Chapter 7:

P7.2-2, 4, 5, 7
P7.3-1, 2, 5
P7.4-1, 4
P7.5-2, 4, 13, 14, 20
P7.6-1, 3
P7.7-2, 6
P7.8-1, 3, 5, 6, 13

Section 7-2: Capacitors

P 7.2-2 The voltage, $v(t)$, across a capacitor and current, $i(t)$, in that capacitor adhere to the passive convention. Determine the current, $i(t)$, when the capacitance is C = 0.125 F and the voltage is $v(t) = 12\cos(2t + 30°)$ V.

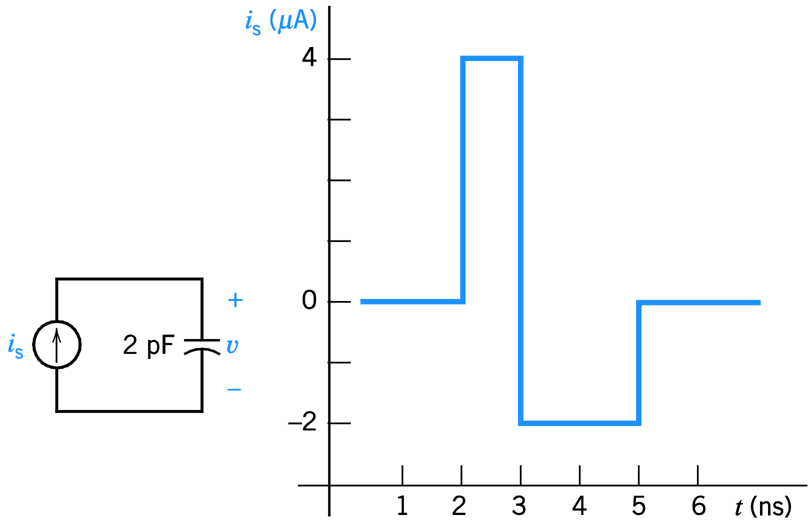
Hint:
$$\frac{d}{dt} A\cos(\omega t+\theta) = -A\sin(\omega t+\theta)\cdot\frac{d}{dt}(\omega t+\theta)$$
$$= -A\omega\sin(\omega t+\theta)$$
$$= A\omega\cos\left(\omega t+\left(\theta+\frac{\pi}{2}\right)\right)$$

Answer: $i(t) = 3\cos(2t + 120°)$ A

Solution:

$$i(t) = C\frac{d}{dt}v(t) = \frac{1}{8}\frac{d}{dt}12\cos\left(2t+30^\circ\right) = \frac{1}{8}(12)(-2)\sin\left(2t + 30^\circ\right) = 3\cos\left(2t + 120^\circ\right) \text{ A}$$

P 7.2-4 Determine $v(t)$ for the circuit shown in Figure P 7.2-4*a* when the $i_s(t)$ is as shown in Figure P 7.2-4*b* and $v_0(0^-) = -1$ mV.



(a) *(b)*

Figure P 7.2-4

Solution:

$$v(t)=\frac{1}{C}\int_0^t i(\tau)\,d\tau+v(0)=\frac{1}{2\times10^{-12}}\int_0^t i(\tau)\,d\tau-10^{-3}$$

$0<t<2\times10^{-9}$ $\quad i_s(t)=0 \quad\Rightarrow\quad v(t)=\frac{1}{2\times10^{-12}}\int_0^t 0\,d\tau-10^{-3}=-10^{-3}$

$2\times10^{-9}<t<3\times10^{-9}$ $\quad i_s(t)=4\times10^{-6}\text{ A}$

$$\Rightarrow\quad v(t)=\frac{1}{2\times10^{-12}}\int_{2\text{ns}}^t \left(4\times10^{-6}\right)d\tau-10^{-3}=-5\times10^{-3}+\left(2\times10^{6}\right)t$$

In particular, $v\left(3\times10^{-9}\right)=-5\times10^{-3}+\left(2\times10^{6}\right)\left(3\times10^{-9}\right)=10^{-3}$

$3\times10^{-9}<t<5\times10^{-9}$ $\quad i_s(t)=-2\times10^{-6}\text{ A}$

$$\Rightarrow\quad v(t)=\frac{1}{2\times10^{-12}}\int_{3\text{ns}}^t \left(-2\times10^{-6}\right)d\tau+10^{-3}=4\times10^{-3}-\left(10^{6}\right)t$$

In particular, $v\left(5\times10^{-9}\right)=4\times10^{-3}-\left(10^{6}\right)\left(5\times10^{-9}\right)=-10^{-3}\text{ V}$

$5\times10^{-9}<t$ $\quad i_s(t)=0 \quad\Rightarrow\quad v(t)=\frac{1}{2\times10^{-12}}\int_{5\text{ns}}^t 0\,d\tau-10^{-3}=-10^{-3}\text{ V}$

P 7.2-5 The voltage, $v(t)$, and current, $i(t)$, of a 1-F capacitor adhere to the passive convention. Also, $v(0)=0$ V and $i(0)=0$ A. (a) Determine $v(t)$ when $i(t)=x(t)$, where $x(t)$ is shown in Figure P 7.2-5 and $i(t)$ has units of A. (b) Determine $i(t)$ when $v(t)=x(t)$, where $x(t)$ is shown in Figure P 7.2-5 and $v(t)$ has units of V.
Hint: $x(t)=4t-4$ when $1<t<2$, and $x(t)=-4t+12$ when $2<t<3$.

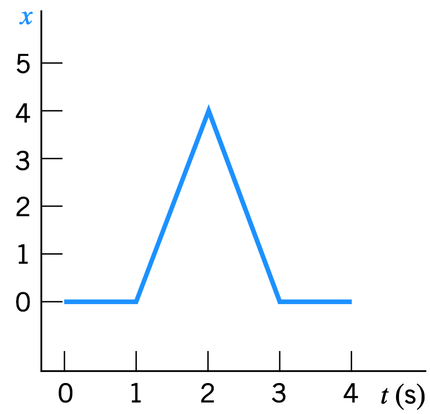


Figure P 7.2-5

Solution:

(b)

$$i(t) = C\frac{d}{dt}v(t) = \begin{cases} 0 & 0 < t < 1 \\ 4 & 1 < t < 2 \\ -4 & 2 < t < 3 \\ 0 & 3 < t \end{cases}$$

(a)

$$v(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau + v(0) = \int_0^t i(\tau)\,d\tau$$

For $0 < t < 1$, $i(t) = 0$ A so $v(t) = \int_0^t 0\,d\tau + 0 = 0$ V

For $1 < t < 2$, $i(t) = (4t - 4)$ A so

$$v(t) = \int_1^t (4\tau - 4)\,d\tau + 0 = \left(2\tau^2 - 4\tau\right)\Big|_1^t = 2t^2 - 4t + 2 \text{ V}$$

$v(2) = 2\left(2^2\right) - 4(2) + 2 = 2$ V. For $2 < t < 3$, $i(t) = (-4t + 12)$ A so

$$v(t) = \int_2^t (-4\tau + 12)\,d\tau + 2 = \left(-2\tau^2 + 12\tau\right)\Big|_1^t + 2 = \left(-2t^2 + 12t - 14\right) \text{ V}$$

$$v(3) = -2\left(3^2\right) + 12(3) - 14 = 4 \text{ V}$$

For $3 < t$, $i(t) = 0$ A so $v(t) = \int_0^t 0\,d\tau + 4 = 4$ V

P 7.2-7 The voltage across a 40-μF capacitor is 25 V at t_0 = 0. If the current through the capacitor as a function of time is given by $i(t) = 6e^{-6t}$ mA for $t < 0$, find $v(t)$ for $t > 0$.
Answer: $v(t) = 50 - 25e^{-6t}$ V

Solution:

$$v(t) = v(0) + \frac{1}{C}\int_0^t i(\tau)\,d\tau = 25 + 2.5\times10^4 \int_0^t \left(6\times10^{-3}\right) e^{-6\tau}\,d\tau$$

$$= 25 + 150\int_0^t e^{-6\tau}\,d\tau$$

$$= 25 + 150\left[-\frac{1}{6}e^{-6\tau}\right]_0^t = \underline{50 - 25e^{-6t} \text{ V}}$$

Section 7-3: Energy Storage in a Capacitor

P 7.3-1 The current, i, through a capacitor is shown in Figure P 7.3-1. When $v(0) = 0$ and C = 0.5 F, determine and plot $v(t)$, $p(t)$, and $w(t)$ for 0 s < t < 6 s.

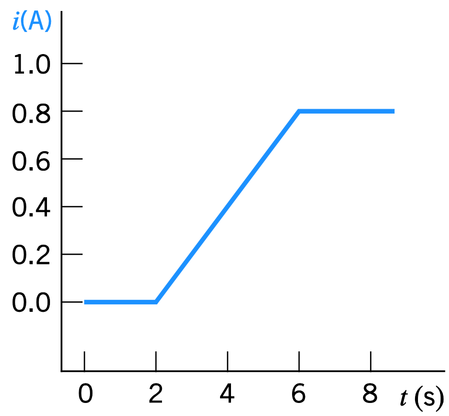


Figure P 7.3-1

Solution:

Given
$$i(t) = \begin{cases} 0 & t < 2 \\ 0.2(t-2) & 2 < t < 6 \\ 0.8 & t > 6 \end{cases}$$

The capacitor voltage is given by

$$v(t) = \frac{1}{0.5}\int_0^t i(\tau)\,d\tau + v(0) = 2\int_0^t i(\tau)\,d\tau + v(0)$$

For $t < 2$
$$v(t) = 2\int_0^t 0\,d\tau + 0 = 0$$

In particular, $v(2) = 0$. For $2 < t < 6$

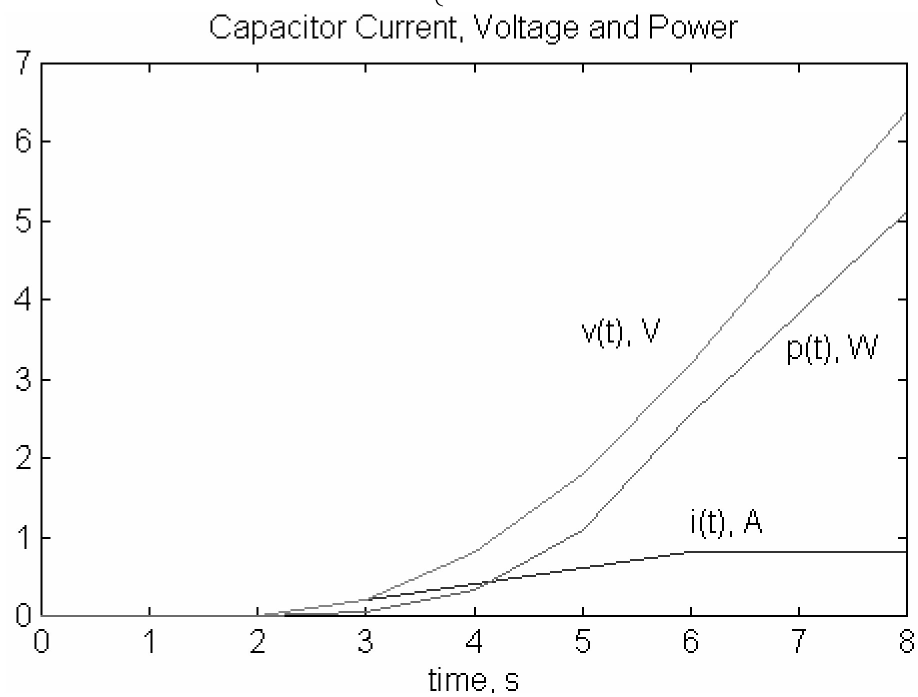
$$v(t) = 2\int_2^t 2(\tau - 2)\,d\tau + 0 = \left(0.2\tau^2 - 0.8\tau\right)\Big|_2^t = \left(0.2t^2 - 0.8t + 0.8\right)\text{ V} = 0.2\left(t^2 - 4t + 4\right)\text{ V}$$

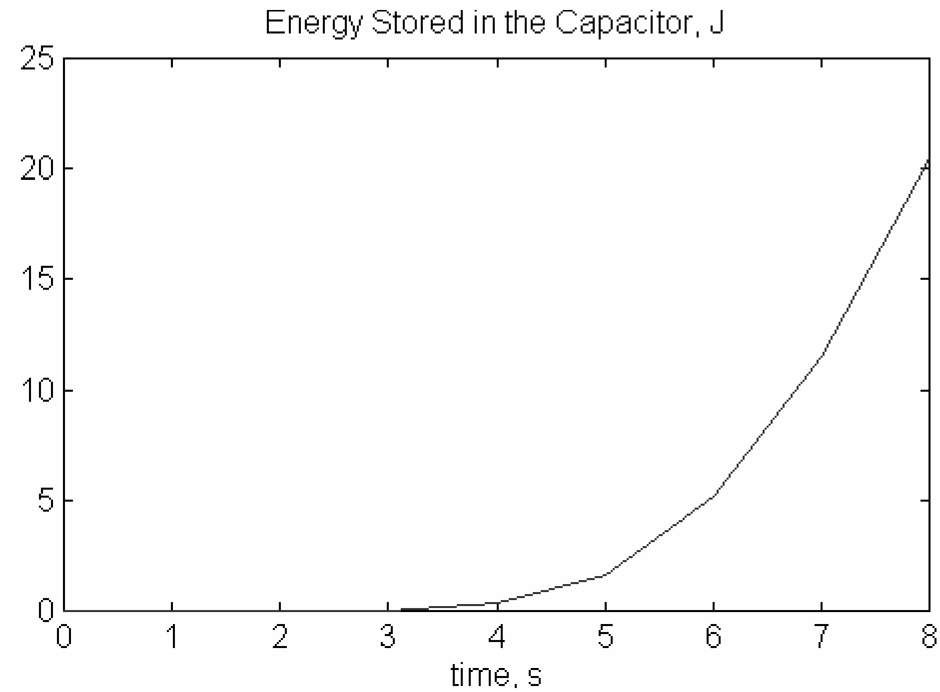
In particular, $v(6) = 3.2$ V. For $6 < t$

$$v(t) = 2\int_6^t 0.8\,d\tau + 3.2 = 1.6\tau\big|_2^t + 3.2 = (1.6t - 6.4)\text{ V} = 1.6(t - 4)\text{ V}$$

Now the power and energy are calculated as

$$p(t) = v(t)\,i(t) = \begin{cases} 0 & t < 2 \\ 0.04(t-2)^2 & 2 < t < 6 \\ 1.28(t-4) & 6 < t \end{cases}$$





Energy Stored in the Capacitor, J
25
20
15
10
5
0
0
1
2
3
4
5
6
7
8
time, s

These plots were produced using three MATLAB scripts:

capvol.m

```
function v = CapVol(t)
   if t<2
      v = 0;
   elseif t<6
      v = 0.2*t*t - .8*t +.8;
   else
      v = 1.6*t - 6.4;
   end
```

capcur.m

```
function i = CapCur(t)
   if t<2
      i=0;
   elseif t<6
      i=.2*t - .4;
   else
      i =.8;
   end
```

c7s4p1.m

```
t=0:1:8;
for k=1:1:length(t)
  i(k)=CapCur(k-1);
  v(k)=CapVol(k-1);
  p(k)=i(k)*v(k);
  w(k)=0.5*v(k)*v(k);
end

 plot(t,i,t,v,t,p)
 text(5,3.6,'v(t), V')
 text(6,1.2,'i(t), A')
 text(6.9,3.4,'p(t), W')
 title('Capacitor Current, Voltage and Power')
 xlabel('time, s')

% plot(t,w)
% title('Energy Stored in the Capacitor, J')
% xlabel('time, s')
```

P 7.3-2 In a pulse power circuit the voltage of a 10-μF capacitor is zero for $t < 0$ and

$$v = 5(1 - e^{-4000t})\text{ V} \quad t \ge 0$$

Determine the capacitor current and the energy stored in the capacitor at t = 0 ms and t = 10 ms.

Solution:

$$i_c = C\frac{dv}{dt} = \left(10\times10^{-6}\right)(-5)(-4000)e^{-4000t} = \underline{0.2e^{-4000t}\text{ A}} \Rightarrow \begin{cases} \underline{i_c(0) = 0.2\ A} \\ \underline{i_c(10ms) = 8.5\times10^{-19}\ A} \end{cases}$$

$$\mathcal{W}(t) = \frac{1}{2}Cv^2(t) \text{ and } v(0) = 5 - 5e^0 = 0 \Rightarrow \underline{\mathcal{W}(0) = 0}$$

$$v\left(10\times10^{-3}\right) = 5 - 5e^{-40} = 5 - 21.2\times10^{-18} \cong 5 \Rightarrow \underline{\mathcal{W}(10) = 1.25\times10^{-4}\text{ J}}$$

P 7.3-5 A capacitor is used in the electronic flash unit of a camera. A small battery with a constant voltage of 6 V is used to charge a capacitor with a constant current of 10 μA. How long does it take to charge the capacitor when C = 10 μF? What is the stored energy?

Solution:

$$\text{Max. charge on capacitor} = C\,v = \left(10\times10^{-6}\right)(6) = 60\ \mu\text{C}$$

$$\Delta t = \frac{\Delta q}{i} = \frac{60\times10^{-6}}{10\times10^{-6}} = \underline{6\text{ sec}} \text{ to charge}$$

$$\text{stored energy} = \mathcal{W} = \frac{1}{2}C\,v^2 = \frac{1}{2}\left(10\times10^{-6}\right)(6)^2 = \underline{180\ \mu\text{J}}$$

Section 7-4: Series and Parallel Capacitors

P 7.4-1 Find the current $i(t)$ for the circuit of Figure P 7.4-1.
Answer: $i(t) = -1.2 \sin 100t$ mA

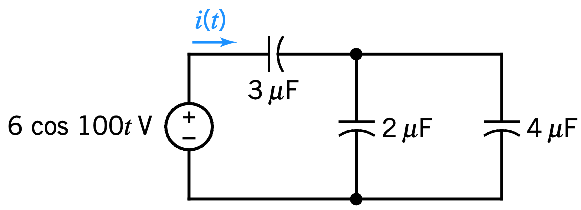


Figure P 7.4-1

Solution:

$$2\mu\text{F} \parallel 4\mu\text{F} = 6\mu\text{F}$$

$$6\mu\text{F in series with } 3\mu\text{F} = \frac{6\mu\text{F}\cdot 3\mu\text{F}}{6\mu\text{F}+3\mu\text{F}} = 2\mu\text{F}$$

$$i(t) = 2\ \mu\text{F}\ \frac{d}{dt}\ (6\cos 100t) = (2\times 10^{-6})\ (6)\ (100)\ \ (-\sin 100t)\ \text{A} = \underline{-1.2\ \sin 100t\ \ \text{mA}}$$

P7.4-4 The circuit shown in Figure P 7.4-4 contains seven capacitors, each having capacitance C. The source voltage is given by

$$v(t) = 4\cos(3t)\ \text{V}$$

Find the current $i(t)$ when C = 1 F.

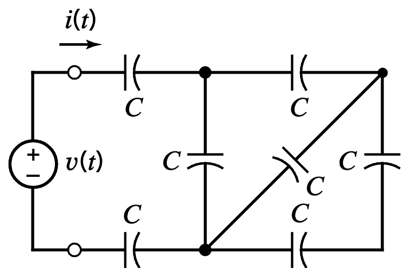
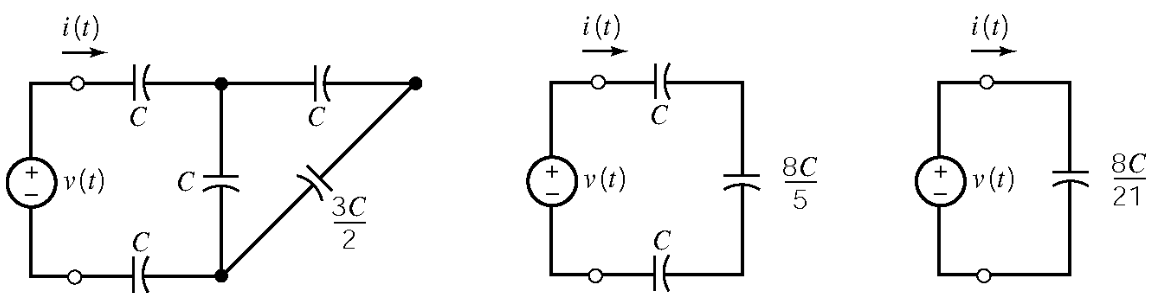


Figure P 7.4-4

Solution: Replacing series and parallel capacitors by equivalent capacitors, the circuit can be reduced as follows:



Then

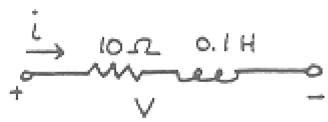
$$i(t) = \frac{8C}{21}\frac{d}{dt}v(t) = \frac{8C}{21}\frac{d}{dt}4\cos(3t) = \frac{8\times 1}{21}\left[-12\sin(3t)\right] = -\frac{32}{7}\sin(3t)\ \ \text{V}$$

Section 7-5: Inductors

P 7.5-2 The model of an electric motor consists of a series combination of a resistor and inductor. A current $i(t) = 4te^{-t}$ A flows through the series combination of a 10-Ω resistor and 0.1-H inductor. Find the voltage across the combination.

Answer: $v(t) = 0.4e^{-t} + 39.6te^{-t}$ V

Solution:



$$v = L\frac{di}{dt} + R\,i = (.1)\ (4e^{-t} - 4te^{-t}) + 10(4te^{-t}) = \underline{\ 0.4\,e^{-t} + 39.6t\,e^{-t}\ \text{V}}$$

P 7.5-4 The voltage, $v(t)$, across an inductor and current, $i(t)$, in that inductor adhere to the passive convention. Determine the voltage, $v(t)$, when the inductance is L = 250 mH and the current is $i(t)$ = 120 sin (500t – 30°) mA.

Hint: $\frac{d}{dt}A\sin(\omega t+\theta) = A\cos(\omega t+\theta)\cdot\frac{d}{dt}(\omega t+\theta) = A\omega\cos(\omega t+\theta) = A\omega\sin\left(\omega t+\left(\theta+\frac{\pi}{2}\right)\right)$

Answer: $v(t)$ = 15 sin(500t + 60°) V

Solution:

$$v(t) = (250\times10^{-3})\frac{d}{dt}(120\times10^{-3})\sin(500t-30^\circ) = (0.25)(0.12)(500)\cos(500\,t-30^\circ)$$

$$= 15\ \cos(500t-30^\circ)$$

P 7.5-13 The inductor current in the circuit shown in Figure P 7.5-13 is given by

$$i(t) = 5 - 3e^{-4t}\ \text{A} \quad \text{for}\ t \ge 0$$

Determine $v(t)$ for t > 0.

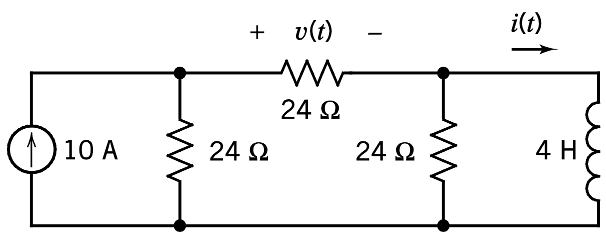
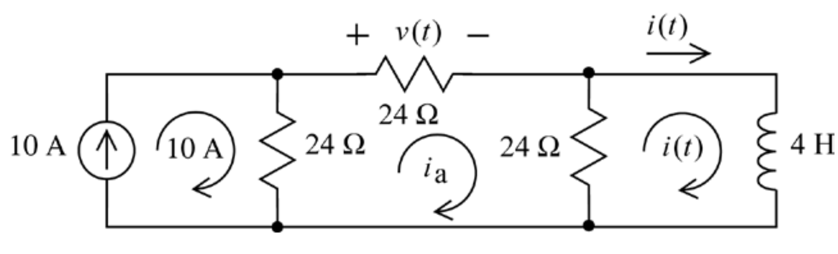


Figure P 7.5-13

Solution:



We'll write and solve a mesh equation. Label the meshes as shown. Apply KVL to the center mesh to get

$$24i_a + 24\left(i_a - i(t)\right) + 24\left(i_a - 10\right) = 0 \quad \Rightarrow \quad i_a = \frac{i(t)+10}{3} = 5 - e^{-4t} \text{ A for } t > 0$$

Then
$$v(t) = 24i_a = 120 - 24e^{-4t} \text{ V for } t > 0$$

(checked: LNAP 6/25/04)

P 7.5-14 The inductor current in the circuit shown in Figure P 7.5-14 is given by

$$i(t) = 3 + 2e^{-3t} \text{ A} \quad \text{for } t \ge 0$$

Determine *v*(*t*) for *t* > 0.

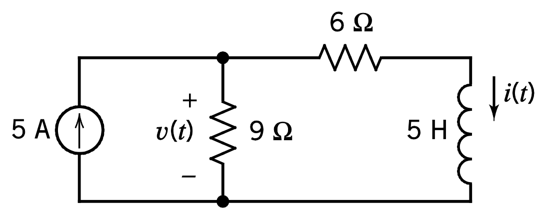


Figure P 7.5-14

Solution: Apply KVL to get

$$v(t) = 6i(t) + 5\frac{d}{dt}i(t) = 6\left(3+2e^{-3t}\right) + 5\frac{d}{dt}\left(3+2e^{-3t}\right) = 18\left(1-e^{-3t}\right) \text{ V for } t > 0$$

P7.5-20 Consider the inductor shown in Figure P7.5-20. The current and voltage are given by

$$i(t) = \begin{cases} 5t - 4.6 & 0 \le t \le 0.2 \\ a\,t + b & 0.2 \le t \le 0.5 \\ c & t \ge 0.5 \end{cases} \quad \text{and} \quad v(t) = \begin{cases} 12.5 & 0 < t < 0.2 \\ 25 & 0.2 < t < 0.5 \\ 0 & t > 0.5 \end{cases}$$

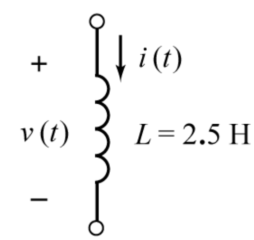


Figure P7.5-20

where *a*, *b* and *c* are real constants. (The current is given in Amps, the voltage in Volts and the time in seconds.) Determine the values of *a*, *b* and *c*.

Answer: *a* = 10 A/s, *b* = –5.6 A and *c* = –0.6 A

Solution: At *t* = 0.2 s

$$i(0.2) = 5(0.2) - 4.6 = -3.6 \text{ A}$$

For $0.2 \le t \le 0.5$

$$i(t) = \frac{1}{2.5}\int_{0.2}^{t} 25\,d\tau - 3.6 = 10\,\tau\,\Big|_{0.2}^{t} - 3.6 = 10(t-0.2) - 3.6 = 10t - 5.6 \text{ A}$$

At t = 0.5 s

$$i(0.5)=10(0.5)-5.6=-0.6 \text{ A}$$

For $t \ge 0.5$

$$i(t)=\frac{1}{2.5}\int_{0.5}^{t} 0\, d\tau - 0.6 = -0.6$$

Checks:

At t = 0.2 s $\quad i(0.2)=10(0.2)-5.6=-3.6 \text{ A} \quad \surd$

For $0.2 \le t \le 0.5$ $\quad v(t)=2.5\frac{d}{dt}i(t)=2.5\frac{d}{dt}(10t-5.6)=2.5(10)=25 \text{ V} \quad \surd$

$$-0.6-(-3.6)=i(0.5)-i(0.2)=\frac{1}{2.5}\int_{0.2}^{0.5} 25\, d\tau = 10(0.5-0.2)=3 \text{ A} \quad \surd$$

Section 7-6: Energy Storage in an Inductor

P 7.6-1 The current, $i(t)$, in a 100-mH inductor connected in a telephone circuit changes according to

$$i(t) = \begin{cases} 0 & t \le 0 \\ 4t & 0 \le t \le 1 \\ 4 & t \ge 1 \end{cases}$$

where the units of time are seconds and the units of current are amperes. Determine the power, $p(t)$, absorbed by the inductor and the energy, $w(t)$, stored in the inductor.

Answer: $p(t) = \begin{cases} 0 & t \le 0 \\ 1.6t & 0 < t < 1 \\ 0 & t \ge 1 \end{cases}$ and $w(t) = \begin{cases} 0 & t \le 0 \\ 0.8t^2 & 0 < t < 1 \\ 0.8 & t \ge 1 \end{cases}$

The units of $p(t)$ are W and the units of $w(t)$ are J.

Solution:

$$v(t) = 100\times10^{-3}\ \frac{d}{dt}\ i(t) = \begin{cases} 0 & \text{t<0} \\ 0.4 & 0\le\text{t}\le1 \\ 0 & \text{t>1} \end{cases}$$

$$p(t) = v(t)\,i(t) = \begin{cases} 0 & t<0 \\ 1.6t & 0\le t\le1 \\ 0 & t>1 \end{cases}$$

$$W(t) = \int_0^t p(\tau)\,d\tau = \begin{cases} 0 & t<0 \\ 0.8t^2 & 0<t<1 \\ 0.8 & t>1 \end{cases}$$

P 7.6-3 The voltage, $v(t)$, across a 25-mH inductor used in a fusion power experiment is

$$v(t) = \begin{cases} 0 & t \le 0 \\ 6\cos 100t & t \ge 0 \end{cases}$$

where the units of time are s and the units of voltage are V. The current in this inductor is zero before the voltage changes at t = 0. Determine the power, $p(t)$, absorbed by the inductor and the energy, $w(t)$, stored in the inductor. ***Hint:*** 2(cos A)(sin B) = sin(A + B) + sin(A – B)

Answer: $p(t)$ = 7.2 sin 200t W and $w(t)$ = 3.6[1 – cos 200t] mJ

Solution:

$$i(t) = \frac{1}{25\times10^{-3}}\int_0^t 6\cos 100\tau\ d\tau + 0 = \frac{6}{(25\times10^{-3})(100)}\ [\sin 100\tau\ |_0^t] = 2.4\sin 100t$$

$$p(t) = v(t)\ i(t) = (6\cos 100\,t)(2.4\sin 100t) = 7.2\ [\ 2(\cos 100\,t)(\sin 100\,t)\ ]$$
$$= 7.2\ [\sin 200\,t + \sin 0] = 7.2\sin 200\,t$$
$$W(t) = \int_0^t p(\tau)\ d\tau = 7.2\int_0^t \sin\ 200\tau\, d\tau = -\frac{7.2}{200}\left[\cos\ 200\tau\Big|_0^t\right]$$
$$= 0.036[1-\cos 200t]\ \text{J} =\ 36\ [1-\cos 200t\ ]\ \text{mJ}$$

Section 7-7: Series and Parallel Inductors

P 7.7-2 Find the voltage $v(t)$ for the circuit of Figure P 7.7-2.
Answer: $v(t) = -6e^{-250t}$ mV

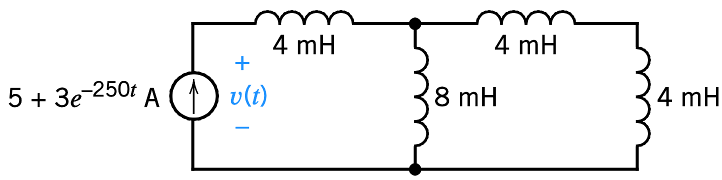


Figure P 7.7-2

Soluton:

$$4\text{ mH} + 4\text{ mH} = 8\text{ mH}\quad ,\quad 8\text{mH}\ \|\ 8\text{mH} = \frac{\left(8\times10^{-3}\right)\times\left(8\times10^{-3}\right)}{8\times10^{-3}+8\times10^{-3}} = 4\text{ mH}$$

and $\quad 4\text{ mH} + 4\text{ mH} = 8\text{ mH}$

$$v(t) = (8\times10^{-3})\ \frac{d}{dt}\ (5+3e^{-250t}) = (8\times10^{-3})(0+3(-250)\,e^{-250t}) = -6\,e^{-250t}\ \text{V}$$

P 7.7-6 Determine the value of the equivalent inductance, L_{eq}, for the circuit shown in Figure P 7.7-6.
Answer: $L_{eq} = 120$ H

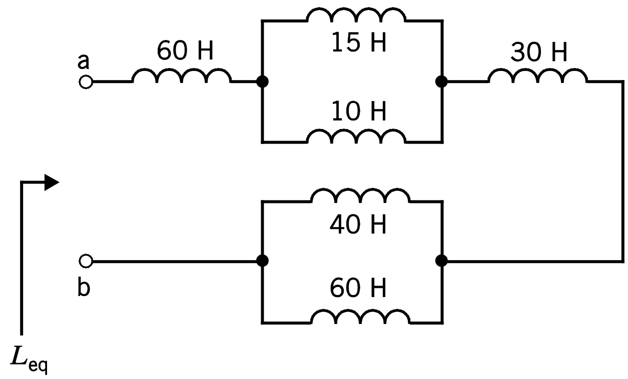


Figure P 7.7-6

Solution:

$$L_{eq} = 60 + \frac{15 \times 10}{15+10} + 30 + \frac{40 \times 60}{40+60} = 60 + 6 + 30 + 24 = 120 \text{ H}$$

Section 7-8: Initial Conditions of Switched Circuits

P 7.8-1 The switch in Figure P 7.8-1 has been open for a long time before closing at time $t = 0$. Find $v_c(0^+)$ and $i_L(0^+)$, the values of the capacitor voltage and inductor current immediately after the switch closes. Let $v_c(\infty)$ and $i_L(\infty)$ denote the values of the capacitor voltage and inductor current after the switch has been closed for a long time. Find $v_c(\infty)$ and $i_L(\infty)$.

Answer: $v_c(0^+) = 12$ V, $i_L(0^+) = 0$, $v_c(\infty) = 4$ V, and $i_L(\infty) = 1$ mA

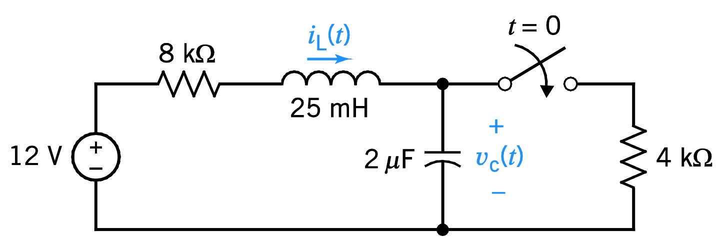
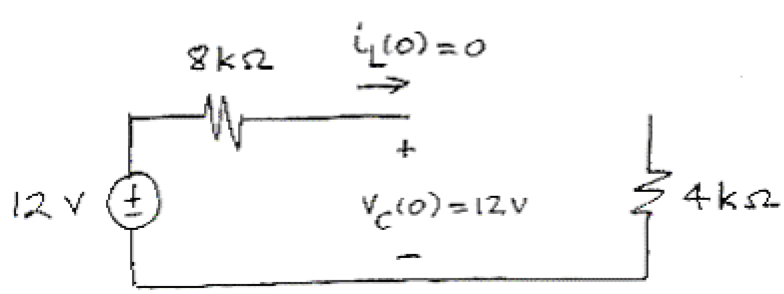


Figure P 7.8-1

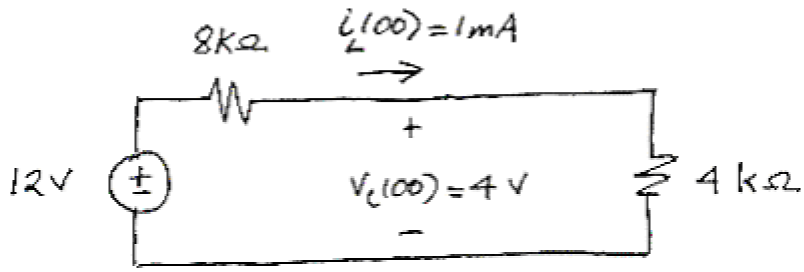
Solution:



Then

$$i_L(0^+) = i_L(0^-) = 0 \quad \text{and} \quad v_C(0^+) = v_C(0^-) = 12 \text{ V}$$

Next



P 7.8-3 The switch in Figure P 7.8-3 has been open for a long time before closing at time $t = 0$. Find $v_c(0^+)$ and $i_L(0^+)$, the values of the capacitor voltage and inductor current immediately after the switch closes. Let $v_c(\infty)$ and $i_L(\infty)$ denote the values of the capacitor voltage and inductor current after the switch has been closed for a long time. Find $v_c(\infty)$ and $i_L(\infty)$.

Answer: $v_c(0^+) = 0$ V, $i_L(0^+) = 0$, $v_c(\infty) = 8$ V, and $i_L(\infty) = 0.5$ mA

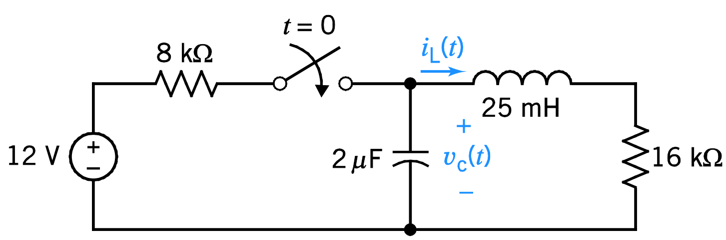
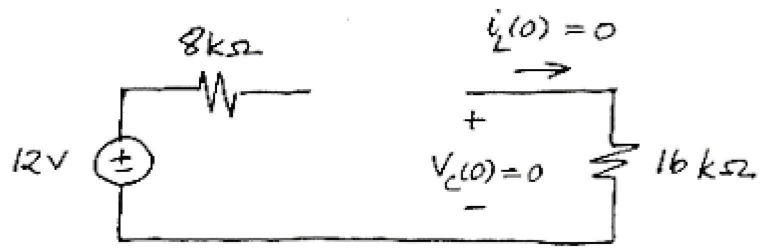


Figure P 7.8-3

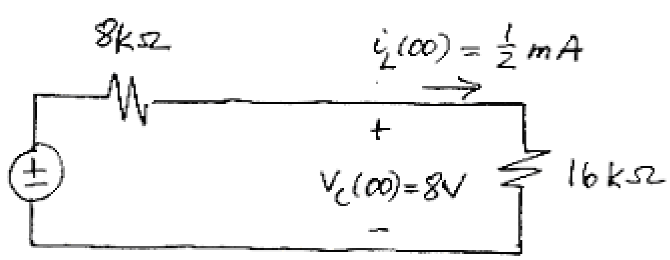
Solution:



Then

$$i_L(0^+) = i_L(0^-) = 0 \quad \text{and} \quad v_C(0^+) = v_C(0^-) = 0 \text{ V}$$

Next



P7.8-5 . The switch in the circuit shown in Figure P7.8-5 has been open for a long time before it closes at time $t = 0$. Determine the values of $i_R(0-)$ and $i_C(0-)$, the current in one of the 20 Ω resistors and in the capacitor immediately before the switch closes and the values of $i_R(0+)$ and $i_C(0+)$, the current in that 20 Ω resistor and in the capacitor immediately after the switch closes.

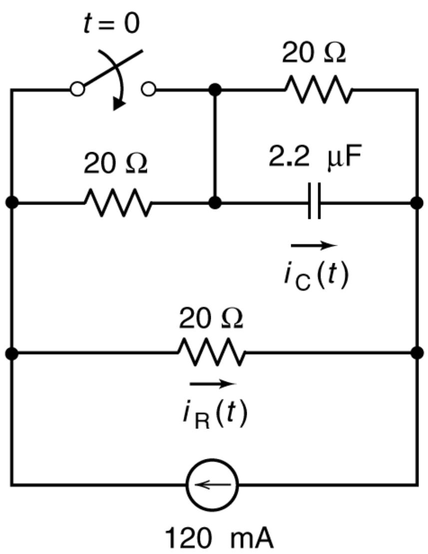
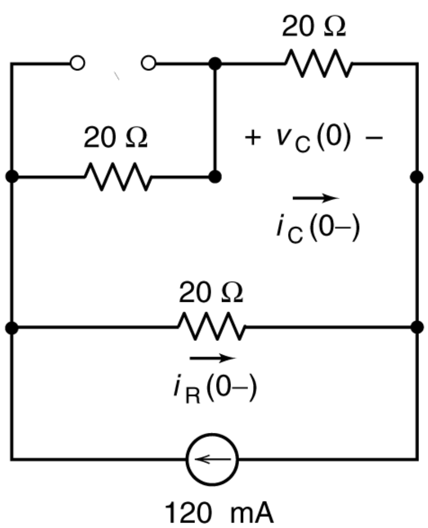


Figure P7.8-5

Solution:

The circuit is at steady state immediately before the switch closes. We have



The capacitor acts like an open circuit so $i_C(0-) = 0$.

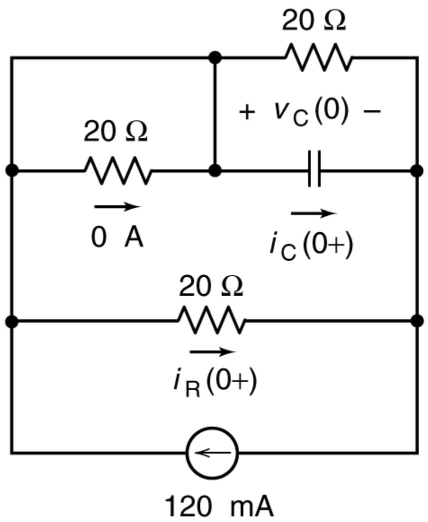
Noticing that two 20 Ω are connected in series and using current division:

$$i_R(0-) = \frac{(20+20)}{20+(20+20)}(120) = \frac{2}{3}(120) = 80 \text{ mA}$$

Using current division and Ohm's law:

$$v_C(0) = \left[\frac{20}{20+(20+20)}(120)\right](20) = 0.8 \text{ V}$$

The capacitor does not change instantaneously so $v_C(0+) = v_C(0-) \cong v_C(0)$. Immediately after the switch closes we have:



Applying KVL to the loop consisting of the closed switch, the capacitor and a 20 Ω resistor gives

$$0+v_C(0)-20\,i_R(0+)=0$$
$$0+0.8=20\,i_R(0+)$$
$$i_R(0+)=40\text{ mA}$$

Applying KCL at the node at the right side of the circuit gives:

$$\frac{v_C(0+)}{20}+i_C(0+)+i_R(0+)=0.120$$
$$\frac{0.8}{20}+i_C(0+)+0.04=0.120$$
$$i_C(0+)=0.04=40\text{ mA}$$

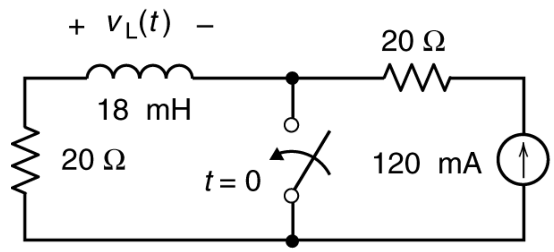
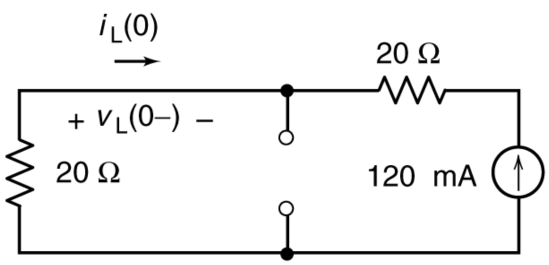


Figure P7.8-6

P7.8-6. The switch in the circuit shown in Figure P7.8-6 has been open for a long time before it closes at time $t = 0$. Determine the values of $v_L(0-)$, the voltage across the inductor immediately before the switch closes and $v_L(0+)$, the voltage across the inductor immediately after the switch closes.

Solution:
The circuit is at steady state immediately before the switch closes. We have

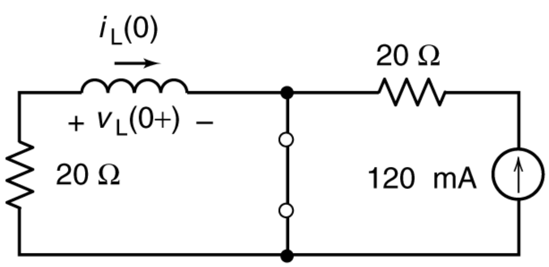


The inductor acts like a short circuit so $v_L(0-) = 0$.

The inductor current is the negative of the current source current:

$$i_L(0) = -120 \text{ mA}$$

The inductor current does not change instantaneously so $i_L(0+) = i_L(0-) \,\square\, i_L(0)$. Immediately after the switch closes we have:



Applying KVL to the left mesh gives:

$$v_L(0+) + 20\,i_L(0) = 0$$
$$v_L(0+) + 20(-0.12) = 0$$
$$v_L(0+) = 2.4$$

P7.8-13

The circuit shown in Figure 7.8-12 has reached steady state before the switch opens at time $t = 0$. Determine the values of $i_L(t)$, $v_C(t)$ and $v_R(t)$ immediately before the switch opens and the value of $v_R(t)$ immediately after the switch opens.

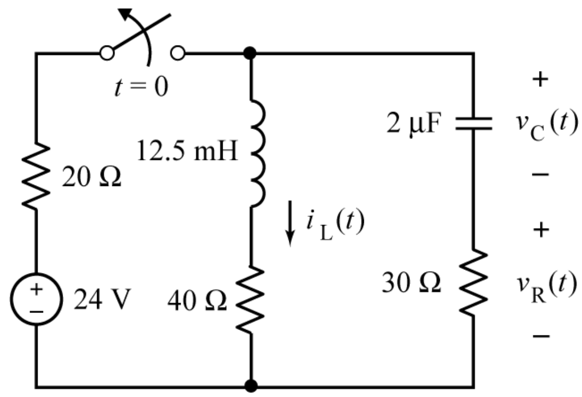
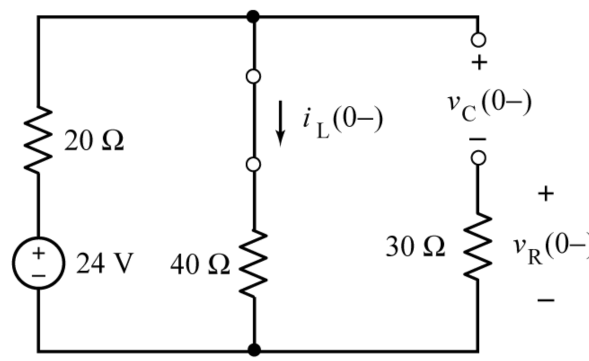


Figure 7.8-13

Answer: $i_L(0-) = 0.4$__A, $v_C(0-) = 16$ V, $v_R(0-) = 0$ V and $v_R(0+) = -12$ V

Solution: Because

- This **circuit has reached steady state** before the switch opens at time $t = 0$.
- The only source is a **constant** voltage **source**.



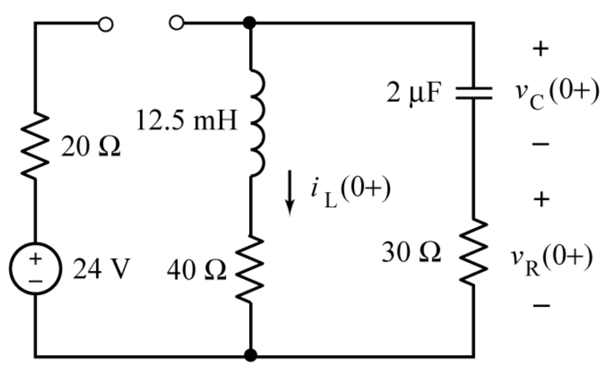
At t=0–, t**he capacitor acts like an open circuit** and **the inductor acts like a short circuit**.

The current in the 30 Ω resistor is zero so $v_R(0-) = 0$ V . Next

$$i_1(0-) = \frac{24}{20+40} = 0.4 \text{ A and}$$

$$v_C(0-) = 40\, i_L(0-) = 16 \text{ V}$$

The **capacitor voltage and inductor current don't change instantaneously** so



$$v_C(0+) = v_C(0-) = 16 \text{ V and}$$

$$i_L(0+) = i_L(0-) = 0.4 \text{ A}$$

Apply KCL at the bottom node and then Ohm's law to get

$$v_R(0+) = -30\, i_L(0+) = -12 \text{ V}$$

(Notice that the resistor voltage did change instantaneously.)