## CHAPTER 7: Similar Figures



Name: $\qquad$

Teacher:

Pd:

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## SWBAT: Use proportions to solve problems

Identify and apply properties of similar polygons to solve problems.
Warm - Up

1) The ratio of the angles in a quadrilateral are $2: 3: 5: 10$, What is the measure of the largest angle?

Figures that are similar ( $\sim$ ) have the same shape but not necessarily the same size.

$\Delta 1$ is similar to $\Delta 2(\Delta 1 \sim \Delta 2)$.

$\Delta 1$ is not similar to $\Delta 3(\Delta 1+\Delta 3)$.

Identify Similar Polygons Similar polygons have the same shape but not necessarily the same size.


Two polygons are similar if and only if their corresponding angles are congruent and corresponding side lengths are proportional.

Example In the diagram below, $A B C D$ is similar to $W X Y Z$.


Corresponding angles
$\angle A \cong \angle W, \angle B \cong \angle X, \angle C \cong \angle Y$, and $\angle D \cong \angle Z$

Corresponding sides

$$
\frac{A B}{W X}=\frac{B C}{X Y}=\frac{C D}{Y Z}=\frac{D A}{Z W}=\frac{3}{1}
$$

Symbols ABCD ~WXYZ

## Example 1:



Given: $\quad \triangle \mathrm{ABC} \sim \Delta \mathrm{DEF}$

- Each pair of corresponding angles are congruent:
$\Varangle A \cong$ $\qquad$ $\Varangle B \cong$ $\qquad$ $\Varangle C \cong$
- The ratios of the measures of all pairs of corresponding sides are equal:



## Check Your Progress

Determine whether the following polygons are similar. If they are, write a similarity statement.
1.

$\Delta$ $\qquad$ $\sim \Delta$ $\qquad$
$\Delta$ $\qquad$ $\sim \Delta$ $\qquad$

## Example 2:

If $\triangle A B C \sim \triangle Z X Y, m \Varangle A=60^{\circ}$, and $m \Varangle B=85^{\circ}$, what is $m \Varangle Y$ ?

## You Try It!

a. If $\triangle D E F \sim \Delta \mathrm{GHI}, \mathrm{m} \Varangle G=110^{\circ}$, and $\mathrm{m} \Varangle E=25^{\circ}$, what is $\mathrm{m} \Varangle F$ ?
b. Given that $\triangle J H K \sim \triangle P O M$ and $\mathrm{m} \Varangle H=90^{\circ}, \mathrm{m} \Varangle J=40^{\circ}, \Varangle m=x+5, \Varangle O=\frac{1}{2} y$. Find the values of $x$ and $y$.

## Example 3:

Given: ABCD ~ EFGH
Part a) Find:
$\mathrm{FG}=$ $\qquad$
GH = $\qquad$
$\mathrm{EH}=$ $\qquad$
$m \nleftarrow F=$ $\qquad$

$m \nleftarrow H=$ $\qquad$

$m \Varangle C=$ $\qquad$
$m \measuredangle E=$ $\qquad$

Part b) Find the ratio of the perimeter of $A B C D$ and the perimeter of $E F G H$.
$\frac{\text { Perimeter of } A B C D}{\text { Perimeter of } E F G H}=$
$\square$

## You Try It!

Two similar squares have a scale factor of $3: 2$. The perimeter of the small rectangle is 50 feet. Find the perimeter of the large rectangle.

## Challenge

The two triangles are similar below. Solve for x .


## SUMMARY

You can use properties of similar polygons to solve problems.

## Rectangle DEFG ~ rectangle HJKL. What is the length of HJKL?



$$
\begin{aligned}
\frac{\text { length of } D E F G}{\text { length of } H J K L} & =\frac{\text { width of } D E F G}{\text { width of } H J K L} \\
\frac{40}{x} & =\frac{27}{18} \\
40(18) & =27(x) \\
720 & =27 x \\
26 \frac{2}{3} & =x
\end{aligned}
$$

Write a proportion.
Substitute the known values.
Cross Products Property
Simplify.
Divide both sides by 27 .

The length of $H J K L$ is $26 \frac{2}{3}$ in.

## Exit Ticket

The triangles below are similar. Find the length of $x$.

[A] 3.3
[B] 28.1
[C] 72
[D] 67.5

## Homework

If quadrilateral $A B C D$ is similar to quadrilateral $E F G H$, find each of the following.

1) Scale factor of $A B C D$ to $E F G H$ ?
2) $E F=$ $\qquad$ 3) $\boldsymbol{F G}=$ $\qquad$
3) $\boldsymbol{G H}=$ $\qquad$
4) Perimeter of $A B C D$ ?
5) Perimeter of EFGH?
6) Ratio of perimeter of ABCD to EFGH ?


Each pair of polygons is similar. Find the values of " $x$ " and " $y$ ".
8)


9)


10) Triangle $A B C$ is similar to triangle DEF find the value of " $x$ " and " $y$ ". What is the sum of the perimeters of the triangles?

11) Quadrilateral $A B C D$ is similar to quadrilateral $E F G H$. Find the value of " $x$ ", " $y$ ", and " $z$ ". What is the perimeter of each figure?

12) Two rectangles are similar. The length of small rectangle is 4 and the length of the big rectangle is 12 . If the perimeter of the smaller rectangle is 28 , then what is the perimeter of the larger rectangle?
13) If two similar polygons have the perimeter of 36 and 21 inches. If the length of the side of the larger rectangle is 4 inches, then what is the product of the lengths of the polygons?
14) The ratio of the height of $\triangle C D E$ to the height of similar triangle $\triangle F G H$ is $3: 5$. The perimeter of $\triangle F G H$ is 25 cm . Find the perimeter of $\triangle C D E$.
15) Given: $\triangle N P R \sim \triangle S T V$, $\mathrm{m} \Varangle P=90^{\circ}, \mathrm{m} \Varangle R=60^{\circ}$, $S V=15, N R=20, R P=10$


Find: $\mathrm{m} \Varangle V, \mathrm{~m} \Varangle S$, and VT
16) Given: $\triangle A B C \sim \triangle D F E$

Find: $m \angle D$ and $\overline{E F}$


## Day 2 - Chapter 7-3: Triangle Proportionality Theorem

SWBAT: Apply Three Theorems frequently used to establish proportionality
Warm - Up

1. If $\triangle A B C \sim \Delta P Q R$, find $x$ and $y$.


$$
\begin{aligned}
& x= \\
& y= \\
&
\end{aligned}
$$

2. The ratio of two sides of similar triangles is $1: 3$. The perimeter of the smaller triangle is 22 cm , find the perimeter of the larger triangle.

If a line is parallel to one side of a triangle and intersects the other two sides, it divides those two sides proportionally. (Side-Splitter Theorem)

Given: $\overleftrightarrow{\mathrm{BE}} \| \overleftrightarrow{\mathrm{CD}}$


Example 1: Given: $\overline{P Q} \| \overline{B C}$. Find the measure of $\overline{C Q}$.


Example 2:


## You Try It!

Solve for $x$.
$D C=18, A D=6, A E=12, E B=x-3$


Example 3: In the diagram below of $\triangle A C D, E$ is a point on $\overline{A D}$ and $B$ is a point on $\overline{A C}$, such that $\overline{E B} \| \overline{D C}$. If $\frac{A E}{E B}=3, E D=6$, and $D C=15$, find the length of

| Triangle | First two <br> letters | Last two <br> letters | First and <br> last letters |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

## You Try It!

In $\triangle A B C, \mathrm{D}$ is a point on $\overline{A B}$ and E is a point on $\overline{B C}$, such that $\overline{D E} \| \overline{A C}$.
If $\mathrm{BE}=4, \mathrm{EC}=8$, and $\mathrm{DE}=6$, find the length of $\overline{A C}$.

| Triangle | First two <br> letters | Last two <br> letters | First and <br> last letters |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

## Parallels Proportion Theorem

If three or more parallel lines are intersected by two transversals, the parallel lines divide the transversals proportionally.
Given: If $\overline{A E}\|\overline{B F}\| \overline{C G}$


Example 4: Find $x$.


You Try It!


## Angle Bisector Proportionality Theorem

If a ray bisects an angle of a triangle, it divides the opposite side into segments that are proportional to the adjacent sides. (Angle Bisector Theorem)

Given: $\xrightarrow[A \mathrm{ABD}]{\triangle \mathrm{ABD}}$;
$\overrightarrow{\mathrm{AC}}$ bisects $\angle \mathrm{BAD}$.

Conclusion:


Ex 5. Given: $\angle \mathrm{RVS} \cong \angle S V T$, lengths as shown
Find: ST


You Try It!

## Find the value of $x$.



18

## Challenge

Solve for x .


SUMMARY

| Triangle Proportionality Theorem | Example |
| :--- | :--- |
| If a line parallel to a side of a triangle <br> intersects the other two sides, then it <br> divides those sides proportionally. | $\boxed{X Y} \\| \overline{A C}$ |

You can use the Triangle Proportionality Theorem to find lengths of segments in triangles.

## Find $E G$.

$$
\begin{aligned}
\frac{E G}{G F} & =\frac{D H}{H F} \\
\frac{E G}{6} & =\frac{7.5}{5} \\
E G(5) & =6(7.5) \\
5(E G) & =45 \\
E G & =9
\end{aligned}
$$

Triangle Proportionality Theorem
Substitute the known values.


Cross Products Property
Simplify.
Divide both sides by 5 .

| Triangle Angle Bisector Theorem | Example |  |
| :--- | :---: | :---: |
| An angle bisector of a triangle divides the |  |  |
| opposite side into two segments whose |  |  |
| lengths are proportional to the lengths of |  |  |
| the other two sides. |  |  |
| ( $\triangle \angle$ Bisector Thm.) |  |  |

Find $L P$ and $L M$.
$\frac{L P}{P N}=\frac{M L}{N M}$
$\triangle \angle$ Bisector Thm.
$\frac{x}{6}=\frac{x+3}{10}$
Substitute the given values.
$x(10)=6(x+3)$
Cross Products Property
$10 x=6 x+18$
Distributive Property

$4 x=18$
Simplify.
$x=4.5 \quad$ Divide both sides by 4 .
$L P=x=4.5$
$L M=x+3=4.5+3=7.5$

## Exit Ticket

In the figure shown, $\overline{B C} \| \overline{D E}, A B=2$ yards, $B C=9$ yards, $A E=36$ yards, and $D E=36$ yards. Find $B D$.

[A] 9 yd
[B] 8 yd
[C] 6 yd
[D] 27 yd

Solve for all of the missing sides. Show the proportions used.
1)

2)

3)

4) Solve for $x$.

5) Solve for $x$.

6) Solve for $x$.

7) CHALLENGE!!!!! Solve for $x$.

8) Determine whether the given information implies $\overline{B C} \| \overline{D E}$. Exp/ain.

9) In $\triangle D E F, \mathrm{M}$ is a point on $\overline{D F}$ and N is a point on $\overline{F E}$, such that $\overline{M N} \| \overline{D E}$. If $\mathrm{FN}=3, \mathrm{NE}=6$, and $\mathrm{MN}=4$, find the length of $\overline{D E}$.
10) In $\triangle D E F, \mathrm{M}$ is a point on $\overline{D F}$ and N is a point on $\overline{F E}$, such that $\overline{M N} \| \overline{D E}$.

If $\mathrm{MF}=4, \mathrm{DF}=8$, and $\mathrm{DE}=10$, find the length of $\overline{M N}$.

## Day 3 - Similarities in Right Triangles

## Warm - Up

Solve for x .


Geometric Mean When the means of a proportion are the same number, that number is called the geometric mean of the extremes. The geometric mean between two numbers is the positive square root of their product.

$$
\begin{aligned}
\text { extreme } & \rightarrow \frac{a}{x}=\frac{x}{b} \leftarrow \text { mean } \\
\quad \text { mean } & \rightarrow \text { extreme }
\end{aligned}
$$

Words The geometric mean of two positive numbers $a$ and $b$ is the number $x$ such that $\frac{a}{x}=\frac{x}{b}$. So, $x^{2}=a b$ and $x=\sqrt{a b}$.

Example The geometric mean of $a=9$ and $b=4$ is 6 , because $6=\sqrt{9 \cdot 4}$.

Find the geometric mean between each pair of numbers.

1A. 5 and 45


1B. 12 and 15


Geometric Means in Right Triangles In a right triangle, an altitude drawn from the vertex of the right angle to the hypotenuse forms two additional right triangles. These three right triangles share a special relationship.

## Similarities in Right Triangles



$$
\Delta_{ـ} \sim \Delta_{ـ} \sim \Delta_{ـ} \sim \Delta_{ـ}
$$

From above, you know that altitude $\overline{C D}$ drawn to the hypotenuse of right triangle $A B C$ forms three similar triangles: $\triangle A C B \sim \triangle A D C \sim \triangle C D B$. By the definition of similar polygons, you can write the following proportions comparing the side lengths of these triangles.

$\frac{\text { shorter leg }}{\text { longer leg }}=\frac{\boldsymbol{b}}{\boldsymbol{a}}=\frac{x}{h}=\frac{\boldsymbol{h}}{y} \frac{\text { hypotenuse }}{\text { shorter leg }}=\frac{\boldsymbol{c}}{\boldsymbol{b}}=\frac{\boldsymbol{b}}{x}=\frac{\boldsymbol{a}}{\boldsymbol{h}} \frac{\text { hypotenuse }}{\text { longer leg }}=\frac{\boldsymbol{c}}{\boldsymbol{a}}=\frac{\boldsymbol{b}}{\boldsymbol{h}}=\frac{a}{y}$
Notice that the circled relationships involve geometric means. This leads to the theorems at the top of the next page.

Rules:
Rule \#1: Altitude Rule

$\frac{\text { part of hypotenuse }}{\text { altitude }}=\frac{\text { altitude }}{\text { other part of hypotenuse }}$

So,

$\underbrace{\text { Altitude }}_{x}$ is the geometric mean of the separate parts of the hypotenuse

Example 1: Solve for x .


You Try It!
 You Try It!


Rule \#2: Leg Rule


$$
\frac{\text { hypotenuse }}{\text { leg }}=\frac{\text { leg }}{\text { projection of the leg }}
$$

So,


A leg is the geometric mean of the hypotenuse and the part of hypotenuse adjacent to the leg

## Example 2: Solve for x .



## Example 3:

In the diagram below of right triangle $A B C, \overline{C D}$ is the altitude to hypotenuse $\overline{A B}, C B=6$, and $A D=5$.


What is the length of $\overline{B D}$ ?

## 



## Challenge: Solve for $x$



## SUMMARY



## Examples:

1. Find $x$ :


Solution:
Examine the diagram to see what is given. This problem needs the Altitude Rule.

$$
\begin{aligned}
\frac{4}{8} & =\frac{8}{x} \\
4 x & =64 \\
x & =16
\end{aligned}
$$

2. Find $x$ :


Solution:
Examine the diagram to see what is given. This problem needs the Leg Rule (there is no value on the altitude).
$\frac{8}{x}=\frac{x}{2}$
$x^{2}=16$
$x=4$
(lengths are positive)

## Exit Ticket

1. In the diagram below of right triangle $A B C$, altitude
$\overline{B D}$ is drawn to hypotenuse $\overline{A C}, A C=16$, and $C D=7$.


What is the length of $\overline{B D}$ ?

1) $3 \sqrt{7}$
2) $4 \sqrt{7}$
3) $7 \sqrt{3}$
4) 12
2. In the diagram below of right triangle $A C B$, altitude $\overline{C D}$ is drawn to hypotenuse $\overline{A B}$.


If $A B=36$ and $A C=12$, what is the length of $\overline{A D}$ ?

1) 32 2) 6 3) 34$) 4$

## Day 3 - HW Similarities in Right Triangles

Geometric Mean - given two numbers "a" and " $b$ ", use the following formula to find the geometric mean " $x$ ".

$$
\frac{a}{x}=\frac{x}{b}
$$

1) Find the geometric mean of 2 and 8 .
2) Find the geometric mean of 4 and 5 .
3) If one number is 6 and the geometric mean of the two numbers is 4 . What is the other number?
4) If an altitude is drawn to the hypotenuse of triangle BAN below, then name and redraw the 3 similar triangles created.

5) Find the value of $x$.

6) How far is it across the lake?

7) Find the length of the altitude of $\triangle P Q R$.

8) Find the length of HK of right triangle GHK.

9) Challenge!!!! Solve for $x$.

10) The altitude, $\overline{X R}$, to the hypotenuse of right $\triangle W X Y$ divides the hypotenuse into segments that are 8 and 10 cm long. Find the length of the altitude.
11) Find the lengths of GH and HK.


## Day 4 - Similarities in Right Triangles - Day 2

Warm - Up
Find the geometric mean of the following numbers.

1. 12 and 6

Find $x$
2.

3.

Find x


## Practice

Example 1: In the accompanying diagram, $\triangle A B C$ is a right triangle and $\overline{C D}$ is the altitude to hypotenuse $\overline{A B}$.
If $A D=4$ and $D B=16$, find the length of $\overline{C D}$.


Leg or Alt?

Example 2: In the accompanying diagram of right triangle $\mathrm{ABC}, \overline{\mathrm{CD}}$ is drawn perpendicular to hypotenuse $\overline{\mathrm{AB}}$. If $A B=16$ and $D B=4$, find $B C$.


Leg or Alt?

Example 3: In right triangle ABC , altitude $\overline{\mathrm{CD}}$ is drawn to hypotenuse $\overline{\mathrm{AB}}$.
Find BC.
Leg or Alt?


## Regents Level Questions

4. The altitude to the hypotenuse of right triangle $A B C$ separates the hypotenuse into two segments. The length of one segment is 5 inches more than the measure of the other. If the length of the altitude is 6 inches, find the length of the hypotenuse.

## 5. You Try It!

In right triangle $A B C, \overline{C D}$ is the altitude to hypotenuse $\overline{A B}$. If $C D=3 \mathrm{~cm}$, and if $D B$ exceeds $A D$ by 8 cm , find $A D$ and $D B$.
6) In right triangle $A B C, C D$ is the altitude to the hypotenuse, $A B$. The segments of the hypotenuse, $A B$, are in the ratio of $2: 5$. The altitude is 8 . Find the two segments of the hypotenuse.
7) If the ratio of the lengths of the segments is $1: 9$ and the length of the altitude is 6 meters, find the lengths of the two segments.
8) In the diagram below, the length of the legs $\overline{A C}$ and $\overline{B C}$ of right triangle $A B C$ are 6 cm and 8 cm , respectively. Altitude $\overline{C D}$ is drawn to the hypotenuse of $\triangle A B C$.

What is the length of $\overline{A D}$ to the nearest tenth of $a$
 centimeter?

## SUMMARY: Solving a Quadratic Equation with Similar Right Triangles

In right triangle $A B C$, altitude $\overline{C D}$ is drawn to hypotenuse $\overline{A B}$. If $C D=12 \mathrm{in}$. and $A D$ exceeds $D B$ by 7 in ., find $D B$ and $A D$.

## Solution

(1) Let $x=$ the length of $\overline{D B}$.

Then, $x+7=$ the length of $\overline{A D}$.
(2) Since $\overline{C D} \perp$ hypotenuse $\overline{A B}$ in right $\triangle A B C$ : $\frac{\text { part of hypotenuse }}{\text { altitude }}=\frac{\text { altitude }}{\text { other part of hypotenuse }}$

$$
\frac{A D}{C D}=\frac{C D}{D B}
$$

(3) Substitute.

$$
\begin{array}{rl}
\frac{x+7}{12} & =\frac{12}{x} \\
x(x+7) & =12(12) \\
x^{2}+7 x & =144 \\
x^{2}+7 x-144 & =0 \\
(x-9)(x+16) & =0 \\
x-9=0 & x+16
\end{array}=0 \begin{aligned}
& x=-16 \\
& x=9 \\
& \text { Reject the negative value. }
\end{aligned}
$$

(4) Solve for $x$.
(5) Then, $x+7=9+7=16$.

Answer: $D B=9$ in., and $A D=16 \mathrm{in}$.

## Section 1: Similar Polygons

Determine whether each pair of figures is similar. If so, write a similarity statement and scale factor. If not explain your reasoning.

3.


4. Delroy's sailboat has two sails that are similar triangles. The larger sail has sides of 10 feet, 24 feet, and 26 feet. If the shortest side of the smaller sail measures 6 feet, what is the perimeter of the smaller sail?
5. On a scale drawing of a new school playground, a triangular area has sides with lengths of 8 centimeters, 15 centimeters, and 17 centimeters. If the triangular area located on the playground has a perimeter of 120 meters, what is the length of its longest side?

Given: $\triangle A B C \sim \triangle D E F$
6. If $\mathrm{BC}=24, \mathrm{EF}=9, \mathrm{AC}=\mathrm{y}+10$, and $\mathrm{DF}=\mathrm{y}$, find AC .
7. If $\mathrm{AB}=\mathrm{x}+1, \mathrm{BC}=5 \mathrm{x}+3, \mathrm{DE}=\mathrm{x}$, and $\mathrm{EF}=4 \mathrm{x}$, find DE .

Section 2: Proportional Parts in Similar Triangles
8. Find the length of each segment. $\overline{S T}$


9. Find $x$
$\overline{E B} \| \overline{D C}$.

10. Solve for y .

11. In the figure below, $\stackrel{\mathrm{YA}}{\|}\|\overrightarrow{\mathrm{OE}}\| \overrightarrow{\mathrm{BR}}$.

Find the values of $x$ and $y$

$$
\text { if } Y O=4, E R=16, \text { and } A R=24
$$



$$
\begin{aligned}
& x= \\
& y=
\end{aligned}
$$


12. Solve for $x$.


Section 3: Geometric Mean and Similarities in Right Triangles
13. Find the geometric mean between each pair of numbers.

## 8 and 12


14. Find $x$.


15. Find $x$

16. Find $x$

17. Find x


## 18. Extended Response Regents Level Question.

The drawing for a right triangular roof truss, represented by $\triangle A B C$, is shown in the accompanying diagram. If $\angle A B C$ is a right angle, altitude $B D=4$ meters, and $\overline{D C}$ is 6 meters longer than $\overline{A D}$, find the length of base $\overline{A C}$ in meters.


