$
y-intercept = –3

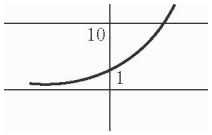


Domain R
Range $(-\infty, 1)$

4 a $y = 3^{x-2}$
When $x = 2$, $y = 3^0$
$= 1$ as required
When $x = 3$, $y = 3^1$
$= 3$ as required
The answer is **A**

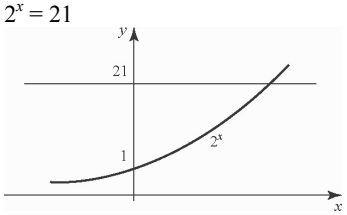
b Asymptote $y = -3$
When $x = 1$, $y = 2^{-1} - 3$
$y = 2^0 - 3$
$= 1 - 3$
$= -2$ as required
The answer is **E**

5 a $2^x = 10$



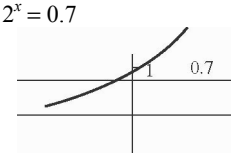
$x = 3.32$

b $2^x = 21$



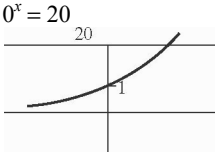
$x = 4.39$

c $2^x = 0.7$



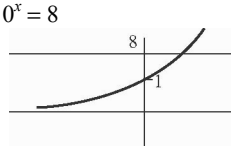
$x = -0.51$

d $10^x = 20$



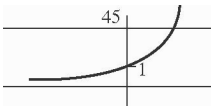
$x = 1.30$

e $10^x = 8$



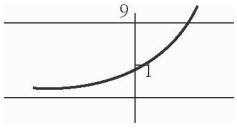
$x = 0.90$

f $10^x = 45$



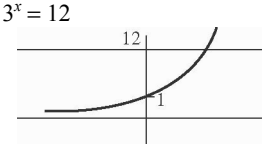
$x = 1.65$

g $5^x = 9$



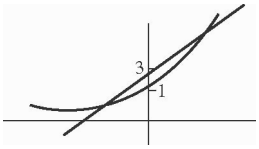
$x = 1.37$

h $3^x = 12$



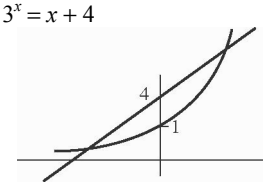
$x = 2.26$

i $2^x = x + 3$



$x = 2.44, -2.86$

j $3^x = x + 4$



$x = 1.56, -3.99$

Exercise 7E — Logarithms

1 a $2^3 = 8$
$\log_2 8 = 3$

b $3^5 = 243$
$\log_3 243 = 5$

c $5^0 = 1$
$\log_5 1 = 0$

d $0.01 = 10^{-2}$
$\log_{10} 0.01 = -2$

e $b^n = a$
$\log_b a = n$

f $2^{-4} = \frac{1}{16}$
$\log_2 \frac{1}{16} = -4$

2 a $\log_4 16 = 2$
$16 = 4^2$

b $\log_{10} 1\,000\,000 = 6$
$1\,000\,000 = 10^6$

c $\log_2 \frac{1}{2} = -1$
$\frac{1}{2} = 2^{-1}$

d $\log_3 27 = 3$
$27 = 3^3$

e $\log_5 625 = 4$
$625 = 5^4$

f $\log_2 128 = 7$
$128 = 2^7$

g $\log_3 \frac{1}{9} = -2$
$\frac{1}{9} = 3^{-2}$

h $\log_b a = x$
$a = b^x$

3 $\log_5 25 = \log_5 5^2$
$= 2 \log_5 5$
$= 2$
The answer is **D**

4 $8^3 = 512$
$\log_8 512 = 3$
The answer is **C**

5 $\log_2 10\,000 = 4$
$10\,000 = 10^4$
The answer is **A**

6 a $\log_2 16 = \log_2 2^4$
$= 4 \log_2 2$
$= 4$

b $\log_3 81 = \log_3 3^4$
$= 4 \log_3 3$
$= 4$

c $\log_5 125 = \log_5 5^3$
$= 3 \log_5 5$
$= 3$

d $\log_2 \frac{1}{4} = \log_2 2^{-2}$
$= -2 \log_2 2$
$= -2$

e $\log_{10} 1000 = \log_{10} 10^3$
$= 3 \log_{10} 10$
$= 3$

f $\log_{10} (0.000\,01) = \log_{10} 10^{-5}$
$= -5 \log_{10} 10$
$= -5$

g $\log_2 0.25 = \log_2 \frac{1}{4}$
$= \log_2 2^{-2}$
$= -2 \log_2 2$
$= -2$

h $\log_3 \frac{1}{243} = \log_3 3^{-5}$
$= -5 \log_3 3$
$= -5$

i $\log_2 32 = \log_2 2^5$
$= 5 \log_2 2$
$= 5$

j $\log_2 \frac{1}{64} = \log_2 2^{-6}$
$= -6 \log_2 2$
$= -6$

k $\log_3 (-3)$ = undefined

l $\log_n n^5 = 5 \log_n n$
$= 5$

7 a $\log_2 8 + \log_2 10 = \log_2 (8 \times 10)$
$= \log_2 80$

b $\log_3 7 + \log_3 15 = \log_3 (7 \times 15)$
$= \log_3 105$

c $\log_{10} 20 + \log_{10} 5 = \log_{10} (20 \times 5)$
$= \log_{10} 100$
$= \log_{10} 10^2$
$= 2 \log_{10} 10$
$= 2$

d $\log_6 8 + \log_6 7 = \log_6 (8 \times 7)$
$= \log_6 56$

e $\log_2 20 - \log_2 5 = \log_2 (20 \div 5)$
$= \log_2 4$
$= \log_2 2^2$
$= 2 \log_2 2$
$= 2$

f $\log_3 36 - \log_3 12 = \log_3 (36 \div 12)$
$= \log_3 3$
$= 1$

g $\log_5 100 - \log_5 8 = \log_5 (100 \div 8)$
$= \log_5 12.5$

h $\log_2 \frac{1}{3} + \log_2 9 = \log_2 (\frac{1}{3} \times 9)$
$= \log_2 3$

i $\log_4 25 + \log_4 \frac{1}{5} = \log_4 (25 \times \frac{1}{5})$
$= \log_4 5$

j $\log_{10} 5 - \log_{10} 20 = \log_{10} (5 \div 20)$
$= \log_{10} \frac{1}{4}$
$= \log_{10} 2^{-2}$
$= -2 \log_{10} 2$

k $\log_3 \frac{4}{5} - \log_3 \frac{1}{5} = \log_3 (\frac{4}{5} \div \frac{1}{5})$
$= \log_3 4$
$= \log_3 2^2$
$= 2 \log_3 2$

l $\log_2 9 + \log_2 4 - \log_2 12$
$= \log_2 (9 \times 4) - \log_2 12$
$= \log_2 36 - \log_2 12$
$= \log_2 (36 \div 12)$
$= \log_2 3$

m $\log_3 8 - \log_3 2 + \log_2 5$
$= \log_3 (8 \div 2) + \log_2 5$
$= \log_3 4 + \log_2 5$
$= \log_3 (4 \times 5)$
$= \log_3 20$

n $\log_4 24 - \log_4 2 - \log_4 6$
$= \log_4 (24 \div 2) - \log_4 6$
$= \log_4 12 - \log_4 6$
$= \log_4 (12 \div 6)$
$= \log_4 2$

8 a $3 \log_{10} 5 + \log_{10} 2$
$= \log_{10} 5^3 + \log_{10} 2$
$= \log_{10} 125 + \log_{10} 2$
$= \log_{10} (125 \times 2)$
$= \log_{10} 250$

b $2 \log_2 8 + 3 \log_2 3$
$= \log_2 8^2 + \log_2 3^3$
$= \log_2 64 + \log_2 27$
$= \log_2 (64 \times 27)$
$= \log_2 1728$

c $2 \log_3 2 + 3 \log_3 1$
$= \log_3 2^2 + \log_3 1^3$
$= \log_3 4 + \log_3 1$
$= \log_3 4$

d $\log_5 12 - 2 \log_5 2$
$= \log_5 12 - \log_5 2^2$
$= \log_5 12 - \log_5 4$
$= \log_5 (12 \div 4)$
$= \log_5 3$

e $4 \log_{10} 2 - 2 \log_{10} 8$
$= \log_{10} 2^4 - \log_{10} 8^2$
$= \log_{10} 16 - \log_{10} 64$
$= \log_{10} (16 \div 64)$
$= \log_{10} \frac{1}{4}$

f $\log_3 4^2 - 3 \log_3 2$
$= \log_3 16 - \log_3 2^3$
$= \log_3 16 - \log_3 8$
$= \log_3 (16 \div 8)$
$= \log_3 2$

g $\frac{1}{3} \log_2 27 - \frac{1}{2} \log_2 36$
$= \log_2 27^{\frac{1}{3}} - \log_2 36^{\frac{1}{2}}$
$= \log_2 3 - \log_2 6$
$= \log_2 (3 \div 6)$
$= \log_2 \frac{1}{2}$
$= \log_2 2^{-1}$
$= -1$

h $\log_2 (x-4) + 3 \log_2 x$
$= \log_2 (x-4) + \log_2 x^3$
$= \log_2 (x-4)x^3$
$= \log_2 (x^4 - 4x^3)$

i $\frac{1}{2} \log_3 16 + 2 \log_3 4$
$= \log_3 16^{\frac{1}{2}} + \log_3 4^2$
$= \log_3 4 + \log_3 16$
$= \log_3 (4 \times 16)$
$= \log_3 64$

j $2 \log_{10} (x+3) - \log_{10} (x-2)$
$= \log_{10} (x+3)^2 - \log_{10} (x-2)$
$= \log_{10} \frac{(x+3)^2}{x-2}$

9 a $\frac{\log_3 25}{\log_3 125} = \frac{\log_3 5^2}{\log_3 5^3}$
$= \frac{2 \log_3 5}{3 \log_3 5}$
$= \frac{2}{3}$

b $\frac{\log_2 81}{\log_2 9} = \frac{\log_2 9^2}{\log_2 9}$
$= \frac{2 \log_2 9}{\log_2 9}$
$= 2$

c $\frac{\log_4 36}{\log_4 6} = \frac{\log_4 6^2}{\log_4 6}$
$= \frac{2 \log_4 6}{\log_4 6}$
$= 2$

d $\frac{2 \log_{10} 8}{\log_{10} 16} = \frac{2 \log_{10} 2^3}{\log_{10} 2^4}$
$= \frac{2 \times 3 \log_{10} 2}{4 \log_{10} 2}$
$= \frac{6}{4}$
$= \frac{3}{2}$

e $\frac{3 \log_5 27}{(2 \log_5 9)} = \frac{3 \log_5 3^3}{2 \log_5 3^2}$
$= \frac{3 \times 3 \log_5 3}{2 \times 2 \log_5 3}$
$= \frac{9}{4}$

f $\frac{4 \log_3 32}{(5 \log_3 4)} = \frac{4 \log_3 2^5}{5 \log_3 2^2}$
$= \frac{5 \times 4 \log_3 2}{2 \times 5 \log_3 2}$
$= \frac{5 \times 4}{2 \times 5}$
$= 2$

g $\frac{\log_3 x^6}{\log_3 x^2} = \frac{6 \log_3 x}{2 \log_3 x}$
$= \frac{6}{2}$
$= 3$

h $\frac{\log_{10} x^3}{\log_{10} \sqrt{x}} = \frac{\log_{10} x^3}{\log_{10} x^{\frac{1}{2}}}$
$= \frac{3 \log_{10} x}{\frac{1}{2} \log_{10} x}$
$= \frac{3}{\frac{1}{2}}$
$= 6$

i $\frac{\log_5 x^{\frac{3}{2}}}{\log_5 \sqrt{x}} = \frac{\log_5 x^{\frac{3}{2}}}{\log_5 x^{\frac{1}{2}}}$
$= \frac{\frac{3}{2} \log_5 x}{\frac{1}{2} \log_5 x}$
$= \frac{\frac{3}{2}}{\frac{1}{2}}$
$= 3$

j $\frac{2 \log_2 (x+1)^3}{\log_2 (x+1)}$
$= \frac{3 \times 2 \log_2 (x+1)}{\log_2 (x+1)}$
$= 3 \times 2$
$= 6$

10 $\log_{10} xy = \log_{10} x + \log_{10} y$
The answer is **E**

11 $\log_5 x^y = y \log_5 x$
The answer is **B**

12 $\frac{1}{3} \log_2 64 + \log_2 5$
$= \log_2 64^{\frac{1}{3}} + \log_2 5$
$= \log_2 (4^3)^{\frac{1}{3}} + \log_2 5$
$= \log_2 4 + \log_2 5$
$= \log_2 (4 \times 5)$
$= \log_2 20$
The answer is **D**

13 $\frac{\log_4 x^5}{\log_4 x^2} = \frac{5 \log_4 x}{2 \log_4 x}$
$= \frac{5}{2}$
The answer is **C**

14 a $\log_3 27 + 1 = \log_3 3^3 + 1$
$= 3 \log_3 3 + 1$
$= 3 + 1$
$= 4$

b $\log_4 16 + 3 = \log_4 4^2 + 3$
$= 2 \log_4 4 + 3$
$= 2 + 3$
$= 5$

c $3 \log_5 2 - 2 = \log_5 2^3 - 2$
$= \log_5 8 - 2 \log_5 5$
$= \log_5 8 - \log_5 5^2$
$= \log_5 8 - \log_5 25$
$= \log_5 (8 \div 25)$
$= \log_5 \frac{8}{25}$

d $2 + 3 \log_{10} x = 2 \log_{10} 10 + \log_{10} x^3$
$= \log_{10} 10^2 + \log_{10} x^3$
$= \log_{10} 100 + \log_{10} x^3$
$= \log_{10} (100 x^3)$

e $2 \log_2 5 - 3 = \log_2 5^2 - 3 \log_2 2$
$= \log_2 25 - \log_2 2^3$
$= \log_2 25 - \log_2 8$
$= \log_2 \frac{25}{8}$

f $4 \log_3 2 - 2 \log_3 6 + 2$
$= 4 \log_3 2 - 2 \log_3 6 + 2 \log_3 3$
$= \log_3 2^4 - \log_3 6^2 + \log_3 3^2$
$= \log_3 16 - \log_3 36 + \log_3 9$
$= \log_3 (16 \div 36 \times 9)$
$= \log_3 4$

g $2 \log_6 6 - \log_6 4 = \log_6 6^2 - \log_6 4$
$= \log_6 36 - \log_6 4$
$= \log_6 (36 \div 4)$
$= \log_6 9$

h $\frac{1}{2} + 3 \log_{10} x^2$
$= \frac{1}{2} \log_{10} 10 + 3 \log_{10} x^2$
$= \log_{10} 10^{\frac{1}{2}} + \log_{10} (x^2)^3$
$= \log_{10} \sqrt{10} + \log_{10} x^6$
$= \log_{10} (\sqrt{10} \times x^6)$
$= \log_{10} (\sqrt{10} x^6)$

Exercise 7F — Solving logarithmic equations

1 a $\log_2 4 = x$
$\log_2 4 = \log_2 2^2$
$= 2\log_2 2$
$= 2$
$x = 2$

b $\log_9 1 = x$
$\log_9 1 = 0$
$x = 0$

c $\log_3 27 = x$
$\log_3 27 = \log_3 3^3$
$= 3\log_3 3$
$= 3$
$x = 3$

d $\log_4 256 = x$
$\log_4 256 = \log_4 4^4$
$= 4\log_4 4$
$= 4$
$x = 4$

e $\log_{10} \frac{1}{10} = x$
$\log_{10} \frac{1}{10} = \log_{10} 10^{-1}$
$= -1\log_{10} 10$
$= -1$
$x = -1$

f $\log_3 \frac{1}{9} = x$
$\log_3 \frac{1}{9} = \log_3 3^{-2}$
$= -2\log_3 3$
$= -2$
$x = -2$

g $2\log_2 8 = x$
$2\log_2 8 = 2\log_2 2^3$
$= 3 \times 2\log_2 2$
$= 6$
$x = 6$

h $\log_3 81 = 2x$
$\log_3 81 = \log_3 3^4$
$= 4\log_3 3$
$= 4$
$2x = 4$
$x = 2$

i $\log_{10} 1000 = 2x - 1$
$\log_{10} 1000 = \log_{10} 10^3$
$= 3\log_{10} 10$
$= 3$
$2x - 1 = 3$
$2x = 4$
$x = 2$

j $2\log_2 32 = 3x + 1$
$2\log_2 32 = 2\log_2 2^5$
$= 5 \times 2\log_2 2$
$= 10$
$3x + 1 = 10$
$3x = 9$
$x = 3$

2 a $\log_2 x = 3$
$x = 2^3$
$x = 8$

b $\log_3 x = 2$
$x = 3^2$
$x = 9$

c $\log_5 x = 4$
$x = 5^4$
$x = 625$

d $\log_{10} x = 1$
$x = 10^1$
$x = 10$

e $\log_8 x = -1$
$x = 8^{-1}$
$x = \frac{1}{8}$

f $\log_3 x = -3$
$x = 3^{-3}$
$x = \frac{1}{27}$

g $\log_2 x = 6$
$x = 2^6$
$x = 64$

h $\log_{10} x = 4$
$x = 10^4$
$x = 10\,000$

i $\log_3 (x - 3) = 3$
$(x - 3) = 3^3$
$x - 3 = 27$
$x = 30$

j $\log_2 (3x + 1) = 4$
$(3x + 1) = 2^4$
$3x + 1 = 16$
$3x = 15$
$x = 5$

k $\log_{10} (2x) = 1$
$(2x) = 10^1$
$2x = 10$
$x = 5$

l $2\log_6 (3x) = 1$
$\log_6 (3x) = \frac{1}{2}$
$3x = 6^{\frac{1}{2}}$
$3x = \sqrt{6}$
$x = \frac{\sqrt{6}}{3}$

m $\log_5 x = \log_5 4 + \log_5 6$
$\log_5 x = \log_5 24$
$x = 24$

n $\log_3 5 - \log_3 4 = \log_3 x - \log_3 8$
$\log_3 \frac{5}{4} = \log_3 \frac{x}{8}$
$\frac{5}{4} = \frac{x}{8}$
$x = \frac{5 \times 8}{4}$
$= 10$

3 a $\log_x 36 = 2$
$x^2 = 36$
$x^2 = 6^2$
$x = 6$

b $\log_x 125 = 3$
$x^3 = 125$
$x^3 = 5^3$
$x = 5$

c $3\log_x 16 = 6$
$\log_x 16 = 2$
$x^2 = 16$
$x^2 = 4^2$
$x = 4$

d $-2\log_x \frac{1}{100} = 4$
$\log_x \frac{1}{100} = -2$
$x^{-2} = \frac{1}{100}$
$x^{-2} = 10^{-2}$
$x = 10$

e $\frac{1}{2}\log_x 64 = 3$
$\log_x 64 = 6$
$x^6 = 64$
$x^6 = 2^6$
$x = 2$

f $5\log_x 625 = 10$
$\log_x 625 = 2$
$x^2 = 625$
$x^2 = 25^2$
$x = 25$

g $\log_{x+1} 27 = 3$
$(x + 1)^3 = 27$
$(x + 1)^3 = 3^3$
$x + 1 = 3$
$x = 2$

h $-\log_{3x-1} \frac{1}{32} = 5$
$\log_{3x-1} \frac{1}{32} = -5$
$(3x - 1)^{-5} = \frac{1}{32}$
$(3x - 1)^{-5} = 2^{-5}$
$3x - 1 = 2$
$3x = 3$
$x = 1$

4 a $\log_7 343 = x$
$7^x = 343$
$7^x = 7^3$
$x = 3$
The answer is **B**

b $\log_8 x = 4$
$x = 8^4$
$= 4096$
The answer is **A**

c $\log_x 3 = \frac{1}{2}$
$x^{\frac{1}{2}} = 3$
$x = 9$
The answer is **E**

d $\log_3 x - 2 = \log_3 (x - 8)$
$\log_3 x - 2\log_3 3 = \log_3 (x - 8)$
$\log_3 x - \log_3 3^2 = \log_3 (x - 8)$
$\log_3 x - \log_3 9 = \log_3 (x - 8)$
$\log_3 \left(\frac{x}{9}\right) = \log_3 (x - 8)$
$\frac{x}{9} = x - 8$
$x - \frac{x}{9} = 8$
$\frac{8x}{9} = 8$
$x = 8 \times \frac{9}{8}$
$x = 9$
The answer is **C**

5 a $2^x = 11$
$\log_{10} 2^x = \log_{10} 11$
$x\log_{10} 2 = \log_{10} 11$
$x = \frac{\log_{10} 11}{\log_{10} 2}$
$= \frac{1.0414}{0.3010}$
$= 3.459$

b $2^x = 0.6$
$\log_{10} 2^x = \log_{10} 0.6$
$x \log_{10} 2 = \log_{10} 0.6$
$x = \dfrac{\log_{10} 0.6}{\log_{10} 2}$
$= \dfrac{-0.2218}{0.3010}$
$= -0.737$

c $3^x = 20$
$\log_{10} 3^x = \log_{10} 20$
$x \log_{10} 3 = \log_{10} 20$
$x = \dfrac{\log_{10} 20}{\log_{10} 3}$
$= \dfrac{1.3010}{0.4771}$
$= 2.727$

d $3^x = 1.7$
$\log_{10} 3^x = \log_{10} 1.7$
$x \log_{10} 3 = \log_{10} 1.7$
$x = \dfrac{\log_{10} 1.7}{\log_{10} 3}$
$= \dfrac{0.2304}{0.4771}$
$= 0.483$

e $5^x = 8$
$\log_{10} 5^x = \log_{10} 8$
$x \log_{10} 5 = \log_{10} 8$
$x = \dfrac{\log_{10} 8}{\log_{10} 5}$
$= \dfrac{0.9031}{0.6990}$
$= 1.292$

f $0.7^x = 3$
$\log_{10} 0.7^x = \log_{10} 3$
$x \log_{10} 0.7 = \log_{10} 3$
$x = \dfrac{\log_{10} 3}{\log_{10} 0.7}$
$= \dfrac{0.4771}{-0.1549}$
$= -3.080$

g $10^{x-1} = 18$
$\log_{10} 10^{x-1} = \log_{10} 18$
$(x-1) \log_{10} 10 = \log_{10} 18$
$x - 1 = \log_{10} 18$
$x - 1 = 1.2552$
$x = 2.255$

h $3^{x+2} = 12$
$\log_{10} 3^{x+2} = \log_{10} 12$
$(x+2) \log_{10} 3 = \log_{10} 12$
$(x+2) = \dfrac{\log_{10} 12}{\log_{10} 3}$
$= \dfrac{1.0792}{0.4771}$
$= 2.262$
$x = 0.262$

i $2^{2x+1} = 5$
$\log_{10} 2^{2x+1} = \log_{10} 5$
$(2x+1) \log_{10} 2 = \log_{10} 5$
$2x + 1 = \dfrac{\log_{10} 5}{\log_{10} 2}$
$2x + 1 = \dfrac{0.6990}{0.3010}$
$2x + 1 = 2.3222$
$2x = 1.3222$
$x = 0.661$

j $4^{3x+1} = 24$
$\log_{10} 4^{3x+1} = \log_{10} 24$
$(3x+1) \log_{10} 4 = \log_{10} 24$
$(3x+1) = \dfrac{\log_{10} 24}{\log_{10} 4}$
$3x + 1 = \dfrac{1.3802}{0.6021}$
$3x + 1 = 2.2923$
$3x = 1.2923$
$x = 0.431$

k $10^{-2x} = 7$
$\log_{10} 10^{-2x} = \log_{10} 7$
$(-2x) \log_{10} 10 = \log_{10} 7$
$-2x = \log_{10} 7$
$-2x = 0.8451$
$x = -0.423$

l $8^{2-x} = 0.75$
$\log_{10} 8^{2-x} = \log_{10} 0.75$
$(2-x) \log_{10} 8 = \log_{10} 0.75$
$(2-x) = \dfrac{\log_{10} 0.75}{\log_{10} 8}$
$2 - x = \dfrac{-0.1249}{0.9031}$
$2 - x = -0.1381$
$x = 2.138$

6 $4^x = 5$
$\log_{10} 4^x = \log_{10} 5$
$x = \dfrac{\log_{10} 5}{\log_{10} 4}$
$x = 1.16$
The answer is **B**

7 $0.6^{2x-1} = 2$
$\log_{10} 0.6^{2x-1} = \log_{10} 2$
$(2x-1) \log_{10} 0.6 = \log_{10} 2$
$(2x-1) = \dfrac{\log_{10} 2}{\log_{10} 0.6}$
$2x - 1 = \dfrac{0.301}{-0.222}$
$2x - 1 = -1.356$
$2x = -0.356$
$x = -0.18$
The answer is **C**

Exercise 7G — Applications of exponential and logarithmic functions

1 a $P(t) = 10\,000\,(2^t)$ where t is in months.

b i $P(3) = 10\,000\,(2^3)$
$= 80\,000$

ii $P(6) = 10\,000\,(2^6)$
$= 640\,000$

c $100\,000 = 10\,000\,(2^t)$
$2^t = 10$
$\log_{10} 2^t = \log_{10} 10$
$t \log_{10} 2 = 1$
$t = \dfrac{1}{\log_{10} 2}$
$= \dfrac{1}{0.301}$
$= 3.32$ months.

2 a $N = 15\,000\,(2^{0.01t})$
When $t = 0$.
$N = 15\,000\,(2^{0.01 \times 0})$
$= 15\,000\,(2^0)$
$= 15\,000$ population in 1990

b i When $t = 5$,
$N = 15\,000\,(2^{0.01 \times 5})$
$= 15\,000\,(2^{0.05})$
$= 15\,529$ population in 1995

ii When $t = 10$,
$N = 15\,000\,(2^{0.01 \times 10})$
$= 15\,000\,(2^{0.1})$
$= 16\,077$ population in 2000

c When $t = 25$
$N = 15\,000\,(2^{0.01 \times 25})$
$= 15\,000\,(2^{0.25})$
$= 17\,838$ predicted population in 2015

d $20\,000 = 15\,000\,(2^{0.01t})$
$2^{0.01t} = 1.333$
$\log_{10} 2^{0.01t} = \log_{10} 1.333$
$0.01t \log_{10} 2 = \log_{10} 1.333$
$0.01t = \dfrac{\log_{10} 1.333}{\log_{10} 2}$
$0.01t = 0.415$
$t = 41.5$
≈ 42 years
$1990 + 42 = 2032$

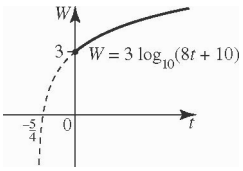
3 a When $t = 0$,
$W = 3 \log_{10} (8 \times 0 + 10)$
$= 3 \log_{10} 10$
$= 3$ kg

b i When $t = 1$,
$W = 3 \log_{10} (8 \times 1 + 10)$
$= 3 \log_{10} 18$
$= 3.77$ kg

ii When $t = 5$,
$W = 3 \log_{10} (8 \times 5 + 10)$
$= 3 \log_{10} 50$
$= 5.10$ kg

iii When $t = 10$,
$W = 3 \log_{10} (8 \times 10 + 10)$
$= 3 \log_{10} 90$
$= 5.86$ kg

c



d $7 = 3 \log_{10} (8t + 10)$
$2.333 = \log_{10} (8t + 10)$
$8t + 10 = 10^{2.333}$
$8t + 10 = 215.278$
$8t = 205.278$
$t = 25.66$
≈ 26 weeks

4 a $A = P(1 + \frac{5}{100})^n$
$A = P(1.05)^n$

b $A = 10\,000\,(1.05)^{10}$
$= \$16\,288.95$

c $26\,500 = 10\,000\,(1.05)^n$
$1.05^n = 2.65$
$\log_{10}1.05^n = \log_{10}2.65$
$n\log_{10}1.05 = \log_{10}2.65$
$n = \dfrac{\log_{10}2.65}{\log_{10}1.05}$
$= 19.97$
≈ 20 years

5 a $V = 25\,000\left(\frac{2}{5}\right)^{0.1t}$
When $t = 0$
$V = 25\,000\left(\frac{2}{5}\right)^{0.1\times 0}$
$= 25\,000\left(\frac{2}{5}\right)^{0}$
= \$25 000 price of car when new

b When $t = 6$
$V = 25\,000\left(\frac{2}{5}\right)^{0.1\times 6}$
$= 25\,000\left(\frac{2}{5}\right)^{0.6}$
= \$14 427

c $10\,000 = 25\,000\left(\frac{2}{5}\right)^{0.1t}$
$\left(\frac{2}{5}\right)^{0.1t} = 0.4$
$\log_{10}\left(\frac{2}{5}\right)^{0.1t} = \log_{10}0.4$
$0.1t\log_{10}\left(\frac{2}{5}\right) = \log_{10}0.4$
$0.1t = \dfrac{\log_{10}0.4}{\log_{10}\left(\frac{2}{5}\right)}$
$0.1t = 1$
$t = 10$ years

6 a $T = 90(3^{-0.5t})$
When $t = 0$,
$T = 90\,(3^{-0.05\times 0})$
$= 90\,(3^0)$
= 90°C initial temperature

b i When $t = 3$,
$T = 90\,(3^{-0.05\times 3})$
$= 90\,(3^{-0.15})$
= 76.3°C

ii When $t = 6$,
$T = 90\,(3^{-0.05\times 6})$
$= 90\,(3^{-0.30})$
= 64.7°C

c $45 = 90(3^{-0.05t})$
$3^{-0.05t} = 0.5$
$\log_{10}3^{-0.05t} = \log_{10}0.5$
$-0.05t\log_{10}3 = \log_{10}0.5$
$-0.05t = \dfrac{\log_{10}0.5}{\log_{10}3}$
$-0.05t = -0.631$
$t = 12.62$ mins or 12 mins 37 s

7 a $N = 120(1.1^t)$
When $t = 0$,
$N = 120\,(1.1^0)$
= 120 initial population of deer

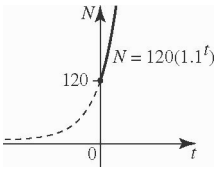
b i When $t = 2$,
$N = 120\,(1.1^2)$
$= 145$

ii When $t = 4$,
$N = 120\,(1.1^4)$
$= 176$

iii When $t = 6$,
$N = 120\,(1.1^6)$
$= 213$

c $3\times 120 = 120(1.1^t)$
$1.1^t = \dfrac{3\times 120}{120}$
$1.1^t = 3$
$\log_{10}1.1^t = \log_{10}3$
$t\log_{10}1.1 = \log_{10}3$
$t = \dfrac{\log_{10}3}{\log_{10}1.1}$
$t = 11.5$ years

d



e The population cannot increase indefinitely owing to limiting factors such as food, predators, disease and space.

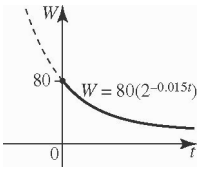
8 a $W = 80(2^{-0.015t})$
When $t = 0$,
$W = 80\,(2^{-0.015t\times 0})$
$= 80\,(2^0)$
= 80 kg before recycling

b i When $t = 10$,
$W = 80\,(2^{-0.015\times 10})$
$= 80\,(2^{-0.15})$
= 72.1 kg

ii When $t = 40$,
$W = 80\,(2^{-0.015\times 40})$
$= 80\,(2^{-0.6})$
= 52.8 kg

c $40 = 80(2^{-0.015t})$
$2^{-0.015t} = 0.5$
$\log_{10}2^{-0.015t} = \log_{10}0.5$
$-0.015t\log_{10}2 = \log_{10}0.5$
$-0.015t = \dfrac{\log_{10}0.5}{\log_{10}2}$
$-0.015t = -1$
$t = 66.7$
≈ 67 weeks

d i



ii In 10 years or 520 weeks, the model predicts negligible rubbish which is unrealistic.

9 a $N = 40\log_{10}(500t + 1)$

i When $t = 1$,
$N = 40\log_{10}(501)$
= 108 hectares

ii When $t = 2$,
$N = 40\log_{10}(500\times 2 + 1)$
$= 40\log_{10}1001$
= 120 hectares

iii When $t = 10$,
$N = 40\log_{10}(500\times 10 + 1)$
$= 40\log_{10}5001$
= 148 hectares

b $155 = 40\log_{10}(500t + 1)$
$\log_{10}(500t + 1) = 3.875$
$500t + 1 = 10^{3.875}$
$500t + 1 = 7498.9$
$500t = 7497.9$
$t = 15$ hours

10 a $d = 50 + \log_{10}(15n)$

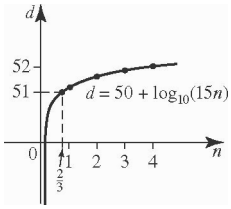
i When $n = 1$,
$d = 50 + \log_{10}15$
= 51.18 metres

ii When $n = 3$,
$d = 50 + \log_{10}(15\times 3)$
$= 50 + \log_{10}45$
= 51.65 metres

iii When $n = 6$,
$d = 50 + \log_{10}(15\times 6)$
$= 50 + \log_{10}90$
= 51.95 metres

iv When $n = 10$,
$d = 50 + \log_{10}(15\times 10)$
$= 50 + \log_{10}150$
= 52.18 metres

b



c $53 = 50 + \log_{10}(15n)$
$\log_{10}(15n) = 3$
$15n = 10^3$
$15n = 1000$
$n = 66.67$
≈ 67th competition

11 a $P = 400(10^{0.08t})$
When $t = 0$,
$P = 400(10^0)$
$= 400$

b i When $t = 5$,
$P = 400\,(10^{0.08\times 5})$
$= 400\,(10^{0.40})$
$= 1005$

ii When $t = 15$,
$P = 400\,(10^{0.08\times 15})$
$= 400\,(10^{1.2})$
$= 6340$

iii When $t = 25$,
$P = 15\,000 + 924\log_{10}[10(25 - 19)]$
$= 15\,000 + 924\log_{10}60$
$= 16\,643$

iv When $t = 40$
$P = 15\,000 + 924\log_{10}[10(40 - 19)]$
$= 15\,000 + 924\log_{10}210$
$= 17\,146$

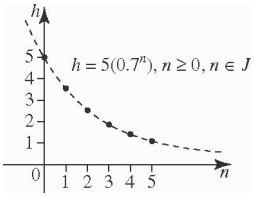
c $10\,000 = 400\,(10^{0.08t})$
$10^{0.08t} = 25$
$\log_{10} 10^{0.08t} = \log_{10} 25$
$0.08t \log_{10} = \log_{10} 25$
$0.08t = \log_{10} 25$
$0.08t = 1.398$
$t = 17.48$ months

12 a $h = 5\left(\frac{7}{10}\right)^n$
$h = 5(0.7)^n$ is the required rule.

b **i** When $n = 4$,
$h = 5\,(0.7)^4$
$= 1.20$ m

ii When $n = 8$,
$h = 5\,(0.7)^8$
$= 0.29$ m

c



13 a $V = 5000\,(1 + \frac{10}{100})^t$
$V = 5000\,(1.1)^t$ is the required rule.

b When $t = 6$,
$V = 5000\,(1.1)^6$
$= \$8857.81$

c $10\,000 = 5000\,(1.1^t)$
$1.1^t = 2$
$\log_{10} 1.1^t = 2$
$t \log_{10} 1.1 = \log_{10} 2$
$t = \dfrac{\log_{10} 2}{\log_{10} 1.1}$
$t = 7.27$ years
Number of years to double value = 8

Chapter review

1 $\dfrac{(2xy^3)^2}{7x^3} \times \dfrac{3x^5y^2}{4y}$
$= \dfrac{4x^2y^6}{7x^3} \times \dfrac{3x^5y^2}{4y}$
$= \dfrac{4 \times 3 \times x^2 \times x^5 \times y^6 \times y^2}{7 \times 4x^3y}$
$= \dfrac{3x^7y^8}{7x^3y^1}$
$= \dfrac{3x^4y^7}{7}$
The answer is **B**

2 $\dfrac{5m^4p^2}{2m^3p} \div \dfrac{(5m^2p^6)^3}{3m^7p}$
$= \dfrac{5m^4p^2}{2m^3p} \times \dfrac{3m^7p}{(5m^2p^6)^3}$
$= \dfrac{5m^4p^2}{2m^3p} \times \dfrac{3m^7p}{125m^6p^{18}}$
$= \dfrac{5 \times 3 \times m^4 \times m^7 \times p^2 \times p^1}{2 \times 125 \times m^3 \times m^6 \times p^1 \times p^{18}}$
$= \dfrac{3m^{11}p^3}{50m^9p^{19}}$
$= \dfrac{3m^2}{50p^{16}}$
The answer is **C**

3 $(16x^{-6}y^{10})^{\frac{1}{2}} \div \sqrt[3]{(27x^3y^9)}$
$= (4^2x^6y^{10})^{\frac{1}{2}} \div (27x^3y^9)^{\frac{1}{3}}$
$= (4x^{-3}y^5) \div (3^3x^3y^9)^{\frac{1}{3}}$
$= 4x^{-3}y^5 \div 3x^1y^3$
$= \dfrac{4x^{-3}y^5}{3x^1y^3}$
$= \dfrac{4y^2}{3x^4}$

4 $5^{-2}(\frac{64}{125})^{-\frac{1}{3}} = 5^{-2}\left(\dfrac{4^3}{5^3}\right)^{-\frac{1}{3}}$
$= 5^{-2}\left(\dfrac{4^{-1}}{5^{-1}}\right)$
$= 5^{-2}\left(\dfrac{5^1}{4^1}\right)$
$= \dfrac{1}{5 \times 4}$
$= \frac{1}{20}$
The answer is **A**

5 $25^{2-x} = 125$
$(5^2)^{2-x} = 5^3$
$5^{4-2x} = 5^3$
$4 - 2x = 3$
$2x = 1$
$x = \frac{1}{2}$
The answer is **B**

6 $4^{2x} - 17(4^x) + 16 = 0$
$(4^x)^2 - 17(4^x) + 16 = 0$
Let $y = 4^x$
$y^2 - 17y + 16 = 0$
$(y - 16)(y - 1) = 0$
$y = 16$ or $y = 1$
$4^x = 16$ or $4^x = 1$
$4^x = 4^2$ or $4^x = 4^0$
$x = 2$ or $x = 0$
The answer is **E**

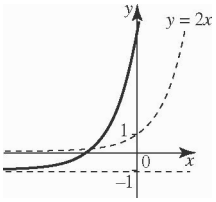
7 a $2x^5 = 100$
$x^5 = 50$
$x = 50^{\frac{1}{5}}$
$x = 2.187$
using graphics calculator.

b $8^{x+1} \times 2^{2x} = 4^{3x-1}$
$(2^3)^{x+1} \times 2^{2x} = (2^2)^{3x-1}$
$2^{3x+3} \times 2^{2x} = 2^{6x-2}$
$2^{5x+3} = 2^{6x-2}$
$5x + 3 = 6x - 2$
$x = 5$

8 $9^x - 5(3^x) + 6 = 0$
$(3^2)^x - 5(3^x) + 6 = 0$
$(3^x)^2 - 5(3^x) + 6 = 0$
Let $y = 3^x$
$y^2 - 5y + 6 = 0$
$(y - 2)(y - 3) = 0$
$y = 2$ or $y = 3$
$3^x = 2$ or $3^x = 3^1$
$3^x - 2 = 0$ or $x = 1$
$x = 0.63$ or 1
using graphics calculator.

9 The graph is a translation of the graph of $y = 3^x$, 2 units down.
Thus required rule is
$y = 3^x - 2$
The answer is **C**

10 The graph of $y = 2^{x+3} - 1$ is the graph of $y = 2^x$, translated 3 units left and 1 unit down.



The answer is **A**

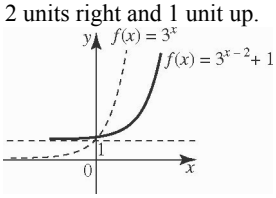
11 Domain R
The answer is **D**

12 Range $(-1, \infty)$
The answer is **E**

13 a $f(x) = 3^{x-2} + 1$
$f(0) = 3^{0-2} + 1$
$= 3^{-2} + 1$
$= \frac{1}{9} + 1$
$= 1\frac{1}{9}$ is the y-intercept

b Asymptote $y = 1$

c The graph of $f(x) = 3^{x-2} + 1$ is the graph of $f(x) = 3^x$ translated 2 units right and 1 unit up.



d Domain R
Range $(1, \infty)$

14 $5^x = 250$
$\log_5 250 = x$
The answer is **C**

15 $\log_7 49 + 3 \log_2 8 - 4$
$= \log_7 7^2 + 3 \log_2 2^3 - 4$
$= 2 \log_7 7 + 3 \times 3 \log_2 2 - 4$
$= 2 + 9 - 4$
$= 7$
The answer is **B**

16 a $\log_3 \frac{1}{27} = \log_3 3^{-3}$
$= -3 \log_3 3$
$= -3$

b $\log_{10} x + \log_{10} y = 2 \log_{10} (x + 1)$
$\log_{10} y = 2 \log_{10} (x + 1) - \log_{10} x$
$\log_{10} y = \log_{10} (x + 1)^2 - \log_{10} x$
$\log_{10} y = \log_{10} \dfrac{(x+1)^2}{x}$
$y = \dfrac{(x+1)^2}{x}$

17 a $3\log_4 5 - 2\log_4 6$
$= \log_4 5^3 - \log_4 6^2$
$= \log_4 125 - \log_4 36$
$= \log_4 \frac{125}{36}$

b $\frac{2\log_5 x^2}{\frac{1}{3}\log_5 x} = \frac{2\times 2\log_5 x}{\frac{1}{3}\log_5 x}$
$= \frac{2\times 2}{\frac{1}{3}}$
$= 2\times 2\times 3$
$= 12$

18 $\frac{\log_3 25}{\log_3 5} = \frac{\log_3 5^2}{\log_3 5}$
$= \frac{2\log_3 5}{\log_3 5}$
$= 2$
The answer is **C**

19 $\frac{\log_7 x^{\frac{5}{4}}}{\log_7 \sqrt{x}} = \frac{\log_7 x^{\frac{5}{4}}}{\log_7 x^{\frac{1}{2}}}$
$= \frac{\frac{5}{4}\log_7 x}{\frac{1}{2}\log_7 x}$
$= \frac{\frac{5}{4}}{\frac{1}{2}}$
$= \frac{5}{4}\times\frac{2}{1}$
$= \frac{5}{2}$
The answer is **A**

20 $\log_5 x = 4$
$x = 5^4$
$x = 625$
The answer is **D**

21 $2\log_x 343 = 6$
$\log_x 343 = 3$
$343 = x^3$
$7^3 = x^3$
$x = 7$
The answer is **B**

22 $\log_3(2x-1) + \log_3 2 = 2$
$\log_3 2(2x-1) = 2$
$\log_3(4x-2) = 2$
$4x - 2 = 3^2$
$4x - 2 = 9$
$4x = 11$
$x = \frac{11}{4}$
The answer is **E**

23 $4^{3-2x} = 12$
$\log_{10} 4^{3-2x} = \log_{10} 12$
$(3-2x)\log_{10} 4 = \log_{10} 12$
$0.602(3-2x) = 1.079$
$1.806 - 1.204x = 1.079$
$1.204x = 0.727$
$x = 0.604$
The answer is **C**

24 a $\log_6 x = 3$
$x = 6^3$
$x = 216$

b $2\log_x 125 = 6$
$\log_x 125 = 3$
$x^3 = 125$
$x^3 = 5^3$
$x = 5$

c $\log_2(3x+6) - \log_2 5 = 2$
$\log_2 \frac{(3x+6)}{5} = 2$
$\frac{(3x+6)}{5} = 2^2$
$\frac{(3x+6)}{5} = 4$
$3x + 6 = 20$
$3x = 14$
$x = \frac{14}{3}$ or $4\frac{2}{3}$

25 $5^x = 4$
$\log_{10} 5^x = \log_{10} 4$
$x\log_{10} 5 = \log_{10} 4$
$x = \frac{\log_{10} 4}{\log_{10} 5}$
$= 0.861$

26 $10^{2-3x} = 8$
$\log_{10} 10^{2-3x} = \log_{10} 8$
$(2-3x)\log_{10} 10 = \log_{10} 8$
$2 - 3x = \log_{10} 8$
$2 - 3x = 0.903$
$-3x = -1.097$
$\frac{-3x}{-3} = \frac{-1.097}{-3}$,
$x = 0.366$

27 a $N = 1500(2^{0.18t})$
When $t = 0$,
$N = 1500(2^{0.18\times 0})$
$N = 1500(2^0)$
$= 1500$

b i When $t = 5$,
$N = 1500(2^{0.18\times 5})$
$N = 1500(2^{0.9})$
$= 2799$
≈ 2800 to nearest 100

ii When $t = 10$,
$N = 1500(2^{0.18\times 10})$
$N = 1500(2^{1.8})$
$= 5223$
≈ 5200 to nearest 100

c $9000 = 1500(2^{0.18t})$
$2^{0.18t} = 6$
$\log_{10} 2^{0.18t} = \log_{10} 6$
$0.18t\log_{10} 2 = \log_{10} 6$
$0.18t = \frac{\log_{10} 6}{\log_{10} 2}$
$0.18t = \frac{0.778}{0.301}$
$0.18t = 2.585$
$t = 14.36$ days

Modelling and problem solving

1 a i $L = 20(10^{0.1t})$
When $t = 0$,
$L = 20(10^{0.1\times 0})$
$= 20(10^0)$
$= 20$ lions

ii $C = 25(10^{0.05t})$
When $t = 0$,
$C = 25(10^{0.05\times 0})$
$= 25(10^0)$
$= 25$ cheetahs

b i When $t = 1$,
$L = 20(10^{0.1})$
$= 25$ lions
and
$C = 25(10^{0.05})$
$= 28$ cheetahs

ii 18 months = 1.5 years
When $t = 1.5$,
$L = 25(10^{0.1\times 1.5})$
$= 25(10^{0.15})$
$= 28$ lions
and
$C = 25(10^{0.05\times 1.5})$
$= 25(10^{0.075})$
$= 30$ cheetahs

c $40 = 20(10^{0.1t})$
$10^{0.1t} = 2$
$\log_{10} 10^{0.01t} = \log_{10} 2$
$0.1t\log_{10} 10 = \log_{10} 2$
$0.1t = \log_{10} 2$
$0.1t = 0.301$
$t = 3.01$ years for lions
Now $40 = 25(10^{0.05t})$
$10^{0.05t} = 1.6$
$\log_{10} 10^{0.05t} = \log_{10} 1.6$
$0.05t\log_{10} 10 = \log_{10} 1.6$
$0.05t = \log_{10} 1.6$
$0.05t = 0.204$
$t = 4.08$ years for cheetahs
Lions reach 40 first by 1.07 years or 1 year 1 month.

d $20(10^{0.1t}) = 25(10^{0.05t})$
using graphics calculator
$t = 1$ year 11 months
$L = C = 31$

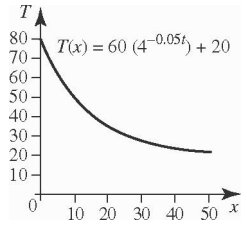
2 $T = 60(4^{-0.05t}) + 20$

a $t = 0$
$T = 60\times 4^0 + 20$
$T = 80°C$

b i $T = 60(4^{-0.05\times 2}) + 20$
$= 72.2°C$

ii $T = 60(4^{-0.05\times 25}) + 20$
$= 30.6°C$

c



d $30 = 60(4^{-0.05\times t}) + 20$
$\frac{10}{60} = 4^{-0.05t}$
$\log_{10}\left(\frac{1}{6}\right) = \log_{10} 4^{-0.05t}$
$\log_{10}\left(\frac{1}{6}\right) = -0.05t\log_{10} 4$
$-0.05t = \frac{\log_{10}\left(\frac{1}{6}\right)}{\log_{10} 4}$
$t = 25.850$ min

$$45 = 60(4^{-0.05t}) + 20$$
$$\frac{25}{60} = 4^{-0.05t}$$
$$\log_{10}\left(\frac{5}{12}\right) = -0.05t \log_{10}4$$
$$-0.05t = \frac{\log_{10}\left(\frac{5}{12}\right)}{\log_{10}4}$$
$$t = 12.630 \text{ min}$$

Time when coffee can be drunk is 13.22 minutes.

e Asymptote: $T = 20$

Temperature will settle to 20°C.

3 $N = 12\,000(2^{0.125t})$

a $t = 0$

$N = 12\,000 \times 2^0$

$N = 12\,000$ initially

b **i** $N = 12\,000(2^{0.125 \times 4})$

$N = 16\,970$

ii $N = 12\,000(2^{0.125 \times 14})$

$= 40\,363$

c $32\,000 = 12\,000(2^{0.125 \times t})$

$$\frac{8}{3} = 2^{0.125t}$$
$$\log_{10}\left(\frac{8}{3}\right) = 0.125t \log_{10}2$$
$$0.125t = \frac{\log_{10}\left(\frac{8}{3}\right)}{\log_{10}2}$$
$$t = 11.32 \text{ days}$$

d $D = N_0 \times 3^{-0.789t}$

$1000 = 32\,000 \times 3^{-0.789t}$

$$\frac{1}{32} = 3^{-0.789t}$$
$$\log_{10}\left(\frac{1}{32}\right) = \log_{10}3^{-0.789t}$$
$$-0.789t = \frac{\log_{10}\left(\frac{1}{32}\right)}{\log_{10}3}$$
$$t = 3.998$$

(4 days)

e $N = 12\,000(2^{0.125 \times 42})$

$N = 456\,655.5$

$N \approx 456\,656$

$D = 456\,656 \times 3^{-0.789t}$

$1000 = 456\,656 \times 3^{-0.789t}$

$3^{-0.789t} = 0.002\,19$

$-0.789t \times \log_{10}3 = \log_{10}0.002\,19$

$$-0.789 \times t = \frac{\log_{10}0.002\,19}{\log_{10}3}$$
$$t = 7.06 \text{ days}$$

3.06 days more

Require whole days, so 4 more days.