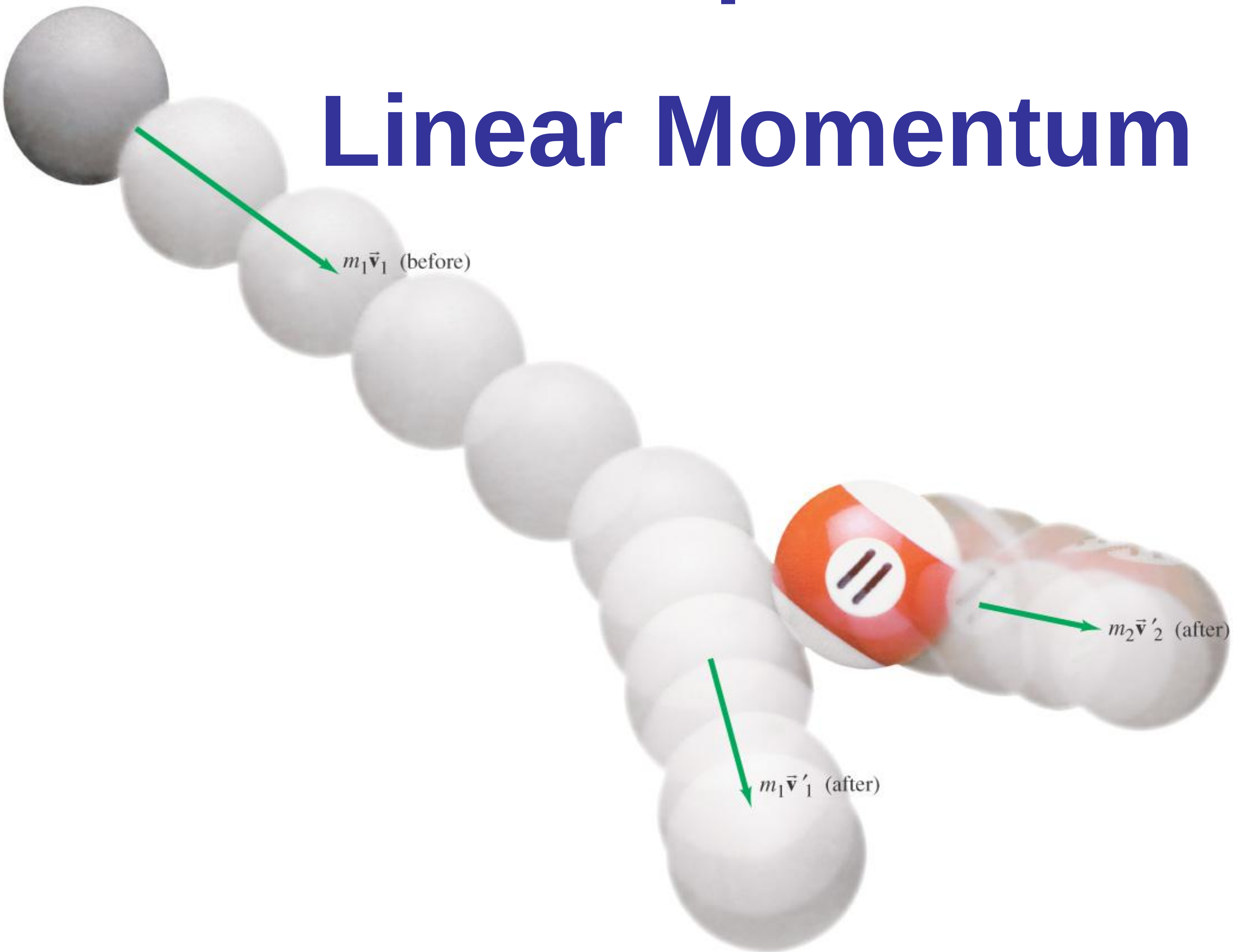
Chapter 7

Linear Momentum



Units of Chapter 7

- Momentum and Its Relation to Force
- Conservation of Momentum
- Collisions and Impulse
- Conservation of Energy and Momentum in Collisions
- Elastic Collisions in One Dimension

Units of Chapter 7

- **Inelastic Collisions**
- **Center of Mass (CM)**
- **CM for the Human Body**
- **Center of Mass and Translational Motion**
- **Torque**

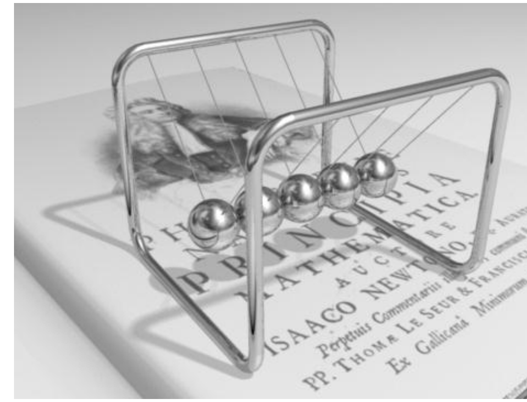
Momentum

- The sports announcer says, "Going into the all-star break, the Chicago White Sox have the momentum." The headlines declare "Chicago Bulls Gaining Momentum." The coach *pumps* up his team at half-time, saying "You have the momentum; the critical need is that you use that momentum and bury them in this third quarter."

Momentum in Sports

- **Momentum** is a commonly used term in sports. A team that has the momentum is on *the move* and is going to take some effort to stop. A team that has a lot of momentum is really *on the move* and is going to be *hard to stop*.
- Momentum refers to the quantity of motion that an object has.

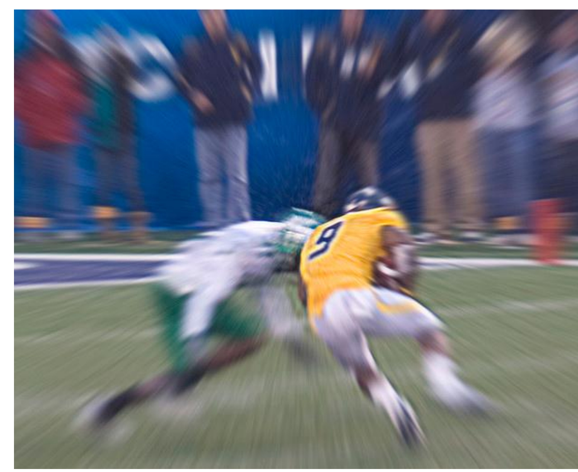
Momentum

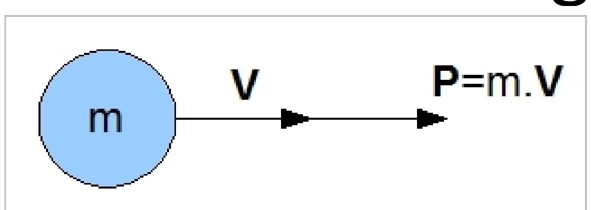


- Momentum can be defined as "**mass in motion**.“
- The **momentum** of an object is equal to the **mass** of the object times the **velocity** of the object.
- **Momentum = mass x velocity**

$$\vec{p} = m\vec{v}$$

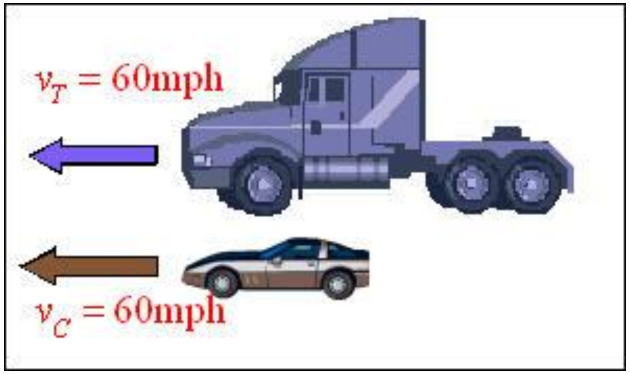
- p - momentum
- Units of momentum   kg x m/s



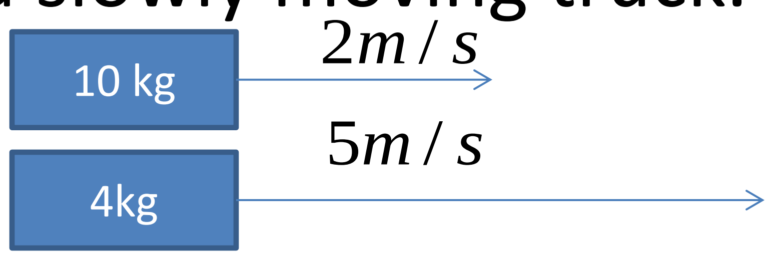


Momentum, Mass and Speed

- An object has a large momentum if either its **mass** or its **velocity** is large.



- If a car and a truck move at the same speed, the truck has **more momentum** since it has **larger** mass.
- But a fast car can have more momentum than a slowly moving truck.



$$\vec{p} = 20 \text{ kg m/s}$$

$$\vec{p} = 20 \text{ kg m/s}$$

- And the momentum of a truck at rest = 0.

Momentum and Direction

- Momentum is a **vector quantity**.
- It has magnitude and direction.
- The direction of the momentum vector is the same as the direction of the velocity of the ball.
- What is the momentum of a 2-kg bowling ball moving eastward at 5 m/s?
- p=10 kg x m/s, eastward

Practice Problems

- 1. Determine the momentum of a ...
- a. 60-kg halfback moving eastward at 9 m/s.
- b. 1000-kg car moving northward at 20 m/s.
- c. 40-kg freshman moving southward at 2 m/s.
- a. $p = mv = (60kg)(9m/s) = 540$ kg m/s, eastward
- b. $p = mv = (1000kg)(20m/s) = 20000$ kg m/s, northward
- c. $p = mv = (40\ kg)(2\ m/s) = 80$ kg m/s, southward

2. A car possesses 20 000 units of momentum. What would be the car's new momentum if ...

a. its velocity was doubled.

b. its velocity was tripled.

c. its mass was doubled (by adding more passengers and a greater load)

d. both its velocity was doubled and its mass was doubled.

a. p = 40,000 kg m/s

b. p = 60,000 kg m/s

c. p = 40, 000 kg m/s

d. p = 80,000 kg m/s

Questions:

- 1. Define momentum.
- 2. What letter represents momentum?
- 3. What units of measurement does momentum have?
- 4. Is momentum a vector or scalar quantity?
- 5. How can you increase the momentum of an object?
- 6. What has a greater momentum a 2.0-kg cart moving at a speed of 5.0 m/s, or a 1.0- kg cart moving at a speed of 8.0 m/s.

Conservation Law

Momentum: $\vec{p} = m\vec{v}$

Closed isolated system: No particles enter or leave

No net external force

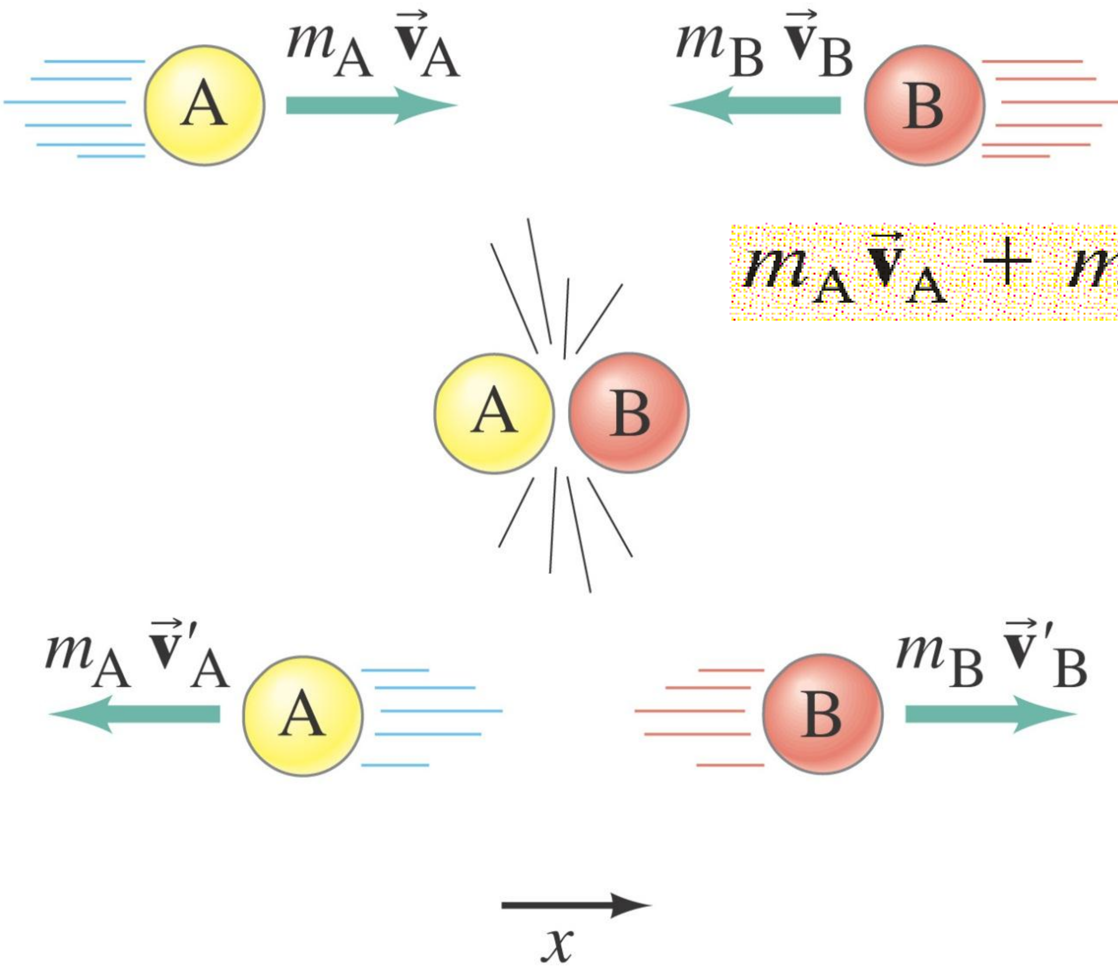
In a closed isolated system the total momentum is constant

$$\sum \vec{p} = const$$

$$\vec{p}_{total} = const$$

Conservation of Momentum

During a collision, measurements show that the total momentum does not change:



$$m_A \vec{v}_A + m_B \vec{v}_B = m_A \vec{v}'_A + m_B \vec{v}'_B$$

(7-3)

Conservation of Momentum

$$m_A \vec{\mathbf{v}}_A + m_B \vec{\mathbf{v}}_B = m_A \vec{\mathbf{v}}'_A + m_B \vec{\mathbf{v}}'_B$$

$$\vec{p}_A + \vec{p}_B = \vec{p}'_A + \vec{p}'_B$$

$$\vec{p}_B - \vec{p}'_B = \vec{p}'_A - \vec{p}_A$$

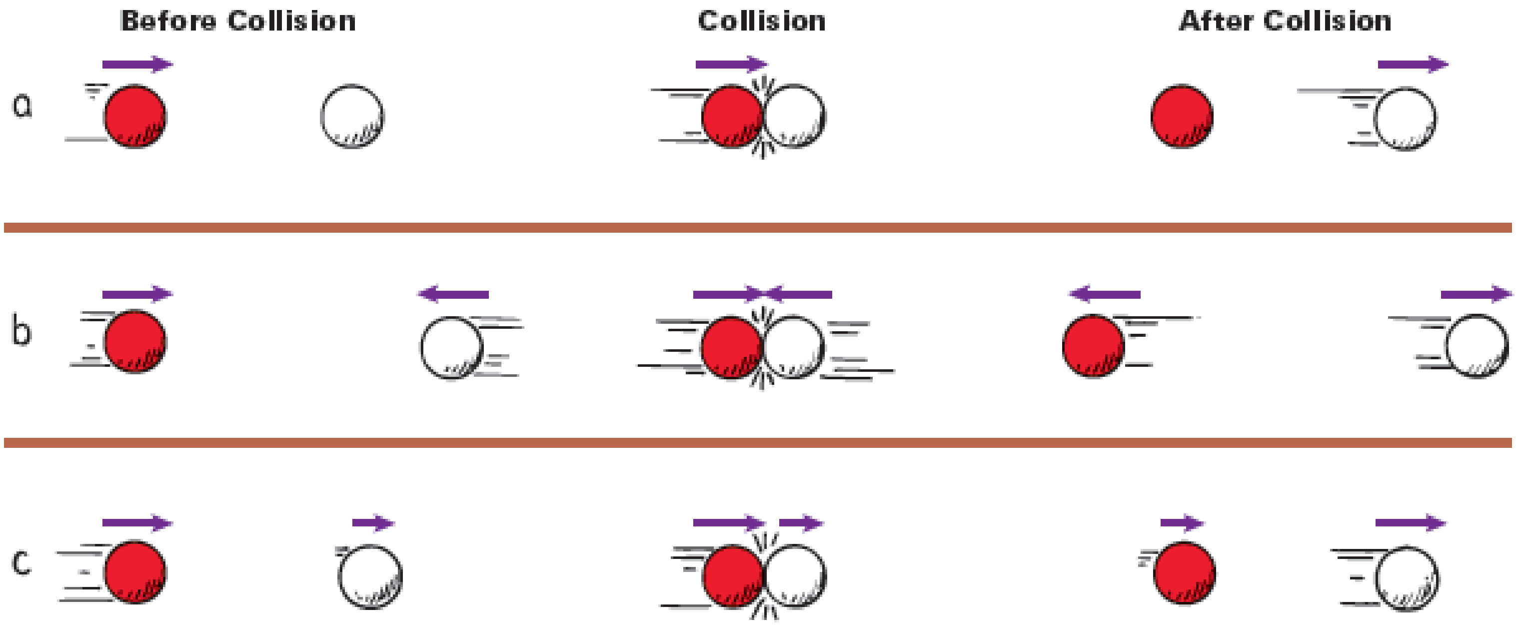
$$-(\vec{p}'_B - \vec{p}_B) = \vec{p}'_A - \vec{p}_A$$

$$-\Delta\vec{p}_B = \Delta\vec{p}_A$$

Momentum is transferred.

Conservation of Momentum

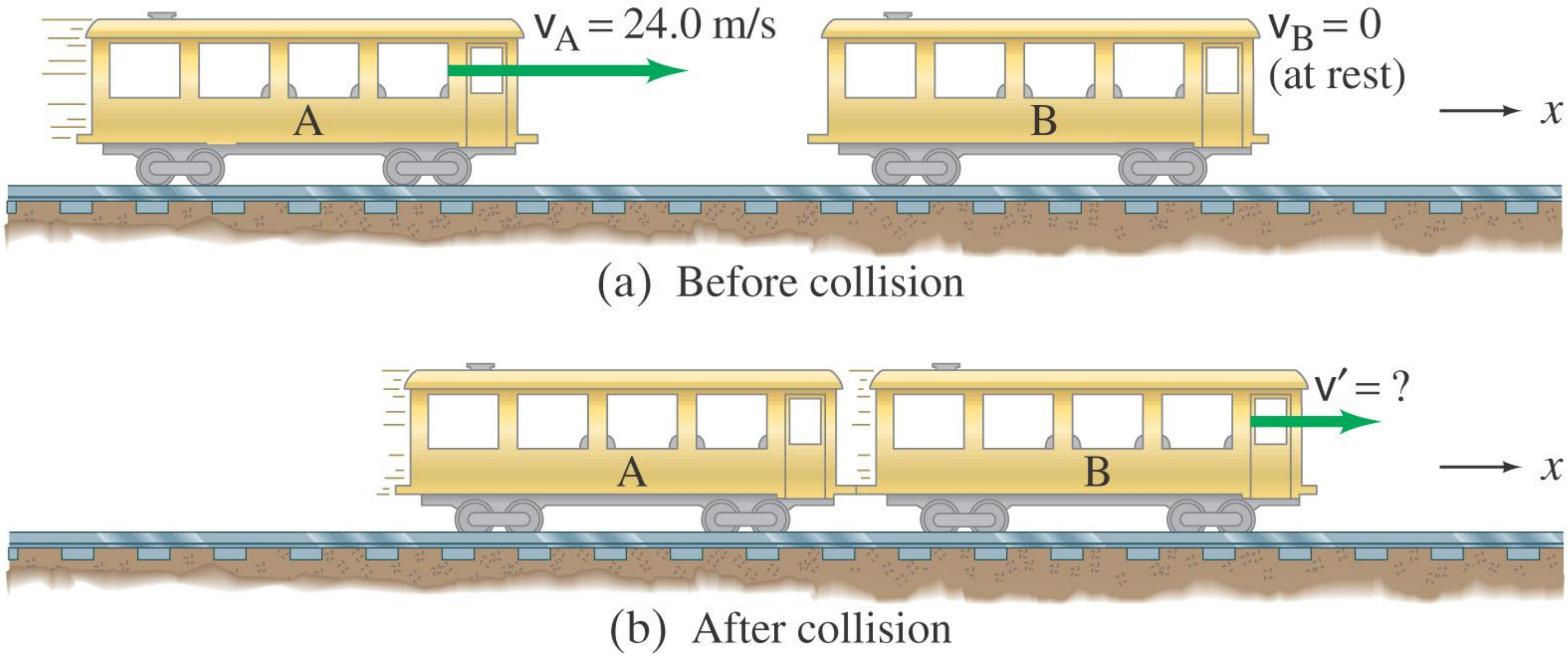
During a collision, the reason that the total momentum is conserved is that momentum is transferred from one object to the other!



7-2 Conservation of Momentum

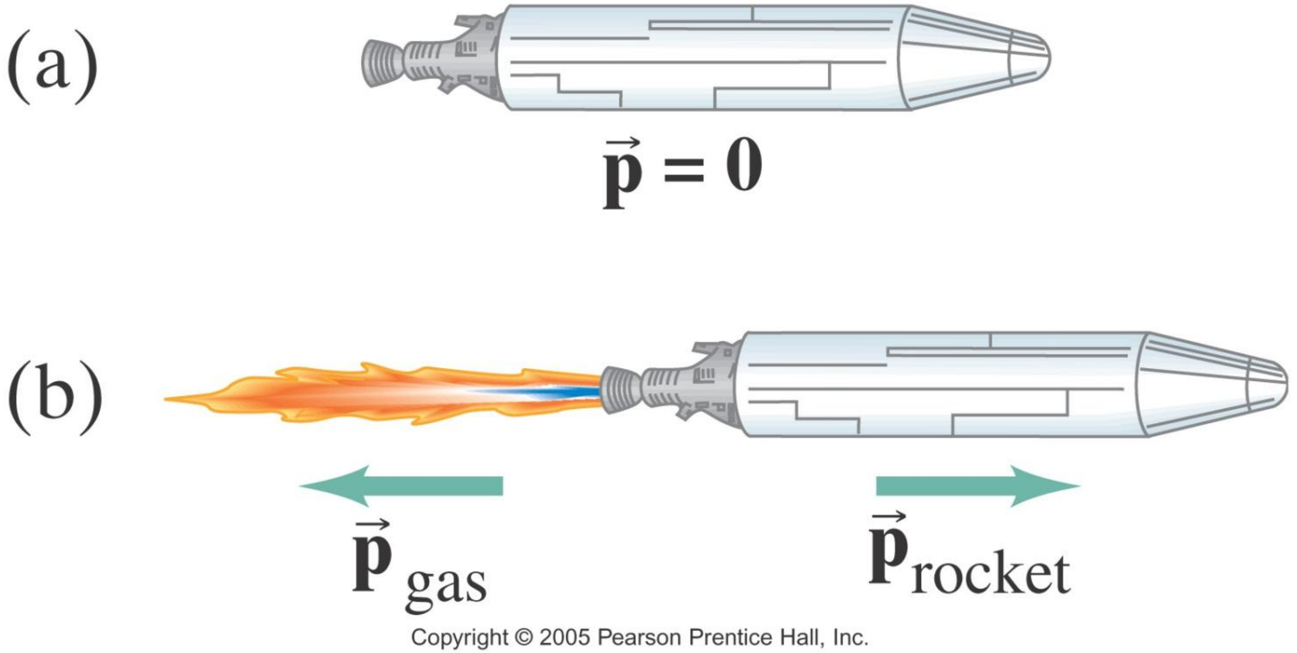
More formally, the law of **conservation** of momentum states:

The total momentum of an isolated system of objects remains constant.



7-2 Conservation of Momentum

Momentum conservation works for a **rocket** as long as we consider the rocket and its **fuel** to be one system, and account for the **mass loss** of the rocket.



Demonstration - Impulse and Change in Momentum

1. Describe the velocity of the cart after you have given it a push.

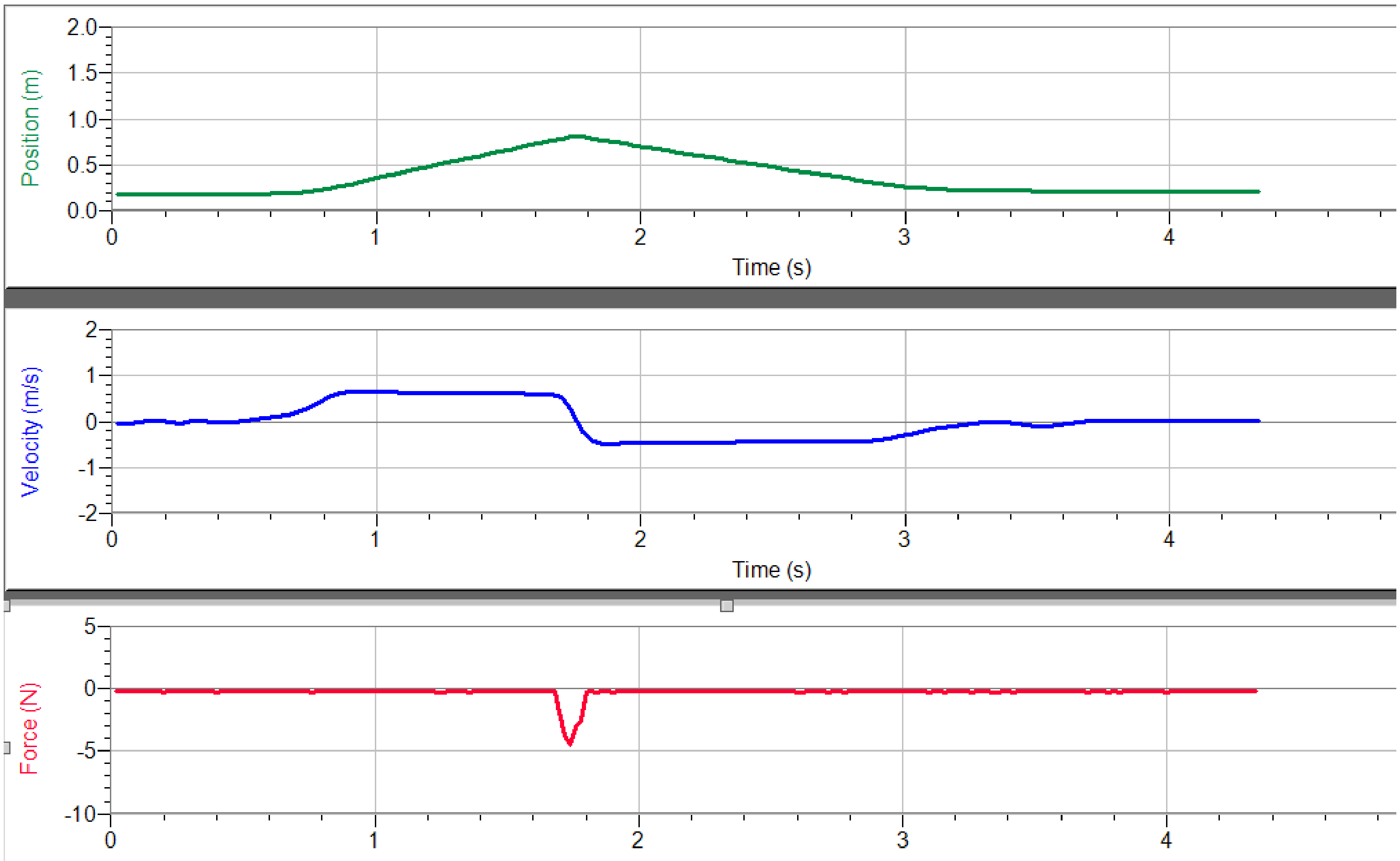
Neglecting friction, the cart will roll at constant velocity.

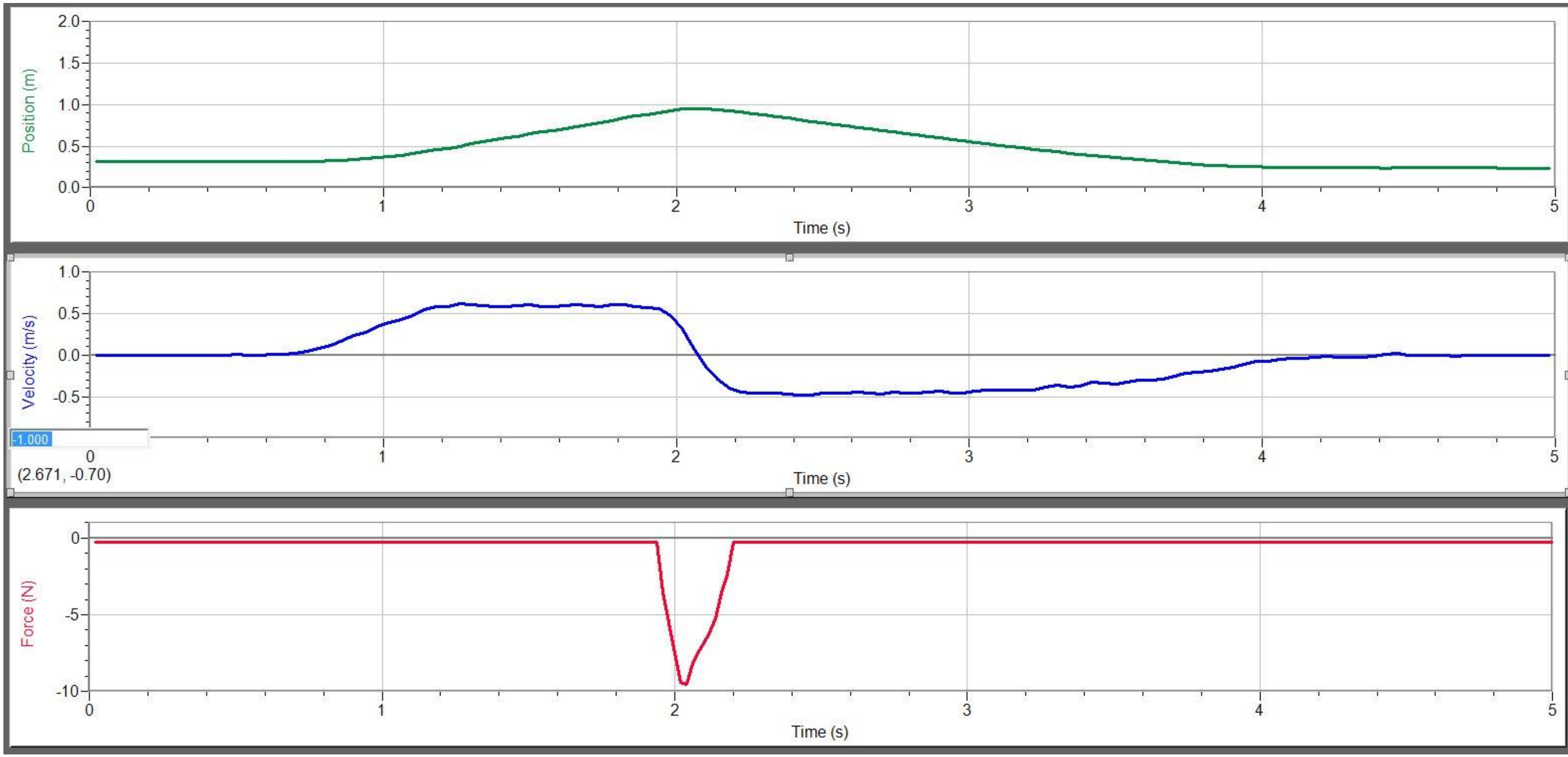
2. How the velocity will change when its plunger strikes the fixed force sensor.

The cart will reverse direction.

The cart will accelerate smoothly while the plunger is in contact with the sensor and return to its original position with a relatively constant, lower speed.

Increasing Momentum





2.0
1.5
1.0
0.5
0.0
Position (m)
0
1
2
3
4
5
Time (s)
1.0
0.5
0.0
-0.5
Velocity (m/s)
-1.000
(2.671, -0.70)
Time (s)
0
-5
-10
Force (N)
Time (s)

Impulse- Momentum Change Theorem

$$\vec{F}_{net} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t}$$

$$\Delta t \times \vec{F}_{net} = m\frac{\Delta\vec{v}}{\Delta t} \times \Delta t$$

$$\vec{F}_{net}\Delta t = m\Delta\vec{v}$$

$$\vec{F}_{net}\Delta t = \Delta\vec{p}$$

Impulse = change in momentum

The quantity ***force x time*** is called impulse.

7-3 Collisions and Impulse

During a collision, objects are **deformed** due to the large forces involved.

Since $\vec{\mathbf{F}} = \dfrac{\Delta \vec{\mathbf{p}}}{\Delta t}$, we can write

$$\vec{\mathbf{F}}\,\Delta t = \Delta \vec{\mathbf{p}} \qquad \text{(7-5)}$$

The definition of impulse:

$$\text{Impulse} = \vec{\mathbf{F}}\,\Delta t$$

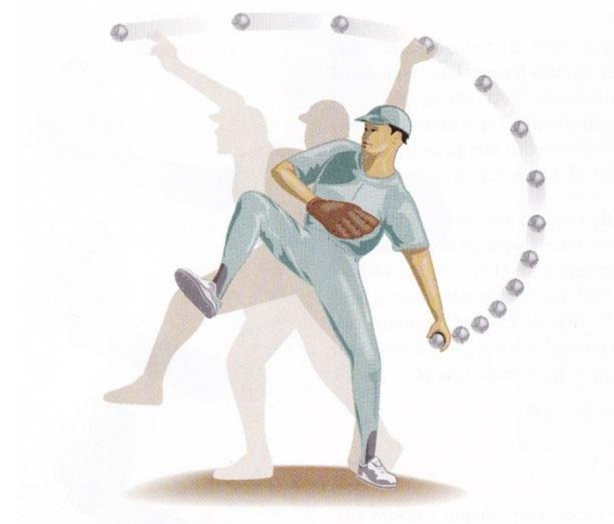
$$\vec{J} = \vec{F}\Delta t$$

$$\vec{J} - impulse$$

Impulse

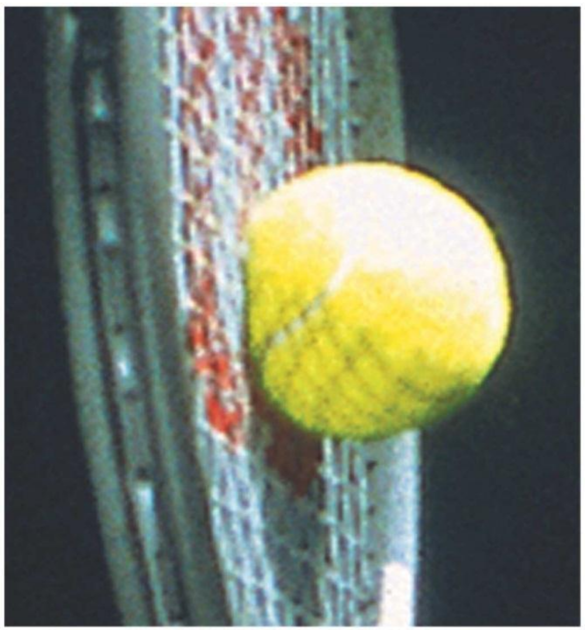
- To change an object's momentum requires an outside force which will must act for some time period.
- $J = Ft = \Delta p = p_{final} - p_{initial}$
- So to cause a greater change in an object's momentum you need either apply a bigger force or apply this force for a longer period of time or both.
- This idea has applications in the world of sports and also the world of firearms.

Increasing Momentum



- The pitcher exerts a force on a ball over as long a distance and for as time as possible.
  The greater Ft, the larger Δp, higher the release speed.
- The pitcher exerts a force on a ball over as long a distance and for as long as possible.
  Hitting a ball.
- *In order to increase the momentum of the ball off the tee the most, the hitter should follow through when hitting the ball off the tee.*

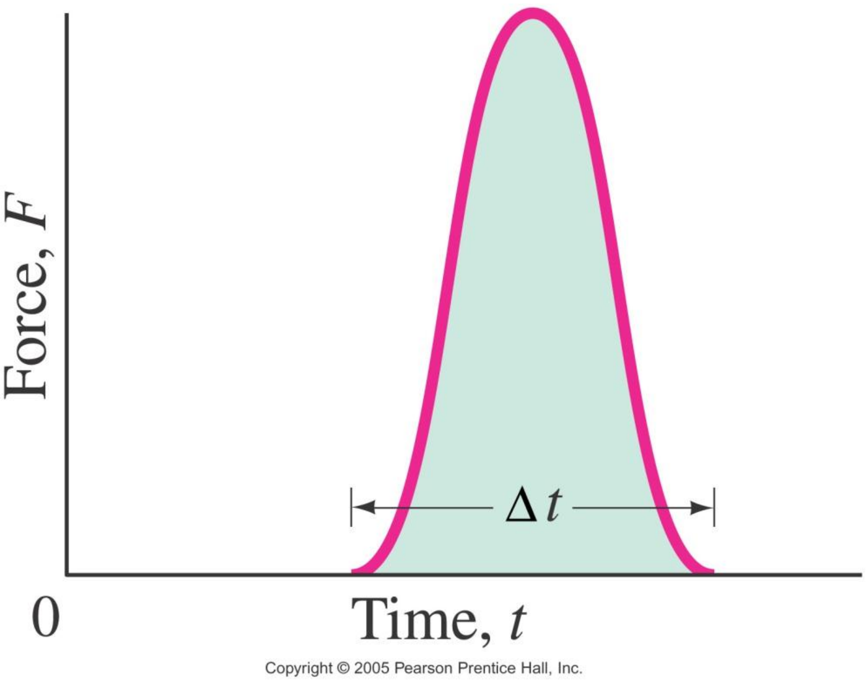
Collisions and Impulse

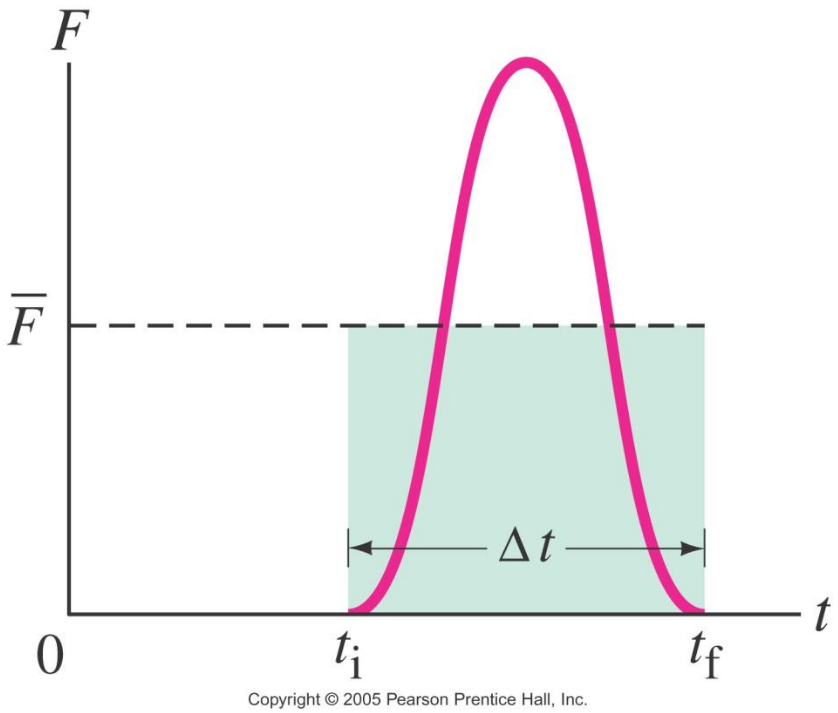


- During collision, objects deform due to the forces involved.
- The impact force changes during the contact time, so we will speak about average force of impact.
- Impact – force
- Impulse – impact force x time

7-3 Collisions and Impulse

Since the time of the collision is very short, we need not worry about the exact time dependence of the force, and can use the average force.

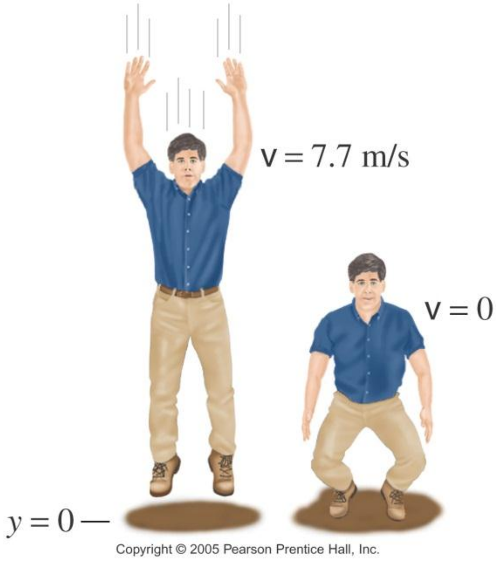




7-3 Collisions and Impulse

The **impulse** tells us that we can get the same **change** in momentum with

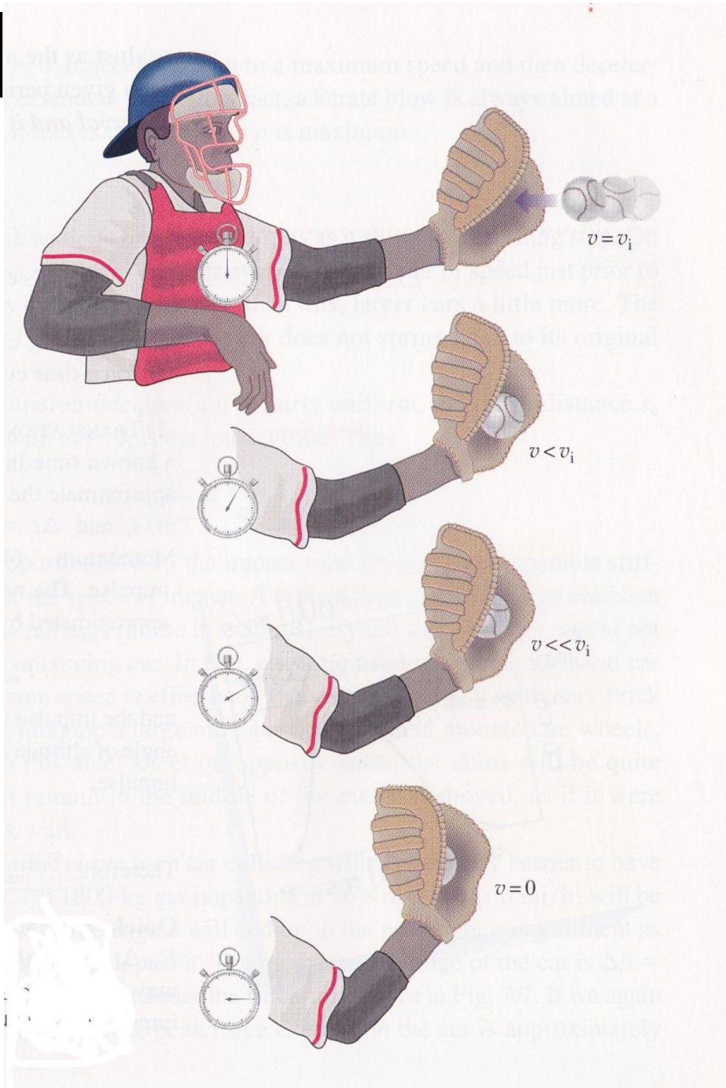
(LARGE FORCE x short time) **large** force acting for a **short** time,
or
(**small** force x LONG TIME) small force acting for a **longer** time.



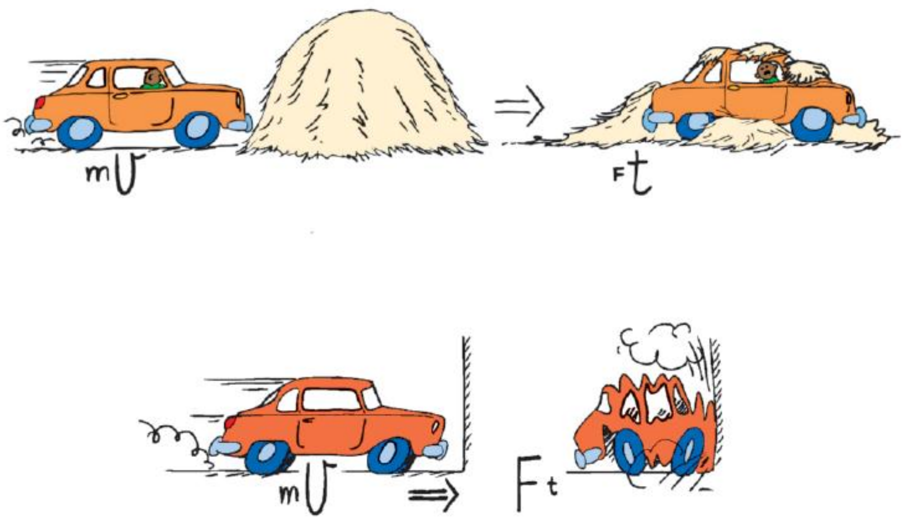
This is why you should **bend** your knees when you land; why **airbags** work; and why landing on a **pillow** hurts less than landing on **concrete**.

Catching Baseball

- For a given change in momentum, the longer the force acts (t), the smaller it will have to be.
- Your hand provides stopping force for a baseball. The momentum changes to 0.
- If you draw the glove back toward you, time will be longer and the stopping force smaller .



Decreasing Momentum



- By hitting a stack instead of the wall, you extend the impact time – time during which the momentum is brought to 0.
- The longer impact reduces the force of impact.

Impulse Example

a. What is the momentum of a 8-kg bowling ball rolling at 2 m/s?

b. If the bowling ball rolls into a pillow and stops in 0.5 s, calculate the average force the pillow exerts on the ball.

c. What average force does the ball exert on the pillow?

a. $p = mv = (8kg)(2m/s) = 16$ kg m/s

b. $Ft = \Delta p$ $\Delta p = 0-16=-16$ kg m/s

$F = \Delta p/t = -16/0.5= -32N$

c. 32N

Practice Problem

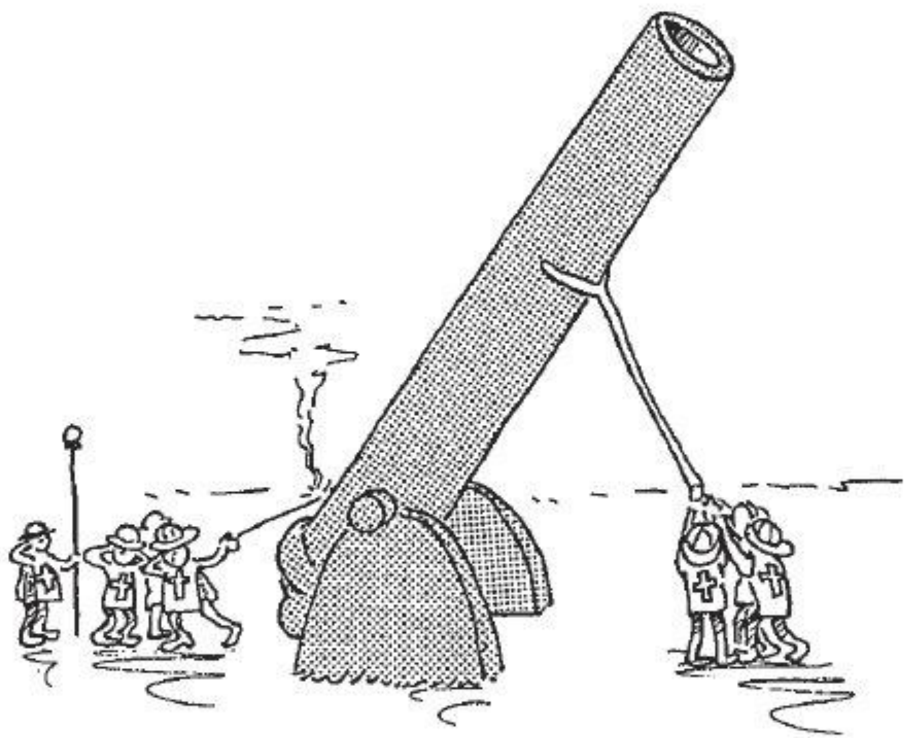
A 100-kg car, moving at 50 m/s is slowed to a stop in 25.0 s. What was the impulse applied to the car? What force was required?

$Ft = \Delta p$

$\Delta p = p_{final} - p_{initial} = 0 - mv = 0 – 100(50) = -5000$ kg m/s

$F = \Delta p/t = -5000/25.0 = -200N$

FOR THE SAME FORCE, WHY IS THE SPEED OF A CANNONBALL GREATER WHEN SHOT FROM A CANNON WITH A LONGER BARREL?



Question:

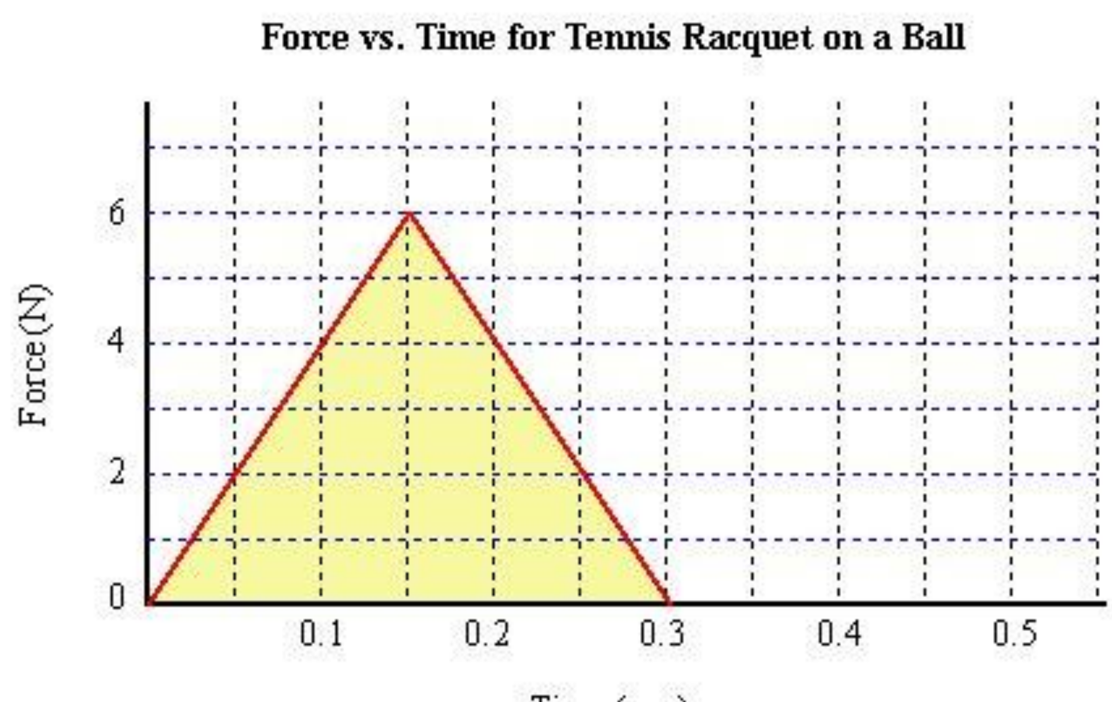
- You can’t throw a raw egg against a wall without braking it, but you can throw it at the same speed into a sagging sheet without it. Explain.

Questions:

1. Define impulse.
2. State the impulse-momentum theorem.
3. How can you increase the change in momentum of an object? Where does that apply?
4. How can you decrease the impact force on an object? Where does that apply?

Do Now

A 0.05 kg. tennis ball is moving to the left at 5 m/s when it is hit by a tennis racquet. The force vs. time graph for this collision is shown to the right. After the collision with the racquet, the ball is moving to the right with what velocity?



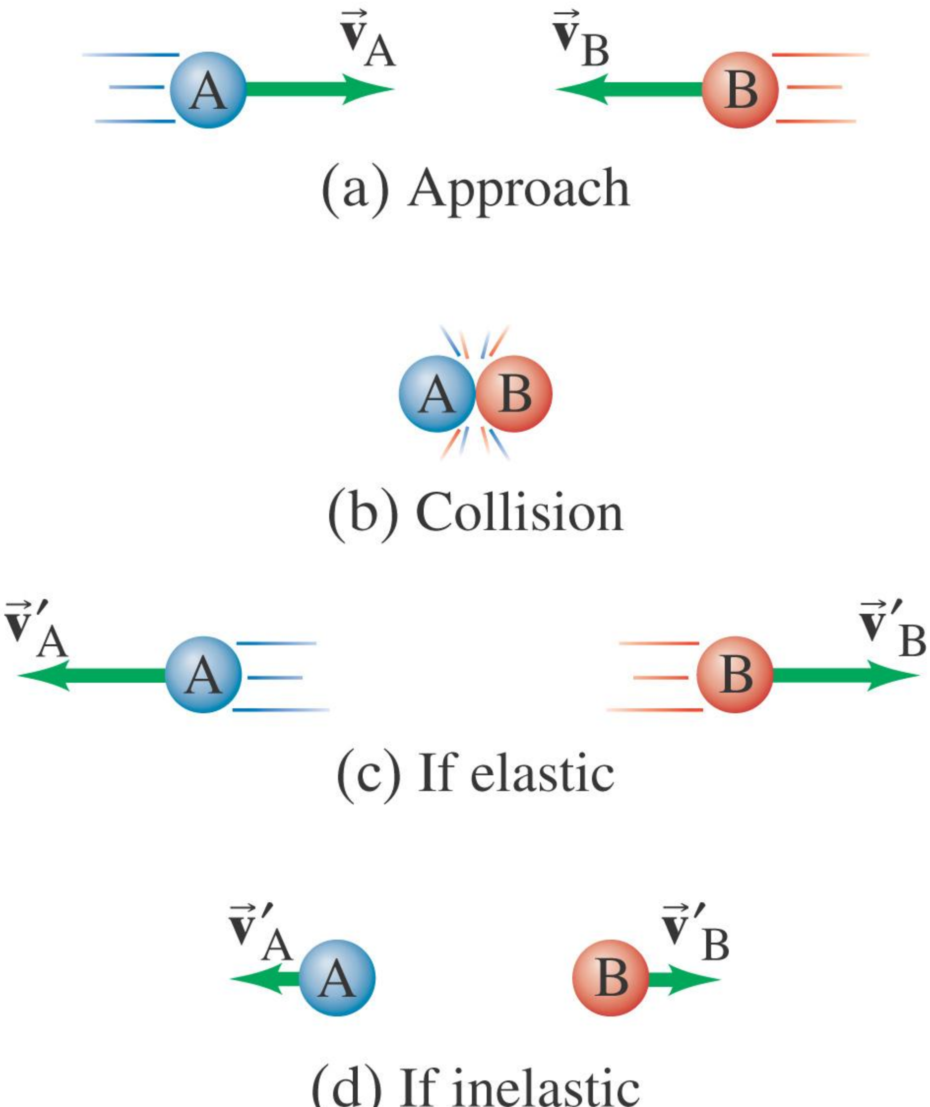
Questions:

1. Newton's Second law states that if no net force is exerted on the system, no acceleration occurs. Does it follow that no change in momentum occurs?
2. Newton's Third Law states that the force a cannon exerts on a cannonball is equal and opposite to the force the cannonball exerts on a cannon. Does it follow that

$$\vec{F}_{ConCB} = -\vec{F}_{CBonC}$$

$$\vec{J}_{ConCB} = -\vec{J}_{CBonC}$$

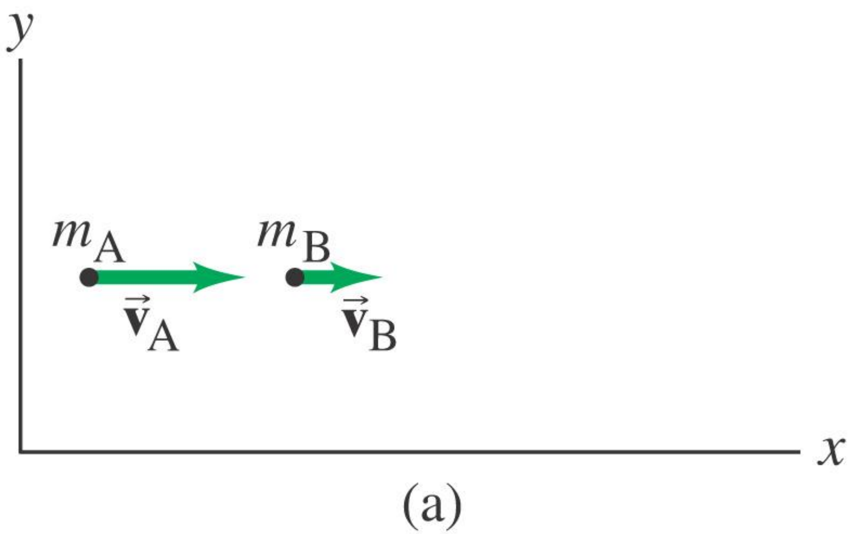
7-4 Conservation of Energy and Momentum in Collisions



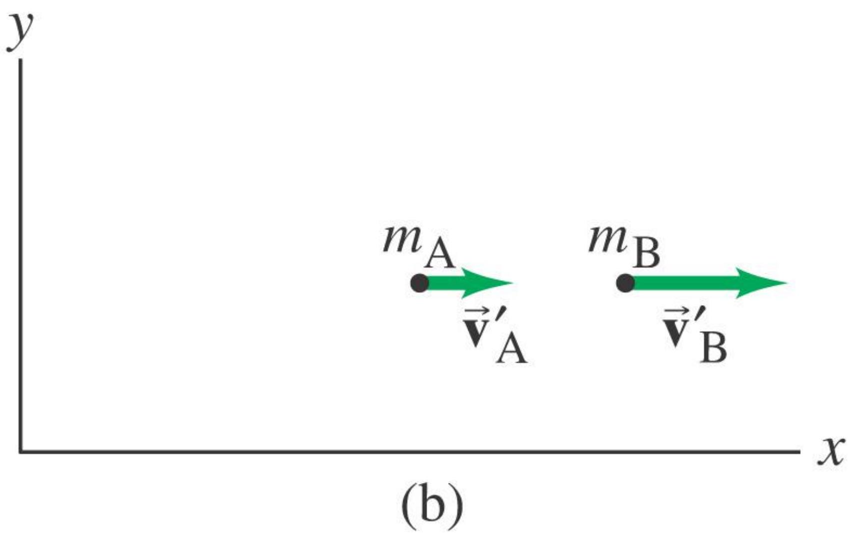
Momentum is **conserved in all collisions**.

Collisions in which **kinetic energy** is conserved as well are called **elastic** collisions, and those in which it is not are called **inelastic**.

7-5 Elastic Collisions in One Dimension



(a)

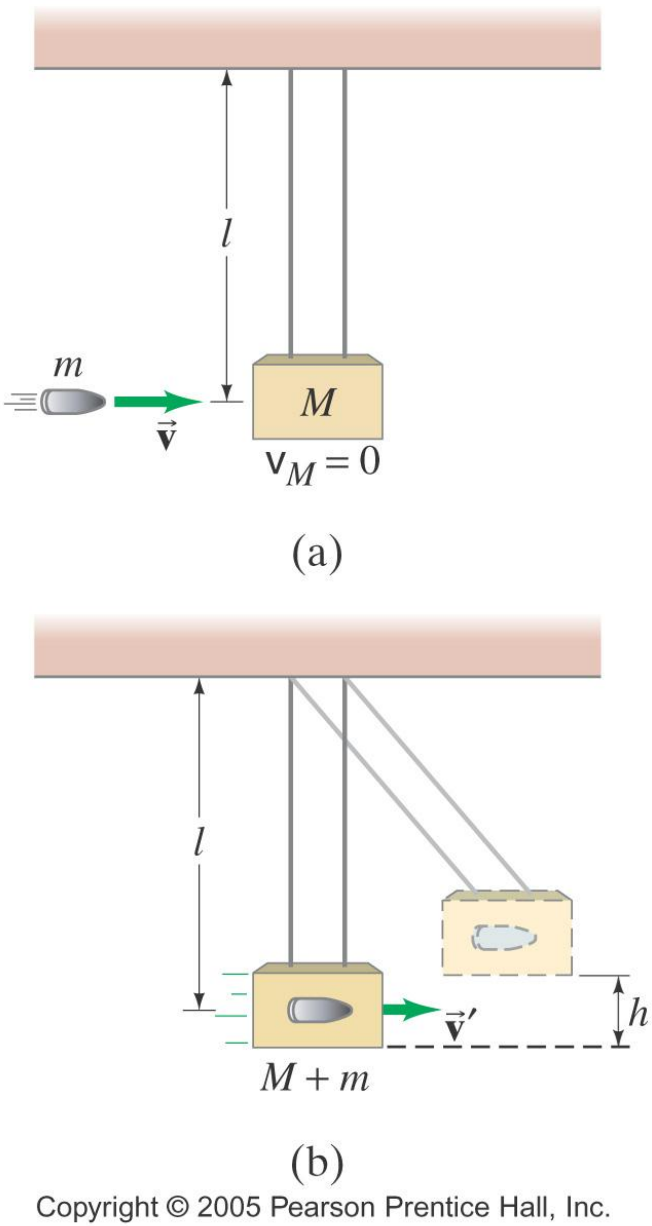


(b)

Here we have two objects colliding **elastically**. We know the masses and the initial speeds.

Since both **momentum** and **kinetic energy** are conserved, we can write **two** equations. This allows us to solve for the **two** unknown final speeds.

7-6 Inelastic Collisions

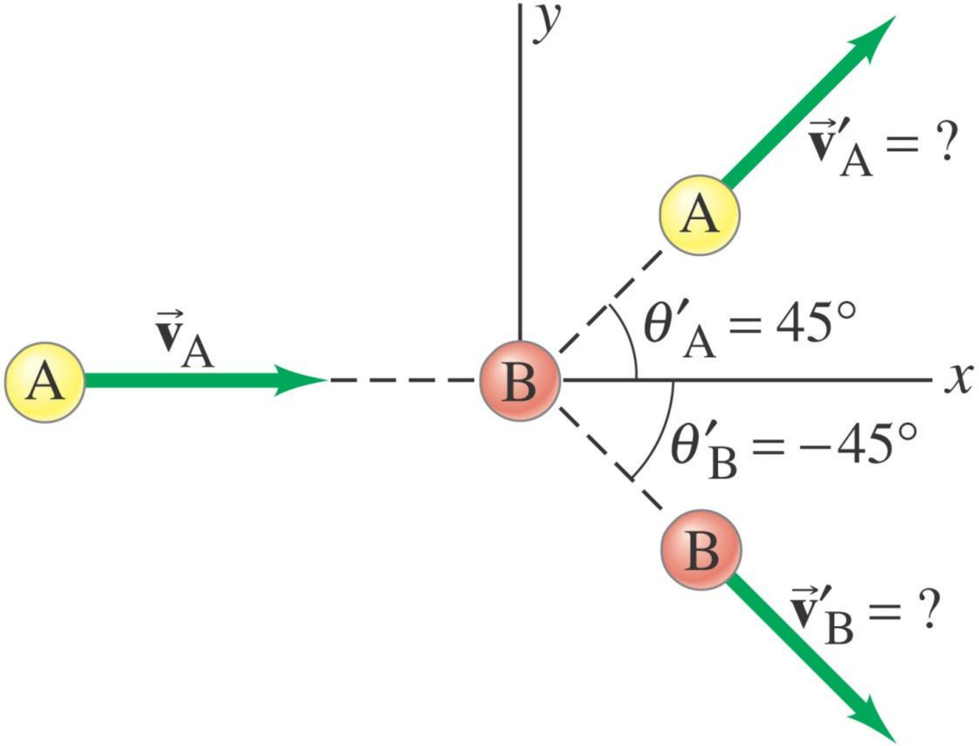


With **inelastic** collisions, some of the initial kinetic energy is lost to **thermal** or **potential** energy. It may also be gained during **explosions**, as there is the addition of **chemical** or **nuclear** energy.

A **completely inelastic** collision is one where the objects **stick together** afterwards, so there is only one final velocity.

7-7 Collisions in Two or Three Dimensions

Conservation of energy and momentum can also be used to analyze collisions in two or three dimensions, but unless the situation is very simple, the math quickly becomes unwieldy.



Here, a moving object collides with an object initially at rest. Knowing the masses and initial velocities is not enough; we need to know the angles as well in order to find the final velocities.

7-7 Collisions in Two or Three Dimensions

Problem solving:

1. **Choose the system. If it is complex, subsystems may be chosen where one or more conservation laws apply.**
2. **Is there an external force? If so, is the collision time short enough that you can ignore it?**
3. **Draw diagrams of the initial and final situations, with momentum vectors labeled.**
4. **Choose a coordinate system.**

7-7 Collisions in Two or Three Dimensions

5. Apply **momentum conservation**; there will be one equation for each dimension.

6. If the collision is **elastic**, apply conservation of **kinetic energy** as well.

7. **Solve**.

8. **Check** units and magnitudes of result.