PH 221-3A Fall 2009

Kinetic Energy and Work

Lecture 10-11

Chapter 7

(Halliday/Resnick/Walker, Fundamentals of Physics 8th edition)

Chapter 7

Kinetic Energy and Work

In this chapter we will introduce the following concepts:
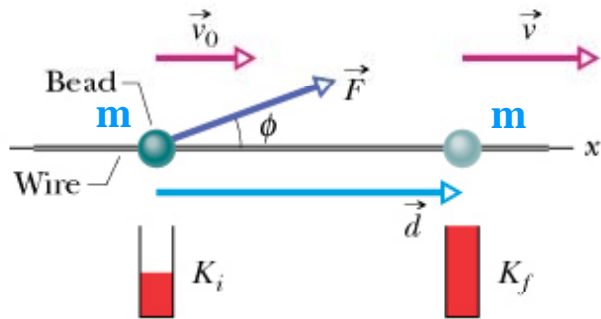
Kinetic energy of a moving object
Work done by a force
Power

In addition we will develop the work-kinetic energy theorem and apply it to solve a variety of problems

This approach is alternative approach to mechanics.   It uses scalars such as work and kinetic energy rather than vectors such as velocity and acceleration. Therefore it simpler to apply.

Kinetic Energy:
We define a new physical parameter to describe the state of motion of an object of mass m and speed v

We define its kinetic energy K as:

$$K = \frac{mv^2}{2}$$

We can use the equation above to define the SI unit for work (the joule, symbol: **J**).   An object of mass $m = 1\text{kg}$ that moves with speed $v = 1$ m/s has a kinetic energy $K = 1\text{J}$

Work: (symbol W)
If a force F is applied to an object of mass m it can accelerate it and increase its speed v and kinetic energy K. Similarly F can decelerate m and decrease its kinetic energy.
We account for these changes in K by saying that F has transferred energy W to or from the object. If energy it transferred **to** m (its K increases) we say that work was done by F **on** the object ($W > 0$). If on the other hand.
If on the other hand energy its transferred **from** the object (its K decreases) we say that work was done **by** m ($W < 0$)

Problem 5. A father racing his son has half the kinetic energy of the son, who has half the mass of the father. The father speeds up by 1.0 m/s and then has the same kinetic energy as the son. What are the original speeds of (a) the father and (b) the son?

We denote the mass of the father as m and his initial speed v_i. The initial kinetic energy of the father is

$$K_i = \frac{1}{2} K_{son}$$

and his final kinetic energy (when his speed is $v_f = v_i + 1.0$ m/s) is $K_f = K_{son}$. We use these relations along with definition of kinetic energy in our solution.

(a) We see from the above that $K_i = \frac{1}{2} K_f$ which (with SI units understood) leads to

$$\frac{1}{2} m v_i^2 = \frac{1}{2} \left[\frac{1}{2} m \ (v_i + 1.0 \ \text{m/s})^2 \right].$$

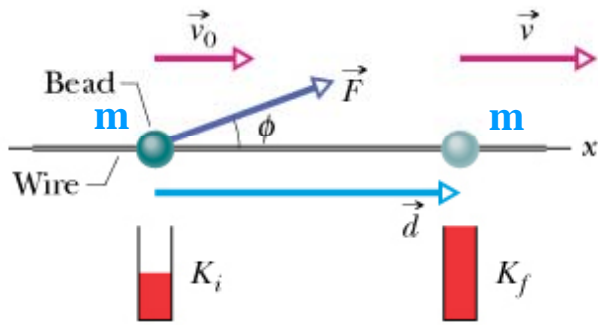The mass cancels and we find a second-degree equation for v_i:

$$\frac{1}{2} v_i^2 - v_i - \frac{1}{2} = 0.$$

The positive root (from the quadratic formula) yields $v_i = 2.4$ m/s.

(b) From the first relation above $\left(K_i = \frac{1}{2} K_{son} \right)$, we have

$$\frac{1}{2} m v_i^2 = \frac{1}{2} \left(\frac{1}{2} \ (m/2) \ v_{son}^2 \right)$$

and (after canceling m and one factor of 1/2) are led to $v_{son} = 2 v_i = 4.8 \ \text{m/s}.$

Finding an expression for Work:

Consider a bead of mass m that can move without friction along a straight wire along the x-axis. A constant force $\vec{F}$ applied at an angle ϕ to the wire is acting on the bead

We apply Newton's second law: $F_x = ma_x$ We assume that the bead had an initial velocity $\vec{v}_o$ and after it has travelled a displacement $\vec{d}$ its velocity is $\vec{v}$. We apply the third equation of kinematics: $v^2 - v_o^2 = 2a_x d$ We multiply both sides by $m/2 \rightarrow$
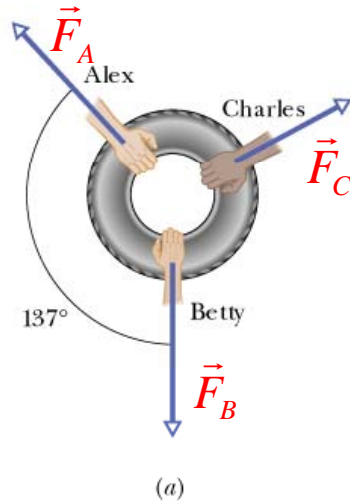
$$\frac{m}{2}v^2 - \frac{m}{2}v_o^2 = \frac{m}{2}2a_x d = \frac{m}{2}2\frac{F_x}{m}d = F_x d = F\cos\phi d \qquad K_i = \frac{m}{2}v_o^2$$

$$K_f = \frac{m}{2}v^2 \rightarrow \text{ The change in kinetic energy } K_f - K_i = Fd\cos\varphi$$

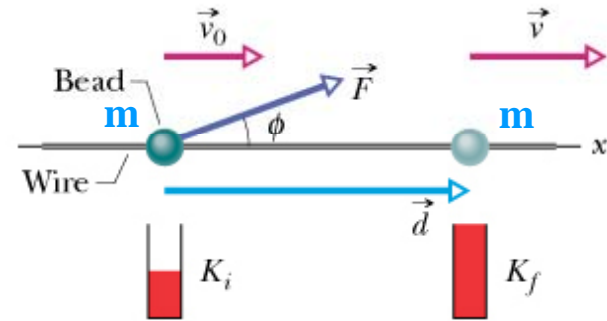Thus the work W done by the force on the bead is given by: $W = F_x d = Fd\cos\varphi$

$$\boxed{W = Fd\cos\varphi} \qquad \boxed{W = \vec{F}\cdot\vec{d}}$$

$$W = Fd \cos \varphi$$

$$W = \vec{F} \cdot \vec{d}$$

The unit of W is the same as that of K i.e. joules

Note 1: The expressions for work we have developed apply when F is constant

Note 2: We have made the implicit assumption that the moving object is point-like

Note 3: $W > 0$ if $0 < \phi < 90°$, $W < 0$ if $90° < \phi < 180°$

Net Work: If we have several forces acting on a body (say three as in the picture) there are two methods that can be used to calculate the net work W_{net}

Method 1: First calculate the work done by each force: W_A by force $\vec{F}_A$, W_B by force $\vec{F}_B$, and W_C by force $\vec{F}_C$. Then determine $W_{net} = W_A + W_B + W_C$

Method 2: Calculate first $\vec{F}_{net} = \vec{F}_A + \vec{F}_B + \vec{F}_C$; Then determine $W_{net} = \vec{F} \cdot \vec{d}$

Accelerating a Crate

A 120kg crate on the flatbed of a truck is moving with an acceleration a = +1.5m/s² along the positive x axis. The crate does not slip with respect to the truck, as the truck undergoes a displacement s = 65m. What is the total work done on the crate by all the forces acting on it?
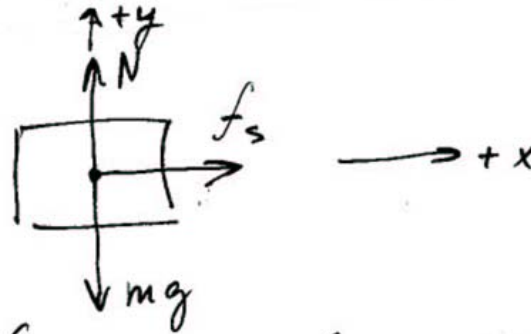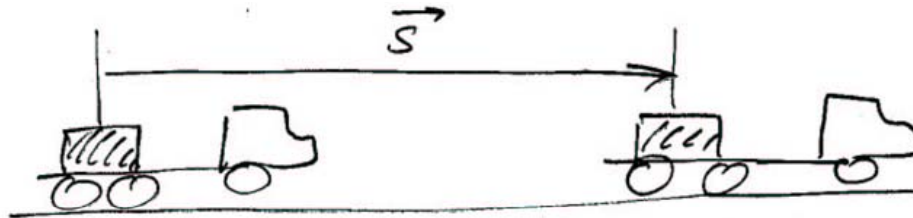
a) free body diagram
   for the crate.

• N & mg are $\perp$ to s
 they do no work.

• only f_s does work

b) 2nd N. law: $f_s = ma = (120\,kg)(1.5\,m/s^2) = 180\,N$

e) WORK calc.

$W = (f_s \cos\theta)\,s = (180N)(\cos 0°)(65m) = \underline{1.2\text{x}10^4\ N}$

Work

Problem 11: A 1200kg car is being driven up a 5.0° hill. The frictional force is directed opposite to the motion of the car and has a magnitude of $f_k = 5.0 \times 10^2 N$. The force F is applied to the car by the road and propels the car forward. In addition to those two forces, two other forces act on the car: its weight W, and the normal force N directed perpendicular to the road surface. The length of the hill is $3.0 \times 10^2 m$. What should be the magnitude of F so that the net work done by all the forces acting on the car is +150,000 J?

Given: $\theta = 5.0°$
$m = 1200 kg$
$f_k = 5.0 \times 10^2 N$
$S = 3.0 \times 10^2 m$
$W = 150000 J$
_____
$F = ?$
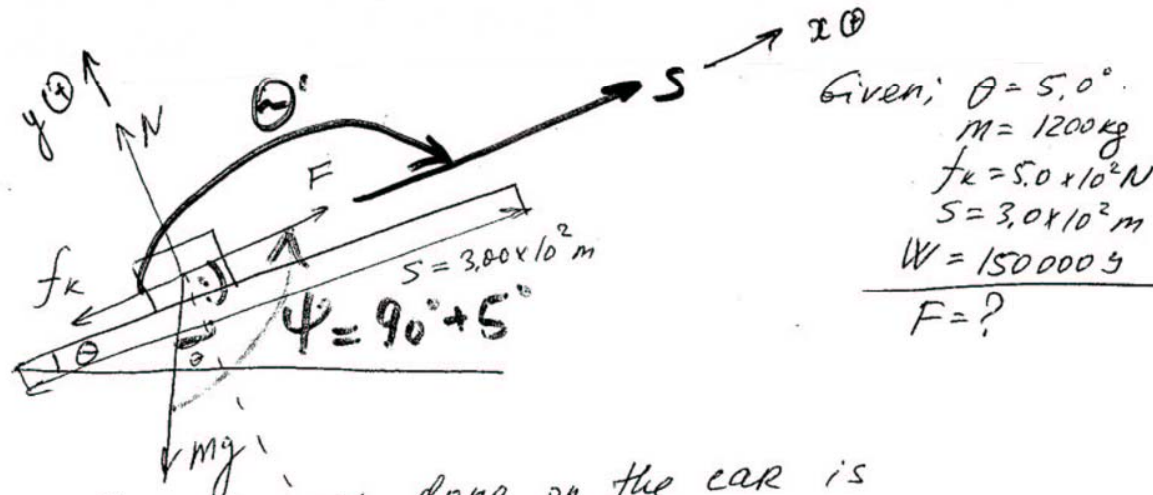
- the net work done on the car is

$$W = W_F + W_{f_k} + W_g + W_N =$$

$$F \cdot S \times \cos 0° + f_k \cdot S \cos 180° + mg \cdot S \cdot \cos(5+90°) + N \cdot S \cos 90°$$

$$F = \frac{W + f_k S - mg \cos 95.0° \cdot S}{S} = \frac{(1.5 \times 10^5 J)}{(3.0 \times 10^2 m)} + (5.0 \times 10^2 N) -$$

$$- (1200 kg)(9.80 \, m/s^2) \cdot \cos 95° =$$
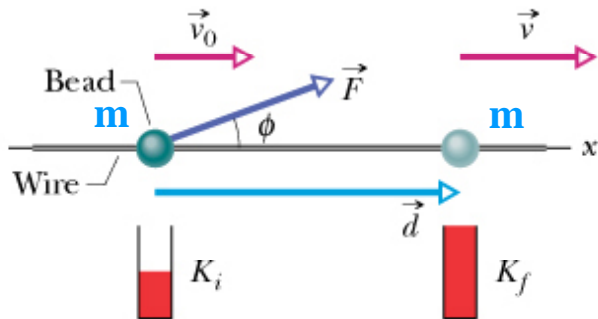
$$= \boxed{2.0 \times 10^3 N}$$

Work-Kinetic Energy Theorem

We have seen earlier that: $K_f - K_i = W_{net}$.

We define the change in kinetic energy as:

$\Delta K = K_f - K_i$. The equation above becomes

the work-kinetic energy theorem

$$\boxed{\Delta K = K_f - K_i = W_{net}}$$

$$\begin{bmatrix} \text{Change in the kinetic} \\ \text{energy of a particle} \end{bmatrix} = \begin{bmatrix} \text{net work done on} \\ \text{the particle} \end{bmatrix}$$

The work-energy theorem: when a net external force does work W on an object, the kinetic energy of the object changes from initial value KE_0 to the final value KE_f, the difference between the two values being equal to the work $W = K_f - K_0 = 1/2mv_f^2 - 1/2mv_0^2$

The work-kinetic energy theorem holds for both positive and negative values of W_{net}

If $\qquad W_{net} > 0 \quad \rightarrow \quad K_f - K_i > 0 \rightarrow K_f > K_i$

If $\qquad W_{net} < 0 \quad \rightarrow \quad K_f - K_i < 0 \rightarrow K_f < K_i$
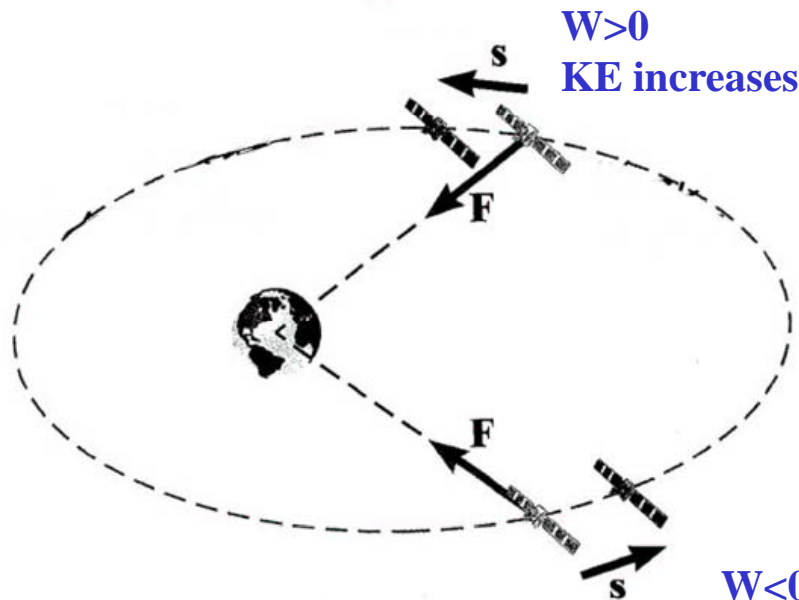
Work and Kinetic Energy
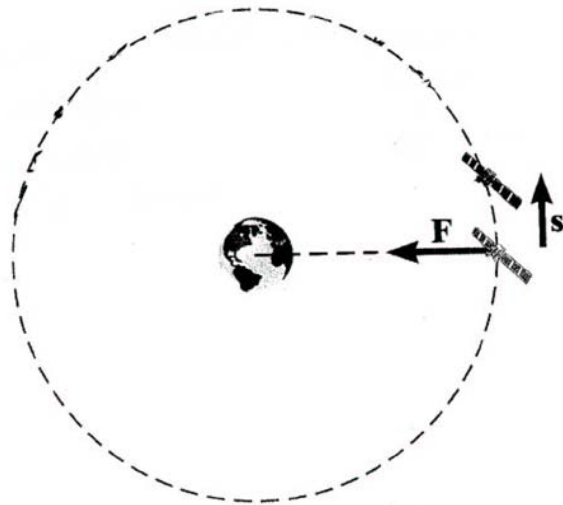
(a)

In a circular orbit the gravitational force F is always perpendicular to the displacement s of the satellite and does no work

KE = constant

W>0
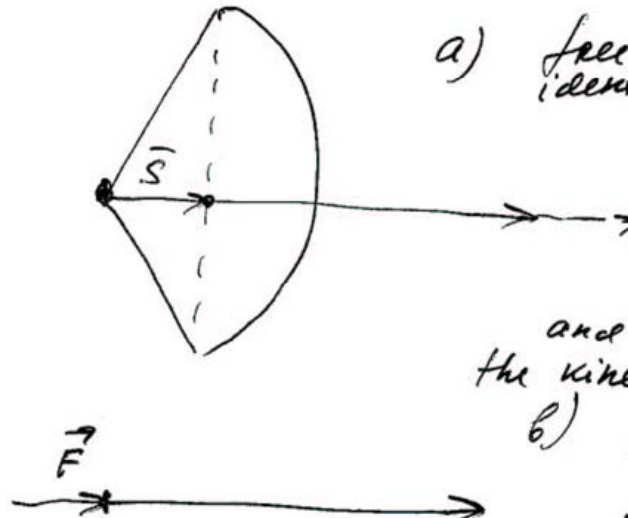**KE increases**

In an elliptical orbit, there can be a component of the force along the displacement

Work is done

(b)

W<0
KE decreases

The Work-Energy Theorem and Kinetic Energy

Problem : A 0.075kg arrow is fired horizontally. The bowstring exerts an average force of 65 N on the arrow over a distance of 0.90 m. With what speed does the arrow leave the bow?

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a) free body diagram.
identify net force.
Since the force $\vec{F}$ is the only force acting on the arrow, it is the net external force and the work it does causes the kinetic energy of the arrow to change.
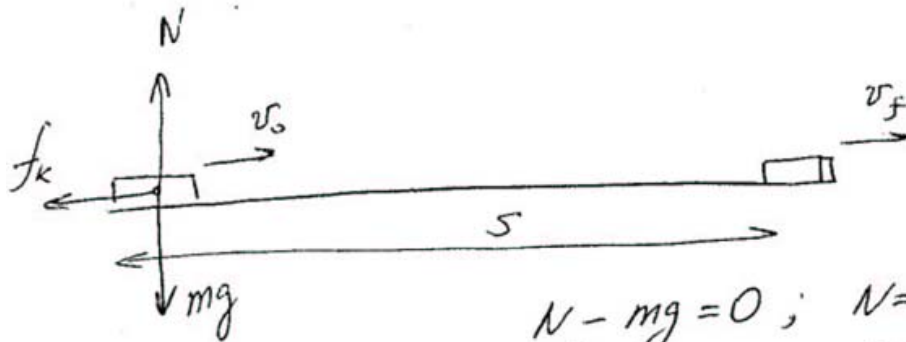
b) Apply work - energy theorem.

$$W = KE_f - KE_o = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_o^2$$

where $W = F\cos 0° \cdot s = Fs$

$$Fs = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_o^2$$

$$\frac{1}{2}mv_f^2 = Fs + \frac{1}{2}mv_o^2$$

$$v_f^2 = \frac{2Fs}{m} + v_o^2$$

$$v_f = \sqrt{\frac{2Fs}{m} + v_o^2} = \sqrt{\frac{2(65N)(0.90m)}{(75\times10^{-3}kg)} + (0\,m/s)^2}$$

$$= \boxed{39 \; m/s}$$

The Work-Energy Theorem and Kinetic Energy

The speed of a hockey puck decreases from 45.00 to 44.67m/s in coasting 16m across the ice. Find the coefficient of kinetic friction between the puck and the ice.

Given
$$v_o = 45.00 \, m/s$$
$$v_f = 44.67 \, m/s$$
$$S = 16 \, m$$

$$N - mg = 0; \quad N = mg$$

$$f_k = \mu_k N = \mu_k mg$$

- The work done by friction in slowing the puck is
$$W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_o^2$$

- The definition of work gives
$$W = f_k \cos 180° \cdot S = -\mu_k mg S$$

- Equating the expressions
$$\frac{1}{2} \cancel{m} v_f^2 - \frac{1}{2} \cancel{m} v_o^2 = -\mu_k \cancel{m} g S$$

$$\mu_k = \frac{1}{2} \frac{(v_o^2 - v_f^2)}{g S} = \frac{\frac{1}{2}[(45.00 \, m/s)^2 - (44.67 \, m/s)^2]}{(9.80 \, m/s^2)(16m)} = \boxed{0.094}$$

The work-energy theorem deals with the work done by the net external force. The work-energy theorem does not apply to the work done by an individual force. If W>0 then KE increases; if W<0 then KE decreases; if W=0 then KE remains constant.

Downhill Skiing: A 58kg skier is coasting down a 25° slope. A kinetic friction force f_k=70N opposes her motion. Near the top of the slope, the skier's speed is v_0=3.6m/s. Ignoring air resistance, determine the speed v_f at a point that is displaced 57m downhill.

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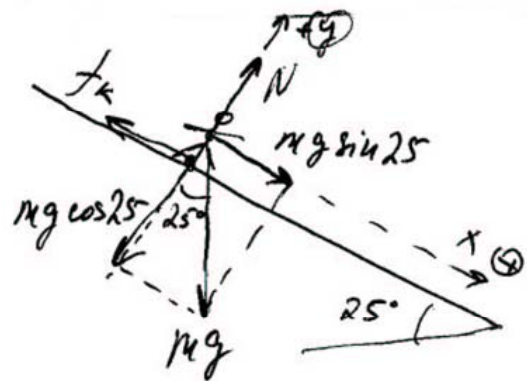
a) Free body diagram & identification of the net external force

$$\Sigma F = mg\sin 25° - f_k =$$
$$= (58kg)(9.80 \, m/s^2)\sin 25° - 70N =$$
$$= + 170 \, N$$

b) the work done by the net external force
$$W = (\Sigma F \cos\theta)s = [(170N)\cos 0°](57m) = 9,700 \, J$$

c) Work-Energy theorem
$$W = KE_f - KE_0$$
$$KE_f = W + KE_0 = 9,700 \, J + \tfrac{1}{2}(58kg)(3.6 \, m/s)^2 =$$
$$= 10,100 \, J$$

d) $KE_f = \tfrac{1}{2}mv_f^2$ => $v_f = \sqrt{\dfrac{2KE_f}{m}} = \sqrt{\dfrac{2(10,100J)}{(58kg)}} = 19 \, m/s$

Problem 15. A 12.0 N force with a fixed orientation does work on a particle as the particle moves through displacement $\vec{d} = (2.00\hat{i} - 4.00\hat{j} + 3.00\hat{k})$ m. What is the angle between the force and the displacement if the charge in the particle's kinetic energy is (a) +30.0J and (b) -30.0J?

Using the work-kinetic energy theorem, we have
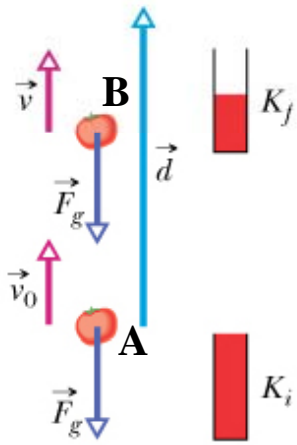
$$\Delta K = W = \vec{F} \cdot \vec{d} = Fd \cos\phi$$

In addition, $F = 12$ N and $d = \sqrt{(2.00 \text{ m})^2 + (-4.00 \text{ m})^2 + (3.00 \text{ m})^2} = 5.39$ m.

(a) If $\Delta K = +30.0$ J, then

$$\phi = \cos^{-1}\left(\frac{\Delta K}{Fd}\right) = \cos^{-1}\left(\frac{30.0 \text{ J}}{(12.0 \text{ N})(5.39 \text{ m})}\right) = 62.3°.$$

(b) $\Delta K = -30.0$ J, then
$$\phi = \cos^{-1}\left(\frac{\Delta K}{Fd}\right) = \cos^{-1}\left(\frac{-30.0 \text{ J}}{(12.0 \text{ N})(5.39 \text{ m})}\right) = 118°$$

Work Done by the Gravitational Force:

Consider a tomato of mass m that is thrown upwards at point A with initial speed v_o. As the tomato rises, it slows down by the gravitational force F_g so that at point B its has a smaller speed v. The work $W_g\left(A \rightarrow B\right)$ done by the gravitational force on the tomato as it travels from point A to point B is:

$$W_g\left(A \rightarrow B\right) = mgd \cos 180° = -mgd$$

The work $W_g\left(B \rightarrow A\right)$ done by the gravitational force on the tomato as it travels from point B to point A is:

$$W_g\left(B \rightarrow A\right) = mgd \cos 0° = mgd$$

B •

$\vec{d}$

$\vec{F}$

Object

A • m

$\vec{F}_g$

(a)
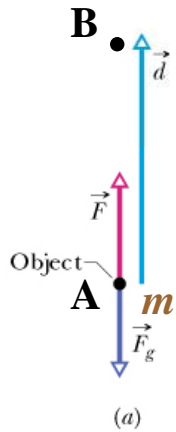
Work done by a force in Lifting an object:

Consider an object of mass m that is lifted by a force F form point A to point B. The object starts from rest at A and arrives at B with zero speed. The force F is not necessarily constant during the trip.

The work-kinetic energy theorem states that: $\Delta K = K_f - K_i = W_{net}$

We also have that $K_i = K_f \rightarrow \Delta K = 0 \rightarrow W_{net} = 0$ There are two forces acting on the object: The gravitational force F_g and the applied force F that lifts the object. $W_{net} = W_a(A \rightarrow B) + W_g(A \rightarrow B) = 0 \rightarrow$

$W_a(A \rightarrow B) = -W_g(A \rightarrow B)$

$W_g(A \rightarrow B) = mgd\cos180° = -mgd \rightarrow W_a(A \rightarrow B) = mgd$

Work done by a force in Lowering an object:

In this case the object moves from B to A

$W_g(B \rightarrow A) = mgd\cos0° = mgd \qquad W_a(B \rightarrow A) = -W_g(B \rightarrow A) = -mgd$

Problem 17. A helicopter lifts a 72 kg astronaut 15 m vertically from the ocean by means of a cable. The acceleration of the astronaut is g/10. How much work is done on the astronaut by (a) the force from the helicopter and (b0 the gravitational force on her? Just before she reaches the helicopter, what are her (c) kinetic energy and (d) speed?

(a) We use $\vec{F}$ to denote the upward force exerted by the cable on the astronaut. The force of the cable is upward and the force of gravity is mg downward. Furthermore, the acceleration of the astronaut is $g/10$ upward. According to Newton's second law, $F - mg = mg/10$, so $F = 11\ mg/10$. Since the force $\vec{F}$ and the displacement $\vec{d}$ are in the same direction, the work done by $\vec{F}$ is

$$W_F = Fd = \frac{11 m g d}{10} = \frac{11\ (72\ \text{kg})(9.8\ \text{m/s}^2)(15\ \text{m})}{10} = 1.164 \times 10^4\ \text{J}$$

which (with respect to significant figures) should be quoted as 1.2×10^4 J.

(b) The force of gravity has magnitude mg and is opposite in direction to the displacement. Thus the work done by gravity is

$$W_g = -m g d = -(72\ \text{kg})(9.8\ \text{m/s}^2)(15\ \text{m}) = -1.058 \times 10^4\ \text{J}$$

which should be quoted as -1.1×10^4 J.

(c) The total work done is $W = 1.164 \times 10^4\ \text{J} - 1.058 \times 10^4\ \text{J} = 1.06 \times 10^3\ \text{J}$. Since the astronaut started from rest, the work-kinetic energy theorem tells us that this (which we round to $1.1 \times 10^3\ \text{J}$) is her final kinetic energy.

(d) Since $K = \frac{1}{2}m v^2$, her final speed is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.06 \times 10^3\ \text{J})}{72\ \text{kg}}} = 5.4\ \text{m/s}.$$

Problem 24. A cave rescue team lifts an injured spelunker directly upward and out of a sinkhole by means of a motor-driven cable. The lift is performed in three stages, each requiring a vertical distance of 10.0 m: (a) the initially stationary spelunker is accelerated to a speed of 5.00 m/s; (b0 he is then lifted at the constant speed of 5.00 m/s; © finally he is decelerated to zero speed. How much work is done on the 80.0kg rescuee by the force lifting him during each stage?

We use d to denote the magnitude of the spelunker's displacement during each stage. The mass of the spelunker is $m = 80.0$ kg. The work done by the lifting force is denoted W_i where $i = 1, 2, 3$ for the three stages. We apply the work-energy theorem, K_f-K_i=W_a+W_g

(a) For stage 1, $W_1 - mgd = \Delta K_1 = \frac{1}{2}mv_1^2$, where $v_1 = 5.00$ m/s. This gives

$$W_1 = mgd + \frac{1}{2}mv_1^2 = (80.0 \text{ kg})(9.80 \text{ m/s}^2)(10.0 \text{ m}) + \frac{1}{2}(80.0 \text{ kg})(5.00 \text{ m/s})^2 = 8.84 \times 10^3 \text{ J}.$$

(b) For stage 2, $W_2 - mgd = \Delta K_2 = 0$, which leads to
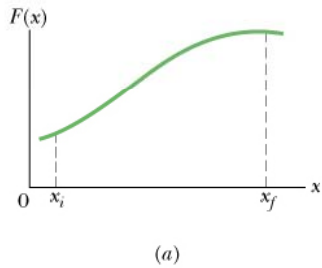
$$W_2 = mgd = (80.0 \text{ kg})(9.80 \text{ m/s}^2)(10.0 \text{ m}) = 7.84 \times 10^3 \text{ J}.$$

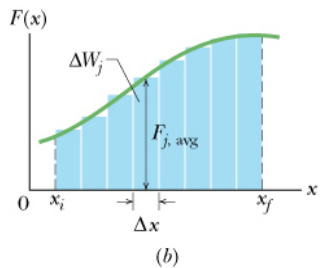(c) For stage 3, $W_3 - mgd = \Delta K_3 = -\frac{1}{2}mv_1^2$. We obtain

$$W_3 = mgd - \frac{1}{2}mv_1^2 = (80.0 \text{ kg})(9.80 \text{ m/s}^2)(10.0 \text{ m}) - \frac{1}{2}(80.0 \text{ kg})(5.00 \text{ m/s})^2 = 6.84 \times 10^3 \text{ J}.$$

Work done by a variable force $F(x)$ acting along the x-axis:

A force F that is not constant but instead varies as function of x is shown in fig.a. We wish to calculate the work W that F does on an object it moves from position x_i to position x_f.
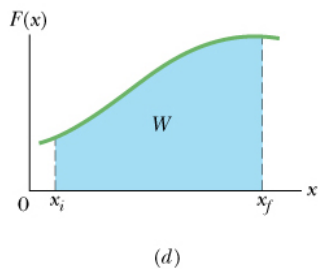
We partition the interval (x_i, x_f) into N "elements" of length Δx each as is shown in fig.b. The work done by F in the j-th interval is: $\Delta W_j = F_{j,avg} \Delta x$ Where $F_{j,avg}$ is the average value of F over the j-th element. $W = \sum_{j=1}^{N} F_{j,avg} \Delta x$ We then take the limit of the sum as $\Delta x \to 0$, (or equivalently $N \to \infty$)
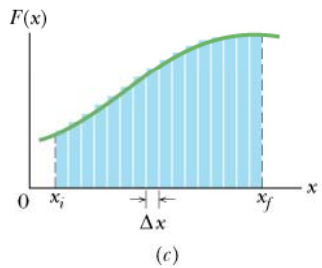
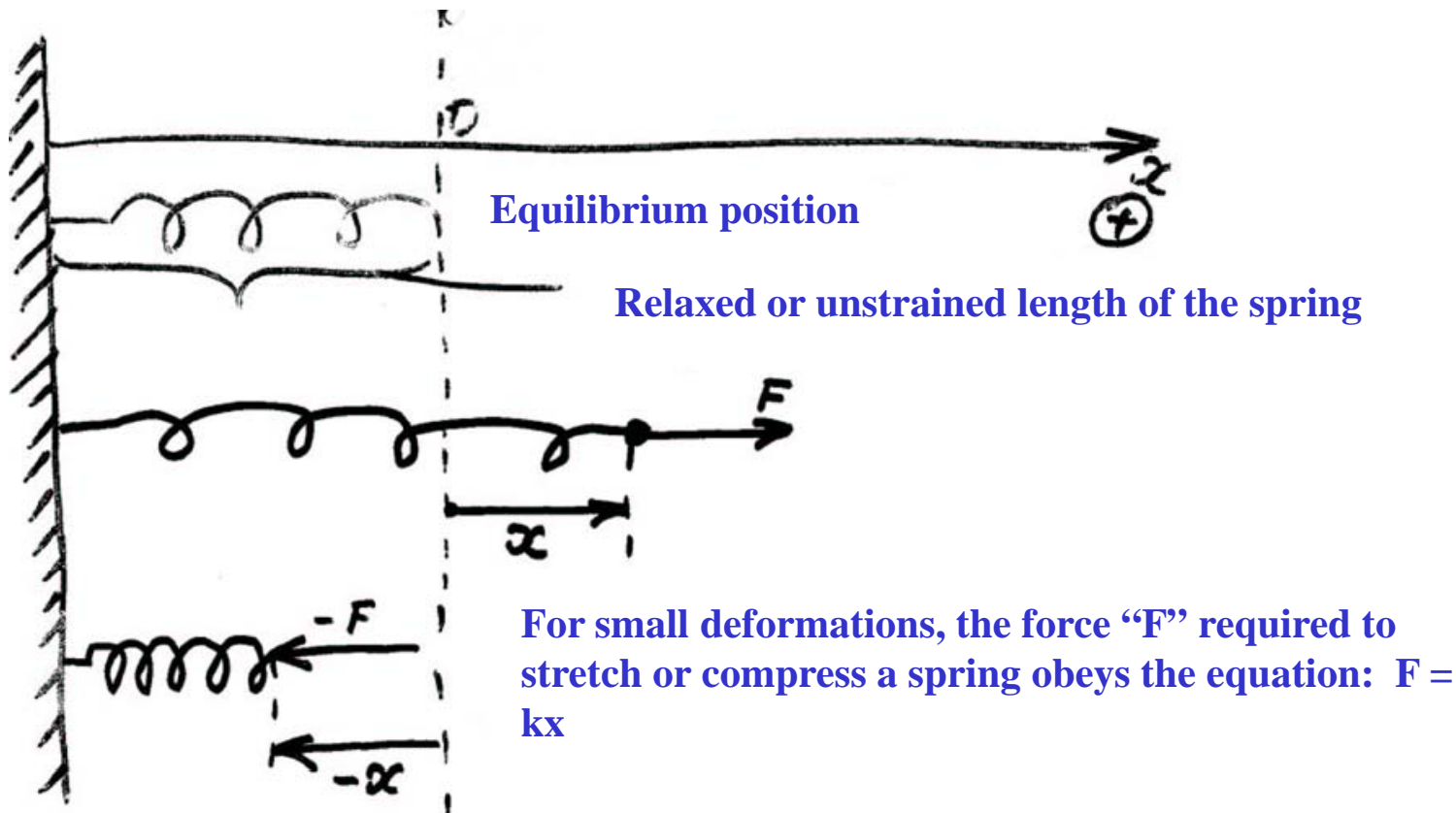$$W = \lim \sum_{j=1}^{N} F_{j,avg} \Delta x = \int_{x_i}^{x_f} F(x)dx$$ Geometrically, W is the area between $F(x)$ curve and the x-axis, between x_i and x_f (shaded blue in fig.d)

$$\boxed{W = \int_{x_i}^{x_f} F(x)dx}$$

The Ideal Spring

Springs are objects that exhibit elastic behavior. It will return back to its original length after being stretched or compressed.

Equilibrium position

Relaxed or unstrained length of the spring

For small deformations, the force "F" required to stretch or compress a spring obeys the equation: $F = kx$

- x - <u>displacement</u> of the spring from its unstrained length
- <u>k – spring constant</u> [N/m] unit
- A spring that behaves according to the relationship <u>$F = kx$</u> it is said to be an <u>ideal spring</u>

Restoring Force
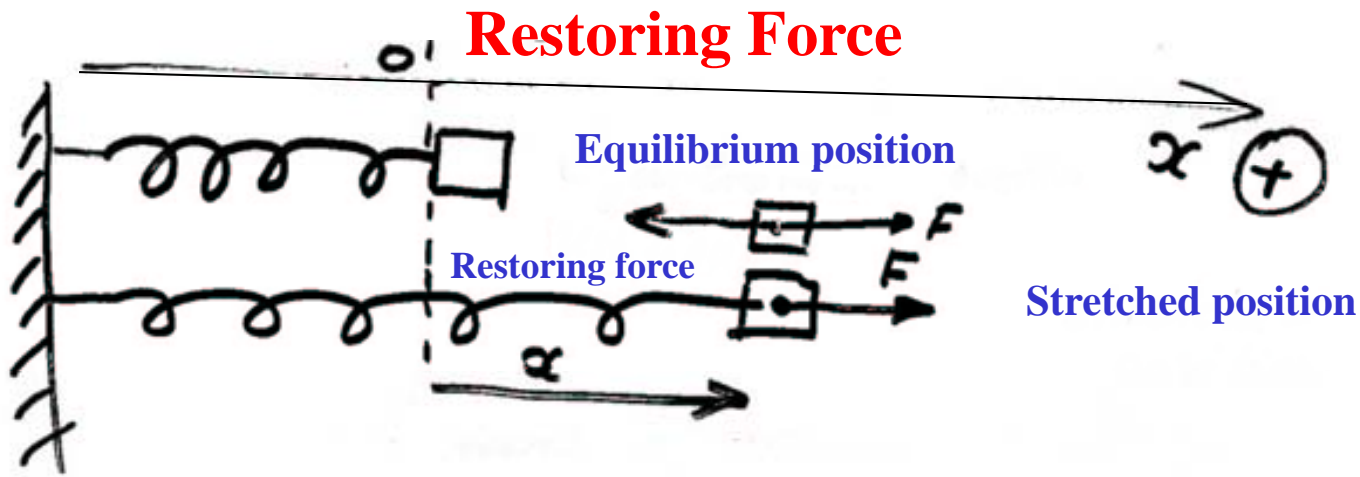
- To stretch or compress a spring a force F must be applied
- Newton's 3rd Law: Every action has an equal in magnitude and opposite reaction.

The reaction force that is applied by the spring to the agent that does the pulling or pushing is called restoring force

The restoring force is always opposite to the displacement of the spring

$x = 0$
$F_x = 0$
Block attached to spring

0

(a)

x positive
F_x negative $\vec{F_s}$
$\vec{d}$

0 — x →

(b)

$\vec{d}$
$\vec{F_s}$
x negative
F_x positive

— x — 0

(c)

The Spring Force:

Fig.a shows a spring in its relaxed state. In fig.b we pull one end of the spring and stretch it by an amount d. The spring resits by exerting a force F on our hand in the opposite direction.

In fig.c we push one end of the spring and compress it by an amount d. Again the spring resists by exerting a force F on our hand in the opposite direction
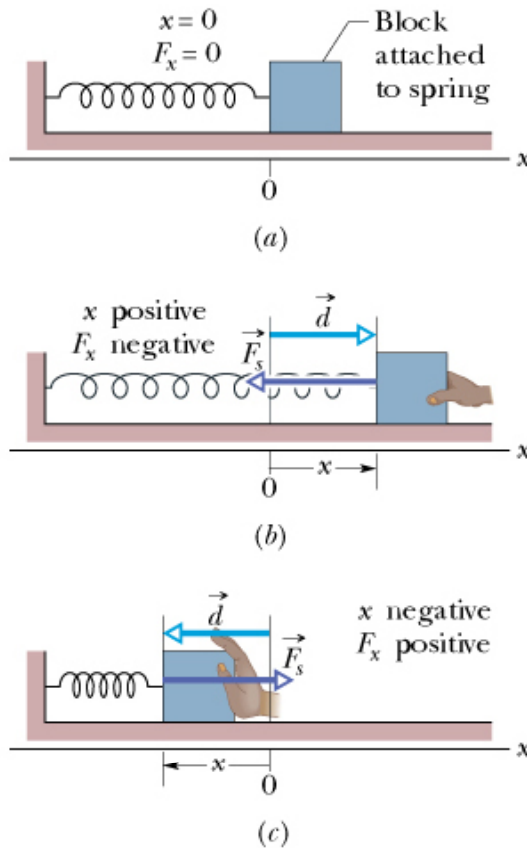
The force F exerted by the spring on whatever agent (in the picture our hand) is trying to change its natural length either by extending or by compressing it is given by the equation: $F = -kx$ Here x is the amount by which the spring has been extended or compressed. This equation is known as "Hookes law" k is known as "spring constant"

$$F = -kx$$

An object is attached to the lower end of a 100-coil spring that is hanging from the ceiling. The spring stretches by 0.160 m. The spring is then cut into two identical springs of 50 coils each. As the drawing shows, each spring is attached between the ceiling and the object. By how much does each spring stretch?
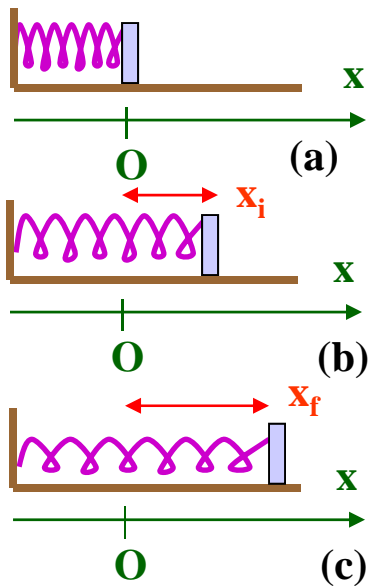

$mg = kx$

before the spring is cut

$mg = k'x' + k'x'$

after it is cut

$$\Rightarrow kx = 2k'x'$$

Shorter springs are stiffer springs
$$k' = 2k$$
$F_1 = k_1 l_{cm})$

$$\Rightarrow x' = \frac{x}{4} = \frac{0.160m}{4} = 0.040m$$

$F_2 = k_2 l_{cm})$

1) a coil is compressed by 1 cm

2) one coil is compressed by 0.5cm $\Rightarrow F_1 = 2F_2$

$2F_2 = k_1 x \cdot 1$

$F_2 = k_2 x 1$ $\Rightarrow \boxed{k_1 = 2k_2}$ shorter springs are stiffer

Work Done by a Spring Force

Consider the relaxed spring of spring constant k shown in (a)
By applying an external force we change the spring's
length from x_i (see b) to x_f (see c). We will
calculate the work W_s done by the spring on the external agent
(in this case our hand) that changed the spring length. We
assume that the spring is massless and that it obeys Hooke's law

We will use the expression: $W_s = \int\limits_{x_i}^{x_f} F(x)dx = \int\limits_{x_i}^{x_f} -kxdx = -k\int\limits_{x_i}^{x_f} xdx$

$W_s = -k\left[\dfrac{x^2}{2}\right]_{x_i}^{x_f} = \dfrac{kx_i^2}{2} - \dfrac{kx_f^2}{2}$ Quite often we start with a relaxed

spring ($x_i = 0$) and we either stretch or compress the spring by an

amount x ($x_f = \pm x$). In this case $W_s = -\dfrac{kx^2}{2}$

Problem 29. The only force acting on a 2.0 kg body as it moves along a positive x axis has an x component F_x=-6x N, with x in meters. The velocity at x= 3.0 m is 8.0 m/s. (a) What is the velocity of the body at x=4.0 m? (b) At what positive value of x will the body have a velocity of 5.0 m/s?

(a) As the body moves along the x axis from $x_i = 3.0$ m to $x_f = 4.0$ m the work done by the force is

$$W = \int_{x_i}^{x_f} F_x \ dx = \int_{x_i}^{x_f} -6x \ dx = -3(x_f^2 - x_i^2) = -3 \ (4.0^2 - 3.0^2) = -21 \ \text{J}.$$

According to the work-kinetic energy theorem, this gives the change in the kinetic energy:

$$W = \Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

where v_i is the initial velocity (at x_i) and v_f is the final velocity (at x_f). The theorem yields

$$v_f = \sqrt{\frac{2W}{m} + v_i^2} = \sqrt{\frac{2(-21 \ \text{J})}{2.0 \ \text{kg}} + (8.0 \ \text{m/s})^2} = 6.6 \ \text{m/s}.$$

(b) The velocity of the particle is $v_f = 5.0$ m/s when it is at $x = x_f$. The work-kinetic energy theorem is used to solve for x_f. The net work done on the particle is $W = -3\left(x_f^2 - x_i^2\right)$, so the theorem leads to

$$-3\left(x_f^2 - x_i^2\right) = \frac{1}{2}m\left(v_f^2 - v_i^2\right).$$

Thus,

$$x_f = \sqrt{-\frac{m}{6}\left(v_f^2 - v_i^2\right) + x_i^2} = \sqrt{-\frac{2.0 \ \text{kg}}{6 \ \text{N/m}}\left((5.0 \ \text{m/s})^2 - (8.0 \ \text{m/s})^2\right) + (3.0 \ \text{m})^2} = 4.7 \ \text{m}.$$

Three dimensional Analysis:

In the general case the force $\vec{F}$ acts in three dimensional space and moves an object on a three dimensional path from an initial point A to a final point B
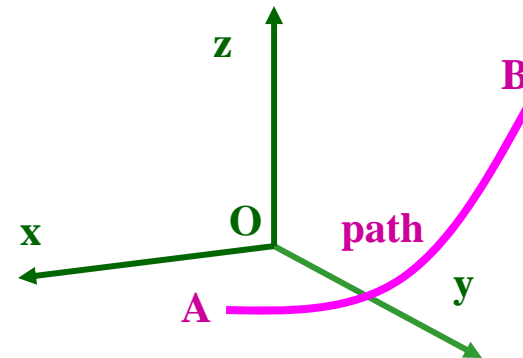
The force has the form: $\vec{F} = F_x(x, y, z)\hat{i} + F_y(x, y, z)\hat{j} + F_z(x, y, z)\hat{k}$

Points A and B have coordinates (x_i, y_i, z_i) and (x_f, y_f, z_f), respectively

$$dW = \vec{F} \cdot d\vec{r} = F_x dx + F_y dy + F_z dz$$

$$W = \int_A^B dW = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$$

$$W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$$

Work-Kinetic Energy Theorem with a Variable Force:

Conside a variable force F(x) which moves an object of mass m from point $A(x = x_i)$

to point $B(x = x_f)$. We apply Newton's second law: $F = ma = m\dfrac{dv}{dt}$ We then

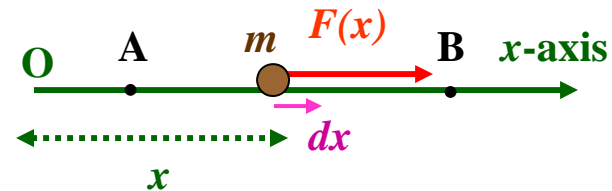multiply both sides of the last equation with dx and get: $Fdx = m\dfrac{dv}{dt}dx$

We integrate both sides over dx from x_i to x_f : $\displaystyle\int_{x_i}^{x_f} Fdx = \int_{x_i}^{x_f} m\dfrac{dv}{dt}dx$

$\dfrac{dv}{dt} = \dfrac{dv}{dx}\dfrac{dx}{dt} \rightarrow \dfrac{dv}{dt}dx = \dfrac{dv}{dx}\dfrac{dx}{dt}dx = vdv$ Thus the integral becomes:

$W = m\displaystyle\int_{x_i}^{x_f} vdv = \dfrac{m}{2}\left[v^2\right]_{x_i}^{x_f} = \dfrac{mv_f^2}{2} - \dfrac{mv_i^2}{2} = K_f - K_i = \Delta K$

Note: The work-kinetic energy theorem has exactly the same form as in the case

when F is constant!

$$\boxed{W = K_f - K_i = \Delta K}$$

Power

We define "power" P as the rate at which work is done by a force F.

If F does work W in a time interval Δt then we define as the average power as:

$$P_{avg} = \frac{W}{\Delta t}$$

The instantaneous power is defined as: $\quad P = \dfrac{dW}{dt}$

Unit of P: The SI unit of power is the watt. It is defined as the power

of an engine that does work $W = 1$ J in a time $t = 1$ second

A commonly used non-SI power unit is the horsepower (hp) defined as:

1 hp = 746 W

The kilowatt-hour   The kilowatt-hour (kWh) is a unit of work. It is defined

as the work performed by an engine of power $P = 1000$ W in a time $t = 1$ hour

$W = Pt = 1000 \times 3600 = 3.60 \times 10^{6}$ J   The kWh is used by electrical utility

companies (check your latest electric bill)

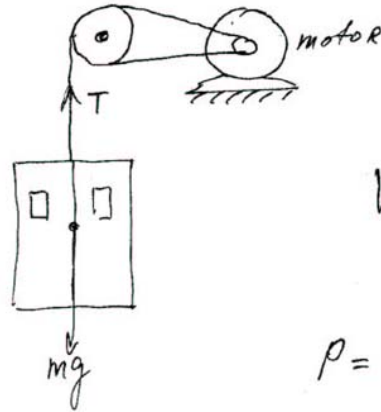Consider a force F acting on a particle at an angle ϕ to the motion. The rate at which F does work is given by: $P = \dfrac{dW}{dt} = \dfrac{F\cos\phi \, dx}{dt} = F\cos\phi \dfrac{dx}{dt} = Fv\cos\phi$

$$\boxed{P = Fv\cos\phi = \vec{F} \cdot \vec{v}}$$
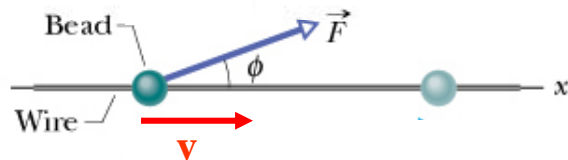
Example:
An elevator cage has a mass of 1000kg. How many horsepower must the motor deliver to the elevator if it is to raise the elevator cage at the rate of 2.0m/s?

Given: $M = 1000 \text{ kg}$
$a = 0$
$v = 2.0 \text{ m/s}$
$P = ?$

$T - mg = 0 \; ; \; T = mg$
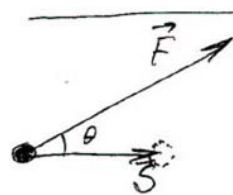
$W = T \cdot S$

S - distance the elevator moves up during Δt.

$P = \dfrac{W}{\Delta t} = \dfrac{T \cdot S}{\Delta t} = T \times v = mg \, v$

$v = \dfrac{S}{\Delta t}$ - Speed of the elevator.

$\Rightarrow P = mg \, v = (1000 \text{ kg})(9.80 \text{ m/s}^2)(2 \text{ m/s}) =$
$= (2.0 \times 10^4 \text{ W}) = (2.0 \times 10^4 \text{ W})\left(\dfrac{1 hp}{746 W}\right) =$
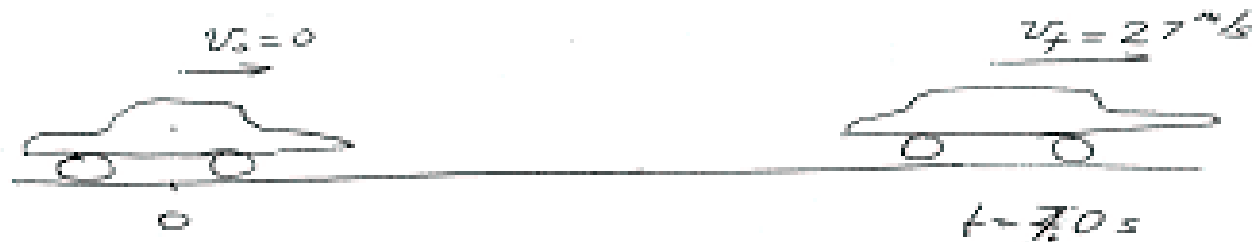$= 27 hp.$

if the force makes an angle θ with the displacement $\vec{S}$

$W = F \cdot \cos\theta \cdot S$

$P = \dfrac{W}{\Delta t} = \dfrac{F \cdot \cos\theta \cdot S}{\Delta t} = F \cos\theta \cdot v$

$$\boxed{P = F \cos\theta \cdot v}$$

A car accelerates uniformly from rest to 27 m/s in 7.0s along a level stretch of road. Ignoring friction, determine the average power required to accelerate the car if
a) The weight of the car is 1.2x10⁴N, and b) weight is 1.6x10⁴N

$v_0 = 0$

$v_f = 27 \text{ m/s}$

$t = 7.0 \text{ s}$

○ The work done on the car by the engine is

$$W = KE_f - KE_0 = \tfrac{1}{2} m v_f^2$$

○ The average power developed by the engine

$$P = \frac{W}{t}$$

(a) for $mg = 1.2 \times 10^4 N$; $\Rightarrow M = \frac{1.2 \times 10^4 N}{9.80 \text{ m/s}^2}$

$$P = \frac{1}{2} \frac{m v_f^2}{t} = \frac{1}{2} \frac{\left(\frac{1.2 \times 10^4 N}{9.80 \text{ m/s}^2} \right)(27 \text{ m/s})^2}{7.0 s} = \boxed{6.4 \times 10^4 W}$$

(b) for $mg = 1.6 \times 10^4 N$

$$P = \frac{1}{2} \frac{m v_f^2}{t} = \frac{1}{2} \frac{\left(\frac{1.6 \times 10^4 N}{9.80 \text{ m/s}^2} \right)(27 \text{ m/s})^2}{7.0 s} = \boxed{8.5 \times 10^4 W}$$

Problem 48. (a) At a certain instant, a particle-like obkect is acted on by a force
$\vec{F} = (4.0\,N\,)\hat{i} - (2.0\,N\,)\hat{j} + (9.0\,N\,)\hat{k}$ while the objects velocity is $\vec{v} = -(2.0m\,/\,s\,)\hat{i} + (4.0m\,/\,s\,)\hat{k}$
What is the instantaneous rate at which the force does work on the object? (b)
AT some other time, the velocity consists of only a y component. if the force is unchanged
and the instantaneous power is -12W, what is the velocity of the object?

(a) we obtain

$$P = \vec{F} \cdot \vec{v} = (4.0\,\text{N})(-2.0\text{ m/s}) + (9.0\text{ N})(4.0\text{ m/s}) = 28\text{ W}.$$

(b) with a one-component velocity: $\vec{v} = v\hat{j}$.

$$P = \vec{F} \cdot \vec{v} \Rightarrow -12\text{ W} = (-2.0\text{ N})v.$$

which yields $v = 6$ m/s.