CHAPTER 7
Systems of Equations and Inequalities

C H A P T E R 7
Systems of Equations and Inequalities

Section 7.1 Linear and Nonlinear Systems of Equations

- You should be able to solve systems of equations by the method of substitution.
  1. Solve one of the equations for one of the variables.
  2. Substitute this expression into the other equation and solve.
  3. Back-substitute into the first equation to find the value of the other variable.
  4. Check your answer in each of the original equations.
- You should be able to find solutions graphically. (See Example 5 in textbook.)

Vocabulary Check

1. system of equations

2. solution

3. solving

4. substitution

5. point of intersection

6. break-even

1. $\begin{cases} 4x - y = 1 \\ 6x + y = -6 \end{cases}$

(a) $4(0) - (-3) \neq 1$

$(0, -3)$ *is not* a solution.

(b) $4(-1) - (-4) \neq 1$

$(-1, -4)$ *is not* a solution.

(c) $4\left(-\frac{3}{2}\right) - (-2) \neq 1$

$\left(-\frac{3}{2}, -2\right)$ *is not* a solution.

(d) $4\left(-\frac{1}{2}\right) - (-3) = 1$

$6\left(-\frac{1}{2}\right) + (-3) = -6$

$\left(-\frac{1}{2}, -3\right)$ *is* a solution.

2. $\begin{cases} 4x^2 + y = 3 \\ -x - y = 11 \end{cases}$

(a) $4(2)^2 + (-13) \stackrel{?}{=} 3$

$16 - 13 = 3$

$-2 - (-13) \stackrel{?}{=} 11$

$-2 + 13 = 11$

$(2, -13)$ *is* a solution.

(b) $4(2)^2 + (-9) \stackrel{?}{=} 3$

$16 - 9 \neq 3$

$(2, -9)$ *is not* a solution.

(c) $4\left(-\frac{3}{2}\right)^2 + \left(-\frac{31}{3}\right) \stackrel{?}{=} 3$

$\frac{36}{4} - \frac{31}{3} \neq 3$

$\left(-\frac{3}{2}, -\frac{31}{3}\right)$ *is not* a solution.

(d) $4\left(-\frac{7}{4}\right)^2 + \left(-\frac{37}{4}\right) \stackrel{?}{=} 3$

$\frac{49}{4} - \frac{37}{4} = 3$

$-\left(-\frac{7}{4}\right) - \left(-\frac{37}{4}\right) \stackrel{?}{=} 11$

$\frac{7}{4} + \frac{37}{4} = 11$

$\left(-\frac{7}{4}, -\frac{37}{4}\right)$ *is* a solution.

3. $\begin{cases} y = -2e^x \\ 3x - y = 2 \end{cases}$

(a) $0 \neq -2e^{-2}$

$(-2, 0)$ *is not* a solution.

(b) $-2 = -2e^0$

$3(0) - (-2) = 2$

$(0, -2)$ *is* a solution.

(c) $-3 \neq -2e^0$

$(0, -3)$ *is not* a solution.

(d) $2 \neq -2e^{-1}$

$(-1, 2)$ *is not* a solution.

4. $\begin{cases} -\log x + 3 = y \\ \frac{1}{9}x + y = \frac{28}{9} \end{cases}$

(a) $-\log 9 + 3 \neq \frac{37}{9}$

$\left(9, \frac{37}{9}\right)$ *is not* a solution.

(b) $-\log 10 + 3 = 2$

$\frac{1}{9}(10) + 2 = \frac{28}{9}$

$(10, 2)$ *is* a solution.

(c) $-\log(1) + 3 = 3$

$\frac{1}{9}(1) + 3 = \frac{28}{9}$

$(1, 3)$ *is* a solution.

(d) $-\log 2 + 3 \neq 4$

$(2, 4)$ *is not* a solution.

5. $\begin{cases} 2x + y = 6 & \text{Equation 1} \\ -x + y = 0 & \text{Equation 2} \end{cases}$

Solve for y in Equation 1: $y = 6 - 2x$

Substitute for y in Equation 2: $-x + (6 - 2x) = 0$

Solve for x: $-3x + 6 = 0 \implies x = 2$

Back-substitute $x = 2$: $y = 6 - 2(2) = 2$

Solution: $(2, 2)$

6. $\begin{cases} x - y = -4 & \text{Equation 1} \\ x + 2y = 5 & \text{Equation 2} \end{cases}$

Solve for x in Equation 1: $x = y - 4$

Substitute for x in Equation 2: $(y - 4) + 2y = 5$

Solve for y: $3y - 4 = 5 \implies y = 3$

Back-substitute $y = 3$: $x = 3 - 4 = -1$

Solution: $(-1, 3)$

7. $\begin{cases} x - y = -4 & \text{Equation 1} \\ x^2 - y = -2 & \text{Equation 2} \end{cases}$

Solve for y in Equation 1: $y = x + 4$

Substitute for y in Equation 2: $x^2 - (x + 4) = -2$

Solve for x: $x^2 - x - 2 = 0 \implies (x + 1)(x - 2) = 0 \implies x = -1, 2$

Back-substitute $x = -1$: $y = -1 + 4 = 3$

Back-substitute $x = 2$: $y = 2 + 4 = 6$

Solutions: $(-1, 3), (2, 6)$

8. $\begin{cases} 3x + y = 2 & \text{Equation 1} \\ x^3 - 2 + y = 0 & \text{Equation 2} \end{cases}$

Solve for y in Equation 1: $y = 2 - 3x$

Substitute for y in Equation 2: $x^3 - 2 + (2 - 3x) = 0$

$$x^3 - 3x = 0$$

Solve for x: $x^3 - 3x = 0 \implies x(x^2 - 3) = 0 \implies x = 0, \pm\sqrt{3}$

Back-substitute $x = 0$: $y = 2 - 3(0) = 2$

Back-substitute $x = \sqrt{3}$: $y = 2 - 3\sqrt{3}$

Back-substitute $x = -\sqrt{3}$: $y = 2 - 3\left(-\sqrt{3}\right) = 2 + 3\sqrt{3}$

Solutions: $(0, 2), \left(\sqrt{3}, 2 - 3\sqrt{3}\right), \left(-\sqrt{3}, 2 + 3\sqrt{3}\right)$

9. $\begin{cases} -2x + y = -5 & \text{Equation 1} \\ x^2 + y^2 = 25 & \text{Equation 2} \end{cases}$

Solve for y in Equation 1: $y = 2x - 5$

Substitute for y in Equation 2: $x^2 + (2x - 5)^2 = 25$

Solve for x:

$$5x^2 - 20x = 0 \implies 5x(x - 4) = 0 \implies x = 0, 4$$

Back-substitute $x = 0$: $y = 2(0) - 5 = -5$

Back-substitute $x = 4$: $y = 2(4) - 5 = 3$

Solutions: $(0, -5), (4, 3)$

10. $\begin{cases} x + y = 0 & \text{Equation 1} \\ x^3 - 5x - y = 0 & \text{Equation 2} \end{cases}$

Solve for y in Equation 1: $y = -x$

Substitute for y in Equation 2: $x^3 - 5x - (-x) = 0$

Solve for x:

$$x^3 - 4x = 0 \implies x(x^2 - 4) = 0 \implies x = 0, \pm 2$$

Back-substitute $x = 0$: $y = -0 = 0$

Back-substitute $x = 2$: $y = -2$

Back-substitute $x = -2$: $y = -(-2) = 2$

Solutions: $(0, 0), (2, -2), (-2, 2)$

11. $\begin{cases} x^2 + y = 0 & \text{Equation 1} \\ x^2 - 4x - y = 0 & \text{Equation 2} \end{cases}$

Solve for y in Equation 1: $y = -x^2$

Substitute for y in Equation 2: $x^2 - 4x - (-x^2) = 0$

Solve for x: $2x^2 - 4x = 0 \implies 2x(x - 2) = 0 \implies x = 0, 2$

Back-substitute $x = 0$: $y = -0^2 = 0$

Back-substitute $x = 2$: $y = -2^2 = -4$

Solutions: $(0, 0), (2, -4)$

12. $\begin{cases} y = -2x^2 + 2 & \text{Equation 1} \\ y = 2(x^4 - 2x^2 + 1) & \text{Equation 2} \end{cases}$

Substitute for y in Equation 1:

$$2(x^4 - 2x^2 + 1) = -2x^2 + 2$$

Solve for x: $x^4 - 2x^2 + 1 + x^2 - 1 = 0$

$$x^4 - x^2 = 0$$

$$x^2(x^2 - 1) = 0 \implies x = 0, \pm 1$$

Back-substitute $x = 0$: $y = -2(0)^2 + 2 = 2$

Back-substitute $x = 1$: $y = -2(1)^2 + 2 = 0$

Back-substitute $x = -1$: $y = -2(-1)^2 + 2 = 0$

Solutions: $(0, 2), (1, 0), (-1, 0)$

13. $\begin{cases} y = x^3 - 3x^2 + 1 & \text{Equation 1} \\ y = x^2 - 3x + 1 & \text{Equation 2} \end{cases}$

Substitute for y in Equation 2:

$$x^3 - 3x^2 + 1 = x^2 - 3x + 1$$

$$x^3 - 4x^2 + 3x = 0$$

$$x(x - 1)(x - 3) = 0 \implies x = 0, 1, 3$$

Back-substitute $x = 0$: $y = 0^3 - 3(0)^2 + 1 = 1$

Back-substitute $x = 1$: $y = 1^3 - 3(1)^2 + 1 = -1$

Back-substitute $x = 3$: $y = 3^3 - 3(3)^2 + 1 = 1$

Solutions: $(0, 1), (1, -1), (3, 1)$

14. $\begin{cases} y = x^3 - 3x^2 + 4 & \text{Equation 1} \\ y = -2x + 4 & \text{Equation 2} \end{cases}$

Substitute for y in Equation 1: $-2x + 4 = x^3 - 3x^2 + 4$

Solve for x: $0 = x^3 - 3x^2 + 2x$

$$0 = x(x^2 - 3x + 2)$$

$$0 = x(x - 2)(x - 1) \implies x = 0, 1, 2$$

Back-substitute $x = 0$: $y = -2(0) + 4 = 4$

Back-substitute $x = 1$: $y = -2(1) + 4 = 2$

Back-substitute $x = 2$: $y = -2(2) + 4 = 0$

Solutions: $(0, 4), (1, 2), (2, 0)$

15. $\begin{cases} x - y = 0 & \text{Equation 1} \\ 5x - 3y = 10 & \text{Equation 2} \end{cases}$

Solve for y in Equation 1: $y = x$

Substitute for y in Equation 2: $5x - 3x = 10$

Solve for x: $2x = 10 \implies x = 5$

Back-substitute in Equation 1: $y = x = 5$

Solution: $(5, 5)$

16. $\begin{cases} x + 2y = 1 & \text{Equation 1} \\ 5x - 4y = -23 & \text{Equation 2} \end{cases}$

Solve for x in Equation 1: $x = 1 - 2y$

Substitute for x in Equation 2: $5(1 - 2y) - 4y = -23$

Solve for y: $-14y = -28 \implies y = 2$

Back-substitute $y = 2$: $x = 1 - 2y = 1 - 2(2) = -3$

Solution: $(-3, 2)$

17. $\begin{cases} 2x - y + 2 = 0 & \text{Equation 1} \\ 4x + y - 5 = 0 & \text{Equation 2} \end{cases}$

Solve for y in Equation 1: $y = 2x + 2$

Substitute for y in Equation 2: $4x + (2x + 2) - 5 = 0$

Solve for x: $6x - 3 = 0 \implies x = \frac{1}{2}$

Back-substitute $x = \frac{1}{2}$: $y = 2x + 2 = 2\left(\frac{1}{2}\right) + 2 = 3$

Solution: $\left(\frac{1}{2}, 3\right)$

18. $\begin{cases} 6x - 3y - 4 = 0 & \text{Equation 1} \\ x + 2y - 4 = 0 & \text{Equation 2} \end{cases}$

Solve for x in Equation 2: $x = 4 - 2y$

Substitute for x in Equation 1: $6(4 - 2y) - 3y - 4 = 0$

Solve for y: $24 - 12y - 3y - 4 = 0 \implies -15y = -20 \implies y = \frac{4}{3}$

Back-substitute $y = \frac{4}{3}$: $x = 4 - 2y = 4 - 2\left(\frac{4}{3}\right) = \frac{4}{3}$

Solution: $\left(\frac{4}{3}, \frac{4}{3}\right)$

19. $\begin{cases} 1.5x + 0.8y = 2.3 & \text{Equation 1} \\ 0.3x - 0.2y = 0.1 & \text{Equation 2} \end{cases}$

Multiply the equations by 10.

$15x + 8y = 23$ Revised Equation 1

$3x - 2y = 1$ Revised Equation 2

Solve for y in revised Equation 2: $y = \frac{3}{2}x - \frac{1}{2}$

Substitute for y in revised Equation 1: $15x + 8\left(\frac{3}{2}x - \frac{1}{2}\right) = 23$

Solve for x: $15x + 12x - 4 = 23 \implies 27x = 27 \implies x = 1$

Back-substitute $x = 1$: $y = \frac{3}{2}(1) - \frac{1}{2} = 1$

Solution: $(1, 1)$

20. $\begin{cases} 0.5x + 3.2y = 9.0 & \text{Equation 1} \\ 0.2x - 1.6y = -3.6 & \text{Equation 2} \end{cases}$

Multiply the equations by 10.

$5x + 32y = 90$ Revised Equation 1

$2x - 16y = -36$ Revised Equation 2

Solve for x in revised Equation 2: $x = 8y - 18$

Substitute for x in revised Equation 1: $5(8y - 18) + 32y = 90$

Solve for y: $40y - 90 + 32y = 90 \implies 72y = 180$

$\implies y = \frac{5}{2}$

Back-substitute $y = \frac{5}{2}$: $x = 8\left(\frac{5}{2}\right) - 18 = 2$

Solution: $\left(2, \frac{5}{2}\right)$

21. $\begin{cases} \frac{1}{5}x + \frac{1}{2}y = 8 & \text{Equation 1} \\ x + y = 20 & \text{Equation 2} \end{cases}$

Solve for x in Equation 2: $x = 20 - y$

Substitute for x in Equation 1: $\frac{1}{5}(20 - y) + \frac{1}{2}y = 8$

Solve for y: $4 + \frac{3}{10}y = 8 \implies y = \frac{40}{3}$

Back-substitute $y = \frac{40}{3}$: $x = 20 - y = 20 - \frac{40}{3} = \frac{20}{3}$

Solution: $\left(\frac{20}{3}, \frac{40}{3}\right)$

22. $\begin{cases} \frac{1}{2}x + \frac{3}{4}y = 10 & \text{Equation 1} \\ \frac{3}{4}x - y = 4 & \text{Equation 2} \end{cases}$

Solve for y in Equation 2: $y = \frac{3}{4}x - 4$

Substitute for y in Equation 1: $\frac{1}{2}x + \frac{3}{4}\left(\frac{3}{4}x - 4\right) = 10$

Solve for x: $\frac{1}{2}x + \frac{9}{16}x - 3 = 10 \implies \frac{17}{16}x = 13 \implies x = \frac{208}{17}$

Back-substitute $x = \frac{208}{17}$: $y = \frac{3}{4}\left(\frac{208}{17}\right) - 4 = \frac{88}{17}$

Solution: $\left(\frac{208}{17}, \frac{88}{17}\right)$

23. $\begin{cases} 6x + 5y = -3 & \text{Equation 1} \\ -x - \frac{5}{6}y = -7 & \text{Equation 2} \end{cases}$

Solve for x in Equation 2: $x = 7 - \frac{5}{6}y$

Substitute for x in Equation 1: $6\left(7 - \frac{5}{6}y\right) + 5y = -3$

Solve for y: $42 - 5y + 5y = -3 \implies 42 = -3$ (False)

No solution

24. $\begin{cases} -\frac{2}{3}x + y = 2 & \text{Equation 1} \\ 2x - 3y = 6 & \text{Equation 2} \end{cases}$

Solve for y in Equation 1: $y = \frac{2}{3}x + 2$

Substitute for y in Equation 2: $2x - 3\left(\frac{2}{3}x + 2\right) = 6$

Solve for x: $2x - 2x - 6 = 6 \implies 0 = 12$ Inconsistent

No solution

25. $\begin{cases} x^2 - y = 0 & \text{Equation 1} \\ 2x + y = 0 & \text{Equation 2} \end{cases}$

Solve for y in Equation 2: $y = -2x$

Substitute for y in Equation 1: $x^2 - (-2x) = 0$

Solve for x: $x^2 + 2x = 0 \implies x(x + 2) = 0 \implies x = 0, -2$

Back-substitute $x = 0$: $y = -2(0) = 0$

Back-substitute $x = -2$: $y = -2(-2) = 4$

Solutions: $(0, 0), (-2, 4)$

26. $\begin{cases} x - 2y = 0 & \text{Equation 1} \\ 3x - y^2 = 0 & \text{Equation 2} \end{cases}$

Solve for x in Equation 1: $x = 2y$

Substitute for x in Equation 2: $3(2y) - y^2 = 0$

Solve for y: $6y - y^2 = 0 \implies y(6 - y) = 0 \implies y = 0, 6$

Back-substitute $y = 0$: $x = 2(0) = 0$

Back-substitute $y = 6$: $x = 2(6) = 12$

Solutions: $(0, 0), (12, 6)$

27. $\begin{cases} x - y = -1 & \text{Equation 1} \\ x^2 - y = -4 & \text{Equation 2} \end{cases}$

Solve for y in Equation 1: $y = x + 1$

Substitute for y in Equation 2: $x^2 - (x + 1) = -4$

Solve for x: $x^2 - x - 1 = -4 \implies x^2 - x + 3 = 0$

The Quadratic Formula yields no real solutions.

28. $\begin{cases} y = -x & \text{Equation 1} \\ y = x^3 + 3x^2 + 2x & \text{Equation 2} \end{cases}$

Substitute for y in Equation 2: $-x = x^3 + 3x^2 + 2x$

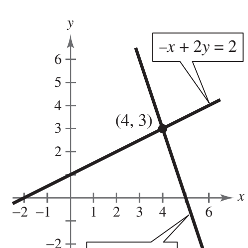
Solve for x: $x^3 + 3x^2 + 3x = 0 \implies x(x^2 + 3x + 3) = 0$

$$\implies x = 0, \frac{-3 \pm i\sqrt{3}}{2}$$

Back-substitute $x = 0$: $y = 0$

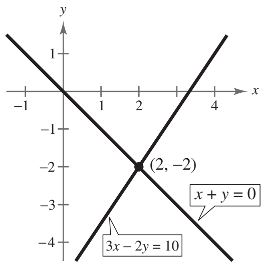
The only real solution is $(0, 0)$.

29. $\begin{cases} -x + 2y = 2 \implies y = \dfrac{x + 2}{2} \\ 3x + y = 15 \implies y = -3x + 15 \end{cases}$



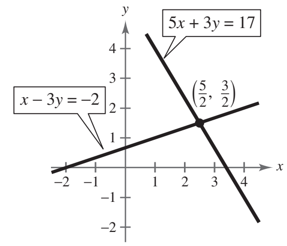
Point of intersection: $(4, 3)$

30. $\begin{cases} x + y = 0 \\ 3x - 2y = 10 \end{cases}$



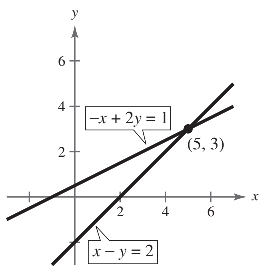
Point of intersection: $(2, -2)$

31. $\begin{cases} x - 3y = -2 \implies y = \frac{1}{3}(x + 2) \\ 5x + 3y = 17 \implies y = \frac{1}{3}(-5x + 17) \end{cases}$



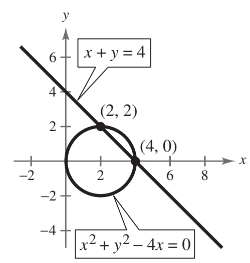
Point of intersection: $\left(\frac{5}{2}, \frac{3}{2}\right)$

32. $\begin{cases} -x + 2y = 1 \\ x - y = 2 \end{cases}$



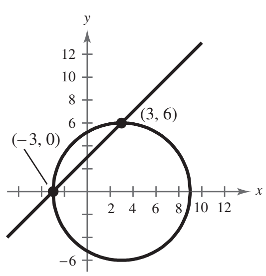
Point of intersection: $(5, 3)$

33. $\begin{cases} x + y = 4 \implies y = -x + 4 \\ x^2 + y^2 - 4x = 0 \implies (x - 2)^2 + y^2 = 4 \end{cases}$



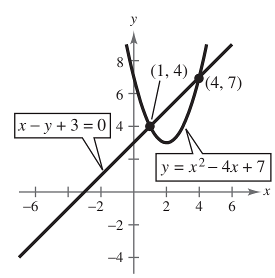
Points of intersection: $(2, 2), (4, 0)$

34. $\begin{cases} -x + y = 3 \\ x^2 - 6x - 27 + y^2 = 0 \end{cases}$



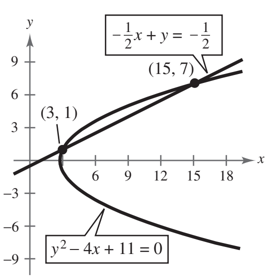
Points of intersection: $(-3, 0), (3, 6)$

35. $\begin{cases} x - y + 3 = 0 \implies y = x + 3 \\ y = x^2 - 4x + 7 \implies y = (x - 2)^2 + 3 \end{cases}$



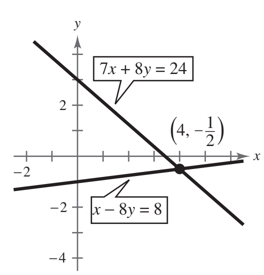
Points of intersection: $(1, 4), (4, 7)$

36. $\begin{cases} y^2 - 4x + 11 = 0 \\ -\frac{1}{2}x + y = -\frac{1}{2} \end{cases}$



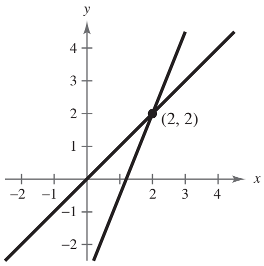
Points of intersection: $(3, 1), (15, 7)$

37. $\begin{cases} 7x + 8y = 24 \implies y = -\frac{7}{8}x + 3 \\ x - 8y = 8 \implies y = \frac{1}{8}x - 1 \end{cases}$



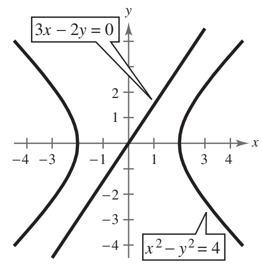
Point of intersection: $\left(4, -\frac{1}{2}\right)$

38. $\begin{cases} x - y = 0 \\ 5x - 2y = 6 \end{cases}$



Point of intersection: $(2, 2)$

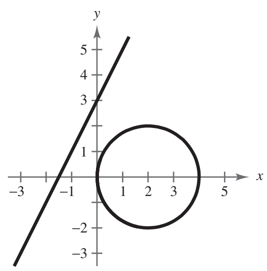
39. $\begin{cases} 3x - 2y = 0 \implies y = \dfrac{3}{2}x \\ x^2 - y^2 = 4 \implies \dfrac{x^2}{4} - \dfrac{y^2}{4} = 1 \end{cases}$



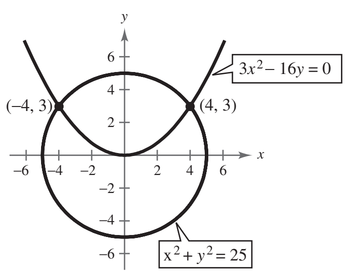
No points of intersection $\implies$ No solution

40. $\begin{cases} 2x - y + 3 = 0 \\ x^2 + y^2 - 4x = 0 \end{cases}$

No points of intersection so, no solution



41. $\begin{cases} x^2 + y^2 = 25 \\ 3x^2 - 16y = 0 \implies y = \frac{3}{16}x^2 \end{cases}$



Points of intersection: $(-4, 3)$ and $(4, 3)$

Algebraically we have:

$$x^2 = 25 - y^2$$
$$\tfrac{16}{3}y = 25 - y^2$$
$$16y = 75 - 3y^2$$
$$3y^2 + 16y - 75 = 0$$
$$(3y + 25)(y - 3) = 0$$

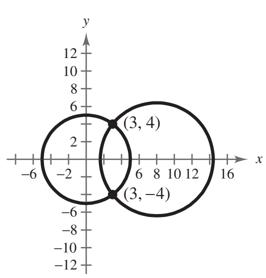
$y = -\frac{25}{3} \implies x^2 = -\frac{400}{9}$, No real solution

$y = 3 \implies x^2 = 16$

Solutions: $(\pm 4, 3)$

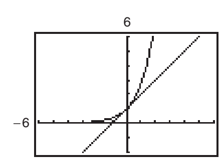
42. $\begin{cases} x^2 + y^2 = 25 \\ (x - 8)^2 + y^2 = 41 \end{cases}$

Points of intersection: $(3, 4), (3, -4)$



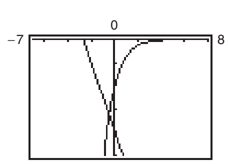
43. $\begin{cases} y = e^x \\ x - y + 1 = 0 \implies y = x + 1 \end{cases}$

Point of intersection: $(0, 1)$

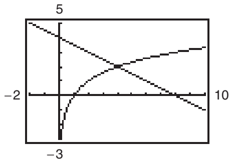


44. $\begin{cases} y = -4e^{-x} \\ y + 3x + 8 = 0 \end{cases}$

Point of intersection: $(-0.49, -6.53)$

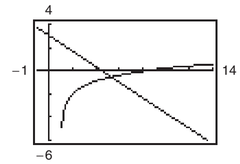


45. $\begin{cases} x + 2y = 8 & \Rightarrow \ y = -\frac{1}{2}x + 4 \\ \quad\ \ y = \log_2 x & \Rightarrow \ y = \frac{\ln x}{\ln 2} \end{cases}$



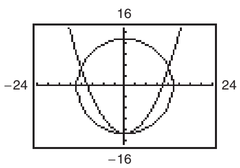
Point of intersection: $(4, 2)$

46. $\begin{cases} \quad\ \ y = -2 + \ln(x - 1) \\ 3y + 2x = 9 \end{cases}$



Point of intersection: $(5.31, -0.54)$

47. $\begin{cases} x^2 + y^2 = 169 \ \Rightarrow \ y_1 = \sqrt{169 - x^2} \text{ and } y_2 = -\sqrt{169 - x^2} \\ x^2 - 8y = 104 \ \Rightarrow \ y_3 = \ \frac{1}{8}x^2 - 13 \end{cases}$

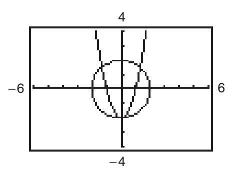


Points of intersection: $(0, -13), (\pm 12, 5)$

48. $\begin{cases} \ x^2 + y^2 = 4 \ \Rightarrow \ y_1 = \sqrt{4 - x^2}, y_2 = -\sqrt{4 - x^2} \\ 2x^2 - y \ = 2 \ \Rightarrow \ y_3 = 2x^2 - 2 \end{cases}$

Points of intersection:

$(0, -2), (1.32, 1.5), (-1.32, 1.5)$



49. $\begin{cases} y = 2x & \text{Equation 1} \\ y = x^2 + 1 & \text{Equation 2} \end{cases}$

Substitute for y in Equation 2: $2x = x^2 + 1$

Solve for x: $x^2 - 2x + 1 = (x - 1)^2 = 0 \Rightarrow x = 1$

Back-substitute $x = 1$ in Equation 1: $y = 2x = 2$

Solution: $(1, 2)$

50. $\begin{cases} \ x + y = 4 & \text{Equation 1} \\ x^2 + y = 2 & \text{Equation 2} \end{cases}$

Solve for y in Equation 1: $y = 4 - x$

Substitute for y in Equation 2: $x^2 + (4 - x) = 2$

Solve for x: $x^2 - x + 2 = 0$

No real solutions because the discriminant in the Quadratic Formula is negative.

Inconsistent; no solution

51. $\begin{cases} 3x - 7y + 6 = 0 & \text{Equation 1} \\ \qquad x^2 - y^2 = 4 & \text{Equation 2} \end{cases}$

Solve for y in Equation 1: $y = \dfrac{3x + 6}{7}$

Substitute for y in Equation 2: $x^2 - \left(\dfrac{3x + 6}{7}\right)^2 = 4$

Solve for x: $x^2 - \left(\dfrac{9x^2 + 36x + 36}{49}\right) = 4$

$$49x^2 - (9x^2 + 36x + 36) = 196$$

$$40x^2 - 36x - 232 = 0$$

$$4(10x - 29)(x + 2) = 0 \ \Rightarrow \ x = \frac{29}{10}, -2$$

Back-substitute $x = \dfrac{29}{10}$: $y = \dfrac{3x + 6}{7} = \dfrac{3(29/10) + 6}{7} = \dfrac{21}{10}$

Back-substitute $x = -2$: $y = \dfrac{3x + 6}{7} = \dfrac{3(-2) + 6}{7} = 0$

Solutions: $\left(\dfrac{29}{10}, \dfrac{21}{10}\right), (-2, 0)$

52. $\begin{cases} x^2 + y^2 = 25 & \text{Equation 1} \\ 2x + y = 10 & \text{Equation 2} \end{cases}$

Solve for y in Equation 2: $y = 10 - 2x$

Substitute for y in Equation 1: $x^2 + (10 - 2x)^2 = 25$

Solve for x: $x^2 + 100 - 40x + 4x^2 = 25 \implies x^2 - 8x + 15 = 0$

$\implies (x - 5)(x - 3) = 0 \implies x = 3, 5$

Back-substitute $x = 3$: $y = 10 - 2(3) = 4$

Back-substitute $x = 5$: $y = 10 - 2(5) = 0$

Solutions: $(3, 4), (5, 0)$

53. $\begin{cases} x - 2y = 4 & \text{Equation 1} \\ x^2 - y = 0 & \text{Equation 2} \end{cases}$

Solve for y in Equation 2: $y = x^2$

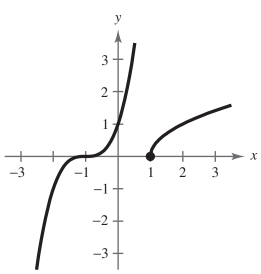
Substitute for y in Equation 1: $x - 2x^2 = 4$

Solve for x: $0 = 2x^2 - x + 4 \implies x = \dfrac{1 \pm \sqrt{1 - 4(2)(4)}}{2(2)} \implies x = \dfrac{1 \pm \sqrt{-31}}{4}$

The discriminant in the Quadratic Formula is negative.

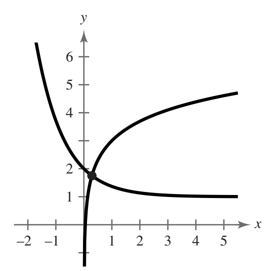
No real solution

54. $\begin{cases} y = (x + 1)^3 \\ y = \sqrt{x - 1} \end{cases}$



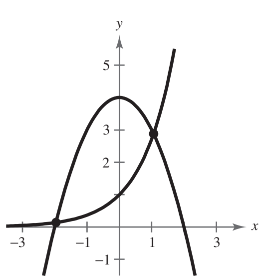
No points of intersection, so no solution

55. $\begin{cases} y - e^{-x} = 1 \implies y = e^{-x} + 1 \\ y - \ln x = 3 \implies y = \ln x + 3 \end{cases}$



Point of intersection: approximately $(0.287, 1.751)$

56. $\begin{cases} x^2 + y = 4 \implies y = 4 - x^2 \\ e^x - y = 0 \implies y = e^x \end{cases}$



Points of intersection (solutions):

approximately $(-1.96, 0.14), (1.06, 2.88)$

57. $\begin{cases} y = x^4 - 2x^2 + 1 & \text{Equation 1} \\ y = 1 - x^2 & \text{Equation 2} \end{cases}$

Substitute for y in Equation 1: $1 - x^2 = x^4 - 2x^2 + 1$

Solve for x: $x^4 - x^2 = 0 \implies x^2(x^2 - 1) = 0$

$\implies x = 0, \pm 1$

Back-substitute $x = 0$: $1 - x^2 = 1 - 0^2 = 1$

Back-substitute $x = 1$: $1 - x^2 = 1 - 1^2 = 0$

Back-substitute $x = -1$: $1 - x^2 = 1 - (-1)^2 = 0$

Solutions: $(0, 1), (\pm 1, 0)$

58. $\begin{cases} y = x^3 - 2x^2 + x - 1 & \text{Equation 1} \\ y = -x^2 + 3x - 1 & \text{Equation 2} \end{cases}$

Substitute for y in Equation 1:

$-x^2 + 3x - 1 = x^3 - 2x^2 + x - 1$

Solve for x: $0 = x^3 - x^2 - 2x$

$0 = x(x^2 - x - 2)$

$0 = x(x - 2)(x + 1) \implies x = 0, 2, -1$

Back-substitute $x = 0$ in Equation 2:

$y = -0^2 + 3(0) - 1 = -1$

Back-substitute $x = 2$ in Equation 2:

$y = -2^2 + 3(2) - 1 = 1$

Back-substitute $x = -1$ in Equation 2:

$y = -(-1)^2 + 3(-1) - 1 = -5$

Solutions: $(0, -1), (2, 1), (-1, -5)$

59. $\begin{cases} xy - 1 = 0 & \text{Equation 1} \\ 2x - 4y + 7 = 0 & \text{Equation 2} \end{cases}$

Solve for y in Equation 1: $y = \dfrac{1}{x}$

Substitute for y in Equation 2: $2x - 4\left(\dfrac{1}{x}\right) + 7 = 0$

Solve for x:

$2x^2 - 4 + 7x = 0 \implies (2x - 1)(x + 4) = 0$

$\implies x = \dfrac{1}{2}, -4$

Back-substitute $x = \dfrac{1}{2}$: $y = \dfrac{1}{1/2} = 2$

Back-substitute $x = -4$: $y = \dfrac{1}{-4} = -\dfrac{1}{4}$

Solutions: $\left(\dfrac{1}{2}, 2\right), \left(-4, -\dfrac{1}{4}\right)$

60. $\begin{cases} x - 2y = 1 & \text{Equation 1} \\ y = \sqrt{x - 1} & \text{Equation 2} \end{cases}$

Substitute for y in Equation 1: $x - 2\sqrt{x - 1} = 1$

Solve for x: $x - 1 = 2\sqrt{x - 1}$

$(x - 1)^2 = 4(x - 1)$

$x^2 - 2x + 1 = 4x - 4$

$x^2 - 6x + 5 = 0$

$(x - 1)(x - 5) = 0 \implies x = 1, 5$

Back-substitute $x = 1$: $y = \sqrt{1 - 1} = 0$

Back-substitute $x = 5$: $y = \sqrt{5 - 1} = 2$

Solutions: $(1, 0), (5, 2)$

61. $C = 8650x + 250{,}000$, $R = 9950x$

$R = C$

$9950x = 8650x + 250{,}000$

$1300x = 250{,}000$

$x \approx 192$ units

62. $C = 5.5\sqrt{x} + 10{,}000$, $R = 3.29x$

$R = C$

$3.29x = 5.5\sqrt{x} + 10{,}000$

$3.29x - 5.5\sqrt{x} - 10{,}000 = 0$

Let $u = \sqrt{x}$.

$3.29u^2 - 5.5u - 10{,}000 = 0$

$$u = \frac{5.5 \pm \sqrt{(-5.5)^2 - 4(3.29)(-10{,}000)}}{2(3.29)}$$

$$u = \frac{5.5 \pm \sqrt{131{,}630.25}}{6.58}$$

$u \approx 55.974, -54.302$

Choosing the positive value for u, we have

$x = u^2 \implies x = (55.974)^2 \approx 3133$ units.

63. $C = 35.45x + 16{,}000,\ R = 55.95x$

(a) $R = C$

$55.95x = 35.45x + 16{,}000$

$20.50x = 16{,}000$

$x \approx 781$ units

(b) $P = R - C$

$60{,}000 = 55.95x - (35.45x + 16{,}000)$

$60{,}000 = 20.50x - 16{,}000$

$76{,}000 = 20.50x$

$x \approx 3708$ units

64. $C = 2.16x + 5000,\ R = 3.49x$

(a) $R = C$

$2.16x + 5000 = 3.49x$

$5000 = 1.33x$

$x \approx 3760$

3760 items must be sold to break even.

(b) $P = R - C$

$8500 = 3.49x - (2.16x + 5000)$

$8500 = 1.33x - 5000$

$13{,}500 = 1.33x$

$10{,}151 \approx x$

10,151 items must be sold to make a profit of $8500.

65. $\begin{cases} R = 360 - 24x & \text{Equation 1} \\ R = 24 + 18x & \text{Equation 2} \end{cases}$

(a) Substitute for R in Equation 2: $360 - 24x = 24 + 18x$

Solve for x: $336 = 42x \implies x = 8$ weeks

(b)

Weeks	1	2	3	4	5	6	7	8	9	10
$R = 360 - 24x$	336	312	288	264	240	216	192	168	144	120
$R = 24 + 18x$	42	60	78	96	114	132	150	168	186	204

The rentals are equal when $x = 8$ weeks.

66. (a) $\begin{cases} S = 25x + 100 & \text{Rock CD} \\ S = -50x + 475 & \text{Rap CD} \end{cases}$

$25x + 100 = -50x + 475$

$75x + 100 = 475$

$75x = 375$

$x = 5$

Conclusion: It takes 5 weeks for the sales of the two CDs to become equal.

(b)

Number of weeks, x	0	1	2	3	4	5	6
Sales, S (rock)	100	125	150	175	200	225	250
Sales, S (rap)	475	425	375	325	275	225	175

By inspecting the table, we can see that the two sales figures are equal when $x = 5$.

67. $0.06x = 0.03x + 350$

$0.03x = 350$

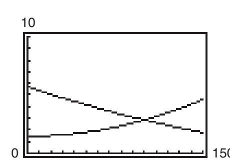
$x \approx \$11{,}666.67$

To make the straight commission offer the better offer, you would have to sell more than $11,666.67 per week.

68. $p = 1.45 + 0.00014x^2$

$p = (2.388 - 0.007x)^2$

The market equilibrium (point of intersection) is approximately (99.99, 2.85).



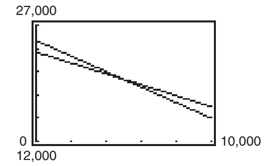
69. (a) $\begin{cases} x + y = 25{,}000 \\ 0.06x + 0.085y = 2000 \end{cases}$

(b) $y_1 = 25{,}000 - x$

$$y_2 = \frac{2000 - 0.06x}{0.085}$$

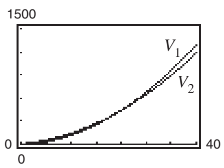
As the amount at 6% increases, the amount at 8.5% decreases. The amount of interest is fixed at $2000.

(c) The point of intersection occurs when $x = 5000$, so the most that can be invested at 6% and still earn $2000 per year in interest is $5000.



70. $\begin{cases} V = (D - 4)^2, & 5 \le D \le 40 & \text{Doyle Log Rule} \\ V = 0.79D^2 - 2D - 4, & 5 \le D \le 40 & \text{Scribner Log Rule} \end{cases}$

(a)



(b) The graphs intersect when $D \approx 24.7$ inches.

(c) For large logs, the Doyle Log Rule gives a greater volume for a given diameter.

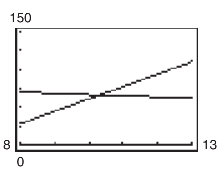
71.

t	8	9	10	11	12	13
Solar	70	69	66	65	64	63
Wind	31	46	57	68	105	108

(a) Solar: $C \approx 0.1429t^2 - 4.46t + 96.8$

Wind: $C \approx 16.371t - 102.7$

(b)



(c) Point of intersection: $(10.3, 66.01)$

During the year 2000, the consumption of solar energy will equal the consumption of wind energy.

(d) $0.1429t^2 - 4.46t + 96.8 = 16.371t - 102.7$

$0.1429t^2 - 20.831t + 199.5 = 0$

By the Quadratic Formula we obtain $t \approx 10.3$ and $t \approx 135.47$.

(e) The results are the same for $t \approx 10.3$. The other "solution", $t \approx 135.47$, is too large to consider as a reasonable answer.

(f) Answers will vary.

72. (a) For Alabama, $P = 17.4t + 4273.2$.

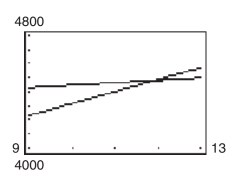
For Colorado, $P = 84.9t + 3467.9$.

(b) The lines appear to intersect at (11.93, 4480.79).

Colorado's population exceeded Alabama's just after this point.

(c) Using the equations from part (a),

$$17.4t + 4273.2 = 84.9t + 3467.9$$
$$4273.2 = 67.5t + 3467.9$$
$$805.3 = 67.5t$$
$$11.93 = t.$$



73. $2l + 2w = 30 \implies l + w = 15$

$l = w + 3 \implies (w + 3) + w = 15$

$2w = 12$

$w = 6$

$l = w + 3 = 9$

Dimensions: 6×9 meters

74. $2l + 2w = 280 \implies l + w = 140$

$w = l - 20 \implies l + (l - 20) = 140$

$2l = 160$

$l = 80$

$w = l - 20 = 80 - 20 = 60$

Dimensions: 60×80 centimeters

75. $2l + 2w = 42 \implies l + w = 21$

$w = \frac{3}{4}l \implies l + \frac{3}{4}l = 21$

$\frac{7}{4}l = 21$

$l = 12$

$w = \frac{3}{4}l = 9$

Dimensions: 9×12 inches

76. $2l + 2w = 210 \implies l + w = 105$

$l = \frac{3}{2}w \implies \frac{3}{2}w + w = 105$

$\frac{5}{2}w = 105$

$w = 42$

$l = \frac{3}{2}(42) = 63$

Dimensions: 42×63 feet

77. $2l + 2w = 40 \implies l + w = 20 \implies w = 20 - l$

$lw = 96 \implies l(20 - l) = 96$

$20l - l^2 = 96$

$0 = l^2 - 20l + 96$

$0 = (l - 8)(l - 12)$

$l = 8$ or $l = 12$

If $l = 8$, then $w = 12$.

If $l = 12$, then $w = 8$.

Since the length is supposed to be greater than the width, we have $l = 12$ kilometers and $w = 8$ kilometers.
Dimensions: 8×12 kilometers

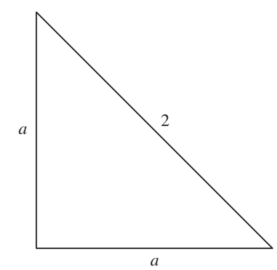
78. $A = \frac{1}{2}bh$

$1 = \frac{1}{2}a^2$

$a^2 = 2$

$a = \sqrt{2}$

The dimensions are $\sqrt{2} \times \sqrt{2} \times 2$ inches.



79. False. To solve a system of equations by substitution, you can solve for either variable in one of the two equations and then back-substitute.

80. False. The system can have at most four solutions because a parabola and a circle can intersect at most four times.

81. To solve a system of equations by substitution, use the following steps.

1. Solve one of the equations for one variable in terms of the other.
2. Substitute this expression into the other equation to obtain an equation in one variable.
3. Solve this equation.
4. Back-substitute the value(s) found in Step 3 into the expression found in Step 1 to find the value(s) of the other variable.
5. Check your solution(s) in each of the original equations.

82. For a linear system the result will be a contradictory equation such as $0 = N$, where N is a nonzero real number. For a nonlinear system there may be an equation with imaginary solutions.

83. $y = x^2$

(a) Line with two points of intersection

$y = 2x$

$(0, 0)$ and $(2, 4)$

(b) Line with one point of intersection

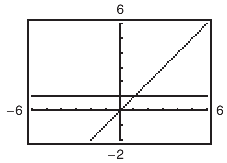
$y = 0$

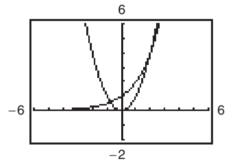
$(0, 0)$

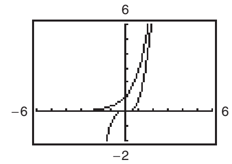
(c) Line with no points of intersection

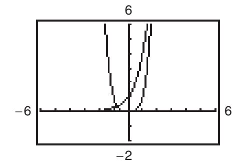
$y = x - 2$

84. (a) $b = 1$ $b = 2$ $b = 3$ $b = 4$









(b) Three

85. $(-2, 7), (5, 5)$

$$m = \frac{5 - 7}{5 - (-2)} = -\frac{2}{7}$$

$$y - 7 = -\frac{2}{7}(x - (-2))$$

$$7y - 49 = -2x - 4$$

$$2x + 7y - 45 = 0$$

86. $(3.5, 4), (10, 6)$

$$m = \frac{6 - 4}{10 - 3.5} = \frac{2}{6.5}$$

$$y - 6 = \frac{2}{6.5}(x - 10)$$

$$6.5y - 39 = 2x - 20$$

$$2x - 6.5y + 19 = 0$$

87. $(6, 3), (10, 3)$

$$m = \frac{3 - 3}{10 - 6} = 0 \implies \text{The line is horizontal.}$$

$$y = 3$$

$$y - 3 = 0$$

88. $(4, -2), (4, 5)$

$$x = 4$$

$$x - 4 = 0$$

89. $\left(\frac{3}{5}, 0\right), (4, 6)$

$$m = \frac{6 - 0}{4 - (3/5)} = \frac{6}{17/5} = \frac{30}{17}$$

$$y - 6 = \frac{30}{17}(x - 4)$$

$$17y - 102 = 30x - 120$$

$$0 = 30x - 17y - 18$$

$$30x - 17y - 18 = 0$$

90. $\left(-\frac{7}{3}, 8\right), \left(\frac{5}{2}, \frac{1}{2}\right)$

$$m = \frac{8 - (1/2)}{-(7/3) - (5/2)} = \frac{15/2}{-29/6} = -\frac{45}{29}$$

$$y - \frac{1}{2} = -\frac{45}{29}\left(x - \frac{5}{2}\right)$$

$$29y - \frac{29}{2} = -45x + \frac{225}{2}$$

$$45x + 29y - 127 = 0$$

91. $f(x) = \dfrac{5}{x - 6}$

Domain: All real numbers except $x = 6$

Horizontal asymptote: $y = 0$

Vertical asymptote: $x = 6$

92. $f(x) = \dfrac{2x - 7}{3x + 2}$

Domain: All real numbers except $x = -\dfrac{2}{3}$

Horizontal asymptote: $y = \dfrac{2}{3}$

Vertical asymptote: $x = -\dfrac{2}{3}$

93. $f(x) = \dfrac{x^2 + 2}{x^2 - 16}$

Domain: All real numbers except $x = \pm 4$.

Horizontal asymptote: $y = 1$

Vertical asymptotes: $x = \pm 4$

94. $f(x) = 3 - \dfrac{2}{x^2}$

Domain: All real numbers except $x = 0$

Horizontal asymptote: $y = 3$

Vertical asymptote: $x = 0$

Section 7.2 Two-Variable Linear Systems

- You should be able to solve a linear system by the method of elimination.
  1. Obtain coefficients for either x or y that differ only in sign. This is done by multiplying all the terms of one or both equations by appropriate constants.
  2. Add the equations to eliminate one of the variables and then solve for the remaining variable.
  3. Use back-substitution into either original equation and solve for the other variable.
  4. Check your answer.
- You should know that for a system of two linear equations, one of the following is true.
  1. There are infinitely many solutions; the lines are identical. The system is consistent. The slopes are equal.
  2. There is no solution; the lines are parallel. The system is inconsistent. The slopes are equal.
  3. There is one solution; the lines intersect at one point. The system is consistent. The slopes are not equal.

Vocabulary Check

1. elimination

2. equivalent

3. consistent; inconsistent

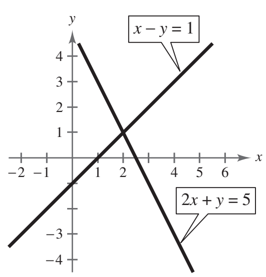
4. equilibrium price

1. $\begin{cases} 2x + y = 5 & \text{Equation 1} \\ x - y = 1 & \text{Equation 2} \end{cases}$

Add to eliminate y: $3x = 6 \implies x = 2$

Substitute $x = 2$ in Equation 2: $2 - y = 1 \implies y = 1$

Solution: $(2, 1)$



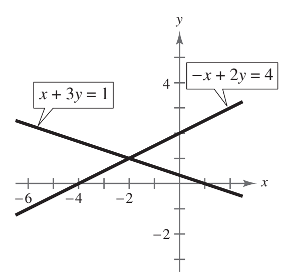
2. $\begin{cases} x + 3y = 1 & \text{Equation 1} \\ -x + 2y = 4 & \text{Equation 2} \end{cases}$

Add to eliminate x:

$$\begin{array}{r} x + 3y = 1 \\ -x + 2y = 4 \\ \hline 5y = 5 \implies y = 1 \end{array}$$

Substitute $y = 1$ in Equation 1: $x + 3(1) = 1 \implies x = -2$

Solution: $(-2, 1)$



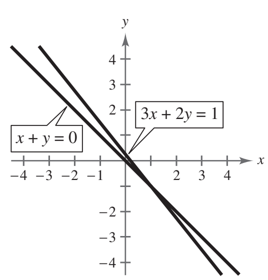
3. $\begin{cases} x + y = 0 & \text{Equation 1} \\ 3x + 2y = 1 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by -2: $-2x - 2y = 0$

Add this to Equation 2 to eliminate y: $x = 1$

Substitute $x = 1$ in Equation 1: $1 + y = 0 \implies y = -1$

Solution: $(1, -1)$



4. $\begin{cases} 2x - y = 3 & \text{Equation 1} \\ 4x + 3y = 21 & \text{Equation 2} \end{cases}$

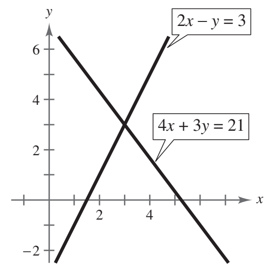
Multiply Equation 1 by 3: $6x - 3y = 9$

Add this to Equation 2 to eliminate y:

$$\begin{array}{rcl} 6x - 3y &=& 9 \\ 4x + 3y &=& 21 \\ \hline 10x &=& 30 \\ \Rightarrow x &=& 3 \end{array}$$

Substitute $x = 3$ in Equation 1: $2(3) - y = 3 \Rightarrow y = 3$

Solution: $(3, 3)$

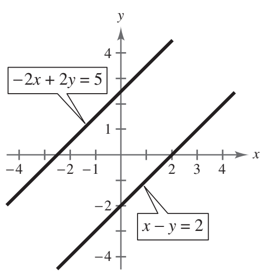


5. $\begin{cases} x - y = 2 & \text{Equation 1} \\ -2x + 2y = 5 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by 2: $2x - 2y = 4$

Add this to Equation 2: $0 = 9$

There are no solutions.



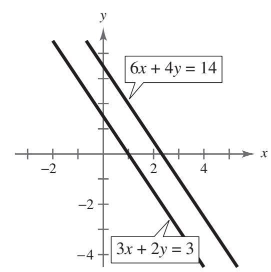
6. $\begin{cases} 3x + 2y = 3 & \text{Equation 1} \\ 6x + 4y = 14 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by -2: $-6x - 4y = -6$

Add this to Equation 2:

$$\begin{array}{rcl} -6x - 4y &=& -6 \\ 6x + 4y &=& 14 \\ \hline 0 &=& 8 \end{array}$$

There are no solutions.



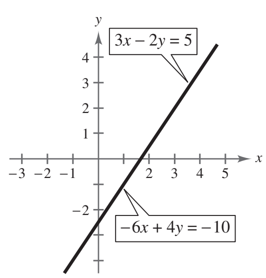
7. $\begin{cases} 3x - 2y = 5 & \text{Equation 1} \\ -6x + 4y = -10 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by 2 and add to Equation 2: $0 = 0$

The equations are dependent. There are infinitely many solutions.

Let $x = a$, then $y = \dfrac{3a - 5}{2} = \dfrac{3}{2}a - \dfrac{5}{2}$.

Solution: $\left(a, \dfrac{3}{2}a - \dfrac{5}{2}\right)$ where a is any real number.



8. $\begin{cases} 9x - 3y = -15 & \text{Equation 1} \\ -3x + y = 5 & \text{Equation 2} \end{cases}$

Multiply Equation 2 by 3: $-9x + 3y = 15$

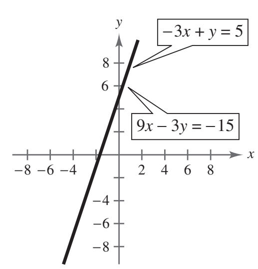
Add this to Equation 1:

$$\begin{array}{rcl} 9x - 3y &=& -15 \\ -9x + 3y &=& 15 \\ \hline 0 &=& 0 \end{array}$$

There are infinitely many solutions. Let $x = a$.

$-3a + y = 5 \Rightarrow y = 3a + 5$

Solution: $(a, 3a + 5)$, where a is any real number.



9. $\begin{cases} 9x + 3y = 1 & \text{Equation 1} \\ 3x - 6y = 5 & \text{Equation 2} \end{cases}$

Multiply Equation 2 by (-3): $9x + 3y = 1$

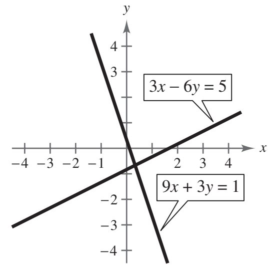
$-9x + 18y = -15$

Add to eliminate x: $21y = -14 \implies y = -\frac{2}{3}$

Substitute $y = -\frac{2}{3}$ in Equation 1: $9x + 3\left(-\frac{2}{3}\right) = 1$

$x = \frac{1}{3}$

Solution: $\left(\frac{1}{3}, -\frac{2}{3}\right)$



10. $\begin{cases} 5x + 3y = -18 & \text{Equation 1} \\ 2x - 6y = 1 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by 2: $10x + 6y = -36$

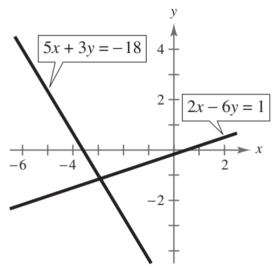
Add this to Equation 2 to eliminate y:

$$\begin{array}{r} 10x + 6y = -36 \\ 2x - 6y = 1 \\ \hline 12x \quad = -35 \end{array} \implies x = -\frac{35}{12}$$

Substitute $x = -\frac{35}{12}$ in Equation 2:

$2\left(-\frac{35}{12}\right) - 6y = 1 \implies y = -\frac{41}{36}$

Solution: $\left(-\frac{35}{12}, -\frac{41}{36}\right)$



11. $\begin{cases} x + 2y = 4 & \text{Equation 1} \\ x - 2y = 1 & \text{Equation 2} \end{cases}$

Add to eliminate y:

$2x = 5$

$x = \frac{5}{2}$

Substitute $x = \frac{5}{2}$ in Equation 1:

$\frac{5}{2} + 2y = 4 \implies y = \frac{3}{4}$

Solution: $\left(\frac{5}{2}, \frac{3}{4}\right)$

12. $\begin{cases} 3x - 5y = 2 & \text{Equation 1} \\ 2x + 5y = 13 & \text{Equation 2} \end{cases}$

Add to eliminate y:
$$\begin{array}{r} 3x - 5y = 2 \\ 2x + 5y = 13 \\ \hline 5x \quad = 15 \end{array} \implies x = 3$$

Substitute $x = 3$ in Equation 1: $3(3) - 5y = 2 \implies y = \frac{7}{5}$

Solution: $\left(3, \frac{7}{5}\right)$

13. $\begin{cases} 2x + 3y = 18 & \text{Equation 1} \\ 5x - y = 11 & \text{Equation 2} \end{cases}$

Multiply Equation 2 by 3: $15x - 3y = 33$

Add this to Equation 1 to eliminate y:

$17x = 51 \implies x = 3$

Substitute $x = 3$ in Equation 1:

$6 + 3y = 18 \implies y = 4$

Solution: (3, 4)

14. $\begin{cases} x + 7y = 12 & \text{Equation 1} \\ 3x - 5y = 10 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by -3: $-3x - 21y = -36$

Add this to Equation 2 to eliminate x:

$$\begin{array}{r} -3x - 21y = -36 \\ 3x - 5y = 10 \\ \hline -26y = -26 \\ \implies y = 1 \end{array}$$

Substitute $y = 1$ in Equation 1: $x + 7 = 12 \implies x = 5$

Solution: (5, 1)

15. $\begin{cases} 3x + 2y = 10 & \text{Equation 1} \\ 2x + 5y = 3 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by 2 and Equation 2 by (-3):

$$\begin{cases} 6x + 4y = 20 \\ -6x - 15y = -9 \end{cases}$$

Add to eliminate x: $-11y = 11 \implies y = -1$

Substitute $y = -1$ in Equation 1:

$3x - 2 = 10 \implies x = 4$

Solution: $(4, -1)$

16. $\begin{cases} 2r + 4s = 5 & \text{Equation 1} \\ 16r + 50s = 55 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by (-8): $-16r - 32s = -40$

Add this to Equation 2 to eliminate r:

$$\begin{aligned} -16r - 32s &= -40 \\ 16r + 50s &= 55 \\ \hline 18s &= 15 \\ \implies s &= \tfrac{5}{6} \end{aligned}$$

Substitute $s = \frac{5}{6}$ in Equation 1:

$2r + 4\left(\frac{5}{6}\right) = 5 \implies r = \frac{5}{6}$

Solution: $\left(\frac{5}{6}, \frac{5}{6}\right)$

17. $\begin{cases} 5u + 6v = 24 & \text{Equation 1} \\ 3u + 5v = 18 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by 5 and Equation 2 by -6:

$$\begin{cases} 25u + 30v = 120 \\ -18u - 30v = -108 \end{cases}$$

Add to eliminate v: $7u = 12 \implies u = \frac{12}{7}$

Substitute $u = \frac{12}{7}$ in Equation 1:

$5\left(\frac{12}{7}\right) + 6v = 24 \implies 6v = \frac{108}{7} \implies v = \frac{18}{7}$

Solution: $\left(\frac{12}{7}, \frac{18}{7}\right)$

18. $\begin{cases} 3x + 11y = 4 & \text{Equation 1} \\ -2x - 5y = 9 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by 2 and Equation 2 by 3:

$$\begin{cases} 6x + 22y = 8 \\ -6x - 15y = 27 \end{cases}$$

Add to eliminate x:

$$\begin{aligned} 6x + 22y &= 8 \\ -6x - 15y &= 27 \\ \hline 7y &= 35 \implies y = 5 \end{aligned}$$

Substitute $y = 5$ in Equation 1: $3x + 11(5) = 4$

$\implies x = -17$

Solution: $(-17, 5)$

19. $\begin{cases} \frac{9}{5}x + \frac{6}{5}y = 4 & \text{Equation 1} \\ 9x + 6y = 3 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by 10 and Equation 2 by -2:

$$\begin{cases} 18x + 12y = 40 \\ -18x - 12y = -6 \end{cases}$$

Add to eliminate x and y: $0 = 34$

Inconsistent

No solution

20. $\begin{cases} \frac{3}{4}x + y = \frac{1}{8} & \text{Equation 1} \\ \frac{9}{4}x + 3y = \frac{3}{8} & \text{Equation 2} \end{cases}$

Multiply Equation 1 by -3:

$$\begin{cases} -\frac{9}{4}x - 3y = -\frac{3}{8} \\ \frac{9}{4}x + 3y = \frac{3}{8} \end{cases}$$

Add these two together to obtain $0 = 0$.

The original equations are dependent. They have infinitely many solutions.

Set $x = a$ in $\frac{3}{4}x + y = \frac{1}{8}$ and solve for y.

The points on the line have the form $\left(a, \frac{1}{8} - \frac{3}{4}a\right)$.

21. $\begin{cases} \dfrac{x}{4} + \dfrac{y}{6} = 1 & \text{Equation 1} \\ x - y = 3 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by 6: $\dfrac{3}{2}x + y = 6$

Add this to Equation 2 to eliminate y:

$\dfrac{5}{2}x = 9 \implies x = \dfrac{18}{5}$

Substitute $x = \dfrac{18}{5}$ in Equation 2:

$\dfrac{18}{5} - y = 3$

$y = \dfrac{3}{5}$

Solution: $\left(\dfrac{18}{5}, \dfrac{3}{5}\right)$

22. $\begin{cases} \frac{2}{3}x + \frac{1}{6}y = \frac{2}{3} & \text{Equation 1} \\ 4x + y = 4 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by (-6): $-4x - y = -4$

Add this to Equation 2: $\begin{aligned} -4x - y &= -4 \\ 4x + y &= 4 \\ \hline 0 &= 0 \end{aligned}$

There are infinitely many solutions. Let $x = a$.

$4a + y = 4 \implies y = 4 - 4a$

Solution: $(a, 4 - 4a)$ where a is any real number

23. $\begin{cases} -5x + 6y = -3 & \text{Equation 1} \\ 20x - 24y = 12 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by 4:

$\begin{cases} -20x + 24y = -12 \\ 20x - 24y = 12 \end{cases}$

Add to eliminate x and y: $0 = 0$

The equations are dependent. There are infinitely many solutions.

Let $x = a$, then

$-5a + 6y = -3 \implies y = \dfrac{5a - 3}{6} = \dfrac{5}{6}a - \dfrac{1}{2}.$

Solution: $\left(a, \dfrac{5}{6}a - \dfrac{1}{2}\right)$ where a is any real number

24. $\begin{cases} 7x + 8y = 6 & \text{Equation 1} \\ -14x - 16y = -12 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by 2:

$\begin{cases} 14x + 16y = 12 \\ -14x - 16y = -12 \end{cases}$

Add these two together to obtain $0 = 0$.

The original equations are dependent. They have infinitely many solutions.

Set $x = a$ in $7x + 8y = 6$ and solve for y.

The points on the line have the form $\left(a, \frac{3}{4} - \frac{7}{8}a\right)$.

25. $\begin{cases} 0.05x - 0.03y = 0.21 & \text{Equation 1} \\ 0.07x + 0.02y = 0.16 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by 200 and Equation 2 by 300:

$\begin{cases} 10x - 6y = 42 \\ 21x + 6y = 48 \end{cases}$

Add to eliminate y: $31x = 90$

$x = \frac{90}{31}$

Substitute $x = \frac{90}{31}$ in Equation 2:

$0.07\left(\frac{90}{31}\right) + 0.02y = 0.16$

$y = -\frac{67}{31}$

Solution: $\left(\frac{90}{31}, -\frac{67}{31}\right)$

26. $\begin{cases} 0.2x - 0.5y = -27.8 & \text{Equation 1} \\ 0.3x + 0.4y = 68.7 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by 4 and Equation 2 by 5:

$\begin{cases} 0.8x - 2y = -111.2 \\ 1.5x + 2y = 343.5 \end{cases}$

Add these to eliminate y: $\begin{aligned} 0.8x - 2y &= -111.2 \\ 1.5x + 2y &= 343.5 \\ \hline 2.3x &= 232.3 \\ \implies x &= 101 \end{aligned}$

Substitute $x = 101$ in Equation 1:

$0.2(101) - 0.5y = -27.8 \implies y = 96$

Solution: $(101, 96)$

27. $\begin{cases} 4b + 3m = 3 & \text{Equation 1} \\ 3b + 11m = 13 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by 3 and Equation 2 by (-4):

$\begin{cases} 12b + 9m = 9 \\ -12b - 44m = -52 \end{cases}$

Add to eliminate b: $-35m = -43$

$m = \frac{43}{35}$

Substitute $m = \frac{43}{35}$ in Equation 1:

$4b + 3\left(\frac{43}{35}\right) = 3 \implies b = -\frac{6}{35}$

Solution: $\left(-\frac{6}{35}, \frac{43}{35}\right)$

28. $\begin{cases} 2x + 5y = 8 & \text{Equation 1} \\ 5x + 8y = 10 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by 5 and Equation 2 by (-2):

$$\begin{cases} 10x + 25y = 40 \\ -10x - 16y = -20 \end{cases}$$

Add to eliminate x:

$$\begin{array}{r} 10x + 25y = 40 \\ -10x - 16y = -20 \\ \hline 9y = 20 \implies y = \frac{20}{9} \end{array}$$

Substitute $y = \frac{20}{9}$ in Equation 1: $2x + 5\left(\frac{20}{9}\right) = 8$

$$\implies x = -\frac{14}{9}$$

Solution: $\left(-\frac{14}{9}, \frac{20}{9}\right)$

29. $\begin{cases} \dfrac{x+3}{4} + \dfrac{y-1}{3} = 1 & \text{Equation 1} \\ 2x - y = 12 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by 12 and Equation 2 by 4:

$$\begin{cases} 3x + 4y = 7 \\ 8x - 4y = 48 \end{cases}$$

Add to eliminate y: $11x = 55 \implies x = 5$

Substitute $x = 5$ into Equation 2:

$2(5) - y = 12 \implies y = -2$

Solution: $(5, -2)$

30. $\begin{cases} \dfrac{x-1}{2} + \dfrac{y+2}{3} = 4 & \text{Equation 1} \\ x - 2y = 5 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by 6:

$3(x - 1) + 2(y + 2) = 24 \implies 3x + 2y = 23$

Add this to Equation 2 to eliminate y:

$$\begin{array}{l} 3x + 2y = 23 \\ x - 2y = 5 \\ \hline 4x = 28 \\ \implies x = 7 \end{array}$$

Substitute $x = 7$ in Equation 2: $7 - 2y = 5 \implies y = 1$

Solution: $(7, 1)$

31. $\begin{cases} 2x - 5y = 0 \\ x - y = 3 \end{cases}$

Multiply Equation 2 by -5:

$$\begin{cases} 2x - 5y = 0 \\ -5x + 5y = -15 \end{cases}$$

Add to eliminate y: $-3x = -15 \implies x = 5$

Matches graph (b).

Number of solutions: One

Consistent

32. $\begin{cases} -7x + 6y = -4 \\ 14x - 12y = 8 \end{cases}$

$-7x + 6y = -4 \implies 6y = 7x - 4 \implies y = \frac{7}{6}x - \frac{2}{3}$;

The graph contains $\left(0, -\frac{2}{3}\right)$ and $(4, 4)$.

$14x - 12y = 8 \implies -12y = -14x + 8 \implies y = \frac{7}{6}x - \frac{2}{3}$;

The graph is the same as the previous graph.

The graph of the system matches (a).

Number of solutions: Infinite

Consistent

33. $\begin{cases} 2x - 5y = 0 \\ 2x - 3y = -4 \end{cases}$

Multiply Equation 1 by -1:

$$\begin{cases} -2x + 5y = 0 \\ 2x - 3y = -4 \end{cases}$$

Add to eliminate x: $2y = -4 \implies y = -2$

Matches graph (c).

Number of solutions: One

Consistent

34. $\begin{cases} 7x - 6y = -6 \\ -7x + 6y = -4 \end{cases}$

$7x - 6y = -6 \implies -6y = -7x - 6 \implies y = \frac{7}{6}x + 1;$

The graph contains (0, 1) and $\left(3, \frac{9}{2}\right)$.

$-7x + 6y = -4 \implies 6y = 7x - 4 \implies y = \frac{7}{6}x - \frac{2}{3};$

The graph contains $\left(0, -\frac{2}{3}\right)$ and is parallel to the previous graph.

The graph of the system matches (d).

Number of solutions: None

Inconsistent

35. $\begin{cases} 3x - 5y = 7 & \text{Equation 1} \\ 2x + y = 9 & \text{Equation 2} \end{cases}$

Multiply Equation 2 by 5:

$10x + 5y = 45$

Add this to Equation 1:

$13x = 52 \implies x = 4$

Back-substitute $x = 4$ into Equation 2:

$2(4) + y = 9 \implies y = 1$

Solution: (4, 1)

36. $\begin{cases} -x + 3y = 17 & \text{Equation 1} \\ 4x + 3y = 7 & \text{Equation 2} \end{cases}$

Subtract Equation 2 from Equation 1 to eliminate y:

$$\begin{aligned} -x + 3y &= 17 \\ -4x - 3y &= -7 \\ \hline -5x \quad &= 10 \implies x = -2 \end{aligned}$$

Substitute $x = -2$ in Equation 1:
$-(-2) + 3y = 17 \implies y = 5$

Solution: $(-2, 5)$

37. $\begin{cases} y = 2x - 5 & \text{Equation 1} \\ y = 5x - 11 & \text{Equation 2} \end{cases}$

Since both equations are solved for y, set them equal to one another and solve for x.

$2x - 5 = 5x - 11$

$6 = 3x$

$2 = x$

Back-substitute $x = 2$ into Equation 1:

$y = 2(2) - 5 = -1$

Solution: $(2, -1)$

38. $\begin{cases} 7x + 3y = 16 & \text{Equation 1} \\ y = x + 2 & \text{Equation 2} \end{cases}$

Substitute for y in Equation 1:

$7x + 3(x + 2) = 16$

$7x + 3x + 6 = 16$

$10x = 10 \implies x = 1$

Substitute $x = 1$ in Equation 2: $y = 1 + 2 = 3$

Solution: (1, 3)

39. $\begin{cases} x - 5y = 21 & \text{Equation 1} \\ 6x + 5y = 21 & \text{Equation 2} \end{cases}$

Add the equations: $7x = 42 \implies x = 6$

Back-substitute $x = 6$ into Equation 1:

$6 - 5y = 21 \implies -5y = 15 \implies y = -3$

Solution: $(6, -3)$

40. $\begin{cases} y = -3x - 8 & \text{Equation 1} \\ y = 15 - 2x & \text{Equation 2} \end{cases}$

Since both equations are solved for y, set them equal to one another and solve for x:

$-3x - 8 = 15 - 2x$

$-x = 23$

$x = -23$

Back-substitute $x = -23$ into Equation 1:

$y = -3(-23) - 8 = 61$

Solution: $(-23, 61)$

41. $\begin{cases} -2x + 8y = 19 & \text{Equation 1} \\ y = x - 3 & \text{Equation 2} \end{cases}$

Substitute the expression for y from Equation 2 into Equation 1.

$-2x + 8(x - 3) = 19 \implies -2x + 8x - 24 = 19$

$6x = 43$

$x = \frac{43}{6}$

Back-substitute $x = \frac{43}{6}$ into Equation 2:

$y = \frac{43}{6} - 3 \implies y = \frac{25}{6}$

Solution: $\left(\frac{43}{6}, \frac{25}{6}\right)$

42. $\begin{cases} 4x - 3y = 6 & \text{Equation 1} \\ -5x + 7y = -1 & \text{Equation 2} \end{cases}$

Multiply Equation 1 by 5 and Equation 2 by 4:

$$\begin{cases} 20x - 15y = 30 \\ -20x + 28y = -4 \end{cases}$$

Add to eliminate x:

$$\begin{array}{r} 20x - 15y = 30 \\ -20x + 28y = -4 \\ \hline 13y = 26 \end{array} \implies y = 2$$

Back-substitute $y = 2$ into Equation 1:

$$4x - 3(2) = 6 \implies x = 3$$

Solution: $(3, 2)$

43. Let $r_1 =$ the air speed of the plane
and $r_2 =$ the wind air speed.

$$\begin{array}{lll} 3.6(r_1 - r_2) = 1800 & \text{Equation 1} \implies & r_1 - r_2 = 500 \\ 3(r_1 + r_2) = 1800 & \text{Equation 2} \implies & r_1 + r_2 = 600 \\ & & 2r_1 = 1100 \quad \text{Add the equations.} \\ & & r_1 = 550 \\ & & 550 + r_2 = 600 \\ & & r_2 = 50 \end{array}$$

The air speed of the plane is 550 mph and the speed of the wind is 50 mph.

44. Let $x =$ the speed of the plane that leaves first and $y =$ the speed of the plane that leaves second.

$\begin{cases} y - x = 80 & \text{Equation 1} \\ 2x + \frac{3}{2}y = 3200 & \text{Equation 2} \end{cases}$

$$\begin{array}{r} -2x + 2y = 160 \\ 2x + \frac{3}{2}y = 3200 \\ \hline \frac{7}{2}y = 3360 \\ y = 960 \end{array}$$

$960 - x = 80$

$x = 880$

Solution: First plane: 880 kilometers per hour; Second plane: 960 kilometers per hour

45. $50 - 0.5x = 0.125x$

$50 = 0.625x$

$x = 80$ units

$p = \$10$

Solution: $(80, 10)$

46. Supply = Demand

$25 + 0.1x = 100 - 0.05x$

$0.15x = 75$

$x = 500$

$p = 75$

Equilibrium point: $(500, 75)$

47. $140 - 0.00002x = 80 + 0.00001x$

$60 = 0.00003x$

$x = 2{,}000{,}000$ units

$p = \$100.00$

Solution: (2,000,000, 100)

48. Supply = Demand

$225 + 0.0005x = 400 - 0.0002x$

$0.0007x = 175$

$x = 250{,}000$

$p = 350$

Equilibrium point: (250,000, 350)

49. Let x = number of calories in a cheeseburger

y = number of calories in a small order of french fries

$$\begin{cases} 2x + y = 850 & \text{Equation 1} \\ 3x + 2y = 1390 & \text{Equation 2} \end{cases}$$

Multiply Equation 1 by -2:

$$\begin{cases} -4x - 2y = -1700 \\ 3x + 2y = 1390 \end{cases}$$

$-x = -310$ Add the equations.

$x = 310$

$y = 230$

Solution: The cheeseburger contains 310 calories and the fries contain 230 calories.

50. Let x = Vitamin C in a glass of apple juice

y = Vitamin C in a glass of orange juice.

$$\begin{cases} x + y = 185 & \text{Equation 1} \\ 2x + 3y = 452 & \text{Equation 2} \end{cases}$$

Multiply Equation 1 by -2; then add the equations:

$$\begin{cases} -2x - 2y = -370 \\ 2x + 3y = 452 \end{cases}$$

$y = 82$

Then $x = 185 - 82 = 103$.

The point (103, 82) is the solution of the system.

Apple juice has 103mg of Vitamin C, while orange juice has 82 mg.

51. Let x = the number of liters at 20%

Let y = the number of liters at 50%.

(a) $$\begin{cases} x + y = 10 \\ 0.2x + 0.5y = 0.3(10) \end{cases}$$

$-2 \cdot$ Equation 1: $-2x - 2y = -20$

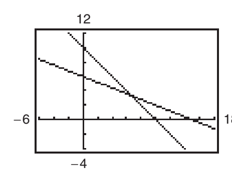
$10 \cdot$ Equation 2: $2x + 5y = 30$

$3y = 10$

$y = \frac{10}{3}$

$x + \frac{10}{3} = 10$

$x = \frac{20}{3}$

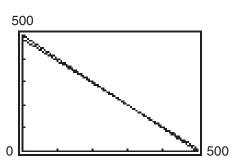
(b) 

As x increases, y decreases.

(c) In order to obtain the specified concentration of the final mixture, $6\frac{2}{3}$ liters of the 20% solution and $3\frac{1}{3}$ liters of the 50% solution are required.

52. Let x = the number of gallons of 87 octane gasoline; y = the number of gallons of 92 octane gasoline.

(a) $$\begin{cases} x + y = 500 & \text{Equation 1} \\ 87x + 92y = 44{,}500 & \text{Equation 2} \end{cases}$$

(b) 

As the amount of 87 octane gasoline increases, the amount of 92 octane gasoline decreases.

(c) (-87)Equation 1: $-87x - 87y = -43{,}500$

Equation 2: $87x + 92y = 44{,}500$

$5y = 1000$

$y = 200$

$x + 200 = 500$

$x = 300$

Solution: 87 octane: 300 gallons; 92 octane: 200 gallons

53. Let $x =$ amount invested at 7.5%

$y =$ amount invested at 9%.

$$\begin{cases} x + y = 12{,}000 & \text{Equation 1} \\ 0.075x + 0.09y = 990 & \text{Equation 2} \end{cases}$$

Multiply Equation 1 by 9 and Equation 2 by -100.

$$\begin{cases} 9x + 9y = 108{,}000 \\ -7.5x - 9y = -99{,}000 \end{cases}$$

$$1.5x = 9{,}000 \qquad \text{Add the equations.}$$

$$x = \$6000$$

$$y = \$6000$$

The most that can be invested at 7.5% is \$6000.

54. Let $x =$ the amount invested at 5.75%; $y =$ the amount invested at 6.25%.

$$\begin{cases} x + y = 32{,}000 & \text{Equation 1} \Longrightarrow (-5.75)\text{Equation 1:} & -5.75x - 5.75y = -184{,}000 \\ 0.0575x + 0.0625y = 1900 & \text{Equation 2} \Longrightarrow (100)\text{Equation 2:} & 5.75x + 6.25y = 190{,}000 \end{cases}$$

$$0.5y = 6000$$

$$y = 12{,}000$$

$$x + 12{,}000 = 32{,}000$$

$$x = 20{,}000$$

The amount that should be invested in the bond that pays 5.75% interest is \$20,000.

55. Let $x =$ number of student tickets

$y =$ number of adult tickets.

$$\begin{cases} x + y = 1435 & \text{Equation 1} \\ 1.50x + 5.00y = 3552.50 & \text{Equation 2} \end{cases}$$

Multiply Equation 1 by -1.50.

$$\begin{cases} -1.50x - 1.50y = -2152.50 \\ 1.50x + 5.00y = 3552.50 \end{cases}$$

$$3.50y = 1400.00 \qquad \text{Add the equations.}$$

$$y = 400$$

$$x = 1035$$

Solution: 1035 student tickets and 400 adult tickets were sold.

56. Let $x =$ the number of jackets sold before noon; $y =$ the number of jackets sold after noon.

$$\begin{cases} x + y = 214 & \text{Equation 1} \Longrightarrow (-31.95)\text{Equation 1:} & -31.95x - 31.95y = -6837.30 \\ 31.95x + 18.95y = 5108.30 & \text{Equation 2} \Longrightarrow \text{Equation 2:} & 31.95x + 18.95y = 5108.30 \end{cases}$$

$$-13y = -1729$$

$$y = 133$$

$$x + 133 = 214$$

$$x = 81$$

So, 81 jackets were sold before noon and 133 jackets were sold after noon.

57. $\begin{cases} 5b + 10a = 20.2 \Rightarrow -10b - 20a = -40.4 \\ 10b + 30a = 50.1 \Rightarrow 10b + 30a = 50.1 \end{cases}$

$$\begin{aligned} 10a &= 9.7 \\ a &= 0.97 \\ b &= 2.10 \end{aligned}$$

Least squares regression line: $y = 0.97x + 2.10$

58. $\begin{cases} 5b + 10a = 11.7 \Rightarrow -10b - 20a = -23.4 \\ 10b + 30a = 25.6 \Rightarrow 10b + 30a = 25.6 \end{cases}$

$$\begin{aligned} 10a &= 2.2 \\ a &= 0.22 \\ 5b + 10(0.22) &= 11.7 \\ b &= 1.9 \end{aligned}$$

Least squares regression line: $y = 0.22x + 1.9$

59. $\begin{cases} 7b + 21a = 35.1 \Rightarrow -21b - 63a = -105.3 \\ 21b + 91a = 114.2 \Rightarrow 21b + 91a = 114.2 \end{cases}$

$$\begin{aligned} 28a &= 8.9 \\ a &= \tfrac{89}{280} \\ b &= \tfrac{1137}{280} \end{aligned}$$

Least squares regression line: $y = \frac{1}{280}(89x + 1137)$

$y \approx 0.32x + 4.1$

60. $\begin{cases} 6b + 15a = 23.6 \Rightarrow -15b - 37.5a = -59 \\ 15b + 55a = 48.8 \Rightarrow 15b + 55a = 48.8 \end{cases}$

$$\begin{aligned} 17.5a &= -10.2 \\ a &\approx -0.583 \\ b &\approx 5.390 \end{aligned}$$

Least squares regression line: $y = -0.583x + 5.390$

61. (0, 4), (1, 3), (1, 1), (2, 0)

$n = 4, \sum_{i=1}^{4} x_i = 4, \sum_{i=1}^{4} y_i = 8, \sum_{i=1}^{4} x_i^2 = 6, \sum_{i=1}^{4} x_i y_i = 4$

$\begin{cases} 4b + 4a = 8 \Rightarrow 4b + 4a = 8 \\ 4b + 6a = 4 \Rightarrow -4b - 6a = -4 \end{cases}$

$$\begin{aligned} -2a &= 4 \\ a &= -2 \\ b &= 4 \end{aligned}$$

Least squares regression line: $y = -2x + 4$

62. $\begin{cases} 8b + 28a = 8 \Rightarrow -224b - 784a = -224 \\ 28b + 116a = 37 \Rightarrow 224b + 928a = 296 \end{cases}$

$$\begin{aligned} 144a &= 72 \\ a &= \tfrac{1}{2} \\ 8b + 28\left(\tfrac{1}{2}\right) &= 8 \\ b &= -\tfrac{3}{4} \end{aligned}$$

Least squares regression line: $y = \frac{1}{2}x - \frac{3}{4}$

63. (5, 66.65), (6, 70.93), (7, 75.31), (8, 78.62), (9, 81.33), (10, 85.89), (11, 88.27)

(a) $n = 7, \sum_{i=1}^{7} x_i = 56, \sum_{i=1}^{7} x_i^2 = 476, \sum_{i=1}^{7} y_i = 547, \sum_{i=1}^{7} x_i y_i = 4476.8$

$\begin{cases} 7b + 56a = 547 \\ 56b + 476a = 4476.8 \end{cases}$

Multiply Equation 1 by -8.

$\begin{cases} -56b - 448a = -4376 \\ 56b + 476a = 4476.8 \end{cases}$

$$\begin{aligned} 28a &= 100.8 \\ a &= 3.6 \\ b &\approx 49.343 \end{aligned}$$

Least squares regression line: $y \approx 3.6t + 49.343$

(b) $y \approx 3.6t + 49.343$, This agrees with part (a).

(c)

t	Actual room rate	Model approximation
5	$66.65	$67.34
6	$70.93	$70.94
7	$75.31	$74.54
8	$78.62	$78.14
9	$81.33	$81.74
10	$85.89	$85.34
11	$88.27	$88.94

The model is a good fit to the data.

(d) When $t = 12$: $y \approx \$92.54$

This is a little off from the actual rate.

(e) $3.6t + 49.343 = 100$

$$\begin{aligned} 3.6t &= 50.657 \\ t &\approx 14.1 \end{aligned}$$

According to the model, room rates will average $100.00 during the year 2004.

64. (a) (1.0, 32), (1.5, 41), (2.0, 48), (2.5, 53)

$$\begin{cases} 4b + 7a = 174 \\ 7b + 13.5a = 322 \end{cases} \begin{array}{l} \Longrightarrow -7b - 12.25a = -304.5 \\ \Longrightarrow 7b + 13.5a = 322 \end{array}$$

$$1.25a = 17.5$$

$$a = 14$$

$$4b + 98 = 174$$

$$b = 19$$

Least squares regression line: $y = 14x + 19$

(b) When $x = 1.6$: $y = 14(1.6) + 19 = 41.4$ bushels per acre.

65. False. Two lines that coincide have infinitely many points of intersection.

66. False. Solving a system of equations algebraically will always give an exact solution.

67. No, it is not possible for a consistent system of linear equations to have exactly two solutions. Either the lines will intersect once or they will coincide and then the system would have infinite solutions.

68. Answers will vary.

(a) No solution

$$\begin{cases} x + y = 10 \\ x + y = 20 \end{cases}$$

(b) Infinite number of solutions

$$\begin{cases} x + y = 3 \\ 2x + 2y = 6 \end{cases}$$

69. $\begin{cases} 100y - x = 200 & \text{Equation 1} \\ 99y - x = -198 & \text{Equation 2} \end{cases}$

Subtract Equation 2 from Equation 1 to eliminate x:

$$\begin{array}{r} 100y - x = 200 \\ -99y + x = 198 \\ \hline y = 398 \end{array}$$

Substitute $y = 398$ into Equation 1:

$100(398) - x = 200 \Longrightarrow x = 39{,}600$

Solution: (39,600, 398)

The lines are not parallel. The scale on the axes must be changed to see the point of intersection.

70. $\begin{cases} 21x - 20y = 0 & \text{Equation 1} \\ 13x - 12y = 120 & \text{Equation 2} \end{cases}$

Multiply Equation 2 by $\left(-\frac{5}{3}\right)$: $-\frac{65}{3}x + 20y = -200$

Add this to Equation 1 to eliminate y:

$-\frac{2}{3}x = -200 \Longrightarrow x = 300$

Back-substitute $x = 300$ in Equation 1:

$21(300) - 20y = 0 \Longrightarrow y = 315$

Solution: (300, 315)

The lines are not parallel. It is necessary to change the scale on the axes to see the point of intersection.

71. $\begin{cases} 4x - 8y = -3 & \text{Equation 1} \\ 2x + ky = 16 & \text{Equation 2} \end{cases}$

Multiply Equation 2 by -2: $-4x - 2ky = -32$

Add this to Equation 1:

$$\begin{array}{r} 4x - 8y = -3 \\ -4x - 2ky = -32 \\ \hline -8y - 2ky = -35 \end{array}$$

The system is inconsistent if $-8y - 2ky = 0$. This occurs when $k = -4$.

72. $\begin{cases} 15x + 3y = 6 \\ -10x + ky = 9 \end{cases} \begin{array}{l} \Longrightarrow 30x + 6y = 12 \\ \Longrightarrow -30x + 3ky = 27 \end{array}$

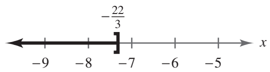
$$(6 + 3k)y = 39$$

If $k = -2$, then we would have $0 = 39$ and the system would be inconsistent.

73. $-11 - 6x \geq 33$

$-6x \geq 44$

$x \leq -\frac{22}{3}$

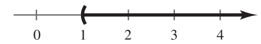


74. $2(x - 3) > -5x + 1$

$2x - 6 > -5x + 1$

$7x > 7$

$x > 1$

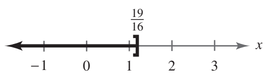


75. $8x - 15 \leq -4(2x - 1)$

$8x - 15 \leq -8x + 4$

$16x \leq 19$

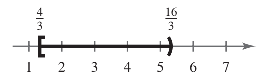
$x \leq \frac{19}{16}$



76. $-6 \leq 3x - 10 < 6$

$4 \leq 3x < 16$

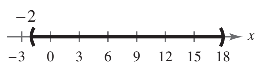
$\frac{4}{3} \leq x < \frac{16}{3}$



77. $|x - 8| < 10$

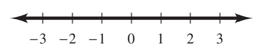
$-10 < x - 8 < 10$

$-2 < x < 18$



78. $|x + 10| \geq -3$

All real numbers x



79. $2x^2 + 3x - 35 < 0$

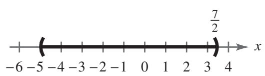
$(2x - 7)(x + 5) < 0$

Critical numbers: $x = -5, \frac{7}{2}$

Test intervals: $(-\infty, -5), \left(-5, \frac{7}{2}\right), \left(\frac{7}{2}, \infty\right)$

Test: Is $(2x - 7)(x + 5) < 0$?

Solution: $-5 < x < \frac{7}{2}$



80. $3x^2 + 12x > 0$

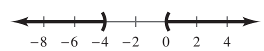
$3x(x + 4) > 0$

Critical numbers: $x = 0, -4$

Test Intervals: $(-\infty, -4), (-4, 0), (0, \infty)$

Test: Is $3x(x + 4) > 0$?

Solution: $x < -4,\ x > 0$



81. $\ln x + \ln 6 = \ln(6x)$

82. $\ln x - 5 \ln(x + 3) = \ln x - \ln(x + 3)^5$

$$= \ln \frac{x}{(x + 3)^5}$$

83. $\log_9 12 - \log_9 x = \log_9\left(\frac{12}{x}\right)$

84. $\frac{1}{4}\log_6 3x = \log_6 \sqrt[4]{3x}$

85. $\begin{cases} 2x - y = 4 \implies y = 2x - 4 \\ -4x + 2y = -12 \end{cases}$

$-4x + 2(2x - 4) = -12$

$-4x + 4x - 8 = -12$

$-8 = -12$

There are no solutions.

86. $30x - 40y - 33 = 0$

$10x + 20y - 21 = 0 \implies y = -\frac{1}{2}x + \frac{21}{20}$

$30x - 40\left(-\frac{1}{2}x + \frac{21}{20}\right) - 33 = 0$

$30x + 20x - 42 - 33 = 0$

$50x = 75$

$x = \frac{3}{2}$

$y = -\frac{1}{2}\left(\frac{3}{2}\right) + \frac{21}{20} = \frac{6}{20} = \frac{3}{10}$

Solution: $\left(\frac{3}{2}, \frac{3}{10}\right)$

87. Answers will vary.

Section 7.3 Multivariable Linear Systems

- You should know the operations that lead to equivalent systems of equations:
  (a) Interchange any two equations.
  (b) Multiply all terms of an equation by a nonzero constant.
  (c) Replace an equation by the sum of itself and a constant multiple of any other equation in the system.
- You should be able to use the method of Gaussian elimination with back-substitution.

Vocabulary Check

1. row-echelon
2. ordered triple
3. Gaussian
4. row operation
5. nonsquare
6. position

1. $\begin{cases} 3x - y + z = 1 \\ 2x - 3z = -14 \\ 5y + 2z = 8 \end{cases}$

(a) $3(2) - (0) + (-3) \neq 1$

$(2, 0, -3)$ *is not* a solution.

(b) $3(-2) - (0) + 8 \neq 1$

$(-2, 0, 8)$ *is not* a solution.

(c) $3(0) - (-1) + 3 \neq 1$

$(0, -1, 3)$ *is not* a solution.

(d) $\begin{aligned} 3(-1) - (0) + 4 &= 1 \\ 2(-1) - 3(4) &= -14 \\ 5(0) + 2(4) &= 8 \end{aligned}$

$(-1, 0, 4)$ *is* a solution.

2. $\begin{cases} 3x + 4y - z = 17 \\ 5x - y + 2z = -2 \\ 2x - 3y + 7z = -21 \end{cases}$

(a) $3(3) + 4(-1) - 2 \neq 17$

$(3, -1, 2)$ *is not* a solution.

(b) $\begin{aligned} 3(1) + 4(3) - (-2) &= 17 \\ 5(1) - 3 + 2(-2) &= -2 \\ 2(1) - 3(3) + 7(-2) &= -21 \end{aligned}$

$(1, 3, -2)$ *is* a solution.

(c) $3(4) + 4(1) - (-3) \neq 17$

$(4, 1, -3)$ *is not* a solution.

(d) $3(1) + 4(-2) - 2 \neq 17$

$(1, -2, 2)$ *is not* a solution.

3. $\begin{cases} 4x + y - z = 0 \\ -8x - 6y + z = -\frac{7}{4} \\ 3x - y = -\frac{9}{4} \end{cases}$

(a) $4\left(\frac{1}{2}\right) + \left(-\frac{3}{4}\right) - \left(-\frac{7}{4}\right) \neq 0$

$\left(\frac{1}{2}, -\frac{3}{4}, -\frac{7}{4}\right)$ *is not* a solution.

(b) $4\left(-\frac{3}{2}\right) + \left(\frac{5}{4}\right) - \left(-\frac{5}{4}\right) \neq 0$

$\left(-\frac{3}{2}, \frac{5}{4}, -\frac{5}{4}\right)$ *is not* a solution.

(c) $\begin{aligned} 4\left(-\frac{1}{2}\right) + \left(\frac{3}{4}\right) - \left(-\frac{5}{4}\right) &= 0 \\ -8\left(-\frac{1}{2}\right) - 6\left(\frac{3}{4}\right) + \left(-\frac{5}{4}\right) &= -\frac{7}{4} \\ 3\left(-\frac{1}{2}\right) - \left(\frac{3}{4}\right) &= -\frac{9}{4} \end{aligned}$

$\left(-\frac{1}{2}, \frac{3}{4}, -\frac{5}{4}\right)$ *is* a solution.

(d) $4\left(-\frac{1}{2}\right) + \left(\frac{1}{6}\right) - \left(-\frac{3}{4}\right) \neq 0$

$\left(-\frac{1}{2}, \frac{1}{6}, -\frac{3}{4}\right)$ *is not* a solution.

4. $\begin{cases} -4x - y - 8z = -6 \\ y + z = 0 \\ 4x - 7y = 6 \end{cases}$

(a) $-4(-2) - (-2) - 8(2) = -6$

$-2 + 2 = 0$

$4(-2) - 7(-2) = 6$

$(-2, -2, 2)$ *is* a solution.

(b) $-4\left(-\frac{33}{2}\right) - (-10) - 8(10) \neq -6$

$\left(-\frac{33}{2}, -10, 10\right)$ *is not* a solution.

(c) $-4\left(\frac{1}{8}\right) - \left(-\frac{1}{2}\right) - 8\left(\frac{1}{2}\right) \neq -6$

$\left(\frac{1}{8}, -\frac{1}{2}, \frac{1}{2}\right)$ *is not* a solution.

(d) $-4\left(-\frac{11}{2}\right) - (-4) - 8(4) = -6$

$-4 + 4 = 0$

$4\left(-\frac{11}{2}\right) - 7(-4) = 6$

$\left(-\frac{11}{2}, -4, 4\right)$ *is* a solution.

5. $\begin{cases} 2x - y + 5z = 24 & \text{Equation 1} \\ y + 2z = 6 & \text{Equation 2} \\ z = 4 & \text{Equation 3} \end{cases}$

Back-substitute $z = 4$ into Equation 2:

$y + 2(4) = 6$

$y = -2$

Back-substitute $y = -2$ and $z = 4$ into Equation 1:

$2x - (-2) + 5(4) = 24$

$2x + 22 = 24$

$x = 1$

Solution: $(1, -2, 4)$

6. $\begin{cases} 4x - 3y - 2z = 21 & \text{Equation 1} \\ 6y - 5z = -8 & \text{Equation 2} \\ z = -2 & \text{Equation 3} \end{cases}$

Back-substitute $z = -2$ in Equation 2:

$6y - 5(-2) = -8$

$y = -3$

Back-substitute $z = -2$ and $y = -3$ in Equation 1:

$4x - 3(-3) - 2(-2) = 21$

$4x + 13 = 21$

$x = 2$

Solution: $(2, -3, -2)$

7. $\begin{cases} 2x + y - 3z = 10 & \text{Equation 1} \\ y + z = 12 & \text{Equation 2} \\ z = 2 & \text{Equation 3} \end{cases}$

Substitute $z = 2$ into Equation 2: $y + (2) = 12 \implies y = 10$

Substitute $y = 10$ and $z = 2$ into Equation 1:

$2x + (10) - 3(2) = 10$

$2x + 4 = 10$

$2x = 6$

$x = 3$

Solution: $(3, 10, 2)$

8. $\begin{cases} x - y + 2z = 22 & \text{Equation 1} \\ 3y - 8z = -9 & \text{Equation 2} \\ z = -3 & \text{Equation 3} \end{cases}$

Back-substitute $z = -3$ in Equation 2:

$3y - 8(-3) = -9$

$3y + 24 = -9$

$y = -11$

Back-substitute $z = -3$ and $y = -11$ in Equation 1:

$x - (-11) + 2(-3) = 22$

$x + 5 = 22$

$x = 17$

Solution: $(17, -11, -3)$

9. $\begin{cases} 4x - 2y + z = 8 & \text{Equation 1} \\ -y + z = 4 & \text{Equation 2} \\ z = 2 & \text{Equation 3} \end{cases}$

Substitute $z = 2$ into Equation 2:

$-y + (2) = 4 \implies y = -2$

Substitute $y = -2$ and $z = 2$ into Equation 1:

$4x - 2(-2) + (2) = 8$

$4x + 6 = 8$

$4x = 2$

$x = \frac{1}{2}$

Solution: $\left(\frac{1}{2}, -2, 2\right)$

10. $\begin{cases} 5x \quad - 8z = 22 \\ 3y - 5z = 10 \\ z = -4 \end{cases}$

Back-substitute $z = -4$ in Equation 2:

$3y - 5(-4) = 10 \implies y = -\frac{10}{3}$

Back-substitute $z = -4$ in Equation 1:

$5x - 8(-4) = 22 \implies x = -2$

Solution: $\left(-2, -\frac{10}{3}, -4\right)$

11. $\begin{cases} x - 2y + 3z = 5 & \text{Equation 1} \\ -x + 3y - 5z = 4 & \text{Equation 2} \\ 2x \quad - 3z = 0 & \text{Equation 3} \end{cases}$

Add Equation 1 to Equation 2:

$\begin{cases} x - 2y + 3z = 5 \\ y - 2z = 9 \\ 2x \quad - 3z = 0 \end{cases}$

This is the first step in putting the system in row-echelon form.

12. $\begin{cases} x - 2y + 3z = 5 & \text{Equation 1} \\ -x + 3y - 5z = 4 & \text{Equation 2} \\ 2x \quad - 3z = 0 & \text{Equation 3} \end{cases}$

Add -2 times Equation 1 to Equation 3:

$\begin{cases} x - 2y + 3z = 5 \\ -x + 3y - 5z = 4 \\ 4y - 9z = -10 \end{cases}$

This is the first step in putting the system in row-echelon form.

13. $\begin{cases} x + y + z = 6 & \text{Equation 1} \\ 2x - y + z = 3 & \text{Equation 2} \\ 3x \quad - z = 0 & \text{Equation 3} \end{cases}$

$\begin{cases} x + y + z = 6 & \\ -3y - z = -9 & -2\text{Eq.1} + \text{Eq.2} \\ -3y - 4z = -18 & -3\text{Eq.1} + \text{Eq.3} \end{cases}$

$\begin{cases} x + y + z = 6 & \\ -3y - z = -9 & \\ -3z = -9 & -\text{Eq.2} + \text{Eq.3} \end{cases}$

$\begin{cases} x + y + z = 6 & \\ -3y - z = -9 & \\ z = 3 & -\frac{1}{3}\text{Eq.3} \end{cases}$

$-3y - 3 = -9 \implies y = 2$

$x + 2 + 3 = 6 \implies x = 1$

Solution: $(1, 2, 3)$

14. $\begin{cases} x + y + z = 3 & \text{Equation 1} \\ x - 2y + 4z = 5 & \text{Equation 2} \\ 3y + 4z = 5 & \text{Equation 3} \end{cases}$

$\begin{cases} x + y + z = 3 & \\ -3y + 3z = 2 & (-1)\text{Eq.1} + \text{Eq.2} \\ 3y + 4z = 5 & \end{cases}$

$\begin{cases} x + y + z = 3 & \\ -3y + 3z = 2 & \\ 7z = 7 & \text{Eq.2} + \text{Eq.3} \end{cases}$

$\begin{cases} x + y + z = 3 & \\ y - z = -\frac{2}{3} & \left(-\frac{1}{3}\right)\text{Eq.2} \\ z = 1 & \left(\frac{1}{7}\right)\text{Eq.3} \end{cases}$

$y - 1 = -\frac{2}{3} \implies y = \frac{1}{3}$

$x + \frac{1}{3} + 1 = 3 \implies x = \frac{5}{3}$

Solution: $\left(\frac{5}{3}, \frac{1}{3}, 1\right)$

15. $\begin{cases} 2x \quad + 2z = 2 \\ 5x + 3y \quad = 4 \\ 3y - 4z = 4 \end{cases}$

$\begin{cases} x \quad + z = 1 & \frac{1}{2}\text{Eq.1} \\ 5x + 3y \quad = 4 & \\ 3y - 4z = 4 & \end{cases}$

$\begin{cases} x \quad + z = 1 & \\ 3y - 5z = -1 & -5\text{Eq.1} + \text{Eq.2} \\ 3y - 4z = 4 & \end{cases}$

$\begin{cases} x \quad + z = 1 & \\ 3y - 5z = -1 & \\ z = 5 & -\text{Eq.2} + \text{Eq.3} \end{cases}$

$3y - 5(5) = -1 \implies y = 8$

$x + 5 = 1 \implies x = -4$

Solution: $(-4, 8, 5)$

16. $\begin{cases} x + y - z = -1 \\ 2x + 4y + z = 1 \\ x - 2y - 3z = 2 \end{cases}$ Interchange equations.

$\begin{cases} x + y - z = -1 \\ 2y + 3z = 3 & (-2)\text{Eq.1} + \text{Eq.2} \\ -3y - 2z = 3 & (-1)\text{Eq.1} + \text{Eq.3} \end{cases}$

$\begin{cases} x + y - z = -1 \\ 2y + 3z = 3 \\ -6y - 4z = 6 & 2\text{ Eq.3} \end{cases}$

$\begin{cases} x + y - z = -1 \\ 2y + 3z = 3 \\ 5z = 15 & 3\text{Eq.2} + \text{Eq.3} \end{cases}$

$\begin{cases} x + y - z = -1 \\ y + \frac{3}{2}z = \frac{3}{2} & \left(\frac{1}{2}\right)\text{Eq.2} \\ z = 3 & \left(\frac{1}{5}\right)\text{Eq.3} \end{cases}$

$y + \frac{3}{2}(3) = \frac{3}{2} \implies y = -3$

$x - 3 - 3 = -1 \implies x = 5$

Solution: $(5, -3, 3)$

17. $\begin{cases} 3x + 3y = 9 \\ 2x - 3z = 10 \\ 6y + 4z = -12 \end{cases}$ Interchange equations.

$\begin{cases} x + y = 3 & \frac{1}{3}\text{Eq.1} \\ 2x - 3z = 10 \\ 6y + 4z = -12 \end{cases}$

$\begin{cases} x + y = 3 \\ -2y - 3z = 4 & -2\text{Eq.1} + \text{Eq.2} \\ 6y + 4z = -12 \end{cases}$

$\begin{cases} x + y = 3 \\ -2y - 3z = 4 \\ -5z = 0 & 3\text{Eq.2} + \text{Eq.3} \end{cases}$

$\begin{cases} x + y = 3 \\ -2y - 3z = 4 \\ z = 0 & -\frac{1}{5}\text{Eq.3} \end{cases}$

$-2y - 3(0) = 4 \implies y = -2$

$x - 2 = 3 \implies x = 5$

Solution: $(5, -2, 0)$

18. $\begin{cases} x + 4y + z = 0 \\ 2x + 4y - z = 7 \\ 2x - 4y + 2z = -6 \end{cases}$ Interchange equations.

$\begin{cases} x + 4y + z = 0 \\ -4y - 3z = 7 & (-2)\text{Eq.1} + \text{Eq.2} \\ -12y = -6 & (-2)\text{Eq.1} + \text{Eq.3} \end{cases}$

$\begin{cases} x + 4y + z = 0 \\ -4y - 3z = 7 \\ 9z = -27 & (-3)\text{Eq.2} + \text{Eq.3} \end{cases}$

$\begin{cases} x + 4y + z = 0 \\ y + \frac{3}{4}z = -\frac{7}{4} & \left(-\frac{1}{4}\right)\text{Eq.2} \\ z = -3 & \left(\frac{1}{9}\right)\text{Eq.3} \end{cases}$

$y + \frac{3}{4}(-3) = -\frac{7}{4} \implies y = \frac{1}{2}$

$x + 4\left(\frac{1}{2}\right) + (-3) = 0 \implies x = 1$

Solution: $\left(1, \frac{1}{2}, -3\right)$

19. $\begin{cases} x - 2y + 2z = -9 \\ 2x + y - z = 7 \\ 3x - y + z = 5 \end{cases}$ Interchange equations.

$\begin{cases} x - 2y + 2z = -9 \\ 5y - 5z = 25 & -2\text{Eq.1} + \text{Eq.2} \\ 5y - 5z = 32 & -3\text{Eq.1} + \text{Eq.3} \end{cases}$

$\begin{cases} x - 2y + 2z = -9 \\ 5y - 5z = 25 \\ 0 = 7 & -\text{Eq.2} + \text{Eq.3} \end{cases}$

Inconsistent, no solution

20. $\begin{cases} x - 11y + 4z = 3 \\ 5x - 3y + 2z = 3 \\ 2x + 4y - z = 7 \end{cases}$ Interchange equations.

$\begin{cases} x - 11y + 4z = 3 \\ 52y - 18z = -12 & (-5)\text{Eq.1} + \text{Eq.2} \\ 26y - 9z = 1 & (-2)\text{Eq.1} + \text{Eq.3} \end{cases}$

$\begin{cases} x - 11y + 4z = 3 \\ 52y - 18z = -12 \\ 0 = 7 & \left(-\frac{1}{2}\right)\text{Eq.2} + \text{Eq.3} \end{cases}$

Inconsistent, no solution

21.

$$\begin{cases} 3x - 5y + 5z = 1 \\ 5x - 2y + 3z = 0 \\ 7x - y + 3z = 0 \end{cases}$$

$$\begin{cases} 6x - 10y + 10z = 2 & \text{2Eq.1} \\ 5x - 2y + 3z = 0 \\ 7x - y + 3z = 0 \end{cases}$$

$$\begin{cases} x - 8y + 7z = 2 & -\text{Eq.2} + \text{Eq.1} \\ 5x - 2y + 3z = 0 \\ 7x - y + 3z = 0 \end{cases}$$

$$\begin{cases} x - 8y + 7z = 2 \\ 38y - 32z = -10 & -5\text{Eq.1} + \text{Eq.2} \\ 55y - 46z = -14 & -7\text{Eq.1} + \text{Eq.3} \end{cases}$$

$$\begin{cases} x - 8y + 7z = 2 \\ 2090y - 1760z = -550 & \text{55Eq.2} \\ -2090y + 1748z = 532 & -38\text{Eq.3} \end{cases}$$

$$\begin{cases} x - 8y + 7z = 2 \\ 2090y - 1760z = -550 \\ -12z = -18 & \text{Eq.2} + \text{Eq.3} \end{cases}$$

$$-12z = -18 \Rightarrow z = \tfrac{3}{2}$$

$$38y - 32\left(\tfrac{3}{2}\right) = -10 \Rightarrow y = 1$$

$$x - 8(1) + 7\left(\tfrac{3}{2}\right) = 2 \Rightarrow x = -\tfrac{1}{2}$$

Solution: $\left(-\frac{1}{2}, 1, \frac{3}{2}\right)$

22.

$$\begin{cases} 2x + y + 3z = 1 & \text{Equation 1} \\ 2x + 6y + 8z = 3 & \text{Equation 2} \\ 6x + 8y + 18z = 5 & \text{Equation 3} \end{cases}$$

$$\begin{cases} 2x + y + 3z = 1 \\ 5y + 5z = 2 & (-1)\text{Eq.1} + \text{Eq.2} \\ 5y + 9z = 2 & (-3)\text{Eq.1} + \text{Eq.3} \end{cases}$$

$$\begin{cases} 2x + y + 3z = 1 \\ 5y + 5z = 2 \\ 4z = 0 & (-1)\text{Eq.2} + \text{Eq.3} \end{cases}$$

$$\begin{cases} x + \frac{1}{2}y + \frac{3}{2}z = \frac{1}{2} & \left(\frac{1}{2}\right)\text{Eq.1} \\ y + z = \frac{2}{5} & \left(\frac{1}{5}\right)\text{Eq.2} \\ z = 0 & \left(\frac{1}{4}\right)\text{Eq.3} \end{cases}$$

$$y + 0 = \tfrac{2}{5} \Rightarrow y = \tfrac{2}{5}$$

$$x + \tfrac{1}{2}\left(\tfrac{2}{5}\right) + \tfrac{3}{2}(0) = \tfrac{1}{2} \Rightarrow x = \tfrac{3}{10}$$

Solution: $\left(\frac{3}{10}, \frac{2}{5}, 0\right)$

23.

$$\begin{cases} x + 2y - 7z = -4 \\ 2x + y + z = 13 \\ 3x + 9y - 36z = -33 \end{cases}$$

$$\begin{cases} x + 2y - 7z = -4 \\ -3y + 15z = 21 & -2\text{Eq.1} + \text{Eq.2} \\ 3y - 15z = -21 & -3\text{Eq.1} + \text{Eq.3} \end{cases}$$

$$\begin{cases} x + 2y - 7z = -4 \\ -3y + 15z = 21 \\ 0 = 0 & \text{Eq.2} + \text{Eq.3} \end{cases}$$

$$\begin{cases} x + 2y - 7z = -4 \\ y - 5z = -7 & -\frac{1}{3}\text{Eq.2} \end{cases}$$

$$\begin{cases} x + 3z = 10 & -2\text{Eq.2} + \text{Eq.1} \\ y - 5z = -7 \end{cases}$$

Let $z = a$, then:

$$y = 5a - 7$$

$$x = -3a + 10$$

Solution: $(-3a + 10, 5a - 7, a)$

24.

$$\begin{cases} 2x + y - 3z = 4 & \text{Equation 1} \\ 4x + 2z = 10 & \text{Equation 2} \\ -2x + 3y - 13z = -8 & \text{Equation 3} \end{cases}$$

$$\begin{cases} 2x + y - 3z = 4 & (-2)\text{Eq.1} + \text{Eq.2} \\ -2y + 8z = 2 & \text{Eq.1} + \text{Eq.3} \\ 4y - 16z = -4 \end{cases}$$

$$\begin{cases} 2x + y - 3z = 4 \\ -2y + 8z = 2 \\ 0 = 0 & \text{2Eq.2} + \text{Eq.3} \end{cases}$$

$$\begin{cases} 2x + z = 5 & \left(\frac{1}{2}\right)\text{Eq.2} + \text{Eq.1} \\ -2y + 8z = 2 \end{cases}$$

$$\begin{cases} x + z/2 = \frac{5}{2} & \left(\frac{1}{2}\right)\text{Eq.1} \\ y - 4z = -1 & \left(-\frac{1}{2}\right)\text{Eq. 2} \end{cases}$$

$z = a$

$$y - 4a = -1 \Rightarrow y = 4a - 1$$

$$x + \tfrac{1}{2}a = \tfrac{5}{2} \Rightarrow x = -\tfrac{1}{2}a + \tfrac{5}{2}$$

Solution: $\left(-\frac{1}{2}a + \frac{5}{2}, 4a - 1, a\right)$

25.

$$\begin{cases} 3x - 3y + 6z = 6 \\ x + 2y - z = 5 \\ 5x - 8y + 13z = 7 \end{cases}$$

$$\begin{cases} x - y + 2z = 2 & \frac{1}{3}\text{Eq.1} \\ x + 2y - z = 5 \\ 5x - 8y + 13z = 7 \end{cases}$$

$$\begin{cases} x - y + 2z = 2 \\ 3y - 3z = 3 & -\text{Eq.1} + \text{Eq.2} \\ -3y + 3z = -3 & -5\text{Eq.1} + \text{Eq.3} \end{cases}$$

$$\begin{cases} x - y + 2z = 2 \\ y - z = 1 & \frac{1}{3}\text{Eq.2} \\ 0 = 0 & \text{Eq.2} + \text{Eq.3} \end{cases}$$

$$\begin{cases} x + z = 3 & \text{Eq.2} + \text{Eq.1} \\ y - z = 1 \end{cases}$$

Let $z = a$, then:

$$y = a + 1$$

$$x = -a + 3$$

Solution: $(-a + 3, a + 1, a)$

26. $\begin{cases} x + 2z = 5 & \text{Equation 1} \\ 3x - y - z = 1 & \text{Equation 2} \\ 6x - y + 5z = 16 & \text{Equation 3} \end{cases}$

$\begin{cases} x + 2z = 5 & \\ -y - 7z = -14 & (-3)\text{Eq.1} + \text{Eq.2} \\ -y - 7z = -14 & (-6)\text{Eq.1} + \text{Eq.3} \end{cases}$

$\begin{cases} x + 2z = 5 & \\ -y - 7z = -14 & \\ 0 = 0 & (-1)\text{Eq.2} + \text{Eq.3} \end{cases}$

$\begin{cases} x + 2z = 5 & \\ y + 7z = 14 & (-1)\text{Eq.2} \end{cases}$

$z = a$

$y + 7a = 14 \implies y = -7a + 14$
$x + 2a = 5 \implies x = -2a + 5$

Solution: $(-2a + 5, -7a + 14, a)$

27. $\begin{cases} x - 2y + 5z = 2 \\ 4x - z = 0 \end{cases}$

Let $z = a$, then $x = \frac{1}{4}a$.

$\frac{1}{4}a - 2y + 5a = 2$
$a - 8y + 20a = 8$
$-8y = -21a + 8$
$y = \frac{21}{8}a - 1$

Answer: $\left(\frac{1}{4}a, \frac{21}{8}a - 1, a\right)$

To avoid fractions, we could go back and let $z = 8a$, then $4x - 8a = 0 \implies x = 2a$.

$2a - 2y + 5(8a) = 2$

$-2y + 42a = 2$

$y = 21a - 1$

Solution: $(2a, 21a - 1, 8a)$

28. $\begin{cases} x - 3y + 2z = 18 & \text{Equation 1} \\ 5x - 13y + 12z = 80 & \text{Equation 2} \end{cases}$

$\begin{cases} x - 3y + 2z = 18 & \\ 2y + 2z = -10 & (-5)\text{Eq.1} + \text{Eq.2} \end{cases}$

$\begin{cases} x - 3y + 2z = 18 & \\ y + z = -5 & \left(\frac{1}{2}\right)\text{Eq.2} \end{cases}$

$\begin{cases} x + 5z = 3 & 3\text{Eq.2} + \text{Eq.1} \\ y + z = -5 & \end{cases}$

Let $z = a$, then: $y + a = -5 \implies y = -a - 5$
$x + 5a = 3 \implies x = -5a + 3$

Solution: $(-5a + 3, -a - 5, a)$

29. $\begin{cases} 2x - 3y + z = -2 \\ -4x + 9y = 7 \end{cases}$

$\begin{cases} 2x - 3y + z = -2 & \\ 3y + 2z = 3 & 2\text{Eq.1} + \text{Eq.2} \end{cases}$

$\begin{cases} 2x + 3z = 1 & \text{Eq.2} + \text{Eq.1} \\ 3y + 2z = 3 & \end{cases}$

Let $z = a$, then:

$y = -\frac{2}{3}a + 1$

$x = -\frac{3}{2}a + \frac{1}{2}$

Solution: $\left(-\frac{3}{2}a + \frac{1}{2}, -\frac{2}{3}a + 1, a\right)$

30. $\begin{cases} 2x + 3y + 3z = 7 & \text{Equation 1} \\ 4x + 18y + 15z = 44 & \text{Equation 2} \end{cases}$

$\begin{cases} 2x + 3y + 3z = 7 & \\ 12y + 9z = 30 & (-2)\text{Eq.1} + \text{Eq.2} \end{cases}$

$\begin{cases} 2x + \frac{3}{4}z = -\frac{1}{2} & \left(-\frac{1}{4}\right)\text{Eq.2} + \text{Eq.1} \\ 12y + 9z = 30 & \end{cases}$

$\begin{cases} x + \frac{3}{8}z = -\frac{1}{4} & \left(\frac{1}{2}\right)\text{Eq.1} \\ y + \frac{3}{4}z = \frac{5}{2} & \left(\frac{1}{12}\right)\text{Eq.2} \end{cases}$

Let $z = a$, then:

$y + \frac{3}{4}a = \frac{5}{2} \implies y = -\frac{3}{4}a + \frac{5}{2}$

$x + \frac{3}{8}a = -\frac{1}{4} \implies x = -\frac{3}{8}a - \frac{1}{4}$

Solution: $\left(-\frac{3}{8}a - \frac{1}{4}, -\frac{3}{4}a + \frac{5}{2}, a\right)$

31. $\begin{cases} x + 3w = 4 \\ 2y - z - w = 0 \\ 3y - 2w = 1 \\ 2x - y + 4z = 5 \end{cases}$

$\begin{cases} x + 3w = 4 \\ 2y - z - w = 0 \\ 3y - 2w = 1 \\ -y + 4z - 6w = -3 & -2\text{Eq.1} + \text{Eq.4} \end{cases}$

$\begin{cases} x + 3w = 4 \\ y - 4z + 6w = 3 & -\text{Eq.4 and interchange the equations.} \\ 2y - z - w = 0 \\ 3y - 2w = 1 \end{cases}$

$\begin{cases} x + 3w = 4 \\ y - 4z + 6w = 3 \\ 7z - 13w = -6 & -\text{Eq.2} + \text{Eq.3} \\ 12z - 20w = -8 & -3\text{Eq.2} + \text{Eq.4} \end{cases}$

$\begin{cases} x + 3w = 4 \\ y - 4z + 6w = 3 \\ z - 3w = -2 & -\frac{1}{2}\text{Eq.4} + \text{Eq.3} \\ 12z - 20w = -8 \end{cases}$

$\begin{cases} x + 3w = 4 \\ y - 4z + 6w = 3 \\ z - 3w = -2 \\ 16w = 16 & -12\text{Eq.3} + \text{Eq.4} \end{cases}$

$16w = 16 \implies w = 1$

$z - 3(1) = -2 \implies z = 1$

$y - 4(1) + 6(1) = 3 \implies y = 1$

$x + 3(1) = 4 \implies x = 1$

Solution: $(1, 1, 1, 1)$

32. $\begin{cases} x + y + z + w = 6 & \text{Equation 1} \\ 2x + 3y - w = 0 & \text{Equation 2} \\ -3x + 4y + z + 2w = 4 & \text{Equation 3} \\ x + 2y - z + w = 0 & \text{Equation 4} \end{cases}$

$\begin{cases} x + y + z + w = 6 \\ y - 2z - 3w = -12 & (-2)\text{Eq.1} + \text{Eq.2} \\ 7y + 4z + 5w = 22 & 3\text{Eq.1} + \text{Eq.3} \\ y - 2z = -6 & (-1)\text{Eq.1} + \text{Eq.4} \end{cases}$

$\begin{cases} x + y + z + w = 6 \\ y - 2z - 3w = -12 \\ 18z + 26w = 106 & (-7)\text{Eq.2} + \text{Eq.3} \\ 3w = 6 & (-1)\text{Eq.2} + \text{Eq.4} \end{cases}$

$\begin{cases} x + y + z + w = 6 \\ y - 2z - 3w = -12 \\ z + \frac{13}{9}w = \frac{53}{9} & \left(\frac{1}{18}\right)\text{Eq.3} \\ w = 2 & \left(\frac{1}{3}\right)\text{Eq.4} \end{cases}$

$z + \frac{13}{9}(2) = \frac{53}{9} \implies z = 3$

$y - 2(3) - 3(2) = -12 \implies y = 0$

$x + 0 + 3 + 2 = 6 \implies x = 1$

Solution: $(1, 0, 3, 2)$

33. $\begin{cases} x + 4z = 1 \\ x + y + 10z = 10 \\ 2x - y + 2z = -5 \end{cases}$

$\begin{cases} x + 4z = 1 \\ y + 6z = 9 & -\text{Eq.1} + \text{Eq.2} \\ -y - 6z = -7 & -2\text{Eq.1} + \text{Eq.3} \end{cases}$

$\begin{cases} x + 4z = 1 \\ y + 6z = 9 \\ 0 = 2 & \text{Eq.2} + \text{Eq.3} \end{cases}$

No solution, inconsistent

34. $\begin{cases} 2x - 2y - 6z = -4 & \text{Equation 1} \\ -3x + 2y + 6z = 1 & \text{Equation 2} \\ x - y - 5z = -3 & \text{Equation 3} \end{cases}$

$\begin{cases} x - y - 5z = -3 \\ -3x + 2y + 6z = 1 & \text{Interchange equations.} \\ 2x - 2y - 6z = -4 \end{cases}$

$\begin{cases} x - y - 5z = -3 \\ -y - 9z = -8 & 3\text{Eq.1} + \text{Eq.2} \\ 4z = 2 & (-2)\text{Eq.1} + \text{Eq.3} \end{cases}$

$\begin{cases} x - y - 5z = -3 \\ y + 9z = 8 & (-1)\text{Eq.2} \\ z = \frac{1}{2} & \left(\frac{1}{4}\right)\text{Eq.3} \end{cases}$

$y + 9\left(\frac{1}{2}\right) = 8 \implies y = \frac{7}{2}$

$x - \frac{7}{2} - 5\left(\frac{1}{2}\right) = -3 \implies x = 3$

Solution: $\left(3, \frac{7}{2}, \frac{1}{2}\right)$

35. $\begin{cases} 2x + 3y = 0 \\ 4x + 3y - z = 0 \\ 8x + 3y + 3z = 0 \end{cases}$

$\begin{cases} 2x + 3y = 0 \\ -3y - z = 0 & -2\text{Eq.}1 + \text{Eq.}2 \\ -9y + 3z = 0 & -4\text{Eq.}1 + \text{Eq.}3 \end{cases}$

$\begin{cases} 2x + 3y = 0 \\ -3y - z = 0 \\ 6z = 0 & -3\text{Eq.}2 + \text{Eq.}3 \end{cases}$

$6z = 0 \implies z = 0$

$-3y - 0 = 0 \implies y = 0$

$2x + 3(0) = 0 \implies x = 0$

Solution: (0, 0, 0)

36. $\begin{cases} 4x + 3y + 17z = 0 \\ 5x + 4y + 22z = 0 \\ 4x + 2y + 19z = 0 \end{cases}$

$\begin{cases} 5x + 4y + 22z = 0 \\ 4x + 3y + 17z = 0 \\ 4x + 2y + 19z = 0 \end{cases}$ Interchange equations.

$\begin{cases} x + y + 5z = 0 & (-1)\text{Eq.}2 + \text{Eq.}1 \\ 4x + 3y + 17z = 0 \\ 4x + 2y + 19z = 0 \end{cases}$

$\begin{cases} x + y + 5z = 0 \\ -y - 3z = 0 & (-4)\text{Eq.}1 + \text{Eq.}2 \\ -2y - z = 0 & (-4)\text{Eq.}1 + \text{Eq.}3 \end{cases}$

$\begin{cases} x + y + 5z = 0 \\ y + 3z = 0 & (-1)\text{Eq.}2 \\ 5z = 0 & (-2)\text{Eq.}2 + \text{Eq.}3 \end{cases}$

$\begin{cases} x + y + 5z = 0 \\ y + 3z = 0 \\ z = 0 & \left(\frac{1}{5}\right)\text{Eq.}3 \end{cases}$

$y + 3(0) = 0 \implies y = 0$

$x + 0 + 5(0) = 0 \implies x = 0$

Solution: (0, 0, 0)

37. $\begin{cases} 12x + 5y + z = 0 \\ 23x + 4y - z = 0 \end{cases}$

$\begin{cases} 24x + 10y + 2z = 0 & 2\text{Eq.}1 \\ 23x + 4y - z = 0 \end{cases}$

$\begin{cases} x + 6y + 3z = 0 & -\text{Eq.}2 + \text{Eq.}1 \\ 23x + 4y - z = 0 \end{cases}$

$\begin{cases} x + 6y + 3z = 0 \\ -134y - 70z = 0 & -23\text{Eq.}1 + \text{Eq.}2 \end{cases}$

$\begin{cases} x + 6y + 3z = 0 \\ -67y - 35z = 0 & \frac{1}{2}\text{Eq.}2 \end{cases}$

To avoid fractions, let $z = 67a$, then:

$-67y - 35(67a) = 0$

$y = -35a$

$x + 6(-35a) + 3(67a) = 0$

$x = 9a$

Solution: $(9a, -35a, 67a)$

38. $\begin{cases} 2x - y - z = 0 & \text{Equation 1} \\ -2x + 6y + 4z = 2 & \text{Equation 2} \end{cases}$

$\begin{cases} 2x - y - z = 0 \\ 5y + 3z = 2 & \text{Eq.}1 + \text{Eq.}2 \end{cases}$

$\begin{cases} x - \frac{1}{2}y - \frac{1}{2}z = 0 & \left(\frac{1}{2}\right)\text{Eq.}1 \\ y + \frac{3}{5}z = \frac{2}{5} & \left(\frac{1}{5}\right)\text{Eq.}2 \end{cases}$

Let $z = a$, then:

$y + \frac{3}{5}a = \frac{2}{5} \implies y = -\frac{3}{5}a + \frac{2}{5}$

$x - \frac{1}{2}\left(-\frac{3}{5}a + \frac{2}{5}\right) - \frac{1}{2}a = 0 \implies x = \frac{1}{5}a + \frac{1}{5}$

Solution: $\left(\frac{1}{5}a + \frac{1}{5}, -\frac{3}{5}a + \frac{2}{5}, a\right)$

39. $s = \frac{1}{2}at^2 + v_0t + s_0$

(1, 128), (2, 80), (3, 0)

$128 = \frac{1}{2}a + v_0 + s_0 \implies a + 2v_0 + 2s_0 = 256$

$80 = 2a + 2v_0 + s_0 \implies 2a + 2v_0 + s_0 = 80$

$0 = \frac{9}{2}a + 3v_0 + s_0 \implies 9a + 6v_0 + 2s_0 = 0$

Solving this system yields $a = -32$, $v_0 = 0$, $s_0 = 144$.

Thus, $s = \frac{1}{2}(-32)t^2 + (0)t + 144 = -16t^2 + 144$.

40. $s = \frac{1}{2}at^2 + v_0t + s_0$

$(1, 48), (2, 64), (3, 48)$

$$\begin{cases} 48 = \frac{1}{2}a + v_0 + s_0 \\ 64 = 2a + 2v_0 + s_0 \\ 48 = \frac{9}{2}a + 3v_0 + s_0 \end{cases}$$

$$\begin{cases} a + 2v_0 + 2s_0 = 96 & \text{2Eq. 1} \\ 2a + 2v_0 + s_0 = 64 & \\ 9a + 6v_0 + 2s_0 = 96 & \text{2Eq. 3} \end{cases}$$

$$\begin{cases} a + 2v_0 + 2s_0 = 96 & \\ -2v_0 - 3s_0 = -128 & (-2)\text{Eq.1} + \text{Eq.2} \\ -12v_0 - 16s_0 = -768 & (-9)\text{Eq.1} + \text{Eq.3} \end{cases}$$

$$\begin{cases} a + 2v_0 + 2s_0 = 96 & \\ -2v_0 - 3s_0 = -128 & \\ 2s_0 = 0 & (-6)\text{Eq.2} + \text{Eq.3} \end{cases}$$

$$\begin{cases} a + 2v_0 + 2s_0 = 96 & \\ v_0 + 1.5s_0 = 64 & (-0.5)\text{Eq.2} \\ s_0 = 0 & (0.5)\text{Eq.3} \end{cases}$$

$$v_0 + 1.5(0) = 64 \implies v_0 = 64$$
$$a + 2(64) + 2(0) = 96 \implies a = -32$$

Thus, $s = \frac{1}{2}(-32)t^2 + 64t + 0$

$= -16t^2 + 64t.$

41. $s = \frac{1}{2}at^2 + v_0t + s_0$

$(1, 452), (2, 372), (3, 260)$

$452 = \frac{1}{2}a + v_0 + s_0 \implies a + 2v_0 + 2s_0 = 904$

$372 = 2a + 2v_0 + s_0 \implies 2a + 2v_0 + s_0 = 372$

$260 = \frac{9}{2}a + 3v_0 + s_0 \implies 9a + 6v_0 + 2s_0 = 520$

Solving this system yields $a = -32$, $v_0 = -32$, $s_0 = 500$.

Thus, $s = \frac{1}{2}(-32)t^2 + (-32)t + 500$

$= -16t^2 - 32t + 500.$

42. $s = \frac{1}{2}at^2 + v_0t + s_0$

$(1, 132), (2, 100), (3, 36)$

$$\begin{cases} 132 = \frac{1}{2}a + v_0 + s_0 \\ 100 = 2a + 2v_0 + s_0 \\ 36 = \frac{9}{2}a + 3v_0 + s_0 \end{cases}$$

$$\begin{cases} a + 2v_0 + 2s_0 = 264 & \text{2Eq. 1} \\ 2a + 2v_0 + s_0 = 100 & \\ 9a + 6v_0 + 2s_0 = 72 & \text{2Eq. 3} \end{cases}$$

$$\begin{cases} a + 2v_0 + 2s_0 = 264 & \\ -2v_0 - 3s_0 = -428 & (-2)\text{Eq.1} + \text{Eq.2} \\ -12v_0 - 16s_0 = -2304 & (-9)\text{Eq.1} + \text{Eq.3} \end{cases}$$

$$\begin{cases} a + 2v_0 + 2s_0 = 264 & \\ -2v_0 - 3s_0 = -428 & \\ 2s_0 = 264 & (-6)\text{Eq.2} + \text{Eq.3} \end{cases}$$

$$\begin{cases} a + 2v_0 + 2s_0 = 264 & \\ v_0 + 1.5s_0 = 214 & (-0.5)\text{Eq.2} \\ s_0 = 132 & (0.5)\text{Eq.3} \end{cases}$$

$$v_0 + 1.5(132) = 214 \implies v_0 = 16$$
$$a + 2(16) + 2(132) = 264 \implies a = -32$$

Thus, $s = \frac{1}{2}(-32)t^2 + 16t + 132$

$= -16t^2 + 16t + 132.$

43. $y = ax^2 + bx + c$ passing through $(0, 0), (2, -2), (4, 0)$

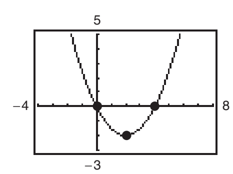
$(0, 0)$: $0 = c$

$(2, -2)$: $-2 = 4a + 2b + c \implies -1 = 2a + b$

$(4, 0)$: $0 = 16a + 4b + c \implies 0 = 4a + b$

Solution: $a = \frac{1}{2}, b = -2, c = 0$

The equation of the parabola is $y = \frac{1}{2}x^2 - 2x$.



44. $y = ax^2 + bx + c$ passing through $(0, 3)$, $(1, 4)$, $(2, 3)$

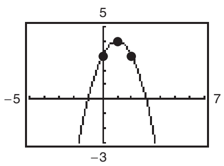
$(0, 3)$: $3 = c$

$(1, 4)$: $4 = a + b + c \implies 1 = a + b$

$(2, 3)$: $3 = 4a + 2b + c \implies 0 = 2a + b$

Solution: $a = -1, b = 2, c = 3$

The equation of the parabola is $y = -x^2 + 2x + 3$.



45. $y = ax^2 + bx + c$ passing through $(2, 0)$, $(3, -1)$, $(4, 0)$

$(2, 0)$: $0 = 4a + 2b + c$

$(3, -1)$: $-1 = 9a + 3b + c$

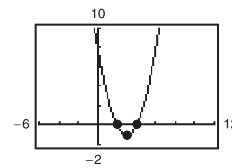
$(4, 0)$: $0 = 16a + 4b + c$

$$\begin{cases} 0 = 4a + 2b + c & \\ -1 = 5a + b & -\text{Eq.1} + \text{Eq.2} \\ 0 = 12a + 2b & -\text{Eq.1} + \text{Eq.3} \end{cases}$$

$$\begin{cases} 0 = 4a + 2b + c & \\ -1 = 5a + b & \\ 2 = 2a & -2\text{Eq.2} + \text{Eq.3} \end{cases}$$

Solution: $a = 1, b = -6, c = 8$

The equation of the parabola is $y = x^2 - 6x + 8$.



46. $y = ax^2 + bx + c$ passing through $(1, 3)$, $(2, 2)$, $(3, -3)$

$(1, 3)$: $3 = a + b + c$

$(2, 2)$: $2 = 4a + 2b + c$

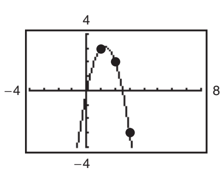
$(3, -3)$: $-3 = 9a + 3b + c$

$$\begin{cases} a + b + c = 3 & \\ 3a + b = -1 & (-1)\text{Eq.1} + \text{Eq.2} \\ 8a + 2b = -6 & (-1)\text{Eq.1} + \text{Eq.3} \end{cases}$$

$$\begin{cases} a + b + c = 3 & \\ 3a + b = -1 & \\ 2a = -4 & (-2)\text{Eq.2} + \text{Eq.3} \end{cases}$$

Solution: $a = -2, b = 5, c = 0$

The equation of the parabola is $y = -2x^2 + 5x$.



47. $x^2 + y^2 + Dx + Ey + F = 0$ passing through $(0, 0)$, $(2, 2)$, $(4, 0)$

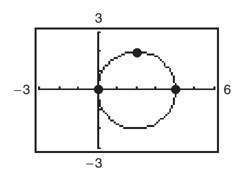
$(0, 0)$: $F = 0$

$(2, 2)$: $8 + 2D + 2E + F = 0 \implies D + E = -4$

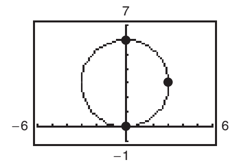
$(4, 0)$: $16 + 4D + F = 0 \implies D = -4$ and $E = 0$

The equation of the circle is $x^2 + y^2 - 4x = 0$.

To graph, let $y_1 = \sqrt{4x - x^2}$ and $y_2 = -\sqrt{4x - x^2}$.



48. $x^2 + y^2 + Dx + Ey + F = 0$ passing through $(0, 0)$, $(0, 6)$, $(3, 3)$



$(0, 0)$: $F = 0$

$(0, 6)$: $36 + 6E + F = 0 \implies E = -6$

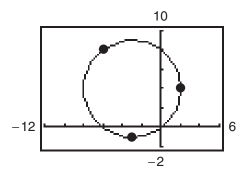
$(3, 3)$: $18 + 3D + 3E + F = 0 \implies D = 0$

The equation of the circle is $x^2 + y^2 - 6y = 0$. To graph, complete the square first, then solve for y.

$$x^2 + (y^2 - 6y + 9) = 9$$
$$x^2 + (y - 3)^2 = 9$$
$$(y - 3)^2 = 9 - x^2$$
$$y - 3 = \pm\sqrt{9 - x^2}$$
$$y = 3 \pm \sqrt{9 - x^2}$$

Let $y_1 = 3 + \sqrt{9 - x^2}$ and $y_2 = 3 - \sqrt{9 - x^2}$.

49. $x^2 + y^2 + Dx + Ey + F = 0$ passing through $(-3, -1)$, $(2, 4)$, $(-6, 8)$



$(-3, -1)$: $10 - 3D - E + F = 0 \implies 10 = 3D + E - F$

$(2, 4)$: $20 + 2D + 4E + F = 0 \implies 20 = -2D - 4E - F$

$(-6, 8)$: $100 - 6D + 8E + F = 0 \implies 100 = 6D - 8E - F$

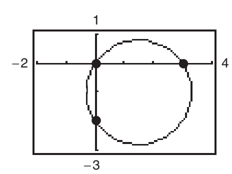
Solution: $D = 6, E = -8, F = 0$

The equation of the circle is $x^2 + y^2 + 6x - 8y = 0$. To graph, complete the squares first, then solve for y.

$$(x^2 + 6x + 9) + (y^2 - 8y + 16) = 0 + 9 + 16$$
$$(x + 3)^2 + (y - 4)^2 = 25$$
$$(y - 4)^2 = 25 - (x + 3)^2$$
$$y - 4 = \pm\sqrt{25 - (x + 3)^2}$$
$$y = 4 \pm \sqrt{25 - (x + 3)^2}$$

Let $y_1 = 4 + \sqrt{25 - (x + 3)^2}$ and $y_2 = 4 - \sqrt{25 - (x + 3)^2}$.

50. $x^2 + y^2 + Dx + Ey + F = 0$ passing through $(0, 0)$, $(0, -2)$, $(3, 0)$



$(0, 0)$: $F = 0$

$(0, -2)$: $4 - 2E + F = 0 \implies E = 2$

$(3, 0)$: $9 + 3D + F = 0 \implies D = -3$

The equation of the circle is $x^2 + y^2 - 3x + 2y = 0$. To graph, complete the squares first, then solve for y.

$$\left(x^2 - 3x + \tfrac{9}{4}\right) + (y^2 + 2y + 1) = \tfrac{9}{4} + 1$$
$$\left(x - \tfrac{3}{2}\right)^2 + (y + 1)^2 = \tfrac{13}{4}$$
$$(y + 1)^2 = \tfrac{13}{4} - \left(x - \tfrac{3}{2}\right)^2$$
$$y + 1 = \pm\sqrt{\tfrac{13}{4} - \left(x - \tfrac{3}{2}\right)^2}$$
$$y = -1 \pm \sqrt{\tfrac{13}{4} - \left(x - \tfrac{3}{2}\right)^2}$$

Let $y_1 = -1 + \sqrt{\tfrac{13}{4} - \left(x - \tfrac{3}{2}\right)^2}$ and $y_2 = -1 - \sqrt{\tfrac{13}{4} - \left(x - \tfrac{3}{2}\right)^2}$.

51. Let x = number of touchdowns.

Let y = number of extra-point kicks.

Let z = number of field goals.

$$\begin{cases} x + y + z = 13 \\ 6x + y + 3z = 45 \\ x - y = 0 \\ x - 6z = 0 \end{cases}$$

$$\begin{cases} x + y + z = 13 & \\ -5y - 3z = -33 & -6\text{Eq.}1 + \text{Eq.}2 \\ -2y - z = -13 & -\text{Eq.}1 + \text{Eq.}3 \\ -y - 7z = -13 & -\text{Eq.}1 + \text{Eq.}4 \end{cases}$$

$$\begin{cases} x + y + z = 13 & \\ -y - 7z = -13 & \text{Interchange Eq.2 and Eq.4.} \\ -2y - z = -13 & \\ -5y - 3z = -33 & \end{cases}$$

$$\begin{cases} x + y + z = 13 & \\ y + 7z = 13 & -\text{Eq.}2 \\ -2y - z = -13 & \\ -5y - 3z = -33 & \end{cases}$$

$$\begin{cases} x + y + z = 13 & \\ y + 7z = 13 & \\ 13z = 13 & 2\text{Eq.}2 + \text{Eq.}3 \\ 32z = 32 & 5\text{Eq.}2 + \text{Eq.}4 \end{cases}$$

$z = 1$

$y + 7(1) = 13 \implies y = 6$

$x + 6 + 1 = 13 \implies x = 6$

Thus, 6 touchdowns, 6 extra-point kicks, and 1 field goal were scored.

52. Let x = number of 2-point baskets.

Let y = number of 3-point baskets.

Let z = number of free throws.

$$\begin{cases} 2x + 3y + z = 70 \\ x - z = 2 \\ -2y + z = 1 \end{cases}$$

Add Equation 2 to Equation 3, and then add Equation 1 to Equation 2:

$$\begin{cases} 2x + 3y + z = 70 \\ 3x + 3y = 72 \\ x - 2y = 3 \end{cases}$$

Divide Equation 2 by 3:

$$\begin{cases} 2x + 3y + z = 70 \\ x + y = 24 \\ x - 2y = 3 \end{cases}$$

Subtract Equation 3 from Equation 2: $3y = 21 \implies y = 7$

Back-substitute into Equation 2: $x = 24 - 7 = 17$

Back-substitute into Equation 1: $z = 70 - 2(17) - 3(7) = 15$

There were 17 two-point baskets, 7 three-pointers, and 15 free-throws.

53. Let x = amount at 8%.

Let y = amount at 9%.

Let z = amount at 10%.

$$\begin{cases} x + y + z = 775{,}000 \\ 0.08x + 0.09y + 0.10z = 67{,}500 \\ x = 4z \end{cases}$$

$$y + 5z = 775{,}000$$

$0.09y + 0.42z = 67{,}500$

$z = 75{,}000$

$y = 775{,}000 - 5z = 400{,}000$

$x = 4z = 300{,}000$

\$300,000 was borrowed at 8%.

\$400,000 was borrowed at 9%.

\$75,000 was borrowed at 10%.

54. Let x = amount at 8%.

Let y = amount at 9%.

Let z = amount at 10%.

$$\begin{cases} x + y + z = 800{,}000 \\ 0.08x + 0.09y + 0.10z = 67{,}000 \\ x = 5z \end{cases}$$

$$\begin{cases} y + 6z = 800{,}000 \\ 0.09y + 0.5z = 67{,}000 \end{cases}$$

$z = 125{,}000$

$y = 800{,}000 - 6(125{,}000) = 50{,}000$

$x = 5(125{,}000) = 625{,}000$

Solution: x = \$625,000 at 8%

y = \$50,000 at 9%

z = \$125,000 at 10%

55. Let C = amount in certificates of deposit.

Let M = amount in municipal bonds.

Let B = amount in blue-chip stocks.

Let G = amount in growth or speculative stocks.

$$\begin{cases} C + M + B + G = 500{,}000 \\ 0.10C + 0.08M + 0.12B + 0.13G = 0.10(500{,}000) \\ B + G = \frac{1}{4}(500{,}000) \end{cases}$$

This system has infinitely many solutions.

Let $G = s$, then $B = 125{,}000 - s$

$M = 125{,}000 + \frac{1}{2}s$

$C = 250{,}000 - \frac{1}{2}s$

One possible solution is to let $s = 50{,}000$.

Certificates of deposit: \$225,000

Municipal bonds: \$150,000

Blue-chip stocks: \$75,000

Growth or speculative stocks: \$50,000

56. Let C = amount in certificates of deposit.

Let M = amount in municipal bonds.

Let B = amount in blue-chip stocks.

Let G = amount in growth or speculative stocks.

$$\begin{cases} C + M + B + G = 500{,}000 \\ 0.09C + 0.05M + 0.12B + 0.14G = 0.10(500{,}000) \\ B + G = \frac{1}{4}(500{,}000) \end{cases}$$

This system has infinitely many solutions.

Let $G = s$, then $B = 125{,}000 - s$

$M = \frac{1}{2}s - 31{,}250$

$C = 406{,}250 - \frac{1}{2}s.$

Solution:

$406{,}250 - \frac{1}{2}s$ in certificates of deposit,

$-31{,}250 + \frac{1}{2}s$ in municipal bonds,

$125{,}000 - s$ in blue-chip stocks,

s in growth stocks

One possible solution is to let s = \$100,000.

Certificates of deposit: \$356,250

Municipal bonds: \$18,750

Blue-chip stocks: \$25,000

Growth or speculative stocks: \$100,000

57. Let x = pounds of brand X.

Let y = pounds of brand Y.

Let z = pounds of brand Z.

Fertilizer A: $\frac{1}{3}y + \frac{2}{9}z = 5$

Fertilizer B: $\frac{1}{2}x + \frac{2}{3}y + \frac{5}{9}z = 13$

Fertilizer C: $\frac{1}{2}x + \frac{2}{9}z = 4$

$$\begin{cases} \frac{1}{2}x + \frac{2}{3}y + \frac{5}{9}z = 13 \\ \frac{1}{3}y + \frac{2}{9}z = 5 \\ \frac{1}{2}x + \frac{2}{9}z = 4 \end{cases} \quad \text{Interchange Eq.1 and Eq.2.}$$

$$\begin{cases} \frac{1}{2}x + \frac{2}{3}y + \frac{5}{9}z = 13 \\ \frac{1}{3}y + \frac{2}{9}z = 5 \\ -\frac{2}{3}y - \frac{1}{3}z = -9 \quad -\text{Eq.1} + \text{Eq.3} \end{cases}$$

$$\begin{cases} \frac{1}{2}x + \frac{2}{3}y + \frac{5}{9}z = 13 \\ \frac{1}{3}y + \frac{2}{9}z = 5 \\ \frac{1}{9}z = 1 \quad 2\text{Eq.2} + \text{Eq.3} \end{cases}$$

$z = 9$

$\frac{1}{3}y + \frac{2}{9}(9) = 5 \implies y = 9$

$\frac{1}{2}x + \frac{2}{3}(9) + \frac{5}{9}(9) = 13 \implies x = 4$

4 pounds of brand X, 9 pounds of brand Y, and 9 pounds of brand Z are needed to obtain the desired mixture.

58. Let x = liters of spray X.

Let y = liters of spray Y.

Let z = liters of spray Z.

$$\left.\begin{array}{l} \text{Chemical A: } \frac{1}{5}x + \frac{1}{2}z = 12 \\ \text{Chemical B: } \frac{2}{5}x + \frac{1}{2}z = 16 \end{array}\right\} \implies x = 20, z = 16$$

Chemical C: $\frac{2}{5}x + y = 26 \implies y = 18$

20 liters of spray X, 18 liters of spray Y, and 16 liters of spray Z are needed to get the desired mixture.

59. Let x = pounds of Vanilla coffee.

Let y = pounds of Hazelnut coffee.

Let z = pounds of French Roast coffee.

$$\begin{cases} x + y + z = 10 \\ 2x + 2.50y + 3z = 26 \\ y - z = 0 \end{cases}$$

$$\begin{cases} x + y + z = 10 \\ 0.5y + z = 6 \quad -2\text{Eq.1} + \text{Eq.2} \\ y - z = 0 \end{cases}$$

$$\begin{cases} x + y + z = 10 \\ 0.5y + z = 6 \\ -3z = -12 \quad -2\text{Eq.2} + \text{Eq.3} \end{cases}$$

$z = 4$

$0.5y + 4 = 6 \implies y = 4$

$x + 4 + 4 = 10 \implies x = 2$

2 pounds of Vanilla coffee, 4 pounds of Hazelnut coffee, and 4 pounds of French Roast coffee are needed to obtain the desired mixture.

60. Each centerpiece costs \$30.

Let x = number of roses in a centerpiece.

Let y = number of lilies.

Let z = number of irises.

$$\begin{cases} x + y + z = 12 \\ 2.5x + 4y + 2z = 30 \\ x - 2y - 2z = 0 \end{cases}$$

$$\begin{cases} x + y + z = 12 \\ 3.5x + 2y = 30 \quad \text{Eq.3} + \text{Eq.2} \\ 3x = 24 \quad 2\text{Eq.1} + \text{Eq.3} \end{cases}$$

$3x = 24 \implies x = 8$

$3.5x + 2y = 30 \implies y = \frac{1}{2}(30 - 3.5(8))$

$= \frac{1}{2}(30 - 28) = \frac{1}{2}(2) = 1$

$x + y + z = 12 \implies z = 12 - 8 - 1 = 3$

The point (8, 1, 3) is the solution of the system of equations.

Each centerpiece should contain 8 roses, 1 lily, and 3 irises.

61. Let x = number of television ads.

Let y = number of radio ads.

Let z = number of local newspaper ads.

$$\begin{cases} x + y + z = 60 \\ 1000x + 200y + 500z = 42{,}000 \\ x - y - z = 0 \end{cases}$$

$$\begin{cases} x + y + z = 60 \\ -800y - 500z = -18{,}000 & -1000\text{Eq.1} + \text{Eq.2} \\ -2y - 2z = -60 & -\text{Eq.1} + \text{Eq.3} \end{cases}$$

$$\begin{cases} x + y + z = 60 \\ -2y - 2z = -60 & \text{Interchange} \\ -800y - 500z = -18{,}000 & \text{Eq.2 and Eq.3.} \end{cases}$$

$$\begin{cases} x + y + z = 60 \\ -2y - 2z = -60 \\ 300z = 6000 & -400\text{Eq.2} + \text{Eq.3} \end{cases}$$

$z = 20$

$-2y - 2(20) = -60 \implies y = 10$

$x + 10 + 20 = 60 \implies x = 30$

30 television ads, 10 radio ads, and 20 newspaper ads can be run each month.

62. Let x = number of rock songs.

Let y = number of dance songs.

Let z = number of pop songs.

$$\begin{cases} x + y + z = 32 \\ x - 2z = 0 \\ y - z = -4 \end{cases}$$

$$\begin{cases} x + y + z = 32 \\ -y - 3z = -32 & (-1)\text{Eq.1} + \text{Eq.2} \\ y - z = -4 \end{cases}$$

$$\begin{cases} x + y + z = 32 \\ -y - 3z = -32 \\ -4z = -36 & \text{Eq.2} + \text{Eq.3} \end{cases}$$

$-4z = -36 \implies z = 9$

$-y - 3(9) = -32 \implies y = 5$

$x + 5 + 9 = 32 \implies x = 18$

Play 18 rock songs, 5 dance songs, and 9 pop songs.

63. (a) To use 2 liters of the 50% solution:

Let x = amount of 10% solution.

Let y = amount of 20% solution.

$x + y = 8 \implies y = 8 - x$

$$x(0.10) + y(0.20) + 2(0.50) = 10(0.25)$$
$$0.10x + 0.20(8 - x) + 1 = 2.5$$
$$0.10x + 1.6 - 0.20x + 1 = 2.5$$
$$-0.10x = -0.1$$
$$x = 1 \text{ liter of 10\% solution}$$
$$y = 7 \text{ liters of 20\% solution}$$

Given: 2 liters of 50% solution

(b) To use as little of the 50% solution as possible, the chemist should use no 10% solution.

Let x = amount of 20% solution.

Let y = amount of 50% solution.

$x + y = 10 \implies y = 10 - x$

$$x(0.20) + y(0.50) = 10(0.25)$$
$$x(0.20) + (10 - x)(0.50) = 10(0.25)$$
$$x(0.20) + 5 - 0.50x = 2.5$$
$$-0.30x = -2.5$$
$$x = 8\tfrac{1}{3} \text{ liters of 20\% solution}$$
$$y = 1\tfrac{2}{3} \text{ liters of 50\% solution}$$

(c) To use as much of the 50% solution as possible, the chemist should use no 20% solution.

Let x = amount of 10% solution.

Let y = amount of 50% solution.

$x + y = 10 \implies y = 10 - x$

$$x(0.10) + y(0.50) = 10(0.25)$$
$$0.10x + 0.50(10 - x) = 2.5$$
$$0.10x + 5 - 0.50x = 2.5$$
$$-0.40x = -2.5$$
$$x = 6\tfrac{1}{4} \text{ liters of 10\% solution}$$
$$y = 3\tfrac{3}{4} \text{ liters of 50\% solution}$$

64. Let x = amount of 10% solution.

Let y = amount of 15% solution.

Let z = amount of 25% solution.

$$\begin{cases} x + y + z = 12 \\ 0.10x + 0.15y + 0.25z = 0.20 \cdot 12 \end{cases}$$

$$\begin{cases} x + y + z = 12 \\ 2x + 3y + 5z = 48 & 20\text{Eq.}2 \end{cases}$$

(a) If $z = 4$,

$$\begin{cases} x + y + 4 = 12 \\ 2x + 3y + 20 = 48 \end{cases}$$

$$\begin{cases} x + y = 8 \\ 2x + 3y = 28 \end{cases}$$

$$\begin{cases} x + y = 8 \\ y = 12 & \text{Eq.}2 + (-2)\text{Eq.}1 \end{cases}$$

$y = 12 \implies x = 8 - 12 = -4$, but $x \geq 0$.

There is no solution; 4 gallons of the 25% solution is not enough.

(b) $$\begin{cases} x + y + z = 12 \\ 2x + 3y + 5z = 48 \end{cases}$$

Minimize z while $x \geq 0$, $y \geq 0$, and $z \geq 0$.

$$\begin{cases} x + y + z = 12 \\ -x + 2z = 12 & (-3)\text{Eq.}1 + \text{Eq.}2 \end{cases}$$

$-x + 2z = 12 \implies z = 6 + \frac{1}{2}x \implies z$ is smallest when $x = 0$.

$x = 0$ and $z = 6 \implies y = 6$

The 12-gallon mixture using the least amount of the 25% solution is made using none of the 10% solution and 6 gallons each of the 15% and 25% solution.

(c) $$\begin{cases} x + y + z = 12 \\ 2x + 3y + 5z = 48 \end{cases}$$

$$\begin{cases} x + y + z = 12 \\ y + 3z = 24 & (-2)\text{Eq.}1 + \text{Eq.}2 \end{cases}$$

$y + 3z = 24 \implies z = 8 - \frac{1}{3}y \implies z$ is largest when $y = 0$.

$y = 0$ and $z = 8 \implies x = 12 - 0 - 8 = 4$.

The 12-gallon mixture made with the largest portion of the 25% solution contains 4 gallons of the 10% solution, none of the 15% solution, and 8 gallons of the 25% solution.

65. $$\begin{cases} I_1 - I_2 + I_3 = 0 & \text{Equation 1} \\ 3I_1 + 2I_2 = 7 & \text{Equation 2} \\ 2I_2 + 4I_3 = 8 & \text{Equation 3} \end{cases}$$

$$\begin{cases} I_1 - I_2 + I_3 = 0 \\ 5I_2 - 3I_3 = 7 & (-3)\text{Eq.}1 + \text{Eq.}2 \\ 2I_2 + 4I_3 = 8 \end{cases}$$

$$\begin{cases} I_1 - I_2 + I_3 = 0 \\ 10I_2 - 6I_3 = 14 & 2\text{Eq.}2 \\ 10I_2 + 20I_3 = 40 & 5\text{Eq.}3 \end{cases}$$

$$\begin{cases} I_1 - I_2 + I_3 = 0 \\ 10I_2 - 6I_3 = 14 \\ 26I_3 = 26 & (-1)\text{Eq.}2 + \text{Eq.}3 \end{cases}$$

$26I_3 = 26 \implies I_3 = 1$
$10I_2 - 6(1) = 14 \implies I_2 = 2$
$I_1 - 2 + 1 = 0 \implies I_1 = 1$

Solution: $I_1 = 1, I_2 = 2, I_3 = 1$

66. (a) $\begin{cases} t_1 - 2t_2 = 0 \\ t_1 - 2a = 128 \\ t_2 + a = 32 \end{cases}$ $\Rightarrow$ $2t_2 - 2a = 128$, $\Rightarrow$ $-2t_2 - 2a = -64$

$$-4a = 64$$
$$a = -16$$
$$t_2 = 48$$
$$t_1 = 96$$

So, $t_1 = 96$ pounds

$t_2 = 48$ pounds

$a = -16$ feet per second squared.

(b) $\begin{cases} t_1 - 2t_2 = 0 & \text{Equation 1} \\ t_1 - 2a = 128 & \text{Equation 2} \\ t_2 + 2a = 64 & \text{Equation 3} \end{cases}$

$\begin{cases} t_1 - 2t_2 = 0 \\ 2t_2 - 2a = 128 & (-1)\text{Eq.1} + \text{Eq.2} \\ t_2 + 2a = 64 \end{cases}$

$\begin{cases} t_1 - 2t_2 = 0 \\ 2t_2 - 2a = 128 \\ 3a = 0 & \left(-\frac{1}{2}\right)\text{Eq.2} + \text{Eq.3} \end{cases}$

$3a = 0 \Rightarrow a = 0$

$2t_2 - 2(0) = 128 \Rightarrow t_2 = 64$

$t_1 - 2(64) = 0 \Rightarrow t_1 = 128$

Solution: $a = 0$ ft/sec^2

$t_1 = 128$ lb

$t_2 = 64$ lb

The system is stable.

67. $(-4, 5), (-2, 6), (2, 6), (4, 2)$

$n = 4, \sum_{i=1}^{4} x_i = 0, \sum_{i=1}^{4} x_i^2 = 40, \sum_{i=1}^{4} x_i^3 = 0, \sum_{i=1}^{4} x_i^4 = 544, \sum_{i=1}^{4} y_i = 19, \sum_{i=1}^{4} x_i y_i = -12, \sum_{i=1}^{4} x_i^2 y_i = 160$

$\begin{cases} 4c + 40a = 19 \\ 40b = -12 \\ 40c + 544a = 160 \end{cases}$

$\begin{cases} 4c + 40a = 19 \\ 40b = -12 \\ 144a = -30 & -10\text{Eq.1} + \text{Eq.3} \end{cases}$

$144a = -30 \Rightarrow a = -\frac{5}{24}$

$40b = -12 \Rightarrow b = -\frac{3}{10}$

$4c + 40\left(-\frac{5}{24}\right) = 19 \Rightarrow c = \frac{41}{6}$

Least squares regression parabola: $y = -\frac{5}{24}x^2 - \frac{3}{10}x + \frac{41}{6}$

68. $\begin{cases} 5c + 10a = 8 \\ 10b = 12 \\ 10c + 34a = 22 \end{cases}$

$\begin{cases} 5c + 10a = 8 \\ 10b = 12 \\ 14a = 6 & (-2)\text{Eq.1} + \text{Eq.3} \end{cases}$

$14a = 6 \Rightarrow a = \frac{3}{7}$

$10b = 12 \Rightarrow b = \frac{6}{5}$

$5c + 10\left(\frac{3}{7}\right) = 8 \Rightarrow c = \frac{26}{35}$

Least squares regression parabola: $y = \frac{3}{7}x^2 + \frac{6}{5}x + \frac{26}{35}$

69. (0, 0), (2, 2), (3, 6), (4, 12)

$n = 4, \sum_{i=1}^{4} x_i = 9, \sum_{i=1}^{4} x_i^2 = 29, \sum_{i=1}^{4} x_i^3 = 99, \sum_{i=1}^{4} x_i^4 = 353, \sum_{i=1}^{4} y_i = 20, \sum_{i=1}^{4} x_i y_i = 70, \sum_{i=1}^{4} x_i^2 y_i = 254$

$$\begin{cases} 4c + 9b + 29a = 20 \\ 9c + 29b + 99a = 70 \\ 29c + 99b + 353a = 254 \end{cases}$$

$$\begin{cases} 9c + 29b + 99a = 70 & \text{Interchange equations.} \\ 4c + 9b + 29a = 20 \\ 29c + 99b + 353a = 254 \end{cases}$$

$$\begin{cases} c + 11b + 41a = 30 & -2\text{Eq.}2 + \text{Eq.}1 \\ -35b - 135a = -100 & -4\text{Eq.}1 + \text{Eq.}2 \\ -220b - 836a = -616 & -29\text{Eq.}1 + \text{Eq.}3 \end{cases}$$

$$\begin{cases} c + 11b + 41a = 30 \\ 1540b + 5940a = 4400 & -44\text{Eq.}2 \\ -1540b - 5852a = -4312 & 7\text{Eq.}3 \end{cases}$$

$$\begin{cases} c + 11b + 41a = 30 \\ 1540b + 5940a = 4400 \\ 88a = 88 & \text{Eq.}2 + \text{Eq.}3 \end{cases}$$

$88a = 88 \implies a = 1$

$1540b + 5940(1) = 4400 \implies b = -1$

$c + 11(-1) + 41(1) = 30 \implies c = 0$

Least squares regression parabola: $y = x^2 - x$

70.
$$\begin{cases} 4c + 6b + 14a = 25 \\ 6c + 14b + 36a = 21 \\ 14c + 36b + 98a = 33 \end{cases}$$

$$\begin{cases} 4c + 6b + 14a = 25 \\ -10b - 30a = 33 & 3\text{Eq.}1 - 2\text{Eq.}2 \\ -60b - 196a = 218 & 14\text{Eq.}1 - 4\text{Eq.}3 \end{cases}$$

$$\begin{cases} 4c + 6b + 14a = 25 \\ -10b - 30a = 33 \\ -16a = 20 & (-6)\text{Eq.}2 + \text{Eq.}3 \end{cases}$$

$-16a = 20 \implies a = -\frac{5}{4}$

$-10b - 30\left(-\frac{5}{4}\right) = 33 \implies b = \frac{9}{20}$

$4c + 6\left(\frac{9}{20}\right) + 14\left(-\frac{5}{4}\right) = 25 \implies c = \frac{199}{20}$

Least squares regression parabola: $y = -\frac{5}{4}x^2 + \frac{9}{20}x + \frac{199}{20}$

71. (a) (100, 75), (120, 68), (140, 55)

$$n = 3,\ \sum_{i=1}^{3} x_i = 360,\ \sum_{i=1}^{3} x_i^2 = 44{,}000,\ \sum_{i=1}^{3} x_i^3 = 5{,}472{,}000$$

$$\sum_{i=1}^{3} x_i^4 = 691{,}520{,}000,\ \sum_{i=1}^{3} y_i = 198,\ \sum_{i=1}^{3} x_i y_i = 23{,}360,$$

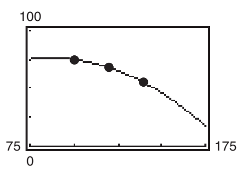
$$\sum_{i=1}^{3} x_i^2 y_i = 2{,}807{,}200$$

$$\begin{aligned} 3c + 360b + 44{,}000a &= 198 \\ 360c + 44{,}000b + 5{,}472{,}000a &= 23{,}360 \\ 44{,}000c + 5{,}472{,}000b + 691{,}520{,}000a &= 2{,}807{,}200 \end{aligned}$$

Solving this system yields $a = -0.0075$, $b = 1.3$ and $c = 20$.

Least squares regression parabola:
$y = -0.0075x^2 + 1.3x + 20$

(b)



(c)

x	Actual Percent y	Model Approximation y
100	75	75
120	68	68
140	55	55

The model is a good fit to the actual data. The values are the same.

(d) For $x = 170$:

$y = -0.0075(170)^2 + 1.3(170) + 20$

$= 24.25\%$

(e) For $y = 40$:

$$40 = -0.0075x^2 + 1.3x + 20$$

$0.0075x^2 - 1.3x + 20 = 0$

By the Quadratic Formula we have $x \approx 17$ or $x \approx 156$.

Choosing the value that fits with our data, we have 156 females.

72. (30, 55), (40, 105), (50, 188)

(a) $\begin{cases} 3c + 120b + 5000a = 348 \\ 120c + 5000b + 216{,}000a = 15{,}250 \\ 5000c + 216{,}000b + 9{,}620{,}000a = 687{,}500 \end{cases}$

$\begin{cases} 3c + 120b + 5000a = 348 \\ 200b + 16{,}000a = 1330 & (-40)\text{Eq.}1 + \text{Eq.}2 \\ 48{,}000b + 3{,}860{,}000a = 322{,}500 & (-5000)\text{Eq.}1 + (3)\text{Eq.}3 \end{cases}$

$\begin{cases} 3c + 120b + 5000a = 348 \\ 200b + 16{,}000a = 1330 \\ 20{,}000a = 3300 & (-240)\text{Eq.}2 + \text{Eq.}3 \end{cases}$

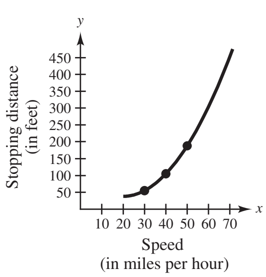
$$20{,}000a = 3300 \implies a = 0.165$$

$$200b + 16{,}000(0.165) = 1330 \implies b = -6.55$$

$3c + 120(-6.55) + 5000(0.165) = 348 \implies c = 103$

Least-squares regression parabola: $y = 0.165x^2 - 6.55x + 103$

(b)



(c) When $x = 70$, $y = 453$ feet.

73. Let x = number of touchdowns.

Let y = number of extra-point kicks.

Let z = number of two-point conversions.

Let w = number of field goals.

$$\begin{cases} x + y + z + w = 16 \\ 6x + y + 2z + 3w = 32 + 29 \\ x - 4w = 0 & \Longrightarrow x = 4w \\ 2z - w = 0 & \Longrightarrow z = \frac{1}{2}w \end{cases}$$

$$\begin{cases} 4w + y + \frac{1}{2}w + w = 16 \Longrightarrow 5.5w + y = 16 \\ 6(4w) + y + 2\left(\frac{1}{2}\right)w + 3w = 61 \Longrightarrow 28w + y = 61 \end{cases}$$

$$\begin{array}{r} 28w + y = 61 \\ -5.5w - y = -16 \\ \hline 22.5w \quad = 45 \end{array}$$

$$w = 2$$

$$y = 5$$

$$x = 4w = 8$$

$$z = \tfrac{1}{2}w = 1$$

Thus, 8 touchdowns, 5 extra-point kicks, 1 two-point conversion, and 2 field goals were scored.

74. Let t = number of touchdowns.

Let x = number of extra-points.

Let f = number of field goals.

Let s = number of safeties.

$$\begin{cases} t + x + f + s = 22 \\ 6t + x + 3f + 2s = 74 \\ t - x = 0 \\ f - 3s = 0 \end{cases}$$

$$\begin{cases} 2t + f + s = 22 & \text{Eq.1 + Eq.3} \\ 7t + 3f + 2s = 74 & \text{Eq.2 + Eq.3} \\ t - x = 0 \\ f - 3s = 0 \end{cases}$$

$$\begin{cases} 2t + 4s = 22 & \text{Eq.1} - \text{Eq.4} \\ 7t + 3f + 2s = 74 \\ t - x = 0 \\ f - 3s = 0 \end{cases}$$

$$\begin{cases} 2t + 4s = 22 & \text{Eq.2} - (3)\text{Eq.4} \\ 7t + 11s = 74 \\ t - x = 0 \\ f - 3s = 0 \end{cases}$$

$$\begin{cases} t + 2s = 11 & \left(\frac{1}{2}\right)\text{Eq.1} \\ 7t + 11s = 74 \\ t - x = 0 \\ f - 3s = 0 \end{cases}$$

$$\begin{cases} t + 2s = 11 \\ -3s = -3 & (-7)\text{Eq.1} + \text{Eq.2} \\ t - x = 0 \\ f - 3s = 0 \end{cases}$$

$-3s = -3 \Longrightarrow s = 1$

$t + 2(1) = 11 \Longrightarrow t = 9$

$9 - x = 0 \Longrightarrow x = 9$

$f - 3(1) = 0 \Longrightarrow f = 3$

There were 9 touchdowns, each with an extra point; and there were 3 field goals and 1 safety.

75. $$\begin{cases} y + \lambda = 0 \\ x + \lambda = 0 \\ x + y - 10 = 0 \end{cases} \begin{array}{l} \left.\right\} \Longrightarrow x = y = -\lambda \\ \Longrightarrow 2x - 10 = 0 \end{array}$$

$$x = 5$$

$$y = 5$$

$$\lambda = -5$$

76. $$\begin{cases} 2x + \lambda = 0 \\ 2y + \lambda = 0 \\ x + y - 4 = 0 \end{cases} \begin{array}{l} \left.\right\} x = y = -\dfrac{\lambda}{2} \\ \Longrightarrow 2x - 4 = 0 \end{array}$$

$$2x = 4$$

$$x = 2$$

$$y = 2$$

$$\lambda = -4$$

77. $\begin{cases} 2x - 2x\lambda = 0 \implies 2x(1-\lambda) = 0 \implies \lambda = 1 \text{ or } x = 0 \\ -2y + \lambda = 0 \\ y - x^2 = 0 \end{cases}$

If $\lambda = 1$:

$$2y = \lambda \implies y = \frac{1}{2}$$

$$x^2 = y \implies x = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$$

If $x = 0$:

$$x^2 = y \implies y = 0$$

$$2y = \lambda \implies \lambda = 0$$

Solution: $x = \pm\frac{\sqrt{2}}{2}$ or $x = 0$

$y = \frac{1}{2}$ $\quad$ $y = 0$

$\lambda = 1$ $\quad$ $\lambda = 0$

78. $\begin{cases} 2 + 2y + 2\lambda = 0 \\ 2x + 1 + \lambda = 0 \implies \lambda = -2x - 1 \\ 2x + y - 100 = 0 \end{cases}$

$$2 + 2y + 2(-2x - 1) = 0 \implies -4x + 2y = 0 \implies -4x + 2y = 0$$

$$2x + y - 100 = 0 \implies 2x + y = 100 \implies 4x + 2y = 200$$

$$4y = 200$$

$$y = 50$$

$$x = 25$$

$$\lambda = -2(25) - 1 = -51$$

79. False. Equation 2 does not have a leading coefficient of 1.

80. True. If a system of three linear equations is inconsistent, then it has no points in common to all three equations.

81. No, they are not equivalent. There are two arithmetic errors. The constant in the second equation should be -11 and the coefficient of z in the third equation should be 2.

82. When using Gaussian elimination to solve a system of linear equations, a system has no solution when there is a row representing a contradictory equation such as $0 = N$, where N is a nonzero real number.

For instance:

$x + y = 3$ $\quad$ Equation 1

$-x - y = 3$ $\quad$ Equation 2

$x + y = 0$

$0 = 6$ $\quad$ Eq.1 + Eq.2

No solution

83. There are an infinite number of linear systems that have $(4, -1, 2)$ as their solution. Two such systems are as follows:

$$\begin{cases} 3x + y - z = 9 \\ x + 2y - z = 0 \\ -x + y + 3z = 1 \end{cases} \qquad \begin{cases} x + y + z = 5 \\ x - 2z = 0 \\ 2y + z = 0 \end{cases}$$

84. There are an infinite number of linear systems that have $(-5, -2, 1)$ as their solution. Two systems are:

$$\begin{cases} x + y + z = -6 \\ -2x - y + 3z = 15 \\ x + 4y - z = -14 \end{cases} \qquad \begin{cases} 2x - y - z = -9 \\ -x + 2y + 2z = 3 \\ -3x + y - 2z = 11 \end{cases}$$

85. There are an infinite number of linear systems that have $\left(3, -\frac{1}{2}, \frac{7}{4}\right)$ as their solution. Two such systems are as follows:

$$\begin{cases} x + 2y - 4z = -5 \\ -x - 4y + 8z = 13 \\ x + 6y + 4z = 7 \end{cases} \qquad \begin{cases} x + 2y + 4z = 9 \\ y + 2z = 3 \\ x - 4z = -4 \end{cases}$$

86. There are an infinite number of linear systems that have $\left(-\frac{3}{2}, 4, -7\right)$ as their solution. Two systems are:

$$\begin{cases} 2x - y + 3z = -28 \\ -6x + 4y + z = 18 \\ -4x - 2y - 3z = 19 \end{cases} \qquad \begin{cases} 4x + y - 2z = 12 \\ 4y + 2z = 2 \\ -2x + y + z = 0 \end{cases}$$

87. $(0.075)(85) = 6.375$

88.
$$225 = \frac{x}{100}(150)$$
$$225 = 1.5x$$
$$150\% = x$$

89.
$$(0.005)n = 400$$
$$n = 80{,}000$$

90.
$$(0.48)n = 132$$
$$n = 275$$

91. $(7 - i) + (4 + 2i) = (7 + 4) + (-i + 2i) = 11 + i$

92.
$$(-6 + 3i) - (1 + 6i) = (-6 - 1) + (3 - 6)i$$
$$= -7 - 3i$$

93.
$$(4 - i)(5 + 2i) = 20 + 8i - 5i - 2i^2 = 20 + 3i + 2$$
$$= 22 + 3i$$

94.
$$(1 + 2i)(3 - 4i) = 3 - 4i + 6i - 8i^2$$
$$= 3 + 2i - 8(-1) = 11 + 2i$$

95.
$$\frac{i}{1 + i} + \frac{6}{1 - i} = \frac{i(1 - i) + 6(1 + i)}{(1 + i)(1 - i)}$$
$$= \frac{i - i^2 + 6 + 6i}{1 - i^2}$$
$$= \frac{7 + 7i}{2}$$
$$= \frac{7}{2} + \frac{7}{2}i$$

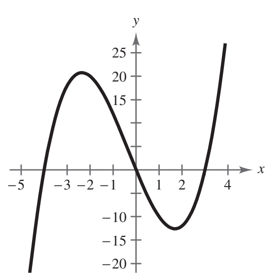
96.
$$\frac{i}{4 + i} - \frac{2i}{8 - 3i} = \frac{i}{4 + i}\left(\frac{4 - i}{4 - i}\right) - \frac{2i}{8 - 3i}\left(\frac{8 + 3i}{8 + 3i}\right)$$
$$= \frac{1 + 4i}{17} - \frac{-6 + 16i}{73}$$
$$= \frac{73(1 + 4i) - 17(-6 + 16i)}{17(73)}$$
$$= \frac{73 + 292i + 102 - 272i}{1241}$$
$$= \frac{175}{1241} + \frac{20}{1241}i$$

97. $f(x) = x^3 + x^2 - 12x$

(a)
$$x^3 + x^2 - 12x = 0$$
$$x(x^2 + x - 12) = 0$$
$$x(x + 4)(x - 3) = 0$$

Zeros: $x = -4, 0, 3$

(b)

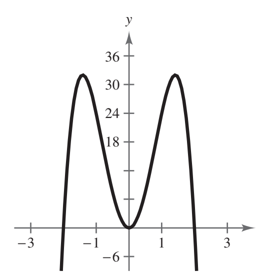


98. $f(x) = -8x^4 + 32x^2$

(a)
$$-8x^4 + 32x^2 = 0$$
$$-8x^2(x^2 - 4) = 0$$

Zeros: $x = 0, \pm 2$

(b)



99. $f(x) = 2x^3 + 5x^2 - 21x - 36$

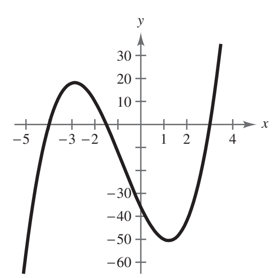
(a) $2x^3 + 5x^2 - 21x - 36 = 0$

3	2	5	−21	−36
		6	33	36
	2	11	12	0

$$f(x) = (x - 3)(2x^2 + 11x + 12)$$
$$= (x - 3)(x + 4)(2x + 3)$$

Zeros: $x = -4, -\frac{3}{2}, 3$

(b)



100. $f(x) = 6x^3 - 29x^2 - 6x + 5$

(a) $6x^3 - 29x^2 - 6x + 5 = 0$

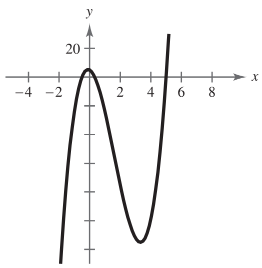
$$\begin{array}{r|rrrr} 5 & 6 & -29 & -6 & 5 \\ & & 30 & 5 & -5 \\ \hline & 6 & 1 & -1 & 0 \end{array}$$

$f(x) = (x - 5)(6x^2 + x - 1)$

$= (x - 5)(3x - 1)(2x + 1)$

Zeros: $x = 5, \frac{1}{3}, -\frac{1}{2}$

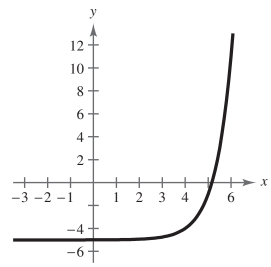
(b)



101. $y = 4^{x-4} - 5$

x	-2	0	2	4	5
y	-4.9998	-4.996	-4.938	-4	-1

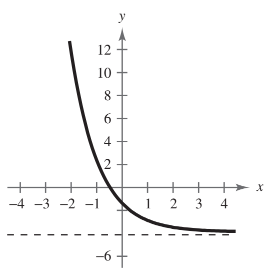
Horizontal asymptote: $y = -5$



102. $y = \left(\frac{5}{2}\right)^{-x+1} - 4$

Horizontal asymptote: $y = -4$

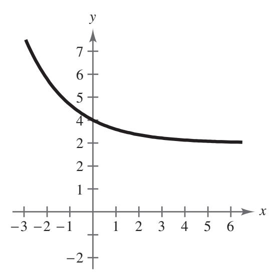
x	y
-2	11.625
-1	2.25
0	-1.5
1	-3
2	-3.6



103. $y = 1.9^{-0.8x} + 3$

x	-2	-1	0	1	2
y	5.793	4.671	4	3.598	3.358

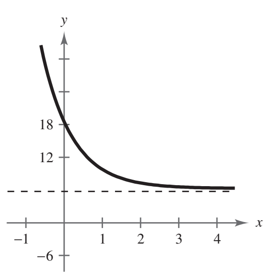
Horizontal asymptote: $y = 3$



104. $y = 3.5^{-x+2} + 6$

Horizontal asymptote: $y = 6$

x	y
$-\frac{1}{2}$	28.918
0	18.25
$\frac{1}{2}$	12.548
1	9.5
2	7



105. $\begin{cases} 2x + y = 120 & \text{Equation 1} \\ x + 2y = 120 & \text{Equation 2} \end{cases}$

$$\begin{array}{rll} 2x + y = & 120 & \\ -2x - 4y = & -240 & (-2)\text{Eq.2} \\ \hline -3y = & -120 & \\ y = & 40 & \end{array}$$

$x + 2(40) = 120 \implies x = 40$

Solution: $(40, 40)$

106. $\begin{cases} 6x - 5y = 3 & \text{Equation 1} \\ 10x - 12y = 5 & \text{Equation 2} \end{cases}$

$$\begin{cases} 72x - 60y = 36 & 12\text{Eq.}1 \\ -50x + 60y = -25 & (-5)\text{Eq.}2 \end{cases}$$

$$22x = 11$$

$$x = \tfrac{1}{2}$$

$6\left(\frac{1}{2}\right) - 5y = 3 \implies y = 0$

Solution: $\left(\frac{1}{2}, 0\right)$

107. Answers will vary.

Section 7.4 Partial Fractions

- You should know how to decompose a rational function $\dfrac{N(x)}{D(x)}$ into partial fractions.
  - (a) If the fraction is improper, divide to obtain

    $$\frac{N(x)}{D(x)} = p(x) + \frac{N_1(x)}{D(x)}$$

    where $p(x)$ is a polynomial.
  - (b) Factor the denominator completely into linear and irreducible quadratic factors.
  - (c) For each factor of the form $(px + q)^m$, the partial fraction decomposition includes the terms

    $$\frac{A_1}{(px + q)} + \frac{A_2}{(px + q)^2} + \cdots + \frac{A_m}{(px + q)^m}.$$
  - (d) For each factor of the form $(ax^2 + bx + c)^n$, the partial fraction decomposition includes the terms

    $$\frac{B_1x + C_1}{ax^2 + bx + c} + \frac{B_2x + C_2}{(ax^2 + bx + c)^2} + \cdots + \frac{B_nx + C_n}{(ax^2 + bx + c)^n}.$$
- You should know how to determine the values of the constants in the numerators.
  - (a) Set $\dfrac{N_1(x)}{D(x)} =$ partial fraction decomposition.
  - (b) Multiply both sides by $D(x)$ to obtain the basic equation.
  - (c) For distinct linear factors, substitute the zeros of the distinct linear factors into the basic equation.
  - (d) For repeated linear factors, use the coefficients found in part (c) to rewrite the basic equation. Then use other values of x to solve for the remaining coefficients.
  - (e) For quadratic factors, expand the basic equation, collect like terms, and then equate the coefficients of like terms.

Vocabulary Check

1. partial fraction decomposition
2. improper
3. m; n; irreducible
4. basic equation

1. $\dfrac{3x - 1}{x(x - 4)} = \dfrac{A}{x} + \dfrac{B}{x - 4}$

Matches (b).

2. $\dfrac{3x - 1}{x^2(x - 4)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x - 4}$

Matches (c).

3. $\dfrac{3x - 1}{x(x^2 + 4)} = \dfrac{A}{x} + \dfrac{Bx + C}{x^2 + 4}$

Matches (d).

4. $\dfrac{3x-1}{x(x^2-4)} = \dfrac{3x-1}{x(x-2)(x+2)} = \dfrac{A}{x} + \dfrac{B}{x-2} + \dfrac{C}{x+2}$

Matches (a).

5. $\dfrac{7}{x^2-14x} = \dfrac{7}{x(x-14)} = \dfrac{A}{x} + \dfrac{B}{x-14}$

6. $\dfrac{x-2}{x^2+4x+3} = \dfrac{x-2}{(x+3)(x+1)} = \dfrac{A}{x+3} + \dfrac{B}{x+1}$

7. $\dfrac{12}{x^3-10x^2} = \dfrac{12}{x^2(x-10)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-10}$

8. $\dfrac{x^2-3x+2}{4x^3+11x^2} = \dfrac{x^2-3x+2}{x^2(4x+11)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{4x+11}$

9. $\dfrac{4x^2+3}{(x-5)^3} = \dfrac{A}{x-5} + \dfrac{B}{(x-5)^2} + \dfrac{C}{(x-5)^3}$

10. $\dfrac{6x+5}{(x+2)^4} = \dfrac{6x+5}{(x+2)(x+2)(x+2)(x+2)}$

$= \dfrac{A}{x+2} + \dfrac{B}{(x+2)^2} + \dfrac{C}{(x+2)^3} + \dfrac{D}{(x+2)^4}$

11. $\dfrac{2x-3}{x^3+10x} = \dfrac{2x-3}{x(x^2+10)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+10}$

12. $\dfrac{x-6}{2x^3+8x} = \dfrac{x-6}{2x(x^2+4)} = \dfrac{A}{2x} + \dfrac{Bx+C}{x^2+4}$

13. $\dfrac{x-1}{x(x^2+1)^2} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1} + \dfrac{Dx+E}{(x^2+1)^2}$

14. $\dfrac{x+4}{x^2(3x-1)^2} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{3x-1} + \dfrac{D}{(3x-1)^2}$

15. $\dfrac{1}{x^2-1} = \dfrac{A}{x+1} + \dfrac{B}{x-1}$

$1 = A(x-1) + B(x+1)$

Let $x = -1$: $1 = -2A \implies A = -\dfrac{1}{2}$

Let $x = 1$: $1 = 2B \implies B = \dfrac{1}{2}$

$\dfrac{1}{x^2-1} = \dfrac{1/2}{x-1} - \dfrac{1/2}{x+1} = \dfrac{1}{2}\left(\dfrac{1}{x-1} - \dfrac{1}{x+1}\right)$

16. $\dfrac{1}{4x^2-9} = \dfrac{A}{2x+3} + \dfrac{B}{2x-3}$

$1 = A(2x-3) + B(2x+3)$

Let $x = -\dfrac{3}{2}$: $1 = -6A \implies A = -\dfrac{1}{6}$

Let $x = \dfrac{3}{2}$: $1 = 6B \implies B = \dfrac{1}{6}$

$\dfrac{1}{4x^2-9} = \dfrac{1}{6}\left(\dfrac{1}{2x-3} - \dfrac{1}{2x+3}\right)$

17. $\dfrac{1}{x^2+x} = \dfrac{A}{x} + \dfrac{B}{x+1}$

$1 = A(x+1) + Bx$

Let $x = 0$: $1 = A$

Let $x = -1$: $1 = -B \implies B = -1$

$\dfrac{1}{x^2+x} = \dfrac{1}{x} - \dfrac{1}{x+1}$

18. $\dfrac{3}{x^2-3x} = \dfrac{A}{x-3} + \dfrac{B}{x}$

$3 = Ax + B(x-3)$

Let $x = 3$: $3 = 3A \implies A = 1$

Let $x = 0$: $3 = -3B \implies B = -1$

$\dfrac{3}{x^2-3x} = \dfrac{1}{x-3} - \dfrac{1}{x}$

19. $\dfrac{1}{2x^2+x} = \dfrac{A}{2x+1} + \dfrac{B}{x}$

$1 = Ax + B(2x+1)$

Let $x = -\dfrac{1}{2}$: $1 = -\dfrac{1}{2}A \implies A = -2$

Let $x = 0$: $1 = B$

$\dfrac{1}{2x^2+x} = \dfrac{1}{x} - \dfrac{2}{2x+1}$

20. $\dfrac{5}{x^2+x-6} = \dfrac{A}{x+3} + \dfrac{B}{x-2}$

$5 = A(x-2) + B(x+3)$

Let $x = -3$: $5 = -5A \implies A = -1$

Let $x = 2$: $5 = 5B \implies B = 1$

$\dfrac{5}{x^2+x-6} = \dfrac{1}{x-2} - \dfrac{1}{x+3}$

21. $\dfrac{3}{x^2+x-2} = \dfrac{A}{x-1} + \dfrac{B}{x+2}$

$3 = A(x+2) + B(x-1)$

Let $x = 1$: $3 = 3A \implies A = 1$

Let $x = -2$: $3 = -3B \implies B = -1$

$\dfrac{3}{x^2+x-2} = \dfrac{1}{x-1} - \dfrac{1}{x+2}$

22. $\dfrac{x+1}{x^2+4x+3} = \dfrac{x+1}{(x+3)(x+1)} = \dfrac{1}{x+3}, x \neq -1$

23. $\dfrac{x^2+12x+12}{x^3-4x} = \dfrac{A}{x} + \dfrac{B}{x+2} + \dfrac{C}{x-2}$

$x^2 + 12x + 12 = A(x+2)(x-2) + Bx(x-2) + Cx(x+2)$

Let $x = 0$: $12 = -4A \implies A = -3$

Let $x = -2$: $-8 = 8B \implies B = -1$

Let $x = 2$: $40 = 8C \implies C = 5$

$\dfrac{x^2+12x+12}{x^3-4x} = -\dfrac{3}{x} - \dfrac{1}{x+2} + \dfrac{5}{x-2}$

24. $\dfrac{x+2}{x(x-4)} = \dfrac{A}{x} + \dfrac{B}{x-4}$

$x + 2 = A(x-4) + Bx$

Let $x = 0$: $2 = -4A \implies A = -\dfrac{1}{2}$

Let $x = 4$: $6 = 4B \implies B = \dfrac{3}{2}$

$\dfrac{x+2}{x(x-4)} = \dfrac{1}{2}\left(\dfrac{3}{x-4} - \dfrac{1}{x}\right)$

25. $\dfrac{4x^2+2x-1}{x^2(x+1)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+1}$

$4x^2 + 2x - 1 = Ax(x+1) + B(x+1) + Cx^2$

Let $x = 0$: $-1 = B$

Let $x = -1$: $1 = C$

Let $x = 1$: $5 = 2A + 2B + C$

$5 = 2A - 2 + 1$

$6 = 2A$

$3 = A$

$\dfrac{4x^2+2x-1}{x^2(x+1)} = \dfrac{3}{x} - \dfrac{1}{x^2} + \dfrac{1}{x+1}$

26. $\dfrac{2x-3}{(x-1)^2} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2}$

$2x - 3 = A(x-1) + B$

Let $x = 1$: $-1 = B$

Let $x = 0$: $-3 = -A + B$

$-3 = -A - 1$

$2 = A$

$\dfrac{2x-3}{(x-1)^2} = \dfrac{2}{x-1} - \dfrac{1}{(x-1)^2}$

27. $\dfrac{3x}{(x-3)^2} = \dfrac{A}{x-3} + \dfrac{B}{(x-3)^2}$

$3x = A(x-3) + B$

Let $x = 3$: $9 = B$

Let $x = 0$: $0 = -3A + B$

$0 = -3A + 9$

$3 = A$

$\dfrac{3x}{(x-3)^2} = \dfrac{3}{x-3} + \dfrac{9}{(x-3)^2}$

28. $\dfrac{6x^2+1}{x^2(x-1)^2} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-1} + \dfrac{D}{(x-1)^2}$

$6x^2 + 1 = Ax(x-1)^2 + B(x-1)^2 + Cx^2(x-1) + Dx^2$

Let $x = 0 : 1 = B$

Let $x = 1 : 7 = D$

Substitute B and D into the equation, expand the binomials, collect like terms, and equate the coefficients of like terms.

$-2x^2 + 2x = (A + C)x^3 + (-2A - C)x^2 + Ax$

$A = 2$

$-2A - C = -2 \implies C = -2$ or

$A + C = 0 \implies C = -2$

$\dfrac{6x^2+1}{x^2(x-1)^2} = \dfrac{2}{x} + \dfrac{1}{x^2} - \dfrac{2}{x-1} + \dfrac{7}{(x-1)^2}$

29. $\dfrac{x^2-1}{x(x^2+1)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1}$

$$\begin{aligned} x^2 - 1 &= A(x^2+1) + (Bx+C)x \\ &= Ax^2 + A + Bx^2 + Cx \\ &= (A+B)x^2 + Cx + A \end{aligned}$$

Equating coefficients of like terms gives

$1 = A + B$, $0 = C$, and $-1 = A$.

Therefore, $A = -1$, $B = 2$, and $C = 0$.

$\dfrac{x^2-1}{x(x^2+1)} = -\dfrac{1}{x} + \dfrac{2x}{x^2+1}$

30. $\dfrac{x}{(x-1)(x^2+x+1)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+x+1}$

$$\begin{aligned} x &= A(x^2+x+1) + (Bx+C)(x-1) \\ &= Ax^2 + Ax + A + Bx^2 - Bx + Cx - C \\ &= (A+B)x^2 + (A-B+C)x + (A-C) \end{aligned}$$

Equating coefficients of like powers gives $0 = A + B$, $1 = A - B + C$, and $0 = A - C$. Substituting $-A$ for B and A for C in the second equation gives $1 = 3A$, so $A = \frac{1}{3}$, $B = -\frac{1}{3}$, and $C = \frac{1}{3}$.

$\dfrac{x}{(x-1)(x^2+x+1)} = \dfrac{1}{3}\left(\dfrac{1}{x-1} - \dfrac{x-1}{x^2+x+1}\right)$

31. $\dfrac{x}{x^3 - x^2 - 2x + 2} = \dfrac{x}{(x-1)(x^2-2)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2-2}$

$$\begin{aligned} x &= A(x^2-2) + (Bx+C)(x-1) \\ &= Ax^2 - 2A + Bx^2 - Bx + Cx - C \\ &= (A+B)x^2 + (C-B)x - (2A+C) \end{aligned}$$

Equating coefficients of like terms gives $0 = A + B$, $1 = C - B$, and $0 = 2A + C$. Therefore, $A = -1$, $B = 1$, and $C = 2$.

$\dfrac{x}{x^3 - x^2 - 2x + 2} = \dfrac{-1}{x-1} + \dfrac{x+2}{x^2-2}$

32. $\dfrac{x+6}{x^3 - 3x^2 - 4x + 12} = \dfrac{x+6}{(x+2)(x-2)(x-3)} = \dfrac{A}{x+2} + \dfrac{B}{x-2} + \dfrac{C}{x-3}$

$x + 6 = A(x-2)(x-3) + B(x+2)(x-3) + C(x+2)(x-2)$

Let $x = 3 :\quad 9 = 5C \implies \frac{9}{5} = C$

Let $x = -2 : 4 = 20A \implies \frac{1}{5} = A$

Let $x = 2 :\quad 8 = -4B \implies -2 = B$

$\dfrac{x+6}{x^3 - 3x^2 - 4x + 12} = \dfrac{\frac{1}{5}}{x+2} + \dfrac{-2}{x-2} + \dfrac{\frac{9}{5}}{x-3} = \dfrac{1}{5}\left(\dfrac{1}{x+2} - \dfrac{10}{x-2} + \dfrac{9}{x-3}\right)$

33. $\dfrac{x^2}{x^4 - 2x^2 - 8} = \dfrac{x^2}{(x^2 - 4)(x^2 + 2)} = \dfrac{x^2}{(x + 2)(x - 2)(x^2 + 2)}$

$$= \frac{A}{x + 2} + \frac{B}{x - 2} + \frac{Cx + D}{x^2 + 2}$$

$$x^2 = A(x - 2)(x^2 + 2) + B(x + 2)(x^2 + 2) + (Cx + D)(x + 2)(x - 2)$$

$$= A(x^3 - 2x^2 + 2x - 4) + B(x^3 + 2x^2 + 2x + 4) + (Cx + D)(x^2 - 4)$$

$$= Ax^3 - 2Ax^2 + 2Ax - 4A + Bx^3 + 2Bx^2 + 2Bx + 4B + Cx^3 + Dx^2 - 4Cx - 4D$$

$$= (A + B + C)x^3 + (-2A + 2B + D)x^2 + (2A + 2B - 4C)x + (-4A + 4B - 4D)$$

Equating coefficients of like terms gives

$0 = A + B + C$, $1 = -2A + 2B + D$, $0 = 2A + 2B - 4C$, and $0 = -4A + 4B - 4D$.

Using the first and third equation, we have $A + B + C = 0$ and $A + B - 2C = 0$; by subtraction, $C = 0$. Using the second and fourth equation, we have $-2A + 2B + D = 1$ and $-2A + 2B - 2D = 0$; by subtraction, $3D = 1$, so $D = \frac{1}{3}$. Substituting 0 for C and $\frac{1}{3}$ for D in the first and second equations, we have

$A + B = 0$ and $-2A + 2B = \frac{2}{3}$, so $A = -\frac{1}{6}$ and $B = \frac{1}{6}$.

$$\frac{x^2}{x^4 - 2x^2 - 8} = \frac{-\frac{1}{6}}{x + 2} + \frac{\frac{1}{6}}{x - 2} + \frac{\frac{1}{3}}{x^2 + 2}$$

$$= \frac{1}{3(x^2 + 2)} - \frac{1}{6(x + 2)} + \frac{1}{6(x - 2)}$$

$$= \frac{1}{6}\left(\frac{2}{x^2 + 2} - \frac{1}{x + 2} + \frac{1}{x - 2}\right)$$

34. $\dfrac{2x^2 + x + 8}{(x^2 + 4)^2} = \dfrac{Ax + B}{x^2 + 4} + \dfrac{Cx + D}{(x^2 + 4)^2}$

$2x^2 + x + 8 = (Ax + B)(x^2 + 4) + Cx + D$

$2x^2 + x + 8 = Ax^3 + Bx^2 + (4A + C)x + (4B + D)$

Equating coefficients of like powers:

$0 = A$

$2 = B$

$1 = 4A + C \implies C = 1$

$8 = 4B + D \implies D = 0$

$$\frac{2x^2 + x + 8}{(x^2 + 4)^2} = \frac{2}{x^2 + 4} + \frac{x}{(x^2 + 4)^2}$$

35. $\dfrac{x}{16x^4 - 1} = \dfrac{x}{(4x^2 - 1)(4x^2 + 1)} = \dfrac{x}{(2x + 1)(2x - 1)(4x^2 + 1)}$

$$= \frac{A}{2x + 1} + \frac{B}{2x - 1} + \frac{Cx + D}{4x^2 + 1}$$

$$x = A(2x - 1)(4x^2 + 1) + B(2x + 1)(4x^2 + 1) + (Cx + D)(2x + 1)(2x - 1)$$

$$= A(8x^3 - 4x^2 + 2x - 1) + B(8x^3 + 4x^2 + 2x + 1) + (Cx + D)(4x^2 - 1)$$

$$= 8Ax^3 - 4Ax^2 + 2Ax - A + 8Bx^3 + 4Bx^2 + 2Bx + B + 4Cx^3 + 4Dx^2 - Cx - D$$

$$= (8A + 8B + 4C)x^3 + (-4A + 4B + 4D)x^2 + (2A + 2B - C)x + (-A + B - D)$$

—CONTINUED—

35. —CONTINUED—

Equating coefficients of like terms gives $0 = 8A + 8B + 4C$, $0 = -4A + 4B + 4D$, $1 = 2A + 2B - C$, and $0 = -A + B - D$.

Using the first and third equations, we have $2A + 2B + C = 0$ and $2A + 2B - C = 1$; by subtraction, $2C = -1$, so $C = -\frac{1}{2}$.

Using the second and fourth equations, we have $-A + B + D = 0$ and $-A + B - D = 0$; by subtraction $2D = 0$, so $D = 0$.

Substituting $-\frac{1}{2}$ for C and 0 for D in the first and second equations, we have $8A + 8B = 2$ and $-4A + 4B = 0$, so $A = \frac{1}{8}$ and $B = \frac{1}{8}$.

$$\frac{x}{16x^4 - 1} = \frac{\frac{1}{8}}{2x + 1} + \frac{\frac{1}{8}}{2x - 1} + \frac{\left(-\frac{1}{2}\right)x}{4x^2 + 1}$$

$$= \frac{1}{8(2x + 1)} + \frac{1}{8(2x - 1)} - \frac{x}{2(4x^2 + 1)}$$

$$= \frac{1}{8}\left(\frac{1}{2x + 1} + \frac{1}{2x - 1} - \frac{4x}{4x^2 + 1}\right)$$

36. $\dfrac{x + 1}{x^3 + x} = \dfrac{A}{x} + \dfrac{Bx + C}{x^2 + 1}$

$= (A + B)x^2 + Cx + A$

Equating coefficients of like powers gives $0 = A + B$, $1 = C$, and $1 = A$. Therefore, $A = 1$, $B = -1$, and $C = 1$.

$$\frac{x + 1}{x^3 + x} = \frac{1}{x} - \frac{x - 1}{x^2 + 1}$$

37. $\dfrac{x^2 + 5}{(x + 1)(x^2 - 2x + 3)} = \dfrac{A}{x + 1} + \dfrac{Bx + C}{x^2 - 2x + 3}$

$$x^2 + 5 = A(x^2 - 2x + 3) + (Bx + C)(x + 1)$$

$$= Ax^2 - 2Ax + 3A + Bx^2 + Bx + Cx + C$$

$$= (A + B)x^2 + (-2A + B + C)x + (3A + C)$$

Equating coefficients of like terms gives $1 = A + B$, $0 = -2A + B + C$, and $5 = 3A + C$.

Subtracting both sides of the second equation from the first gives $1 = 3A - C$; combining this with the third equation gives $A = 1$ and $C = 2$. Since $A + B = 1$, we also have $B = 0$.

$$\frac{x^2 + 5}{(x + 1)(x^2 - 2x + 3)} = \frac{1}{x + 1} + \frac{2}{x^2 - 2x + 3}$$

38. $\dfrac{x^2 - 4x + 7}{(x + 1)(x^2 - 2x + 3)} = \dfrac{A}{x + 1} + \dfrac{Bx + C}{x^2 - 2x + 3}$

$$x^2 - 4x + 7 = A(x^2 - 2x + 3) + (Bx + C)(x + 1)$$

$$= Ax^2 - 2Ax + 3A + Bx^2 + Bx + Cx + C$$

$$= (A + B)x^2 + (-2A + B + C)x + (3A + C)$$

Equating coefficients of like terms gives $1 = A + B$, $-4 = -2A + B + C$, and $7 = 3A + C$. Adding the second and third equations, and subtracting the first, gives $2 = 2C$, so $C = 1$. Therefore, $A = 2$, $B = -1$, and $C = 1$.

$$\frac{x^2 - 4x + 7}{(x + 1)(x^2 - 2x + 3)} = \frac{2}{x + 1} - \frac{x - 1}{x^2 - 2x + 3}$$

39. $\dfrac{x^2 - x}{x^2 + x + 1} = 1 + \dfrac{-2x - 1}{x^2 + x + 1} = 1 - \dfrac{2x + 1}{x^2 + x + 1}$

40. $\dfrac{x^2 - 4x}{x^2 + x + 6}$

Using long division gives $\dfrac{x^2 - 4x}{x^2 + x + 6} = 1 - \dfrac{5x + 6}{x^2 + x + 6}$.

41. $\dfrac{2x^3 - x^2 + x + 5}{x^2 + 3x + 2} = 2x - 7 + \dfrac{18x + 19}{(x + 1)(x + 2)}$

$$\frac{18x + 19}{(x + 1)(x + 2)} = \frac{A}{x + 1} + \frac{B}{x + 2}$$

$$18x + 19 = A(x + 2) + B(x + 1)$$

Let $x = -1$: $1 = A$

Let $x = -2$: $-17 = -B \implies B = 17$

$$\frac{2x^3 - x^2 + x + 5}{x^2 + 3x + 2} = 2x - 7 + \frac{1}{x + 1} + \frac{17}{x + 2}$$

42. $\dfrac{x^3 + 2x^2 - x + 1}{x^2 + 3x - 4}$

Using long division gives:

$$\frac{x^3 + 2x^2 - x + 1}{x^2 + 3x - 4} = x - 1 + \frac{6x - 3}{x^2 + 3x - 4}$$

$$\frac{x^3 + 2x^2 - x + 1}{x^2 + 3x - 4} - x + 1 = \frac{6x - 3}{x^2 + 3x - 4} = \frac{6x - 3}{(x + 4)(x - 1)} = \left(\frac{A}{x + 4} + \frac{B}{x - 1}\right)$$

$$\frac{6x - 3}{(x + 4)(x - 1)} = \left(\frac{A}{x + 4} + \frac{B}{x - 1}\right)$$

$$6x - 3 = A(x - 1) + B(x + 4)$$

$$6x - 3 = (A + B)x + (4B - A)$$

$$A + B = 6 \implies A = 6 - B$$

$$4B - A = -3 \implies 4B - 6 + B = -3$$

$$5B - 6 = -3$$

$$5B = 3$$

$$B = \frac{3}{5}$$

$$A = 6 - \frac{3}{5} = \frac{30 - 3}{5} = \frac{27}{5}$$

$$\frac{x^3 + 2x^2 - x + 1}{x^2 + 3x - 4} = x - 1 + \left(\frac{\frac{27}{5}}{x + 4} + \frac{\frac{3}{5}}{x - 1}\right) = x - 1 + \frac{1}{5}\left(\frac{27}{x + 4} + \frac{3}{x - 1}\right)$$

43. $$\frac{x^4}{(x-1)^3} = \frac{x^4}{x^3 - 3x^2 + 3x - 1} = x + 3 + \frac{6x^2 - 8x + 3}{(x-1)^3}$$

$$\frac{6x^2 - 8x + 3}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$$

$$6x^2 - 8x + 3 = A(x-1)^2 + B(x-1) + C$$

Let $x = 1$: $1 = C$

$$\left.\begin{aligned} &\text{Let } x = 0\text{: } 3 = A - B + 1 \\ &\text{Let } x = 2\text{: } 11 = A + B + 1 \end{aligned}\right\} \begin{aligned} A - B &= 2 \\ A + B &= 10 \end{aligned}$$

So, $A = 6$ and $B = 4$.

$$\frac{x^4}{(x-1)^3} = x + 3 + \frac{6}{x-1} + \frac{4}{(x-1)^2} + \frac{1}{(x-1)^3}$$

44. $$\frac{16x^4}{(2x-1)^3} = \frac{16x^4}{8x^3 - 12x^2 + 6x - 1} = 2x + 3 + \frac{24x^2 - 16x + 3}{(2x-1)^3}$$

$$\frac{24x^2 - 16x + 3}{(2x-1)^2} = \frac{A}{2x-1} + \frac{B}{(2x-1)^2} + \frac{C}{(2x-1)^3}$$

$$24x^2 - 16x + 3 = A(2x-1)^2 + B(2x-1) + C$$

Let $x = \frac{1}{2}$: $1 = C$

$$24x^2 - 16x + 3 = 4Ax^2 - 4Ax + A + 2Bx - B + 1$$

$$24x^2 - 16x + 3 = 4Ax^2 + (-4A + 2B)x + (A - B + 1)$$

Equating coefficients of like powers:

$6 = A$, $3 = A - B + 1$

$3 = 6 - B + 1$

$4 = B$

$$\frac{16x^4}{(2x-1)^3} = 2x + 3 + \frac{6}{2x-1} + \frac{4}{(2x-1)^2} + \frac{1}{(2x-1)^3}$$

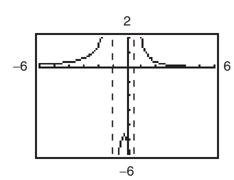
45. $$\frac{5 - x}{2x^2 + x - 1} = \frac{A}{2x-1} + \frac{B}{x+1}$$

$$-x + 5 = A(x+1) + B(2x-1)$$

Let $x = \frac{1}{2}$: $\frac{9}{2} = \frac{3}{2}A \implies A = 3$

Let $x = -1$: $6 = -3B \implies B = -2$

$$\frac{5 - x}{2x^2 + x - 1} = \frac{3}{2x-1} - \frac{2}{x+1}$$



46. $$\frac{3x^2 - 7x - 2}{x^3 - x} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$$

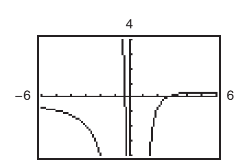
$$3x^2 - 7x - 2 = A(x^2 - 1) + Bx(x-1) + Cx(x+1)$$

Let $x = 0$: $-2 = -A \implies A = 2$

Let $x = -1$: $8 = 2B \implies B = 4$

Let $x = 1$: $-6 = 2C \implies C = -3$

$$\frac{3x^2 - 7x - 2}{x^3 - x} = \frac{2}{x} + \frac{4}{x+1} - \frac{3}{x-1}$$



47. $\dfrac{x-1}{x^3+x^2} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+1}$

$x - 1 = Ax(x+1) + B(x+1) + Cx^2$

Let $x = -1$: $-2 = C$

Let $x = 0$: $-1 = B$

Let $x = 1$: $0 = 2A + 2B + C$

$0 = 2A - 2 - 2$

$2 = A$

$\dfrac{x-1}{x^3+x^2} = \dfrac{2}{x} - \dfrac{1}{x^2} - \dfrac{2}{x+1}$

48. $\dfrac{4x^2-1}{2x(x+1)^2} = \dfrac{A}{2x} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$

$4x^2 - 1 = A(x+1)^2 + 2Bx(x+1) + 2Cx$

Let $x = 0$: $-1 = A$

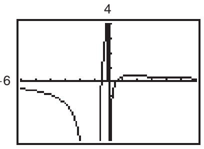
Let $x = -1$: $3 = -2C \implies C = -\dfrac{3}{2}$

Let $x = 1$: $3 = 4A + 4B + 2C$

$3 = -4 + 4B - 3$

$\dfrac{5}{2} = B$

$\dfrac{4x^2-1}{2x(x+1)^2} = \dfrac{1}{2}\left[-\dfrac{1}{x} + \dfrac{5}{x+1} - \dfrac{3}{(x+1)^2}\right]$



49. $\dfrac{x^2+x+2}{(x^2+2)^2} = \dfrac{Ax+B}{x^2+2} + \dfrac{Cx+D}{(x^2+2)^2}$

$x^2 + x + 2 = (Ax+B)(x^2+2) + Cx + D$

$x^2 + x + 2 = Ax^3 + Bx^2 + (2A+C)x + (2B+D)$

Equating coefficients of like powers:

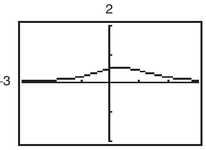
$0 = A$

$1 = B$

$1 = 2A + C \implies C = 1$

$2 = 2B + D \implies D = 0$

$\dfrac{x^2+x+2}{(x^2+2)^2} = \dfrac{1}{x^2+2} + \dfrac{x}{(x^2+2)^2}$



50. $\dfrac{x^3}{(x+2)^2(x-2)^2} = \dfrac{A}{x+2} + \dfrac{B}{(x+2)^2} + \dfrac{C}{x-2} + \dfrac{D}{(x-2)^2}$

$x^3 = A(x+2)(x-2)^2 + B(x-2)^2 + C(x+2)^2(x-2) + D(x+2)^2$

Let $x = -2$: $-8 = 16B \implies B = -\dfrac{1}{2}$

Let $x = 2$: $8 = 16D \implies D = \dfrac{1}{2}$

$x^3 = A(x+2)(x-2)^2 - \dfrac{1}{2}(x-2)^2 + C(x+2)^2(x-2) + \dfrac{1}{2}(x+2)^2$

$x^3 - 4x = (A+C)x^3 + (-2A+2C)x^2 + (-4A-4C)x + (8A-8C)$

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50. —CONTINUED—

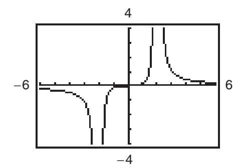
Equating coefficients of like powers:

$0 = -2A + 2C \implies A = C$

$1 = A + C$

$1 = 2A \implies A = \frac{1}{2} \implies C = \frac{1}{2}$

$$\frac{x^3}{(x+2)^2(x-2)^2} = \frac{1}{2}\left[\frac{1}{x+2} - \frac{1}{(x+2)^2} + \frac{1}{x-2} + \frac{1}{(x-2)^2}\right]$$



51. $$\frac{2x^3 - 4x^2 - 15x + 5}{x^2 - 2x - 8} = 2x + \frac{x+5}{(x+2)(x-4)}$$

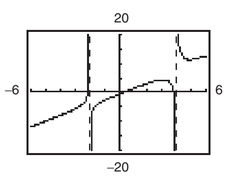
$$\frac{x+5}{(x+2)(x-4)} = \frac{A}{x+2} + \frac{B}{x-4}$$

$$x + 5 = A(x-4) + B(x+2)$$

Let $x = -2$: $3 = -6A \implies A = -\frac{1}{2}$

Let $x = 4$: $9 = 6B \implies B = \frac{3}{2}$

$$\frac{2x^3 - 4x^2 - 15x + 5}{x^2 - 2x - 8} = 2x + \frac{1}{2}\left(\frac{3}{x-4} - \frac{1}{x+2}\right)$$



52. $$\frac{x^3 - x + 3}{x^2 + x - 2} = x - 1 + \frac{2x+1}{(x+2)(x-1)}$$

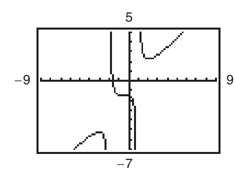
$$\frac{2x+1}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$$

$$2x + 1 = A(x-1) + B(x+2)$$

Let $x = -2$: $-3 = -3A \implies A = 1$

Let $x = 1$: $3 = 3B \implies B = 1$

$$\frac{x^3 - x + 3}{x^2 + x - 2} = x - 1 + \frac{1}{x+2} + \frac{1}{x-1}$$



53. (a) $\frac{x-12}{x(x-4)} = \frac{A}{x} + \frac{B}{x-4}$

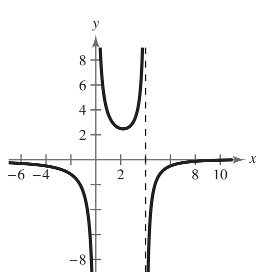
$x - 12 = A(x-4) + Bx$

Let $x = 0$: $-12 = -4A \implies A = 3$

Let $x = 4$: $-8 = 4B \implies B = -2$

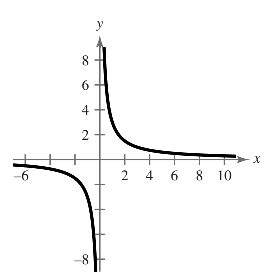
$\frac{x-12}{x(x-4)} = \frac{3}{x} - \frac{2}{x-4}$

(b) $y = \frac{x-12}{x(x-4)}$



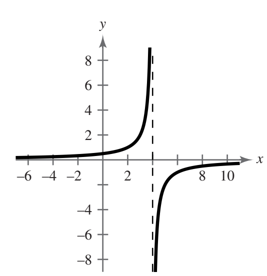
Vertical asymptotes: $x = 0$ and $x = 4$

$y = \frac{3}{x}$



Vertical asymptote: $x = 0$

$y = -\frac{2}{x-4}$



Vertical asymptote: $x = 4$

(c) The combination of the vertical asymptotes of the terms of the decomposition are the same as the vertical asymptotes of the rational function.

54. (a) $y = \dfrac{2(x+1)^2}{x(x^2+1)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1}$

$2(x+1)^2 = A(x^2+1) + Bx^2 + Cx$

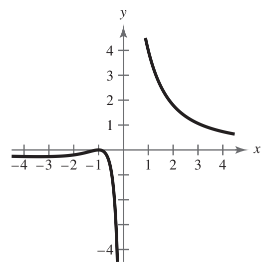
$2x^2 + 4x + 2 = (A+B)x^2 + Cx + A$

Equating coefficients of like powers gives $2 = A + B$, $4 = C$, and $2 = A$.

Therefore, $A = 2$, $B = 0$, and $C = 4$.

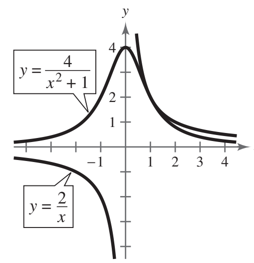
$\dfrac{2(x+1)^2}{x(x^2+1)} = \dfrac{2}{x} + \dfrac{4}{x^2+1}$

(b) $\dfrac{2(x+1)^2}{x(x^2+1)}$



Vertical asymptote at $x = 0$

$y = \dfrac{2}{x}$ and $y = \dfrac{4}{x^2+1}$



$y = \dfrac{2}{x}$ has vertical asymptote $x = 0$.

(c) The vertical asymptote of $y = \dfrac{2}{x}$ is the same as the vertical asymptote of the rational function.

55. (a) $\dfrac{2(4x-3)}{x^2-9} = \dfrac{A}{x-3} + \dfrac{B}{x+3}$

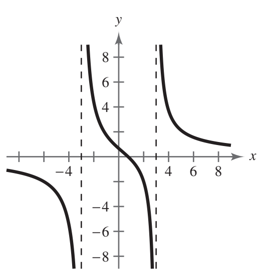
$2(4x-3) = A(x+3) + B(x-3)$

Let $x = 3$: $18 = 6A \implies A = 3$

Let $x = -3$: $-30 = -6B \implies B = 5$

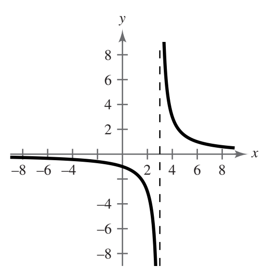
$\dfrac{2(4x-3)}{x^2-9} = \dfrac{3}{x-3} + \dfrac{5}{x+3}$

(b) $y = \dfrac{2(4x-3)}{x^2-9}$



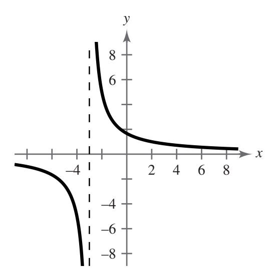
Vertical asymptotes: $x = \pm 3$

$y = \dfrac{3}{x-3}$



Vertical asymptote: $x = 3$

$y = \dfrac{5}{x+3}$



Vertical asymptote: $x = -3$

(c) The combination of the vertical asymptotes of the terms of the decomposition are the same as the vertical asymptotes of the rational function.

56. (a) $y = \dfrac{2(4x^2 - 15x + 39)}{x^2(x^2 - 10x + 26)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx + D}{x^2 - 10x + 26}$

$2(4x^2 - 15x + 39) = Ax(x^2 - 10x + 26) + B(x^2 - 10x + 26) + Cx^3 + Dx^2$

$8x^2 - 30x + 78 = Ax^3 - 10Ax^2 + 26Ax + Bx^2 - 10Bx + 26B + Cx^3 + Dx^2$

$= (A + C)x^3 + (-10A + B + D)x^2 + (26A - 10B)x + 26B$

Equating coefficients of like powers gives $0 = A + C$, $8 = -10A + B + D$, $-30 = 26A - 10B$, and $78 = 26B$. Since $78 = 26B$, $B = 3$. Therefore, $A = 0$, $B = 3$, $C = 0$, and $D = 5$.

$\dfrac{2(4x^2 - 15x + 39)}{x^2(x^2 - 10x + 26)} = \dfrac{3}{x^2} + \dfrac{5}{x^2 - 10x + 26}$

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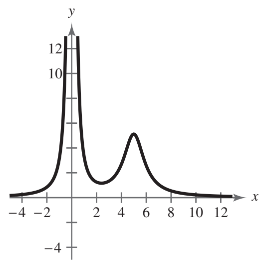
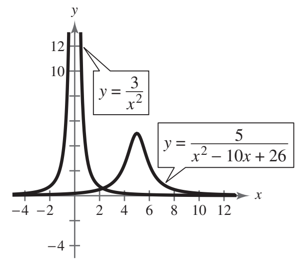
56. —CONTINUED—

(b) $\dfrac{2(4x^2 - 15x + 39)}{x^2(x^2 - 10x + 26)}$



Vertical asymptote is $x = 0$.

$\dfrac{3}{x^2}$ and $\dfrac{5}{x^2 - 10x + 26}$



$y = \dfrac{3}{x^2}$ has vertical asymptote $x = 0$.

(c) The vertical asymptote of $y = \dfrac{3}{x^2}$ is the same as the vertical asymptote of the rational function.

57. (a) $\dfrac{2000(4 - 3x)}{(11 - 7x)(7 - 4x)} = \dfrac{A}{11 - 7x} + \dfrac{B}{7 - 4x}, \; 0 < x \le 1$

$$2000(4 - 3x) = A(7 - 4x) + B(11 - 7x)$$

Let $x = \dfrac{11}{7}$: $-\dfrac{10,000}{7} = \dfrac{5}{7}A \implies A = -2000$

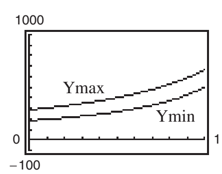
Let $x = \dfrac{7}{4}$: $-2500 = -\dfrac{5}{4}B \implies B = 2000$

$$\frac{2000(4 - 3x)}{(11 - 7x)(7 - 4x)} = \frac{-2000}{11 - 7x} + \frac{2000}{7 - 4x} = \frac{2000}{7 - 4x} - \frac{2000}{11 - 7x}, \; 0 < x \le 1$$

(b) $Y_{\max} = \left|\dfrac{2000}{7 - 4x}\right|$

$Y_{\min} = \left|\dfrac{2000}{11 - 7x}\right|$

(c)



(d) $Y_{\max}(0.5) = 400°\text{F}$

$Y_{\min}(0.5) \approx 266.7°\text{F}$

58. One way to find the constants is to choose values of the variable that eliminate one or more of the constants in the basic equation so that you can solve for another constant. If necessary, you can then use these constants with other chosen values of the variable to solve for any remaining constants. Another way is to expand the basic equation and collect like terms. Then you can equate coefficients of the like terms on each side of the equation to obtain simple equations involving the constants. If necessary, you can solve these equations using substitution.

59. False. The partial fraction decomposition is

$$\frac{A}{x + 10} + \frac{B}{x - 10} + \frac{C}{(x - 10)^2}.$$

60. False. The expression is an improper rational expression, so you must first divide before applying partial fraction decomposition.

61. $\dfrac{1}{a^2 - x^2} = \dfrac{A}{a + x} + \dfrac{B}{a - x}$, a is a constant.

$$1 = A(a - x) + B(a + x)$$

Let $x = -a$: $1 = 2aA \implies A = \dfrac{1}{2a}$

Let $x = a$: $1 = 2aB \implies B = \dfrac{1}{2a}$

$$\frac{1}{a^2 - x^2} = \frac{1}{2a}\left(\frac{1}{a + x} + \frac{1}{a - x}\right)$$

62. $\dfrac{1}{x(x + a)} = \dfrac{A}{x} + \dfrac{B}{x + a}$, a is a constant.

$$1 = A(x + a) + Bx$$

Let $x = 0$: $1 = aA \implies A = \dfrac{1}{a}$

Let $x = -a$: $1 = -aB \implies B = -\dfrac{1}{a}$

$$\frac{1}{x(x + a)} = \frac{1}{a}\left(\frac{1}{x} - \frac{1}{x + a}\right)$$

63. $\dfrac{1}{y(a-y)} = \dfrac{A}{y} + \dfrac{B}{a-y}$

$$1 = A(a-y) + By$$

Let $y = 0$: $1 = aA \implies A = \dfrac{1}{a}$

Let $y = a$: $1 = aB \implies B = \dfrac{1}{a}$

$\dfrac{1}{y(a-y)} = \dfrac{1}{a}\left(\dfrac{1}{y} + \dfrac{1}{a-y}\right)$

64. $\dfrac{1}{(x+1)(a-x)} = \dfrac{A}{x+1} + \dfrac{B}{a-x}$, a is a positive integer.

$$1 = A(a-x) + B(x+1)$$

Let $x = -1$: $1 = A(a+1) \implies A = \dfrac{1}{a+1}$

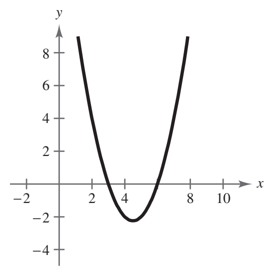
Let $x = a$: $1 = B(a+1) \implies B = \dfrac{1}{a+1}$

$\dfrac{1}{(x+1)(a-x)} = \dfrac{1}{a+1}\left(\dfrac{1}{x+1} + \dfrac{1}{a-x}\right)$

65. $f(x) = x^2 - 9x + 18 = (x-6)(x-3)$

Intercepts:
$(0, 18), (3, 0), (6, 0)$

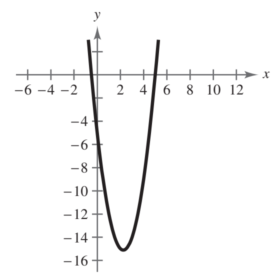
Graph rises to the left and rises to the right.



66. $f(x) = 2x^2 - 9x - 5 = (2x+1)(x-5)$
$= 2\left(x - \frac{9}{4}\right)^2 - \frac{121}{8}$

Vertex: $\left(\frac{9}{4}, -\frac{121}{8}\right)$

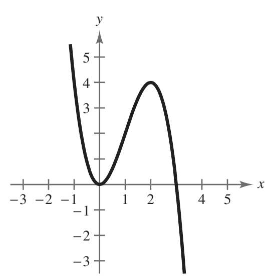
x-intercepts:
$\left(-\frac{1}{2}, 0\right), (5, 0)$



67. $f(x) = -x^2(x-3)$

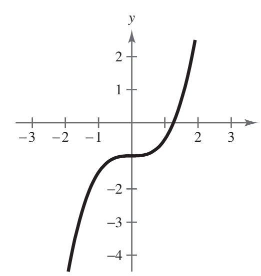
Intercepts: $(0, 0), (3, 0)$

Graph rises to the left and falls to the right.



68. $f(x) = \frac{1}{2}x^3 - 1$

Intercepts: $(0, -1), \left(\sqrt[3]{2}, 0\right)$



69. $f(x) = \dfrac{x^2 + x - 6}{x + 5}$

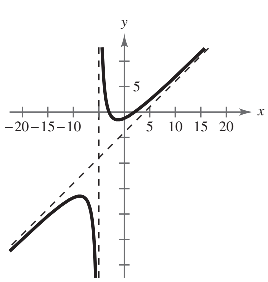
x-intercepts: $(-3, 0), (2, 0)$

y-intercept: $\left(0, -\frac{6}{5}\right)$

Vertical asymptote: $x = -5$

Slant asymptote: $y = x - 4$

No horizontal asymptote.

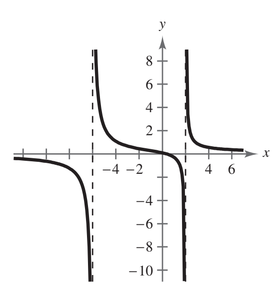


70. $f(x) = \dfrac{3x - 1}{x^2 + 4x - 12} = \dfrac{3x - 1}{(x+6)(x-2)}$

x-intercept: $\left(\frac{1}{3}, 0\right)$

Vertical asymptotes: $x = -6$ and $x = 2$

Horizontal asymptote: $y = 0$



Section 7.5 Systems of Inequalities

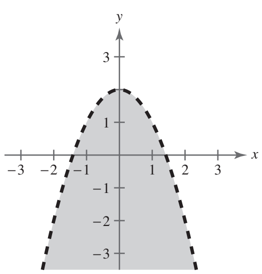
- You should be able to sketch the graph of an inequality in two variables.
  - (a) Replace the inequality with an equal sign and graph the equation. Use a dashed line for $<$ or $>$, a solid line for $\le$ or $\ge$.
  - (b) Test a point in each region formed by the graph. If the point satisfies the inequality, shade the whole region.
- You should be able to sketch systems of inequalities.

Vocabulary Check

1. solution
2. graph
3. linear
4. solution
5. consumer surplus

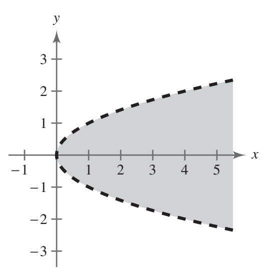
1. $y < 2 - x^2$

Using a dashed line, graph $y = 2 - x^2$ and shade inside the parabola.



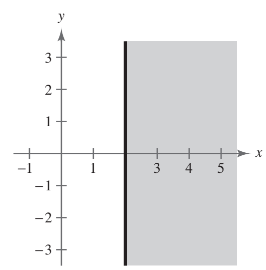
2. $y^2 - x < 0$

Using a dashed line, graph the parabola $y^2 - x = 0$, and shade the region inside this parabola. (Use $(1, 0)$ as a test point.)



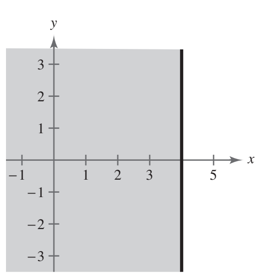
3. $x \ge 2$

Using a solid line, graph the vertical line $x = 2$ and shade to the right of this line.



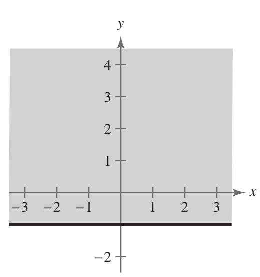
4. $x \le 4$

Using a solid line, graph the vertical line $x = 4$, and shade to the left of this line.



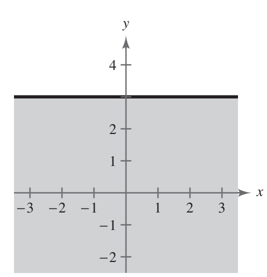
5. $y \ge -1$

Using a solid line, graph the horizontal line $y = -1$ and shade above this line.



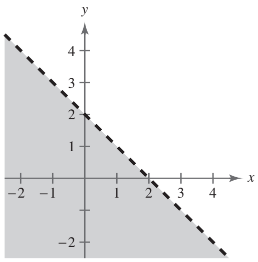
6. $y \le 3$

Using a solid line, graph the horizontal line $y = 3$, and shade below this line.



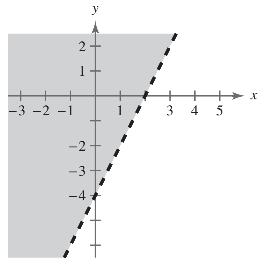
7. $y < 2 - x$

Using a dashed line, graph $y = 2 - x$, and then shade below the line. (Use $(0, 0)$ as a test point.)



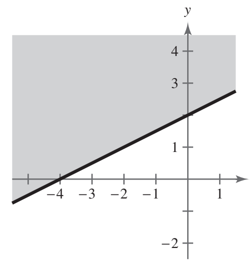
8. $y > 2x - 4$

Using a dashed line, graph $y = 2x - 4$, and shade above the line. (Use $(0, 0)$ as a test point.)



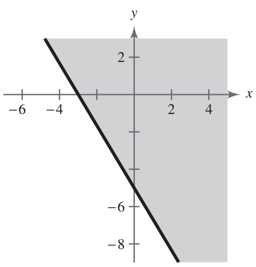
9. $2y - x \geq 4$

Using a solid line, graph $2y - x = 4$, and then shade above the line. (Use $(0, 0)$ as a test point.)



10. $5x + 3y \geq -15$

Using a solid line, graph $5x + 3y = -15$, and shade above the line. (Use $(0, 0)$ as a test point.)



11. $(x + 1)^2 + (y - 2)^2 < 9$

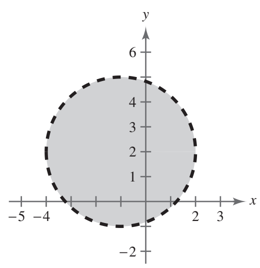
Using a dashed line, sketch the circle $(x + 1)^2 + (y - 2)^2 = 9$.

Center: $(-1, 2)$

Radius: 3

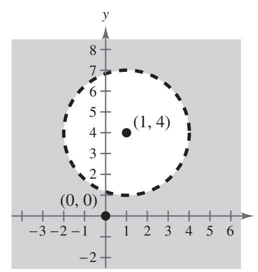
Test point: $(0, 0)$

Shade the inside of the circle.



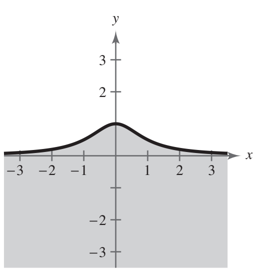
12. $(x - 1)^2 + (y - 4)^2 > 9$

Using a dashed line, graph the circle $(x - 1)^2 + (y - 4)^2 = 9$ and shade the exterior. The circle has center $(1, 4)$ and radius 3, so the origin could serve as a test point.



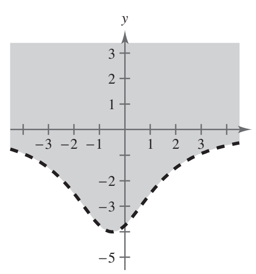
13. $y \leq \dfrac{1}{1 + x^2}$

Using a solid line, graph $y = \dfrac{1}{1 + x^2}$, and then shade below the curve. (Use $(0, 0)$ as a test point.)

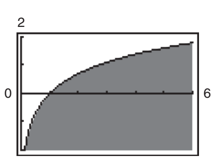


14. $y > \dfrac{-15}{x^2 + x + 4}$

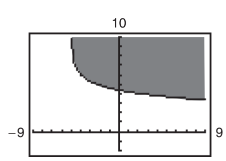
Using a dashed line, graph $y = \dfrac{-15}{x^2 + x + 4}$ and then shade above the curve. (Use $(0, 0)$ as a test point.)



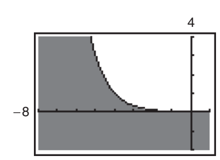
15. $y < \ln x$



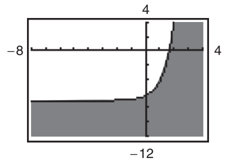
16. $y \geq 6 - \ln(x + 5)$



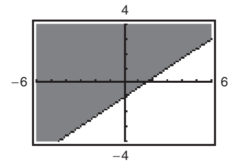
17. $y < 3^{-x-4}$



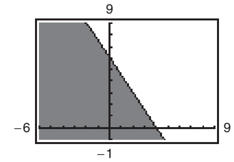
18. $y \le 2^{2x-0.5} - 7$



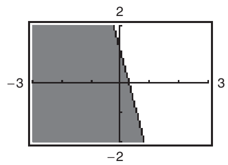
19. $y \ge \frac{2}{3}x - 1$



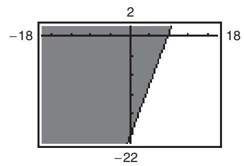
20. $y \le 6 - \frac{3}{2}x$



21. $y < -3.8x + 1.1$

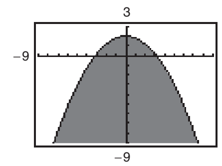


22. $y \ge -20.74 + 2.66x$



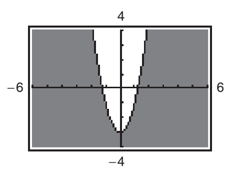
23. $x^2 + 5y - 10 \le 0$

$$y \le 2 - \frac{x^2}{5}$$



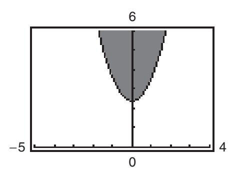
24. $2x^2 - y - 3 > 0$

$$y < 2x^2 - 3$$



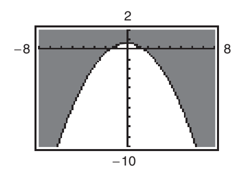
25. $\frac{5}{2}y - 3x^2 - 6 \ge 0$

$$y \ge \frac{2}{5}(3x^2 + 6)$$



26. $-\frac{1}{10}x^2 - \frac{3}{8}y < -\frac{1}{4}$

$$y > \frac{2}{3} - \frac{4}{15}x^2$$



27. The line through $(-4, 0)$ and $(0, 2)$ is $y = \frac{1}{2}x + 2$. For the shaded region below the line, we have $y \le \frac{1}{2}x + 2$.

28. The parabola through $(-2, 0)$, $(0, -4)$, $(2, 0)$ is $y = x^2 - 4$. For the shaded region inside the parabola, we have $y \ge x^2 - 4$.

29. The line through $(0, 2)$ and $(3, 0)$ is $y = -\frac{2}{3}x + 2$. For the shaded region above the line, we have

$$y \ge -\frac{2}{3}x + 2.$$

30. The circle shown is $x^2 + y^2 = 9$. For the shaded region inside the circle, we have $x^2 + y^2 \le 9$.

31. $\begin{cases} x \ge -4 \\ y > -3 \\ y \le -8x - 3 \end{cases}$

(a) $0 \le -8(0) - 3$, False

$(0, 0)$ *is not* a solution.

(b) $-3 > -3$, False

$(-1, -3)$ *is not* a solution.

(c) $-4 \ge -4$, True

$0 > -3$, True

$0 \le -8(-4) - 3$, True

$(-4, 0)$ *is* a solution.

(d) $-3 \ge -4$, True

$11 > -3$, True

$11 < -8(-3) - 3$, True

$(-3, 11)$ *is* a solution.

32. $\begin{cases} -2x + 5y \geq 3 \\ y < 4 \\ -4x + 2y < 7 \end{cases}$

(a) $-2(0) + 5(2) \geq 3$, True

$2 < 4$, True

$-4(0) + 2(2) < 7$, True

$(0, 2)$ *is* a solution

(b) $-2(-6) + 5(4) \geq 3$, True

$4 < 4$, False

$(-6, 4)$ *is not* a solution.

(c) $-2(-8) + 5(-2) \geq 3$, True

$-2 < 4$, True

$-4(-8) + 2(-2) < 7$, False

$(-8, -2)$ *is not* a solution.

(d) $-2(-3) + 5(2) \geq 3$, True

$2 < 4$, True

$-4(-3) + 2(2) < 7$, False

$(-3, 2)$ *is not* a solution.

33. $\begin{cases} 3x + y > 1 \\ -y - \frac{1}{2}x^2 \leq -4 \\ -15x + 4y > 0 \end{cases}$

(a) $3(0) + (10) > 1$, True

$-10 - \frac{1}{2}(0)^2 \leq -4$, True

$-15(0) + 4(10) > 0$, True

$(0, 10)$ *is* a solution.

(b) $3(0) + (-1) > 1$, False $\Longrightarrow$ $(0, -1)$ *is not* a solution.

(c) $3(2) + (9) > 1$, True

$-9 - \frac{1}{2}(2)^2 \leq -4$, True

$-15(2) + 4(9) > 0$, True

$(2, 9)$ *is* a solution.

(d) $3(-1) + 6 > 1$, True

$-6 - \frac{1}{2}(-1)^2 \leq -4$, True

$-15(-1) + 4(6) > 0$, True

$(-1, 6)$ *is* a solution.

34. $\begin{cases} x^2 + y^2 \geq 36 \\ -3x + y \leq 10 \\ \frac{2}{3}x - y \geq 5 \end{cases}$

(a) $(-1)^2 + 7^2 \geq 36$, True

$-3(-1) + 7 \leq 10$, True

$\frac{2}{3}(-1) - 7 \geq 5$, False

$(-1, 7)$ *is not* a solution.

(b) $(-5)^2 + 1^2 \geq 36$, False

$(-5, 1)$ *is not* a solution.

(c) $6^2 + 0^2 \geq 36$, True

$-3(6) + 0 \leq 0$, True

$\frac{2}{3}(6) - 0 \geq 5$, False

$(6, 0)$ *is not* a solution.

(d) $4^2 + (-8)^2 \geq 36$, True

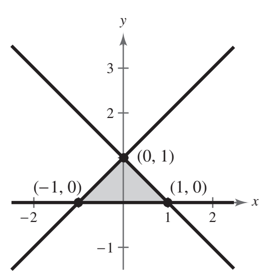
$-3(4) - 8 \leq 10$, True

$\frac{2}{3}(4) - (-8) \geq 5$, True

$(4, -8)$ *is* a solution.

35. $\begin{cases} x + y \leq 1 \\ -x + y \leq 1 \\ y \geq 0 \end{cases}$

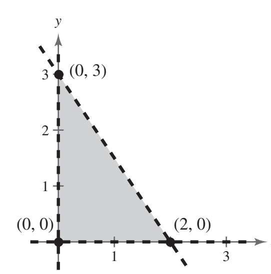
First, find the points of intersection of each pair of equations.



Vertex A	Vertex B	Vertex C
$x + y = 1$	$x + y = 1$	$-x + y = 1$
$-x + y = 1$	$y = 0$	$y = 0$
$(0, 1)$	$(1, 0)$	$(-1, 0)$

36. $\begin{cases} 3x + 2y < 6 \\ x > 0 \\ y > 0 \end{cases}$

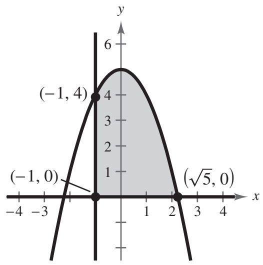
First, find the points of intersection of each pair of equations.



Vertex A	Vertex B	Vertex C
$3x + 2y = 6$	$x = 0$	$3x + 2y = 6$
$x = 0$	$y = 0$	$y = 0$
$(0, 3)$	$(0, 0)$	$(2, 0)$

37. $\begin{cases} x^2 + y \le 5 \\ x \ge -1 \\ y \ge 0 \end{cases}$

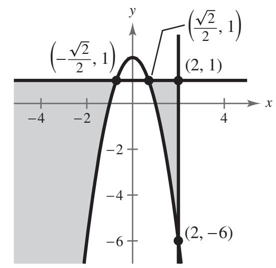
First, find the points of intersection of each pair of equations.



Vertex A	Vertex B	Vertex C
$x^2 + y = 5$	$x^2 + y = 5$	$x = -1$
$x = -1$	$y = 0$	$y = 0$
$(-1, 4)$	$(\pm\sqrt{5}, 0)$	$(-1, 0)$

38. $\begin{cases} 2x^2 + y \ge 2 \\ x \le 2 \\ y \le 1 \end{cases}$

First, find the points of intersection of each pair of equations.

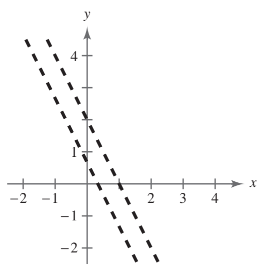


Vertex A	Vertex B	Vertex C
$2x + y = 2$	$x = 2$	$2x^2 + y = 2$
$x = 2$	$y = 1$	$y = 1$
$(2, -6)$	$(2, 1)$	

39. $\begin{cases} 2x + y > 2 \\ 6x + 3y < 2 \end{cases}$

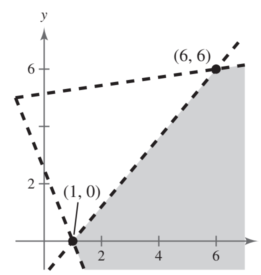
The graphs of $2x + y = 2$ and $6x + 3y = 2$ are parallel lines. The first inequality has the region above the line shaded. The second inequality has the region below the line shaded. There are no points that satisfy both inequalities.

No solution



40. $\begin{cases} x - 7y > -36 \\ 5x + 2y > 5 \\ 6x - 5y > 6 \end{cases}$

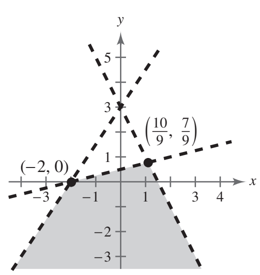
First, find the points of intersection of each pair of equations.



Vertex A	Vertex B	Vertex C
$x - 7y = -36$	$5x + 2y = 5$	$x - 7y = -36$
$5x + 2y = 5$	$6x - 5y = 6$	$6x - 5y = 6$
$(-1, 5)$	$(1, 0)$	$(6, 6)$

41. $\begin{cases} -3x + 2y < 6 \\ x - 4y > -2 \\ 2x + y < 3 \end{cases}$

First, find the points of intersection of each pair of equations.

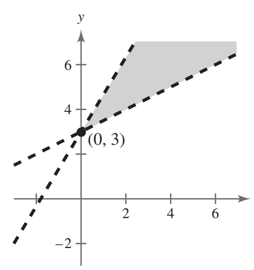


Vertex A	Point B	Vertex C
$-3x + 2y = 6$	$-3x + 2y = 6$	$x - 4y = -2$
$x - 4y = -2$	$2x + y = 3$	$2x + y = 3$
$(-2, 0)$	$(0, 3)$	$\left(\frac{10}{9}, \frac{7}{9}\right)$

Note that B is not a vertex of the solution region.

42. $\begin{cases} x - 2y < -6 \\ 5x - 3y > -9 \end{cases}$

Point of intersection: $(0, 3)$



43. $\begin{cases} x > y^2 \\ x < y + 2 \end{cases}$

Points of intersection:

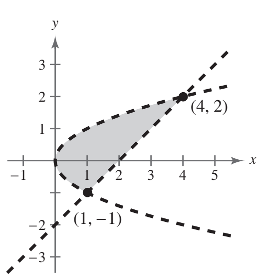
$$y^2 = y + 2$$

$$y^2 - y - 2 = 0$$

$$(y + 1)(y - 2) = 0$$

$$y = -1, 2$$

$(1, -1), (4, 2)$



44. $\begin{cases} x - y^2 > 0 \\ x - y > 2 \end{cases}$

Points of intersection:

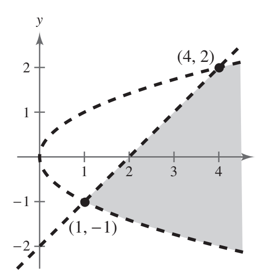
$$y^2 = y + 2$$

$$y^2 - y - 2 = 0$$

$$(y + 1)(y - 2) = 0$$

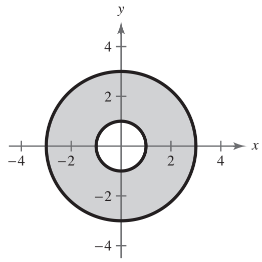
$$y = -1, 2$$

$(1, -1), (4, 2)$



45. $\begin{cases} x^2 + y^2 \le 9 \\ x^2 + y^2 \ge 1 \end{cases}$

There are no points of intersection. The region common to both inequalities is the region between the circles.



46. $\begin{cases} x^2 + y^2 \le 25 \\ 4x - 3y \le 0 \end{cases}$

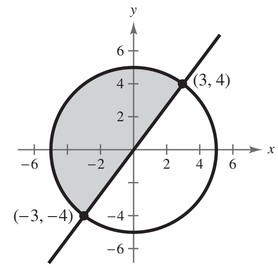
Points of intersection:

$x^2 + \left(\frac{4}{3}x\right)^2 = 25$

$\frac{25}{9}x^2 = 25$

$x = \pm 3$

$(-3, -4), (3, 4)$



47. $3x + 4 \ge y^2$

$x - y < 0$

Points of intersection:

$x - y = 0 \Rightarrow y = x$

$3y + 4 = y^2$

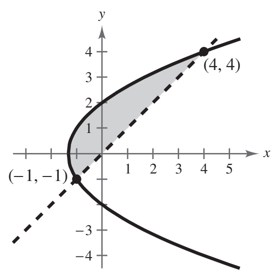
$0 = y^2 - 3y - 4$

$0 = (y - 4)(y + 1)$

$y = 4$ or $y = -1$

$x = 4 \qquad x = -1$

$(4, 4)$ and $(-1, -1)$



48. $\begin{cases} x < 2y - y^2 \\ 0 < x + y \end{cases}$

Points of intersection:

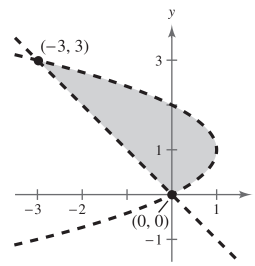
$-y = 2y - y^2$

$y^2 - 3y = 0$

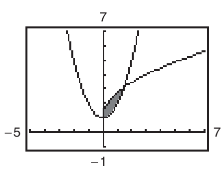
$y(y - 3) = 0$

$y = 0, 3$

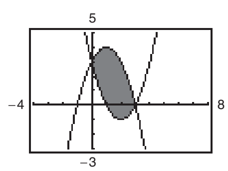
$(0, 0), (-3, 3)$



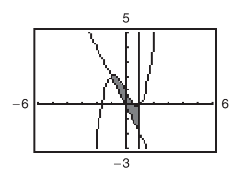
49. $\begin{cases} y \le \sqrt{3x} + 1 \\ y \ge x^2 + 1 \end{cases}$



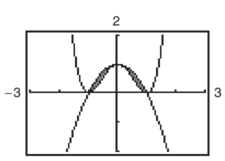
50. $\begin{cases} y < -x^2 + 2x + 3 \\ y > x^2 - 4x + 3 \end{cases}$



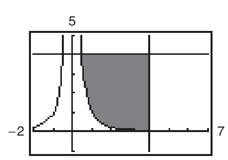
51. $\begin{cases} y < x^3 - 2x + 1 \\ y > -2x \\ x \le 1 \end{cases}$



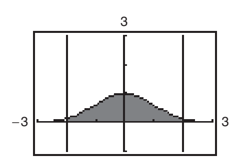
52. $\begin{cases} y \ge x^4 - 2x^2 + 1 \\ y \le 1 - x^2 \end{cases}$



53. $\begin{cases} x^2y \ge 1 \Rightarrow y \ge \dfrac{1}{x^2} \\ 0 < x \le 4 \\ y \le 4 \end{cases}$



54. $\begin{cases} y \le e^{-x^2/2} \\ y \ge 0 \\ -2 \le x \le 2 \end{cases}$



55. $\begin{cases} y \le 4 - x \\ x \ge 0 \\ y \ge 0 \end{cases}$

56. $(0, 6), (3, 0), (0, -3)$

$\begin{cases} y < 6 - 2x \\ y \ge x - 3 \\ x \ge 1 \end{cases}$

57. Line through points (0, 4) and (4, 0): $y = 4 - x$

Line through points (0, 2) and (8, 0): $y = 2 - \frac{1}{4}x$

$$\begin{cases} y \geq 4 - x \\ y \geq 2 - \frac{1}{4}x \\ x \geq 0 \\ y \geq 0 \end{cases}$$

58. Circle: $x^2 + y^2 > 4$

59. $$\begin{cases} x^2 + y^2 \leq 16 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

60. $(0, 0), (0, 4), \left(\sqrt{8}, \sqrt{8}\right)$

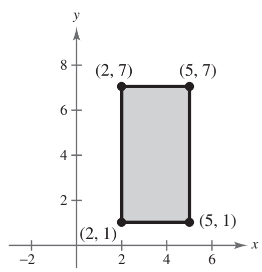
$$\begin{cases} x^2 + y^2 \leq 16 \\ x \leq y \\ x \geq 0 \end{cases}$$

61. Rectangular region with vertices at (2, 1), (5, 1), (5, 7), and (2, 7)

$$\begin{cases} x \geq 2 \\ x \leq 5 \\ y \geq 1 \\ y \leq 7 \end{cases}$$

This system may be written as:

$$\begin{cases} 2 \leq x \leq 5 \\ 1 \leq y \leq 7 \end{cases}$$



62. Parallelogram with vertices at (0, 0), (4, 0), (1, 4), (5, 4)

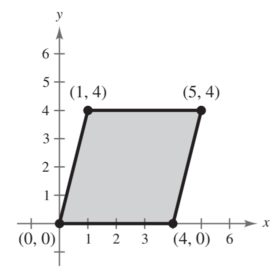
(0, 0), (4, 0): $y \geq 0$

(4, 0), (5, 4): $4x - y \leq 16$

(1, 4), (5, 4): $y \leq 4$

(0, 0), (1, 4): $4x - y \geq 0$

$$\begin{cases} 4x - y \geq 0 \\ 4x - y \leq 16 \\ 0 \leq y \leq 4 \end{cases}$$



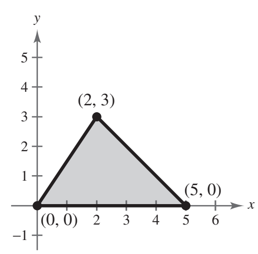
63. Triangle with vertices at (0, 0), (5, 0), (2, 3)

(0, 0), (5, 0) Line: $y = 0$

(0, 0), (2, 3) Line: $y = \frac{3}{2}x$

(2, 3), (5, 0) Line: $y = -x + 5$

$$\begin{cases} y \leq \frac{3}{2}x \\ y \leq -x + 5 \\ y \geq 0 \end{cases}$$



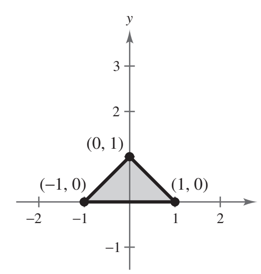
64. Triangle with vertices at $(-1, 0)$, (1, 0), (0, 1)

$(-1, 0)$, (1, 0): $y \geq 0$

$(-1, 0)$, (0, 1): $y \leq x + 1$

(0, 1), (1, 0): $y \leq -x + 1$

$$\begin{cases} y \leq x + 1 \\ y \leq -x + 1 \\ y \geq 0 \end{cases}$$



65. (a) Demand = Supply

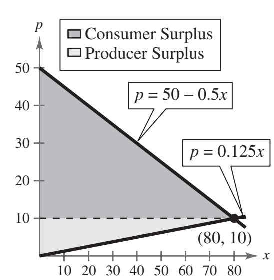
$50 - 0.5x = 0.125x$

$50 = 0.625x$

$80 = x$

$10 = p$

Point of equilibrium: (80, 10)



(b) The consumer surplus is the area of the triangular region defined by

$$\begin{cases} p \leq 50 - 0.5x \\ p \geq 10 \\ x \geq 0. \end{cases}$$

Consumer surplus $= \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(80)(40) = \$1600$

The producer surplus is the area of the triangular region defined by

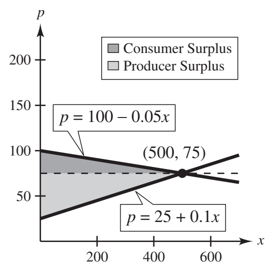
$$\begin{cases} p \geq 0.125x \\ p \leq 10 \\ x \geq 0. \end{cases}$$

Producer surplus $= \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(80)(10) = \$400$

66. (a) Demand = Supply

$$100 - 0.05x = 25 + 0.1x$$
$$75 = 0.15x$$
$$500 = x$$
$$75 = p$$

Point of equilibrium: $(500, 75)$



(b) The consumer surplus is the area of the triangular region defined by

$$\begin{cases} p \le 100 - 0.05x \\ p \ge 75 \\ x \ge 0. \end{cases}$$

Consumer surplus $= \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(500)(25) = 6250$

The producer surplus is the area of the triangular region defined by

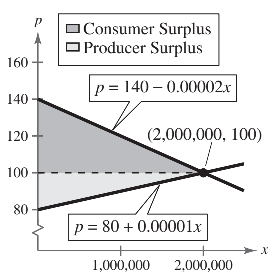
$$\begin{cases} p \le 25 + 0.1x \\ p \le 75 \\ x \le 0. \end{cases}$$

Producer surplus $= \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(500)(50) = 12{,}500$

67. (a) Demand = Supply

$$140 - 0.00002x = 80 + 0.00001x$$
$$60 = 0.00003x$$
$$2{,}000{,}000 = x$$
$$100 = p$$

Point of equilibrium: $(2{,}000{,}000, 100)$



(b) The consumer surplus is the area of the triangular region defined by

$$\begin{cases} p \le 140 - 0.00002x \\ p \ge 100 \\ x \ge \quad 0. \end{cases}$$

Consumer surplus $= \frac{1}{2}(\text{base})(\text{height})$

$= \frac{1}{2}(2{,}000{,}000)(40)$

$= \$40{,}000{,}000$ or $40 million

The producer surplus is the area of the triangular region defined by

$$\begin{cases} p \ge \quad 80 + 0.00001x \\ p \le 100 \\ x \ge \quad 0. \end{cases}$$

Producer surplus $= \frac{1}{2}(\text{base})(\text{height})$

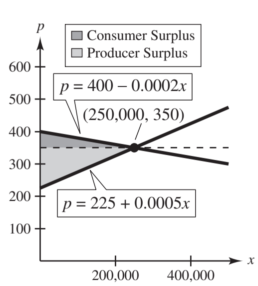
$= \frac{1}{2}(2{,}000{,}000)(20)$

$= \$20{,}000{,}000$ or $20 million

68. (a) Demand = Supply

$$400 - 0.0002x = 225 + 0.0005x$$
$$175 = 0.0007x$$
$$250{,}000 = x$$
$$350 = p$$

Point of equilibrium: $(250{,}000, 350)$



(b) The consumer surplus is the area of the triangular region defined by

$$\begin{cases} p \le 400 - 0.0002x \\ p \ge 350 \\ x \ge 0. \end{cases}$$

Consumer surplus
$= \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(250{,}000)(50) = 6{,}250{,}000$

The producer surplus is the area of the triangular region defined by

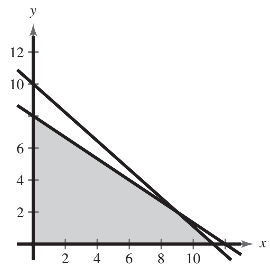
$$\begin{cases} p \ge 225 + 0.0005x \\ p \le 350 \\ x \ge 0. \end{cases}$$

Producer surplus
$= \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(250{,}000)(125) = 15{,}625{,}000$

69. $x =$ number of tables

$y =$ number of chairs

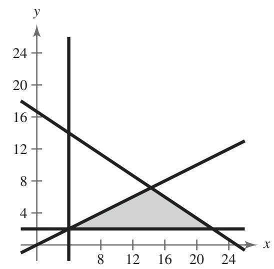
$$\begin{cases} x + \frac{3}{2}y \le 12 & \text{Assembly center} \\ \frac{4}{3}x + \frac{3}{2}y \le 15 & \text{Finishing center} \\ x \ge 0 \\ y \ge 0 \end{cases}$$



70. $x =$ number of model A

$y =$ number of model B

$$\begin{cases} x \ge 2y \\ 8x + 12y \le 200 \\ x \ge 4 \\ y \ge 2 \end{cases}$$

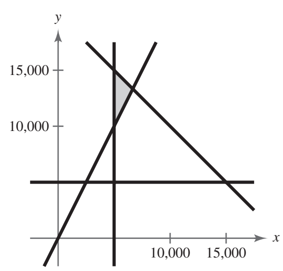


71. $x =$ amount in smaller account

$y =$ amount in larger account

Account constraints:

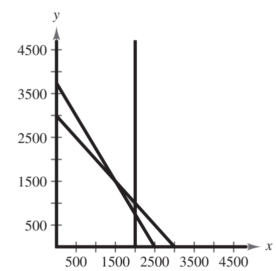
$$\begin{cases} x + y \le 20{,}000 \\ y \ge 2x \\ x \ge 5{,}000 \\ y \ge 5{,}000 \end{cases}$$



72. $x =$ number of \$30 tickets

$y =$ number of \$20 tickets

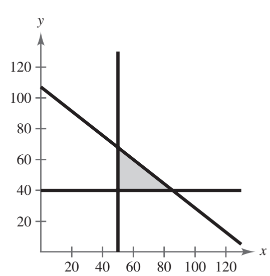
$$\begin{cases} x + y \le 3000 \\ 30x + 20y \ge 75{,}000 \\ x \le 2000 \\ x \ge 0 \\ y \ge 0 \end{cases}$$



73. $x =$ number of packages of gravel

$y =$ number of bags of stone

$$\begin{cases} 55x + 70y \le 7500 & \text{Weight} \\ x \ge 50 \\ y \ge 40 \end{cases}$$

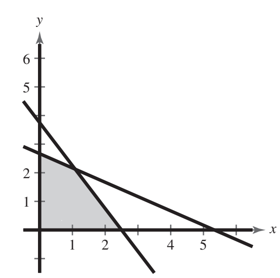


74. Let $x =$ number of large trucks.

Let $y =$ number of medium trucks.

The delivery requirements are:

$$\begin{cases} 6x + 4y \ge 15 \\ 3x + 6y \ge 16 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

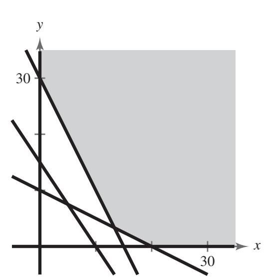


75. (a) $x =$ number of ounces of food X

$y =$ number of ounces of food Y

$$\begin{cases} 20x + 10y \ge 300 & \text{(calcium)} \\ 15x + 10y \ge 150 & \text{(iron)} \\ 10x + 20y \ge 200 & \text{(vitamin B)} \\ x \ge 0 \\ y \ge 0 \end{cases}$$

(b)



(c) Answers will vary. Some possible solutions which would satisfy the minimum daily requirements for calcium, iron, and vitamin B:

$(0, 30) \Rightarrow$ 30 ounces of food Y

$(20, 0) \Rightarrow$ 20 ounces of food X

$\left(13\frac{1}{3}, 3\frac{1}{3}\right) \Rightarrow 13\frac{1}{3}$ ounces of food X and $3\frac{1}{3}$ ounces of food Y

76. (a) Let $y =$ heart rate.

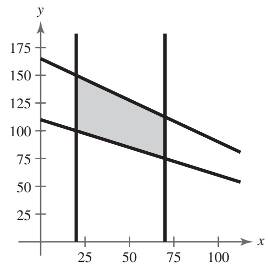
$y \geq 0.5(220 - x)$

$y \leq 0.75(220 - x)$

$x \geq 20$

$x \leq 70$

(b)

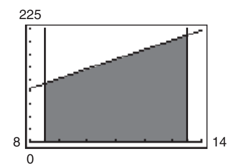


(c) Answers will vary. For example, the points (24, 98) and (24, 147) are on the boundary of the solution set; a person aged 24 should have a heart rate between 98 and 147.

77. (a) (9, 125.8), (10, 145.6), (11, 164.1), (12, 182.7), (13, 203.1)

Linear model:
$y = 19.17t - 46.61$

(b) $$\begin{cases} y \leq 19.17t - 46.61 \\ t \geq 8.5 \\ t \leq 13.5 \\ y \geq 0 \end{cases}$$



(c) Area of a trapezoid: $A = \dfrac{h}{2}(a + b)$

$h = 13.5 - 8.5 = 5$

$a = 19.17(8.5) - 46.61 = 116.335$

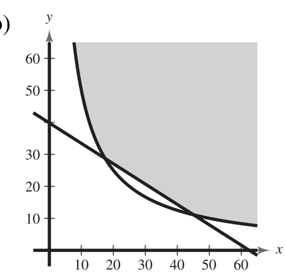
$b = 19.17(13.5) - 46.61 = 212.185$

$A = \dfrac{5}{2}(116.335 + 212.185)$

$= \$821.3$ billion

78. (a) $$\begin{cases} xy \geq 500 & \text{Body-building space} \\ 2x + \pi y \geq 125 & \text{Track (Two semi–circles and two lengths)} \\ x \geq 0 \\ y \geq 0 \end{cases}$$

(b)



79. True. The figure is a rectangle with length of 9 units and width of 11 units.

80. False. The graph shows the solution of the system

$$\begin{cases} y < 6 \\ -4x - 9y < 6 \\ 3x + y^2 \geq 2. \end{cases}$$

81. The graph is a half-line on the real number line; on the rectangular coordinate system, the graph is a half-plane.

82. Test a point on either side of the boundary.

83. $x =$ radius of smaller circle

$y =$ radius of larger circle

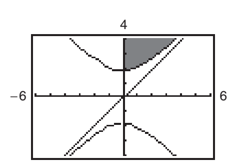
(a) Constraints on circles:

$\pi y^2 - \pi x^2 \geq 10$

$y > x$

$x > 0$

(b)



(c) The line is an asymptote to the boundary. The larger the circles, the closer the radii can be and the constraint still be satisfied.

84. (a) The boundary would be included in the solution.

(b) The solution would be the half-plane on the opposite side of the boundary.

85. $\begin{cases} x^2 + y^2 \le 16 \implies \text{region inside the circle} \\ x + y \ge 4 \implies \text{region above the line} \end{cases}$

Matches graph (d).

86. $x^2 + y^2 \le 16 \implies$ region inside the circle

$x + y \le 4 \implies$ region below the line

Matches graph (b).

87. $\begin{cases} x^2 + y^2 \ge 16 \implies \text{region outside the circle} \\ x + y \ge 4 \implies \text{region above the line} \end{cases}$

Matches graph (c).

88 . $x^2 + y^2 \ge 16 \implies$ region outside the circle

$x + y \le 4 \implies$ region below the line

Matches graph (a).

89. $(-2, 6), (4, -4)$

$$m = \frac{-4 - 6}{4 - (-2)} = \frac{-10}{6} = -\frac{5}{3}$$

$$y - (-4) = -\frac{5}{3}(x - 4)$$

$$3y + 12 = -5x + 20$$

$$5x + 3y - 8 = 0$$

90. $(-8, 0), (3, -1)$

$$m = \frac{-1 - 0}{3 - (-8)} = -\frac{1}{11}$$

$$y - 0 = -\frac{1}{11}(x - (-8))$$

$$y = -\frac{1}{11}x - \frac{8}{11}$$

$$11y = -x - 8$$

$$x + 11y + 8 = 0$$

91. $\left(\frac{3}{4}, -2\right), \left(-\frac{7}{2}, 5\right)$

$$m = \frac{5 - (-2)}{-\frac{7}{2} - \frac{3}{4}} = \frac{7}{-\frac{17}{4}} = -\frac{28}{17}$$

$$y - (-2) = -\frac{28}{17}\left(x - \frac{3}{4}\right)$$

$$17y + 34 = -28x + 21$$

$$28x + 17y + 13 = 0$$

92. $\left(-\frac{1}{2}, 0\right), \left(\frac{11}{2}, 12\right)$

$$m = \frac{12 - 0}{\frac{11}{2} - \left(\frac{1}{2}\right)} = \frac{12}{6} = 2$$

$$y - 0 = 2\left(x - \left(-\frac{1}{2}\right)\right)$$

$$y = 2x + 1$$

$$2x - y + 1 = 0$$

93. $(3.4, -5.2), (-2.6, 0.8)$

$$m = \frac{0.8 - (-5.2)}{-2.6 - 3.4} = \frac{6}{-6} = -1$$

$$y - 0.8 = -1(x - (-2.6))$$

$$y - 0.8 = -x - 2.6$$

$$x + y + 1.8 = 0$$

94. $(-4.1, -3.8), (2.9, 8.2)$

$$m = \frac{8.2 - (-3.8)}{2.9 - (-4.1)} = \frac{12}{7}$$

$$y + 3.8 = \frac{12}{7}(x + 4.1)$$

$$y + 3.8 = \frac{12}{7}x + \frac{246}{35}$$

$$y = \frac{12}{7}x + \frac{113}{35}$$

$$35y = 60x + 113$$

$$60x - 35y + 113 = 0$$

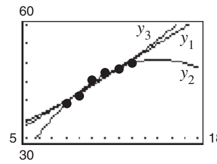
95. (a) (8, 39.43), (9, 41.24), (10, 45.27), (11, 47.37), (12, 48.40), (13, 49.91)

Linear model: $y \approx 2.17t + 22.5$

Quadratic model: $y \approx -0.241t^2 + 7.23t - 3.4$

Exponential model: $y \approx 27(1.05^t)$

(b)



(c) The quadratic model is the best fit for the actual data.

(d) For 2008, use $t = 18$: $y \approx -0.241(18)^2 + 7.23(18) - 3.4 \approx \$48.66$

96. $A = P\left(1 + \dfrac{r}{t}\right)^{nt}$

$A = 4000\left(1 + \dfrac{0.06}{12}\right)^{5 \cdot 12}$

$= 4000(1.005)^{60}$

$= 5395.40061$

The amount after 5 years is \$5395.40.

Section 7.6 Linear Programming

- To solve a linear programming problem:
  1. Sketch the solution set for the system of constraints.
  2. Find the vertices of the region.
  3. Test the objective function at each of the vertices.

Vocabulary Check

1. optimization

2. linear programming

3. objective

4. constraints; feasible solutions

5. vertex

1. $z = 4x + 3y$

At (0, 5): $z = 4(0) + 3(5) = 15$

At (0, 0): $z = 4(0) + 3(0) = 0$

At (5, 0): $z = 4(5) + 3(0) = 20$

The minimum value is 0 at (0, 0).

The maximum value is 20 at (5, 0).

2. $z = 2x + 8y$

At (0, 4): $z = 2(0) + 8(4) = 32$

At (0, 0): $z = 2(0) + 8(0) = 0$

At (2, 0): $z = 2(2) + 8(0) = 4$

The minimum value is 0 at (0, 0).

The maximum value is 32 at (0, 4).

3. $z = 3x + 8y$

At (0, 5): $z = 3(0) + 8(5) = 40$

At (0, 0): $z = 3(0) + 8(0) = 0$

At (5, 0): $z = 3(5) + 8(0) = 15$

The minimum value is 0 at (0, 0).

The maximum value is 40 at (0, 5).

4. $z = 7x + 3y$

At (0, 4): $z = 7(0) + 3(4) = 12$

At (0, 0): $z = 7(0) + 3(0) = 0$

At (2, 0): $z = 7(2) + 3(0) = 14$

The minimum value is 0 at (0, 0).

The maximum value is 14 at (2, 0).

5. $z = 3x + 2y$

At (0, 5): $z = 3(0) + 2(5) = 10$

At (4, 0): $z = 3(4) + 2(0) = 12$

At (3, 4): $z = 3(3) + 2(4) = 17$

At (0, 0): $z = 3(0) + 2(0) = 0$

The minimum value is 0 at (0, 0).

The maximum value is 17 at (3, 4).

6. $z = 4x + 5y$

At (0, 2): $z = 4(0) + 5(2) = 10$

At (0, 4): $z = 4(0) + 5(4) = 20$

At (3, 0): $z = 4(3) + 5(0) = 12$

At (4, 3): $z = 4(4) + 5(3) = 31$

The minimum value is 10 at (0, 2).

The maximum value is 31 at (4, 3).

7. $z = 5x + 0.5y$

At (0, 5): $z = 5(0) + \frac{5}{2} = \frac{5}{2}$

At (4, 0): $z = 5(4) + \frac{0}{2} = 20$

At (3, 4): $z = 5(3) + \frac{4}{2} = 17$

At (0, 0): $z = 5(0) + \frac{0}{2} = 0$

The minimum value is 0 at (0, 0).

The maximum value is 20 at (4, 0).

8. $z = 2x + y$

At (0, 2): $z = 2(0) + 2 = 2$

At (0, 4): $z = 2(0) + 4 = 4$

At (3, 0): $z = 2(3) + 0 = 6$

At (4, 3): $z = 2(4) + 3 = 11$

The minimum value is 2 at (0, 2).

The maximum value is 11 at (4, 3).

9. $z = 10x + 7y$

At (0, 45): $z = 10(0) + 7(45) = 315$

At (30, 45): $z = 10(30) + 7(45) = 615$

At (60, 20): $z = 10(60) + 7(20) = 740$

At (60, 0): $z = 10(60) + 7(0) = 600$

At (0, 0): $z = 10(0) + 7(0) = 0$

The minimum value is 0 at (0, 0).

The maximum value is 740 at (60, 20).

10. $z = 25x + 35y$

At (0, 400): $z = 25(0) + 35(400) = 14{,}000$

At (0, 800): $z = 25(0) + 35(800) = 28{,}000$

At (450, 0): $z = 25(450) + 35(0) = 11{,}250$

At (900, 0): $z = 25(900) + 35(0) = 22{,}500$

The minimum value is 11,250 at (450, 0).

The maximum value is 28,000 at (0, 800).

11. $z = 25x + 30y$

At (0, 45): $z = 25(0) + 30(45) = 1350$

At (30, 45): $z = 25(30) + 30(45) = 2100$

At (60, 20): $z = 25(60) + 30(20) = 2100$

At (60, 0): $z = 25(60) + 30(0) = 1500$

At (0, 0): $z = 25(0) + 30(0) = 0$

The minimum value is 0 at (0, 0).

The maximum value is 2100 at any point along the line segment connecting (30, 45) and (60, 20).

12. $z = 15x + 20y$

At (0, 400): $z = 15(0) + 20(400) = 8000$

At (0, 800): $z = 15(0) + 20(800) = 16{,}000$

At (450, 0): $z = 15(450) + 20(0) = 6750$

At (900, 0): $z = 15(900) + 20(0) = 13{,}500$

The minimum value is 6750 at (450, 0).

The maximum value is 16,000 at (0, 800).

13. $z = 6x + 10y$

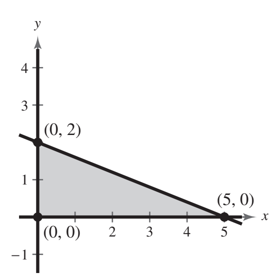
At (0, 2): $z = 6(0) + 10(2) = 20$

At (5, 0): $z = 6(5) + 10(0) = 30$

At (0, 0): $z = 6(0) + 10(0) = 0$

The minimum value is 0 at (0, 0).

The maximum value is 30 at (5, 0).



14. $z = 7x + 8y$

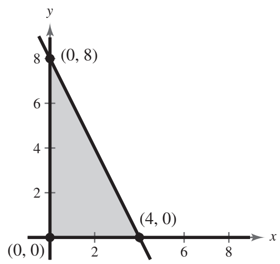
At $(0, 8)$: $z = 7(0) + 8(8) = 64$

At $(4, 0)$: $z = 7(4) + 8(0) = 28$

At $(0, 0)$: $z = 7(0) + 8(0) = 0$

The minimum value is 0 at $(0, 0)$.

The maximum value is 64 at $(0, 8)$.



15. $z = 9x + 24y$

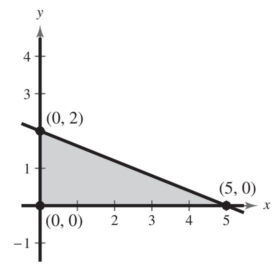
At $(0, 2)$: $z = 9(0) + 24(2) = 48$

At $(5, 0)$: $z = 9(5) + 24(0) = 45$

At $(0, 0)$: $z = 9(0) + 24(0) = 0$

The minimum value is 0 at $(0, 0)$.

The maximum value is 48 at $(0, 2)$.



16. $z = 7x + 2y$

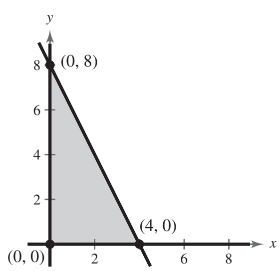
At $(0, 8)$: $z = 7(0) + 2(8) = 16$

At $(4, 0)$: $z = 7(4) + 2(0) = 28$

At $(0, 0)$: $z = 7(0) + 2(0) = 0$

The minimum value is 0 at $(0, 0)$.

The maximum value is 28 at $(4, 0)$.



17. $z = 4x + 5y$

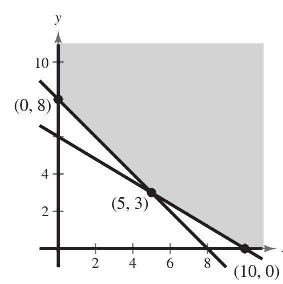
At $(10, 0)$: $z = 4(10) + 5(0) = 40$

At $(5, 3)$: $z = 4(5) + 5(3) = 35$

At $(0, 8)$: $z = 4(0) + 5(8) = 40$

The minimum value is 35 at $(5, 3)$.

The region is unbounded. There is no maximum.



18. $z = 4x + 5y$

At $(0, 0)$: $z = 4(0) + 5(0) = 0$

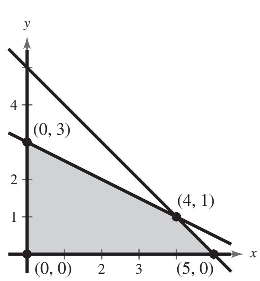
At $(5, 0)$: $z = 4(5) + 5(0) = 20$

At $(4, 1)$: $z = 4(4) + 5(1) = 21$

At $(0, 3)$: $z = 4(0) + 5(3) = 15$

The minimum value is 0 at $(0, 0)$.

The maximum value is 21 at $(4, 1)$.



19. $z = 2x + 7y$

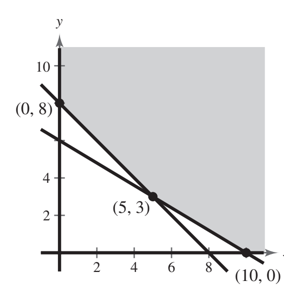
At $(10, 0)$: $z = 2(10) + 7(0) = 20$

At $(5, 3)$: $z = 2(5) + 7(3) = 31$

At $(0, 8)$: $z = 2(0) + 7(8) = 56$

The minimum value is 20 at $(10, 0)$.

The region is unbounded. There is no maximum.



20. $z = 2x - y$

At $(0, 0)$: $z = 2(0) - 0 = 0$

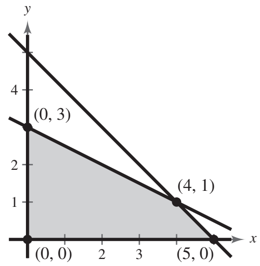
At $(5, 0)$: $z = 2(5) - 0 = 10$

At $(4, 1)$: $z = 2(4) - 1 = 7$

At $(0, 3)$: $z = 2(0) - 3 = -3$

The minimum value is -3 at $(0, 3)$.

The maximum value is 10 at $(5, 0)$.



21. $z = 4x + y$

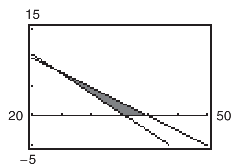
At $(36, 0)$: $z = 4(36) + 0 = 144$

At $(40, 0)$: $z = 4(40) + 0 = 160$

At $(24, 8)$: $z = 4(24) + 8 = 104$

The minimum value is 104 at $(24, 8)$.

The maximum value is 160 at $(40, 0)$.



22. $z = x$

At $(0, 0)$: $z = 0$

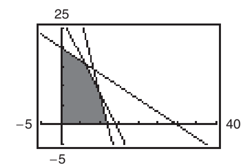
At $(12, 0)$: $z = 12$

At $(10, 8)$: $z = 10$

At $(6, 16)$: $z = 6$

At $(0, 20)$: $z = 0$

The minimum value is 0 at any point along the line segment connecting $(0, 0)$ and $(0, 20)$. The maximum value is 12 at $(12, 0)$.



23. $z = x + 4y$

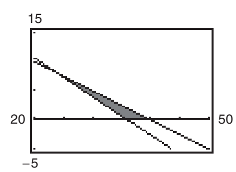
At $(36, 0)$: $z = 36 + 4(0) = 36$

At $(40, 0)$: $z = 40 + 4(0) = 40$

At $(24, 8)$: $z = 24 + 4(8) = 56$

The minimum value is 36 at $(36, 0)$.

The maximum value is 56 at $(24, 8)$.



24. $z = y$

At $(0, 0)$: $z = 0$

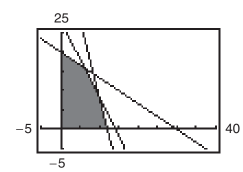
At $(12, 0)$: $z = 0$

At $(10, 8)$: $z = 8$

At $(6, 16)$: $z = 16$

At $(0, 20)$: $z = 20$

The minimum value is 0 at any point along the line segment connecting $(0, 0)$ and $(12, 0)$.The maximum value is 20 at $(0, 20)$.



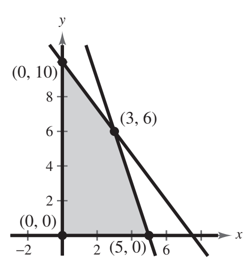


Figure for Exercises 25–28

25. $z = 2x + y$

At $(0, 10)$: $z = 2(0) + (10) = 10$

At $(3, 6)$: $z = 2(3) + (6) = 12$

At $(5, 0)$: $z = 2(5) + (0) = 10$

At $(0, 0)$: $z = 2(0) + (0) = 0$

The maximum value is 12 at $(3, 6)$.

26. $z = 5x + y$

At $(0, 10)$: $z = 5(0) + 10 = 10$

At $(3, 6)$: $z = 5(3) + 6 = 21$

At $(5, 0)$: $z = 5(5) + 0 = 25$

At $(0, 0)$: $z = 5(0) + 0 = 0$

The maximum value is 25 at $(5, 0)$.

27. $z = x + y$

At $(0, 10)$: $z = (0) + (10) = 10$

At $(3, 6)$: $z = (3) + (6) = 9$

At $(5, 0)$: $z = (5) + (0) = 5$

At $(0, 0)$: $z = (0) + (0) = 0$

The maximum value is 10 at $(0, 10)$.

28. $z = 3x + y$

At $(0, 10)$: $z = 3(0) + 10 = 10$

At $(3, 6)$: $z = 3(3) + 6 = 15$

At $(5, 0)$: $z = 3(5) + 0 = 15$

At $(0, 0)$: $z = 3(0) + 0 = 0$

The maximum value is 15 at any point along the line segment connecting $(3, 6)$ and $(5, 0)$.

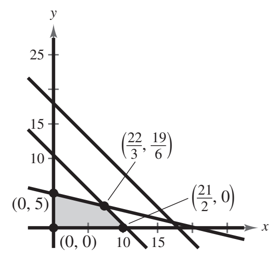


Figure for Exercises 29–32

29. $z = x + 5y$

At $(0, 5)$: $z = 0 + 5(5) = 25$

At $\left(\frac{22}{3}, \frac{19}{6}\right)$: $z = \frac{22}{3} + 5\left(\frac{19}{6}\right) = \frac{139}{6}$

At $\left(\frac{21}{2}, 0\right)$: $z = \frac{21}{2} + 5(0) = \frac{21}{2}$

At $(0, 0)$: $z = 0 + 5(0) = 0$

The maximum value is 25 at $(0, 5)$.

30. $z = 2x + 4y$

At $(0, 5)$: $z = 2(0) + 4(5) = 20$

At $\left(\frac{22}{3}, \frac{19}{6}\right)$: $z = 2\left(\frac{22}{3}\right) + 4\left(\frac{19}{6}\right) = \frac{82}{3}$

At $\left(\frac{21}{2}, 0\right)$: $z = 2\left(\frac{21}{2}\right) + 4(0) = 21$

At $(0, 0)$: $z = 2(0) + 4(0) = 0$

The maximum value is $\frac{82}{3}$ at $\left(\frac{22}{3}, \frac{19}{6}\right)$.

31. $z = 4x + 5y$

At $(0, 5)$: $z = 4(0) + 5(5) = 25$

At $\left(\frac{22}{3}, \frac{19}{6}\right)$: $z = 4\left(\frac{22}{3}\right) + 5\left(\frac{19}{6}\right) = \frac{271}{6}$

At $\left(\frac{21}{2}, 0\right)$: $z = 4\left(\frac{21}{2}\right) + 5(0) = 42$

At $(0, 0)$: $z = 4(0) + 5(0) = 0$

The maximum value is $\frac{271}{6}$ at $\left(\frac{22}{3}, \frac{19}{6}\right)$.

32. $z = 4x + y$

At $(0, 5)$: $z = 4(0) + 5 = 5$

At $\left(\frac{22}{3}, \frac{19}{6}\right)$: $z = 4\left(\frac{22}{3}\right) + \frac{19}{6} = \frac{65}{2}$

At $\left(\frac{21}{2}, 0\right)$: $z = 4\left(\frac{21}{2}\right) + 0 = 42$

At $(0, 0)$: $z = 4(0) + 0 = 0$

The maximum value is 42 at $\left(\frac{21}{2}, 0\right)$.

33. Objective function: $z = 2.5x + y$

Constraints: $x \geq 0$, $y \geq 0$, $3x + 5y \leq 15$, $5x + 2y \leq 10$

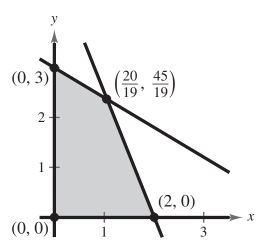
At $(0, 0)$: $z = 0$

At $(2, 0)$: $z = 5$

At $\left(\frac{20}{19}, \frac{45}{19}\right)$: $z = \frac{95}{19} = 5$

At $(0, 3)$: $z = 3$

The maximum value of 5 occurs at any point on the line segment connecting $(2, 0)$ and $\left(\frac{20}{19}, \frac{45}{19}\right)$.



34. Objective function: $z = x + y$

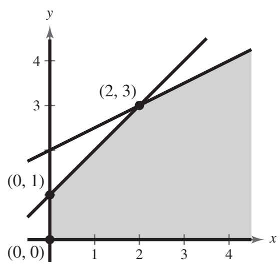
Constraints: $x \geq 0, y \geq 0, -x + y \leq 1, -x + 2y \leq 4$

At $(0, 0)$: $z = 0 + 0 = 0$

At $(0, 1)$: $z = 0 + 1 = 1$

At $(2, 3)$: $z = 2 + 3 = 5$

The constraints do not form a closed set of points. Therefore, $z = x + y$ is unbounded.



35. Objective function: $z = -x + 2y$

Constraints: $x \geq 0, y \geq 0, x \leq 10, x + y \leq 7$

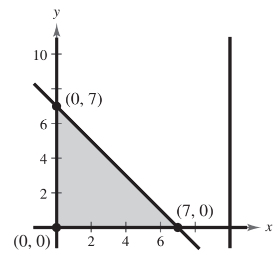
At $(0, 0)$: $z = -0 + 2(0) = 0$

At $(0, 7)$: $z = -0 + 2(7) = 14$

At $(7, 0)$: $z = -7 + 2(0) = -7$

The constraint $x \leq 10$ is extraneous.

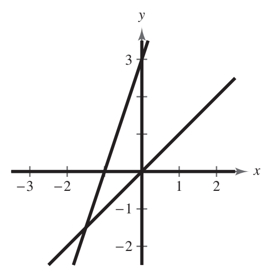
The maximum value of 14 occurs at $(0, 7)$.



36. Objective function: $z = x + y$

Constraints: $x \geq 0, y \geq 0, -x + y \leq 0, -3x + y \geq 3$

The feasible set is empty.



37. Objective function: $z = 3x + 4y$

Constraints: $x \geq 0, y \geq 0, x + y \leq 1, 2x + y \leq 4$

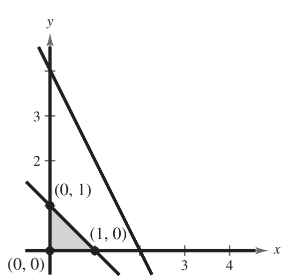
At $(0, 0)$: $z = 3(0) + 4(0) = 0$

At $(0, 1)$: $z = 3(0) + 4(1) = 4$

At $(1, 0)$: $z = 3(1) + 4(0) = 3$

The constraint $2x + y \leq 4$ is extraneous.

The maximum value of 4 occurs at $(0, 1)$.



38. Objective function: $z = x + 2y$

Constraints: $x \geq 0, y \geq 0, x + 2y \leq 4, 2x + y \leq 4$

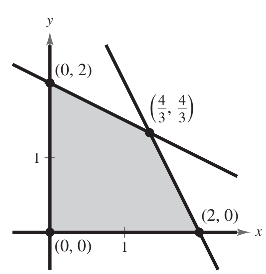
At $(0, 0)$: $z = 0 + 2(0) = 0$

At $(0, 2)$: $z = 0 + 2(2) = 4$

At $\left(\frac{4}{3}, \frac{4}{3}\right)$: $z = \frac{4}{3} + 2\left(\frac{4}{3}\right) = 4$

At $(2, 0)$: $z = 2 + 2(0) = 2$

The maximum value is 4 at any point along the line segment connecting $(0, 2)$ and $\left(\frac{4}{3}, \frac{4}{3}\right)$.



39. $x =$ number of Model A

$y =$ number of Model B

Constraints:
$$\begin{aligned} 2x + 2.5y &\le 4000 \\ 4x + y &\le 4800 \\ x + 0.75y &\le 1500 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

Objective function: $P = 45x + 50y$

Vertices:

(0, 0), (0, 1600), (750, 1000), (1050, 600), (1200, 0)

At (0, 0): $P = 45(0) + 50(0) = 0$

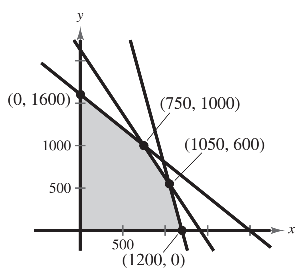
At (0, 1600): $P = 45(0) + 50(1600) = 80{,}000$

At (750, 1000): $P = 45(750) + 50(1000) = 83{,}750$

At (1050, 600): $P = 45(1050) + 50(600) = 77{,}250$

At (1200, 0): $P = 45(1200) + 50(0) = 54{,}000$

The optimal profit of \$83,750 occurs when 750 units of Model A and 1000 units of Model B are produced.



40. $x =$ number of Model A

$y =$ number of Model B

Constraints:
$$\begin{aligned} 2.5x + 3y &\le 4000 \\ 2x + y &\le 2500 \\ 0.75x + 1.25y &\le 1500 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

Objective function: $P = 50x + 52y$

Vertices:
(0, 0), (0, 1200), $\left(\frac{4000}{7}, \frac{6000}{7}\right)$, (1000, 500), (1250, 0)

At (0, 0): $P = 50(0) + 52(0) = 0$

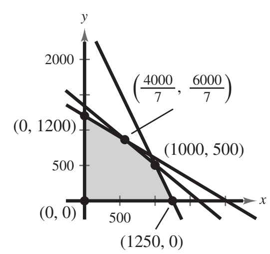
At (0, 1200): $P = 50(0) + 52(1200) = 62{,}400$

At $\left(\frac{4000}{7}, \frac{6000}{7}\right)$: $P = 50\left(\frac{4000}{7}\right) + 52\left(\frac{6000}{7}\right) \approx 73{,}142.86$

At (1000, 500): $P = 50(1000) + 52(500) = 76{,}000$

At (1250, 0): $P = 50(1250) + 52(0) = 62{,}500$

The optimal profit of \$76,000 occurs when 1000 units of Model A and 500 units of Model B are produced.



41. $x =$ number of \$250 models

$y =$ number of \$300 models

Constraints:
$$\begin{aligned} 250x + 300y &\le 65{,}000 \\ x + y &\le 250 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

Objective function: $P = 25x + 40y$

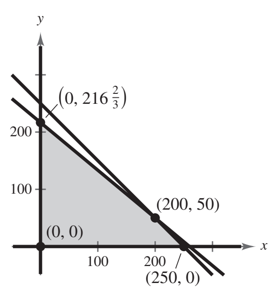
Vertices: (0, 0), (250, 0), (200, 50), $\left(0, 216\frac{2}{3}\right)$

At (0, 0): $P = 25(0) + 40(0) = 0$

At (250, 0): $P = 25(250) + 40(0) = 6250$

At (200, 50): $P = 25(200) + 40(50) = 7000$

At $\left(0, 216\frac{2}{3}\right)$: $P = 25(0) + 40\left(216\frac{2}{3}\right) \approx 8666.67$



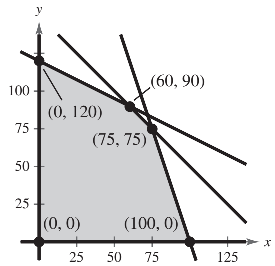
An optimal profit of \$8640 occurs when 0 units of the \$250 model and 216 units of the \$300 model are stocked in inventory. (**Note:** A merchant cannot sell $\frac{2}{3}$ of a unit.)

42. x = number of acres for crop A

y = number of acres for crop B

Constraints:

$$\begin{aligned} x + y &\le 150 \\ x + 2y &\le 240 \\ 0.3x + 0.1y &\le 30 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$



Objective function:
$P = 140x + 235y$

Vertices: (0, 0), (100, 0), (0, 120), (60, 90), (75, 75)

At (0, 0): $P = 140(0) + 235(0) = 0$

At (100, 0): $P = 140(100) + 235(0) = 14{,}000$

At (0, 120): $P = 140(0) + 235(120) = 28{,}200$

At (60, 90): $P = 140(60) + 235(90) = 29{,}550$

At (75, 75): $P = 140(75) + 235(75) = 28{,}125$

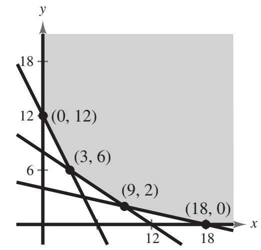
To optimize the profit, the fruit grower should plant 60 acres of crop A and 90 acres of crop B. The optimal profit is $29,550.

43. x = number of bags of Brand X

y = number of bags of Brand Y

Constraints:

$$\begin{aligned} 2x + y &\ge 12 \\ 2x + 9y &\ge 36 \\ 2x + 3y &\ge 24 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$



Objective function:
$C = 25x + 20y$

Vertices: (0, 12), (3, 6), (9, 2), (18, 0)

At (0, 12): $C = 25(0) + 20(12) = 240$

At (3, 6): $C = 25(3) + 20(6) = 195$

At (9, 2): $C = 25(9) + 20(2) = 265$

At (18, 0): $C = 25(18) + 20(0) = 450$

To optimize cost, use three bags of Brand X and six bags of Brand Y for an optimal cost of $195.

44. (a) x = the proportion of regular gasoline

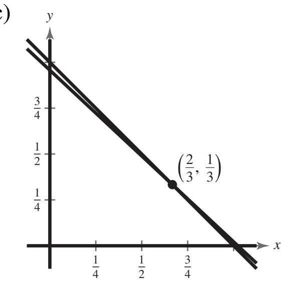
y = the proportion of premium

$C = 1.84x + 2.03y$

(b) The constraints are:

$$\begin{aligned} x + y &= 1 \\ 87x + 93y &\ge 89 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

(c)



(d) Actually the only points of the plane which satisfy all the constraints are the points of the line segment connecting (0, 1) and $\left(\frac{2}{3}, \frac{1}{3}\right)$. Evaluate $C = 1.84x + 2.03y$ at the two endpoints to find that the lower cost occurs at $\left(\frac{2}{3}, \frac{1}{3}\right)$.

(e) The optimal cost is $C = 1.84\left(\frac{2}{3}\right) + 2.03\left(\frac{1}{3}\right) = \$1.90$.

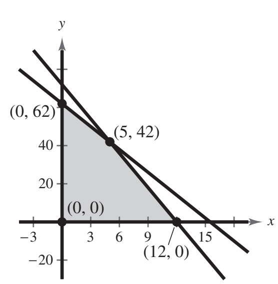
(f) This is lower than the national average of $1.96.

45. x = number of audits

y = number of tax returns

Constraints:

$$\begin{aligned} 75x + 12.5y &\le 900 \\ 10x + 2.5y &\le 155 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$



Objective function:
$R = 2500x + 350y$

Vertices: (0, 0), (12, 0), (5, 42), (0, 62)

At (0, 0): $R = 2500(0) + 350(0) = 0$

At (12, 0): $R = 2500(12) + 350(0) = 30{,}000$

At (5, 42): $R = 2500(5) + 350(42) = 27{,}200$

At (0, 62): $R = 2500(0) + 350(62) = 21{,}700$

The revenue will be optimal if 12 audits and 0 tax returns are done each week. The optimal revenue is $30,000.

46. The modified objective function is $R = 2000x + 350y$.

The vertices of the region are at (0, 0), (0, 62), (5, 42), and (12, 0).

At (0, 0): $R = 2000(0) + 350(0) = 0$

At (0, 62): $R = 2000(0) + 350(62) = 21{,}700$

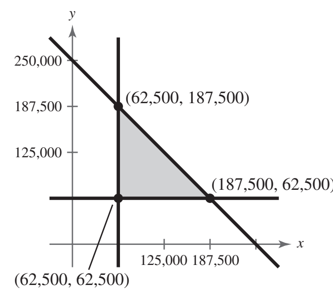
At (5, 42): $R = 2000(5) + 350(42) = 24{,}700$

At (12, 0): $R = 2000(12) + 350(0) = 24{,}000$

The optimal revenue of $24,700 occurs with 5 audits and 42 tax returns.

47. $x =$ amount of Type A

$y =$ amount of Type B



Constraints:
$$\begin{aligned} x + y &\le 250{,}000 \\ x &\ge \tfrac{1}{4}(250{,}000) \\ y &\ge \tfrac{1}{4}(250{,}000) \end{aligned}$$

Objective Function: $P = 0.08x + 0.10y$

Vertices: (62,500, 62,500), (62,500, 187,500), (187,500, 62,500)

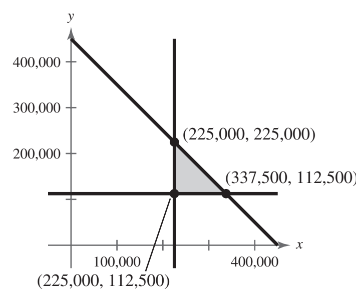
At (62,500, 62,500): $P = 0.08(62{,}500) + 0.10(62{,}500) = \$11{,}250$

At (62,500, 187,500): $P = 0.08(62{,}500) + 0.10(187{,}500) = \$23{,}750$

At (187,500, 62,500): $P = 0.08(187{,}500) + 0.10(62{,}500) = \$21{,}250$

To obtain an optimal return the investor should allocate $62,500 to Type A and $187,500 to Type B. The optimal return is $23,750.

48. $x =$ amount in investment of Type A; $y =$ amount in investment of Type B



Constraints:
$$\begin{aligned} x + y &\le 450{,}000 \\ x &\ge 225{,}000 \\ y &\ge 112{,}500 \end{aligned}$$

Objective function: $R = 0.06x + 0.1y$

Vertices: (225,000, 112,500), (337,500, 112,500), (225,000, 225,000)

At (225,000, 112,500): $R = 0.06(225{,}000) + 0.1(112{,}500) = 24{,}750$

At (337,500, 112,500): $R = 0.06(337{,}500) + 0.1(112{,}500) = 31{,}500$

At (225,000, 225,000): $R = 0.06(225{,}000) + 0.1(225{,}000) = 36{,}000$

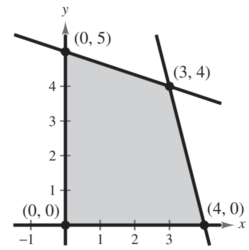
The optimal return of $36,000 occurs for an investment of $225,000 to Type A and $225,000 to Type B.

49. True. The objective function has a maximum value at any point on the line segment connecting the two vertices. Both of these points are on the line $y = -x + 11$ and lie between (4, 7) and (8, 3).

50. True. If an objective function has a maximum value at more than one vertex, then any point on the line segment connecting the points will produce the maximum value.

51. Constraints: $x \geq 0, y \geq 0, x + 3y \leq 15, 4x + y \leq 16$

Vertex	Value of $z = 3x + ty$
(0, 0)	$z = 0$
(0, 5)	$z = 5t$
(3, 4)	$z = 9 + 4t$
(4, 0)	$z = 12$



(a) For the maximum value to be at (0, 5), $z = 5t$ must be greater than or equal to $z = 9 + 4t$ and $z = 12$.

$5t \geq 9 + 4t$ and $5t \geq 12$

$t \geq 9$ $\quad$ $t \geq \frac{12}{5}$

Thus, $t \geq 9$.

(b) For the maximum value to be at (3, 4), $z = 9 + 4t$ must be greater than or equal to $z = 5t$ and $z = 12$.

$9 + 4t \geq 5t$ and $9 + 4t \geq 12$

$9 \geq t$ $\quad$ $4t \geq 3$

$t \geq \frac{3}{4}$

Thus, $\frac{3}{4} \leq t \leq 9$.

52. Constraints: $x \geq 0, y \geq 0, x + 2y \leq 4, x - y \leq 1$

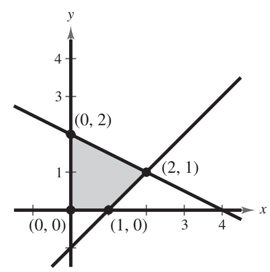
$z = 3x + ty$

At (0, 0): $z = 3(0) + t(0) = 0$

At (1, 0): $z = 3(1) + t(0) = 3$

At (2, 1): $z = 3(2) + t(1) = 6 + t$

At (0, 2): $z = 3(0) + t(2) = 2t$



(a) For the maximum value to be at (2, 1), $z = 6 + t$ must be greater than or equal to $z = 2t$ and $z = 3$.

$6 + t \geq 2t$ and $6 + t \geq 3$

$6 \geq t$ $\quad$ $t \geq -3$

Thus, $-3 \leq t \leq 6$.

(b) For maximum value to be at (0, 2), $z = 2t$ must be greater than or equal to $z = 6 + t$ and $z = 3$.

$2t \geq 6 + t$ and $2t \geq 3$

$t \geq 6$ $\quad$ $t \geq \frac{3}{2}$

Thus, $t \geq 6$.

53. There are an infinite number of objective functions that would have a maximum at (0, 4). One such objective function is $z = x + 5y$.

54. There are an infinite number of objective functions that would have a maximum at (4, 3). One such objective function is $z = x + y$.

55. There are an infinite number of objective functions that would have a maximum at (5, 0). One such objective function is $z = 4x + y$.

56. There are an infinite number of objective functions that would have a minimum at (5, 0). One such objective function is $z = -10x + y$.

57. $$\frac{\frac{9}{x}}{\left(\frac{6}{x} + 2\right)} = \frac{\frac{9}{x}}{\frac{6 + 2x}{x}} = \frac{9}{x} \cdot \frac{x}{2(3 + x)} = \frac{9}{2(3 + x)} = \frac{9}{2(x + 3)}, \quad x \neq 0$$

58. $$\frac{\left(1 + \frac{2}{x}\right)}{x - \frac{4}{x}} = \frac{\frac{x + 2}{x}}{\frac{x^2 - 4}{x}} = \frac{x + 2}{x} \cdot \frac{x}{x^2 - 4} = \frac{x + 2}{x} \cdot \frac{x}{(x + 2)(x - 2)} = \frac{1}{x - 2}, \quad x \neq 0, -2$$

59. $$\frac{\left(\frac{4}{x^2-9}+\frac{2}{x-2}\right)}{\left(\frac{1}{x+3}+\frac{1}{x-3}\right)} = \frac{\frac{4(x-2)+2(x^2-9)}{(x-2)(x^2-9)}}{\frac{(x-3)+(x+3)}{x^2-9}}$$

$$= \frac{2x^2+4x-26}{(x-2)\cancel{(x^2-9)}} \cdot \frac{\cancel{x^2-9}}{2x}$$

$$= \frac{2(x^2+2x-13)}{(x-2)(2x)}$$

$$= \frac{x^2+2x-13}{x(x-2)}, \quad x \neq \pm 3$$

60. $$\frac{\left(\frac{1}{x+1}+\frac{1}{2}\right)}{\left(\frac{3}{2x^2+4x+2}\right)} = \frac{\frac{x+3}{2(x+1)}}{\frac{3}{2(x+1)^2}}$$

$$= \frac{x+3}{2(x+1)} \cdot \frac{2(x+1)^2}{3}$$

$$= \frac{(x+3)(x+1)}{3}, x \neq -1$$

61. $e^{2x} + 2e^x - 15 = 0$

$(e^x + 5)(e^x - 3) = 0$

$e^x = -5$ or $e^x = 3$

No real solution. $\quad x = \ln 3$

$x \approx 1.099$

62. $e^{2x} - 10e^x + 24 = 0$

$(e^x - 4)(e^x - 6) = 0$

$e^x = 4$ or $e^x = 6$

$x = \ln 4 \quad x = \ln 6$

$x \approx 1.386 \quad x \approx 1.792$

63. $8(62 - e^{x/4}) = 192$

$62 - e^{x/4} = 24$

$-e^{x/4} = -38$

$e^{x/4} = 38$

$\frac{x}{4} = \ln 38$

$x = 4 \ln 38$

$x \approx 14.550$

64. $\frac{150}{e^{-x} - 4} = 75$

$150 = 75e^{-x} - 300$

$75e^{-x} = 450$

$e^{-x} = 6$

$-x = \ln 6$

$x = -\ln 6$

$x \approx -1.792$

65. $7 \ln 3x = 12$

$\ln 3x = \frac{12}{7}$

$3x = e^{12/7}$

$x = \frac{e^{12/7}}{3}$

$x \approx 1.851$

66. $\ln(x + 9)^2 = 2$

$2 \ln(x + 9) = 2$

$\ln(x + 9) = 1$

$x + 9 = e$

$x = e - 9$

$x \approx -6.282$

67. $$\begin{cases} -x - 2y + 3z = -23 \\ 2x + 6y - z = 17 \\ 5y + z = 8 \end{cases}$$

$$\begin{cases} -x - 2y + 3z = -23 \\ 2y + 5z = -29 \\ 5y + z = 8 \end{cases} \quad 2\text{Eq.}1 + \text{Eq.}2$$

$$\begin{cases} -x - 2y + 3z = -23 \\ 2y + 5z = -29 \\ -\frac{23}{2}z = \frac{161}{2} \end{cases} \quad -\tfrac{5}{2}\text{Eq.}2 + \text{Eq.}3$$

$-\frac{23}{2}z = \frac{161}{2} \implies z = -7$

$2y + 5(-7) = -29 \implies y = 3$

$-x - 2(3) + 3(-7) = -23 \implies -x - 27 = -23$

$\implies x = -4$

Solution: $(-4, 3, -7)$

68. $$\begin{cases} 7x - 3y + 5z = -28 \\ 4x + 4z = -16 \\ 7x + 2y - z = 0 \end{cases}$$

$$\begin{cases} 7x - 3y + 5z = -28 \\ 12y + 8z = 0 \\ 5y - 6z = 28 \end{cases} \quad \begin{matrix} (-4)\text{Eq.}1 + 7\text{Eq.}2 \\ (-1)\text{Eq.}1 + \text{Eq.}3 \end{matrix}$$

$$\begin{cases} 7x - 3y + 5z = -28 \\ 12y + 8z = 0 \\ -112z = 336 \end{cases} \quad (-5)\text{Eq.}2 + 12\text{Eq.}3$$

$$\begin{cases} 7x - 3y + 5z = -28 \\ 3y + 2z = 0 \\ z = -3 \end{cases} \quad \begin{matrix} \left(\frac{1}{4}\right)\text{Eq.}2 \\ \left(-\frac{1}{112}\right)\text{Eq.}3 \end{matrix}$$

$3y + 2(-3) = 0 \implies y = 2$

$7x - 3(2) + 5(-3) = -28 \implies x = -1$

Solution: $(-1, 2, -3)$

Review Exercises for Chapter 7

1. $\begin{cases} x + y = 2 \\ x - y = 0 \implies x = y \end{cases}$

$x + x = 2$

$2x = 2$

$x = 1$

$y = 1$

Solution: $(1, 1)$

2. $\begin{cases} 2x - 3y = 3 \\ x - y = 0 \implies x = y \end{cases}$

$2y - 3y = 3$

$-y = 3$

$y = -3$

$x = -3$

Solution: $(-3, -3)$

3. $\begin{cases} 0.5x + y = 0.75 \implies y = 0.75 - 0.5x \\ 1.25x - 4.5y = -2.5 \end{cases}$

$1.25x - 4.5(0.75 - 0.5x) = -2.5$

$1.25x - 3.375 + 2.25x = -2.5$

$3.50x = 0.875$

$x = 0.25$

$y = 0.625$

Solution: $(0.25, 0.625)$

4. $\begin{cases} -x + \frac{2}{5}y = \frac{3}{5} \\ -x + \frac{1}{5}y = -\frac{4}{5} \end{cases}$

Multiply both equations by 5 to clear the denominators.

$\begin{cases} -5x + 2y = 3 \\ -5x + y = -4 \implies -5x = -4 - y \end{cases}$

$(-4 - y) + 2y = 3$

$-4 + y = 3$

$y = 7$

$-5x = -4 - 7$

$-5x = -11$

$x = \frac{11}{5}$

Solution: $\left(\frac{11}{5}, 7\right)$

5. $\begin{cases} x^2 - y^2 = 9 \\ x - y = 1 \implies x = y + 1 \end{cases}$

$(y + 1)^2 - y^2 = 9$

$2y + 1 = 9$

$y = 4$

$x = 5$

Solution: $(5, 4)$

6. $\begin{cases} x^2 + y^2 = 169 \\ 3x + 2y = 39 \implies x = \frac{1}{3}(39 - 2y) \end{cases}$

$\left[\frac{1}{3}(39 - 2y)\right]^2 + y^2 = 169$

$\frac{1}{9}(1521 - 156y + 4y^2) + y^2 = 169$

$1521 - 156y + 4y^2 + 9y^2 = 1521$

$13y^2 - 156y = 0$

$13y(y - 12) = 0 \implies y = 0, 12$

$y = 0$: $x = \frac{1}{3}(39 - 2(0)) = 13$

$y = 12$: $x = \frac{1}{3}(39 - 2(12)) = 5$

Solution: $(13, 0), (5, 12)$

7. $\begin{cases} y = 2x^2 \\ y = x^4 - 2x^2 \implies 2x^2 = x^4 - 2x^2 \end{cases}$

$0 = x^4 - 4x^2$
$0 = x^2(x^2 - 4)$
$0 = x^2(x + 2)(x - 2)$
$x = 0, x = -2, x = 2$
$y = 0, y = 8, y = 8$

Solutions: $(0, 0), (-2, 8), (2, 8)$

8. $\begin{cases} x = y + 3 \\ x = y^2 + 1 \end{cases}$

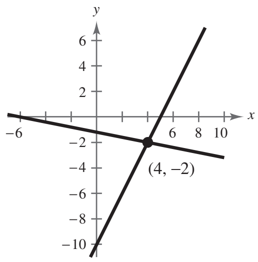
$y + 3 = y^2 + 1$
$0 = y^2 - y - 2$
$0 = (y - 2)(y + 1) \implies y = 2, -1$

$y = 2$: $x = 2 + 3 = 5$
$y = -1$: $x = -1 + 3 = 2$

Solution: $(5, 2), (2, -1)$

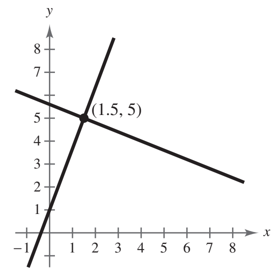
9. $\begin{cases} 2x - y = 10 \\ x + 5y = -6 \end{cases}$

Point of intersection: $(4, -2)$



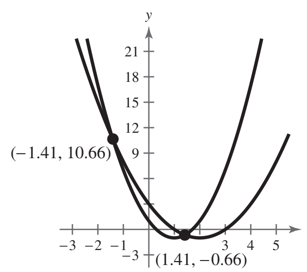
10. $\begin{cases} 8x - 3y = -3 \\ 2x + 5y = 28 \end{cases}$

The point of intersection appears to be at $(1.5, 5)$.



11. $\begin{cases} y = 2x^2 - 4x + 1 \\ y = x^2 - 4x + 3 \end{cases}$

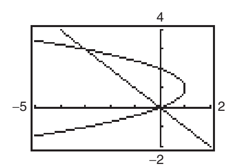
Point of intersection: $(1.41, -0.66), (-1.41, 10.66)$



12. $y^2 - 2y + x = 0 \implies (y - 1)^2 = 1 - x \implies y = 1 \pm \sqrt{1 - x}$

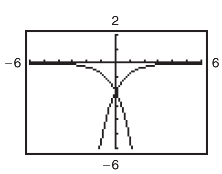
$x + y = 0 \implies y = -x$

Points of intersection: $(0, 0)$ and $(-3, 3)$



13. $\begin{cases} \phantom{2e^x + {}} y = -2e^{-x} \\ 2e^x + y = 0 \implies y = -2e^x \end{cases}$

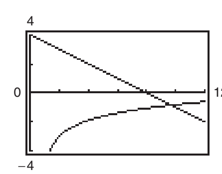
Point of intersection: $(0, -2)$



14. $y = \ln(x - 1) - 3$

$y = 4 - \frac{1}{2}x$

Point of intersection:
$(9.68, -0.84)$



15. Let $x =$ number of kits.

$C = 12x + 50{,}000$

$R = 25x$

Break-even: $R = C$

$25x = 12x + 50{,}000$

$13x = 50{,}000$

$x \approx 3846.15$

You would need to sell 3847 kits to cover your costs.

16. $\begin{cases} y = 35{,}000 + 0.015x \\ y = 32{,}500 + 0.02x \end{cases}$

$35{,}000 + 0.015x = 32{,}500 + 0.02x$

$2500 = 0.005x$

$\$500{,}000 = x$

For the second offer to be better, you would have to sell more than \$500,000 per year.

17.
$$2l + 2w = 480$$
$$l = 1.50w$$
$$2(1.50w) + 2w = 480$$
$$5w = 480$$
$$w = 96$$
$$l = 144$$

The dimensions are 96×144 meters.

18. $\begin{cases} 2l + 2w = 68 \\ w = \frac{8}{9}l \end{cases}$

$$2l + 2\left(\tfrac{8}{9}\right)l = 68$$
$$\tfrac{34}{9}l = 68$$
$$l = 18$$
$$w = \tfrac{8}{9}l = 16$$

The width of the rectangle is 16 feet, and the length is 18 feet.

19. $\begin{cases} 2x - y = 2 \Longrightarrow 16x - 8y = 16 \\ 6x + 8y = 39 \Longrightarrow \underline{6x + 8y = 39} \end{cases}$

$$22x = 55$$
$$x = \tfrac{55}{22} = \tfrac{5}{2}$$

Back-substitute $x = \frac{5}{2}$ into Equation 1.

$$2\left(\tfrac{5}{2}\right) - y = 2$$
$$y = 3$$

Solution: $\left(\frac{5}{2}, 3\right)$

20. $\begin{cases} 40x + 30y = 24 \Longrightarrow 40x + 30y = 24 \\ 20x - 50y = -14 \Longrightarrow \underline{-40x + 100y = 28} \end{cases}$

$$130y = 52$$
$$y = \tfrac{2}{5}$$

Back-substitute $y = \frac{2}{5}$ in Equation 1.

$$40x + 30\left(\tfrac{2}{5}\right) = 24$$
$$40x = 12$$
$$x = \tfrac{3}{10}$$

Solution: $\left(\frac{3}{10}, \frac{2}{5}\right)$

21. $\begin{cases} 0.2x + 0.3y = 0.14 \Longrightarrow 20x + 30y = 14 \Longrightarrow 20x + 30y = 14 \\ 0.4x + 0.5y = 0.20 \Longrightarrow 4x + 5y = 2 \Longrightarrow \underline{-20x - 25y = -10} \end{cases}$

$$5y = 4$$
$$y = \tfrac{4}{5}$$

Back-substitute $y = \frac{4}{5}$ into Equation 2.

$$4x + 5\left(\tfrac{4}{5}\right) = 2$$
$$4x = -2$$
$$x = -\tfrac{1}{2}$$

Solution: $\left(-\frac{1}{2}, \frac{4}{5}\right) = (-0.5, 0.8)$

22. $\begin{cases} 12x + 42y = -17 \Longrightarrow 36x + 126y = -51 \\ 30x - 18y = 19 \Longrightarrow \underline{210x - 126y = 133} \end{cases}$

$$246x = 82$$
$$x = \tfrac{1}{3}$$

Back-substitute $x = \frac{1}{3}$ in Equation 1.

$$12\left(\tfrac{1}{3}\right) + 42y = -17$$
$$42y = -21$$
$$y = -\tfrac{1}{2}$$

Solution: $\left(\frac{1}{3}, -\frac{1}{2}\right)$

23. $\begin{cases} 3x - 2y = 0 \Longrightarrow 3x - 2y = 0 \\ 3x + 2(y + 5) = 10 \Longrightarrow \underline{3x + 2y = 0} \end{cases}$

$$6x = 0$$
$$x = 0$$

Back-substitute $x = 0$ into Equation 1.

$$3(0) - 2y = 0$$
$$2y = 0$$
$$y = 0$$

Solution: $(0, 0)$

24. $\begin{cases} 7x + 12y = 63 \\ 2x + 3(y + 2) = 21 \end{cases}$

$\begin{cases} 7x + 12y = 63 \Longrightarrow -7x - 12y = -63 \\ 2x + 3y = 15 \Longrightarrow \underline{8x + 12y = 60} \end{cases}$

$$x = -3$$

Back-substitute $x = -3$ in Equation 1.

$$7(-3) + 12y = 63$$
$$12y = 84$$
$$y = 7$$

Solution: $(-3, 7)$

25. $\begin{cases} 1.25x - 2y = 3.5 \Rightarrow & 5x - 8y = 14 \\ 5x - 8y = 14 \Rightarrow & -5x + 8y = -14 \end{cases}$

$$0 = 0$$

There are infinitely many solutions.

Let $y = a$, then $5x - 8a = 14 \Rightarrow x = \frac{8}{5}a + \frac{14}{5}$.

Solution: $\left(\frac{8}{5}a + \frac{14}{5}, a\right)$ where a is any real number.

26. $\begin{cases} 1.5x + 2.5y = 8.5 \Rightarrow & 3x + 5y = 17 \\ 6x + 10y = 24 \Rightarrow & -3x - 5y = -12 \end{cases}$

$$0 = 5$$

The system is inconsistent. There is no solution.

27. $\begin{cases} x + 5y = 4 \Rightarrow & x + 5y = 4 \\ x - 3y = 6 \Rightarrow & -x + 3y = -6 \end{cases}$

$$8y = -2 \Rightarrow y = -\tfrac{1}{4}$$

Matches graph (d). The system has one solution and is consistent.

28. $\begin{cases} -3x + y = -7 \\ 9x - 3y = 21 \end{cases}$

$-3x + y = -7 \Rightarrow y = 3x - 7$;
The graph contains $(0, -7)$ and $(2, -1)$.

$9x - 3y = 21 \Rightarrow -3y = -9x + 21 \Rightarrow y = 3x - 7$;
The graph is the same as the previous graph.

The graph of the system matches (c). The system has infinitely many solutions and is consistent.

29. $\begin{cases} 3x - y = 7 \Rightarrow & 6x - 2y = 14 \\ -6x + 2y = 8 \Rightarrow & -6x + 2y = 8 \end{cases}$

$$0 \neq 22$$

Matches graph (b). The system has no solution and is inconsistent.

30. $\begin{cases} 2x - y = -3 \\ x + 5y = 4 \end{cases}$

$2x - y = -3 \Rightarrow -y = -2x - 3 \Rightarrow y = 2x + 3$;
The graph contains $(0, 3)$ and $(-2, -1)$.

$x + 5y = 4 \Rightarrow 5y = -x + 4 \Rightarrow y = -\frac{1}{5}x + \frac{4}{5}$;

The graph contains $\left(0, \frac{4}{5}\right)$ and $(4, 0)$.

The graph of the system matches (a). The system has one solution and is consistent.

31. $37 - 0.0002x = 22 + 0.00001x$

$$15 = 0.00021x$$

$$x = \frac{500{,}000}{7}, p = \frac{159}{7}$$

Point of equilibrium: $\left(\dfrac{500{,}000}{7}, \dfrac{159}{7}\right)$

32. $45 + 0.0002x = 120 - 0.0001x$

$$0.0003x = 75$$

$$x = 250{,}000 \text{ units}$$

$$p = \$95.00$$

Point of equilibrium: $(250{,}000, 95)$

33. $\begin{cases} x - 4y + 3z = 3 \\ -y + z = -1 \\ z = -5 \end{cases}$

$-y + (-5) = -1 \Rightarrow y = -4$

$x - 4(-4) + 3(-5) = 3 \Rightarrow x = 2$

Solution: $(2, -4, -5)$

34. $\begin{cases} x - 7y + 8z = 85 \\ y - 9z = -35 \\ z = 3 \end{cases}$

$y - 9(3) = -35 \Rightarrow y = -8$

$x - 7(-8) + 8(3) = 85 \Rightarrow x = 5$

Solution: $(5, -8, 3)$

35.
$$\begin{cases} x + 2y + 6z = 4 \\ -3x + 2y - z = -4 \\ 4x + 2z = 16 \end{cases}$$

$$\begin{cases} x + 2y + 6z = 4 & \\ 8y + 17z = 8 & 3\text{Eq.}1 + \text{Eq.}2 \\ -8y - 22z = 0 & -4\text{Eq.}1 + \text{Eq.}3 \end{cases}$$

$$\begin{cases} x + 2y + 6z = 4 & \\ 8y + 17z = 8 & \\ -5z = 8 & \text{Eq.}2 + \text{Eq.}3 \end{cases}$$

$$\begin{cases} x + 2y + 6z = 4 & \\ 8y + 17z = 8 & \\ z = -\frac{8}{5} & -\frac{1}{5}\text{Eq.}3 \end{cases}$$

$$8y + 17\left(-\tfrac{8}{5}\right) = 8 \implies y = \tfrac{22}{5}$$

$$x + 2\left(\tfrac{22}{5}\right) + 6\left(-\tfrac{8}{5}\right) = 4 \implies x = \tfrac{24}{5}$$

Solution: $\left(\frac{24}{5}, \frac{22}{5}, -\frac{8}{5}\right)$

36.
$$\begin{cases} x + 3y - z = 13 & \text{Equation 1} \\ 2x - 5z = 23 & \text{Equation 2} \\ 4x - y - 2z = 14 & \text{Equation 3} \end{cases}$$

$$\begin{cases} x + 3y - z = 13 & \\ -6y - 3z = -3 & (-2)\text{Eq.}1 + \text{Eq.}2 \\ -13y + 2z = -38 & (-4)\text{Eq.}1 + \text{Eq.}3 \end{cases}$$

$$\begin{cases} x + 3y - z = 13 & \\ -6y - 3z = -3 & \\ \frac{17}{2}z = -\frac{63}{2} & \left(-\frac{13}{6}\right)\text{Eq.}2 + \text{Eq.}3 \end{cases}$$

$$\begin{cases} x + 3y - z = 13 & \\ y + \frac{1}{2}z = \frac{1}{2} & \left(-\frac{1}{6}\right)\text{Eq.}2 \\ z = -\frac{63}{17} & \left(\frac{2}{17}\right)\text{Eq.}3 \end{cases}$$

$$y + \tfrac{1}{2}\left(-\tfrac{63}{17}\right) = \tfrac{1}{2} \implies y = \tfrac{40}{17}$$

$$x + 3\left(\tfrac{40}{17}\right) - \left(-\tfrac{63}{17}\right) = 13 \implies x = \tfrac{38}{17}$$

Solution: $\left(\frac{38}{17}, \frac{40}{17}, -\frac{63}{17}\right)$

37.
$$\begin{cases} x - 2y + z = -6 \\ 2x - 3y = -7 \\ -x + 3y - 3z = 11 \end{cases}$$

$$\begin{cases} x - 2y + z = -6 & \\ y - 2z = 5 & -2\text{Eq.}1 + \text{Eq.}2 \\ y - 2z = 5 & \text{Eq.}1 + \text{Eq.}3 \end{cases}$$

$$\begin{cases} x - 2y + z = -6 & \\ y - 2z = 5 & \\ 0 = 0 & -\text{Eq.}2 + \text{Eq.}3 \end{cases}$$

Let $z = a$, then:

$$y = 2a + 5$$

$$x - 2(2a + 5) + a = -6$$

$$x - 3a - 10 = -6$$

$$x = 3a + 4$$

Solution: $(3a + 4, 2a + 5, a)$ where a is any real number.

38.
$$\begin{cases} 2x + 6z = -9 & \text{Equation 1} \\ 3x - 2y + 11z = -16 & \text{Equation 2} \\ 3x - y + 7z = -11 & \text{Equation 3} \end{cases}$$

$$\begin{cases} -x + 2y - 5z = 7 & (-1)\text{Eq.}2 + \text{Eq.}1 \\ 3x - 2y + 11z = -16 & \\ 3x - y + 7z = -11 & \end{cases}$$

$$\begin{cases} -x + 2y - 5z = 7 & \\ 4y - 4z = 5 & 3\text{Eq.}1 + \text{Eq.}2 \\ 5y - 8z = 10 & 3\text{Eq.}1 + \text{Eq.}3 \end{cases}$$

$$\begin{cases} -x + 2y - 5z = 7 & \\ 4y - 4z = 5 & \\ -3y = 0 & (-2)\text{Eq.}2 + \text{Eq.}3 \end{cases}$$

$$\begin{cases} -x + 2y - 5z = 7 & \\ y - z = \frac{5}{4} & \left(\frac{1}{4}\right)\text{Eq.}2 \\ y = 0 & \left(-\frac{1}{3}\right)\text{Eq.}3 \end{cases}$$

$$(0) - z = \tfrac{5}{4} \implies z = -\tfrac{5}{4}$$

$$-x + 2(0) - 5\left(-\tfrac{5}{4}\right) = 7 \implies x = -\tfrac{3}{4}$$

Solution: $\left(-\frac{3}{4}, 0, -\frac{5}{4}\right)$

39.
$$\begin{cases} 5x - 12y + 7z = 16 \implies \\ 3x - 7y + 4z = 9 \implies \end{cases} \begin{cases} 15x - 36y + 21z = 48 \\ -15x + 35y - 20z = -45 \end{cases}$$
$$-y + z = 3$$

Let $z = a$. Then $y = a - 3$ and

$$5x - 12(a - 3) + 7a = 16 \implies x = a - 4.$$

Solution: $(a - 4,\ a - 3, a)$ where a is any real number.

40.
$$\begin{cases} 2x + 5y - 19z = 34 \implies & 6x + 15y - 57z = 102 \\ 3x + 8y - 31z = 54 \implies & -6x - 16y + 62z = -108 \end{cases}$$
$$-y + 5z = -6$$

Let $z = a$. Then:

$$-y + 5a = -6 \implies y = 5a + 6$$

$$2x + 5(5a + 6) - 19a = 34 \implies x = -3a + 2$$

Solution: $(-3a + 2, 5a + 6, a)$ where a is any real number.

41. $y = ax^2 + bx + c$ through $(0, -5)$, $(1, -2)$, and $(2, 5)$.

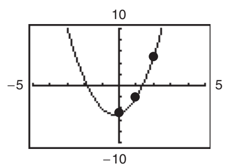
$$\begin{aligned}
(0, -5)&: \; -5 = + c \implies & c &= -5 \\
(1, -2)&: \; -2 = a + b + c \implies & \begin{cases} a + b = 3 \\ 2a + b = 5 \end{cases} \\
(2, 5)&: \; 5 = 4a + 2b + c \implies
\end{aligned}$$

$$\begin{cases} 2a + b = 5 \\ -a - b = -3 \end{cases}$$

$$a = 2$$

$$b = 1$$

The equation of the parabola is $y = 2x^2 + x - 5$.



42. $y = ax^2 + bx + c$ through $(-5, 6)$, $(1, 0)$, $(2, 20)$.

$(-5, 6)$: $6 = 25a - 5b + c$

$(1, 0)$: $0 = a + b + c \implies c = -a - b$

$(2, 20)$: $20 = 4a + 2b + c$

$$\begin{cases} 24a - 6b = 6 \implies 24a - 6b = 6 \\ 3a + b = 20 \implies -24a - 8b = -160 \end{cases}$$

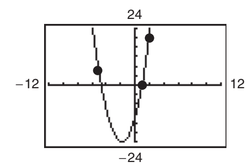
$$-14b = -154$$

$$b = 11$$

$$3a + 11 = 20 \implies a = 3$$

$$c = -3 - 11 \implies c = -14$$

The equation of the parabola is $y = 3x^2 + 11x - 14$.



43. $x^2 + y^2 + Dx + Ey + F = 0$ through $(-1, -2)$, $(5, -2)$ and $(2, 1)$.

$$\begin{aligned}
(-1,-2)&: \; 5 - D - 2E + F = 0 \implies & D + 2E - F &= 5 \\
(5,-2)&: \; 29 + 5D - 2E + F = 0 \implies & 5D - 2E + F &= -29 \\
(2, 1)&: \; 5 + 2D + E + F = 0 \implies & 2D + E + F &= -5
\end{aligned}$$

From the first two equations we have

$$6D = -24$$

$$D = -4.$$

Substituting $D = -4$ into the second and third equations yields:

$$\begin{aligned}
-20 - 2E + F = -29 &\implies \begin{cases} -2E + F = -9 \\ -E - F = -3 \end{cases} \\
-8 + E + F = -5 &\implies
\end{aligned}$$

$$-3E = -12$$

$$E = 4$$

$$F = -1$$

The equation of the circle is $x^2 + y^2 - 4x + 4y - 1 = 0$.

To verify the result using a graphing utility, solve the equation for y.

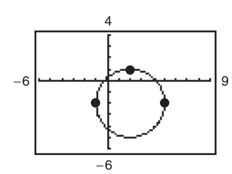
$$(x^2 - 4x + 4) + (y^2 + 4y + 4) = 1 + 4 + 4$$

$$(x - 2)^2 + (y + 2)^2 = 9$$

$$(y + 2)^2 = 9 - (x - 2)^2$$

$$y = -2 \pm \sqrt{9 - (x - 2)^2}$$

Let $y_1 = -2 + \sqrt{9 - (x - 2)^2}$ and $y_2 = -2 - \sqrt{9 - (x - 2)^2}$.



44. $x^2 + y^2 + Dx + Ey + F = 0$ through $(1, 4), (4, 3), (-2, -5)$.

$(1, 4)$: $17 + D + 4E + F = 0$

$(4, 3)$: $25 + 4D + 3E + F = 0$

$(-2, -5)$: $29 - 2D - 5E + F = 0$

$$\begin{cases} D + 4E + F = -17 & \text{Equation 1} \\ 4D + 3E + F = -25 & \text{Equation 2} \\ 2D + 5E - F = 29 & \text{Equation 3} \end{cases}$$

$$\begin{cases} D + 4E + F = -17 & \\ -13E - 3F = 43 & (-4)\text{Eq.1} + \text{Eq.2} \\ -3E - 3F = 63 & (-2)\text{Eq.1} + \text{Eq.3} \end{cases}$$

$$\begin{cases} D + 4E + F = -17 & \\ -3E - 3F = 63 & \text{Interchange equations.} \\ -13E - 3F = 43 & \end{cases}$$

$$\begin{cases} D + 4E + F = -17 & \\ -3E - 3F = 63 & \\ 10F = -230 & \left(-\frac{13}{3}\right)\text{Eq.2} + \text{Eq.3} \end{cases}$$

$F = -23, E = 2, D = -2$

The equation of the circle is $x^2 + y^2 - 2x + 2y - 23 = 0$.

To verify the result using a graphing utility, solve the equation for y.

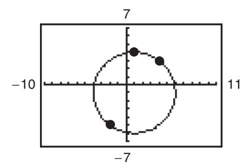
$$(x^2 - 2x + 1) + (y^2 + 2y + 1) = 23 + 1 + 1$$

$$(x - 1)^2 + (y + 1)^2 = 25$$

$$(y + 1)^2 = 25 - (x - 1)^2$$

$$y = -1 \pm \sqrt{25 - (x - 1)^2}$$

Let $y_1 = -1 + \sqrt{25 - (x - 1)^2}$ and $y_2 = -1 - \sqrt{25 - (x - 1)^2}$.



45. (3, 101.7), (4, 108.4), (5, 121.1)

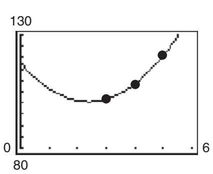
(a) $n = 3, \sum_{i=1}^{3} x_i = 12, \sum_{i=1}^{3} x_i^2 = 50, \sum_{i=1}^{3} x_i^3 = 216, \sum_{i=1}^{3} x_i^4 = 962, \sum_{i=1}^{3} y_i = 331.2, \sum_{i=1}^{3} x_i y_i = 1344.2, \sum_{i=1}^{3} x_i^2 y_i = 5677.2$

$$\begin{cases} 3c + 12b + 50a = 331.2 \\ 12c + 50b + 216a = 1344.2 \\ 50c + 216b + 962a = 5677.2 \end{cases}$$

Solving this system yields $c = 117.6, b = -14.3, a = 3$.

Quadratic model: $y = 3x^2 - 14.3x + 117.6$

(b)



The model is a good fit to the data. The actual points lie on the parabola.

(c) For 2008, use $x = 8$:
$y = 3(8)^2 - 14.3(8) + 117.6$
$= 195.2$ million online shoppers
This answer seems reasonable.

46. From the following chart we obtain our system of equations.

	A	B	C
Mixture X	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$
Mixture Y	0	0	1
Mixture Z	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
Desired Mixture	$\frac{6}{27}$	$\frac{8}{27}$	$\frac{13}{27}$

$$\left.\begin{aligned} \tfrac{1}{5}x + \tfrac{1}{3}z &= \tfrac{6}{27} \\ \tfrac{2}{5}x + \tfrac{1}{3}z &= \tfrac{8}{27} \end{aligned}\right\} x = \tfrac{10}{27}, z = \tfrac{12}{27}$$

$$\tfrac{2}{5}x + y + \tfrac{1}{3}z = \tfrac{13}{27} \implies y = \tfrac{5}{27}$$

To obtain the desired mixture, use 10 gallons of spray X, 5 gallons of spray Y, and 12 gallons of spray Z.

47. Let x = amount invested at 7%

y = amount invested at 9%

z = amount invested at 11%.

$y = x - 3000$ and
$z = x - 5000 \implies y + z = 2x - 8000$

$$\begin{cases} x + y + z = 40{,}000 \\ 0.07x + 0.09y + 0.11z = 3500 \\ y + z = 2x - 8000 \end{cases}$$

$$x + (2x - 8000) = 40{,}000 \implies x = 16{,}000$$
$$y = 16{,}000 - 3000 \implies y = 13{,}000$$
$$z = 16{,}000 - 5000 \implies z = 11{,}000$$

Thus, \$16,000 was invested at 7%, \$13,000 at 9% and \$11,000 at 11%.

48. $s = \frac{1}{2}at^2 + v_0t + s_0$

(a) When $t = 1$: $s = 134$: $\frac{1}{2}a(1)^2 + v_0(1) + s_0 = 134 \implies a + 2v_0 + 2s_0 = 268$

When $t = 2$: $s = 86$: $\frac{1}{2}a(2)^2 + v_0(2) + s_0 = 86 \implies 2a + 2v_0 + s_0 = 86$

When $t = 3$: $s = 6$: $\frac{1}{2}a(3)^2 + v_0(3) + s_0 = 6 \implies 9a + 6v_0 + 2s_0 = 12$

$$\begin{cases} a + 2v_0 + 2s_0 = 268 \\ 2a + 2v_0 + s_0 = 86 \\ 9a + 6v_0 + 2s_0 = 12 \end{cases}$$

$$\begin{cases} a + 2v_0 + 2s_0 = 268 & \\ -2v_0 - 3s_0 = -450 & (-2)\text{Eq.1} + \text{Eq.2} \\ -12v_0 - 16s_0 = -2400 & (-9)\text{Eq.1} + \text{Eq.3} \end{cases}$$

$$\begin{cases} a + 2v_0 + 2s_0 = 268 & \\ -2v_0 - 3s_0 = -450 & \\ 3v_0 + 4s_0 = 600 & \left(-\frac{1}{4}\right)\text{Eq.3} \end{cases}$$

$$\begin{cases} a + 2v_0 + 2s_0 = 268 & \\ -2v_0 - 3s_0 = -450 & \\ -s_0 = -150 & 3\text{Eq.2} + 2\text{Eq.3} \end{cases}$$

$$-s_0 = -150 \implies s_0 = 150$$

$$-2v_0 - 3(150) = -450 \implies v_0 = 0$$

$$a + 2(0) + 2(150) = 268 \implies a = -32$$

The position equation is $s = \frac{1}{2}(-32)t^2 + (0)t + 150$, or $s = -16t^2 + 150$.

(b) When $t = 1$: $s = 184$: $\frac{1}{2}a(1)^2 + v_0(1) + s_0 = 184 \implies a + 2v_0 + 2s_0 = 368$

When $t = 2$: $s = 116$: $\frac{1}{2}a(2)^2 + v_0(2) + s_0 = 116 \implies 2a + 2v_0 + s_0 = 116$

When $t = 3$: $s = 16$: $\frac{1}{2}a(3)^2 + v_0(3) + s_0 = 16 \implies 9a + 6v_0 + 2s_0 = 32$

$$\begin{cases} a + 2v_0 + 2s_0 = 368 \\ 2a + 2v_0 + s_0 = 116 \\ 9a + 6v_0 + 2s_0 = 32 \end{cases}$$

$$\begin{cases} a + 2v_0 + 2s_0 = 368 & \\ -2v_0 - 3s_0 = -620 & (-2)\text{Eq.1} + \text{Eq.2} \\ -12v_0 - 16s_0 = -3280 & (-9)\text{Eq.1} + \text{Eq.3} \end{cases}$$

$$\begin{cases} a + 2v_0 + 2s_0 = 368 & \\ -2v_0 - 3s_0 = -620 & \\ 3v_0 + 4s_0 = 820 & \left(-\frac{1}{4}\right)\text{Eq.3} \end{cases}$$

$$\begin{cases} a + 2v_0 + 2s_0 = 368 & \\ -2v_0 - 3s_0 = -620 & \\ -s_0 = -220 & 3\text{Eq.2} + 2\text{Eq.3} \end{cases}$$

$$-s_0 = -220 \implies s_0 = 220$$

$$-2v_0 - 3(220) = -620 \implies v_0 = -20$$

$$a + 2(-20) + 2(220) = 368 \implies a = -32$$

The position equation is $s = \frac{1}{2}(-32)t^2 + (-20)t + 220$, or $s = -16t^2 - 20t + 220$.

49. $\dfrac{3}{x^2 + 20x} = \dfrac{3}{x(x + 20)} = \dfrac{A}{x} + \dfrac{B}{x + 20}$

50. $\dfrac{x - 8}{x^2 - 3x - 28} = \dfrac{x - 8}{(x - 7)(x + 4)} = \dfrac{A}{x - 7} + \dfrac{B}{x + 4}$

51. $\dfrac{3x - 4}{x^3 - 5x^2} = \dfrac{3x - 4}{x^2(x - 5)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x - 5}$

52. $\dfrac{x - 2}{x(x^2 + 2)^2} = \dfrac{A}{x} + \dfrac{Bx + C}{x^2 + 2} + \dfrac{Dx + E}{(x^2 + 2)^2}$

53. $\dfrac{4 - x}{x^2 + 6x + 8} = \dfrac{A}{x + 2} + \dfrac{B}{x + 4}$

$4 - x = A(x + 4) + B(x + 2)$

Let $x = -2$: $6 = 2A \implies A = 3$

Let $x = -4$: $8 = -2B \implies B = -4$

$\dfrac{4 - x}{x^2 + 6x + 8} = \dfrac{3}{x + 2} - \dfrac{4}{x + 4}$

54. $\dfrac{-x}{x^2 + 3x + 2} = \dfrac{A}{x + 1} + \dfrac{B}{x + 2}$

$-x = A(x + 2) + B(x + 1)$

Let $x = -1$: $1 = A$

Let $x = -2$: $2 = -B \implies B = -2$

$\dfrac{-x}{x^2 + 3x + 2} = \dfrac{1}{x + 1} - \dfrac{2}{x + 2}$

55. $\dfrac{x^2}{x^2 + 2x - 15} = 1 - \dfrac{2x - 15}{x^2 + 2x - 15}$

$\dfrac{-2x + 15}{(x + 5)(x - 3)} = \dfrac{A}{x + 5} + \dfrac{B}{x - 3}$

$-2x + 15 = A(x - 3) + B(x + 5)$

Let $x = -5$: $25 = -8A \implies A = -\dfrac{25}{8}$

Let $x = 3$: $9 = 8B \implies B = \dfrac{9}{8}$

$\dfrac{x^2}{x^2 + 2x - 15} = 1 - \dfrac{25}{8(x + 5)} + \dfrac{9}{8(x - 3)}$

56. $\dfrac{9}{x^2 - 9} = \dfrac{A}{x - 3} + \dfrac{B}{x + 3}$

$9 = A(x + 3) + B(x - 3)$

Let $x = 3$: $9 = 6A \implies A = \dfrac{3}{2}$

Let $x = -3$: $9 = -6B \implies B = -\dfrac{3}{2}$

$\dfrac{9}{x^2 - 9} = \dfrac{1}{2}\left(\dfrac{3}{x - 3} - \dfrac{3}{x + 3}\right)$

57. $\dfrac{x^2 + 2x}{x^3 - x^2 + x - 1} = \dfrac{x^2 + 2x}{(x - 1)(x^2 + 1)} = \dfrac{A}{x - 1} + \dfrac{Bx + C}{x^2 + 1}$

$$\begin{aligned} x^2 + 2x &= A(x^2 + 1) + (Bx + C)(x - 1) \\ &= Ax^2 + A + Bx^2 - Bx + Cx - C \\ &= (A + B)x^2 + (-B + C)x + (A - C) \end{aligned}$$

Equating coefficients of like terms gives $1 = A + B$, $2 = -B + C$, and $0 = A - C$. Adding both sides of all three equations gives $3 = 2A$. Therefore, $A = \frac{3}{2}$, $B = -\frac{1}{2}$, and $C = \frac{3}{2}$.

$$\begin{aligned} \frac{x^2 + 2x}{x^3 - x^2 + x - 1} &= \frac{\frac{3}{2}}{x - 1} + \frac{-\frac{1}{2}x + \frac{3}{2}}{x^2 + 1} \\ &= \frac{1}{2}\left(\frac{3}{x - 1} - \frac{x - 3}{x^2 + 1}\right) \end{aligned}$$

58. $\dfrac{4x}{3(x - 1)^2} = \dfrac{A}{x - 1} + \dfrac{B}{(x - 1)^2}$

$\dfrac{4}{3}x = A(x - 1) + B$

Let $x = 1$: $\dfrac{4}{3} = B$

Let $x = 2$: $\dfrac{8}{3} = A + \dfrac{4}{3} \implies A = \dfrac{4}{3}$

$\dfrac{4x}{3(x - 1)^2} = \dfrac{4}{3(x - 1)} + \dfrac{4}{3(x - 1)^2}$

59. $\dfrac{3x^3 + 4x}{(x^2 + 1)^2} = \dfrac{Ax + B}{x^2 + 1} + \dfrac{Cx + D}{(x^2 + 1)^2}$

$$\begin{aligned} 3x^3 + 4x &= (Ax + B)(x^2 + 1) + Cx + D \\ &= Ax^3 + Bx^2 + (A + C)x + (B + D) \end{aligned}$$

Equating coefficients of like powers:

$3 = A$

$0 = B$

$4 = 3 + C \implies C = 1$

$0 = B + D \implies D = 0$

$\dfrac{3x^3 + 4x}{(x^2 + 1)^2} = \dfrac{3x}{x^2 + 1} + \dfrac{x}{(x^2 + 1)^2}$

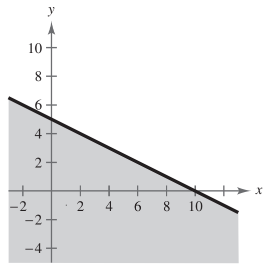
60. $\dfrac{4x^2}{(x - 1)(x^2 + 1)} = \dfrac{A}{x - 1} + \dfrac{Bx + C}{x^2 + 1}$

$$\begin{aligned} 4x^2 &= A(x^2 + 1) + (Bx + C)(x - 1) \\ &= Ax^2 + A + Bx^2 - Bx + Cx - C \\ &= (A + B)x^2 + (-B + C)x + (A - C) \end{aligned}$$

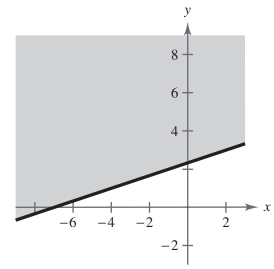
Equating coefficients of like terms gives $4 = A + B$, $0 = -B + C$, and $0 = A - C$. Adding both sides of all three equations gives $4 = 2A$, so $A = 2$. Then $B = 2$ and $C = 2$.

$$\begin{aligned} \frac{4x^2}{(x - 1)(x^2 + 1)} &= \frac{2}{x - 1} + \frac{2x + 2}{x^2 + 1} \\ &= 2\left(\frac{1}{x - 1} + \frac{x + 1}{x^2 + 1}\right) \end{aligned}$$

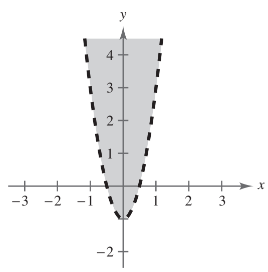
61. $y \le 5 - \frac{1}{2}x$



62. $3y - x \ge 7$

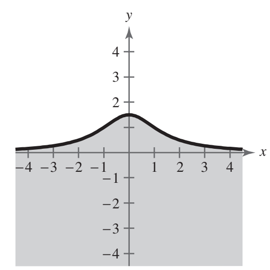


63. $y - 4x^2 > -1$

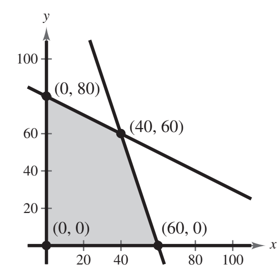


64. $y \le \dfrac{3}{x^2 + 2}$

Using a solid line, graph $y = \dfrac{3}{x^2 + 2}$, and shade below the curve. Use (0, 0) as a test point.



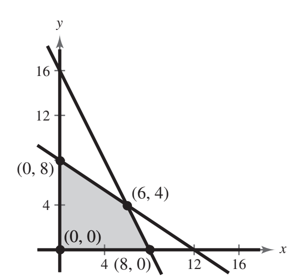
65. $\begin{cases} x + 2y \le 160 \\ 3x + y \le 180 \\ x \ge 0 \\ y \ge 0 \end{cases}$



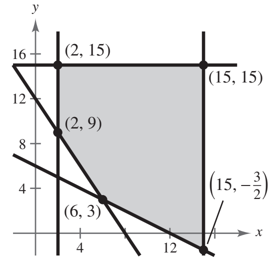
Vertex A	Vertex B	Vertex C	Vertex D
$x + 2y = 160$	$x + 2y = 160$	$3x + y = 180$	$x = 0$
$3x + y = 180$	$x = 0$	$y = 0$	$y = 0$
(40, 60)	(0, 80)	(60, 0)	(0, 0)

66. $\begin{cases} 2x + 3y \le 24 \\ 2x + y \le 16 \\ x \ge 0 \\ y \ge 0 \end{cases}$

Vertices: (0, 0), (0, 8), (6, 4), (8, 0)



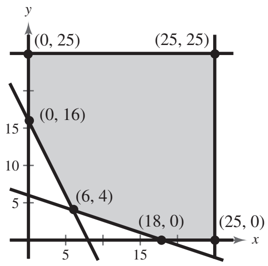
67. $\begin{cases} 3x + 2y \ge 24 \\ x + 2y \ge 12 \\ 2 \le x \le 15 \\ y \le 15 \end{cases}$



Vertex A	Vertex B	Vertex C	Vertex D	Vertex E
$3x + 2y = 24$	$3x + 2y = 24$	$x = 2$	$x = 15$	$x + 2y = 12$
$x + 2y = 12$	$x = 2$	$y = 15$	$y = 15$	$x = 15$
(6, 3)	(2, 9)	(2, 15)	(15, 15)	$\left(15, -\frac{3}{2}\right)$

68. $\begin{cases} 2x + y \ge 16 \\ x + 3y \ge 18 \\ 0 \le x \le 25 \\ 0 \le y \le 25 \end{cases}$

Vertices: (6, 4), (0, 16), (0, 25), (25, 25), (25, 0), (18, 0)



69. $\begin{cases} y < x + 1 \\ y > x^2 - 1 \end{cases}$

Vertices:

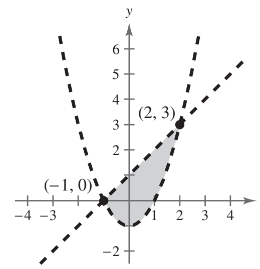
$x + 1 = x^2 - 1$

$0 = x^2 - x - 2 = (x + 1)(x - 2)$

$x = -1$ or $x = 2$

$y = 0 \qquad y = 3$

$(-1, 0) \qquad (2, 3)$



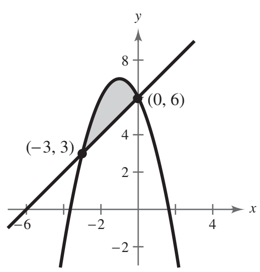
70. $\begin{cases} y \le 6 - 2x - x^2 \\ y \ge x + 6 \end{cases}$

Vertices: $x + 6 = 6 - 2x - x^2$

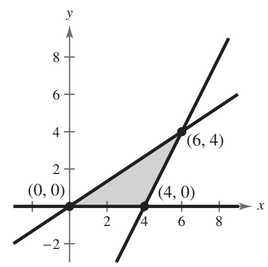
$x^2 + 3x = 0$

$x(x + 3) = 0 \implies x = 0, -3$

$(0, 6), (-3, 3)$



71. $\begin{cases} 2x - 3y \ge 0 \\ 2x - y \le 8 \\ y \ge 0 \end{cases}$



Vertex A	Vertex B	Vertex C
$2x - 3y = 0$	$2x - 3y = 0$	$2x - y = 8$
$2x - y = 8$	$y = 0$	$y = 0$
(6, 4)	(0, 0)	(4, 0)

72. $\begin{cases} x^2 + y^2 \le 9 \implies y^2 \le 9 - x^2 \\ (x - 3)^2 + y^2 \le 9 \implies y^2 \le 9 - (x - 3)^2 \end{cases}$

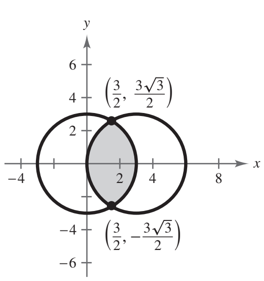
Vertices: $9 - x^2 = 9 - (x - 3)^2$

$(x - 3)^2 - x^2 = 0$

$x^2 - 6x + 9 - x^2 = 0$

$x = \dfrac{3}{2}$

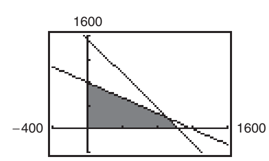
$\left(\dfrac{3}{2}, \pm\dfrac{3\sqrt{3}}{2}\right)$



73. x = number of units of Product I

y = number of units of Product II

$\begin{cases} 20x + 30y \le 24{,}000 \\ 12x + 8y \le 12{,}400 \\ x \ge 0 \\ y \ge 0 \end{cases}$

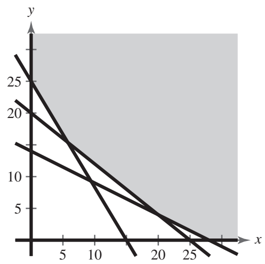


74. (a) Let x = amount of Food X.,

Let y = amount of Food Y.

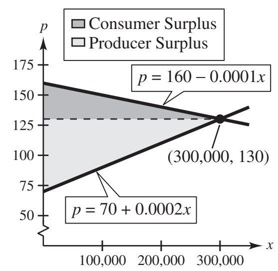
$$\begin{cases} 12x + 15y \ge 300 \\ 10x + 20y \ge 280 \\ 20x + 12y \ge 300 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

(b)



(c) Answers may vary. For example, (15, 8) or (16, 9) represent acceptable quantities (x, y) for Foods X and Y.

75. (a)



$$160 - 0.0001x = 70 + 0.0002x$$

$$90 = 0.0003x$$

$$x = 300{,}000 \text{ units}$$

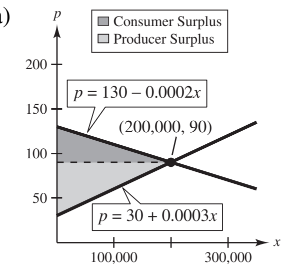
$$p = \$130$$

Point of equilibrium: (300,000, 130)

(b) Consumer surplus: $\frac{1}{2}(300{,}000)(30) = \$4{,}500{,}000$

Producer surplus: $\frac{1}{2}(300{,}000)(60) = \$9{,}000{,}000$

76. (a)



$$130 - 0.0002x = 30 + 0.0003x$$

$$100 = 0.0005x$$

$$x = 200{,}000 \text{ units}$$

$$p = \$90$$

Point of equilibrium: (200,000, 90)

(b) Consumer surplus: $\frac{1}{2}(200{,}000)(40) = \$4{,}000{,}000$

Producer surplus: $\frac{1}{2}(200{,}000)(60) = \$6{,}000{,}000$

77. Objective function: $z = 3x + 4y$

Constraints: $\begin{cases} x \ge 0 \\ y \ge 0 \\ 2x + 5y \le 50 \\ 4x + y \le 28 \end{cases}$

At (0, 0): $z = 0$

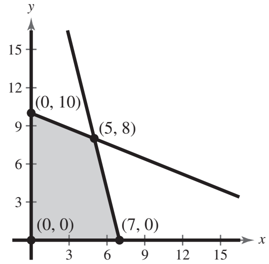
At (0, 10): $z = 40$

At (5, 8): $z = 47$

At (7, 0): $z = 21$

The minimum value is 0 at (0, 0).

The maximum value is 47 at (5, 8).



78. $z = 10x + 7y$

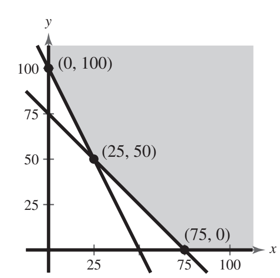
At (0, 100): $z = 10(0) + 7(100) = 700$

At (25, 50): $z = 10(25) + 7(50) = 600$

At (75, 0): $z = 10(75) + 7(0) = 750$

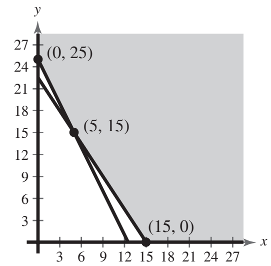
The minimum value is 600 at (25, 50).

There is no maximum value.



79. Objective function: $z = 1.75x + 2.25y$

Constraints: $\begin{cases} x \geq 0 \\ y \geq 0 \\ 2x + y \geq 25 \\ 3x + 2y \geq 45 \end{cases}$



At $(0, 25)$: $z = 56.25$

At $(5, 15)$: $z = 42.5$

At $(15, 0)$: $z = 26.25$

The minimum value is 26.25 at $(15, 0)$.

Since the region in unbounded, there is no maximum value.

80. $z = 50x + 70y$

At $(0, 0)$: $z = 50(0) + 70(0) = 0$

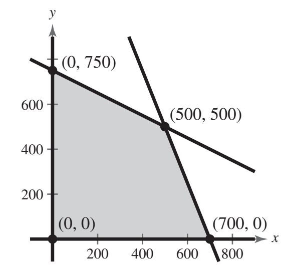
At $(0, 750)$: $z = 50(0) + 70(750) = 52{,}500$

At $(500, 500)$: $z = 50(500) + 70(500) = 60{,}000$

At $(700, 0)$: $z = 50(700) + 70(0) = 35{,}000$

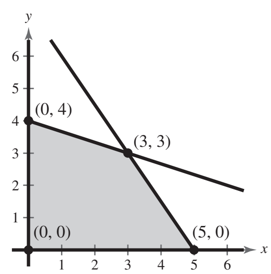
The minimum value is 0 at $(0, 0)$.

The maximum value is 60,000 at $(500, 500)$.



81. Objective function: $z = 5x + 11y$

Constraints: $\begin{cases} x \geq 0 \\ y \geq 0 \\ x + 3y \leq 12 \\ 3x + 2y \leq 15 \end{cases}$



At $(0, 0)$: $z = 0$

At $(5, 0)$: $z = 25$

At $(3, 3)$: $z = 48$

At $(0, 4)$: $z = 44$

The minimum value is 0 at $(0, 0)$.

The maximum value is 48 at $(3, 3)$.

82. $z = -2x + y$

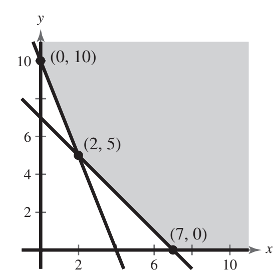
At $(0, 10)$: $z = -2(0) + 10 = 10$

At $(2, 5)$: $z = -2(2) + 5 = 1$

At $(7, 0)$: $z = -2(7) + 0 = -14$

The minimum value is -14 at $(7, 0)$.

There is no maximum value.



83. Let $x =$ number of haircuts

$y =$ number of permanents.

Objective function: Optimize $R = 25x + 70y$ subject to the following constraints:

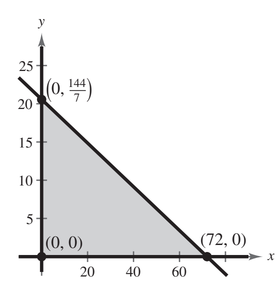
$$\begin{cases} x \geq 0 \\ y \geq 0 \\ \left(\frac{20}{60}\right)x + \left(\frac{70}{60}\right)y \leq 24 \implies 2x + 7y \leq 144 \end{cases}$$

At $(0, 0)$: $R = 0$

At $(72, 0)$: $R = 1800$

At $\left(0, \frac{144}{7}\right)$: $R = 1440$

The revenue is optimal if the student does 72 haircuts and no permanents. The maximum revenue is \$1800.

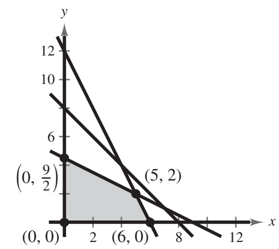


84. x = number of walking shoes

y = number of running shoes

Objective function: Optimize $P = 18x + 24y$ subject to the following constraints:

$$\begin{cases} 4x + 2y \le 24 \\ x + 2y \le 9 \\ x + y \le 8 \\ x \ge 0 \\ y \ge 0 \end{cases}$$



At $(0, 0)$: $P = 18(0) + 24(0) = 0$

At $(6, 0)$: $P = 18(6) + 24(0) = 108$

At $(5, 2)$: $P = 18(5) + 24(2) = 138$

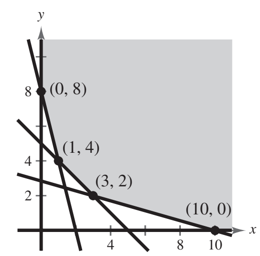
At $\left(0, \frac{9}{2}\right)$: $P = 19(0) + 24\left(\frac{9}{2}\right) = 108$

The optimal profit of \$138 occurs when 5 walking shoes and 2 running shoes are produced.

85. Let x = the number of bags of Brand X, and
y = the number of bags of Brand Y.

Objective function: Optimize $C = 15x + 30y$.

Constraints: $\begin{cases} 8x + 2y \ge 16 \\ x + y \ge 5 \\ 2x + 7y \ge 20 \\ x \ge 0 \\ y \ge 0 \end{cases}$



At $(0, 8)$: $C = 15(0) + 30(8) = 240$

At $(1, 4)$: $C = 15(1) + 30(4) = 135$

At $(3, 2)$: $C = 15(3) + 30(2) = 105$

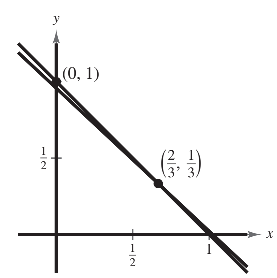
At $(10, 0)$: $C = 15(10) + 30(0) = 150$

To optimize cost, use three bags of Brand X and two bags of Brand Y. The minimum cost is \$105.

86. x = fraction of regular

y = fraction of premium

Constraints: $\begin{cases} 87x + 93y \ge 89 \\ x + y = 1 \\ x \ge 0 \\ y \ge 0 \end{cases}$



Objective function: Minimize $C = 1.63x + 1.83y$.

Note that the "region" defined by the constraints is actually the line segment connecting $(0, 1)$ and $\left(\frac{2}{3}, \frac{1}{3}\right)$.

At $(0, 1)$: $C = 1.63(0) + 1.83(1) = 1.83$

At $\left(\frac{2}{3}, \frac{1}{3}\right)$: $C = 1.63\left(\frac{2}{3}\right) + 1.83\left(\frac{1}{3}\right) = 1.70$

The minimum cost is \$1.70 and occurs with a mixture of $\frac{2}{3}$ gallon of regular and $\frac{1}{3}$ gallon of premium.

87. False. The system $y \le 5$, $y \ge -2$, $y \le \frac{7}{2}x - 9$, and $y \le -\frac{7}{2}x + 26$ represents the region covered by an isosceles trapezoid.

88. False. A linear programming problem either has one optional solution or infinitely many optimal solutions. (However, in real-life situations where the variables must have integer values, it is possible to have exactly ten integer-valued solutions.)

89. There are an infinite number of linear systems with the solution $(-6, 8)$. One possible solution is:

$$\begin{cases} x + y = 2 \\ x - y = -14 \end{cases}$$

90. There are an infinite number of linear systems with the solution $(5, -4)$. One possible system is:

$$\begin{cases} x - y = 9 \\ 3x + y = 11 \end{cases}$$

91. There are infinite linear systems with the solution $\left(\frac{4}{3}, 3\right)$. One possible solution is:

$$\begin{cases} 3x + y = 7 \\ -6x + 3y = 1 \end{cases}$$

92. There are an infinite number of linear systems with the solution $\left(-1, \frac{9}{4}\right)$. One possible system is:

$$\begin{cases} -x + 4y = 10 \\ 3x - 8y = -21 \end{cases}$$

93. There are an infinite number of linear systems with the solution $(4, -1, 3)$. One possible system is as follows:

$$\begin{cases} x + y + z = 6 \\ x + y - z = 0 \\ x - y - z = 2 \end{cases}$$

94. There are an infinite number of linear systems with the solution $(-3, 5, 6)$. One possible system is:

$$\begin{cases} x - 2y + z = -7 \\ 2x + y - 4z = -25 \\ -x + 3y - z = 12 \end{cases}$$

95. There are an infinite number of linear systems with the solution $\left(5, \frac{3}{2}, 2\right)$. One possible solution is:

$$\begin{cases} 2x + 2y - 3z = 7 \\ x - 2y + z = 4 \\ -x + 4y - z = -1 \end{cases}$$

96. There are an infinite number of linear systems with the solution $\left(\frac{3}{4}, -2, 8\right)$. One possible system is:

$$\begin{aligned} 4x + y - z &= -7 \\ 8x + 3y + 2z &= 16 \\ 4x - 2y + 3z &= 31 \end{aligned}$$

97. A system of linear equations is inconsistent if it has no solution.

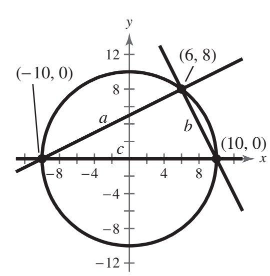
98. The lines are distinct and parallel.

$$\begin{cases} x + 2y = 3 \\ 2x + 4y = 9 \end{cases}$$

99. If the solution to a system of equations is at fractional or irrational values, then the substitution method may yield an exact answer. The graphical method works well when the solution is at integer values, otherwise we can usually only approximate the solution.

Problem Solving for Chapter 7

1. The longest side of the triangle is a diameter of the circle and has a length of 20.



The lines $y = \frac{1}{2}x + 5$ and $y = -2x + 20$ intersect at the point $(6, 8)$.

The distance between $(-10, 0)$ and $(6, 8)$ is:

$$d_1 = \sqrt{(6 - (-10))^2 + (8 - 0)^2} = \sqrt{320} = 8\sqrt{5}$$

The distance between $(6, 8)$ and $(10, 0)$ is:

$$d_2 = \sqrt{(10 - 6)^2 + (0 - 8)^2} = \sqrt{80} = 4\sqrt{5}$$

Since $\left(\sqrt{320}\right)^2 + \left(\sqrt{80}\right)^2 = (20)^2$

$$400 = 400$$

the sides of the triangle satisfy the Pythagorean Theorem. Thus, the triangle is a right triangle.

2. The system will have infinite solutions when the lines coincide, or are identical.

$$\begin{cases} 3x - 5y = 8 \implies 6x - 10y = 16 \\ 2x + k_1 y = k_2 \implies 6x + 3k_1 y = 3k_2 \end{cases}$$

$$3k_1 = -10 \implies k_1 = -\tfrac{10}{3}$$

$$3k_2 = 16 \implies k_2 = \tfrac{16}{3}$$

3. The system will have exactly one solution when the slopes of the line are *not* equal.

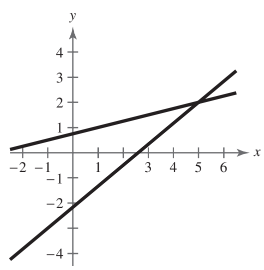
$$\begin{cases} ax + by = e \implies y = -\dfrac{a}{b}x + \dfrac{e}{b} \\ cx + dy = f \implies y = -\dfrac{c}{d}x + \dfrac{f}{d} \end{cases}$$

$$-\frac{a}{b} \neq -\frac{c}{d}$$

$$\frac{a}{b} \neq \frac{c}{d}$$

$$ad \neq bc$$

4. (a) $\begin{cases} x - 4y = -3 & \text{Eq. 1} \\ 5x - 6y = 13 & \text{Eq. 2} \end{cases}$

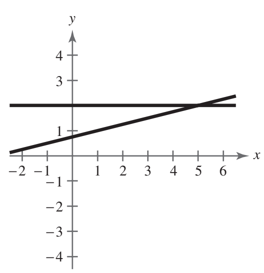


$$\begin{cases} x - 4y = -3 \\ \quad 14y = 28 & -5\text{Eq.}1 + \text{Eq.}2 \end{cases}$$

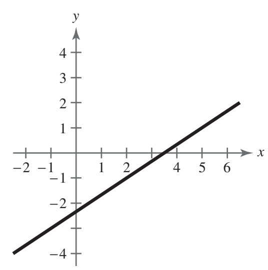
$y = 2$

$x - 4(2) = -3 \implies x = 5$

Solution: $(5, 2)$



(b) $\begin{cases} 2x - 3y = 7 & \text{Eq.1} \\ -4x + 6y = -14 & \text{Eq.2} \end{cases}$



$$\begin{cases} 2x - 3y = 7 \\ \quad 0 = 0 & 2\text{Eq.}1 + \text{Eq.}2 \end{cases}$$

The lines coincide. Infinite solutions.

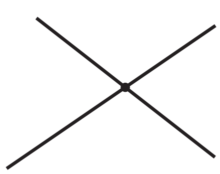
Let $y = a$, then $2x - 3a = 7 \implies x = \dfrac{3a + 7}{2}$

Solution: $\left(\dfrac{3a + 7}{2}, a\right)$

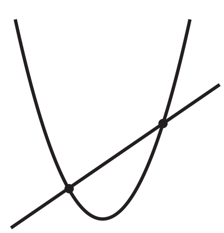
The solution(s) remain the same at each step of the process.

5. There are a finite number of solutions.

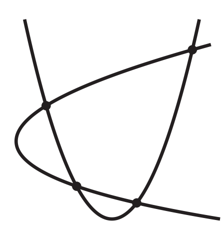
(a) If both equations are linear, then the maximum number of solutions to a finite system is *one*.



(b) If one equation is linear and the other is quadratic, then the maximum number of solutions is *two*.



(c) If both equations are quadratic, then the maximum number of solutions to a finite system is *four*.



6. $B =$ total votes cast for Bush

$K =$ total votes cast for Kerry

$N =$ total votes cast for Nader

$$\begin{cases} B + K + N = 118{,}304{,}000 \\ B - K = 3{,}320{,}000 \\ N = 0.003(118{,}304{,}000) \end{cases}$$

$N = 354{,}912$

$$\begin{cases} B + K + 354{,}912 = 118{,}304{,}000 \\ B - K = 3{,}320{,}000 \end{cases}$$

$$\begin{cases} B + K = 117{,}949{,}088 \\ B - K = 3{,}320{,}000 \end{cases}$$

$2B = 121{,}269{,}088$

$B = 60{,}634{,}544$

$K = 57{,}314{,}544$

Bush: 60,634,544 votes

Kerry: 57,314,544 votes

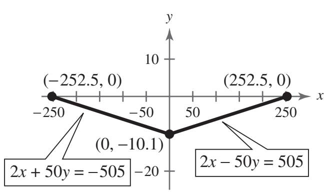
Nader: 354,912 votes

7. The point where the two sections meet is at a depth of 10.1 feet. The distance between $(0, -10.1)$ and $(252.5, 0)$ is:

$d = \sqrt{(252.5 - 0)^2 + (0 - (-10.1))^2} = \sqrt{63858.26}$

$d \approx 252.7$

Each section is approximately 252.7 feet long.



8. Let $C =$ weight of a carbon atom.

Let $H =$ weight of a hydrogen atom.

$$\begin{cases} 2C + 6H = 30.07 \implies & 8C + 24H = 120.28 \\ 3C + 8H = 44.097 \implies & -9C - 24H = -132.291 \end{cases}$$

$$-C = -12.011$$

$$C = 12.011$$

$$H = 1.008$$

Each carbon atom weighs 12.011 u.

Each hydrogen atom weighs 1.008 u.

9. Let $x =$ cost of the cable, per foot.

Let $y =$ cost of a connector.

$$\begin{cases} 6x + 2y = 15.50 \implies & 6x + 2y = 15.50 \\ 3x + 2y = 10.25 \implies & -3x - 2y = -10.25 \end{cases}$$

$$3x = 5.25$$

$$x = 1.75$$

$$y = 2.50$$

For a four-foot cable with a connector on each end the cost should be $4(1.75) + 2(2.50) = \$12.00$

10. Let $t =$ time that the 9:00 A.M. bus is on the road.

Then $t - \frac{1}{4} =$ time that the 9:15 A.M. bus is on the road.

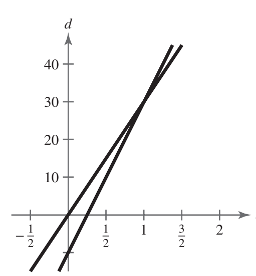
$$\begin{cases} d = 30t \\ d = 40\left(t - \frac{1}{4}\right) \end{cases}$$

$$40\left(t - \tfrac{1}{4}\right) = 30t$$

$$40t - 10 = 30t$$

$$10t = 10$$

$$t = 1$$



The 9:15 A.M. bus will catch up with the 9:00 A.M. bus in **one** hour. At that point both buses have traveled 30 miles and are 5 **miles** from the airport.

11. Let $X = \frac{1}{x}$, $Y = \frac{1}{y}$, and $Z = \frac{1}{z}$.

(a) $$\begin{cases} \frac{12}{x} - \frac{12}{y} = 7 \implies 12X - 12Y = 7 \implies 12X - 12Y = 7 \\ \frac{3}{x} + \frac{4}{y} = 0 \implies 3X + 4Y = 0 \implies \underline{9X + 12Y = 0} \end{cases}$$

$$21X = 7$$

$$X = \frac{1}{3}$$

$$Y = -\frac{1}{4}$$

Thus, $\frac{1}{x} = \frac{1}{3} \implies x = 3$ and $\frac{1}{y} = -\frac{1}{4} \implies y = -4$.

Solution: $(3, -4)$

(b) $$\begin{cases} \frac{2}{x} + \frac{1}{y} - \frac{3}{z} = 4 \implies 2X + Y - 3Z = 4 & \text{Eq.1} \\ \frac{4}{x} + \frac{2}{z} = 10 \implies 4X + 2Z = 10 & \text{Eq.2} \\ -\frac{2}{x} + \frac{3}{y} - \frac{13}{z} = -8 \implies -2X + 3Y - 13Z = -8 & \text{Eq.3} \end{cases}$$

$$\begin{cases} 2X + Y - 3Z = 4 \\ -2Y + 8Z = 2 & -2\text{Eq.1} + \text{Eq.2} \\ 4Y - 16Z = -4 & \text{Eq.1} + \text{Eq.3} \end{cases}$$

$$\begin{cases} 2X + Y - 3Z = 4 \\ -2Y + 8Z = 2 \\ 0 = 0 & 2\text{Eq.2} + \text{Eq.3} \end{cases}$$

The system has infinite solutions.

Let $Z = a$, then $Y = 4a - 1$ and $X = \frac{-a + 5}{2}$.

Then $\frac{1}{z} = a \implies z = \frac{1}{a}$, $\frac{1}{y} = 4a - 1 \implies y = \frac{1}{4a - 1}$

$x = \frac{-a + 5}{2} \implies x = \frac{2}{-a + 5}$.

Solution: $\left(\frac{2}{-a + 5}, \frac{1}{4a - 1}, \frac{1}{a}\right)$, $a \neq 5, \frac{1}{4}, 0$

12. Solution: $(-1, 2, -3)$

$x + 2y - 3z = a \implies (-1) + 2(2) - 3(-3) = 12 = a$

$-x - y + z = b \implies -(-1) - 2 + (-3) = -4 = b$

$2x + 3y - 2z = c \implies 2(-1) + 3(2) - 2(-3) = 10 = c$

Thus, $a = 12$, $b = -4$, and $c = 10$.

13. Solution: $(1, -1, 2)$

$$\begin{cases} 4x - 2y + 5z = 16 & \text{Equation 1} \\ x + y = 0 & \text{Equation 2} \\ -x - 3y + 2z = 6 & \text{Equation 3} \end{cases}$$

(a) $\begin{cases} 4x - 2y + 5z = 16 \\ x + y = 0 \end{cases}$

$$\begin{cases} x + y = 0 \\ 4x - 2y + 5z = 16 \end{cases} \quad \text{Interchange the equations.}$$

$$\begin{cases} x + y = 0 \\ -6y + 5z = 16 \end{cases} \quad -4\text{Eq.}1 + \text{Eq.}2$$

Let $z = a$, then $y = \dfrac{5a - 16}{6}$ and $x = \dfrac{-5a + 16}{6}$.

Solution: $\left(\dfrac{-5a + 16}{6}, \dfrac{5a - 16}{6}, a\right)$

When $a = 2$ we have the original solution.

(c) $\begin{cases} x + y = 0 \\ -x - 3y + 2z = 6 \end{cases}$

$$\begin{cases} x + y = 0 \\ -2y + 2z = 6 \end{cases} \quad \text{Eq.}1 + \text{Eq.}2$$

Let $z = c$, then $y = c - 3$ and $x = -c + 3$

Solution: $(-c + 3, c - 3, c)$

When $c = 2$ we have the original solution.

(b) $\begin{cases} 4x - 2y + 5z = 16 \\ -x - 3y + 2z = 6 \end{cases}$

$$\begin{cases} -x - 3y + 2z = 6 \\ 4x - 2y + 5z = 16 \end{cases} \quad \text{Interchange the equations.}$$

$$\begin{cases} -x - 3y + 2z = 6 \\ -14y + 13z = 40 \end{cases} \quad 4\text{Eq.}1 + \text{Eq.}2$$

Let $z = b$, then $y = \dfrac{13b - 40}{14}$ and $x = \dfrac{-11b + 36}{14}$

Solution: $\left(\dfrac{-11b + 36}{14}, \dfrac{13b - 40}{14}, b\right)$

When $b = 2$ we have the original solution.

(d) Each of these systems has infinite solutions.

14. $$\begin{cases} x_1 - x_2 + 2x_3 + 2x_4 + 6x_5 = 6 \\ 3x_1 - 2x_2 + 4x_3 + 4x_4 + 12x_5 = 14 \\ x_2 - x_3 - x_4 - 3x_5 = -3 \\ 2x_1 - 2x_2 + 4x_3 + 5x_4 + 15x_5 = 10 \\ 2x_1 - 2x_2 + 4x_3 + 4x_4 + 13x_5 = 13 \end{cases}$$

$$\begin{cases} x_1 - x_2 + 2x_3 + 2x_4 + 6x_5 = 6 \\ x_1 = 2 & -2\text{Eq.}1 + \text{Eq.}2 \\ x_2 - x_3 - x_4 - 3x_5 = -3 \\ 2x_1 - 2x_2 + 4x_3 + 5x_4 + 15x_5 = 10 \\ 2x_1 - 2x_2 + 4x_3 + 4x_4 + 13x_5 = 13 \end{cases}$$

$$\begin{cases} x_1 + x_2 = 0 & \text{Eq.}1 + 2\text{Eq.}3 \\ x_1 = 2 \\ x_2 - x_3 - x_4 - 3x_5 = -3 \\ 2x_1 - 2x_2 + 4x_3 + 5x_4 + 15x_5 = 10 \\ 2x_1 - 2x_2 + 4x_3 + 4x_4 + 13x_5 = 13 \end{cases}$$

$$\begin{cases} x_2 = -2 & \text{Eq.}1 - \text{Eq.}2 \\ x_1 = 2 \\ x_2 - x_3 - x_4 - 3x_5 = -3 \\ 2x_1 - 2x_2 + 4x_3 + 5x_4 + 15x_5 = 10 \\ 2x_1 - 2x_2 + 4x_3 + 4x_4 + 13x_5 = 13 \end{cases}$$

—CONTINUED—

14. —CONTINUED—

Substitute into the subsequent equations and simplify:

$$\begin{cases} x_1 = 2 \\ x_2 = -2 \\ (-2) - x_3 - x_4 - 3x_5 = -3 \\ 2(2) - 2(-2) + 4x_3 + 5x_4 + 15x_5 = 10 \\ 2(2) - 2(-2) + 4x_3 + 4x_4 + 13x_5 = 13 \end{cases}$$

$$\begin{cases} x_1 = 2 \\ x_2 = -2 \\ -x_3 - x_4 - 3x_5 = -1 \\ 4x_3 + 5x_4 + 15x_5 = 2 \\ 4x_3 + 4x_4 + 13x_5 = 5 \end{cases}$$

$$\begin{cases} x_1 = 2 \\ x_2 = -2 \\ x_3 + x_4 + 3x_5 = 1 & -\text{Eq.3} \\ x_4 + 3x_5 = -2 & \text{Eq.4} + (4)\text{Eq.3} \\ x_5 = 1 & \text{Eq.5} + (4)\text{Eq.3} \end{cases}$$

$$\begin{cases} x_1 = 2 \\ x_2 = -2 \\ x_3 = 3 & \text{Eq.3} - \text{Eq.4} \\ x_4 = -5 & \text{Eq.4} - (3)\text{Eq.5} \\ x_5 = 1 \end{cases}$$

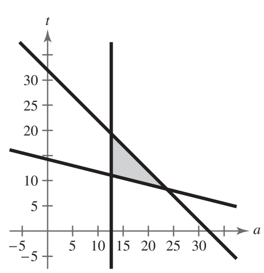
15. $t =$ amount of terrestrial vegetation in kilograms

$a =$ amount of aquatic vegetation in kilograms

$$a + t \le 32$$

$$0.15a \ge 1.9$$

$$193a + 4(193)t \ge 11{,}000$$

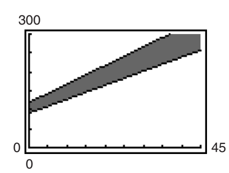


16. $x =$ number of inches by which a person's height exceeds 4 feet 10 inches

$y =$ person's weight in pounds

(a) $\begin{cases} y \quad 91 + 3.7x \\ y \le 119 + 4.8x \\ x \ge 0, \quad y \ge 0 \end{cases}$

(b)



(c) For someone 6 feet tall, $x = 14$ inches.

Minimum weight: $91 + 3.7(14) = 142.8$ pounds

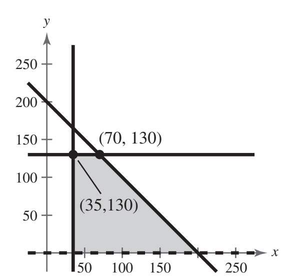
Maximum weight: $119 + 4.8(14) = 186.2$ pounds

17. (a) $x =$ HDL cholesterol (good)

$y =$ LDL cholesterol (bad)

$$\begin{cases} 0 < y < 130 \\ x \ge 35 \\ x + y \le 200 \end{cases}$$

(b)



(c) $y = 120$ *is* in the region since $0 < y < 130$.

$x = 90$ *is* in the region since $35 < x < 200$.

$x + y = 210$ *is not* in the region since $x + y < 200$.

(d) If the LDL reading is 150 and the HDL reading is 40, then $x \ge 35$ and $x + y \le 200$ but $y \nless 130$.

(e) $\dfrac{x + y}{x} < 4$

$$x + y < 4x$$

$$y < 3x$$

The point $(50, 120)$ is in the region and $120 < 3(50)$.

Chapter 7 Practice Test

For Exercises 1–3, solve the given system by the method of substitution.

1. $\begin{cases} x + y = 1 \\ 3x - y = 15 \end{cases}$

2. $\begin{cases} x - 3y = -3 \\ x^2 + 6y = 5 \end{cases}$

3. $\begin{cases} x + y + z = 6 \\ 2x - y + 3z = 0 \\ 5x + 2y - z = -3 \end{cases}$

4. Find the two numbers whose sum is 110 and product is 2800.

5. Find the dimensions of a rectangle if its perimeter is 170 feet and its area is 1500 square feet.

For Exercises 6–8, solve the linear system by elimination.

6. $\begin{cases} 2x + 15y = 4 \\ x - 3y = 23 \end{cases}$

7. $\begin{cases} x + y = 2 \\ 38x - 19y = 7 \end{cases}$

8. $\begin{cases} 0.4x + 0.5y = 0.112 \\ 0.3x - 0.7y = -0.131 \end{cases}$

9. Herbert invests $17,000 in two funds that pay 11% and 13% simple interest, respectively. If he receives $2080 in yearly interest, how much is invested in each fund?

10. Find the least squares regression line for the points $(4, 3)$, $(1, 1)$, $(-1, -2)$, and $(-2, -1)$.

For Exercises 11–12, solve the system of equations.

11. $\begin{cases} x + y = -2 \\ 2x - y + z = 11 \\ 4y - 3z = -20 \end{cases}$

12. $\begin{cases} 3x + 2y - z = 5 \\ 6x - y + 5z = 2 \end{cases}$

13. Find the equation of the parabola $y = ax^2 + bx + c$ passing through the points $(0, -1)$, $(1, 4)$ and $(2, 13)$.

For Exercises 14–15, write the partial fraction decomposition of the rational functions.

14. $\dfrac{10x - 17}{x^2 - 7x - 8}$

15. $\dfrac{x^2 + 4}{x^4 + x^2}$

16. Graph $x^2 + y^2 \geq 9$.

17. Graph the solution of the system.

$$\begin{cases} x + y \leq 6 \\ x \geq 2 \\ y \geq 0 \end{cases}$$

18. Derive a set of inequalities to describe the triangle with vertices (0, 0), (0, 7), and (2, 3).

19. Find the maximum value of the objective function, $z = 30x + 26y$, subject to the following constraints.

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ 2x + 3y \leq 21 \\ 5x + 3y \leq 30 \end{cases}$$

20. Graph the system of inequalities.

$$\begin{cases} x^2 + y^2 \leq 4 \\ (x - 2)^2 + y^2 \geq 4 \end{cases}$$

For Exercises 21–22, write the partial fraction decomposition for the rational expression.

21. $\dfrac{1 - 2x}{x^2 + x}$

22. $\dfrac{6x - 17}{(x - 3)^2}$