CHAPTER 7 NOTES
EXPONENT PROPERTIES REVIEW

Product of Powers: **Multiplying** numbers or variables with the same base	**Product of Powers:** **WHEN YOU SEE MULTIPLICATION, ADD!**	▪ Each common base stays the same and moves directly to the answer. Then exponents are added. ▪ Numbers that do not have a common base are multiplied like normal.
1. $(-3)^4 \cdot (-3)^2$	**2.** $x^3 \cdot x^2$	****3.** $7^{1/2} \bullet 7^{3/2}$
Power of a Power: **Raising a power to another power**	**Power of a Power:** **MULTIPLY A POWER TO A POWER!**	▪ Identify the base and the exponent. Immediately put base into the answer. ▪ Multiply the exponent with the exponent and attach it to the base. ▪ Simplify your answer as much as possible.
4. $(a^2)^4$	**5.** $(3^2)^6$	****6.** $(7^{2/3})^{3/4}$
Quotient of Powers: **Dividing numbers or variables with the same base**	**Quotient of Powers:** **DIVISION IS SUBTRACTION!**	▪ Each common base stays the same and goes to the **numerator** of the answer. Then, the exponents are subtracted. ▪ Numbers that do not have a common base are divided like normal. If they do not divide evenly, reduce them like a fraction.
7. $\frac{x^3}{x^{-2}}$	**8.** $\frac{2x^7}{x^5}$	****9.** $\frac{7^{2/3}}{7^{1/3}}$

<u>Negative</u> Exponents **Flip**	**<u>Negative Exponents</u>:** **Flip the Base when the exponents are negative and make the exponents positive!**	▪ Implies that the base it is attached to must change position from the numerator to the denominator or vice versa ▪ Once the position change occurs, the exponent goes back to being positive
10. $(xy^{-5})^{-3}$	**11.** $\left(\frac{3}{4}\right)^{-2}$	****12.** $(x^{5/6})^{-2}$

ZERO EXPONENTS: Anything raised to the zero power is ALWAYS 1.

Simplify using exponent properties.

13. $x^{\frac{1}{3}} \cdot x^{\frac{1}{2}}$	**14.** $(x^4)^3$
15. $\frac{2^1}{2^{\frac{-1}{5}}}$	**16.** $(54x^4y)^{\frac{1}{3}}$
17. $\frac{11^{\frac{4}{5}}}{11^{\frac{2}{5}}}$	**18.** $\left(54x^4y^{\frac{1}{4}}\right)^0$

19. $\left(7^{\frac{4}{3}}\right)^{\frac{5}{4}}$	**20.** $\dfrac{13^{\frac{3}{7}}}{13^{\frac{5}{7}}}$
21. $\left(\dfrac{x^3}{y^5}\right)^2$	**22.** $\dfrac{3^{5/6}}{3^{1/3}}$
23. $\left(\dfrac{a^9}{b^6}\right)^{\frac{1}{3}}$	**24.** $(36m^4n^{10})^{1/2}$

7.1 Evaluating Rational Exponents on the Calculator

Rational Exponent: We have a rational exponent when there is a fraction in the exponent position. What we'll find is that we can take a fraction in the exponent position and very easily convert it into a radical.

To evaluate the expressions with a calculator:
When you enter in that rational exponent, make sure you put it in as a fraction in parenthesis. As always, all negative bases should be put in parenthesis.

Expression	Keystrokes	Display
a. $9^{1/5}$	9 ^ (1 ÷ 5) ENTER	1.551845574
b. $12^{3/8}$	12 ^ (3 ÷ 8) ENTER	2.539176951

Evaluate the following using your calculator. NO DECIMAL ANSWERS!

25. $49^{\frac{1}{2}}$	**26.** $27^{-\frac{2}{3}}$	**27.** $25^{\frac{3}{2}}$	**28.** $8^{\frac{5}{3}}$

7.1 Radicals with a Larger Index: Simplifying & Solving

Radical: We know radicals as square roots. But really, radicals can be used to express any root: $\sqrt[3]{8}, \quad \sqrt[4]{48}, \quad \sqrt[10]{256}$	**Index:** The index tells us exactly what type of root that it is. To determine whether we are looking for pairs in a square root, or sets of three of something in a cube root, the index tells us the size of the group that we are looking to pull out.

Simplifying radical expressions: After determining the index, use a factor tree looking for groups of whatever the index is. Let's look back at an example with square roots!

1. $\sqrt{112}$ **Index:** ______

Groups of ____

2. $\sqrt{48}$ **Index:** ______

Groups of ____

3. $\sqrt[3]{250}$ **Index:** ______

Groups of ____

4. $\sqrt[5]{128}$ **Index:** ______

Groups of ____

VOCABULARY

NTH ROOT	the nth root of a where a is a REAL number and n is the index. $$\sqrt[n]{a}$$					
NUMBER OF REAL NTH ROOTS	**N**	**N is ODD (3, 5, . . .)**	**N is EVEN (2, 4, 6, . . .)**			
	a	Any real #	Greater than 0 (positive)	0	Less than 0 (negative)	
	# real nth roots	ONE	TWO	ONE	NONE (imaginary)	
	Examples	$\sqrt[3]{8}=2; \sqrt[3]{-1}=-1$	$\sqrt[4]{16}=2; -\sqrt[4]{16}=-2$	$\sqrt{0}=0; \sqrt[4]{0}=0$	$\sqrt[4]{-20}$, no real root	

Solve the equations using nth roots.

5. $3x^4 = 768$

Index: _______

 # Answers: ONE TWO NONE

6. $(x-2)^3 = 16$

Index: _______

 # Answers: ONE TWO NONE

7. $6x^3 = 384$

Index: _______

 # Answers: ONE TWO NONE

8. $8(x+8)^4 = 128$

Index: _______

 # Answers: ONE TWO NONE

7.2 Evaluate Roots and Rational Exponents

Use a calculator to evaluate the following expressions. Circle if each is a radical expression or rational exponent!

1. $\sqrt{16} =$ **Radical Expression** **Rational Exponent**	**2.** $\sqrt[3]{125} =$ **Radical Expression** **Rational Exponent**	**3.** $64^{1/3} =$ **Radical Expression** **Rational Exponent**
4. $25^{3/2} =$ **Radical Expression** **Rational Exponent**	**5.** $\left(\sqrt[3]{27}\right)^2 =$ **Radical Expression** **Rational Exponent**	**6.** $32^{-1/5} =$ **Radical Expression** **Rational Exponent**
7. $-\sqrt[4]{81} =$ **Radical Expression** **Rational Exponent**	**8.** $8^{4/3} =$ **Radical Expression** **Rational Exponent**	**9.** $144^{1/2} =$ **Radical Expression** **Rational Exponent**

VOCABULARY:

Radical: We know radicals as square roots. But really, radicals can be used to express any root:

$$\sqrt[3]{8}, \quad \sqrt[4]{48}, \quad \sqrt[10]{256}$$

Index: The index tells us exactly what type of root that it is. To determine whether we are looking for pairs in a square root, or sets of three of something in a cube root, the index tells us the size of the group that we are looking to pull out.

Rational Exponent: We have a rational exponent when there is a fraction in the exponent position. What we'll find is that we can take a fraction in the exponent position and very easily convert it into a radical.

Every radical can be written as a base with a rational exponent. Look at the examples below. What you see is a radical turning into a base with a rational (fractional) exponent!!

$$\left(\sqrt[2]{25}\right)^1 = 25^{\frac{1}{2}} \qquad \left(\sqrt[3]{8}\right)^1 = 8^{\frac{1}{3}} \qquad \left(\sqrt[3]{x}\right)^2 = x^{\frac{2}{3}}$$

Look at the pattern above:

INDEX of the radical → **numerator** **OR** **denominator** of the rational exponent

EXPONENT with the radical → **numerator** **OR** **denominator** of the rational exponent

HOW DOES ONE CONVERT FROM RADICAL FORM $\left(\sqrt[2]{25}\right)^1$ **TO RATIONAL EXPONENT FORM** $25^{\frac{1}{2}}$ **?**

- ⌘ Take the base under the radical. If it is negative, put it in parenthesis first. The base will then be raised by a fraction created by:
- ⌘ The index of the radical becomes the denominator of the fraction. Think: index – denominator!
- ⌘ The exponent of the expression becomes the numerator of the fraction.

Rewrite the expression using rational exponent notation.		
10. $\left(\sqrt[5]{63}\right)^3$	**11.** $\left(\sqrt[3]{-7}\right)^4$	**12.** $\left(\sqrt[7]{-13}\right)^9$

Rewrite the expression using radical notation.		
13. $39^{\frac{7}{5}}$	**14.** $(-5)^{\frac{11}{6}}$	**15.** $(-24)^{\frac{4}{3}}$

APPLY PROPERTIES OF RADICALS

ADDING AND SUBTRACTING RADICALS	▪ You can add or subtract radicals as long as they have the same index and the same radicand (number underneath the radical). ▪ We will call these "like radicals" because much like "like terms" on the number in front of the radical will be affected, while the radicand will remain the same.	Before you begin any addition or subtraction problem, you should simplify each radical as much as possible. For example: $3\sqrt{20}+5\sqrt{45}$

$2\sqrt[3]{6}+7\sqrt[3]{6}=$	$10\sqrt[4]{2x}-3\sqrt[4]{2x}=$

$12\sqrt[4]{2}-7\sqrt[4]{32}$

MULTIPLYING RADICAL BY RADICALS	• You can multiply a radical by another radical as long as they both have the same index.	To multiply a radical by another radical, multiply the numbers in front, multiply the radicands, and keep the index the same. Then check your answer to see if anything can be pulled out. If not, it is in simplified form. $\left(6\sqrt[3]{3}\right)\left(2\sqrt[3]{7}\right)$

$5\sqrt[4]{64} \cdot 2\sqrt[4]{8} =$	$\sqrt[3]{40x} \cdot \sqrt[3]{5x^4} =$

7.3 Solving Radical Equations

VOCABULARY

Radical Equation	A radical equation is an equation that contains radicals with the variable in the radicand. EXAMPLE: $\sqrt{x+6} = 3$
Extraneous Solution	A solution that does not make the original equation true. YOU MUST CHECK ALL OF YOUR SOLUTIONS TO MAKE SURE THAT THEY ARE NOT EXTRANEOUS SOLUTIONS!
Solving Radical Equations	To solve a radical equation, follow these steps: **Step 1** **Isolate** the radical on __________ of the equation, if necessary. **Step 2** **Raise** each side of the equation to the same ________ to eliminate the radical. **Step 3** **Solve** the resulting __________ using techniques that you learned in previous chapters. **Step 4** **Check** your solution.

Solve each radical equation.

$\sqrt{x+6} = 3$ CHECK YOUR ANSWER(S):	$\sqrt{2x+1} - \sqrt{10-x} = 0$ CHECK YOUR ANSWER(S):

$\sqrt[3]{x-5}+1=-1$

CHECK YOUR ANSWER(S):

$(3x+4)^{2/3}=16$

CHECK YOUR ANSWER(S):

$x-2=\sqrt{x+10}$

CHECK YOUR ANSWER(S):

7.4 Function Operations and Composition Functions

VOCABULARY-REVIEW

Function Notation	You can write an equation in function form by replacing the y with an f(x) or another variable. This is read "f of x".	
Evaluating Functions (REVIEW)	**Evaluate the function for the given value of x.**	
	$f(x) = 6x$ Evaluate $f(2)$	$g(x) = 3x + 5$ Evaluate $g(-3)$

OPERATIONS ON FUNCTIONS

FIND THE FOLLOWING:

GIVEN: $f(x) = 2x + 3$ $g(x) = x - 5$

$f(x) + g(x)$	$f(x) - g(x)$
$g(x) + f(x)$	$g(x) - f(x)$

GIVEN: $f(x) = x^3$ $g(x) = 7x$

$f(x) \cdot g(x)$	$\frac{f(x)}{g(x)}$
$g(x) \cdot f(x)$	$\frac{g(x)}{f(x)}$

COMPOSITION OF FUNCTIONS	The composition of a function f with a function g is found by replacing each variable in f with the expression for g. The composition of f with g is written as $f(g(x))$ or $(f \circ g)(x)$. This is a process for which we substitute an entire function into another function.
COMPOSITION OF FUNCTIONS	The composition of a function f with a function g is found by replacing each variable in f with the expression for g. The composition of f with g is written as $f(g(x))$ or $(f \circ g)(x)$. This is a process for which we substitute an entire function into another function.

$f(x) = 6x$ $g(x) = 3x + 5$

Evaluate $f(g(2))$.	**COMPOSITION**: START ON THE INSIDE **STEPS:** 1) Find $g(2)$ 2) Substitute #1 in for x in the f function. 3) Simplify

$f(x) = 6x$ $\quad$ $g(x) = 3x + 5$	
Evaluate $g(f(2))$.	**COMPOSITION**: START ON THE INSIDE **STEPS:** 1) Find $f(2)$ 2) Substitute #1 in for x in the g function. 3) Simplify
Find $f(g(x))$.	**COMPOSITION**: START ON THE INSIDE **STEPS:** 1) Take the inside function $g(x)$ 2) Substitute #1 in for x in the f function. 3) Simplify (Your answer will be an expression) **Plug the g function into the f function!**
Find $g(f(x))$.	**COMPOSITION**: START ON THE INSIDE **STEPS:** 1) Take the inside function $f(x)$ 2) Substitute #1 in for x in the g function. 3) Simplify (Your answer will be an expression) **Plug the f function into the g function!**

Notice how the order of the functions MATTER when it comes to the final answer!

You go to a hair salon to buy hair care products and you have two coupons. If you can use both, does it make a difference which one you use first? Let's say the total before the coupons is $50.

$10 off first	15% off first

7.5 INVERSE FUNCTIONS

VOCABULARY

INVERSE RELATION

An inverse relation switches the input (x) and output (y) values.

Original relation

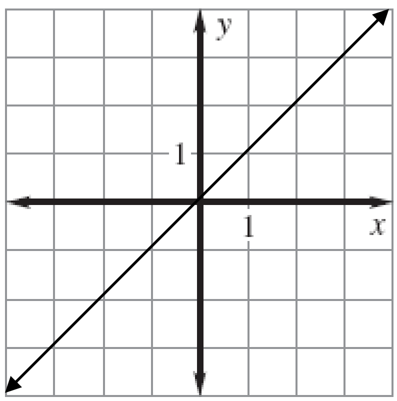
Input x	−2	−1	0	1	2
Output y	−4	−2	0	2	4

Inverse relation

Input x	−4	−2	0	2	4
Output y	−2	−1	0	1	2

You can see how the domain of the original is the range of the inverse and the range of the original is the domain of the inverse.

Graph both the original and the inverse relation on the following coordinate plane.



What is the equation of the line drawn on the graph?

INVERSE FUNCTIONS

If both the original relation and the inverse relation are FUNCTIONS, then the two functions are called inverse functions.

REVIEW: What makes a relation a function?

Find the inverse of each relation:

x	0	1	2	3	4
y	3	5	7	9	11

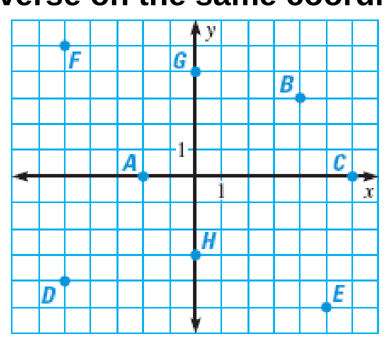
x					
y					

Is the original relation a function? ______

Is the inverse relation a function? ______

Are these inverse functions? _______

Plot the inverse on the same coordinate plane



Is the original relation a function? ______

Is the inverse relation a function? ______

Are these inverse functions? _______

FINDING INVERSE FUNCTIONS

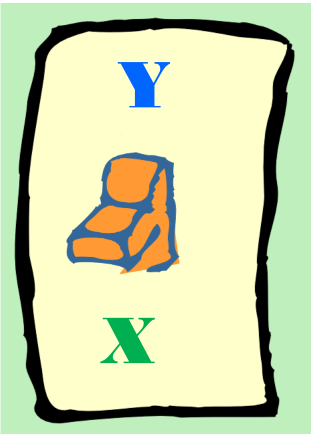
You can find the inverse of a function by following these steps.

Step 1 **Replace** $f(x)$ with ___ (if the function is written using __________ notation).

Step 2 **Switch** x and y.

Step 3 **Solve** for ___.

Find the inverse of each function. DON'T FORGET TO SWITCH X AND Y!!!



$$f(x) = -3x + 6$$

Find the inverse of the following functions.

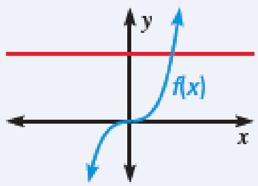
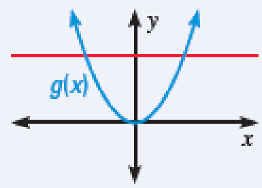
$f(x) = 2x + 10$

$f(x) = x^2 - 3$

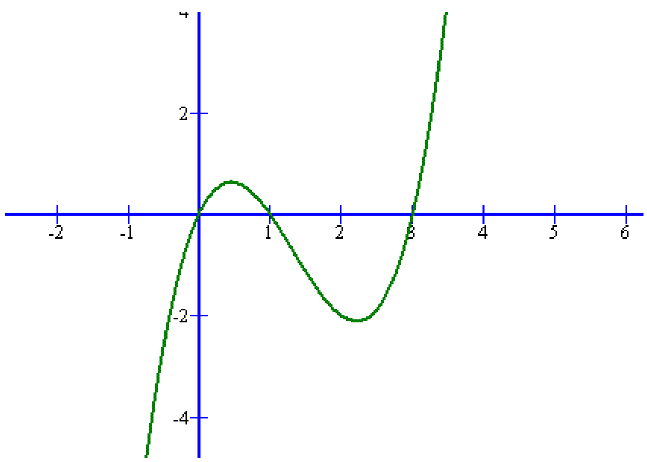
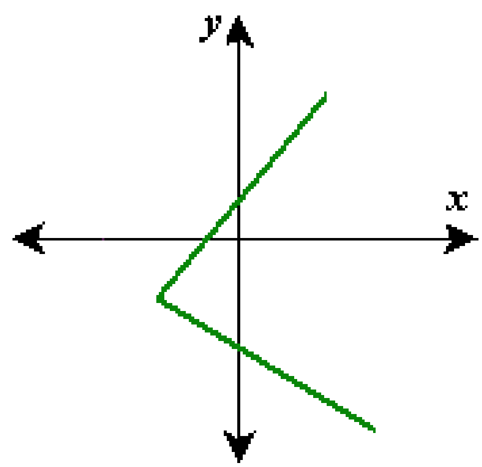
To determine if the graph of the inverse is a function, use the HORIZONTAL LINE TEST.
Rule:

Horizontal Line Test

If no horizontal line intersects the graph of a function *f* more than once, then the inverse of *f* is also a function. Consider the examples below.

The inverse of *f* is a function.	**The inverse of *g* is *not* a function.**
	
No horizontal line intersects the graph of *f* more than once.	**A horizontal line intersects the graph of *g* more than once.**

Determine if each graph is a function and then if the graph's inverse is a function.

 Is the graph a function? YES NO Is the inverse of the graph a function? YES NO	 Is the graph a function? YES NO Is the inverse of the graph a function? YES NO

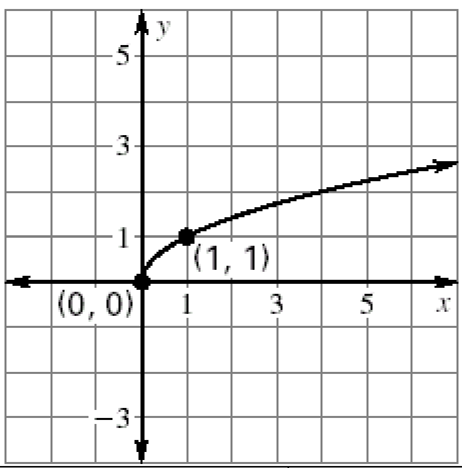
7.6 Graphing Square Root Functions

VOCABULARY

Radical Function	A radical function contains a radical with a variable in the radicand. EXAMPLE:

PARENT SQUARE ROOT FUNCTION

The most basic square root function in the family of all square root functions, called the ______________________, is y = $\sqrt{x}$. The graph of the parent square root function is shown.



x	y
0	
1	
2	
3	

$y = a\sqrt{x-h}+k$ **1.** $y = 3\sqrt{x+2}-1$ $y = \sqrt{x}$	***a* >1** causes a stretch ***a*** between 0 and 1 causes a shrink **-*a*** causes a flip *a* = _______ **SHRINK STRETCH FLIP**	**+ *h*** moves the graph to the right **-*h*** moves the graph to the left *h* = _______ **RIGHT LEFT**	**+ *k*** moves the graph up **-*k*** moves the graph down *k* = _______ **UP DOWN**

x	h	y	a	k
0				
1				
2				
3				

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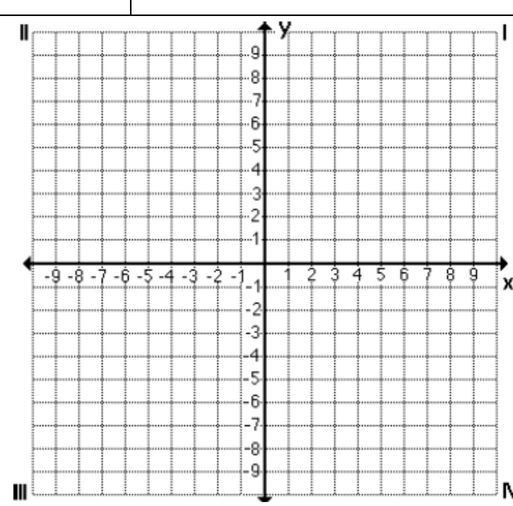

$y = a\sqrt{x-h} + k$ **2.** $y = \frac{1}{2}\sqrt{x-1} + 3$ $y = \sqrt{x}$	***a* >1** causes a stretch ***a*** between 0 and 1 causes a shrink **-*a*** causes a flip ***a*** = _______ **SHRINK STRETCH FLIP**	**+ *h*** moves the graph to the right **-*h*** moves the graph to the left ***h*** = _______ **RIGHT LEFT**	**+ *k*** moves the graph up **-*k*** moves the graph down ***k*** = _______ **UP DOWN**

x	h	y	a	k
0				
1				
2				
3				

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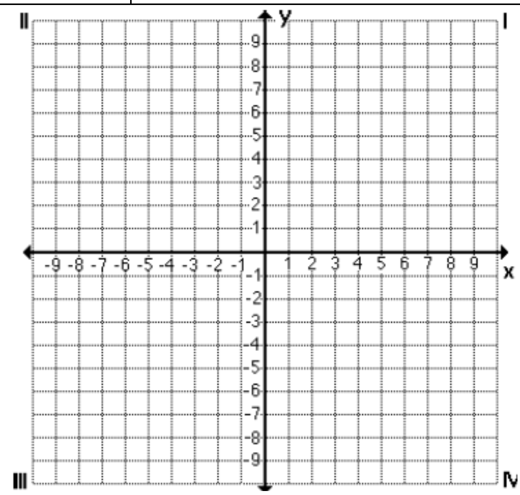

$y = a\sqrt{x-h} + k$ **3.** $y = -4\sqrt{x-3} + 5$ $y = \sqrt{x}$	***a* >1** causes a stretch ***a*** between 0 and 1 causes a shrink **-*a*** causes a flip ***a*** = _______ **SHRINK STRETCH FLIP**	**+ *h*** moves the graph to the right **-*h*** moves the graph to the left ***h*** = _______ **RIGHT LEFT**	**+ *k*** moves the graph up **-*k*** moves the graph down ***k*** = _______ **UP DOWN**

x	h	y	a	k
0				
1				
2				
3				

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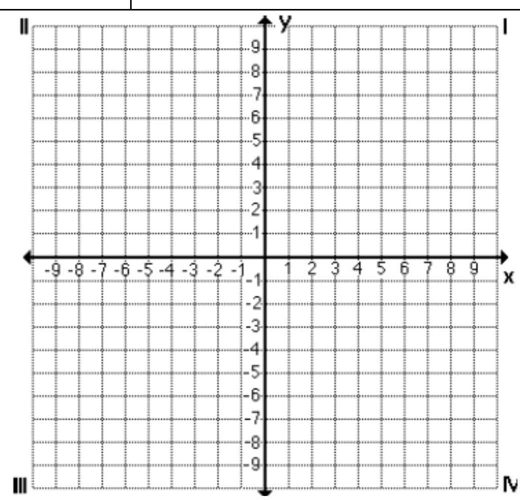

4. $y = 6\sqrt{x+2} - 7$

a = _______ SHRINK STRETCH FLIP

h = _______ RIGHT LEFT

k = _______ UP DOWN

x	*h*	*y*	*a*	*k*
0				
1				
2				
3				

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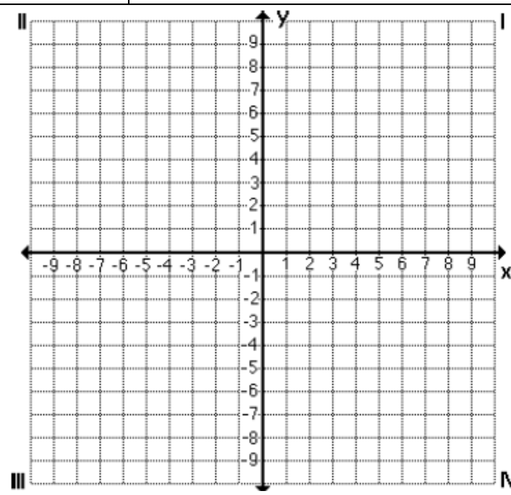
(,)



5. $y = \frac{1}{3}\sqrt{x+6}$

a = _______ SHRINK STRETCH FLIP

h = _______ RIGHT LEFT

k = _______ UP DOWN

x	*h*	*y*	*a*	*k*
0				
1				
2				
3				

(,)

(,)

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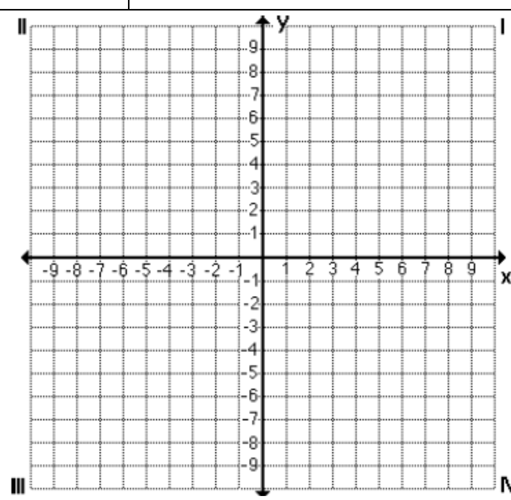

6. $y = -5\sqrt{x} - 1$

a = _______ SHRINK STRETCH FLIP

h = _______ RIGHT LEFT

k = _______ UP DOWN

x	*h*	*y*	*a*	*k*
0				
1				
2				
3				

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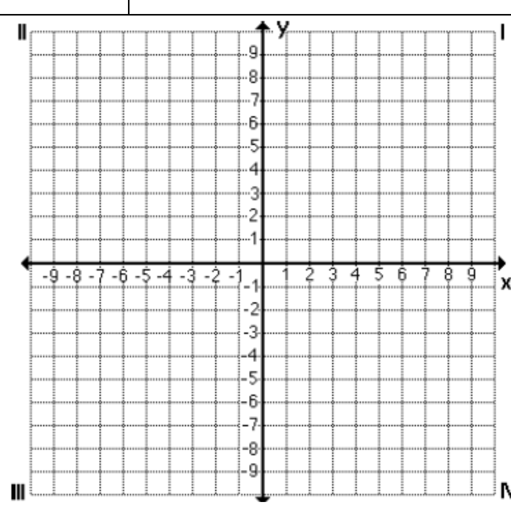
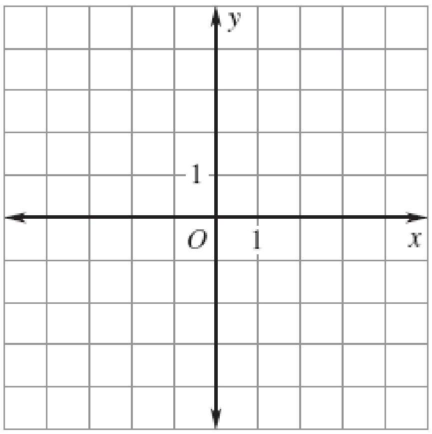
(,)

(,)



7.6 Graphing Cube Root Functions

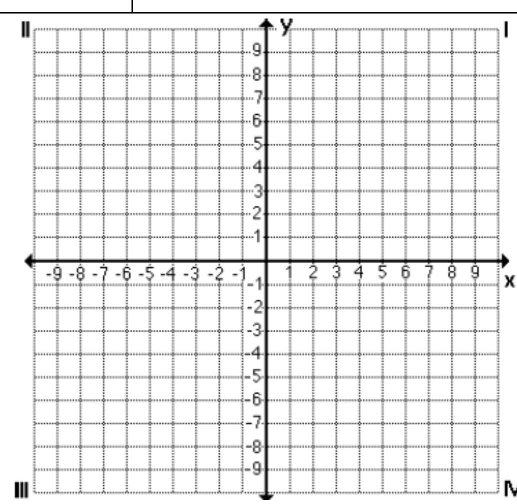
The most basic cube root function in the family of all cube root functions, called the ______________________,
Is $y = \sqrt[3]{x}$. Use the t-chart to help draw the parent graph.



x	y
-1	
0	
1	
2	

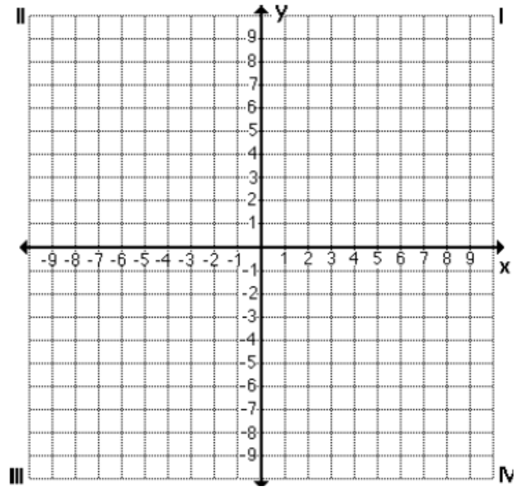
$y = a\sqrt[3]{x-h} + k$ **1.** $y = 3\sqrt[3]{x-2} + 1$ $y = \sqrt[3]{x}$	***a* >1** causes a stretch ***a*** between 0 and 1 causes a shrink **-*a*** causes a flip ***a*** = _______ **SHRINK STRETCH FLIP**	**+ *h*** moves the graph to the right **-*h*** moves the graph to the left ***h*** = _______ **RIGHT LEFT**	**+ *k*** moves the graph up **-*k*** moves the graph down ***k*** = _______ **UP DOWN**

x	h	y	a	k	
-1					(,)
0					(,)
1					(,)
2					(,)



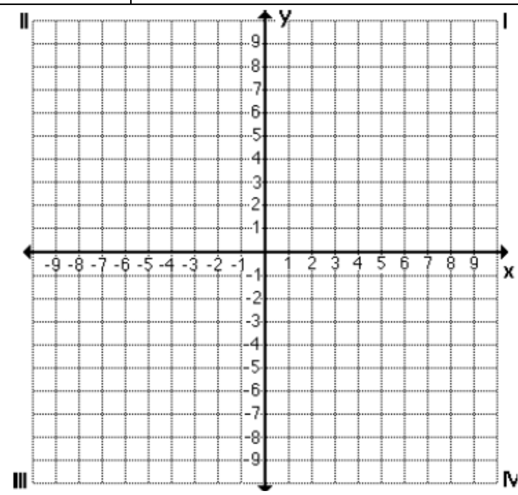
$y = a\sqrt[3]{x-h}+k$ **2.** $y = \frac{1}{2}\sqrt[3]{x+1}+4$ $y=\sqrt[3]{x}$	***a* >*1*** causes a stretch ***a*** between 0 and 1 causes a shrink **-*a*** causes a flip ***a*** = _______ **SHRINK STRETCH FLIP**	**+ *h*** moves the graph to the right **-*h*** moves the graph to the left ***h*** = _______ **RIGHT LEFT**	**+ *k*** moves the graph up **-*k*** moves the graph down ***k*** = _______ **UP DOWN**

x	*h*	*y*	*a*	*k*	
-1					(,)
0					(,)
1					(,)
2					(,)



$y = a\sqrt[3]{x-h}+k$ **3.** $y = -2\sqrt[3]{x+6}-3$ $y=\sqrt[3]{x}$	***a* >*1*** causes a stretch ***a*** between 0 and 1 causes a shrink **-*a*** causes a flip ***a*** = _______ **SHRINK STRETCH FLIP**	**+ *h*** moves the graph to the right **-*h*** moves the graph to the left ***h*** = _______ **RIGHT LEFT**	**+ *k*** moves the graph up **-*k*** moves the graph down ***k*** = _______ **UP DOWN**

x	*h*	*y*	*a*	*k*	
-1					(,)
0					(,)
1					(,)
2					(,)



4. $y = 3\sqrt[3]{x} - 5$	***a*** = _______ **SHRINK STRETCH FLIP**	***h*** = _______ **RIGHT LEFT**	***k*** = _______ **UP DOWN**

x	h	y	a	k
-1				
0				
1				
2				

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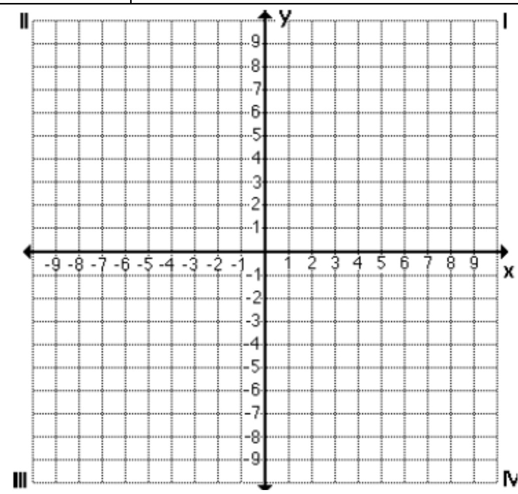

5. $y = -2\sqrt[3]{x-4} + 7$	***a*** = _______ **SHRINK STRETCH FLIP**	***h*** = _______ **RIGHT LEFT**	***k*** = _______ **UP DOWN**

x	h	y	a	k
-1				
0				
1				
2				

(,)

(,)

(,)

(,)