

CHAPTER 7: SIMILAR TRIANGLES AND TRIGONOMETRY

Specific Expectations Addressed in the Chapter

- Verify, through investigation (e.g., using dynamic geometry software, concrete materials), the properties of similar triangles (e.g., given similar triangles, verify the equality of corresponding angles and the proportionality of corresponding sides). **[7.1]**
- Describe and compare the concepts of similarity and congruence. **[7.1]**
- Solve problems involving similar triangles in realistic situations (e.g., shadows, reflections, scale models, surveying). **[7.2]**
- Determine, through investigation (e.g., using dynamic geometry software, concrete materials), the relationship between the ratio of two sides in a right triangle and the ratio of the two corresponding sides in a similar right triangle, and define the sine, cosine, and tangent ratios (e.g., $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$). **[7.3, 7.4]**
- Determine the measures of the sides and angles in right triangles, using the primary trigonometric ratios and the Pythagorean theorem. **[7.4, 7.5, 7.6, Chapter Task]**
- Solve problems involving the measures of sides and angles in right triangles in real life applications (e.g., in surveying, in navigating, in determining the height of an inaccessible object around the school), using the primary trigonometric ratios and the Pythagorean theorem. **[7.5, 7.6, Chapter Task]**

Prerequisite Skills Needed for the Chapter

- Determine the measures of angles formed by parallel lines.
- Determine whether two triangles or two quadrilaterals are congruent.
- Solve problems involving applications of ratio, rate, and proportion.
- Calculate the perimeter and area of a two-dimensional figure.
- Apply the Pythagorean theorem to calculate the length of a side in a right triangle.

What “big ideas” should students develop in this chapter?

Students who have successfully completed the work of this chapter and who understand the essential concepts and procedures will know the following:

- If two triangles are congruent, then they are similar. If two triangles are similar, however, they may or may not be congruent.
- If two pairs of corresponding angles in two triangles are equal, then the triangles are similar. If, in addition, two corresponding sides are equal, then the triangles are congruent.
- Similar triangles can be used to determine lengths that cannot be measured directly.
- The primary trigonometric ratios for $\angle A$ are $\sin A$, $\cos A$, and $\tan A$.
- If $\angle A$ is an acute angle in a right triangle, the primary trigonometric ratios can be determined using the ratios of the sides.
- Trigonometric ratios can be used to calculate unknown side lengths and unknown angle measures in a right triangle. The ratio that is used depends on the information given and the measurement to be calculated.
- A problem that can be represented with a right triangle and involves an unknown side length or angle measure can be solved using trigonometric ratios.

Chapter 7: Planning Chart

Lesson Title	Lesson Goal	Pacing 11 days	Materials/Masters Needed
Getting Started , pp. 370–373	Use concepts and skills developed prior to this chapter.	2 days	grid paper; protractor; ruler; coloured pencils; Diagnostic Test
Lesson 7.1: Congruence and Similarity in Triangles , pp. 374–381	Investigate the relationships between corresponding sides and angles in pairs of congruent and similar triangles.	1 day	dynamic geometry software, or ruler and protractor; Lesson 7.1 Extra Practice
Lesson 7.2: Solving Similar Triangle Problems , pp. 382–388	Solve problems using similar triangle models.	1 day	ruler; Lesson 7.2 Extra Practice
Lesson 7.3: Exploring Similar Right Triangles , pp. 391–393	Explore the connection between the ratios of the sides in the same triangle for similar triangles.	1 day	dynamic geometry software, or ruler and protractor
Lesson 7.4: The Primary Trigonometric Ratios , pp. 394–399	Determine the values of the sine, cosine, and tangent ratios for a specific acute angle in a right triangle.	1 day	Lesson 7.4 Extra Practice
Lesson 7.5: Solving Right Triangles , pp. 400–406	Use primary trigonometric ratios to calculate side lengths and angle measures in right triangles.	1 day	Lesson 7.5 Extra Practice
Lesson 7.6: Solving Right Triangle Problems , pp. 408–414	Use the primary trigonometric ratios to solve problems that involve right triangle models.	1 day	Lesson 7.6 Extra Practice
Mid-Chapter Review , pp. 389–390 Chapter Review , pp. 415–417 Chapter Self-Test , p. 418 Curious Math , p. 407 Chapter Task , p. 419		3 days	Mid-Chapter Review Extra Practice; Chapter Review Extra Practice; Chapter Test

CHAPTER OPENER

Using the Chapter Opener

Introduce the chapter by discussing the photograph on pages 368 and 369 of the Student Book. The photograph shows a view of Earth from space. Ask students to identify familiar places in the photograph. If a map is available, have students point out the same places on the map. Use this context to introduce the idea of direct and indirect measurement. Ask students for examples of distances that can be measured directly and distances that cannot be measured directly. Then ask students if the distance from Earth to the Sun can be measured directly. Encourage them to provide reasons why this distance can only be measured indirectly.

Discuss different strategies for measuring indirectly. Include the Pythagorean theorem in the discussion. Tell students that, in this chapter, they will focus on strategies for measuring distances indirectly, based on the properties of a triangle.

GETTING STARTED

Using the Words You Need to Know

Remind students that they can use the Glossary as a reference, but encourage them to match each term with the example or diagram based on their knowledge. Invite students to describe other examples for the terms or to sketch other diagrams on the board. Ask questions such as these:

- How would you explain the meaning of “proportion” in your own words?
- How do the different examples of congruent triangles vary? How do you know whether triangles are congruent?
- What kind of triangle do you need to have if you want to use the Pythagorean theorem? What can you calculate using the Pythagorean theorem?
- Does every triangle have a hypotenuse? How do you know which side is the hypotenuse?
- What are some measures for an acute angle? How do you know whether an angle is acute?

Using the Skills and Concepts You Need

Work through each of the examples in the Student Book (or similar examples, if you would like students to see more examples), and answer any questions that students have. Discuss the Communication Tip on page 371 of the Student Book. Ask students to explain how rounding is applied in the example on page 371. Encourage discussion about whether students think this strategy for determining the number of decimal places in an answer makes sense. Tell students that they can use this strategy to round their answers throughout the chapter.

Ask students to look over the Practice questions to see if there are any questions they do not know how to solve. Direct their attention to the Study Aid chart. Have students work on the Practice questions in class, and assign any unfinished questions for homework.

Using the Applying What You Know

Have students work on their own or in pairs. Ask them to read the whole activity before beginning their work. Encourage them to make predictions for part C before completing parts A and B. Ask some students to justify their predictions. If they need help making or justifying their predictions, suggest that they refer to the diagrams in the Practice questions.

After students have finished the activity, discuss the answers as a class. Ask students to explain how they knew that the triangles are congruent.

Student Book Pages 370–373

Preparation and Planning

Pacing

5–10 min	Words You Need to Know
40–45 min	Skills and Concepts You Need
45–55 min	Applying What You Know

Materials

- grid paper
- protractor
- ruler
- coloured pencils

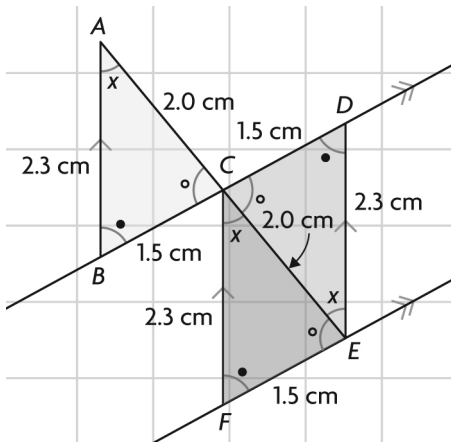
Nelson Website

<http://www.nelson.com/math>

Answers to Applying What You Know

Answers may vary, e.g.,

A.–D.



C. $AB \parallel CF \parallel DE$ because they are vertical line segments. $BD \parallel FE$ because CF and DE represent the distance between BD and FE and are the same length.

D. i) $\angle BAC = \angle DEC$; $\angle ACB = \angle ECD$; $\angle CBA = \angle CDE$

ii) $\angle BAC = \angle FCE$; $\angle ACB = \angle CEF$; $\angle CBA = \angle EFC$

iii) $\angle DEC = \angle FCE$; $\angle ECD = \angle CEF$; $\angle CDE = \angle EFC$

E. $\triangle ACB$, $\triangle CEF$, and $\triangle ECD$ are congruent because corresponding sides are equal and corresponding angles are equal. AB , CF , and ED are equal corresponding sides. So are AC , CE , and EC , as well as BC , FE , and DC . Corresponding angles in $\triangle ACB$, $\triangle CEF$, and $\triangle ECD$ are $\angle BAC = \angle FCE = \angle DEC$, $\angle ACB = \angle CEF = \angle ECD$, and $\angle CBA = \angle EFC = \angle CDE$.

Initial Assessment	What You Will See Students Doing...
<p>When students understand...</p> <p>Students correctly measure the lengths of the sides in the three triangles.</p> <p>Students correctly identify and mark pairs of parallel line segments, and they explain why the line segments are parallel.</p> <p>Students identify and mark the pairs of equal angles.</p> <p>Students identify the congruent triangles and justify their decision using their information about corresponding sides and corresponding angles for parts B and D.</p>	<p>If students misunderstand...</p> <p>Students measure inaccurately or make conclusions based on observations only.</p> <p>Students cannot identify the parallel line segments, or they do not mark the parallel line segments correctly on their diagram. Their explanation is inadequate, possibly based on observations only.</p> <p>Students cannot determine pairs of equal angles because they do not measure the angles or they measure incorrectly, or they do not mark the angles correctly on their diagram.</p> <p>Students do not identify the congruent triangles correctly, or their explanation is inadequate (for example, using corresponding angles but not corresponding sides).</p>

7.1 CONGRUENCE AND SIMILARITY IN TRIANGLES

Lesson at a Glance

GOAL

Investigate the relationships between corresponding sides and angles in pairs of congruent and similar triangles.

Prerequisite Skills/Concepts

- Determine the measures of angles formed by parallel lines.
- Determine whether two triangles or two quadrilaterals are congruent.
- Solve problems involving the application of proportion.

Specific Expectations

- Verify, through investigation (e.g., using dynamic geometry software, concrete materials), the properties of similar triangles (e.g., given similar triangles, verify the equality of corresponding angles and the proportionality of corresponding sides).
- Describe and compare the concepts of similarity and congruence.

Mathematical Process Focus

- Reasoning and Proving
- Selecting Tools and Computational Strategies

Student Book Pages 374–381

Preparation and Planning

Pacing

10 min	Introduction
30–40 min	Teaching and Learning
15–20 min	Consolidation

Materials

- dynamic geometry software, or ruler and protractor

Recommended Practice

Questions 4, 6, 8, 12, 13, 14, 15

Key Assessment Question

Question 6

Extra Practice

Lesson 7.1 Extra Practice

New Vocabulary/Symbols

similar triangles
scale factor

Nelson Website

<http://www.nelson.com/math>

MATH BACKGROUND | LESSON OVERVIEW

- Students should be able to construct triangles using dynamic geometry software or concrete materials.
- Students should be able to measure the angles and sides in two-dimensional figures.
- In this lesson, students investigate the relationships between corresponding sides and corresponding angles in congruent and similar triangles.
- The concept that congruent triangles are similar but similar triangles are not necessarily congruent is developed.
- Students learn that triangles are similar if corresponding angles are equal, and vice versa. They learn that triangles are congruent if corresponding angles are equal and corresponding sides are equal, and vice versa.

1

Introducing the Lesson

(10 min)

Introduce the lesson with a brief discussion about the difference between congruent figures and similar figures.

Have a few students sketch similar triangles for the class. Ask the class to explain why the triangles are similar but not congruent. (They are the same shape. Congruent triangles are the same shape and the same size.)

Use a similar process to discuss congruent triangles. Guide students to realize that congruent triangles are the same shape and the same size.

2

Teaching and Learning

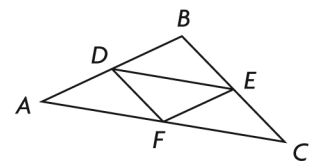
(30 to 40 min)

Investigate the Math

As students work on the investigation, circulate throughout the class and provide assistance and support where needed. Ensure that students are able to match corresponding sides and angles.

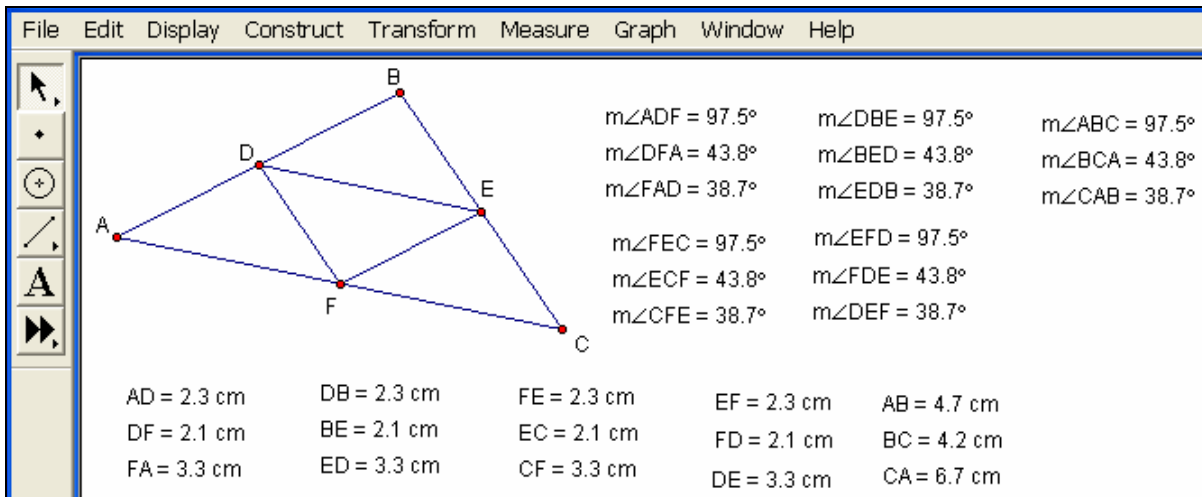
- If students are using dynamic geometry software, try to spread more experienced users throughout the class. Encourage students to ask for help if they are having difficulty. Remind them to review Appendix B. Alternatively, the investigation could be completed as a class demonstration.
- Make sure that students know how to choose units of measurement and decimal precision for units for their constructions and measurements.
- Emphasize that for part C, students use Appendix B-29 for reference, but they select two endpoints of a segment and then choose Distance from the Measure menu.
- Students can use a scientific or graphing calculator to determine the ratios. Alternatively, they can use the calculator built into the dynamic geometry software to create screens that are similar to the screens on the next page. Appendix B-28 presents a sample calculation.

If students are using rulers and protractors for the investigation, have them work in pairs. Emphasize that accurate measurements are necessary. Students should construct their own triangle but discuss their conclusions with their partner. For parts H and I, students could compare their triangles with their partner's triangles to see variations.

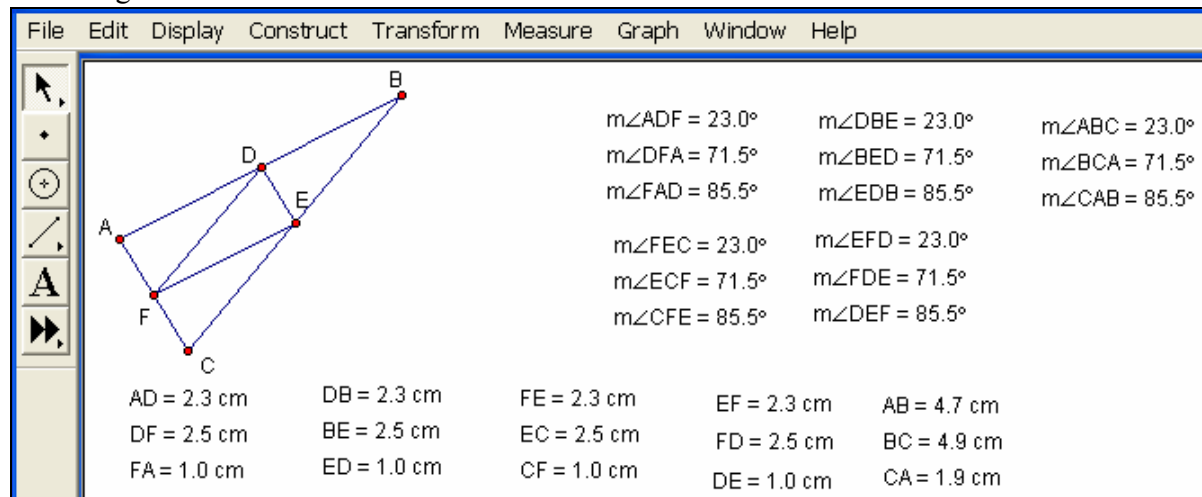


Answers to Investigate the Math

A.–E.



- C. The corresponding angles in all four small triangles are equal. The corresponding sides in all four small triangles are equal.
- D. Yes. The corresponding angles are equal and the corresponding sides are also equal, so the triangles are congruent.
- E. The angles in the large triangle are equal to the corresponding angles in the four small triangles. The sides of the large triangle are twice the length of the corresponding sides in the small triangles.
- F. Yes, there are similar triangles in Colin's design. The large triangle is similar to each of the four small triangles. The corresponding angles are equal, and the sides in the large triangle are exactly two times longer than the corresponding sides in the small triangles. $\triangle ABC$ is similar to $\triangle ADF$, $\triangle DBE$, $\triangle FEC$, and $\triangle EFD$. $\triangle ADF$, $\triangle DBE$, $\triangle FEC$, and $\triangle EFD$ are similar to each other because the corresponding angles are equal.
- G. The scale factor $\frac{1}{2}$ makes sense because each side in the small triangles is half the length of the corresponding side in the large triangle.
- H. i) The four small triangles are still congruent.
 ii) The small and large triangles are still similar. The scale factor does not change.



I. It does not matter which vertex is dragged.

Answers to Reflecting

J. All congruent triangles are similar. The corresponding angles are equal, and the scale factor is 1. All similar triangles may not be congruent because the scale factor is not always 1.

K. The scale factor would be $\frac{1}{2}$. A scale factor of 2 doubles each side length. To get back to the original size, the side lengths would need to be halved.

L. i) I could make the conclusion that all the corresponding angles are equal because the sum of the angles in a triangle is 180° . The third pair of corresponding angles would also be equal.

ii) I could make the conclusion that the triangles are similar because triangles with equal corresponding angles are similar.

iii) I could not make the conclusion that the triangles are congruent. Congruent triangles have sides that are the same length. I would not know this by discovering that two pairs of corresponding angles are equal.

3 Consolidation

(15 to 20 min)

Apply the Math

Using the Solved Examples

Students should understand the difference between congruent triangles and similar triangles. In part a) of *Example 1*, a triangle that is similar to a given triangle is created. Review the method that is used to label the vertices. In part b) of *Example 1*, two given triangles are compared to discover whether they are similar.

In *Example 2*, the scale factor for similar logos is determined. Emphasize the Communication Tip about naming sides and angles in the same order. Ask students to explain how this is done in the examples.

In *Example 3*, two triangles are shown to be similar by using corresponding angles of parallel lines. Proportions are used to determine unknown values.

Answer to the Key Assessment Question

For question 6, students may find a diagram helpful for visualizing the triangles.

6. a) $\angle L$ is 90° because the corresponding vertices are listed in the same order.

b) The length of PR is 36 cm, and the length of QR is 39 cm.

Closing

Have students read question 15. Ask them to work in pairs, and post their flow charts when completed. Ask each pair of students to describe their flow chart to the class. Alternatively, arrange students in small groups of pairs so that each pair can describe their flow chart to the other pairs in the group.

Assessment and Differentiating Instruction	
What You Will See Students Doing...	
<p>When students understand...</p> <p>Students correctly create a triangle similar to a given triangle by multiplying each of the given sides by the same number and using congruent angles.</p> <p>Students determine the scale factor for similar triangles by determining the corresponding sides and calculating the ratio of these sides. Students recognize that the triangles are congruent if the scale factor is 1.</p> <p>Students correctly identify equal angles to determine that two triangles are similar.</p> <p>Students accurately use proportions to calculate the value of unknown sides.</p>	<p>If students misunderstand...</p> <p>Students may forget to consider the angles and focus only on the sides, or vice versa.</p> <p>Students may label a new triangle incorrectly and then make incorrect conclusions about whether two triangles are similar, even though they correctly calculate the ratios of the sides they use.</p> <p>Students may write the vertices of similar triangles in the wrong order and thus conclude that the triangles are not similar.</p> <p>Students may not solve proportions correctly. If one of the unknowns is part of the side of a triangle, they may use the unknown part only and not add the known part.</p>
<p>Key Assessment Question 6</p> <p>Students correctly identify corresponding angles to determine the angle that corresponds to $\angle L$.</p> <p>Students correctly set up and accurately solve the proportion.</p>	<p>Students do not recall that the vertices are listed in the same order, or they do not reach the correct conclusion.</p> <p>Students may set up an incorrect proportion by not identifying the correct corresponding sides, or they may be unable to solve the proportion accurately.</p>
Differentiating Instruction How You Can Respond	
<p>EXTRA SUPPORT</p> <ol style="list-style-type: none"> Some students may have difficulty setting up a problem. To identify the correct pairs of corresponding sides, have them begin by listing the vertices of one triangle. Then have them match each vertex with the vertex that is marked as being equal in the other triangle. This will help them write the vertices in the correct order and obtain the correct proportion. A diagram is necessary for most students. If students have difficulty when two triangles are nested, have them trace the smaller triangle on a separate piece of paper and then move the tracing. 	
<p>EXTRA CHALLENGE</p> <ol style="list-style-type: none"> Have students use dynamic geometry software to experiment with different combinations of congruent angles and corresponding sides to determine what will guarantee similarity in triangles. Ask students to make and then check their own conjectures about similar triangles. Invite them to share their conjectures with classmates. 	

7.2 SOLVING SIMILAR TRIANGLE PROBLEMS

Lesson at a Glance

GOAL

Solve problems using similar triangle models.

Prerequisite Skills/Concepts

- Solve problems involving the application of proportion.
- Calculate the perimeter and area of a two-dimensional figure.

Specific Expectation

- Solve problems involving similar triangles in realistic situations (e.g., shadows, reflections, scale models, surveying).

Mathematical Process Focus

- Problem Solving
- Reasoning and Proving
- Selecting Tools and Computational Strategies

Student Book Pages 382–388

Preparation and Planning

Pacing

5–10 min	Introduction
30–40 min	Teaching and Learning
15–20 min	Consolidation

Materials

- ruler

Recommended Practice

Questions 4, 5, 7, 9, 10, 11, 12, 13

Key Assessment Question

Question 12

Extra Practice

Lesson 7.2 Extra Practice

New Vocabulary/Symbols

angle of elevation (angle of inclination)

Nelson Website

<http://www.nelson.com/math>

MATH BACKGROUND | LESSON OVERVIEW

- In the previous lesson, students investigated the conditions under which two triangles are congruent or similar:
 - If the measures of two pairs of corresponding angles are equal, then the triangles are similar.
 - If the measures of two pairs of corresponding angles are equal and the lengths of two pairs of corresponding sides are equal, then the triangles are congruent.
- In this lesson, students solve problems using similar triangles in realistic situations.
- Students should understand how similar triangles and indirect measurement can be used to determine lengths that cannot be measured directly.

1 | Introducing the Lesson

(5 to 10 min)

Have students summarize, in their own words, what they learned about similar triangles from the investigation in Lesson 7.1. Students can work in pairs and then share their ideas with another pair. The two pairs can combine and summarize their ideas before sharing with the entire class. The summaries could be either posted or read to the class.

Emphasize the importance of the order in which the vertices are listed when naming triangles, sides, and angles. The corresponding vertices, where the equal angles occur, are what determine how similar triangles are named.

2 | Teaching and Learning

(30 to 40 min)

Learn About the Math

In *Example 1*, indirect measurement is used to determine the width of a river. Remind students about the general conventions for naming triangles. The vertices are labelled with upper-case letters. The sides opposite the angles are labelled with the corresponding lower-case letters.

Have students work in pairs. Ask them to study the diagram of the river and the two triangles and consider how they know that the two triangles are similar. Have students also consider why Marnie chose the proportion $\frac{AB}{DE} = \frac{AC}{DC}$. Then discuss the example as a class, inviting pairs to take lead roles.

Students can continue to work in pairs to discuss the Reflecting questions, before the class discussion.

Answers to Reflecting

- A. The surveyors set up the posts so they could use similar triangles to determine the width of the river.
- B. The surveyors had to set up a proportion with the ratios of two pairs of corresponding sides. One of the corresponding sides was the unknown width of the river, AB . The surveyors wanted to determine the width of the river using indirect measurement, instead of measuring across the actual river. They measured the other three sides involved: two in $\triangle DEC$ and one in $\triangle ABC$.
- C. Using similar triangles, the surveyors were able to determine a length that could not be measured directly. They were able to use measurable distances to determine the unknown distance.

3

Consolidation

(15 to 20 min)

Apply the Math

Using the Solved Examples

Example 2 uses similar triangles and a scale factor to determine the dimensions of a yard from a scale diagram. Encourage students to think of the scale as a ratio. Direct their attention to the fact that units need to be converted. After the units are the same, the scale factor can be used to multiply the lengths of the sides in the model triangle and thus determine the corresponding dimensions of the yard.

Example 3 introduces the concept of the angle of elevation, which can also be called the angle of inclination. Ask students to explain the angle of elevation in relation to objects in the classroom. Students should understand why the angles of elevation in the diagram are equal. Some students may need to be reminded about the properties of angles formed by parallel lines. The unknown height could not be measured directly. Students need to understand how the ratios of the corresponding sides are used to determine this height.

Answer to the Key Assessment Question

Students may need to draw their own diagram for question 12 so they can label the dimensions. They may need help setting up the proportion.

12. The window is 16.0 m high.

Closing

Have students work in pairs on question 13. Remind students to begin by writing the proportion statement with the ratios of the corresponding sides. Ask some students to present their solutions to the class.

Assessment and Differentiating Instruction

What You Will See Students Doing...

When students understand...

Students establish, with justification, that triangles are similar and then record the vertices in the correct order.

Students set up a statement of proportionality with the corresponding sides correctly matched.

Students correctly determine unknown side lengths by solving the proportion.

Students determine and correctly use a scale factor to calculate the lengths of unknown sides in similar triangles.

If students misunderstand...

Students may identify similar triangles but not provide justification, or they may incorrectly name the triangles.

Students do not correctly identify corresponding sides, or they do not understand how the corresponding sides fit in the statement about proportion.

Students may omit steps in the solution, causing errors, or they may calculate incorrectly.

Students may identify the scale factor but use it incorrectly to determine unknown side lengths.

Key Assessment Question 12

Students identify the similar triangles and justify why the triangles are similar.

Students set up a statement of proportionality with corresponding sides correctly matched.

Students correctly solve the proportion to determine the height of the window.

Students may identify corresponding sides but set up an incorrect statement of proportionality.

Students may reverse the order in one of the ratios.

Students may make errors when calculating the height of the window, or they may not interpret the results of their calculations correctly.

Differentiating Instruction | How You Can Respond

EXTRA SUPPORT

1. If students have difficulty setting up a proportion, suggest that they mark corresponding vertices on their diagram, using a different colour for each pair. Then they can list the vertices of one of the triangles and match each vertex with a corresponding vertex.
2. Encourage students to redraw triangles, with labels on the vertices, before setting up a proportion. They can use the names of the triangles in the statement before substituting the known lengths and the variable for the unknown.

EXTRA CHALLENGE

1. Have students create their own problem involving similar triangles in the school environment. They can obtain actual measurements of corresponding sides and angles in the similar triangles and identify an unknown length that cannot be measured directly. Have students solve each other's problems.

MID-CHAPTER REVIEW

Big Ideas Covered So Far

- If two triangles are congruent, then they are similar. If two triangles are similar, however, they may or may not be congruent.
- If two pairs of corresponding angles in two triangles are equal, then the triangles are similar. If, in addition, two corresponding sides are equal, then the triangles are congruent.
- Similar triangles can be used to determine lengths that cannot be measured directly.

Using the Frequently Asked Questions

Have students keep their Student Books closed. Display the Frequently Asked Questions on a board. Have students discuss the questions and use the discussion to draw out what the class thinks are good answers. Then have students compare the class answers with the answers on Student Book page 389. Students can refer to the answers to the Frequently Asked Questions as they work through the Practice Questions.

Using the Mid-Chapter Review

Ask students if they have any questions about any of the topics covered so far in the chapter. Review any topics that students would benefit from considering again. Assign Practice Questions for class work and for homework.

To gain greater insight into students' understanding of the material covered so far in the chapter, you may want to ask questions such as the following:

- What does it mean when two figures are congruent? What does it mean when two figures are similar?
- Are all similar triangles congruent? Are all congruent triangles similar? Explain how you know.
- If two triangles are similar, what is the relationship between the sides of the triangles?
- Explain why saying that one triangle is a scale model of another triangle means that the two triangles are similar.
- How can you make sure that you have set up a proportion for similar triangles correctly? What other strategy could you use?

7.3 EXPLORING SIMILAR RIGHT TRIANGLES

Lesson at a Glance

GOAL

Explore the connection between the ratios of the sides in the same triangle for similar triangles.

Prerequisite Skills/Concepts

- Solve problems involving the application of proportion.
- Solve problems involving properties of similar triangles.

Specific Expectation

- Determine, through investigation (e.g., using dynamic geometry software, concrete materials), the relationship between the ratio of two sides in a right triangle and the ratio of the two corresponding sides in a similar right triangle, and define the sine, cosine, and tangent ratios (e.g.,

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}).$$

Mathematical Process Focus

- Reasoning and Proving
- Connecting

Student Book Pages 391–393

Preparation and Planning

Pacing

5–10 min	Introduction
35–45 min	Teaching and Learning
10–15 min	Consolidation

Materials

- dynamic geometry software, or ruler and protractor

Recommended Practice

Questions 1, 2, 3, 4

New Vocabulary/Symbols

opposite side
adjacent side

Nelson Website

<http://www.nelson.com/math>

MATH BACKGROUND | LESSON OVERVIEW

- In Lessons 7.1 and 7.2, students verified properties of similar triangles and solved problems involving similar triangles.
- In this lesson, students explore the relationship between side lengths in similar right triangles.
- Students refer to the sides in a right triangle as follows: the side adjacent to a specific angle, the side opposite this angle, and the hypotenuse.
- Students discover that the ratios $\frac{\text{opposite}}{\text{adjacent}}$, $\frac{\text{opposite}}{\text{hypotenuse}}$, and $\frac{\text{adjacent}}{\text{hypotenuse}}$ are the same for the corresponding acute angles in similar right triangles.

1

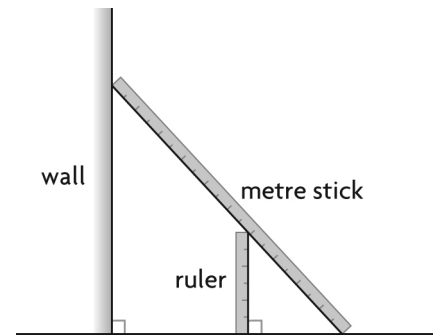
Introducing the Lesson

(5 to 10 min)

Lean a metre stick against a wall to form a right triangle, as shown in the diagram to the right. Place a ruler vertically, so it is touching the metre stick, to form a smaller right triangle. Ask students what they notice about the two triangles. Encourage a variety of answers. Guide students to include

- which triangles are similar, and why
- what is shown by measurements on the metre stick and ruler
- which other lengths they can measure
- how students know that the triangles are right triangles

Tell students that, in this lesson, they will learn about three important ratios used to relate the sides of similar right triangles.



2

Teaching and Learning

(35 to 45 min)

Explore the Math

Ask students to explain what they notice about the triangles formed by the ground, the ramp, and the vertical supports in the pictures. Elicit the following information from students:

- Each triangle has a right angle, so it is a right triangle.
- Each picture has a set of similar triangles.

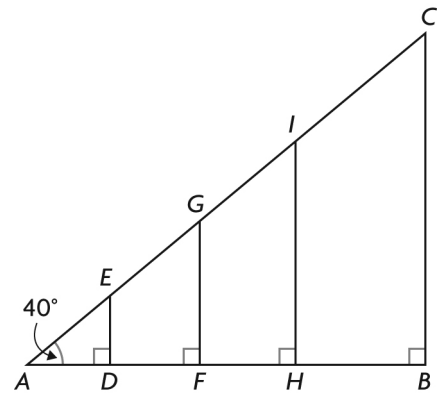
Have students work through the exploration in pairs. Emphasize that the definitions of “opposite” and “adjacent” in the margin of the Student Book require reference to a specified acute angle.

If students are using dynamic geometry software, you may want to pair more experienced users with students who are less experienced.

- Remind students to use Appendix B for reference.
- Make sure that students know how to choose units of measurement and decimal precision for units for their constructions and measurements.
- To construct an angle of 40° and to construct right triangles, students can construct and measure as described in Appendix B-25 and B-26.
- An alternative strategy for constructing an angle of 40° is to construct a segment and rotate it by 40° .
- An alternative strategy for constructing a right angle is to hold down the Shift key while drawing a line segment. This will snap the segment to different angles (for example, 0° , 90° , 180° , 270°) and make it easier to construct a right angle.
- Students can measure lengths of line segments using Appendix B-29 as a reference. However, they measure the distance between two points by selecting both points and then choosing Distance from the Measure menu. Measuring the distance between two points would result in displaying lengths as in the screens on the next page.

- Students can use a scientific or graphing calculator to determine the ratios. Alternatively, they can use the calculator built into the dynamic geometry software, which will create screens that are similar to the following screens. Appendix B-28 presents a sample calculation.
- Results will vary depending on the precision in units.

If students are using a ruler and protractor, have them construct angles to the nearest degree and measure length to the nearest tenth of a centimetre. Make sure that they realize part C says to calculate each ratio to two decimal places. Discuss any differences due to rounding.



Answers to Explore the Math

A.–C.

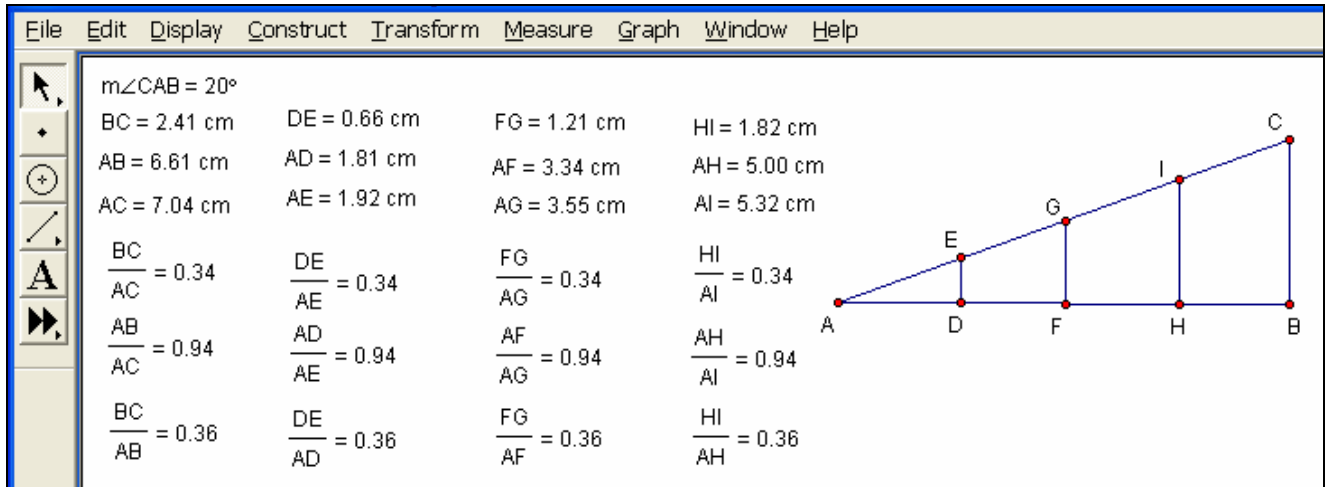
$m\angle CAB = 40^\circ$	$BC = 7.16 \text{ cm}$	$DE = 1.78 \text{ cm}$	$FG = 3.60 \text{ cm}$	$HI = 5.43 \text{ cm}$
$AB = 8.53 \text{ cm}$	$AD = 2.12 \text{ cm}$	$AF = 4.28 \text{ cm}$	$AH = 6.47 \text{ cm}$	
$AC = 11.14 \text{ cm}$	$AE = 2.77 \text{ cm}$	$AG = 5.59 \text{ cm}$	$AI = 8.45 \text{ cm}$	
$\frac{BC}{AC} = 0.64$	$\frac{DE}{AE} = 0.64$	$\frac{FG}{AG} = 0.64$	$\frac{HI}{AI} = 0.64$	
$\frac{AB}{AC} = 0.77$	$\frac{AD}{AE} = 0.77$	$\frac{AF}{AG} = 0.77$	$\frac{AH}{AI} = 0.77$	
$\frac{BC}{AB} = 0.84$	$\frac{DE}{AD} = 0.84$	$\frac{FG}{AF} = 0.84$	$\frac{HI}{AH} = 0.84$	

B. The triangles are similar because each triangle contains two common angles: one 40° angle and one right angle.

C. Answers may vary, e.g.,

Triangle	Side Opposite to $\angle A$	Side Adjacent to $\angle A$	Hypotenuse	Trigonometric Ratios		
				$\frac{\text{opposite}}{\text{hypotenuse}}$	$\frac{\text{adjacent}}{\text{hypotenuse}}$	$\frac{\text{opposite}}{\text{adjacent}}$
$\triangle ABC$	$BC = 7.16 \text{ cm}$	$AB = 8.53 \text{ cm}$	$AC = 11.14 \text{ cm}$	$\frac{BC}{AC} = 0.64$	$\frac{AB}{AC} = 0.77$	$\frac{BC}{AB} = 0.84$
$\triangle ADE$	$DE = 1.78 \text{ cm}$	$AD = 2.12 \text{ cm}$	$AE = 2.77 \text{ cm}$	$\frac{DE}{AE} = 0.64$	$\frac{AD}{AE} = 0.77$	$\frac{DE}{AD} = 0.84$
$\triangle AFG$	$FG = 3.60 \text{ cm}$	$AF = 4.28 \text{ cm}$	$AG = 5.59 \text{ cm}$	$\frac{FG}{AG} = 0.64$	$\frac{AF}{AG} = 0.77$	$\frac{FG}{AF} = 0.84$
$\triangle AHI$	$HI = 5.43 \text{ cm}$	$AH = 6.47 \text{ cm}$	$AI = 8.45 \text{ cm}$	$\frac{HI}{AI} = 0.64$	$\frac{AH}{AI} = 0.77$	$\frac{HI}{AH} = 0.84$

- D. The lengths of the sides in the triangles vary, depending on the size of the triangle, but the ratios $\frac{\text{opposite}}{\text{hypotenuse}}$, $\frac{\text{adjacent}}{\text{hypotenuse}}$, and $\frac{\text{opposite}}{\text{adjacent}}$ are equal for the four triangles.
- E. I think that the relationship I described for part D would not change if $\angle A$ were changed to a different measure. My conjecture is that the relationship would be the same.



The measurements in the table show that my conjecture is correct.

- F. In similar right triangles, the ratios $\frac{\text{opposite}}{\text{hypotenuse}}$, $\frac{\text{adjacent}}{\text{hypotenuse}}$, and $\frac{\text{opposite}}{\text{adjacent}}$ are always equal for corresponding acute angles.

Answers to Reflecting

- G.** The formula for the slope of a line segment is $\frac{\text{rise}}{\text{run}}$ or $\frac{\Delta y}{\Delta x}$. The opposite side is equivalent to the rise of AC , or Δy . The adjacent side is equivalent to the run of AC , or Δx . Therefore, the ratio $\frac{\text{opposite}}{\text{adjacent}}$ is equivalent to the slope of AC .
- H.** Yes, the value of each of the three ratios is constant for an angle of a given size.

3 Consolidation

(10 to 15 min)

Students should understand that the ratios $\frac{\text{opposite}}{\text{hypotenuse}}$, $\frac{\text{adjacent}}{\text{hypotenuse}}$, and $\frac{\text{opposite}}{\text{adjacent}}$ are equivalent for corresponding acute angles in similar right triangles.

Students should also understand that the slope of a line segment is related to the angle between the line segment and the x -axis.

Students should be able to answer the Further Your Understanding questions independently.

7.4 THE PRIMARY TRIGONOMETRIC RATIOS

Lesson at a Glance

GOAL

Determine the values of the sine, cosine, and tangent ratios for a specific acute angle in a right triangle.

Prerequisite Skills/Concepts

- Understand that the ratios $\frac{\text{opposite}}{\text{hypotenuse}}$, $\frac{\text{adjacent}}{\text{hypotenuse}}$, and $\frac{\text{opposite}}{\text{adjacent}}$ are equivalent for corresponding acute angles in similar right triangles.
- Apply the Pythagorean theorem to calculate the length of a side in a right triangle.

Specific Expectations

- Determine, through investigation (e.g., using dynamic geometry software, concrete materials), the relationship between the ratio of two sides in a right triangle and the ratio of the two corresponding sides in a similar right triangle, and define the sine, cosine, and tangent ratios (e.g., $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$).
- Determine the measures of the [sides and] angles in right triangles, using the primary trigonometric ratios [and the Pythagorean theorem].

Mathematical Process Focus

- Reasoning and Proving
- Connecting

Student Book Pages 394–399

Preparation and Planning

Pacing

5–10 min	Introduction
30–40 min	Teaching and Learning
15–20 min	Consolidation

Recommended Practice

Questions 4, 5, 7, 10, 11, 13, 14, 15

Key Assessment Question

Question 5

Extra Practice

Lesson 7.4 Extra Practice

New Vocabulary/Symbols

trigonometry
tangent
sine
cosine
primary trigonometric ratios
inverse

Nelson Website

<http://www.nelson.com/math>

MATH BACKGROUND | LESSON OVERVIEW

- In Lesson 7.3, students explored how the ratios of the sides in similar right triangles are related. They learned that the ratios $\frac{\text{opposite}}{\text{hypotenuse}}$, $\frac{\text{adjacent}}{\text{hypotenuse}}$, and $\frac{\text{opposite}}{\text{adjacent}}$ are equivalent for corresponding acute angles in similar right triangles.
- This lesson extends the exploration in Lesson 7.3 by introducing students to the primary trigonometric ratios: sine, cosine, and tangent.
- Students learn that the three primary trigonometric ratios for an acute angle, represented by $\angle A$, in a right triangle can be written as follows:

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}, \cos A = \frac{\text{adjacent}}{\text{hypotenuse}}, \tan A = \frac{\text{opposite}}{\text{adjacent}}$$
- Students learn how to use technology to calculate the primary trigonometric ratios and the inverses of the primary trigonometric ratios.

1 | Introducing the Lesson

(5 to 10 min)

Discuss the results from Lesson 7.3 with the class. Ask students what they think were the most important results. Display students' responses.

Tell students that the ratios they explored in Lesson 7.3 are called the primary trigonometric ratios. Explain that trigonometry is the branch of mathematics that deals with the properties of triangles and calculations based on these properties.

2 | Teaching and Learning

(30 to 40 min)

Learn About the Math

In *Example 1*, trigonometry is used to calculate the slope of a ski hill, given the angle that the hill makes with the horizontal. Remind students that the opposite and adjacent sides in a triangle are named relative to a specific acute angle.

Have students work in pairs to consider Nadia's solution. Ask them to explain the connection between the rise and the opposite side, and the run and the adjacent side. Discuss the need for the triangles to be right triangles.

When students are using a scientific calculator to calculate trigonometric ratios, remind them to check that the calculator is in degree mode. Ensure that they understand how to change the calculator to degree mode. Students should understand that trigonometric ratios are usually expressed with four digits of accuracy.

Technology-Based Alternative Lesson

If students are using TI-*n*spire calculators, they should be aware that the calculations will be in radians, unless the system settings have been changed. To change to degrees, the document settings must be changed as described in Appendix B-48.

Answers to Reflecting

- A. Nadia knew that the $\frac{\text{rise}}{\text{run}}$ ratio is equivalent to $\frac{\text{opposite}}{\text{adjacent}}$. She also knew that this ratio is the same in any right triangle with an 18° angle.
- B. Nadia used the tangent of the 18° angle instead of the 72° angle because the side opposite the angle corresponds to the rise of the hill and the side adjacent to the angle corresponds to the run of the hill. Therefore, the tangent of 18° gives the slope of the hill.

3

Consolidation

(15 to 20 min)

Apply the Math

Using the Solved Examples

In *Example 2*, the primary trigonometric ratios are determined for the two acute angles in a right triangle. This example provides an opportunity to emphasize the fact that naming the opposite and adjacent sides depends on the angle being used. Discuss with the class how to calculate the primary trigonometric ratios using a scientific calculator. The order of the keystrokes will depend on the calculator.

Example 3 introduces the inverse of the primary trigonometric ratios. Students determine the measure of θ using all three primary trigonometric ratios. Point out that the angle symbol is not used with θ .

Answer to the Key Assessment Question

Some students may need to redraw the diagram as they work on question 5. After students have completed the question, discuss why the ratio in fraction form is equal to the sine, cosine, or tangent, but the ratio in decimal form is approximately equal to the sine, cosine, or tangent. Ask students to explain why some ratios in the answers, such as $\sin C$ and $\cos B$, are equal.

5. a) $\sin C = \frac{8}{17} \doteq 0.4706$

b) $\cos C = \frac{15}{17} \doteq 0.8824$

c) $\tan B = \frac{15}{8} \doteq 1.8750$

d) $\tan C = \frac{8}{15} \doteq 0.5333$

e) $\cos B = \frac{8}{17} \doteq 0.4706$

f) $\sin B = \frac{15}{17} \doteq 0.8824$

Closing

Question 15 provides students with a chance to summarize, in their own words, the conventions for identifying the opposite side, the adjacent side, and the hypotenuse in a right-angled triangle. Have students work in pairs on the question but record their own answers. Explain that the wording of the answers can vary.

Assessment and Differentiating Instruction

What You Will See Students Doing...

When students understand...

Students correctly determine the three primary trigonometric ratios for any given acute angle.

Students determine a primary trigonometric ratio, given the length of at least two sides in a right triangle, and then use the inverse of the primary trigonometric ratio to determine the acute angle.

If students misunderstand...

Students may identify the opposite and adjacent sides incorrectly, or they may write the reciprocals of the ratios by mistake.

Students may use the incorrect inverse primary trigonometric ratio, or they may not know how to determine the angle measure. The exactness of their answer may not be appropriate.

Key Assessment Question 5

Students correctly determine the primary trigonometric ratios for $\angle B$ and $\angle C$.

Students write each ratio as a decimal to four decimal places.

Students may make errors when deciding whether to use the adjacent side, opposite side, or hypotenuse for each ratio, or when identifying the correct side of $\triangle ABC$.

Students may write fewer than four decimal places or make rounding errors.

Differentiating Instruction | How You Can Respond

EXTRA SUPPORT

1. If students are having difficulty remembering the three primary trigonometric ratios, have them create posters for these ratios. The posters can be displayed for reference.
2. Help students understand how to use a calculator for primary trigonometric ratios. Make sure that students understand that different kinds of calculators require different keystroke sequences.
3. Have students create posters or electronic displays of right triangles, with one acute angle identified and the opposite side, adjacent side, and hypotenuse clearly labelled. Make sure that the position of the identified acute angle is different in different posters. This will help students label triangles correctly.

EXTRA CHALLENGE

1. Have students do a research project about trigonometry. Possible topics include the historical origins of trigonometry and different careers that require the use of trigonometry. Students could create a poster, build a scale model, or make a presentation to share their results.

7.5 SOLVING RIGHT TRIANGLES

GOAL

Use primary trigonometric ratios to calculate side lengths and angle measures in right triangles.

Lesson at a Glance

Prerequisite Skills/Concepts

- Determine the values of the sine, cosine, and tangent ratios for a specific acute angle in a right triangle.
- Determine the measure of an acute angle using a primary trigonometric ratio.
- Apply the Pythagorean theorem to calculate the length of a side in a right triangle.

Specific Expectations

- Determine the measures of the sides and angles in right triangles, using the primary trigonometric ratios and the Pythagorean theorem.
- Solve problems involving the measures of sides and angles in right triangles in real life applications (e.g., in surveying, in navigating, in determining the height of an inaccessible object around the school), using the primary trigonometric ratios and the Pythagorean theorem.

Mathematical Process Focus

- Problem Solving
- Representing
- Selecting Tools and Computational Strategies
- Connecting

Student Book Pages 400–407

Preparation and Planning

Pacing

5–10 min	Introduction
30–40 min	Teaching and Learning
15–20 min	Consolidation

Recommended Practice

Questions 5, 7, 9, 10, 11, 12, 13, 15

Key Assessment Question

Question 13

Extra Practice

Lesson 7.5 Extra Practice

Nelson Website

<http://www.nelson.com/math>

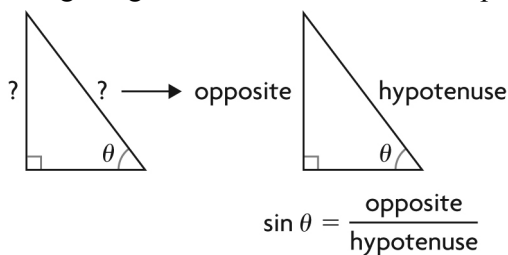
MATH BACKGROUND | LESSON OVERVIEW

- In Lesson 7.4, students applied the primary trigonometric ratios to determine side lengths and angle measures in right triangles.
- In this lesson, students apply the primary trigonometric ratios to solve right triangles (that is, to determine all the unknown angle measures and side lengths) and to solve problems that involve right triangles.
- Students learn that the trigonometric ratio they need to use depends on the information given and the quantity they need to calculate.

1 | Introducing the Lesson

(5 to 10 min)

Provide students with several right triangles that have one acute angle and two sides identified. Alternatively, sketch several triangles on the board for students to copy. Have students work in pairs to label the sides, relative to the identified angle, as opposite, adjacent, or hypotenuse. Ask students to indicate which of the three primary trigonometric ratios could be calculated using the given information. For example:



Have students share their results with the class. Ask students to explain how they decided which trigonometric ratio could be calculated.

2 | Teaching and Learning

(30 to 40 min)

Learn About the Math

Introduce the situation, relating it to students' experiences or knowledge. Have a student read the information about the grain auger, connecting the given measurements to the picture at the right of Student Book page 400.

Students could work in pairs to discuss each step in *Example 1* before the class discussion. When discussing the example with the class, ask these questions:

- Why can you not use the Pythagorean theorem to determine the maximum height that the auger can reach?
- If a silo is less than 10 m high, would the angle of elevation for the auger be 35° , less than 35° , or greater than 35° ? Why?

Have students work in pairs to answer the Reflecting questions before the class discussion.

Answers to Reflecting

- A. If the height of the grain auger is increased, the length of the side opposite the angle of elevation increases and the length of the adjacent side decreases. The hypotenuse remains the same because the length of the auger is 18 m.

The sine of the angle of elevation is $\frac{\text{opposite}}{\text{hypotenuse}}$. If the length of the opposite side increases and the length of the hypotenuse remains the same, the sine ratio increases.

The cosine of the angle of elevation is $\frac{\text{adjacent}}{\text{hypotenuse}}$. If the length of the adjacent side decreases and the length of the hypotenuse remains the same, the cosine ratio decreases.

The tangent of the angle of elevation is $\frac{\text{opposite}}{\text{adjacent}}$. If the length of the opposite side increases and the length of the adjacent side decreases, the tangent ratio increases.

- B.** For the 35° angle, the height of the silo is the opposite side and the length of the auger is the hypotenuse of the right triangle. Therefore, the sine ratio can be used to calculate the height of the silo, by using $\sin 35^\circ$. Since the sum of the angle measures in any triangle is 180° and the right angle is 90° , the angle formed by the auger and the silo is 180° less 90° and less 35° , which gives 55° . For this angle, the height of the silo is the adjacent side and the length of the auger is the hypotenuse, so the cosine ratio can be used to calculate the height of the silo, by using $\cos 55^\circ$.
- C.** The angle of elevation, 35° , is given as the greatest angle of elevation. The sine ratio can be used with this angle measure. If Hong used the cosine ratio, he would need to calculate the angle formed by the auger and the silo by subtracting 90° and 35° from 180° . This would require an extra step. Hong could not use the tangent ratio since he did not know the distance between the auger and the silo.

3 Consolidation

(15 to 20 min)

Apply the Math

Using the Solved Examples

In *Example 2*, the cosine ratio is used to solve for an unknown side length in a right triangle. Emphasize that students need to choose the trigonometric ratio that involves the known side length and the unknown side length. Ask students to explain each step for solving the equation in their own words.

In *Example 3*, the angle is the unknown measure. Have students draw a right triangle that corresponds to the situation and label the sides of the triangle with the given information. Identifying the opposite, adjacent, and hypotenuse will help students understand why the cosine ratio is selected as the most appropriate ratio to use. Review the keystrokes required to determine the measure of an unknown angle using the inverse.

Example 4 introduces students to solving a triangle. It is important for students to realize that this will involve more than one step.

Answer to the Key Assessment Question

Students need to use the primary trigonometric ratios, the Pythagorean theorem, and the angle sum of a triangle to solve the three triangles in question 13. Drawing diagrams with the given and calculated measurements will help. Invite students to share their strategies, and discuss appropriate variations in their strategies. Ensure that students notice the instructions about rounding.

13. a) To the nearest degree, $\angle A = 32^\circ$ and $\angle C = 58^\circ$. To the nearest unit, $b = 9$ mm.
- b) To the nearest degree, $\angle L = 18^\circ$. To the nearest unit, $j = 10$ cm and $l = 3$ cm.
- c) To the nearest degree, $\angle Q = 48^\circ$. To the nearest unit, $q = 14$ km and $r = 19$ km.

Closing

Have students read question 15 and then work in pairs to create their mind maps. When finished, students can post their mind maps to share with the class. Students can use their own mind maps, their classmates' mind maps, or both as reference to answer part b). Discuss answers for part b) with the class after the mind maps have been displayed.

Curious Math

This Curious Math feature provides students with an opportunity to explore the nautical mile. Students use the given formula to calculate the length, in metres, of the original nautical mile for various locations in Canada. Have students work on the questions individually.

Answers to Curious Math

- a) 25.27°
b) $48^\circ 18'$
- a) The nautical mile at Kingston, Ontario, is about 1852.02 m.
b) The nautical mile at Yellowknife, Northwest Territories, is about 1857.68 m.
c) The nautical mile at Alert, Nunavut, is about 1861.40 m.
- The length of the nautical mile varies from the equator to the North Pole. The shortest length, at the equator, is about 1842.8 m. The longest length is about 1861.7 m. The value that was chosen is the integer value that is closest to the mean, or 1852 m. It is approximately the mean length of one minute of latitude, or one minute of arc along a line of longitude.
- The expression $9.45 (\cos 2\theta)$ is the change in distance of the nautical mile depending on the specific latitude. For latitudes less than 45° , the nautical mile becomes shorter. For latitudes greater than 45° , the nautical mile becomes longer.

Assessment and Differentiating Instruction

What You Will See Students Doing...

When students understand...	If students misunderstand...
<p>Students correctly determine the length of a side in a right triangle using trigonometry, given the length of another side and the measure of one of the acute angles.</p> <p>Students correctly determine the measure of an acute angle in a right triangle using trigonometry, given the lengths of two sides.</p> <p>Students use trigonometry and other concepts, such as the Pythagorean theorem and the angle sum of a triangle, to correctly solve a triangle.</p>	<p>Students may have difficulty choosing the correct trigonometric ratio, or they may confuse the trigonometric ratios.</p> <p>Students may choose sides and angles incorrectly, or they may use incorrect keystrokes when determining the inverse.</p> <p>Students may not realize that they need to determine all the unknown measurements, or they may be unable to integrate other concepts to solve the triangle.</p>
<p>Key Assessment Question 13</p> <p>Students clearly identify a reference angle, correctly label the sides of the triangle, and select the appropriate trigonometric ratio to solve.</p> <p>Students correctly use the angle sum of a triangle and the Pythagorean theorem to determine the unknown measurements.</p> <p>Students accurately set up and solve the equations.</p>	<p>Students mislabel the triangle, or they select the incorrect trigonometric ratio to solve.</p> <p>Students may not think of using the angle sum of a triangle and the Pythagorean theorem. They may not remember these concepts correctly, or they may make calculation errors. Students may be able to determine only one unknown measurement.</p> <p>Students may set up the equations incorrectly, or they may make calculation errors.</p>

Differentiating Instruction | How You Can Respond

EXTRA SUPPORT

- If students are having difficulty selecting the correct trigonometric ratio, have them draw several right triangles and show situations in which the sine, cosine, and tangent ratios can be used. They can refer to their drawings as needed.
- Make sure that students understand how to label the opposite side, adjacent side, and hypotenuse.
 - If one side length and an angle measure are given, have them write the trigonometric ratios using the given and unknown measurements and also using the words “opposite,” “adjacent,” and “hypotenuse.” They can then set up equations by choosing the correct ratio.
 - If two side lengths are given and the unknown measurement is an angle, ask students to write the ratio of the two sides along with the corresponding trigonometric ratio with the words “opposite,” “adjacent,” or “hypotenuse.” This will help them choose the correct trigonometric ratio. Some students may need to review using the inverse.

EXTRA CHALLENGE

- Ask students to sketch diagrams of objects in the classroom or other places in the school. Have them measure lengths or angles needed to solve the triangles. They can solve the triangles and then check by measuring. If the objects they measured have lengths or angles that cannot be measured directly, students can trade triangles and compare their solutions to check.

7.6 SOLVING RIGHT TRIANGLE PROBLEMS

Lesson at a Glance

GOAL

Use the primary trigonometric ratios to solve problems that involve right triangle models.

Prerequisite Skills/Concepts

- Apply the Pythagorean theorem to calculate the length of a side in a right triangle.
- Determine the values of the sine, cosine, and tangent ratios for a specific acute angle in a right triangle.
- Determine the measure of an acute angle using a primary trigonometric ratio.
- Determine the length of a side using a primary trigonometric ratio.

Specific Expectations

- Determine the measures of the sides and angles in right triangles, using the primary trigonometric ratios and the Pythagorean theorem.
- Solve problems involving the measures of sides and angles in right triangles in real life applications (e.g., in surveying, in navigating, in determining the height of an inaccessible object around the school), using the primary trigonometric ratios and the Pythagorean theorem.

Mathematical Process Focus

- Problem Solving
- Representing
- Selecting Tools and Computational Strategies
- Connecting

Student Book Pages 408–414

Preparation and Planning

Pacing

5–10 min	Introduction
30–40 min	Teaching and Learning
15–25 min	Consolidation

Recommended Practice

Questions 4, 6, 8, 9, 10, 11, 13, 15

Key Assessment Question

Question 10

Extra Practice

Lesson 7.6 Extra Practice

New Vocabulary/Symbols

angle of depression (angle of declination)

Nelson Website

<http://www.nelson.com/math>

MATH BACKGROUND | LESSON OVERVIEW

- In Lessons 7.4 and 7.5, students used trigonometric ratios to calculate unknown side lengths and unknown angle measures in right triangles.
- In this lesson, students apply their skills to solve real-life problems that can be modelled using right triangles. For example, students calculate the area of a triangle after using the sine ratio to determine the height.

1

Introducing the Lesson

(5 to 10 min)

Have students briefly discuss, in pairs, what they have learned about the primary trigonometric ratios. Then follow up with a class discussion about what they have learned. Guide them to include these ideas:

- The triangle must be a right triangle to use the primary trigonometric ratios.
- The primary trigonometric ratios are sine, cosine, and tangent.
- Opposite and adjacent sides are identified in relation to a specific angle.
- The hypotenuse is opposite the right angle.
- Different ratios can be used to solve a triangle, depending on the known side lengths and angle measures and on the measurements to be calculated.
- Inverse ratios are used to determine angle measures.
- Angle measures can be calculated using the fact that the sum of the angles in a triangle is 180° , in combination with the primary trigonometric ratios.
- Side lengths can be calculated using the Pythagorean theorem, in combination with the primary trigonometric ratios.

2

Teaching and Learning

(30 to 40 min)

Learn About the Math

Have students read the information about drilling a well and then explain it in their own words. Give students an opportunity to work in pairs to discuss *Example 1* before the class discussion. They may find it helpful to draw a labelled diagram showing the lake, the oil deposit, and the drill site.

When going through the example as a class, ensure that students understand how the diagram represents the given information and how it compares with the picture in the margin. It is important for students to understand which angle in the triangle is the angle they need to determine and how the given information provides the lengths of two sides in the triangle. Ask students to explain why the length of the opposite side is the sum of 2300 m and 150 m.

Emphasize that the angle of depression is the angle between the horizontal and the line of sight, not the angle between the vertical and the line of sight. Ask a few students to describe the angle of depression in relation to a few objects in the classroom.

Answers to Reflecting

- A. The angle of depression is the angle between the horizontal and the line of sight when looking down at an object. The angle of elevation is the angle between the horizontal and the line of sight when looking up at an object. Both angles are between the horizontal and the line of sight.

B. Jackie could add the depth of the lake to the distance below the bottom of the lake to calculate the depth of the oil deposit. Then she could use the Pythagorean theorem to determine the distance from the oil deposit to the drill site. By letting x represent the distance, in metres, from the oil deposit to the drill site, Jackie could write and solve the equation $x^2 = 1000^2 + (2300 + 150)^2$. The result would give the distance from the oil deposit to the drill site in metres.

3 Consolidation

(15 to 25 min)

Apply the Math

Using the Solved Examples

In *Example 2*, an angle of elevation is given and the tangent ratio is used to determine the height of the tree. Ask:

- How did Ayesha use a clinometer?
- How do you know that 7.2 m represents the length of the adjacent side in the triangle?
- Why is the distance above ground for Ayesha's eyes added to the length of the opposite side to determine the height of the tree? How does the picture beside the problem show this? How does Joan's diagram show this? How do Joan's calculations show this?

In *Example 3*, the sine ratio is used to calculate the height of a triangle so that the area formula can be used. You could have students work in pairs to talk about the problem and draw a diagram with the given information. It is important for students to realize that they must always identify the quantity they are asked to find. Ask the class to explain why Hugo drew the height of the triangle and how he did this.

In *Example 4*, an acute triangle is divided into two right triangles. Guide students to understand how the two right triangles relate to the angle of elevation and the angle of depression.

Answer to the Key Assessment Question

For question 10, students should draw a diagram with the given dimensions labelled. They need to realize that the actual flight path represents the hypotenuse of the triangle, which is 350 km.

10. The airplane is about 8° off its planned flight path.

Closing

Question 18 provides students with an opportunity to summarize the steps in problem solving. Have them work in pairs to create posters that show the steps required to solve a problem involving a right triangle, using trigonometry. They could share their posters with other pairs and then display their posters in the class.

Assessment and Differentiating Instruction

What You Will See Students Doing...

When students understand...

Students represent a problem with a model that includes a right triangle, creating and correctly labelling a diagram that shows the model.

Students correctly use the appropriate primary trigonometric ratio to calculate an unknown side length or angle measure.

Students add a perpendicular line segment to create a right triangle, which is needed to solve a problem, such as determining the area of a triangle.

If students misunderstand...

Students may have difficulty modelling a problem with a right triangle, or they may create an incorrect drawing or a drawing that lacks information.

Students may use the wrong primary trigonometric ratio, or they may confuse the trigonometric ratios. They may substitute incorrectly or calculate inaccurately.

Students may use the trigonometric ratios for a triangle that is not a right triangle. They may not understand how to draw a perpendicular to represent the height.

Key Assessment Question 10

Students use the given information correctly to set up a right triangle, with the length of the hypotenuse (actual flight path) as 350 km and the opposite side as 48 km.

Students correctly use a primary trigonometric ratio to calculate the number of degrees that the airplane is off course, and they answer the problem appropriately.

Students may identify the length of the adjacent side as 350 km.

Students may use the wrong trigonometric ratio, or they may make calculation errors when using a correct ratio. They may not round the number of degrees correctly, or they may not answer the problem with an appropriate conclusion.

Differentiating Instruction | How You Can Respond

EXTRA SUPPORT

1. Students may benefit from reviewing how to draw trigonometric diagrams, how to select the appropriate primary trigonometric ratio, and how to determine missing values in a right triangle. These concepts could be reviewed with a group of students or individual students. Then students could discuss, with a partner, how to focus on these concepts when solving problems in the lesson.

EXTRA CHALLENGE

1. Have students create their own problems that can be solved using right triangle models. Then have them exchange problems with a partner and solve each other's problems. They could use triangles from this lesson or previous lessons, or they could create their own triangles. Remind them to check that their triangles are right triangles.

CHAPTER REVIEW

Big Ideas Covered So Far

- If two triangles are congruent, then they are similar. If two triangles are similar, however, they may or may not be congruent.
- If two pairs of corresponding angles in two triangles are equal, then the triangles are similar. If, in addition, two corresponding sides are equal, then the triangles are congruent.
- Similar triangles can be used to determine lengths that cannot be measured directly.
- The primary trigonometric ratios for $\angle A$ are $\sin A$, $\cos A$, and $\tan A$.
- If $\angle A$ is an acute angle in a right triangle, the primary trigonometric ratios can be determined using the ratios of the sides.
- Trigonometric ratios can be used to calculate unknown side lengths and unknown angle measures in a right triangle. The ratio that is used depends on the information given and the measurement to be calculated.
- A problem that can be represented with a right triangle and involves an unknown side length or angle measure can be solved using trigonometric ratios.

Using the Frequently Asked Questions

Have students keep their Student Books closed. Display the Frequently Asked Questions on a board. Have students discuss the questions and use the discussion to draw out what the class thinks are good answers. Then have students compare the class answers with the answers on Student Book page 415. Students can refer to the answers to the Frequently Asked Questions as they work through the Practice Questions.

Using the Chapter Review

Ask students if they have any questions about any of the topics covered so far in the chapter. Review any topics that students would benefit from considering again. Assign Practice Questions for class work and for homework.

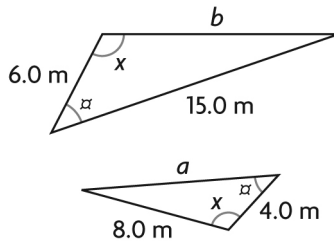
To gain greater insight into students' understanding of the material covered so far in the chapter, you may want to ask questions such as the following:

- What is the minimum amount of information needed to determine the measurements of all the sides and all the angles in a right triangle?
- If you are given a right triangle, how do you know which side is the opposite side, the adjacent side, and the hypotenuse?
- How do you know which primary trigonometric ratio you should use to determine an unknown side length or angle measure?
- When do you use the inverse?
- What does it mean for the slope of a hill to be approximately 0.4? How does slope relate to a primary trigonometric ratio?

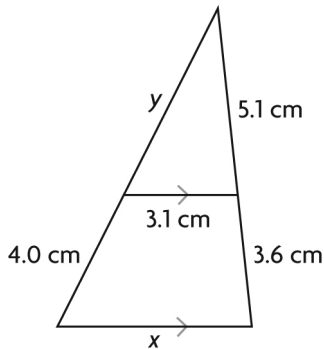
CHAPTER 7 TEST

For further assessment items, please use Nelson's Computerized Assessment Bank.

1. Determine the values of a and b .



2. Determine the values of x and y .



3. A tower, 17.5 m high, has been built on level ground. The tower casts a shadow that is 5.0 m long, when measured from the centre of its base. A woman, 1.5 m tall, is standing close to the tower. Determine the length of the shadow that is cast by the woman.

4. Determine each unknown value to one decimal place.

a) $\sin 40^\circ = \frac{x}{12}$

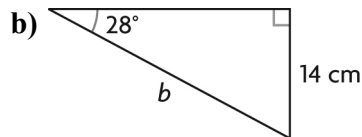
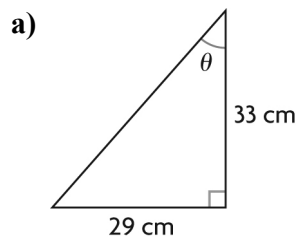
c) $\cos A = 0.7431$

b) $\tan 65^\circ = \frac{30}{y}$

d) $\sin B = \frac{7}{11}$

5. The lengths of the hypotenuse and one other side in a right triangle are known. Is this enough information to determine the measure of any angle or side in the triangle? Justify your answer.

6. Determine the indicated side length or angle measure in each triangle.

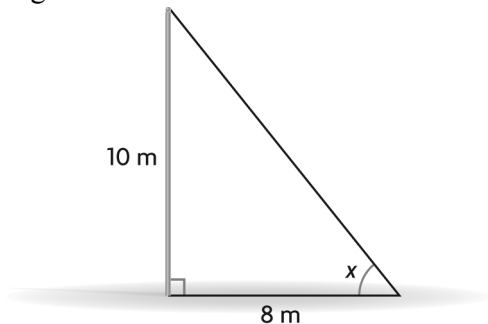


7. Solve each triangle.

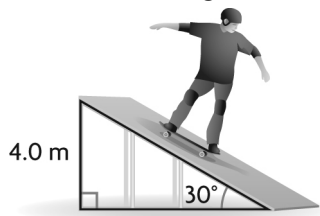
a) In $\triangle ABC$, $\angle A = 90^\circ$, $\angle B = 72^\circ$, and $b = 22$ cm.

b) In $\triangle DEF$, $\angle F = 90^\circ$, $f = 10.2$ cm, and $d = 4.7$ cm.

8. A vertical pole is 10 m tall. A cable is connected from the top of the pole to a point on the ground that is 8 m away from the foot of the pole. Determine, to the nearest degree, the angle of elevation of the cable.



9. A skateboarder wants to skate down a ramp that has a slope angle of 30° and a height of 4.0 m. How long is the ramp?



10. From a window, 26 m above the ground, the angle of elevation to the top of a building is 39° . The angle of depression to the bottom of the building is 29° . How high is the building?

CHAPTER TASK

What's the Height of Your School?

Specific Expectations

- Determine the measures of the sides and angles in right triangles, using the primary trigonometric ratios and the Pythagorean theorem.
- Solve problems involving the measures of sides and angles in right triangles in real life applications (e.g., in surveying, in navigating, in determining the height of an inaccessible object around the school), using the primary trigonometric ratios [and the Pythagorean theorem].

Introducing the Chapter Task (Whole Class)

Introduce the task, relating it to a suitable location in your school. Ask students what measurements they would need if they wanted to calculate the height of the school or another tall structure. Have students create a diagram showing these measurements, and ask how they would obtain all of these measurements. Explain that a clinometer is a tool used to measure the angle of elevation in a right triangle. Remind students about the picture of a clinometer on Student Book page 419. Explain that, in this task, they will make their own clinometer, use their clinometer to determine an angle of elevation, and then determine the height of the school.

Using the Chapter Task

Have students work in pairs on parts A to D. Each pair can design, assemble, and test their own clinometer. For part D, one student can hold the clinometer directed at the top of the school while the other student reads the angle of elevation from the clinometer. For part F, students should prepare their own report.

Remind students to use the Task Checklist to help them produce an excellent report. As students work through the task, observe and/or review them individually to see how they are interpreting and carrying out the task.

Assessing Students' Work

Use the Assessment of Learning chart as a guide for assessing students' work.

Adapting the Task

You can adapt the task in the Student Book to suit the needs of your students. For example:

- Suggest several sources where students can research how to build a clinometer, if needed.
- Have stronger students research the history of the clinometer and its use and then report their findings to the class.

Student Book Page 419

Preparation and Planning

Pacing

5–10 min Introducing the Chapter Task

50–55 min Using the Chapter Task

Materials

- protractor
- drinking straw
- string
- clear tape
- bolt
- tape measure

Nelson Website

<http://www.nelson.com/math>

Assessment of Learning—What to Look for in Student Work...

Assessment Strategy: Interview/Observation and Product Marking

Level of Performance	1	2	3	4
Knowledge and Understanding Knowledge of content	demonstrates limited knowledge of content	demonstrates some knowledge of content	demonstrates considerable knowledge of content	demonstrates thorough knowledge of content
Understanding of mathematical concepts	demonstrates limited understanding of concepts	demonstrates some understanding of concepts	demonstrates considerable understanding of concepts	demonstrates thorough understanding of concepts
Thinking Use of planning skills <ul style="list-style-type: none"> understanding the problem making a plan for solving the problem 	uses planning skills with limited effectiveness (e.g., is unable to design a clinometer)	uses planning skills with some effectiveness (e.g., is able to design a clinometer, with some errors)	uses planning skills with considerable effectiveness (e.g., is able to design a clinometer, with only minor errors)	uses planning skills with a high degree of effectiveness (e.g., is able to design an accurate clinometer)
Use of processing skills <ul style="list-style-type: none"> carrying out a plan looking back at the solution 	uses processing skills with limited effectiveness (e.g., is unable to assemble a proper clinometer from a design or measure an angle of elevation)	uses processing skills with some effectiveness (e.g., is able to assemble a clinometer with some errors and measure an angle of elevation with some accuracy)	uses processing skills with considerable effectiveness (e.g., is able to assemble a clinometer from a design and measure an angle of elevation with an acceptable degree of accuracy)	uses processing skills with a high degree of effectiveness (e.g., is able to assemble a clinometer from a design and measure an angle of elevation with a high degree of accuracy)
Use of critical/creative thinking processes	uses critical/creative thinking processes with limited effectiveness	uses critical/creative thinking processes with some effectiveness	uses critical/creative thinking processes with considerable effectiveness	uses critical/creative thinking processes with a high degree of effectiveness

Assessment of Learning—What to Look for in Student Work...

Assessment Strategy: Interview/Observation and Product Marking

Level of Performance	1	2	3	4
Communication Expression and organization of ideas and mathematical thinking, using oral, visual, and written forms Communication for different audiences and purposes in oral, visual, and written forms Use of conventions, vocabulary, and terminology of the discipline in oral, visual, and written forms	expresses and organizes mathematical thinking with limited effectiveness (e.g., writes a report about how a clinometer works with limited clarity)	expresses and organizes mathematical thinking with some effectiveness (e.g., writes a report about how a clinometer works with some clarity)	expresses and organizes mathematical thinking with considerable effectiveness (e.g., writes a report about how a clinometer works with a fair amount of clarity)	expresses and organizes mathematical thinking with a high degree of effectiveness (e.g., writes a report about how a clinometer works with a high degree of clarity)
	communicates for different audiences and purposes with limited effectiveness	communicates for different audiences and purposes with some effectiveness	communicates for different audiences and purposes with considerable effectiveness	communicates for different audiences and purposes with a high degree of effectiveness
	uses conventions, vocabulary, and terminology of the discipline with limited effectiveness	uses conventions, vocabulary, and terminology of the discipline with some effectiveness	uses conventions, vocabulary, and terminology of the discipline with considerable effectiveness	uses conventions, vocabulary, and terminology of the discipline with a high degree of effectiveness
Application Application of knowledge and skills in familiar contexts Transfer of knowledge and skills to new contexts Making connections within and between various contexts	applies knowledge and skills in familiar contexts with limited effectiveness (e.g., is unable to determine the angle of elevation or the height of the school using a primary trigonometric ratio)	applies knowledge and skills in familiar contexts with some effectiveness (e.g., is able to determine an angle of elevation of the school, but makes some errors when determining the height of the school)	applies knowledge and skills in familiar contexts with considerable effectiveness (e.g., determines the correct angle of elevation of the school, and uses a primary trigonometric ratio to determine the height with only minor errors)	applies knowledge and skills in familiar contexts with a high degree of effectiveness (e.g., correctly determines the angle of elevation of the school, and uses the correct primary trigonometric ratio to determine the height)
	transfers knowledge and skills to new contexts with limited effectiveness	transfers knowledge and skills to new contexts with some effectiveness	transfers knowledge and skills to new contexts with considerable effectiveness	transfers knowledge and skills to new contexts with a high degree of effectiveness
	makes connections within and between various contexts with limited effectiveness	makes connections within and between various contexts with some effectiveness	makes connections within and between various contexts with considerable effectiveness	makes connections within and between various contexts with a high degree of effectiveness