## Addressed or Prepped VA SOL:

G. 9 The student will verify and use properties of quadrilaterals to solve problems, including practical problems.
G. 10 The student will solve problems, including practical problems, involving angles of convex polygons. This will include determining the
a) sum of the interior and/or exterior angles;
b) measure of an interior and/or exterior angle; and
c) number of sides of a regular polygon.

## SOL Progression

## Middle School:

- Compare and Contrast quadrilaterals based on their properties
- Determine unknown side lengths or angle measures in quadrilaterals
- Solve linear equations with rational number coefficients
- Draw polygons in the coordinate plane given vertices and find lengths of sides


## Algebra I:

- Create equations in one variable
- Solve linear equations in one variable
- Graph in the coordinate plane
- Find the slope of a line
- Identify and write equations of parallel and perpendicular lines


## Geometry:

- Find and use the interior and exterior angle measurements of polygons
- Use properties of parallelograms and special parallelograms
- Prove that a quadrilateral is a parallelogram
- Identify and use properties of trapezoids and kites
- Determine angle measurements of a regular polygon in a tessellation



## Chapter 7: Quadrilaterals and Other Polygons

## Section 7-1: Angles of Polygons

SOL: G. 10

## Objective:

Use the interior angle measures of polygons Use the exterior angle measures of polygons

## Vocabulary:

Convex - no line that contains a side of the polygon goes into the interior of the polygon
Diagonal - a segment of a polygon that joins two nonconsecutive vertices
Equilateral polygon - all sides of the polygon are congruent
Equiangular polygon - all interior angles of the polygon are congruent
Exterior angles - angle outside the polygon formed by an extended side
Interior angles - an angle inside the polygon
Regular polygon - convex polygon that is both equilateral and equiangular

## Core Concept:

Vertex $B$ has two diagonals, $\overline{B D}$ and $\overline{B E}$.

In an equilateral polygon, In an equiangular all sides are congruent.

polygon, all angles in the interior of the polygon are congruent.


A regular polygon is a convex polygon that is both equilateral and equiangular.


## G) Theorem

Theorem 7.1 Polygon Interior Angles Theorem
The sum of the measures of the interior angles of a convex $n$-gon is $(n-2) \cdot 180^{\circ}$.
$m \angle 1+m \angle 2+\cdots+m \angle n=(n-2) \cdot 180^{\circ}$

Proof Ex. 42 (for pentagons), p. 329


Note: Sum of interior angles in a polygon is found by $\mathrm{S}=(\mathrm{n}-2) \times 180^{\circ}$

## C. Corollary

## Corollary 7.1 Corollary to the Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a quadrilateral is $360^{\circ}$.
Proof Ex. 43, p. 330

## Chapter 7: Quadrilaterals and Other Polygons

## Theorem

## Theorem 7.2 Polygon Exterior Angles Theorem

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is $360^{\circ}$.

$$
m \angle 1+m \angle 2+\cdots+m \angle n=360^{\circ}
$$

Proof Ex. 51, p. 330


Note: Sum of exterior angles in a polygon is $360^{\circ}$

## Examples:

Example 1:
Find the sum of the measures of the interior angles of the figure.


Example 2:
The sum of the measures of the interior angles of a convex polygon is $1800^{\circ}$. Classify the polygon by the number of sides.

## Example 3:

Find the value of $x$ in the diagram.


## Example 4:

A polygon is shown.
a. Is the polygon regular? Explain your reasoning
b. Find the measures of $\angle B, \angle D, \angle E$, and $\angle G$.


## Example 5:

Find the value of $x$ in the diagram.


## Example 6:

Each face of the dodecahedron is shaped like a regular pentagon.
a. Find the measure of each interior angle of a regular pentagon.

b. Find the measure of each exterior angle of a regular pentagon.

## Concept Summary:

The sum of exterior angles is always $360^{\circ}$ (regardless of number of sides)
The sum of interior angles is given by the formula, $S=(n-2) \times 180$
To find the number of sides use: $n=360 / E x t$
The interior and exterior angles always form a linear pair (sum to 180)

## Khan Academy Videos:

1. Sum of interior angles of a polygon
2. Sum of exterior angles of a polygon

Homework: 7-1SOL Worksheet
Reading Assignment: student notes section 7-2

## Chapter 7: Quadrilaterals and Other Polygons

## Section 7-2: Properties of Parallelograms

SOL: G. 9

## Objectives:

Use properties to find side lengths and angles of parallelograms Use parallelograms in the coordinate plane

## Vocabulary:

Parallelogram - a quadrilateral with both pairs of opposite sides parallel

## Core Concept:

## G Theorems

## Theorem 7.3 Parallelogram Opposite Sides Theorem

If a quadrilateral is a parallelogram, then its opposite sides are congruent.
If $P Q R S$ is a parallelogram, then $\overline{P Q} \cong \overline{R S}$ and $\overline{Q R} \cong \overline{S P}$.


Proof p. 332
Theorem 7.4 Parallelogram Opposite Angles Theorem
If a quadrilateral is a parallelogram, then its opposite angles are congruent.
If $P Q R S$ is a parallelogram, then $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.

Proof Ex. 37, p. 337


## G) Theorems

## Theorem 7.5 Parallelogram Consecutive Angles Theorem

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.
If $P Q R S$ is a parallelogram, then $x^{\circ}+y^{\circ}=180^{\circ}$.
Proof Ex. 38, p. 337


Theorem 7.6 Parallelogram Diagonals Theorem
If a quadrilateral is a parallelogram, then its diagonals bisect each other.
If $P Q R S$ is a parallelogram, then $\overline{Q M} \cong \overline{S M}$
and $\overline{P M} \cong \overline{R M}$.


Proof p. 334

## Examples:

Example 1:
Find the values of $x$ and $y$.


Example 2:
In parallelogram PQRS, $m \angle P$ is four times $m \angle Q$. Find $m \angle P$.

Example 3:
Write a two-column proof.
Given: ABCD and GDEF are parallelograms
Prove: $\angle C \cong \angle G$


| Statements | Reasons |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |

## Example 4:

Find the coordinates of the intersection of the diagonals of parallelogram $A B C D$ with vertices $A(1,0), B(6,0), C(5,3)$, and $D(0,3)$.


## Example 5:



Three vertices of parallelogram $D E F G$ are $D(-1,4), E(2,3)$, and $F(4,-2)$. Find the coordinates of vertex $G$.

## Concept Summary:

Opposite sides are parallel and congruent
Opposite angles are congruent; Consecutive angles are supplementary
Diagonals bisect each other

## Quadrilateral Characteristics Summary



Khan Academy Videos:

1. Introduction to quadrilaterals
2. Quadrilateral properties

Homework: Parallelogram characteristics and problems, Quadrilaterals Worksheet
Reading Assignment: read section 7-3

## Section 7-3: Proving a Quadrilateral is a Parallelogram

SOL: G. 9

## Objective:

Identify and verify parallelograms
Show that a quadrilateral is a parallelogram in the coordinate plane
Vocabulary: None new

## Core Concepts:

## G) Theorems

## Theorem 7.7 Parallelogram Opposite Sides Converse

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
If $\overline{A B} \cong \overline{C D}$ and $\overline{B C} \cong \overline{D A}$, then $A B C D$ is a parallelogram.


## Theorem 7.8 Parallelogram Opposite Angles Converse

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $A B C D$ is a parallelogram.
Proof Ex. 39, p. 347


## (5) Theorems

## Theorem 7.9 Opposite Sides Parallel and Congruent Theorem

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.
If $\overline{B C} \| \overline{A D}$ and $\overline{B C} \cong \overline{A D}$, then $A B C D$ is a parallelogram.


Proof Ex. 40, p. 347

## Theorem 7.10 Parallelogram Diagonals Converse

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
If $\overline{B D}$ and $\overline{A C}$ bisect each other, then $A B C D$ is a parallelogram.


Proof Ex. 41, p. 347

## Examples:

Example 1:
In quadrilateral $A B C D, A B=B C$ and $C D=A D$. Is $A B C D$ a parallelogram? Explain your reasoning.

Example 2:
For what values of $x$ and $y$ is quadrilateral $S T U V$ a parallelogram?


Example 3:
Use the photograph to the right. Explain how you know that $\angle S \cong \angle U$.


Example 4:
For what value of $x$ is quadrilateral $C D E F$ a parallelogram?

## Example 5:

Show that quadrilateral ABCD is a parallelogram.


## Concept Summary:

## Concept Summary

Ways to Prove a Quadrilateral Is a Parallelogram

1. Show that both pairs of opposite sides are parallel. (Definition)

| 2. Show that both pairs of opposite sides are congruent. |
| :--- |
| (Parallelogram Opposite Sides Converse) |
| 3. Show that both pairs of opposite angles are congruent. |
| (Parallelogram Opposite Angles Converse) |
| (Opposite Sides Parallel and Congruent Theorem) |
| 5. Show that the diagonals bisect each other. |
| (Parallelogram Diagonals Converse) |

## Khan Academy Videos:

1. Opposite sides of a parallelogram proof
2. Opposite angles of a parallelogram proof

Homework: Parallelogram characteristics and problems, Quadrilaterals Worksheet
Reading Assignment: section 7-4

## Chapter 7: Quadrilaterals and Other Polygons

## Section 7-4: Properties of Special Parallelograms

SOL: G. 9

## Objective:

Use properties of special parallelograms
Use properties of diagonals of special parallelograms
Use coordinate geometry to identify special types of parallelograms

## Vocabulary:

Rectangle - a parallelogram with four right angles
Rhombus - a parallelogram with four congruent sides
Square - a parallelogram with four congruent sides and four right angles

## Core Concept:

## G) Core Concept

Rhombuses, Rectangles, and Squares


A rhombus is a parallelogram with four congruent sides.


A rectangle is a parallelogram with four right angles.


A square is a parallelogram with four congruent sides and four right angles.


## (5) Theorem

Theorem 7.13 Rectangle Diagonals Theorem
A parallelogram is a rectangle if and only if its diagonals are congruent.
$\square A B C D$ is a rectangle if and only if $\overline{A C} \cong \overline{B D}$.
Proof Exs. 85 and 86, p. 358


## G) Corollaries

## Corollary 7.2 Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.
$A B C D$ is a rhombus if and only if
$\overline{A B} \cong \overline{B C} \cong \overline{C D} \cong \overline{A D}$.


Proof Ex. 79, p. 358

Corollary 7.3 Rectangle Corollary
A quadrilateral is a rectangle if and only if it has four right angles.
$A B C D$ is a rectangle if and only if
$\angle A, \angle B, \angle C$, and $\angle D$ are right angles.


Proof Ex. 80, p. 358

## Corollary 7.4 Square Corollary

A quadrilateral is a square if and only if it is a rhombus and a rectangle.
$A B C D$ is a square if and only if
$\overline{A B} \cong \overline{B C} \cong \overline{C D} \cong \overline{A D}$ and $\angle A, \angle B, \angle C$,

and $\angle D$ are right angles.
Proof Ex. 81, p. 358

## (5) Theorems

## Theorem 7.11 Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular.
$\square A B C D$ is a rhombus if and only if $\overline{A C} \perp \overline{B D}$.
Proof p. 352; Ex. 72, p. 357


Theorem 7.12 Rhombus Opposite Angles Theorem
A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.
$\square A B C D$ is a rhombus if and only if $\overline{A C}$ bisects $\angle B C D$ and $\angle B A D$, and $\overline{B D}$ bisects $\angle A B C$ and $\angle A D C$.


Proof Exs. 73 and 74, p. 357

## Examples:

## Example 1:

For any rectangle $A B C D$, decide whether the statement is always or sometimes true. Explain your reasoning.
a. $A B=B C$
b. $A B=C D$


Example 2:
Classify the special quadrilateral. Explain your reasoning.

## Example 3:

Find the $m \angle A B C$ and $m \angle A C B$ in the rhombus $A B C D$


## Example 4:

Suppose you measure one angle of the window opening and its measure is $90^{\circ}$. Can you conclude that the shape of the opening is a rectangle? Explain.

## Example 5:

In rectangle $A B C D, A C=7 x-15$ and $B D=2 x+25$. Find the lengths of the diagonals of $A B C D$.


## Example 6:

Decide whether quadrilateral $A B C D$ with vertices $A(-2,3)$, $B(2,2), C(1,-2)$, and $D(-3,-1)$ is a rectangle, a rhombus, or a square. Give all names that apply.

## Concept Summary:

- Rectangle: A parallelogram with four right angles and congruent diagonals
- Opposite sides parallel and congruent
- All angles equal $90^{\circ}$
- Diagonals congruent and bisect each other
- Diagonals break figure into two separate congruent isosceles triangles
- Rhombus: A parallelogram with four congruent sides, diagonals that are perpendicular bisectors to each other and angle bisectors of corner angles
- Opposite sides parallel; all sides congruent
- Opposite angles congruent; consecutive angles supplementary
- Diagonals perpendicular, bisect each other and bisect opposite angles
- Diagonals break figure into 4 congruent triangles
- Square: All rectangle and a rhombus characteristics
- Opposite sides parallel; all sides congruent
- All angles equal $90^{\circ}$
- Diagonals perpendicular, bisect each other and bisect opposite angles
- Diagonals break figure into 4 congruent triangles

Khan Academy Videos: none relate
Homework: Characteristics and problems, Quadrilaterals Worksheet
Reading Assignment: section 7-5

## Chapter 7: Quadrilaterals and Other Polygons

## Section 7-5: Properties of Trapezoids and Kites

SOL: G. 9

## Objective:

Use properties of trapezoids
Use the Trapezoid Midsegment Theorem to find distance Use properties of kites Identify quadrilaterals

## Vocabulary:

Bases - parallel sides of a trapezoid


Base angles - consecutive angles whose common side is the base of the trapezoid
Isosceles trapezoid - legs of the trapezoid are congruent
Kite - a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent


Legs - nonparallel sides of the trapezoid
Midsegment of a trapezoid - segment that connects the legs of the trapezoid; parallel to the bases
Trapezoid - a quadrilateral with exactly one pair of parallel sides


## Core Concept:

## Theorems

Theorem 7.14 Isosceles Trapezoid Base Angles Theorem
If a trapezoid is isosceles, then each pair of base angles is congruent.
If trapezoid $A B C D$ is isosceles, then $\angle A \cong \angle D$
and $\angle B \cong \angle C$.
Proof Ex. 39, p. 367


## Theorem 7.15 Isosceles Trapezoid Base Angles Converse

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.
If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$ ), then trapezoid $A B C D$ is isosceles.
Proof Ex. 40, p. 367


## Theorem 7.16 Isosceles Trapezoid Diagonals Theorem

A trapezoid is isosceles if and only if its diagonals are congruent.
Trapezoid $A B C D$ is isosceles if and only if $\overline{A C} \cong \overline{B D}$.
Proof Ex. 51, p. 368


## G Theorem

## Theorem 7.17 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.
If $\overline{M N}$ is the midsegment of trapezoid $A B C D$, then $\overline{M N}\|\overline{A B}, \overline{M N}\| \overline{D C}$, and $M N=\frac{1}{2}(A B+C D)$.

Proof Ex. 49, p. 368


## (5) Theorems

## Theorem 7.18 Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular.
If quadrilateral $A B C D$ is a kite, then $\overline{A C} \perp \overline{B D}$.
Proof p. 363


Theorem 7.19 Kite Opposite Angles Theorem
If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.
If quadrilateral $A B C D$ is a kite and $\overline{B C} \cong \overline{B A}$, then $\angle A \cong \angle C$ and $\angle B \nRightarrow \angle D$.


Proof Ex. 47, p. 368

## Examples:

## Example 1:

Show that $A B C D$ is a trapezoid and decide whether it is isosceles.



Example 2:
$A B C D$ is an isosceles trapezoid, and $m \angle A=42^{\circ}$. Find $m \angle B$, $m \angle C$, and $m \angle D$.

## Chapter 7: Quadrilaterals and Other Polygons

## Example 3:

In the diagram, $\overline{M N}$ is the midsegment of trapezoid $P Q R S$. Find $M N$.


## Example 4:

Find the length of midsegment $\overline{Y Z}$ in trapezoid $P Q R S$

## Example 5:

Find $m \angle C$ in the kite shown.


## Concept Summary:



- In an isosceles trapezoid, both pairs of base angles are congruent and the diagonals are congruent.
- The median of a trapezoid is parallel to the bases and its measure is one-half the sum of the measures of the bases
- Kites have diagonals perpendicular and "arm" angles congruent


## Khan Academy Videos:

1. Kites as a geometric shape

Homework: Quadrilateral Worksheet
Reading Assignment: section 7-6

## Section 7-6: Tessellations

SOL: G. 10

## Objectives:

Determine whether a shape tessellates
Find angle measures in tessellations of polygons
Determine whether a regular polygon tessellates a plane

## Vocabulary:

Regular tessellation - a transformation that enlarges or reduces an image
Tessellation - the covering of a plane with figures so that there are no gaps or overlaps

## Key Concept:

## C) Core Concept

## Angle Measures in a Tessellation of Polygons

The sum of the angle measures around a point of intersection in a tessellation of polygons is $360^{\circ}$.


## C) Core Concept

Regular Tessellations
A regular polygon tessellates a plane if the measure of an interior angle of the polygon is a factor of 360 .

## Examples:

Example 1:
Determine whether each shape tessellates.
a. Rhombus

b. Crescent

## Example 2:

Find x in each tessellation.
a.

b.

## Example 3:



Determine whether each polygon tessellates
a. Equilateral triangle
b. Regular 13-sided polygon
c. Regular 14-sided polygon

## Concept Summary:

- A tessellation is a repetitious pattern that covers a plane without overlaps or gaps
- Only 3 regular polygons tessellate the plane
- Triangle (Equilateral)
- Quadrilateral (Square)
- Hexagon
- Other irregular polygons can tessellate: rectangles, right isosceles triangle

Khan Academy Videos: none relate
Homework: Chapter Quiz Review
Reading Assignment: read section 7-R

## Section 7-R: Chapter Review

SOL: G. 10

## Objectives:

Review chapter material
Vocabulary: none new

## Key Concept:

Angles in convex polygons:

- Interior angle + exterior angle $=180^{\circ}$
- They are a Linear Pair
- Sum of Interior angles, $\mathrm{S}=(\mathrm{n}-2) \times 180^{\circ}$
- One Interior angle $=S / n=(n-2) \times 180^{\circ} / n$
- Sum of Exterior angles $=360^{\circ}$
- Number of sides, $\mathrm{n}=360^{\circ}$ / Exterior angle

Quadrilaterals: Sides, Angles and Diagonals

- Parallelograms:
- Opposite sides parallel and congruent
- Opposite angles congruent
- Consecutive angles supplementary
- Diagonals bisect each other
- Rectangles:
- Angles all $90^{\circ}$
- Diagonals congruent
- Rhombi:
- All sides congruent
- Diagonals perpendicular
- Diagonals bisect opposite angles
- Diagonals divide into 4 congruent triangles
- Squares: Rectangle and Rhombi characteristics
- Trapezoids:
- Bases Parallel
- Legs are not Parallel
- Leg angles are supplementary
- Median is parallel to bases

Median $=1 / 2($ base + base $)$

- Isosceles Trapezoid:
- Legs are congruent
- Base angle pairs congruent
- Diagonals are congruent
- Kites:
- 2 congruent sides (consecutive)
- Diagonals perpendicular
- Diagonals bisect opposite angles
- One diagonal bisected
- One pair of opposite angles congruent ("arm" angles)

Homework: SOL Gateway
Reading Assignment: none
Interior and Exterior always make a linear pair (adds to $180^{\circ}$ )
interior exterior

Interior angle + Exterior angle $=180$
Exterior angle $=180$ - interior angle
To find number of sides: $\mathbf{3 6 0}$ divided by exterior angle

$$
\mathrm{n}=360 / \mathrm{Ext} \angle
$$

Sometimes use Int + Ext = 180 to find Ext angle


| Sides | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Triangles | 1 | 2 | 3 | 4 | 5 | 6 |

Sum of Interior Angles $=(\mathrm{n}-2) \times 180$

Angles with Polygons

| Name | Pic | Nr <br> Sides | Int Sum | Ext <br> Sum | One Int | One Ext |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Triangle |  | 3 | 180 | 360 | 60 | 120 |
| Quadrilateral |  | 4 | 360 | 360 | 90 | 90 |
| Pentagon |  | 5 | 540 | 360 | 108 | 72 |
| Hexagon |  | 6 | 720 | 360 | 120 | 60 |
| Heptagon |  | 7 | 900 | 360 | 128.57 | 51.43 |
| Octagon |  | 8 | 1080 | 360 | 135 | 35 |
| Nonagon |  | 9 | 1260 | 360 | 140 | 40 |
| Decagon |  | 10 | 1440 | 360 | 144 | 36 |
| Eleven-gon | (1) | 11 | 1620 | 360 | 147.27 | 32.72 |
| Dodecagon |  | 12 | 1800 | 360 | 150 | 30 |
| N-gon |  | n | $(\mathrm{n}-2) \times 180$ | 360 | 180-Ext | 360/n |

Exterior angles always sum to 360 (once around a circle).

## Polygon Hierarchy



Polygons are closed figures with line segments as sides Exterior Angles add to 360

Quadrilaterals are 4-sided figures
Interior Angles add to 360

Parallelogram:
Sides: Opposite sides parallel and congruent
Angles: Opposite angles congruent Consecutive angles supplementary
Diagonals: Bisect each other

Rectangle:


Sides: Opposite sides parallel and congruent
Angles: Opposite angles congruent
Consecutive angles supplementary
Corner angles $=90^{\circ}$
Diagonals: Bisect each other

## Rhombus



Sides: Opposite sides parallel and congruent All four sides equal
Angles: Opposite angles congruent
Consecutive angles supplementary
Diagonals: Bisect each other Bisect opposite angles
Perpendicular to each other


Divides into 4 congruent triangles

Square:
Sides: Opposite sides parallel and congruent All four sides equal
Angles: Opposite angles congruent Consecutive angles supplementary Corner angles $=90^{\circ}$
Diagonals: Bisect each other
Congruent
Bisect opposite angles


Perpendicular to each other
Divides into 4 congruent triangles
Trapezoid
Sides:
Bases are parallel Legs are not parallel
Angles: Leg angles supplementary
Diagonals: nothing special
Median: parallel to the bases

connects leg midpoint to other leg midpoint
formula: Median $=\frac{\text { Base } 1+\text { Base } 2}{2}$

## Isosceles Trapezoid

Sides: Bases are parallel Legs are not parallel but are congruent
Angles: Leg angles supplementary


Diagonals: congruent
Median: parallel to the bases connects leg midpoint to other leg midpoint formula: Median $=\frac{\text { Base } 1+\text { Base } 2}{2}$

