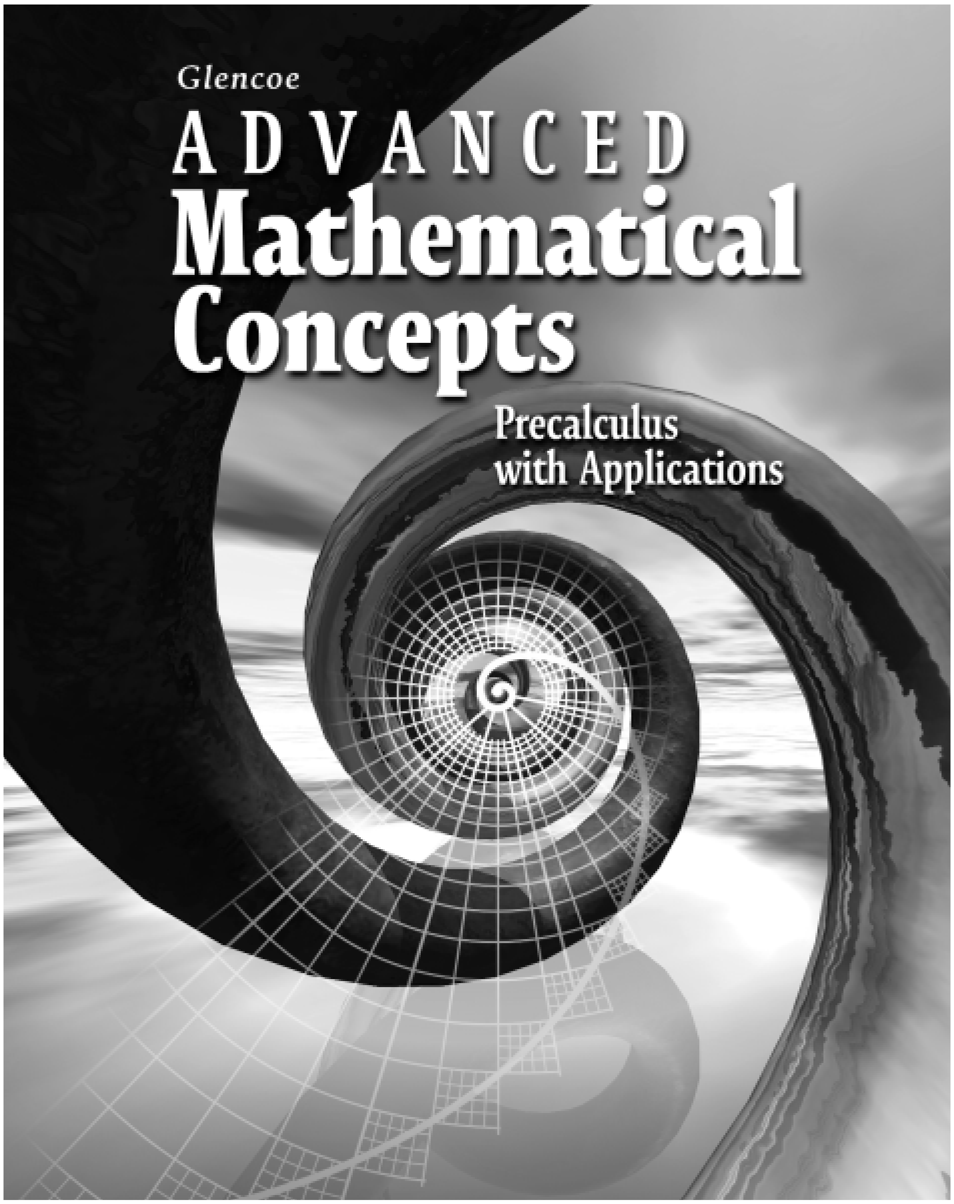
Chapter 7
Resource Masters



Glencoe

New York, New York Columbus, Ohio Woodland Hills, California Peoria, Illinois

StudentWorks™ This CD-ROM includes the entire Student Edition along with the Study Guide, Practice, and Enrichment masters.

TeacherWorks™ All of the materials found in this booklet are included for viewing and printing in the *Advanced Mathematical Concepts TeacherWorks* CD-ROM.

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Advanced Mathematical Concepts
Chapter 7 Resource Masters

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Contents

A Teacher's Guide to Using the Chapter 7 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 7 Resource Masters* include the core materials needed for Chapter 7. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Advanced Mathematical Concepts TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii-viii include a student study tool that presents the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

When to Use Give these pages to students before beginning Lesson 7-1. Remind them to add definitions and examples as they complete each lesson.

Study Guide There is one Study Guide master for each lesson.

When to Use Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for those students who have been absent.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice section of the Student Edition exercises. These exercises are of average difficulty.

When to Use These provide additional practice options or may be used as homework for second day teaching of the lesson.

Enrichment There is one master for each lesson. These activities may extend the concepts in the lesson, offer a historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

When to Use These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment section of the *Chapter 7 Resources Masters* offers a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessments

Chapter Tests

- *Forms 1A, 1B, and 1C* Form 1 tests contain multiple-choice questions. Form 1A is intended for use with honors-level students, Form 1B is intended for use with average-level students, and Form 1C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.
- *Forms 2A, 2B, and 2C* Form 2 tests are composed of free-response questions. Form 2A is intended for use with honors-level students, Form 2B is intended for use with average-level students, and Form 2C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.

  All of the above tests include a challenging Bonus question.
- The **Extended Response Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.

Intermediate Assessment

- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of free-response questions.
- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.

Continuing Assessment

- The **SAT and ACT Practice** offers continuing review of concepts in various formats, which may appear on standardized tests that they may encounter. This practice includes multiple-choice, quantitative-comparison, and grid-in questions. Bubble-in and grid-in answer sections are provided on the master.
- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of advanced mathematics. It can also be used as a test. The master includes free-response questions.

Answers

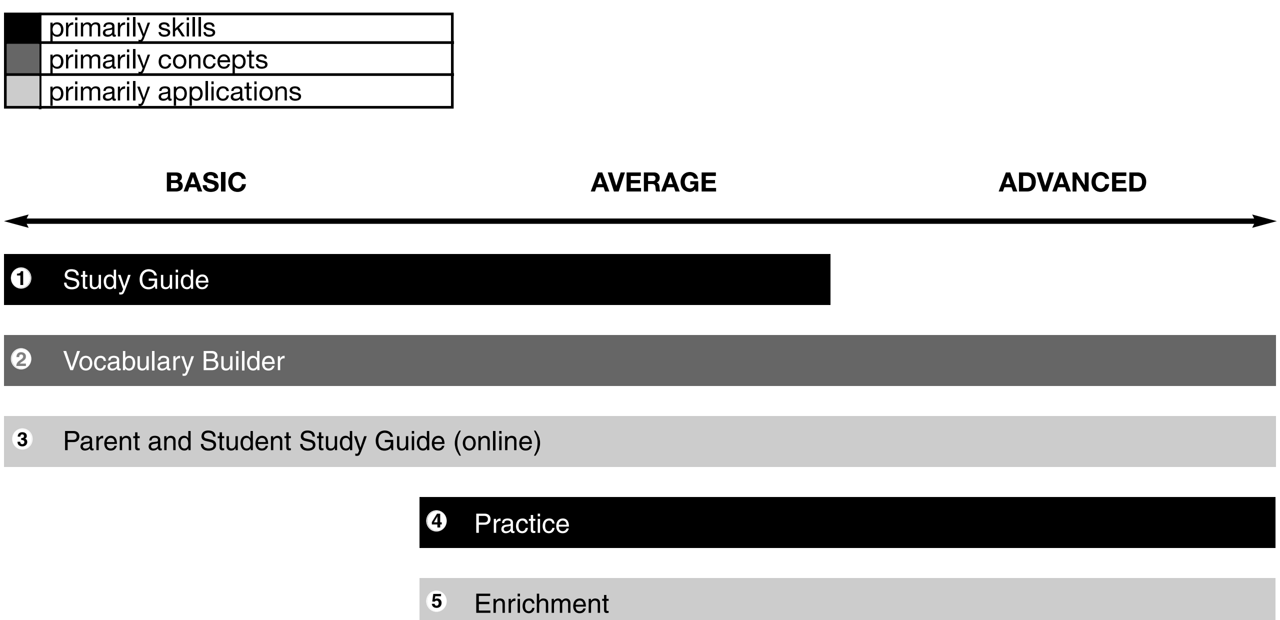
- Page A1 is an answer sheet for the SAT and ACT Practice questions that appear in the Student Edition on page 483. Page A2 is an answer sheet for the SAT and ACT Practice master. These improve students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment options in this booklet.

Chapter 7 Leveled Worksheets

Glencoe's **leveled worksheets** are helpful for meeting the needs of every student in a variety of ways. These worksheets, many of which are found in the **FAST FILE Chapter Resource Masters**, are shown in the chart below.

- **Study Guide** masters provide worked-out examples as well as practice problems.
- Each chapter's **Vocabulary Builder** master provides students the opportunity to write out key concepts and definitions in their own words.
- **Practice** masters provide average-level problems for students who are moving at a regular pace.
- **Enrichment** masters offer students the opportunity to extend their learning.

Five Different Options to Meet the Needs of Every Student in a Variety of Ways



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Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 7. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term.

Vocabulary Term	Found on Page	Definition/Description/Example
counterexample		
difference identity		
double-angle identity		
half-angle identity		
identity		
normal form		
normal line		
opposite-angle identity		
principal value		
Pythagorean identity		

(continued on the next page)

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Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
quotient identity		
reciprocal identity		
reduction identity		
sum identity		
symmetry identity		
trigonometric identity		

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Study Guide

Basic Trigonometric Identities

You can use the **trigonometric identities** to help find the values of trigonometric functions.

Example 1 **If $\sin \theta = \frac{3}{5}$, find $\tan \theta$.**

Use two identities to relate $\sin \theta$ and $\tan \theta$.

$\sin^2 \theta + \cos^2 \theta = 1$ *Pythagorean identity*

$\left(\frac{3}{5}\right)^2 + \cos^2 \theta = 1$ *Substitute $\frac{3}{5}$ for sin θ.*

$\cos^2 \theta = \frac{16}{25}$

$\cos \theta = \pm\sqrt{\frac{16}{25}}$ or $\pm\frac{4}{5}$

Now find $\tan \theta$.

$\tan \theta = \frac{\sin \theta}{\cos \theta}$ *Quotient identity*

$\tan \theta = \frac{\frac{3}{5}}{\pm\frac{4}{5}}$

$\tan \theta = \pm\frac{3}{4}$

To determine the sign of a function value, use the **symmetry identities** for sine and cosine. To use these identities with radian measure, replace 180° with π and 360° with 2π.

Case 1:	$\sin (A + 360k°) = \sin A$	$\cos (A + 360k°) = \cos A$
Case 2:	$\sin [A + 180°(2k - 1)] = -\sin A$	$\cos [A + 180°(2k - 1)] = -\cos A$
Case 3:	$\sin (360k° - A) = -\sin A$	$\cos (360k° - A) = \cos A$
Case 4:	$\sin [180°(2k - 1) - A] = \sin A$	$\cos [180°(2k - 1) - A] = -\cos A$

Example 2 Express $\tan \frac{11\pi}{3}$ as a trigonometric function of an angle in Quadrant I.

The sum of $\frac{11\pi}{3}$ and $\frac{\pi}{3}$, which is $\frac{12\pi}{3}$ or 4π, is a multiple of 2π.

$\frac{11\pi}{3} = 2(2\pi) - \frac{\pi}{3}$ *Case 3, with $A = \frac{\pi}{3}$ and $k = 2$*

$\tan \frac{11\pi}{3} = \frac{\sin \frac{11\pi}{3}}{\cos \frac{11\pi}{3}}$ *Quotient identity*

$= \frac{\sin \left[2(2\pi) - \frac{\pi}{3}\right]}{\cos \left[2(2\pi) - \frac{\pi}{3}\right]}$

$= \frac{-\sin \frac{\pi}{3}}{\cos \frac{\pi}{3}}$ *Symmetry identities*

$= -\tan \frac{\pi}{3}$ *Quotient identity*

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Practice

Basic Trigonometric Identities

Use the given information to determine the exact trigonometric value if $0° < \theta < 90°$.

1. If $\cos \theta = \frac{1}{4}$, find $\tan \theta$.

2. If $\sin \theta = \frac{2}{3}$, find $\cos \theta$.

3. If $\tan \theta = \frac{7}{2}$, find $\sin \theta$.

4. If $\tan \theta = 2$, find $\cot \theta$.

Express each value as a trigonometric function of an angle in Quandrant I.

5. $\cos 892°$

6. $\csc 495°$

7. $\sin \frac{23\pi}{3}$

Simplify each expression.

8. $\cos x + \sin x \tan x$

9. $\frac{\cot A}{\tan A}$

10. $\sin^2 \theta \cos^2 \theta - \cos^2 \theta$

11. ***Kite Flying*** Brett and Tara are flying a kite. When the string is tied to the ground, the height of the kite can be determined by the formula $\frac{L}{H} = \csc \theta$, where L is the length of the string and θ is the angle between the string and the level ground. What formula could Brett and Tara use to find the height of the kite if they know the value of $\sin \theta$?

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Enrichment

The Physics of Soccer

Recall from Lesson 7-1 that the formula for the maximum height h of a projectile is $h = \frac{v_0^2 \sin^2 \theta}{2g}$, where θ is the measure of the angle of elevation in degrees, v_0 is the initial velocity in feet per second, and g is the acceleration due to gravity in feet per second squared.

Solve. Give answers to the nearest tenth.

1. A soccer player kicks a ball at an initial velocity of 60 ft/s and an angle of elevation of 40°. The acceleration due to gravity is 32 ft/s^2. Find the maximum height reached by the ball.

2. With what initial velocity must you kick a ball at an angle of 35° in order for it to reach a maximum height of 20 ft?

The distance d that a projected object travels is given by the formula $d = \frac{2v_0^2 \sin \theta \cos \theta}{g}$.

3. Find the distance traveled by the ball described in Exercise 1.

In order to kick a ball the greatest possible distance at a given initial velocity, a soccer player must maximize $d = \frac{2v_0^2 \sin \theta \cos \theta}{g}$. Since 2, v_0, and g are constants, this means the player must maximize $\sin \theta \cos \theta$.

sin 0°cos 0°	= sin 90°cos 90°	= 0
sin 10°cos 10°	= sin 80°cos 80°	= 0.1710
sin 20°cos 20°	= sin 70°cos 70°	= 0.3214

4. Use the patterns in the table to hypothesize a value of θ for which $\sin \theta \cos \theta$ will be maximal. Use a calculator to check your hypothesis. At what angle should the player kick the ball to achieve the greatest distance?

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Study Guide

Verifying Trigonometric Identities

When verifying trigonometric identities, you cannot add or subtract quantities from each side of the identity. An unverified identity is not an equation, so the properties of equality do not apply.

Example 1 **Verify that $\frac{\sec^2 x - 1}{\sec^2 x} = \sin^2 x$ is an identity.**

Since the left side is more complicated, transform it into the expression on the right.

$$\frac{\sec^2 x - 1}{\sec^2 x} \stackrel{?}{=} \sin^2 x$$

$$\frac{(\tan^2 x + 1) - 1}{\sec^2 x} \stackrel{?}{=} \sin^2 x \qquad \textit{sec}^2\ x = \textit{tan}^2\ x + 1$$

$$\frac{\tan^2 x}{\sec^2 x} \stackrel{?}{=} \sin^2 x \qquad \textit{Simplify.}$$

$$\frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}} \stackrel{?}{=} \sin^2 x \qquad \textit{tan}^2\ x = \frac{\sin^2 x}{\cos^2 x},\ \textit{sec}^2\ x = \frac{1}{\cos^2 x}$$

$$\frac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x \stackrel{?}{=} \sin^2 x$$

$$\sin^2 x = \sin^2 x \qquad \textit{Multiply.}$$

The techniques that you use to verify trigonometric identities can also be used to simplify trigonometric equations.

Example 2 **Find a numerical value of one trigonometric function of x if $\cos x \csc x = 3$.**

You can simplify the trigonometric epression on the left side by writing it in terms of sine and cosine.

$$\cos x \csc x = 3$$

$$\cos x \cdot \frac{1}{\sin x} = 3 \qquad \textit{csc}\ x = \frac{1}{\sin x}$$

$$\frac{\cos x}{\sin x} = 3 \qquad \textit{Multiply.}$$

$$\cot x = 3 \qquad \textit{cot}\ x = \frac{\cos x}{\sin x}$$

Therefore, if $\cos x \csc x = 3$, then $\cot x = 3$.

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7-2

Practice

Verifying Trigonometric Identities

Verify that each equation is an identity.

1. $\frac{\csc x}{\cot x + \tan x} = \cos x$

2. $\frac{1}{\sin y - 1} - \frac{1}{\sin y + 1} = -2 \sec^2 y$

3. $\sin^3 x - \cos^3 x = (1 + \sin x \cos x)(\sin x - \cos x)$

4. $\tan \theta + \frac{\cos \theta}{1 + \sin \theta} = \sec \theta$

Find a numerical value of one trigonometric function of x.

5. $\sin x \cot x = 1$

6. $\sin x = 3 \cos x$

7. $\cos x = \cot x$

8. ***Physics*** The work done in moving an object is given by the formula $W = Fd \cos \theta$, where d is the displacement, F is the force exerted, and θ is the angle between the displacement and the force. Verify that $W = Fd \frac{\cot \theta}{\csc \theta}$ is an equivalent formula.

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Enrichment

Building from 1 = 1

By starting with the most fundamental identity of all, 1 = 1, you can create new identities as complex as you would like them to be.

First, think of ways to write 1 using trigonometric identities. Some examples are the following.

$$1 = \cos A \sec A$$

$$1 = \csc^2 A - \cot^2 A$$

$$1 = \frac{\cos (A + 360°)}{\cos (360° - A)}$$

Choose two such expressions and write a new identity.

$$\cos A \sec A = \csc^2 A - \cot^2 A$$

Now multiply the terms of the identity by the terms of another identity of your choosing, preferably one that will allow some simplification upon multiplication.

$$\begin{array}{rl} & \cos A \sec A = \csc^2 A - \cot^2 A \\ \times & \frac{\sin A}{\cos A} = \tan A \\ \hline & \sin A \sec A = \tan A \csc^2 A - \cot A \end{array}$$

Beginning with 1 = 1, create two trigonometric identities.

1. ______________________________

2. ______________________________

Verify that each of the identities you created is an identity.

3. ______________________ **4.** ______________________

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Study Guide

Sum and Difference Identities

You can use the **sum and difference identities** and the values of the trigonometric functions of common angles to find the values of trigonometric functions of other angles. Notice how the addition and subtraction symbols are related in the sum and difference identities.

Sum and Difference Identities	
Cosine function	$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
Sine function	$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
Tangent function	$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Example 1 **Use the sum or difference identity for cosine to find the exact value of cos 375°.**

$375° = 360° + 15°$

$\cos 375° = \cos 15°$ *Symmetry identity, Case 1*

$\cos 15° = \cos(60° - 45°)$ *60° and 45° are two common angles that differ by 15°.*

$\cos 15° = \cos 60° \cos 45° + \sin 60° \sin 45°$ *Difference identity for cosine*

$\cos 15° = \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$ or $\frac{\sqrt{2}+\sqrt{6}}{4}$

Example 2 **Find the value of $\sin(x + y)$ if $0 < x < \frac{\pi}{2}$, $0 < y < \frac{\pi}{2}$, $\sin x = \frac{3}{5}$, and $\sin y = \frac{12}{37}$.**

In order to use the sum identity for sine, you need to know cos x and cos y. Use a Pythagorean identity to determine the necessary values.

$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow \cos^2\alpha = 1 - \sin^2\alpha.$ *Pythagorean identity*

Since it is given that the angles are in Quadrant I, the values of sine and cosine are positive. Therefore, $\cos\alpha = \sqrt{1 - \sin^2\alpha}$.

$\cos x = \sqrt{1 - \left(\frac{3}{5}\right)^2}$ $\qquad$ $\cos y = \sqrt{1 - \left(\frac{12}{37}\right)^2}$

$= \sqrt{\frac{16}{25}}$ or $\frac{4}{5}$ $\qquad$ $= \sqrt{\frac{1225}{1369}}$ or $\frac{35}{37}$

Now substitute these values into the sum identity for sine.

$\sin(x + y) = \sin x \cos y + \cos x \sin y$

$= \left(\frac{3}{5}\right)\left(\frac{35}{37}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{37}\right)$ or $\frac{153}{185}$

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7-3

Practice

Sum and Difference Identities

Use sum or difference identities to find the exact value of each trigonometric function.

1. $\cos \frac{5\pi}{12}$

2. $\sin(-165°)$

3. $\tan 345°$

4. $\csc 915°$

5. $\tan\left(-\frac{7\pi}{12}\right)$

6. $\sec \frac{\pi}{12}$

Find each exact value if $0 < x < \frac{\pi}{2}$ and $0 < y < \frac{\pi}{2}$.

7. $\cos(x + y)$ if $\sin x = \frac{5}{13}$ and $\sin y = \frac{4}{5}$

8. $\sin(x - y)$ if $\cos x = \frac{8}{17}$ and $\cos y = \frac{3}{5}$

9. $\tan(x - y)$ if $\csc x = \frac{13}{5}$ and $\cot y = \frac{4}{3}$

Verify that each equation is an identity.

10. $\cos(180° - \theta) = -\cos\theta$

11. $\sin(360° + \theta) = \sin\theta$

12. ***Physics*** Sound waves can be modeled by equations of the form $y = 20 \sin(3t + \theta)$. Determine what type of interference results when sound waves modeled by the equations $y = 20 \sin(3t + 90°)$ and $y = 20 \sin(3t + 270°)$ are combined. (*Hint:* Refer to the application in Lesson 7-3.)

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Enrichment

Identities for the Products of Sines and Cosines

By adding the identities for the sines of the sum and difference of the measures of two angles, a new identity is obtained.

$$\begin{aligned} \sin(\alpha + \beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta \\ \sin(\alpha - \beta) &= \sin\alpha\cos\beta - \cos\alpha\sin\beta \\ \hline (\boldsymbol{i})\ \sin(\alpha + \beta) + \sin(\alpha - \beta) &= 2\sin\alpha\cos\beta \end{aligned}$$

This new identity is useful for expressing certain products as sums.

Example **Write sin 3θ cos θ as a sum.**

In the right side of identity (***i***) let $\alpha = 3\theta$ and $\beta = \theta$ so that $2\sin 3\theta\cos\theta = \sin(3\theta + \theta) + \sin(3\theta - \theta)$.
Thus, $\sin 3\theta\cos\theta = \frac{1}{2}\sin 4\theta + \frac{1}{2}\sin 2\theta$.

By subtracting the identities for $\sin(\alpha + \beta)$ and $\sin(\alpha - \beta)$, you obtain a similar identity for expressing a product as a difference.

$$(\boldsymbol{ii})\ \sin(\alpha + \beta) - \sin(\alpha - \beta) = 2\cos\alpha\sin\beta$$

Example **Verify the identity** $\dfrac{\cos 2x\sin x}{\sin 2x\cos x} = \dfrac{(\sin 3x - \sin x)^2}{\sin^2 3x - \sin^2 x}$.

In the right sides of identities (***i***) and (***ii***) let $\alpha = 2x$ and $\beta = x$. Then write the following quotient.

$$\frac{2\cos 2x\sin x}{2\sin 2x\cos x} = \frac{\sin(2x + x) - \sin(2x - x)}{\sin(2x + x) + \sin(2x - x)}$$

By simplifying and multiplying by the conjugate, the identity is verified.

$$\begin{aligned} \frac{\cos 2x\sin x}{\sin 2x\cos x} &= \frac{\sin 3x - \sin x}{\sin 3x + \sin x} \cdot \frac{\sin 3x - \sin x}{\sin 3x - \sin x} \\ &= \frac{(\sin 3x - \sin x)^2}{\sin^2 3x - \sin^2 x} \end{aligned}$$

Complete.

1. Use the identities for $\cos(\alpha + \beta)$ and $\cos(\alpha - \beta)$ to find identities for expressing the products $2\cos\alpha\cos\beta$ and $2\sin\alpha\sin\beta$ as a sum or difference.

2. Find the value of $\sin 105° \cos 75°$ by using the identity above.

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Study Guide

Double-Angle and Half-Angle Identities

Example 1 **If $\sin\theta = \frac{1}{4}$ and θ has its terminal side in the first quadrant, find the exact value of $\sin 2\theta$.**

To use the double-angle identity for $\sin 2\theta$, we must first find $\cos\theta$.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(\frac{1}{4}\right)^2 + \cos^2\theta = 1 \qquad \textit{sin } \theta = \frac{1}{4}$$

$$\cos^2\theta = \frac{15}{16}$$

$$\cos\theta = \frac{\sqrt{15}}{4}$$

Now find $\sin 2\theta$.

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \textit{Double-angle identity for sine}$$

$$= 2\left(\frac{1}{4}\right)\frac{\sqrt{15}}{4} \qquad \textit{sin } \theta = \frac{1}{4}, \textit{ cos } \theta = \frac{\sqrt{15}}{4}$$

$$= \frac{\sqrt{15}}{8}$$

Example 2 **Use a half-angle identity to find the exact value of $\sin\frac{\pi}{12}$.**

$$\sin\frac{\pi}{12} = \sin\frac{\frac{\pi}{6}}{2}$$

$$= \sqrt{\frac{1-\cos\frac{\pi}{6}}{2}}$$

Use $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$. *Since* $\frac{\pi}{12}$ *is in Quadrant I, choose the positive sine value.*

$$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{2-\sqrt{3}}{4}}$$

$$= \frac{\sqrt{2-\sqrt{3}}}{2}$$

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7-4 Practice

Double-Angle and Half-Angle Identities

Use a half-angle identity to find the exact value of each function.

1. $\sin 105°$

2. $\tan \frac{\pi}{8}$

3. $\cos \frac{5\pi}{8}$

Use the given information to find sin 2θ, cos 2θ, and tan 2θ.

4. $\sin \theta = \frac{12}{13}, 0° < \theta < 90°$

5. $\tan \theta = \frac{1}{2}, \pi < \theta < \frac{3\pi}{2}$

6. $\sec \theta = -\frac{5}{2}, \frac{\pi}{2} < \theta < \pi$

7. $\sin \theta = \frac{3}{5}, 0 < \theta < \frac{\pi}{2}$

Verify that each equation is an identity.

8. $1 + \sin 2x = (\sin x + \cos x)^2$

9. $\cos x \sin x = \frac{\sin 2x}{2}$

10. ***Baseball*** A batter hits a ball with an initial velocity v_0 of 100 feet per second at an angle θ to the horizontal. An outfielder catches the ball 200 feet from home plate. Find θ if the range of a projectile is given by the formula $R = \frac{1}{32}v_0^2 \sin 2\theta$.

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Enrichment

Reading Mathematics: Using Examples

Most mathematics books, including this one, use examples to illustrate the material of each lesson. Examples are chosen by the authors to show how to apply the methods of the lesson and to point out places where possible errors can arise.

1. Explain the purpose of Example 1c in Lesson 7-4.

2. Explain the purpose of Example 3 in Lesson 7-4.

3. Explain the purpose of Example 4 in Lesson 7-4.

To make the best use of the examples in a lesson, try following this procedure:

a. When you come to an example, stop. Think about what you have just read. If you don't understand it, reread the previous section.

b. Read the example problem. Then instead of reading the solution, try solving the problem yourself.

c. After you have solved the problem or gone as far as you can go, study the solution given in the text. Compare your method and solution with those of the authors. If necessary, find out where you went wrong. If you don't understand the solution, reread the text or ask your teacher for help.

4. Explain the advantage of working an example yourself over simply reading the solution given in the text.

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7-5

Study Guide

Solving Trigonometric Equations

When you solve trigonometric equations for **principal values** of x, x is in the interval $-90° \leq x \leq 90°$ for $\sin x$ and $\tan x$. For $\cos x$, x is in the interval $0° \leq x \leq 180°$. If an equation cannot be solved easily by factoring, try writing the expressions in terms of only one trigonometric function.

Example 1 **Solve $\tan x \cos x - \cos x = 0$ for principal values of x. Express solutions in degrees.**

$\tan x \cos x - \cos x = 0$

$\cos x (\tan x - 1) = 0$ *Factor.*

$\cos x = 0$ or $\tan x - 1 = 0$ *Set each factor equal to 0.*

$x = 90°$ $\tan x = 1$

$x = 45°$

When $x = 90°$, $\tan x$ is undefined, so the only principal value is 45°.

Example 2 **Solve $2 \tan^2 x - \sec^2 x + 3 = 1 - 2 \tan x$ for $0 \leq x < 2\pi$.**

This equation can be written in terms of $\tan x$ only.

$2 \tan^2 x - \sec^2 x + 3 = 1 - 2 \tan x$

$2 \tan^2 x - (\tan^2 x + 1) + 3 = 1 - 2 \tan x$ *$\sec^2 x = \tan^2 x + 1$*

$\tan^2 x + 2 = 1 - 2 \tan x$ *Simplify.*

$\tan^2 x + 2 \tan x + 1 = 0$

$(\tan x + 1)^2 = 0$ *Factor.*

$\tan x + 1 = 0$ *Take the square root of each side.*

$\tan x = -1$

$x = \frac{3\pi}{4}$ or $x = \frac{7\pi}{4}$

When you solve for all values of x, the solution should be represented as $x + 360°k$ or $x + 2\pi k$ for $\sin x$ and $\cos x$ and $x + 180°k$ or $x + \pi k$ for $\tan x$, where k is any integer.

Example 3 **Solve $\sin x + \sqrt{3} = -\sin x$ for all real values of x.**

$\sin x + \sqrt{3} = -\sin x$

$2 \sin x + \sqrt{3} = 0$

$2 \sin x = -\sqrt{3}$

$\sin x = -\frac{\sqrt{3}}{2}$

$x = \frac{4\pi}{3} + 2\pi k$ or $x = \frac{5\pi}{3} + 2\pi k$, where k is any integer

The solutions are $\frac{4\pi}{3} + 2\pi k$ and $\frac{5\pi}{3} + 2\pi k$.

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Practice

Solving Trigonometric Equations

Solve each equation for principal values of x. Express solutions in degrees.

1. $\cos x = 3 \cos x - 2$

2. $2 \sin^2 x - 1 = 0$

Solve each equation for 0° ≤ x < 360°.

3. $\sec^2 x + \tan x - 1 = 0$

4. $\cos 2x + 3 \cos x - 1 = 0$

Solve each equation for 0 ≤ x < 2π.

5. $4 \sin^2 x - 4 \sin x + 1 = 0$

6. $\cos 2x + \sin x = 1$

Solve each equation for all real values of x.

7. $3 \cos 2x - 5 \cos x = 1$

8. $2 \sin^2 x - 5 \sin x + 2 = 0$

9. $3 \sec^2 x - 4 = 0$

10. $\tan x (\tan x - 1) = 0$

11. ***Aviation*** An airplane takes off from the ground and reaches a height of 500 feet after flying 2 miles. Given the formula $H = d \tan \theta$, where H is the height of the plane and d is the distance (along the ground) the plane has flown, find the angle of ascent θ at which the plane took off.

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Enrichment

The Spectrum

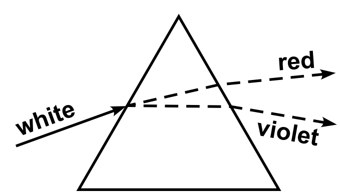
In some ways, light behaves as though it were composed of waves. The wavelength of visible light ranges from about 4×10^{-5} cm for violet light to about 7×10^{-5} cm for red light.

As light passes through a medium, its velocity depends upon the wavelength of the light. The greater the wavelength, the greater the velocity. Since white light, including sunlight, is composed of light of varying wavelengths, waves will pass through the medium at an infinite number of different speeds. The index of refraction n of the medium is defined by $n = \frac{c}{v}$, where c is the velocity of light in a vacuum (3×10^{10} cm/s), and v is the velocity of light in the medium. As you can see, the index of refraction of a medium is not a constant. It depends on the wavelength and the velocity of light passing through it. (The index of refraction of diamond given in the lesson is an average.)

1. For all media, $n > 1$. Is the speed of light in a medium greater than or less than c? Explain.

2. A beam of violet light travels through water at a speed of 2.234×10^{10} cm/s. Find the index of refraction of water for violet light.

The diagram shows why a prism splits white light into a spectrum. Because they travel at different velocities in the prism, waves of light of different colors are refracted different amounts.



3. Beams of red and violet light strike crown glass at an angle of 20°. Use Snell's Law to find the difference between the angles of refraction of the two beams.

violet light: $n = 1.531$ red light: $n = 1.513$

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Study Guide

Normal Form of a Linear Equation

Normal Form	The normal form of a linear equation is $x \cos \phi + y \sin \phi - p = 0$, where p is the length of the normal from the line to the origin and ϕ is the positive angle formed by the positive x-axis and the normal.

You can write the standard form of a linear equation if you are given the values of ϕ and p.

Example 1 **Write the standard form of the equation of a line for which the length of the normal segment to the origin is 5 and the normal makes an angle of 135° with the positive x-axis.**

$$x \cos \phi + y \sin \phi - p = 0 \quad \textit{Normal form}$$
$$x \cos 135° + y \sin 135° - 5 = 0 \quad \phi = 135° \textit{ and } p = 5$$
$$-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - 5 = 0$$
$$\sqrt{2}x - \sqrt{2}y + 10 = 0 \quad \textit{Multiply each side by } -2.$$

The standard form of the equation is $\sqrt{2}x - \sqrt{2}y + 10 = 0$.

The standard form of a linear equation, $Ax + By + C = 0$, can be changed to the normal form by dividing each term of the equation by $\pm\sqrt{A^2 + B^2}$. The sign is chosen opposite the sign of C. You can then find the length of the normal, p units, and the angle ϕ.

Example 2 **Write $3x + 4y - 10 = 0$ in normal form. Then find the length of the normal and the angle it makes with the positive x-axis.**

Since C is negative, use $\sqrt{A^2 + B^2}$ to determine the normal form.

$\sqrt{A^2 + B^2} = \sqrt{3^2 + 4^2}$ or 5

The normal form is $\frac{3}{5}x + \frac{4}{5}y - \frac{10}{5} = 0$ or $\frac{3}{5}x + \frac{4}{5}y - 2 = 0$.

Therefore, $\cos \phi = \frac{3}{5}$, $\sin \phi = \frac{4}{5}$, and $p = 2$.

Since $\cos \phi$ and $\sin \phi$ are both positive, ϕ must lie in Quadrant I.

$$\tan \phi = \frac{\frac{4}{5}}{\frac{3}{5}} \text{ or } \frac{4}{3} \qquad \tan \phi = \frac{\sin \phi}{\cos \phi}$$

$$\phi \approx 53°$$

The normal segment has length 2 units and makes an angle of 53° with the positive x-axis.

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Practice

Normal Form of a Linear Equation

Write the standard form of the equation of each line, given p, the length of the normal segment, and ϕ, the angle the normal segment makes with the positive x-axis.

1. $p = 4, \phi = 30°$

2. $p = 2\sqrt{2}, \phi = \frac{\pi}{4}$

3. $p = 3, \phi = 60°$

4. $p = 8, \phi = \frac{5\pi}{6}$

5. $p = 2\sqrt{3}, \phi = \frac{7\pi}{4}$

6. $p = 15, \phi = 225°$

Write each equation in normal form. Then find the length of the normal and the angle it makes with the positive x-axis.

7. $3x - 2y - 1 = 0$

8. $5x + y - 12 = 0$

9. $4x + 3y - 4 = 0$

10. $y = x + 5$

11. $2x + y + 1 = 0$

12. $x + y - 5 = 0$

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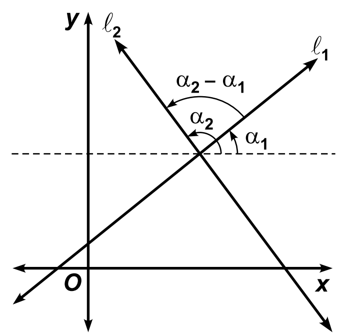
Enrichment

Slopes of Perpendicular Lines

The derivation of the normal form of a linear equation uses this familiar theorem, first stated in Lesson 1-6: Two nonvertical lines are perpendicular if and only if the slope of one is the negative reciprocal of the slope of the other.

You can use trigonometric identities to prove that if lines are perpendicular, then their slopes are negative reciprocals of each other.



ℓ_1 and ℓ_2 are perpendicular lines.
α_1 and α_2 are the angles that ℓ_1 and ℓ_2, respectively, make with the horizontal.
Let m_1 = slope of ℓ_1
m_2 = slope of ℓ_2

Complete the following exercises to prove that $m_1 = -\frac{1}{m_2}$.

1. Explain why $m_1 = \tan \alpha_1$ and $m_2 = \tan \alpha_2$.

2. According to the difference identity for the cosine function, $\cos(\alpha_2 - \alpha_1) = \cos \alpha_2 \cos \alpha_1 + \sin \alpha_2 \sin \alpha_1$. Explain why the left side of the equation is equal to zero.

3. $\cos \alpha_2 \cos \alpha_1 + \sin \alpha_2 \sin \alpha_1 = 0$

$$\sin \alpha_2 \sin \alpha_1 = -\cos \alpha_2 \cos \alpha_1$$

$$\frac{\sin \alpha_1}{\cos \alpha_1} = -\frac{\cos \alpha_2}{\sin \alpha_2}$$

Complete using the tangent function. ____________ = ____________

Complete, using m_1 and m_2. ____________ = ____________

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Study Guide

Distance from a Point to a Line

The distance from a point at (x_1, y_1) to a line with equation $Ax + By + C = 0$ can be determined by using the formula $d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$. The sign of the radical is chosen opposite the sign of C.

Example 1 **Find the distance between $P(3, 4)$ and the line with equation $4x + 2y = 10$.**

First, rewrite the equation of the line in standard form.

$$4x + 2y - 10 = 0$$

Then, use the formula for the distance from a point to a line.

$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$

$d = \frac{4(3) + 2(4) - 10}{\pm\sqrt{4^2 + 2^2}}$ *$A = 4, B = 2, C = -10, x_1 = 3, y_1 = 4$*

$d = \frac{10}{2\sqrt{5}}$ or $\sqrt{5}$ *Since C is negative, use $+\sqrt{A^2 + B^2}$.*

$d \approx 2.24$ units

Therefore, P is approximately 2.24 units from the line with equation $4x + 2y = 10$. Since d is positive, P is on the opposite side of the line from the origin.

You can also use the formula to find the distance between two parallel lines. To do this, choose any point on one of the lines and use the formula to find the distance from that point to the other line.

Example 2 **Find the distance between the lines with equations $2x - 2y = 5$ and $y = x - 1$.**

Since $y = x - 1$ is in slope-intercept form, you can see that it passes through the point at $(0, -1)$. Use this point to find the distance to the other line.

The standard form of the other equation is $2x - 2y - 5 = 0$.

$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$

$d = \frac{2(0) - 2(-1) - 5}{\pm\sqrt{2^2 + (-2)^2}}$ *$A = 2, B = -2, C = -5, x_1 = 0, y_1 = -1$*

$d = -\frac{3}{2\sqrt{2}}$ or $-\frac{3\sqrt{2}}{4}$ *Since C is negative, use $+\sqrt{A^2 + B^2}$.*

≈ -1.06

The distance between the lines is about 1.06 units.

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Practice

Distance From a Point to a Line

Find the distance between the point with the given coordinates and the line with the given equation.

1. $(-1, 5)$, $3x - 4y - 1 = 0$

2. $(2, 5)$, $5x - 12y + 1 = 0$

3. $(1, -4)$, $12x + 5y - 3 = 0$

4. $(-1, -3)$, $6x + 8y - 3 = 0$

Find the distance between the parallel lines with the given equations.

5. $2x - 3y + 4 = 0$
$y = \frac{2}{3}x + 5$

6. $4x - y + 1 = 0$
$4x - y - 8 = 0$

Find equations of the lines that bisect the acute and obtuse angles formed by the lines with the given equations.

7. $x + 2y - 3 = 0$
$x - y + 4 = 0$

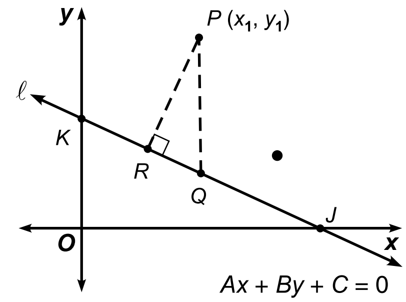
8. $9x + 12y + 10 = 0$
$3x + 2y - 6 = 0$

NAME ______________________ DATE ____________ PERIOD ________

Enrichment

Deriving the Point-Line Distance

Line ℓ has the equation $Ax + By + C = 0$.
Answer these questions to derive the formula given in Lesson 7-7 for the distance from $P(x_1, y_1)$ to ℓ .



1. Use the equation of the line to find the coordinates of J and K, the x- and y-intercepts of ℓ .

2. $\overline{PQ}$ is a vertical segment from P to ℓ. Find the x-coordinate of Q.

3. Since Q is on ℓ, its coordinates must satisfy the equation of ℓ. Use your answer to Exercise 2 to find the y-coordinate of Q.

4. Find PQ by finding the difference between the y-coordinates of P and Q. Write your answer as a fraction.

5. Triangle KJO is a right triangle. Use your answers to Exercise 1 and the Pythagorean Theorem to find KJ. Simplify.

6. Since $\angle Q \cong \angle K$, $\triangle JKO \sim \triangle PQR$.

$$\frac{PR}{OJ} = \frac{PQ}{KJ}$$

Use your answers to Exercises 1, 4, and 5 to find PR, the distance from P to ℓ. Simplify.

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Chapter 7

NAME ______________________ DATE ______________ PERIOD ________

Chapter 7 Test, Form 1A

Write the letter for the correct answer in the blank at the right of each problem.

1. Find an expression equivalent to $\frac{\sec\theta\tan\theta}{\sin\theta}$. **1.** ________

A. $\sec^2\theta$ **B.** $\cot\theta$ **C.** $\tan^2\theta$ **D.** $\cos^2\theta$

2. If $\csc\theta = -\frac{5}{4}$ and $180° < \theta < 270°$, find $\tan\theta$. **2.** ________

A. $-\frac{4}{3}$ **B.** $\frac{3}{4}$ **C.** $\frac{4}{3}$ **D.** $-\frac{4}{5}$

3. Simplify $\frac{\tan^2\theta\csc^2\theta - 1}{\tan^2\theta}$. **3.** ________

A. $\csc^2\theta$ **B.** -1 **C.** $\tan^2\theta$ **D.** 1

4. Simplify $\frac{\cos x}{\sec x - 1} + \frac{\cos x}{\sec x + 1}$. **4.** ________

A. $2\tan^2 x$ **B.** $2\cos x$ **C.** $2\cos^2 x - 1$ **D.** $2\cot^2 x$

5. Find a numerical value of one trigonometric function of x if **5.** ________
$\frac{\tan x}{\cot x} - \frac{\sec x}{\cos x} = \frac{2}{\csc x}$.

A. $\csc x = 1$ **B.** $\sin x = -\frac{1}{2}$ **C.** $\csc x = -1$ **D.** $\sin x = \frac{1}{2}$

6. Use a sum or difference identity to find the exact value of sin 255°. **6.** ________

A. $\frac{-\sqrt{2} - \sqrt{6}}{4}$ **B.** $\frac{\sqrt{6} - \sqrt{2}}{4}$ **C.** $\frac{\sqrt{6} + \sqrt{2}}{4}$ **D.** $\frac{\sqrt{2} - \sqrt{6}}{4}$

7. Find the value of $\tan(\alpha - \beta)$ if $\cos\alpha = -\frac{3}{5}$, $\sin\beta = \frac{5}{13}$, $90° < \alpha < 180°$, and $90° < \beta < 180°$. **7.** ________

A. $\frac{63}{56}$ **B.** $-\frac{63}{56}$ **C.** $-\frac{33}{56}$ **D.** $\frac{33}{56}$

8. Which expression is equivalent to $\cos(\pi - \theta)$? **8.** ________

A. $-\cos\theta$ **B.** $\cos\theta$ **C.** $-\sin\theta$ **D.** $\sin\theta$

9. Which expression is *not* equivalent to $\cos 2\theta$? **9.** ________

A. $\cos^2\theta - \sin^2\theta$ **B.** $2\cos^2\theta - 1$
C. $1 - 2\sin^2\theta$ **D.** $2\sin\theta\cos\theta$

10. If $\cos\theta = 0.8$ and $270° < \theta < 360°$, find the exact value of $\sin 2\theta$. **10.** ________

A. -0.96 **B.** -0.6 **C.** 0.96 **D.** 0.28

11. If $\csc\theta = -\frac{5}{3}$ and θ has its terminal side in Quadrant III, find the exact value of $\tan 2\theta$. **11.** ________

A. $\frac{24}{25}$ **B.** $\frac{7}{25}$ **C.** $\frac{24}{7}$ **D.** $-\frac{7}{25}$

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Chapter 7 Test, Form 1A (continued)

12. Use a half-angle identity to find the exact value of cos 165°. **12.** ________

A. $\frac{1}{2}\sqrt{2 + \sqrt{3}}$ **B.** $-\frac{1}{2}\sqrt{2 + \sqrt{3}}$

C. $\frac{1}{2}\sqrt{2 + \sqrt{2}}$ **D.** $-\frac{1}{2}\sqrt{1 + \sqrt{3}}$

13. Solve $4 \sin^2 x + 4\sqrt{2} \cos x - 6 = 0$ for all real values of x. **13.** ________

A. $\frac{3\pi}{4} + 2\pi k, \frac{5\pi}{4} + 2\pi k$ **B.** $\frac{\pi}{4} + 2\pi k, \frac{7\pi}{4} + 2\pi k$

C. $\frac{\pi}{4} + 2\pi k, \frac{5\pi}{4} + 2\pi k$ **D.** $\frac{3\pi}{4} + 2\pi k, \frac{7\pi}{4} + 2\pi k$

14. Solve $2 \cos^2 x - 5 \cos x + 2 = 0$ for principal values of x. **14.** ________

A. 0° and 30° **B.** 30° **C.** 60° **D.** 60° and 300°

15. Solve $2 \sin x + \sqrt{3} < 0$ for $0 \le x < 2\pi$. **15.** ________

A. $\frac{4\pi}{3} < x < \frac{5\pi}{3}$ **B.** $\frac{2\pi}{3} < x < \frac{4\pi}{3}$

C. $\frac{7\pi}{6} < x < \frac{11\pi}{6}$ **D.** $\frac{5\pi}{6} < x < \frac{7\pi}{6}$

16. Write the equation $2x + 3y - 5 = 0$ in normal form. **16.** ________

A. $\frac{2\sqrt{13}}{13}x + \frac{3\sqrt{13}}{13}y + \frac{5\sqrt{13}}{13} = 0$ **B.** $-\frac{2\sqrt{13}}{13}x - \frac{3\sqrt{13}}{13}y + \frac{5\sqrt{13}}{13} = 0$

C. $-\frac{2\sqrt{13}}{13}x - \frac{3\sqrt{13}}{13}y - \frac{5\sqrt{13}}{13} = 0$ **D.** $\frac{2\sqrt{13}}{13}x + \frac{3\sqrt{13}}{13}y - \frac{5\sqrt{13}}{13} = 0$

17. Write the standard form of the equation of a line for which the length of the normal is 6 and the normal makes an angle of 120° with the positive x-axis. **17.** ________

A. $x - \sqrt{3}y + 12 = 0$ **B.** $x + \sqrt{3}y - 12 = 0$

C. $\sqrt{3}x - y + 12 = 0$ **D.** $\sqrt{3}x + y - 12 = 0$

18. Find the distance between $P(-4, 3)$ and the line with equation $2x - 5y = -7$. **18.** ________

A. $\frac{14\sqrt{29}}{29}$ **B.** 0 **C.** $-\frac{16\sqrt{29}}{29}$ **D.** $\frac{16\sqrt{29}}{29}$

19. Find the distance between the lines with equations $3x - y = 9$ and $y = 3x - 4$. **19.** ________

A. $\frac{5}{4}$ **B.** $\frac{5\sqrt{10}}{2}$ **C.** $\frac{\sqrt{10}}{2}$ **D.** $\frac{13\sqrt{10}}{2}$

20. Find an equation of the line that bisects the obtuse angles formed by the lines with equations $3x - y = 1$ and $x + y = -2$. **20.** ________

A. $(3\sqrt{2} + \sqrt{10})x - (\sqrt{10} + \sqrt{2})y - 2\sqrt{10} + \sqrt{2} = 0$

B. $(3\sqrt{2} - \sqrt{10})x + (\sqrt{10} - \sqrt{2})y + 2\sqrt{10} + \sqrt{2} = 0$

C. $(3\sqrt{2} - \sqrt{10})x - (\sqrt{10} + \sqrt{2})y - 2\sqrt{10} - \sqrt{2} = 0$

D. $(3\sqrt{2} + \sqrt{10})x - (\sqrt{10} + \sqrt{2})y - 2\sqrt{10} - \sqrt{2} = 0$

Bonus If $90° < \theta < 180°$ and $\cos \theta = -\frac{4}{5}$, find $\sin 4\theta$. **Bonus:** ________

A. $-\frac{48}{25}$ **B.** $\frac{48}{25}$ **C.** $\frac{336}{625}$ **D.** $-\frac{336}{625}$

Chapter 7

NAME ______________________ DATE ______________ PERIOD ________

Chapter 7 Test, Form 1B

Write the letter for the correct answer in the blank at the right of each problem.

1. Find an expression equivalent to $\sec \theta \sin \theta \cot \theta \csc \theta$. **1.** ________

A. $\tan \theta$ **B.** $\csc \theta$ **C.** $\sec \theta$ **D.** $\sin \theta$

2. If $\sec \theta = -\frac{5}{4}$ and $180° < \theta < 270°$, find $\tan \theta$. **2.** ________

A. $-\frac{3}{5}$ **B.** $-\frac{4}{5}$ **C.** $\frac{3}{4}$ **D.** $\frac{3}{5}$

3. Simplify $\frac{\tan^2 \theta + 1}{\tan^2 \theta}$. **3.** ________

A. $\csc^2 \theta$ **B.** -1 **C.** $\tan^2 \theta$ **D.** 1

4. Simplify $\frac{\tan x}{\sin x} + \frac{1}{\cos x}$. **4.** ________

A. $2 \tan^2 x$ **B.** $2 \cos x$ **C.** $2 \cos x - 1$ **D.** $2 \sec x$

5. Find a numerical value of one trigonometric function of x if $\sec x \cot x = 4$. **5.** ________

A. $\csc x = \frac{1}{4}$ **B.** $\sec x = 4$ **C.** $\sec x = \frac{1}{4}$ **D.** $\csc x = 4$

6. Use a sum or difference identity to find the exact value of sin 105°. **6.** ________

A. $\frac{-\sqrt{2} - \sqrt{6}}{4}$ **B.** $\frac{\sqrt{6} - \sqrt{2}}{4}$ **C.** $\frac{\sqrt{6} + \sqrt{2}}{4}$ **D.** $\frac{\sqrt{2} - \sqrt{6}}{4}$

7. Find the value of $\tan (\alpha - \beta)$ if $\cos \alpha = \frac{4}{5}$, $\sin \beta = -\frac{5}{13}$, $270° < \alpha < 360°$, and $270° < \beta < 360°$. **7.** ________

A. $\frac{16}{63}$ **B.** $-\frac{16}{63}$ **C.** $-\frac{56}{33}$ **D.** $\frac{56}{33}$

8. Which expression is equivalent to $\cos (\pi + \theta)$? **8.** ________

A. $-\cos \theta$ **B.** $\cos \theta$ **C.** $-\sin \theta$ **D.** $\sin \theta$

9. Which expression is *not* equivalent to $\cos 2\theta$? **9.** ________

A. $2 \cos^2 \theta - 1$ **B.** $1 - 2 \sin^2 \theta$ **C.** $\cos^2 \theta + \sin^2 \theta$ **D.** $\cos^2 \theta - \sin^2 \theta$

10. If $\sin \theta = 0.6$ and $90° < \theta < 180°$, find the exact value of $\sin 2\theta$. **10.** ________

A. -0.6 **B.** -0.96 **C.** 0.96 **D.** 0.28

11. If $\cos \theta = -\frac{3}{5}$ and θ has its terminal side in Quadrant II, find the exact value of $\tan 2\theta$. **11.** ________

A. $\frac{24}{25}$ **B.** $\frac{7}{25}$ **C.** $\frac{24}{7}$ **D.** $-\frac{7}{25}$

12. Use a half-angle identity to find the exact value of cos 75°. **12.** ________

A. $\frac{1}{2}\sqrt{2 + \sqrt{3}}$ **B.** $\frac{1}{2}\sqrt{2 - \sqrt{3}}$ **C.** $\frac{1}{2}\sqrt{2 + \sqrt{2}}$ **D.** $-\frac{1}{2}\sqrt{1 + \sqrt{3}}$

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Chapter 7 Test, Form 1B (continued)

13. Solve $\csc x + 2 = 0$ for $0 \leq x < 2\pi$. **13.** ________

A. $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ **B.** $\frac{\pi}{6}$ and $\frac{7\pi}{6}$ **C.** $\frac{4\pi}{3}$ and $\frac{5\pi}{3}$ **D.** $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$

14. Solve $2\cos^2 x - \cos x - 1 = 0$ for principal values of x. **14.** ________

A. 0° and 120° **B.** 30° **C.** 60° **D.** 0° and 60°

15. Solve $4\sin^2 x - 4\sin x + 1 = 0$ for all real values of x. **15.** ________

A. $\frac{\pi}{6} + 2\pi k, \frac{7\pi}{6} + 2\pi k$ **B.** $\frac{7\pi}{6} + 2\pi k, \frac{11\pi}{6} + 2\pi k$

C. $\frac{5\pi}{6} + 2\pi k, \frac{11\pi}{6} + 2\pi k$ **D.** $\frac{\pi}{6} + 2\pi k, \frac{5\pi}{6} + 2\pi k$

16. Write the equation $3x - 2y + 7 = 0$ in normal form. **16.** ________

A. $\frac{3\sqrt{13}}{13}x + \frac{2\sqrt{13}}{13}y + \frac{7\sqrt{13}}{13} = 0$ **B.** $-\frac{3\sqrt{13}}{13}x + \frac{2\sqrt{13}}{13}y - \frac{7\sqrt{13}}{13} = 0$

C. $-\frac{3\sqrt{13}}{13}x - \frac{2\sqrt{13}}{13}y - \frac{7\sqrt{13}}{13} = 0$ **D.** $\frac{3\sqrt{13}}{13}x + \frac{2\sqrt{13}}{13}y - \frac{7\sqrt{13}}{13} = 0$

17. Write the standard form of the equation of a line for which the length of the normal is 3 and the normal makes an angle of 135° with the positive x-axis. **17.** ________

A. $\sqrt{2}x - \sqrt{2}y + 6 = 0$ **B.** $\sqrt{2}x + \sqrt{2}y - 6 = 0$

C. $\sqrt{2}x - \sqrt{2}y - 6 = 0$ **D.** $\sqrt{2}x + \sqrt{2}y + 6 = 0$

18. Find the distance between $P(-2, 5)$ and the line with equation $x - 3y + 4 = 0$. **18.** ________

A. $\frac{17\sqrt{10}}{10}$ **B.** 0 **C.** $-\frac{17\sqrt{10}}{10}$ **D.** $\frac{13\sqrt{10}}{10}$

19. Find the distance between the lines with equations $5x + 12y = 12$ and $y = -\frac{5}{12}x + 3$. **19.** ________

A. $\frac{24}{13}$ **B.** $\frac{48}{13}$ **C.** $-\frac{48}{13}$ **D.** $\frac{48}{17}$

20. Find an equation of the line that bisects the acute angles formed by the lines with equations $2x + y - 5 = 0$ and $3x - 2y + 6 = 0$. **20.** ________

A. $(2\sqrt{13} - 3\sqrt{5})x + (\sqrt{13} + 2\sqrt{5})y - 5\sqrt{13} - 6\sqrt{5} = 0$

B. $(-2\sqrt{13} - 3\sqrt{5})x + (-\sqrt{13} + 2\sqrt{5})y + 5\sqrt{13} - 6\sqrt{5} = 0$

C. $(-2\sqrt{13} + 3\sqrt{5})x + (-\sqrt{13} - 2\sqrt{5})y + 5\sqrt{13} - 6\sqrt{5} = 0$

D. $(-2\sqrt{13} - 3\sqrt{5})x + (-\sqrt{13} + 2\sqrt{5})y + 5\sqrt{13} + 6\sqrt{5} = 0$

Bonus If $90° < \theta < 180°$, express $\cos\theta$ in terms of $\tan\theta$. **Bonus:** ________

A. $-\sqrt{\frac{1}{1 + \tan^2\theta}}$ **B.** $\sqrt{\frac{1}{1 + \tan^2\theta}}$ **C.** $\sqrt{1 + \tan^2\theta}$ **D.** $-\sqrt{1 + \tan^2\theta}$

NAME ______________________ DATE ______________ PERIOD ________

Chapter 7 Test, Form 1C

Write the letter for the correct answer in the blank at the right of each problem.

1. Find an expression equivalent to $\frac{\cos \theta}{\sin \theta}$. **1.** ________

A. $\tan \theta$ **B.** $\cot \theta$ **C.** $\sec \theta$ **D.** $\csc \theta$

2. If $\sec \theta = \frac{5}{4}$ and $0° < \theta < 90°$, find $\sin \theta$. **2.** ________

A. $-\frac{3}{5}$ **B.** $-\frac{4}{5}$ **C.** $\frac{3}{4}$ **D.** $\frac{3}{5}$

3. Simplify $\frac{1 - \sec^2 \theta}{\tan^2 \theta}$. **3.** ________

A. $\csc^2 \theta$ **B.** -1 **C.** $\tan^2 \theta$ **D.** 1

4. Simplify $\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x}$. **4.** ________

A. $2 \tan^2 x$ **B.** $2 \cos x$ **C.** 1 **D.** $2 \cot^2 x$

5. Find a numerical value of one trigonometric function of x if $\sin x \cot x = \frac{1}{4}$. **5.** ________

A. $\cos x = \frac{1}{4}$ **B.** $\sec x = \frac{1}{4}$ **C.** $\csc x = 4$ **D.** $\cos x = 4$

6. Use a sum or difference identity to find the exact value of sin 15°. **6.** ________

A. $\frac{-\sqrt{2} - \sqrt{6}}{4}$ **B.** $\frac{\sqrt{6} - \sqrt{2}}{4}$ **C.** $\frac{\sqrt{6} + \sqrt{2}}{4}$ **D.** $\frac{\sqrt{2} - \sqrt{6}}{4}$

7. Find the value of $\tan (\alpha - \beta)$ if $\cos \alpha = \frac{3}{5}$, $\sin \beta = \frac{5}{13}$, $0° < \alpha < 90°$, and $0° < \beta < 90°$. **7.** ________

A. $\frac{63}{56}$ **B.** $\frac{63}{16}$ **C.** $\frac{33}{16}$ **D.** $\frac{33}{56}$

8. Which expression is equivalent to $\cos (\theta - 2\pi)$? **8.** ________

A. $-\cos \theta$ **B.** $\sin \theta$ **C.** $\cos \theta$ **D.** $-\sin \theta$

9. Which expression is equivalent to $\cos 2\theta$ for all values of θ? **9.** ________

A. $\cos^2 \theta - \sin^2 \theta$ **B.** $\cos^2 \theta - 1$

C. $1 - \sin^2 \theta$ **D.** $2 \sin \theta \cos \theta$

10. If $\cos \theta = 0.8$ and $0° < \theta < 90°$, find the exact value of $\sin 2\theta$. **10.** ________

A. 9.6 **B.** 2.8 **C.** 0.96 **D.** 0.28

11. If $\sin \theta = \frac{3}{5}$ and θ has its terminal side in Quadrant II, find the exact value of $\tan 2\theta$. **11.** ________

A. $\frac{24}{25}$ **B.** $-\frac{24}{25}$ **C.** $\frac{24}{7}$ **D.** $-\frac{24}{7}$

NAME ______________________ DATE ______________ PERIOD ________

Chapter 7 Test, Form 1C (continued)

12. Use a half-angle identity to find the exact value of sin 105°. **12.** ________

A. $-\frac{1}{2}\sqrt{2+\sqrt{3}}$ **B.** $\frac{1}{2}\sqrt{2+\sqrt{3}}$

C. $\frac{1}{2}\sqrt{2+\sqrt{2}}$ **D.** $-\frac{1}{2}\sqrt{1+\sqrt{3}}$

13. Solve $2\cos x - 1 = 0$ for $0 \le x < 2\pi$. **13.** ________

A. $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ **B.** $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ **C.** $\frac{\pi}{3}$ and $\frac{2\pi}{3}$ **D.** $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$

14. Solve $2\sin^2 x - \sin x = 0$ for principal values of x. **14.** ________

A. 60° and 120° **B.** 0° and 150° **C.** 0° and 30° **D.** 60°

15. Solve $\cos x \tan x - \sin^2 x = 0$ for all real values of x. **15.** ________

A. $\pi k, \frac{\pi}{2} + 2\pi k$ **B.** $\frac{\pi}{2} + \pi k, 2\pi k$

C. $\frac{\pi}{2} + 2\pi k, \frac{3\pi}{2} + 2\pi k$ **D.** $\pi k, \frac{\pi}{4} + 2\pi k$

16. Write the equation $3x + 4y - 7 = 0$ in normal form. **16.** ________

A. $\frac{3}{5}x + \frac{4}{5}y + \frac{7}{5} = 0$ **B.** $-\frac{3}{5}x - \frac{4}{5}y - \frac{7}{5} = 0$

C. $-\frac{3}{5}x - \frac{4}{5}y + \frac{7}{5} = 0$ **D.** $\frac{3}{5}x + \frac{4}{5}y - \frac{7}{5} = 0$

17. Write the standard form of the equation of a line for which the length of the normal is 4 and the normal makes an angle of 45° with the positive x-axis. **17.** ________

A. $\sqrt{2}x + \sqrt{2}y + 8 = 0$ **B.** $2x + 2y - 8 = 0$

C. $\sqrt{2}x + \sqrt{2}y - 8 = 0$ **D.** $2x + 2y + 8 = 0$

18. Find the distance between $P(-2, 1)$ and the line with equation $x - 2y + 4 = 0$. **18.** ________

A. $\frac{4\sqrt{5}}{5}$ **B.** 0 **C.** $-\frac{4\sqrt{5}}{5}$ **D.** $\frac{5}{4}$

19. Find the distance between the lines with equations $3x - 4y = 8$ and $y = \frac{3}{4}x + 4$. **19.** ________

A. $\frac{8}{5}$ **B.** $-\frac{24}{7}$ **C.** $\frac{24}{7}$ **D.** $\frac{24}{5}$

20. Find an equation of the line that bisects the acute angles formed by the lines with equations $4x + y + 3 = 0$ and $x + y - 2 = 0$. **20.** ________

A. $(4\sqrt{2} + \sqrt{17})x + (\sqrt{2} + \sqrt{17})y + 3\sqrt{2} - 2\sqrt{17} = 0$

B. $(\sqrt{2} + \sqrt{17})x - (4\sqrt{2} + \sqrt{17})y + 3\sqrt{2} + 2\sqrt{17} = 0$

C. $(4\sqrt{2} - \sqrt{17})x + (\sqrt{2} - \sqrt{17})y + 3\sqrt{2} + 2\sqrt{17} = 0$

D. $(\sqrt{2} - \sqrt{17})x - (4\sqrt{2} - \sqrt{17})y - 3\sqrt{2} - 2\sqrt{17} = 0$

Bonus If $90° < \theta < 180°$, express $\sin\theta$ in terms of $\cos\theta$. **Bonus:** ________

A. $-\sqrt{1 + \cos^2\theta}$ **B.** $-\sqrt{1 - \cos^2\theta}$

C. $\sqrt{1 + \cos^2\theta}$ **D.** $\sqrt{1 - \cos^2\theta}$

Chapter 7

NAME ______________________ DATE ______________ PERIOD ________

Chapter 7 Test, Form 2A

1. Simplify $(\sec\theta - \tan\theta)(1 + \sin\theta)$. **1.** ________________

2. If $\tan\theta = -\frac{3}{4}$ and $90° < \theta < 180°$, find $\sec\theta$. **2.** ________________

3. Simplify $\frac{\sec^2\theta}{\tan\theta + \cot^2\theta\tan\theta}$. **3.** ________________

4. Simplify $\sin\theta + \cos\theta\tan\theta$. **4.** ________________

5. If $\frac{1 + \tan^2 x}{\sec x} = \sin^2 x + \frac{1}{\sec^2 x}$, find the value of $\cos x$. **5.** ________________

6. Use a sum or difference identity to find the exact value of $\sin 285°$. **6.** ________________

7. Find the value of $\sin(\alpha - \beta)$ if $\cos\alpha = \frac{15}{17}$, $\cot\beta = \frac{24}{7}$, $0° < \alpha < 90°$, and $0° < \beta < 90°$. **7.** ________________

8. Simplify $\cos\left(\frac{\pi}{2} - \theta\right)$. **8.** ________________

9. If $\sec\theta = 4$, find the exact value of $\cos 2\theta$. **9.** ________________

10. If $\cos\theta = 0.6$ and $270° < \theta < 360°$, find the exact value of $\sin 2\theta$. **10.** ________________

11. If $\sin\theta = -\frac{4}{5}$ and θ has its terminal side in Quadrant III, find the exact value of $\tan 2\theta$. **11.** ________________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 7 Test, Form 2A (continued)

12. Use a half-angle identity to find the exact value of $\cos 67.5°$. **12.** ______________

13. Solve $2 \cos x - \sin^2 x + 2 = 0$ for all real values of x. **13.** ______________

14. Solve $2 \cos^2 x = \sqrt{3} \cos x$ for principal values of x. Express the solution(s) in degrees. **14.** ______________

15. Solve $2 \sin x + 1 < 0$ for $0 \leq x < 2\pi$. **15.** ______________

16. Write the equation $3x + 2y - 4 = 0$ in normal form. **16.** ______________

17. Write the standard form of the equation of a line for which the length of the normal is 5 and the normal makes an angle of 120° with the positive x-axis. **17.** ______________

18. Find the distance between $P(3, -2)$ and the line with equation $x + 2y - 3 = 0$. **18.** ______________

19. Find the distance between the lines with equations $3x + y = 7$ and $y = -3x + 4$. **19.** ______________

20. Find an equation of the line that bisects the obtuse angles formed by the lines with equations $2x + y - 5 = 0$ and $3x - 2y + 6 = 0$. **20.** ______________

Bonus If $180° < \theta < 270°$ and $\cos \theta = -\frac{4}{5}$, find $\sin 4\theta$. **Bonus:** ______________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 7 Test, Form 2B

1. Simplify $\cos\theta\tan^2\theta + \cos\theta$. **1.** ______________

2. If $\cot\theta = -\frac{3}{4}$ and $90° < \theta < 180°$, find $\sin\theta$. **2.** ______________

3. Simplify $\csc\theta - \cot\theta\cos\theta$. **3.** ______________

4. Simplify $\frac{1 - \sin^2\theta}{1 - \cos^2\theta}$. **4.** ______________

5. If $\sin^2 x \sec x \cot x = 3$, find the value of $\csc x$. **5.** ______________

6. Use a sum or difference identity to find the exact value of $\cos 255°$. **6.** ______________

7. Find the value of $\sin(\alpha - \beta)$ if $\tan\alpha = \frac{4}{3}$, $\cot\beta = \frac{5}{12}$, $0° < \alpha < 90°$, and $0° < \beta < 90°$. **7.** ______________

8. Simplify $\sin\left(\frac{\pi}{2} - \theta\right)$. **8.** ______________

9. If θ is an angle in the first quadrant and $\csc\theta = 3$, find the exact value of $\cos 2\theta$. **9.** ______________

10. If $\sin\theta = -0.6$ and $180° < \theta < 270°$, find the exact value of $\sin 2\theta$. **10.** ______________

11. If $\cos\theta = \frac{4}{5}$ and θ has its terminal side in Quadrant IV, find the exact value of $\tan 2\theta$. **11.** ______________

NAME ______________________ DATE ______________ PERIOD _________

Chapter 7 Test, Form 2B (continued)

12. Use a half-angle identity to find the exact value of $\cos 105°$. **12.** ______________

13. Solve $\tan x - \sqrt{3} = 0$ for $0 \leq x < 2\pi$. **13.** ______________

14. Solve $4 \sin^2 x - 1 = 0$ for principal values of x. Express the solution(s) in degrees. **14.** ______________

15. Solve $\cos^4 x - 1 = 0$ for all real values of x. **15.** ______________

16. Write the equation $2x + 5y - 3 = 0$ in normal form. **16.** ______________

17. Write the standard form of the equation of a line for which the length of the normal is 7 and the normal makes an angle of 150° with the positive x-axis. **17.** ______________

18. Find the distance between $P(-1, 4)$ and the line with equation $4x - 2y + 3 = 0$. **18.** ______________

19. Find the distance between the lines with equations $x - 2y = 3$ and $y = \frac{1}{2}x - 2$. **19.** ______________

20. Find an equation of the line that bisects the acute angles formed by the lines with equations $3x + y + 6 = 0$ and $2x - y - 1 = 0$. **20.** ______________

Bonus Express $\sqrt{\dfrac{\tan^2 \theta}{\sec^2 \theta + \cot^2 \theta \sec^2 \theta}}$ in terms of $\sin \theta$. **Bonus:** ______________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 7 Test, Form 2C

1. Simplify $\frac{\sin \theta}{\tan \theta}$. **1.** ____________

2. If $\cos \theta = -\frac{4}{5}$ and $90° < \theta < 180°$, find $\cot \theta$. **2.** ____________

3. Simplify $\sec^2 \theta - \tan^2 \theta$. **3.** ____________

4. Simplify $\frac{\sin^2 \theta + \cos^2 \theta}{\tan^2 \theta + 1}$. **4.** ____________

5. If $\tan x \cos x = \frac{1}{2}$, find the value of $\sin x$. **5.** ____________

6. Use a sum or difference identity to find the exact value of $\cos 15°$. **6.** ____________

7. Find the value of $\tan (\alpha + \beta)$ if $\cos \alpha = \frac{5}{13}$, $\sin \beta = \frac{3}{5}$, $0° < \alpha < 90°$, and $0° < \beta < 90°$. **7.** ____________

8. Simplify $\sin (\pi - \theta)$. **8.** ____________

9. If θ is an angle in the first quadrant and $\cos \theta = \frac{1}{2}$, find the exact value of $\cos 2\theta$. **9.** ____________

10. If $\cos \theta = 0.6$ and $0° < \theta < 90°$, find the exact value of $\sin 2\theta$. **10.** ____________

11. If $\cos \theta = -\frac{4}{5}$ and θ has its terminal side in Quadrant II, find the exact value of $\tan 2\theta$. **11.** ____________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 7 Test, Form 2C (continued)

12. Use a half-angle identity to find the exact value of cos 22.5°. **12.** ________________

13. Solve $2 \sin x - 1 = 0$ for $0 \leq x < 2\pi$. **13.** ________________

14. Solve $\tan x + \sqrt{3} = 0$ for principal values of x. Express the solution(s) in degrees. **14.** ________________

15. Solve $\frac{\sec x}{\csc x} - 1 = 0$ for all real values of x. **15.** ________________

16. Write the equation $3x - 2y + 6 = 0$ in normal form. **16.** ________________

17. Write the standard form of the equation of a line for which the length of the normal is 9 and the normal makes an angle of 60° with the positive x-axis. **17.** ________________

18. Find the distance between $P(2, 3)$ and the line with equation $2x - 5y + 4 = 0$. **18.** ________________

19. Find the distance between the lines with equations $2x - 2y = 5$ and $y = x - 1$. **19.** ________________

20. Find an equation of the line that bisects the acute angles formed by the lines with equations $3x + 4y - 5 = 0$ and $5x - 12y - 3 = 0$. **20.** ________________

Bonus How are the lines that bisect the angles formed by the graphs of the equations $3x + y = 6$ and $x - 3y = 1$ related to each other? **Bonus:** ________________

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Chapter 7 Open-Ended Assessment

Instructions: ***Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answer. You may show your solution in more than one way or investigate beyond the requirements of the problem.***

1. a. Verify that $\frac{\cos\theta}{1 - \sin\theta} - \frac{1 + \sin\theta}{\cos\theta} = 0$ is an identity.

b. Why is it usually easier to transform the more complicated side of the equation into the simpler side rather than the other way around?

c. Is the following method for verifying an identity correct? Why or why not? If not, write a correct verification.

$$\sec A \sin A \stackrel{?}{=} \tan A$$

$$\sec A \sin A \stackrel{?}{=} \frac{\sin A}{\cos A}$$

$$\cos A \sec A \sin A \stackrel{?}{=} \sin A$$

$$\cos A \frac{1}{\cos A} \sin A \stackrel{?}{=} \sin A$$

$$\sin A = \sin A$$

2. a. Write the equation $2y - 3x = 6$ in normal form. Then, find the length of the normal and the angle it makes with the positive x-axis. Explain how you determined the angle.

b. Find the distance from a point on the line in part **a** to the line with equation $6x - 4y + 16 = 0$. Tell what the sign of the distance d means.

c. Will the sign of the distance from a point on the line with equation $6x - 4y + 16 = 0$ to the line described in part **a** be the same as in part **b**? Why or why not?

d. When will the sign of the distance between two parallel lines be the same regardless of which line it is measured from?

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Chapter 7 Mid-Chapter Test (Lessons 7-1 through 7-4)

1. If $\csc A = 2$, find the value of $\sin A$. **1.** ______________

2. If $\tan \theta = -\frac{3}{4}$ and $90° < \theta < 180°$, find $\cos \theta$. **2.** ______________

3. Simplify $\csc x - \cos x \cot x$. **3.** ______________

4. Simplify $\frac{\csc \theta \tan \theta}{1 + \tan^2 \theta}$. **4.** ______________

5. If $\tan x \csc x = 3$, find the value of $\cos x$. **5.** ______________

6. Use a sum or difference identity to find the exact value of $\sin 285°$. **6.** ______________

7. Find the value of $\tan (\alpha + \beta)$ if $\csc \alpha = \frac{13}{5}$, $\tan \beta = \frac{3}{4}$, $0° < \alpha < 90°$, and $0° < \beta < 90°$. **7.** ______________

8. If $\tan \theta = \frac{3}{4}$ and $180° < \theta < 270°$, find the exact value of $\sin 2\theta$. **8.** ______________

9. If θ is an angle in the first quadrant and $\csc \theta = 4$, find the exact value of $\cos 2\theta$. **9.** ______________

10. Use a half-angle identity to find the exact value of $\sin 22.5°$. **10.** ______________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 7

Chapter 7, Quiz A (Lessons 7-1 and 7-2)

1. If $\sec \theta = 3$, find the value of $\cos \theta$. **1.** ________

2. If $\cot \theta = \frac{4}{3}$ and $180° < \theta < 270°$, find $\csc \theta$. **2.** ________

3. Simplify $\cot^2 x \sec^2 x$. **3.** ________

4. Simplify $\frac{1 - \cos^2 \theta}{1 + \cot^2 \theta}$. **4.** ________

5. If $\sec \theta \sin \theta = 2$, find the value of $\cot \theta$. **5.** ________

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Chapter 7

Chapter 7, Quiz B (Lessons 7-3 and 7-4)

1. Use a sum or difference identity to find the exact value of $\cos 345°$. **1.** ________

2. Find the value of $\tan(\alpha + \beta)$ if $\sin \alpha = -\frac{5}{13}$, $\cos \beta = \frac{4}{5}$, $270° < \alpha < 360°$, and $270° < \beta < 360°$. **2.** ________

3. If $\sec \theta = -\frac{13}{5}$ and $90° < \theta < 180°$, find the exact value of $\sin 2\theta$. **3.** ________

4. If $\cos \theta = \frac{4}{5}$ and θ has its terminal side in Quadrant IV, find the exact value of $\tan 2\theta$. **4.** ________

5. Use a half-angle identity to find the exact value of $\sin 165°$. **5.** ________

Chapter 7

NAME ______________________ DATE ____________ PERIOD ________

Chapter 7, Quiz C (Lessons 7-5 and 7-6)

1. Solve $2 \sin x + 2 = 0$ for $0 \leq x < 2\pi$. **1.** ____________

2. Solve $4 \cos^2 x - 3 = 0$ for principal values of x. Express the solution(s) in degrees. **2.** ____________

3. Solve $\tan x - 1 = 0$ for all real values of x. **3.** ____________

4. Write the equation $x + 5y + 8 = 0$ in normal form. **4.** ____________

5. Write the standard form of the equation of a line for which the length of the normal is 3 and the normal makes an angle of 240° with the positive x-axis. **5.** ____________

Chapter 7

NAME ______________________ DATE ____________ PERIOD ________

Chapter 7, Quiz D (Lesson 7-7)

1. Find the distance between $P(1, 4)$ and the line with equation $x - 2y + 5 = 0$. **1.** ____________

2. Find the distance between $P(3, 1)$ and the line with equation $2x - 3y - 3 = 0$. **2.** ____________

3. Find the distance between the lines with equations $2x - y = 5$ and $y = 2x + 3$. **3.** ____________

4. Find the distance between the lines with equations $3x + 4y + 18 = 0$ and $y = -\frac{3}{4}x + 3$. **4.** ____________

5. Find an equation of the line that bisects the acute angles formed by the lines with equations $x + 3y - 3 = 0$ and $x - 2y - 2 = 0$. **5.** ____________

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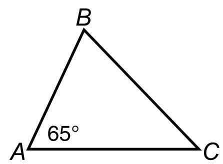
Chapter 7 SAT and ACT Practice

After working each problem, record the correct answer on the answer sheet provided or use your own paper.

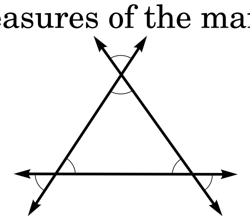
Multiple Choice

1. In the figure below, the measure of $\angle A$ is 65°. If the measure of $\angle C$ is $\frac{4}{5}$ the measure of $\angle A$, what is the measure of $\angle B$?



A 50°
B 52°
C 63°
D 65°
E 68°

2. In the figure below, three lines intersect to form a triangle. Find the sum of the measures of the marked angles.



A 90°
B 180°
C 360°
D 540°
E It cannot be determined from the information given.

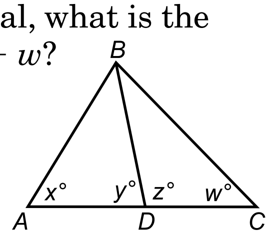
3. If $x = y + z$, and $x + y = 24$ and $x = 10$, then $z =$

A -4 **B** 0
C 4 **D** 8
E 16

4. What are the roots of $x^2 - 169 = 0$?

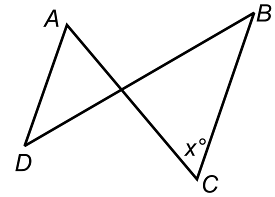
A 0, 169
B 0, 13
C 0, -13
D 169, -169
E 13, -13

5. If $\triangle ABC$ is equilateral, what is the value of $x - (y + z) + w$?



A -60
B 0
C 20
D 60
E It cannot be determined from the information given.

6. In the figure below, $\overline{AD}$ is parallel to $\overline{BC}$. Find the value of x.



A 20
B 40
C 60
D 80
E It cannot be determined from the information given.

7. $\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right) + \text{Cos}^{-1}\left(\frac{\sqrt{3}}{2}\right) =$

A 0
B $\frac{\pi}{6}$
C $\frac{\pi}{3}$
D $\frac{\pi}{2}$
E π

8. The lengths of the sides of a rectangle are 6 inches and 8 inches. Which of the following can be used to find θ, the angle that a diagonal makes with a longer side?

A $\sin \theta = \frac{3}{4}$
B $\cos \theta = \frac{3}{4}$
C $\tan \theta = \frac{3}{4}$
D $\tan \theta = \frac{4}{3}$
E $\cos \theta = \frac{3}{5}$

9. Points $A(-1, -2)$, $B(2, 1)$, and $C(4, -2)$ are vertices of parallelogram $ABCD$. What are the coordinates of D?

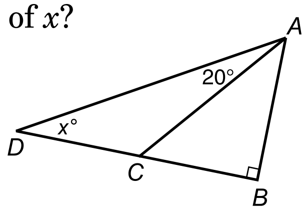
A $(0, -4)$
B $(1, -5)$
C $(-1, -5)$
D $(2, -5)$
E $(2, -4)$

NAME ______________________ DATE ____________ PERIOD ________

Chapter 7 SAT and ACT Practice (continued)

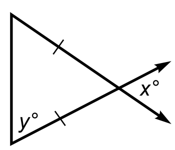
10. The vertices of a triangle are (2, 4), (7, 9), and (8, 2). Which of the following best describes this triangle?
A scalene
B equilateral
C right
D isosceles
E right, isosceles

11. In right $\triangle ABD$, $\overline{CA}$ bisects $\angle DAB$. What is the value of x?



A 20
B 40
C 70
D 80
E None of these

12. In the figure below, what is the value of x in terms of y?



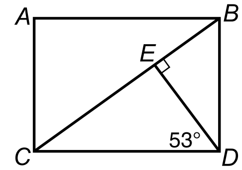
A y
B $2y$
C $180 - 2y$
D $180 - y$
E $360 - 2y$

13. What is the greatest common factor of the terms in the expansion of $2(6x^2y - 9xy^3)(15a^3x + 10ay^2)$?
A 2
B $6y$
C $10a$
D $30ay$
E None of these

14. If $5x + 4y - xy + 8 = 0$ and $x + 3 = 9$, then $3 - y =$
A -19
B -16
C 8
D 19
E 22

15. Which of the following could be lengths of the sides of a triangle?
A 7, 8, 14
B 8, 8, 16
C 8, 9, 20
D 9, 10, 100
E 1, 50, 55

16. In the rectangle $ABDC$ below, what is the measure of $\angle ACB$?



A 63°
B 53°
C 37°
D 45°
E It cannot be determined from the information given.

17–18. Quantitative Comparison
A if the quantity in Column A is greater
B if the quantity in Column B is greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

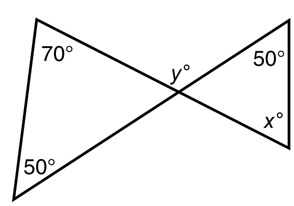
17. Side $\overline{AB}$ of triangle ABC is extended beyond B to point D.

Column A	Column B
The measure of $\angle ABC$	The measure of $\angle DBC$

18. Angles P, Q, and R are the angles of a right triangle.

Column A	Column B
$180° - m\angle P$	90°

19–20. Refer to the figure below.



19. Grid-In What is the value of x?

20. Grid-In What is the value of y?

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Chapter 7

Chapter 7 Cumulative Review (Chapters 1-7)

1. Find the standard form of the equation of the line that passes through $(-1, 2)$ and has a slope of 3. **1.** ______________

2. If $A = \begin{bmatrix} 2 & 0 & 1 \\ -2 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 \\ 1 & 4 \\ 2 & 5 \end{bmatrix}$, find AB. **2.** ______________

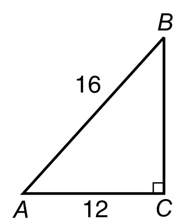
3. Given $f(x) = (x - 2)^2 - 5$, find $f^{-1}(x)$. Then state whether $f^{-1}(x)$ is a function. **3.** ______________

4. If y varies inversely as the square of x and $y = 18$ when $x = 3$, find y when $x = -9$. **4.** ______________

5. Write a polynomial equation of least degree with roots 3, $2\boldsymbol{i}$, and $-2\boldsymbol{i}$. **5.** ______________

6. Use the Remainder Theorem to find the remainder when $x^2 + 5x - 2$ is divided by $x + 5$. State whether the binomial is a factor of the polynomial. **6.** ______________

7. Given the triangle at the right, find $m\angle A$ to the nearest tenth of a degree if $b = 12$ and $c = 16$. **7.** ______________



8. If $a = 8$, $b = 11$, and $c = 13$, find the area of $\triangle ABC$ to the nearest tenth. **8.** ______________

9. State the amplitude, period, and phase shift for the graph of $y = 3 \sin(2x - 4\pi)$. **9.** ______________

10. Find the value of $\text{Cos}^{-1}\left(\tan \frac{\pi}{4}\right)$. **10.** ______________

11. Solve $4 \sin^2 x - 3 = 0$ for principal values of x. Express the solution(s) in degrees. **11.** ______________

12. Find the distance between $P(2, 4)$ and the line with equation $2x - y + 5 = 0$. **12.** ______________

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SAT and ACT Practice Answer Sheet

(10 Questions)

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10

	(/)	(/)	
(.)	(.)	(.)	(.)
	(0)	(0)	(0)
(1)	(1)	(1)	(1)
(2)	(2)	(2)	(2)
(3)	(3)	(3)	(3)
(4)	(4)	(4)	(4)
(5)	(5)	(5)	(5)
(6)	(6)	(6)	(6)
(7)	(7)	(7)	(7)
(8)	(8)	(8)	(8)
(9)	(9)	(9)	(9)

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SAT and ACT Practice Answer Sheet

(20 Questions)

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10 (A) (B) (C) (D) (E)

11 (A) (B) (C) (D) (E)

12 (A) (B) (C) (D) (E)

13 (A) (B) (C) (D) (E)

14 (A) (B) (C) (D) (E)

15 (A) (B) (C) (D) (E)

16 (A) (B) (C) (D) (E)

17 (A) (B) (C) (D) (E)

18 (A) (B) (C) (D) (E)

19

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

20

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

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7-1 Practice

Basic Trigonometric Identities

Use the given information to determine the exact trigonometric value if 0° < θ < 90°.

1. If $\cos\theta = \frac{1}{4}$, find $\tan\theta$.
$\sqrt{15}$

2. If $\sin\theta = \frac{2}{3}$, find $\cos\theta$.
$\frac{\sqrt{5}}{3}$

3. If $\tan\theta = \frac{7}{2}$, find $\sin\theta$.
$\frac{7\sqrt{53}}{53}$

4. If $\tan\theta = 2$, find $\cot\theta$.
$\frac{1}{2}$

Express each value as a trigonometric function of an angle in Quadrant I.

5. cos 892°
− cos 8°

6. csc 495°
csc 45°

7. $\sin\frac{23\pi}{3}$
$-\sin\frac{\pi}{3}$

Simplify each expression.

8. $\cos x + \sin x \tan x$
sec *x*

9. $\frac{\cot A}{\tan A}$
$\cot^2 A$

10. $\sin^2\theta\cos^2\theta - \cos^2\theta$
$-\cos^4\theta$

11. ***Kite Flying*** Brett and Tara are flying a kite. When the string is tied to the ground, the height of the kite can be determined by the formula $\frac{L}{H} = \csc\theta$, where L is the length of the string and θ is the angle between the string and the level ground. What formula could Brett and Tara use to find the height of the kite if they know the value of $\sin\theta$?
$H = L\sin\theta$

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7-1 Enrichment

The Physics of Soccer

Recall from Lesson 7-1 that the formula for the maximum height h of a projectile is $h = \frac{v_0^2\sin^2\theta}{2g}$, where θ is the measure of the angle of elevation in degrees, v_0 is the initial velocity in feet per second, and g is the acceleration due to gravity in feet per second squared.

Solve. Give answers to the nearest tenth.

1. A soccer player kicks a ball at an initial velocity of 60 ft/s and an angle of elevation of 40°. The acceleration due to gravity is 32 ft/s^2. Find the maximum height reached by the ball.
23.2 ft

2. With what initial velocity must you kick a ball at an angle of 35° in order for it to reach a maximum height of 20 ft?
62.4 ft/s

The distance d that a projected object travels is given by the formula $d = \frac{2v_0^2\sin\theta\cos\theta}{g}$.

3. Find the distance traveled by the ball described in Exercise 1.
110.8 ft

In order to kick a ball the greatest possible distance at a given initial velocity, a soccer player must maximize $d = \frac{2v_0^2\sin\theta\cos\theta}{g}$. Since 2, v_0, and g are constants, this means the player must maximize $\sin\theta\cos\theta$.

sin 0°cos 0°	= sin 90°cos 90°	= 0
sin 10°cos 10°	= sin 80°cos 80°	= 0.1710
sin 20°cos 20°	= sin 70°cos 70°	= 0.3214

4. Use the patterns in the table to hypothesize a value of θ for which $\sin\theta\cos\theta$ will be maximal. Use a calculator to check your hypothesis. At what angle should the player kick the ball to achieve the greatest distance?
45°

7-2

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Practice

Verifying Trigonometric Identities

Verify that each equation is an identity.

1. $\dfrac{\csc x}{\cot x + \tan x} = \cos x$

$\dfrac{\csc x}{\cot x + \tan x} = \dfrac{\frac{1}{\sin x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}} \cdot \dfrac{\sin x \cos x}{\sin x \cos x} = \dfrac{\cos x}{\cos^2 x + \sin^2 x} = \dfrac{\cos x}{1} = \cos x$

2. $\dfrac{1}{\sin y - 1} - \dfrac{1}{\sin y + 1} = -2\sec^2 y$

$\dfrac{1}{\sin y - 1} - \dfrac{1}{\sin y + 1} = \dfrac{\sin y + 1 - \sin y + 1}{\sin^2 y - 1} = \dfrac{2}{-\cos^2 y} = -2\sec^2 y$

3. $\sin^3 x - \cos^3 x = (1 + \sin x \cos x)(\sin x - \cos x)$

$\sin^3 x - \cos^3 x = (\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x)$

$= (\sin x - \cos x)(1 + \sin x \cos x)$

$= (1 + \sin x \cos x)(\sin x - \cos x)$

4. $\tan\theta + \dfrac{\cos\theta}{1 + \sin\theta} = \sec\theta$

$\tan\theta + \dfrac{\cos\theta}{1 + \sin\theta} = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1 + \sin\theta} = \dfrac{\sin\theta + \sin^2\theta + \cos^2\theta}{(\cos\theta)(1 + \sin\theta)}$

$= \dfrac{1 + \sin\theta}{(\cos\theta)(1 + \sin\theta)} = \sec\theta$

Find a numerical value of one trigonometric function of x.

5. $\sin x \cot x = 1$
$\cos x = 1$

6. $\sin x = 3\cos x$
$\tan x = 3$

7. $\cos x = \cot x$
$\csc x = 1$ or $\sin x = 1$

8. ***Physics*** The work done in moving an object is given by the formula $W = Fd\cos\theta$, where d is the displacement, F is the force exerted, and θ is the angle between the displacement and the force. Verify that $W = Fd\,\dfrac{\cot\theta}{\csc\theta}$ is an equivalent formula.

$W = Fd\,\dfrac{\cot\theta}{\csc\theta} = Fd\,\dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} = Fd\,\dfrac{\cos\theta}{\sin\theta} \cdot \sin\theta = Fd\cos\theta$

7-2

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Enrichment

Building from 1 = 1

By starting with the most fundamental identity of all, 1 = 1, you can create new identities as complex as you would like them to be.

First, think of ways to write 1 using trigonometric identities. Some examples are the following.

$1 = \cos A \sec A$

$1 = \csc^2 A - \cot^2 A$

$1 = \dfrac{\cos(A + 360°)}{\cos(360° - A)}$

Choose two such expressions and write a new identity.

$\cos A \sec A = \csc^2 A - \cot^2 A$

Now multiply the terms of the identity by the terms of another identity of your choosing, preferably one that will allow some simplification upon multiplication.

$$\begin{array}{rl} & \cos A \sec A = \csc^2 A - \cot^2 A \\ \times & \dfrac{\sin A}{\cos A} = \tan A \\ \hline & \sin A \sec A = \tan A \csc^2 A - \cot A \end{array}$$

Beginning with 1 = 1, create two trigonometric identities. Answers will vary.

1. ______________

2. ______________

Verify that each of the identities you created is an identity.

3. ______________

4. ______________

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7-3 Practice

Sum and Difference Identities

Use sum or difference identities to find the exact value of each trigonometric function.

1. $\cos \frac{5\pi}{12}$ **$\frac{\sqrt{6}-\sqrt{2}}{4}$**

2. $\sin(-165°)$ **$\frac{\sqrt{2}-\sqrt{6}}{4}$**

3. $\tan 345°$ **$\sqrt{3}-2$**

4. $\csc 915°$ **$-\sqrt{6}-\sqrt{2}$**

5. $\tan\left(-\frac{7\pi}{12}\right)$ **$2+\sqrt{3}$**

6. $\sec \frac{\pi}{12}$ **$\sqrt{6}-\sqrt{2}$**

Find each exact value if $0 < x < \frac{\pi}{2}$ and $0 < y < \frac{\pi}{2}$.

7. $\cos(x+y)$ if $\sin x = \frac{5}{13}$ and $\sin y = \frac{4}{5}$ **$\frac{16}{65}$**

8. $\sin(x-y)$ if $\cos x = \frac{8}{17}$ and $\cos y = \frac{3}{5}$ **$\frac{13}{85}$**

9. $\tan(x-y)$ if $\csc x = \frac{13}{5}$ and $\cot y = \frac{4}{3}$ **$-\frac{16}{63}$**

Verify that each equation is an identity.

10. $\cos(180° - \theta) = -\cos\theta$

$\cos(180° - \theta)$
$= \cos 180° \cos\theta + \sin 180° \sin\theta$
$= (-1)\cos\theta + 0 \cdot \sin\theta$
$= -\cos\theta$

11. $\sin(360° + \theta) = \sin\theta$

$\sin(360° + \theta)$
$= \sin 360° \cos\theta + \cos 360° \sin\theta$
$= 0 \cdot \cos\theta + 1 \cdot \sin\theta$
$= \sin\theta$

12. ***Physics*** Sound waves can be modeled by equations of the form $y = 20\sin(3t + \theta)$. Determine what type of interference results when sound waves modeled by the equations $y = 20\sin(3t + 90°)$ and $y = 20\sin(3t + 270°)$ are combined. (*Hint:* Refer to the application in Lesson 7-3.)
The interference is destructive. The waves cancel each other completely.

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7-3 Enrichment

Identities for the Products of Sines and Cosines

By adding the identities for the sines of the sum and difference of the measures of two angles, a new identity is obtained.

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$(i)\ \sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$$

This new identity is useful for expressing certain products as sums.

Example **Write $\sin 3\theta \cos\theta$ as a sum.**

In the right side of identity (*i*) let $\alpha = 3\theta$ and $\beta = \theta$ so that $2\sin 3\theta\cos\theta = \sin(3\theta+\theta) + \sin(3\theta-\theta)$.
Thus, $\sin 3\theta \cos\theta = \frac{1}{2}\sin 4\theta + \frac{1}{2}\sin 2\theta$.

By subtracting the identities for $\sin(\alpha+\beta)$ and $\sin(\alpha-\beta)$, you obtain a similar identity for expressing a product as a difference.

$$(ii)\ \sin(\alpha+\beta) - \sin(\alpha-\beta) = 2\cos\alpha\sin\beta$$

Example **Verify the identity $\frac{\cos 2x \sin x}{\sin 2x \cos x} = \frac{(\sin 3x - \sin x)^2}{\sin^2 3x - \sin^2 x}$.**

In the right sides of identities (*i*) and (*ii*) let $\alpha = 2x$ and $\beta = x$. Then write the following quotient.

$$\frac{2\cos 2x \sin x}{2\sin 2x\cos x} = \frac{\sin(2x+x) - \sin(2x-x)}{\sin(2x+x) + \sin(2x-x)}$$

By simplifying and multiplying by the conjugate, the identity is verified.

$$\frac{\cos 2x\sin x}{\sin 2x\cos x} = \frac{\sin 3x - \sin x}{\sin 3x + \sin x}\cdot\frac{\sin 3x - \sin x}{\sin 3x - \sin x}$$
$$= \frac{(\sin 3x - \sin x)^2}{\sin^2 3x - \sin^2 x}$$

Complete.

1. Use the identities for $\cos(\alpha+\beta)$ and $\cos(\alpha-\beta)$ to find identities for expressing the products $2\cos\alpha\cos\beta$ and $2\sin\alpha\sin\beta$ as a sum or difference.
$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
$2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$
$2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$

2. Find the value of $\sin 105° \cos 75°$ by using the identity above.
$\sin 105° \cos 75° = \frac{1}{2}(\sin 180° + \sin 30°)$
$= \frac{1}{2}\left(0 + \frac{1}{2}\right) = \frac{1}{4}$ or 0.25

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7-4 Practice

Double-Angle and Half-Angle Identities

Use a half-angle identity to find the exact value of each function.

1. $\sin 105°$ $\quad \frac{\sqrt{2+\sqrt{3}}}{2}$

2. $\tan \frac{\pi}{8}$ $\quad \sqrt{2} - 1$

3. $\cos \frac{5\pi}{8}$ $\quad \frac{\sqrt{2-\sqrt{2}}}{2}$

Use the given information to find sin 2θ, cos 2θ, and tan 2θ.

4. $\sin \theta = \frac{12}{13}$, $0° < \theta < 90°$ $\quad \frac{120}{169}, -\frac{119}{169}, -\frac{120}{119}$

5. $\tan \theta = \frac{1}{2}$, $\pi < \theta < \frac{3\pi}{2}$ $\quad \frac{4}{5}, \frac{3}{5}, \frac{4}{3}$

6. $\sec \theta = -\frac{5}{2}$, $\frac{\pi}{2} < \theta < \pi$ $\quad -\frac{4\sqrt{21}}{25}, -\frac{17}{25}, \frac{4\sqrt{21}}{17}$

7. $\sin \theta = \frac{3}{5}$, $0 < \theta < \frac{\pi}{2}$ $\quad \frac{24}{25}, \frac{7}{25}, \frac{24}{7}$

Verify that each equation is an identity.

8. $1 + \sin 2x = (\sin x + \cos x)^2$

$1 + \sin 2x \stackrel{?}{=} (\sin x + \cos x)^2$

$1 + \sin 2x \stackrel{?}{=} \sin^2 x + 2 \sin x \cos x + \cos^2 x$

$1 + \sin 2x \stackrel{?}{=} 1 + 2 \sin x \cos x$

$1 + \sin 2x = 1 + \sin 2x$

9. $\cos x \sin x = \frac{\sin 2x}{2}$

$\cos x \sin x \stackrel{?}{=} \frac{\sin 2x}{2}$

$\cos x \sin x \stackrel{?}{=} \frac{2 \sin x \cos x}{2}$

$\cos x \sin x = \cos x \sin x$

10. ***Baseball*** A batter hits a ball with an initial velocity v_0 of 100 feet per second at an angle θ to the horizontal. An outfielder catches the ball 200 feet from home plate. Find θ if the range of a projectile is given by the formula $R = \frac{1}{32}v_0^2 \sin 2\theta$.

about 20°

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7-4 Enrichment

Reading Mathematics: Using Examples

Most mathematics books, including this one, use examples to illustrate the material of each lesson. Examples are chosen by the authors to show how to apply the methods of the lesson and to point out places where possible errors can arise.

1. Explain the purpose of Example 1c in Lesson 7-4.
to illustrate how to use the double-angle identity for the tangent; to show how to find tan θ from information already known

2. Explain the purpose of Example 3 in Lesson 7-4.
to illustrate how a double-angle identity can be applied to a real-world situation

3. Explain the purpose of Example 4 in Lesson 7-4.
to illustrate the verification of a trigonometric identity involving a double-angle identity

To make the best use of the examples in a lesson, try following this procedure:

a. When you come to an example, stop. Think about what you have just read. If you don't understand it, reread the previous section.

b. Read the example problem. Then instead of reading the solution, try solving the problem yourself.

c. After you have solved the problem or gone as far as you can go, study the solution given in the text. Compare your method and solution with those of the authors. If necessary, find out where you went wrong. If you don't understand the solution, reread the text or ask your teacher for help.

4. Explain the advantage of working an example yourself over simply reading the solution given in the text.
Sample answer: This method checks your understanding of the material rather than your ability to follow the authors' logic. By allowing errors to arise in your solution, it helps you find areas of misunderstanding. Then it gives you a method for correcting your errors and checking your solution.

7-5

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Practice

Solving Trigonometric Equations

Solve each equation for principal values of x. Express solutions in degrees.

1. $\cos x = 3\cos x - 2$
0°

2. $2\sin^2 x - 1 = 0$
±45°

Solve each equation for 0° ≤ x < 360°.

3. $\sec^2 x + \tan x - 1 = 0$
0°, 135°, 180°, 315°

4. $\cos 2x + 3\cos x - 1 = 0$
60°, 300°

Solve each equation for 0 ≤ x < 2π.

5. $4\sin^2 x - 4\sin x + 1 = 0$
$\frac{\pi}{6}, \frac{5\pi}{6}$

6. $\cos 2x + \sin x = 1$
$0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$

Solve each equation for all real values of x.

7. $3\cos 2x - 5\cos x = 1$
$\frac{2\pi}{3} + 2\pi k, \frac{4\pi}{3} + 2\pi k$

8. $2\sin^2 x - 5\sin x + 2 = 0$
$\frac{\pi}{6} + 2\pi k, \frac{5\pi}{6} + 2\pi k$

9. $3\sec^2 x - 4 = 0$
$\frac{\pi}{6} + \pi k, \frac{5\pi}{6} + \pi k$

10. $\tan x(\tan x - 1) = 0$
$\pi k, \frac{\pi}{4} + \pi k$

11. ***Aviation*** An airplane takes off from the ground and reaches a height of 500 feet after flying 2 miles. Given the formula $H = d\tan\theta$, where H is the height of the plane and d is the distance (along the ground) the plane has flown, find the angle of ascent θ at which the plane took off.
about 2.7°

7-5

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Enrichment

The Spectrum

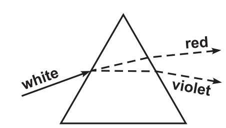
In some ways, light behaves as though it were composed of waves. The wavelength of visible light ranges from about 4×10^{-5} cm for violet light to about 7×10^{-5} cm for red light.

As light passes through a medium, its velocity depends upon the wavelength of the light. The greater the wavelength, the greater the velocity. Since white light, including sunlight, is composed of light of varying wavelengths, waves will pass through the medium at an infinite number of different speeds. The index of refraction n of the medium is defined by $n = \frac{c}{v}$, where c is the velocity of light in a vacuum (3×10^{10} cm/s), and v is the velocity of light in the medium. As you can see, the index of refraction of a medium is not a constant. It depends on the wavelength and the velocity of light passing through it. (The index of refraction of diamond given in the lesson is an average.)

1. For all media, $n > 1$. Is the speed of light in a medium greater than or less than c? Explain.
less than; $v = \frac{c}{n}$. Since $n > 1$, $v < c$.

2. A beam of violet light travels through water at a speed of 2.234×10^{10} cm/s. Find the index of refraction of water for violet light.
1.343

The diagram shows why a prism splits white light into a spectrum. Because they travel at different velocities in the prism, waves of light of different colors are refracted different amounts.



3. Beams of red and violet light strike crown glass at an angle of 20°. Use Snell's Law to find the difference between the angles of refraction of the two beams.
violet light: $n = 1.531$ red light: $n = 1.513$
about 0.16°

7-6

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Practice

Normal Form of a Linear Equation

Write the standard form of the equation of each line, given p, the length of the normal segment, and ϕ, the angle the normal segment makes with the positive x-axis.

1. $p = 4$, $\phi = 30°$
$\sqrt{3}x + y - 8 = 0$

2. $p = 2\sqrt{2}$, $\phi = \frac{\pi}{4}$
$x + y - 4 = 0$

3. $p = 3$, $\phi = 60°$
$x + \sqrt{3}y - 6 = 0$

4. $p = 8$, $\phi = \frac{5\pi}{6}$
$\sqrt{3}x - y + 16 = 0$

5. $p = 2\sqrt{3}$, $\phi = \frac{7\pi}{4}$
$\sqrt{2}x - \sqrt{2}y - 4\sqrt{3} = 0$

6. $p = 15$, $\phi = 225°$
$\sqrt{2}x + \sqrt{2}y + 30 = 0$

Write each equation in normal form. Then find the length of the normal and the angle it makes with the positive x-axis.

7. $3x - 2y - 1 = 0$
$\frac{3\sqrt{13}}{13}x - \frac{2\sqrt{13}}{13}y - \frac{\sqrt{13}}{13} = 0; \frac{\sqrt{13}}{13}; 326°$

8. $5x + y - 12 = 0$
$\frac{5\sqrt{26}}{26}x + \frac{\sqrt{26}}{26}y - \frac{6\sqrt{26}}{13} = 0; \frac{6\sqrt{26}}{13}; 11°$

9. $4x + 3y - 4 = 0$
$\frac{4}{5}x + \frac{3}{5}y - \frac{4}{5} = 0; \frac{4}{5}; 37°$

10. $y = x + 5$
$-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - \frac{5\sqrt{2}}{2} = 0; \frac{5\sqrt{2}}{2}; 135°$

11. $2x + y + 1 = 0$
$-\frac{2\sqrt{5}}{5}x - \frac{\sqrt{5}}{5}y - \frac{\sqrt{5}}{5} = 0; \frac{\sqrt{5}}{5}; 207°$

12. $x + y - 5 = 0$
$\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - \frac{5\sqrt{2}}{2} = 0; \frac{5\sqrt{2}}{2}; 45°$

7-6

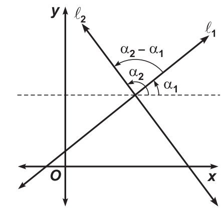
NAME ______________ DATE ________ PERIOD ____

Enrichment

Slopes of Perpendicular Lines

The derivation of the normal form of a linear equation uses this familiar theorem, first stated in Lesson 1-6: Two nonvertical lines are perpendicular if and only if the slope of one is the negative reciprocal of the slope of the other.

You can use trigonometric identities to prove that if lines are perpendicular, then their slopes are negative reciprocals of each other.



ℓ_1 and ℓ_2 are perpendicular lines.
α_1 and α_2 are the angles that ℓ_1 and ℓ_2, respectively, make with the horizontal.
Let m_1 = slope of ℓ_1
m_2 = slope of ℓ_2

Complete the following exercises to prove that $m_1 = -\frac{1}{m_2}$.

1. Explain why $m_1 = \tan \alpha_1$ and $m_2 = \tan \alpha_2$.

$\tan \alpha = \frac{\text{change in } y}{\text{change in } x} = m$

2. According to the difference identity for the cosine function, $\cos(\alpha_2 - \alpha_1) = \cos \alpha_2 \cos \alpha_1 + \sin \alpha_2 \sin \alpha_1$. Explain why the left side of the equation is equal to zero.
$\alpha_2 - \alpha_1 = 90°$ and $\cos 90° = 0$.

3. $\cos \alpha_2 \cos \alpha_1 + \sin \alpha_2 \sin \alpha_1 = 0$

$\sin \alpha_2 \sin \alpha_1 = -\cos \alpha_2 \cos \alpha_1$

$\frac{\sin \alpha_1}{\cos \alpha_1} = -\frac{\cos \alpha_2}{\sin \alpha_2}$

Complete using the tangent function. $\tan \alpha_1 = -\frac{1}{\tan \alpha_2}$

Complete, using m_1 and m_2. $m_1 = -\frac{1}{m_2}$

NAME ______________ DATE ________ PERIOD ______

7-7

Practice

Distance From a Point to a Line

Find the distance between the point with the given coordinates and the line with the given equation.

1. $(-1, 5)$, $3x - 4y - 1 = 0$ **$\frac{24}{5}$**
2. $(2, 5)$, $5x - 12y + 1 = 0$ **$\frac{49}{13}$**
3. $(1, -4)$, $12x + 5y - 3 = 0$ **$\frac{11}{13}$**
4. $(-1, -3)$, $6x + 8y - 3 = 0$ **$\frac{33}{10}$**

Find the distance between the parallel lines with the given equations.

5. $2x - 3y + 4 = 0$; $y = \frac{2}{3}x + 5$ **$\frac{11\sqrt{13}}{13}$**
6. $4x - y + 1 = 0$; $4x - y - 8 = 0$ **$\frac{9\sqrt{17}}{17}$**

Find equations of the lines that bisect the acute and obtuse angles formed by the lines with the given equations.

7. $x + 2y - 3 = 0$; $x - y + 4 = 0$
   $(\sqrt{2} + \sqrt{5})x + (2\sqrt{2} - \sqrt{5})y - 3\sqrt{2} + 4\sqrt{5} = 0$;
   $(\sqrt{2} - \sqrt{5})x + (2\sqrt{2} + \sqrt{5})y - 3\sqrt{2} - 4\sqrt{5} = 0$

8. $9x + 12y + 10 = 0$; $3x + 2y - 6 = 0$
   $(45 + 9\sqrt{13})x + (30 + 12\sqrt{13})y - 90 + 10\sqrt{13} = 0$;
   $(9\sqrt{13} - 45)x + (12\sqrt{13} - 30)y + 90 + 10\sqrt{13} = 0$

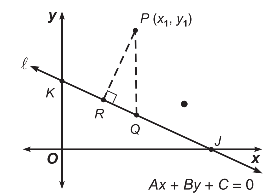
NAME ______________ DATE ________ PERIOD ______

7-7

Enrichment

Deriving the Point-Line Distance

Line ℓ has the equation $Ax + By + C = 0$. Answer these questions to derive the formula given in Lesson 7-7 for the distance from $P(x_1, y_1)$ to ℓ.



1. Use the equation of the line to find the coordinates of J and K, the x- and y-intercepts of ℓ.
   $J\left(-\frac{C}{A}, 0\right), K\left(0, -\frac{C}{B}\right)$
2. $\overline{PQ}$ is a vertical segment from P to ℓ. Find the x-coordinate of Q.
   x_1
3. Since Q is on ℓ, its coordinates must satisfy the equation of ℓ. Use your answer to Exercise 2 to find the y-coordinate of Q.
   $-\frac{A}{B}x_1 - \frac{C}{B}$
4. Find PQ by finding the difference between the y-coordinates of P and Q. Write your answer as a fraction.
   $\left|\frac{Ax_1 + By_1 + C}{B}\right|$
5. Triangle KJO is a right triangle. Use your answers to Exercise 1 and the Pythagorean Theorem to find KJ. Simplify.
   $\left|\frac{C\sqrt{A^2 + B^2}}{AB}\right|$
6. Since $\angle Q \cong \angle K$, $\triangle JKO \sim \triangle PQR$.
   $$\frac{PR}{OJ} = \frac{PQ}{KJ}$$
   Use your answers to Exercises 1, 4, and 5 to find PR, the distance from P to ℓ. Simplify.
   $PR = \frac{|(Ax_1 + By_1 + C)|}{\sqrt{A^2 + B^2}}$

Chapter 7 Answer Key

Form 1A

Page 297

1. A
2. C
3. D
4. D
5. B
6. A
7. C
8. A
9. D
10. A
11. C

Page 298

12. B
13. B
14. C
15. A
16. D
17. A
18. D
19. C
20. C

Bonus: D

Form 1B

Page 299

1. B
2. C
3. A
4. D
5. D
6. C
7. B
8. A
9. C
10. B
11. C
12. B

Page 300

13. D
14. A
15. D
16. B
17. A
18. D
19. A
20. B

Bonus: A

Chapter 7 Answer Key

Form 1C

Page 301

1. B
2. D
3. B
4. C
5. A
6. B
7. D
8. C
9. A
10. C
11. D

Page 302

12. B
13. B
14. C
15. A
16. D
17. C
18. B
19. D
20. A

Bonus: D

Form 2A

Page 303

1. $\cos\theta$
2. $-\frac{5}{4}$
3. $\tan\theta$
4. $2\sin\theta$
5. 1
6. $\frac{-\sqrt{6}-\sqrt{2}}{4}$
7. $\frac{87}{425}$
8. $\sin\theta$
9. $-\frac{7}{8}$
10. $-\frac{24}{25}$ or -0.96
11. $-\frac{24}{7}$

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12. $\frac{1}{2}\sqrt{2-\sqrt{2}}$
13. $\pi + 2\pi k$
14. $30°$ and $90°$
15. $\frac{7\pi}{6} < x < \frac{11\pi}{6}$
16. $\frac{3\sqrt{13}}{13}x + \frac{2\sqrt{13}}{13}y - \frac{4\sqrt{13}}{13} = 0$
17. $x - \sqrt{3}y + 10 = 0$
18. $\frac{4\sqrt{5}}{5}$ units
19. $\frac{3\sqrt{10}}{10}$ units
20. $(2\sqrt{13} - 3\sqrt{5})x + (\sqrt{13} + 2\sqrt{5})y - 5\sqrt{13} - 6\sqrt{5} = 0$

Bonus: $\frac{336}{625}$ or 0.5376

Chapter 7 Answer Key

Form 2B

Page 305

1. $\sec\theta$
2. $\frac{4}{5}$
3. $\sin\theta$
4. $\cot^2\theta$
5. $\frac{1}{3}$
6. $\frac{\sqrt{2}-\sqrt{6}}{4}$
7. $-\frac{16}{65}$
8. $\cos\theta$
9. $\frac{7}{9}$
10. $\frac{24}{25}$ or 0.96
11. $-\frac{24}{7}$

Page 306

12. $-\frac{\sqrt{2-\sqrt{3}}}{2}$
13. $\frac{\pi}{3}, \frac{4\pi}{3}$
14. $-30°, 30°$
15. πk
16. $\frac{2\sqrt{29}}{29}x + \frac{5\sqrt{29}}{29}y - \frac{3\sqrt{29}}{29} = 0$
17. $\sqrt{3}x - y + 14 = 0$
18. $\frac{9\sqrt{5}}{10}$
19. $\frac{\sqrt{5}}{5}$
20. $(3\sqrt{5} + 2\sqrt{10})x + (\sqrt{5} - \sqrt{10})y + 6\sqrt{5} - \sqrt{10} = 0$

Bonus: $\sin^2\theta$

Form 2C

Page 307

1. $\cos\theta$
2. $-\frac{4}{3}$
3. 1
4. $\cos^2\theta$
5. $\frac{1}{2}$
6. $\frac{\sqrt{2}+\sqrt{6}}{4}$
7. $-\frac{63}{16}$
8. $\sin\theta$
9. $-\frac{1}{2}$
10. $\frac{24}{25}$ or 0.96
11. $-\frac{24}{7}$

Page 308

12. $\frac{\sqrt{2+\sqrt{2}}}{2}$
13. $\frac{\pi}{6}, \frac{5\pi}{6}$
14. $-60°$
15. $\frac{\pi}{4} + \pi k$
16. $\frac{-3\sqrt{13}}{13}x + \frac{2\sqrt{13}}{13}y - \frac{6\sqrt{13}}{13} = 0$
17. $x + \sqrt{3}y - 18 = 0$
18. $\frac{7\sqrt{29}}{29}$ units
19. $\frac{3\sqrt{2}}{4}$ units
20. $7x + 56y - 25 = 0$

Bonus: They are perpendicular to each other.

Chapter 7 Answer Key

CHAPTER 7 SCORING RUBRIC

Level	Specific Criteria
3 Superior	• Shows thorough understanding of the concepts *proof, identity, normal to a line,* and *distance from a point to a line.* • Uses appropriate strategies to prove identities and write equations in normal form. • Computations are correct. • Written explanations are exemplary. • Graphs are accurate and appropriate. • Goes beyond requirements of some or all problems.
2 Satisfactory, with Minor Flaws	• Shows understanding of the concepts *proof, identity, normal to a line,* and *distance from a point to a line.* • Uses appropriate strategies to prove identities and write equations in normal form. • Computations are mostly correct. • Written explanations are effective. • Graphs are mostly accurate and appropriate. • Satisfies all requirements of problems.
1 Nearly Satisfactory, with Serious Flaws	• Shows understanding of most of the concepts *proof, identity, normal to a line,* and *distance from a point to a line.* • May not use appropriate strategies to prove identities and write equations in normal form. • Computations are mostly correct. • Written explanations are satisfactory. • Graphs are mostly accurate and appropriate. • Satisfies most requirements of problems.
0 Unsatisfactory	• Shows little or no understanding of the concepts *proof, identity, normal to a line,* and *distance from a point to a line.* • May not use appropriate strategies to prove identities and write equations in normal form. • Computations are incorrect. • Written explanations are not satisfactory. • Graphs are not accurate and appropriate. • Does not satisfy requirements of problems.

Chapter 7 Answer Key

Open-Ended Assessment

Page 309

1a.

$$\frac{\cos\theta}{1-\sin\theta} - \frac{1+\sin\theta}{\cos\theta} \stackrel{?}{=} 0$$

$$\frac{\cos^2\theta - (1-\sin^2\theta)}{(1-\sin\theta)\cos\theta} \stackrel{?}{=} 0$$

$$\frac{\cos^2\theta - \cos^2\theta}{(1-\sin\theta)\cos\theta} \stackrel{?}{=} 0$$

$$\frac{0}{(1-\sin\theta)\cos\theta} \stackrel{?}{=} 0$$

$$0 = 0$$

1b. There are many ways to make a simple expression complicated but few ways to simplify a complicated expression. Thus, it is easier to find the proper simplification.

1c. No, because in the third line of the attempted proof, the expression has been treated as an equality by multiplying both sides by cos A. A correct verifications follows.

$$\sec A \sin A \stackrel{?}{=} \tan A$$

$$\frac{1}{\cos A}\sin A \stackrel{?}{=} \tan A$$

$$\frac{\sin A}{\cos A} \stackrel{?}{=} \tan A$$

$$\tan A = \tan A$$

2a. $-\frac{3}{\sqrt{13}}x + \frac{2}{\sqrt{13}}y - \frac{6}{\sqrt{13}} = 0$ or

$-\frac{3\sqrt{13}}{13}x + \frac{2\sqrt{13}}{13}y - \frac{6\sqrt{13}}{13} = 0$

Length of normal $= \frac{6}{\sqrt{13}}$ or $\frac{6\sqrt{13}}{13}$, $\tan\phi = \frac{\frac{2\sqrt{13}}{13}}{-\frac{3\sqrt{13}}{13}} = -\frac{2}{3}$.

Since $\cos\phi < 0$ and $\sin\phi > 0$, ϕ is in Quadrant II. The angle of the normal with the positive x-axis $= 146°$.

2b. The point at (0, 3) is on the line with equation $2y - 3x = 6$. The distance from the point at (0, 3) to the line with equation $6x - 4y + 16 = 0$ is

$d = \frac{6(0) - 4(3) + 16}{-\sqrt{36 + 16}}$, or $-\frac{2\sqrt{13}}{13}$.

The negative sign indicates that the point and the origin are on the same side of the line.

2c. No, because the origin and any point on the line with equation $6x - 4y + 16 = 0$ are on opposite sides of the line with equation $2y - 3x = 6$.

2d. For parallel lines, d will have the same sign only when the origin is between the lines.

Chapter 7 Answer Key

Mid-Chapter Test
Page 310

1. $\frac{1}{2}$
2. $-\frac{4}{5}$
3. $\sin x$
4. $\cos \theta$
5. $\frac{1}{3}$
6. $\frac{-\sqrt{6} - \sqrt{2}}{4}$
7. $\frac{56}{33}$
8. $\frac{24}{25}$
9. $\frac{7}{8}$
10. $\frac{1}{2}\sqrt{2 - \sqrt{2}}$

Quiz A
Page 311

1. $\frac{1}{3}$
2. $-\frac{5}{3}$
3. $\csc^2 x$
4. $\sin^4 \theta$
5. $\frac{1}{2}$

Quiz B
Page 311

1. $\frac{\sqrt{2} + \sqrt{6}}{4}$
2. $-\frac{56}{33}$
3. $-\frac{120}{169}$
4. $-\frac{24}{7}$
5. $\frac{1}{2}\sqrt{2 - \sqrt{3}}$

Quiz C
Page 312

1. $\frac{3\pi}{2}$
2. 30°, 150°
3. $\frac{\pi}{4} + \pi k$
4. $-\frac{\sqrt{26}}{26}x - \frac{5\sqrt{26}}{26}y - \frac{4\sqrt{26}}{13} = 0$
5. $x + \sqrt{3}y + 6 = 0$

Quiz D
Page 312

1. $\frac{2\sqrt{5}}{5}$ units
2. 0 units
3. $\frac{8\sqrt{5}}{5}$ units
4. 6 units
5. $(\sqrt{5} - \sqrt{10})x + (3\sqrt{5} + 2\sqrt{10})y - 3\sqrt{5} + 2\sqrt{10} = 0$

Chapter 7 Answer Key

SAT/ACT Practice

Page 313

1. C
2. C
3. A
4. E
5. A
6. E
7. D
8. C
9. B

Page 314

10. D
11. E
12. C
13. E
14. B
15. A
16. B
17. D
18. D
19. 70
20. 120

Cumulative Review

Page 315

1. $3x - y + 5 = 0$
2. $\begin{bmatrix} 8 & 5 \\ 5 & 32 \end{bmatrix}$
3. $f^{-1}(x) = 2 \pm \sqrt{x + 5}$; no
4. 2
5. $x^3 - 3x^2 + 4x - 12 = 0$
6. −2; no
7. 41.4°
8. 43.8 units^2
9. 13; π; 2π
10. 0
11. −60°, 60°
12. $\sqrt{5}$

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