Addressed or Prepped VA SOL:

- G.9 The student will verify and use properties of quadrilaterals to solve problems, including practical problems.
- G.10 The student will solve problems, including practical problems, involving angles of convex polygons. This will include determining the
 - a) sum of the interior and/or exterior angles;
 - b) measure of an interior and/or exterior angle; and
 - c) number of sides of a regular polygon.

SOL Progression

Middle School:

- Compare and Contrast quadrilaterals based on their properties
- Determine unknown side lengths or angle measures in quadrilaterals
- Solve linear equations with rational number coefficients
- Draw polygons in the coordinate plane given vertices and find lengths of sides

Algebra I:

- Create equations in one variable
- Solve linear equations in one variable
- Graph in the coordinate plane
- Find the slope of a line
- Identify and write equations of parallel and perpendicular lines

Geometry:

- Find and use the interior and exterior angle measurements of polygons
- Use properties of parallelograms and special parallelograms
- Prove that a quadrilateral is a parallelogram
- Identify and use properties of trapezoids and kites
- Determine angle measurements of a regular polygon in a tessellation



Section 7-1: Angles of Polygons

SOL: G.10

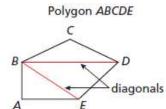
Objective:

Use the interior angle measures of polygons Use the exterior angle measures of polygons

Vocabulary:

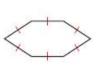
Convex – no line that contains a side of the polygon goes into the interior of the polygon Diagonal – a segment of a polygon that joins two nonconsecutive vertices Equilateral polygon – all sides of the polygon are congruent Equiangular polygon – all interior angles of the polygon are congruent Exterior angles – angle outside the polygon formed by an extended side Interior angles – an angle inside the polygon Regular polygon – convex polygon that is both equilateral and equiangular

Core Concept:

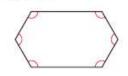


A and B are consecutive vertices. Vertex B has two diagonals, \overline{BD} and \overline{BE} .

In an equilateral polygon, all sides are congruent.



In an equiangular polygon, all angles in the interior of the polygon are congruent.



A regular polygon is a convex polygon that is both equilateral and equiangular.

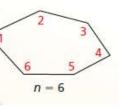


💪 Theorem

Theorem 7.1 Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a convex *n*-gon is $(n - 2) \cdot 180^\circ$. $m \angle 1 + m \angle 2 + \dots + m \angle n = (n - 2) \cdot 180^\circ$

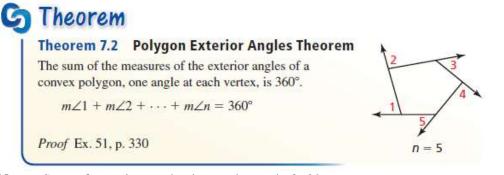
Proof Ex. 42 (for pentagons), p. 329



Note: Sum of interior angles in a polygon is found by $S = (n - 2) \times 180^{\circ}$

Corollary 7.1 Corollary to the Polygon Interior Angles Theorem The sum of the measures of the interior angles of a quadrilateral is 360°. Proof Ex. 43, p. 330



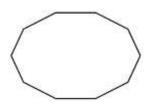


Note: Sum of exterior angles in a polygon is 360°

Examples:

Example 1:

Find the sum of the measures of the interior angles of the figure.

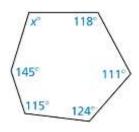


Example 2:

The sum of the measures of the interior angles of a convex polygon is 1800°. Classify the polygon by the number of sides.

Example 3:

Find the value of *x* in the diagram.



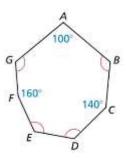
Example 4:

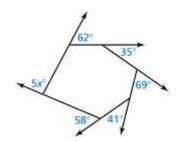
A polygon is shown.

- a. Is the polygon regular? Explain your reasoning
- b. Find the measures of $\angle B$, $\angle D$, $\angle E$, and $\angle G$.

Example 5:

Find the value of *x* in the diagram.





Example 6:

Each face of the dodecahedron is shaped like a regular pentagon.

- a. Find the measure of each interior angle of a regular pentagon.
- b. Find the measure of each exterior angle of a regular pentagon.

Concept Summary:

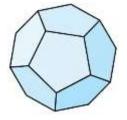
The sum of exterior angles is always 360° (regardless of number of sides) The sum of interior angles is given by the formula, $S = (n - 2) \times 180$ To find the number of sides use: n = 360/ExtThe interior and exterior angles always form a linear pair (sum to 180)

Khan Academy Videos:

- 1. <u>Sum of interior angles</u> of a polygon
- 2. <u>Sum of exterior angles</u> of a polygon

Homework: <u>7-1SOL Worksheet</u>

Reading Assignment: student notes section 7-2



Section 7-2: Properties of Parallelograms

SOL: G.9

Objectives:

Use properties to find side lengths and angles of parallelograms Use parallelograms in the coordinate plane

Vocabulary:

Parallelogram – a quadrilateral with both pairs of opposite sides parallel

Core Concept:

Theorems

Theorem 7.3 Parallelogram Opposite Sides Theorem

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If PQRS is a parallelogram, then $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$.

Proof p. 332

Theorem 7.4 Parallelogram Opposite Angles Theorem

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

If *PQRS* is a parallelogram, then $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.

Proof Ex. 37, p. 337

5 Theorems

Theorem 7.5 Parallelogram Consecutive Angles Theorem

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If *PQRS* is a parallelogram, then $x^{\circ} + y^{\circ} = 180^{\circ}$.

Proof Ex. 38, p. 337

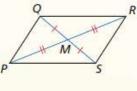
P X° Y° R P S

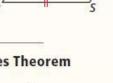
Theorem 7.6 Parallelogram Diagonals Theorem

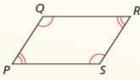
If a quadrilateral is a parallelogram, then its diagonals bisect each other.

If *PQRS* is a parallelogram, then $\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$.

Proof p. 334



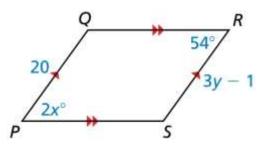




Examples:

Example 1:

Find the values of *x* and *y*.



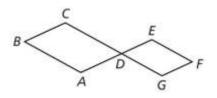
Example 2:

In parallelogram PQRS, $m \angle P$ is four times $m \angle Q$. Find $m \angle P$.

Example 3:

Write a two-column proof.

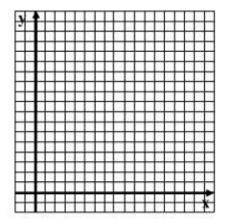
Given: ABCD and GDEF are parallelograms Prove: $\angle C \cong \angle G$



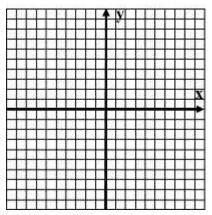
Statements	Reasons

Example 4:

Find the coordinates of the intersection of the diagonals of parallelogram *ABCD* with vertices A(1,0), B(6,0), C(5,3), and D(0,3).



Example 5:

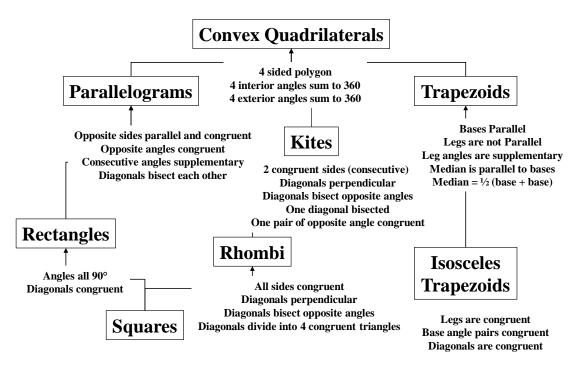


Three vertices of parallelogram *DEFG* are D(-1,4), E(2,3), and F(4,-2). Find the coordinates of vertex *G*.

Concept Summary:

Opposite sides are parallel and congruent Opposite angles are congruent; Consecutive angles are supplementary Diagonals bisect each other

Quadrilateral Characteristics Summary



Khan Academy Videos:

- 1. <u>Introduction</u> to quadrilaterals
- 2. Quadrilateral properties

Homework: Parallelogram characteristics and problems, Quadrilaterals Worksheet

Reading Assignment: read section 7-3

Section 7-3: Proving a Quadrilateral is a Parallelogram

SOL: G.9

Objective:

Identify and verify parallelograms Show that a quadrilateral is a parallelogram in the coordinate plane

Vocabulary: None new

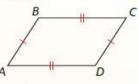
Core Concepts:

Theorems

Theorem 7.7 Parallelogram Opposite Sides Converse

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$, then ABCD is a parallelogram.



C

С

Theorem 7.8 Parallelogram Opposite Angles Converse

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then *ABCD* is a parallelogram.

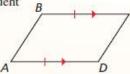
Proof Ex. 39, p. 347

Theorems

Theorem 7.9 Opposite Sides Parallel and Congruent Theorem

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then ABCD is a parallelogram.



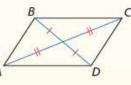
Proof Ex. 40, p. 347

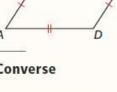
Theorem 7.10 Parallelogram Diagonals Converse

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

If \overline{BD} and \overline{AC} bisect each other, then ABCD is a parallelogram.

Proof Ex. 41, p. 347





B

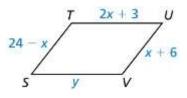
Examples:

Example 1:

In quadrilateral ABCD, AB = BC and CD = AD. Is ABCD a parallelogram? Explain your reasoning.

Example 2:

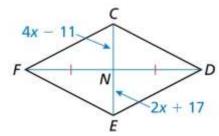
For what values of *x* and *y* is quadrilateral *STUV* a parallelogram?



Example 3:

Use the photograph to the right. Explain how you know that $\angle S \cong \angle U$.



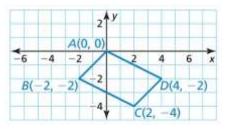


Example 4:

For what value of *x* is quadrilateral *CDEF* a parallelogram?

Example 5:

Show that quadrilateral ABCD is a parallelogram.



Concept Summary:

Concept Summary

Ways to Prove a Quadrilateral Is a Parallelogram

1. Show that both pairs of opposite sides are parallel. (<i>Definition</i>)	
2. Show that both pairs of opposite sides are congruent. (Parallelogram Opposite Sides Converse)	
3. Show that both pairs of opposite angles are congruent. (<i>Parallelogram Opposite Angles Converse</i>)	
4. Show that one pair of opposite sides are congruent and parallel. (<i>Opposite Sides Parallel and Congruent Theorem</i>)	
5. Show that the diagonals bisect each other. (Parallelogram Diagonals Converse)	

Khan Academy Videos:

- 1. <u>Opposite sides</u> of a parallelogram proof
- 2. <u>Opposite angles</u> of a parallelogram proof

Homework: Parallelogram characteristics and problems, <u>Quadrilaterals Worksheet</u>

Reading Assignment: section 7-4

Section 7-4: Properties of Special Parallelograms

SOL: G.9

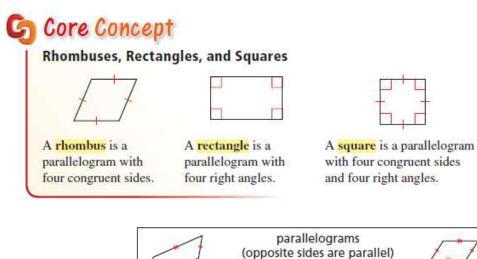
Objective:

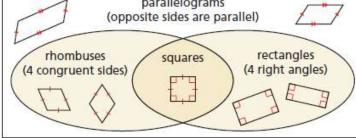
Use properties of special parallelograms Use properties of diagonals of special parallelograms Use coordinate geometry to identify special types of parallelograms

Vocabulary:

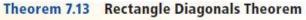
Rectangle – a parallelogram with four right angles Rhombus – a parallelogram with four congruent sides Square – a parallelogram with four congruent sides and four right angles

Core Concept:





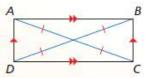
🔄 Theorem



A parallelogram is a rectangle if and only if its diagonals are congruent.

 $\square ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

Proof Exs. 85 and 86, p. 358



G Corollaries

Corollary 7.2 Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.

 $\frac{ABCD}{AB} \stackrel{\text{is a rhombus if and only if}}{\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}.$

Proof Ex. 79, p. 358

Corollary 7.3 Rectangle Corollary

A quadrilateral is a rectangle if and only if it has four right angles.

ABCD is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof Ex. 80, p. 358

Corollary 7.4 Square Corollary

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

ABCD is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof Ex. 81, p. 358

G Theorems

5

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Theorem 7.11 Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

 $\square ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

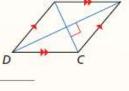
Proof p. 352; Ex. 72, p. 357

Theorem 7.12 Rhombus Opposite Angles Theorem

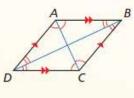
A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

 $\Box ABCD$ is a rhombus if and only if \overline{AC} bisects $\angle BCD$ and $\angle BAD$, and \overline{BD} bisects $\angle ABC$ and $\angle ADC$.

Proof Exs. 73 and 74, p. 357



B







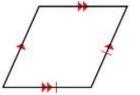


Examples:

Example 1:

For any rectangle *ABCD*, decide whether the statement is always or sometimes true. Explain your reasoning.

- a. AB = BC
- b. AB = CD

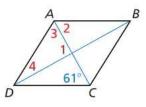


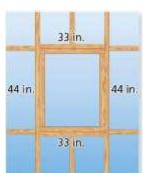
Example 2:

Classify the special quadrilateral. Explain your reasoning.

Example 3:

Find the $m \angle ABC$ and $m \angle ACB$ in the rhombus ABCD



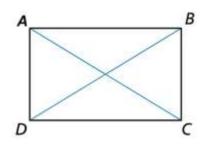


Example 4:

Suppose you measure one angle of the window opening and its measure is 90° . Can you conclude that the shape of the opening is a rectangle? Explain.

Example 5:

In rectangle *ABCD*, AC = 7x - 15 and BD = 2x + 25. Find the lengths of the diagonals of *ABCD*.



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Example 6:

Decide whether quadrilateral *ABCD* with vertices A(-2,3), B(2,2), C(1,-2), and D(-3,-1) is a *rectangle*, a *rhombus*, or a *square*. Give all names that apply.

Concept Summary:

- Rectangle: A parallelogram with four right angles and congruent diagonals
 - Opposite sides parallel and congruent
 - All angles equal 90°
 - Diagonals congruent and bisect each other
 - Diagonals break figure into two separate congruent isosceles triangles
 - Rhombus: A parallelogram with four congruent sides, diagonals that are perpendicular bisectors to each other and angle bisectors of corner angles
 - Opposite sides parallel; all sides congruent
 - Opposite angles congruent; consecutive angles supplementary
 - Diagonals perpendicular, bisect each other and bisect opposite angles
 - Diagonals break figure into 4 congruent triangles
- Square: All rectangle and a rhombus characteristics
 - Opposite sides parallel; all sides congruent
 - All angles equal 90°
 - Diagonals perpendicular, bisect each other and bisect opposite angles
 - Diagonals break figure into 4 congruent triangles

Khan Academy Videos: none relate

Homework: Characteristics and problems, Quadrilaterals Worksheet

Reading Assignment: section 7-5

Section 7-5: Properties of Trapezoids and Kites

SOL: G.9

Objective:

Use properties of trapezoids Use the Trapezoid Midsegment Theorem to find distance Use properties of kites Identify quadrilaterals

Vocabulary:

Bases – parallel sides of a trapezoid

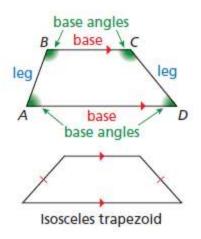
Base angles – consecutive angles whose common side is the base of the trapezoid Isosceles trapezoid – legs of the trapezoid are congruent

Kite – a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent

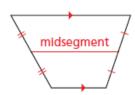
Legs - nonparallel sides of the trapezoid

Midsegment of a trapezoid – segment that connects the legs of the trapezoid; parallel to the bases

Trapezoid – a quadrilateral with exactly one pair of parallel sides







Core Concept:

G Theorems

Theorem 7.14 Isosceles Trapezoid Base Angles Theorem

If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid *ABCD* is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

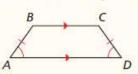
Proof Ex. 39, p. 367

Theorem 7.15 Isosceles Trapezoid Base Angles Converse

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid *ABCD* is isosceles.

Proof Ex. 40, p. 367



Theorem 7.16 Isosceles Trapezoid Diagonals Theorem

A trapezoid is isosceles if and only if its diagonals are congruent.

Trapezoid *ABCD* is isosceles if and only if $\overline{AC} \cong \overline{BD}$.

Proof Ex. 51, p. 368



B

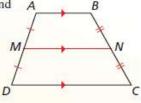
G Theorem

Theorem 7.17 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.

If \overline{MN} is the midsegment of trapezoid ABCD, then $\overline{MN} \parallel \overline{AB}, \overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.

Proof Ex. 49, p. 368



D Theorems

Theorem 7.18 Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular.

If quadrilateral ABCD is a kite, then $\overline{AC} \perp \overline{BD}$.

Proof p. 363

Theorem 7.19 Kite Opposite Angles Theorem

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

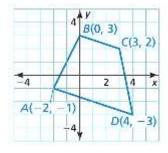
If quadrilateral *ABCD* is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \not\equiv \angle D$.

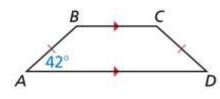
Proof Ex. 47, p. 368

Examples:

Example 1:

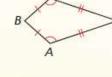
Show that *ABCD* is a trapezoid and decide whether it is isosceles.





Example 2:

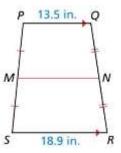
ABCD is an isosceles trapezoid, and $m \angle A = 42^{\circ}$. Find $m \angle B$, $m \angle C$, and $m \angle D$.

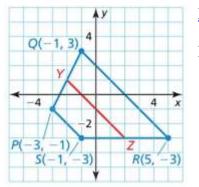


C

Example 3:

In the diagram, \overline{MN} is the midsegment of trapezoid *PQRS*. Find *MN*.



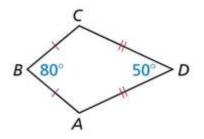


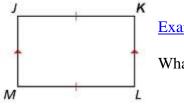
Example 4:

Find the length of midsegment \overline{YZ} in trapezoid PQRS

Example 5:

Find $m \angle C$ in the kite shown.

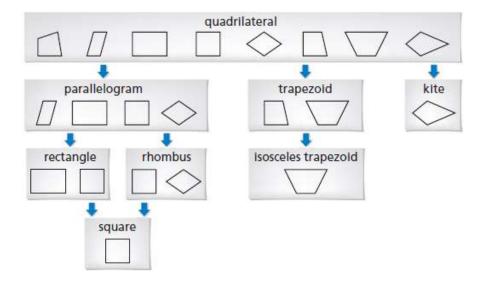




Example 6:

What is the most specific name for quadrilateral JKLM?

Concept Summary:



- In an isosceles trapezoid, both pairs of base angles are congruent and the diagonals are congruent.
- The median of a trapezoid is parallel to the bases and its measure is one-half the sum of the measures of the bases
- Kites have diagonals perpendicular and "arm" angles congruent

Khan Academy Videos:

1. <u>Kites</u> as a geometric shape

Homework: <u>Quadrilateral Worksheet</u>

Reading Assignment: section 7-6

Section 7-6: Tessellations

SOL: G.10

Objectives:

Determine whether a shape tessellates Find angle measures in tessellations of polygons Determine whether a regular polygon tessellates a plane

Vocabulary:

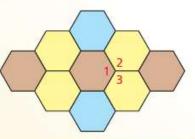
Regular tessellation – a transformation that enlarges or reduces an image *Tessellation* – the covering of a plane with figures so that there are no gaps or overlaps

Key Concept:

🔄 Core Concept

Angle Measures in a Tessellation of Polygons

The sum of the angle measures around a point of intersection in a tessellation of polygons is 360°.



 $m \angle 1 + m \angle 2 + m \angle 3 = 360^{\circ}$

G Core Concept

Regular Tessellations

A regular polygon tessellates a plane if the measure of an interior angle of the polygon is a factor of 360.

Examples:

Example 1:

Determine whether each shape tessellates.

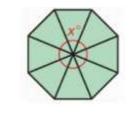
- a. Rhombus
- b. Crescent



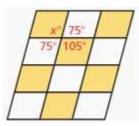
Example 2:

a.

Find x in each tessellation.



b.



Example 3:

Determine whether each polygon tessellates

- a. Equilateral triangle
- b. Regular 13-sided polygon
- c. Regular 14-sided polygon

Concept Summary:

- A tessellation is a repetitious pattern that covers a plane without overlaps or gaps
- Only 3 regular polygons tessellate the plane
 - Triangle (Equilateral)
 - Quadrilateral (Square)
 - Hexagon
- Other irregular polygons can tessellate: rectangles, right isosceles triangle

Khan Academy Videos: none relate

Homework: Chapter <u>Quiz Review</u>

Reading Assignment: read section 7-R

Section 7-R: Chapter Review

SOL: G.10

Objectives:

Review chapter material

Vocabulary: none new

Key Concept:

Angles in convex polygons:

- Interior angle + exterior angle = 180°
 - They are a Linear Pair
- Sum of Interior angles, $S = (n-2) \times 180^{\circ}$
- One Interior angle = $S / n = (n-2) \times 180^{\circ}/n$
- Sum of Exterior angles = 360°
- Number of sides, $n = 360^{\circ}$ / Exterior angle

Quadrilaterals: Sides, Angles and Diagonals

- Parallelograms:
 - Opposite sides parallel and congruent
 - Opposite angles congruent
 - Consecutive angles supplementary
 - Diagonals bisect each other
 - Rectangles:
 - Angles all 90°
 - Diagonals congruent
 - Rhombi:
 - All sides congruent
 - Diagonals perpendicular
 - Diagonals bisect opposite angles
 - Diagonals divide into 4 congruent triangles
 - Squares: Rectangle and Rhombi characteristics
- Trapezoids:
 - o Bases Parallel
 - Legs are not Parallel
 - o Leg angles are supplementary
 - Median is parallel to bases
 - Median = $\frac{1}{2}$ (base + base)
 - Isosceles Trapezoid:
 - Legs are congruent
 - Base angle pairs congruent
 - Diagonals are congruent

- Kites:
 - 2 congruent sides (consecutive)
 - Diagonals perpendicular
 - Diagonals bisect opposite angles
 - One diagonal bisected
 - One pair of opposite angles congruent ("arm" angles)

Homework: SOL Gateway

Reading Assignment: none

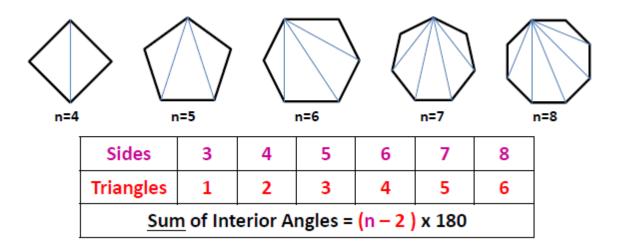
Interior and Exterior always make a linear pair (adds to 180°)



Interior angle + Exterior angle = 180 Exterior angle = 180 – interior angle

To find number of sides: 360 divided by exterior angle n = $360 / Ext \angle$

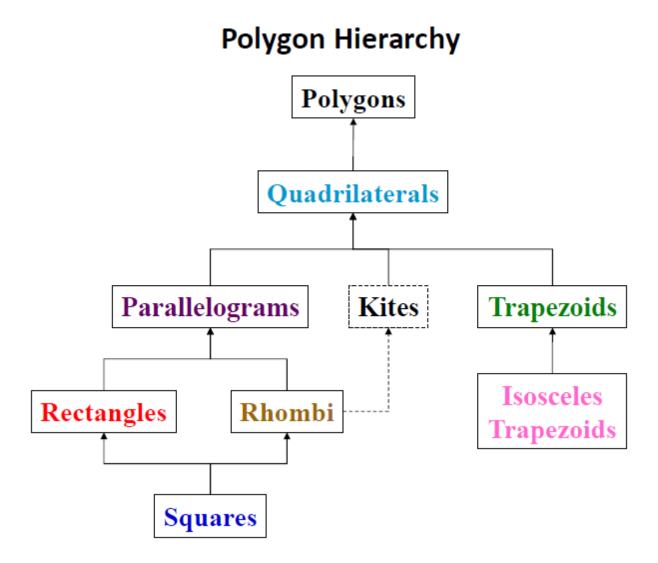
Sometimes use Int + Ext = 180 to find Ext angle



Angles with Polygons

Name	Pic	Nr Sides	Int Sum	Ext Sum	One Int	One Ext
Triangle	\triangle	3	180	360	60	120
Quadrilateral	\diamond	4	360	360	90	90
Pentagon	\bigcirc	5	540	360	108	72
Hexagon	$\langle \rangle$	6	720	360	120	60
Heptagon	\bigcirc	7	900	360	128.57	51.43
Octagon	\bigcirc	8	1080	360	135	35
Nonagon	\bigcirc	9	1260	360	140	40
Decagon	\bigcirc	10	1440	360	144	36
Eleven-gon	\bigcirc	11	1620	360	147.27	32.72
Dodecagon	\bigcirc	12	1800	360	150	30
N-gon	\bigcirc	n	(n-2)×180	360	180-Ext	360/n

Exterior angles always <u>sum</u> to 360 (once around a circle).



Polygons are closed figures with line segments as sides Exterior Angles add to 360

Quadrilaterals are 4-sided figures Interior Angles add to 360

