Chapter 7 The Energy Equation

7.1 Energy, Work, and Power

When matter has *energy*, the matter can be used to do work. A fluid can have several forms of energy. For example a fluid jet has kinetic energy, water behind a dam has gravitational potential energy, and hot steam has thermal energy.

Work is force acting through a distance when the force is parallel to the direction of motion

Work = Force x distance =Torque x Angular displacement

Machine is any device that transmits or modifies energy, typically to perform or assist in a human task.

a *turbine* is a machine that is used to extract energy from a flowing fluid.

a *pump* is a machine that is used to provide energy to a flowing fluid.

Work and energy both have the same primary dimensions, and the same units, and both characterize an amount or quantity

Power: Amount of work per unite time.

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \dot{W}$$

$P = \lim_{\Delta t \to 0} \frac{F\Delta x}{\Delta t} = FV \text{ , } for\ moving\ body$, V: velocity

$P = \lim_{\Delta t \to 0} \frac{T\Delta \theta}{\Delta t} = T\omega \text{ , } for\ rotating\ shaft$, ω: Angular speed

$$P = \dot{W} = FV = T\omega$$

Watt, kWatt, Horsepower

1 Horsepower (Hp) = 746 Watt or 1 Kw = 1.34 Hp

7.2 Energy Equation: General Form

$$\dot{Q} - \dot{W} = \frac{dE}{dt}$$

$$\begin{Bmatrix} Net\ rate\ of \\ Thermal\ Energy \\ entering\ system \end{Bmatrix} - \begin{Bmatrix} Net\ rate\ at\ which \\ system\ works\ on \\ environment \end{Bmatrix} = \begin{Bmatrix} rate\ of\ change \\ of\ Energy\ of\ the \\ matter\ within\ system \end{Bmatrix}$$

$$\dot{Q} - \dot{W} = \frac{d}{dt}\int_{CV} \left(\frac{V^2}{2} + gz + u\right)\rho \ \ d\forall + \int_{CS} \left(\frac{V^2}{2} + gz + u\right)\rho \ \ \boldsymbol{V} \ \ .d\boldsymbol{A}$$

u: internal energy

Shaft and Flow Work

Work = flow work + shaft work

$$\dot{W}_{shaft} = \dot{W}_{Turbine} - \dot{W}_{pump} = \dot{W}_t - \dot{W}_p$$

$$\dot{W}_{flow} = \int_{CS} \left(\frac{p}{\rho}\right)\rho \ \ \boldsymbol{V} \ \ .d\boldsymbol{A}$$

$$\dot{Q}-\dot{W}=\frac{d}{dt}\int_{CV}\left(\frac{V^2}{2}+gz+u\right)\rho\ \ d\forall+\sum_{CS}\dot{m}_o\left(\frac{V_o^2}{2}+gz_o+h_o\right)-\sum_{CS}\dot{m}_i\left(\frac{V_i^2}{2}+gz_i+h_i\right)$$

Where h = u+p/ρ : enthalpy

7.3 Energy Equation: Pipe Flow

Kinetic Energy Correction Factor

$$\alpha=\frac{\text{actual KE / time that crosses a section}}{\text{KE / time by assuming a uniform velocity distribution}}=\frac{\int_A \frac{\rho V^3 dA}{2}}{\frac{\rho\bar{V}^3 A}{2}}$$

$$\alpha=\frac{1}{A}\int_A\left(\frac{V}{\bar{V}}\right)^3 dA$$

In most cases, α takes on a value of 1 or 2. When the velocity profile in a pipe is uniformly distributed, then α = 1. When flow is laminar, the velocity distribution is parabolic and α = 2. When flow is turbulent, the velocity profile is plug-like and α ≈ 1.05. For turbulent flow it is common practice to let α = 1.

A Simplified Form of the Energy Equation

$$\left(\frac{p_1}{\gamma}+z_1+\alpha_1\frac{\bar{V}_1^2}{2g}\right)+h_p=\left(\frac{p_2}{\gamma}+z_2+\alpha_2\frac{\bar{V}_2^2}{2g}\right)+h_t+h_L$$

$$h_p, h_t, h_L\text{: pump head, turbine head and losses head respectively.}$$

$$\begin{pmatrix}\text{head}\\ \text{carried by}\\ \text{flow into the cv}\end{pmatrix}+\begin{pmatrix}\text{head}\\ \text{added by}\\ \text{pumps}\end{pmatrix}=\begin{pmatrix}\text{head}\\ \text{carried by}\\ \text{flow out of the cv}\end{pmatrix}+\begin{pmatrix}\text{head}\\ \text{extracted by}\\ \text{turbines}\end{pmatrix}+\begin{pmatrix}\text{head}\\ \text{loss due to}\\ \text{viscous effects}\end{pmatrix}$$

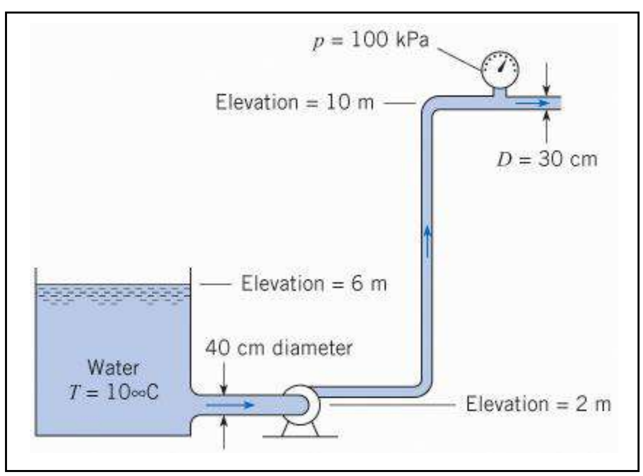
$$\text{head}\approx\frac{\text{energy / time or work / time}}{\text{weight / time of flowing fluid}}$$

$$h_p=\frac{\dot{W}_p}{\dot{m}g},\ h_t=\frac{\dot{W}_t}{\dot{m}g}$$

$$h_L=\frac{f(L/D)V^2}{2g},\ f\text{: friction factor}$$

Example

Water (10°C) is flowing at a rate of 0.35 m3/s, and it is assumed that hL= $2V^2/2g$ **from the reservoir to the gage, where V is the velocity in the 30-cm pipe.** What power must the pump supply? Assume α= 1.0 at all locations



$$
\begin{aligned}
V &= \frac{Q}{A} \\
&= \frac{0.35}{(\pi/4)\times(0.3\,\mathrm{m})^2} \\
&= 4.95\ \mathrm{m/s} \\
\frac{V_2^2}{2g} &= 1.250\ \mathrm{m}
\end{aligned}
$$

Energy equation (locate 1 on the reservoir surface; locate 2 at the pressure gage)

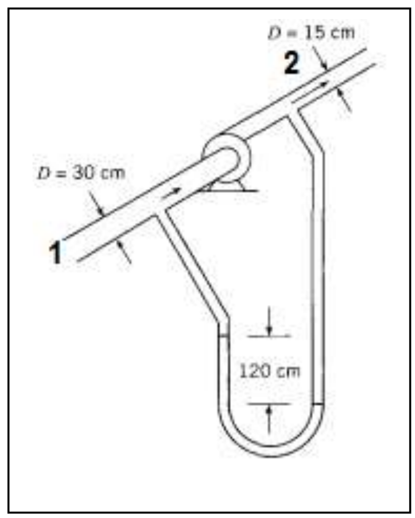
$$
\begin{aligned}
0+0+6\,\mathrm{m}+h_p &= \frac{100000\,\mathrm{Pa}}{9810\,\mathrm{N/m^3}}+1.25\,\mathrm{m}+10\,\mathrm{m}+2.0\,(1.25\,\mathrm{m}) \\
h_p &= 17.94\,\mathrm{m}
\end{aligned}
$$

Power equation:

$$
\begin{aligned}
P &= Q\gamma h_p \\
&= \left(0.35\,\mathrm{m^3/s}\right)\left(9810\,\mathrm{N/m^3}\right)(17.94\,\mathrm{m}) \\
& \boxed{P = 61.6\,\mathrm{kW}}
\end{aligned}
$$

Example

In the pump test shown, the rate of flow is 0.16 m^3/s of oil (S = 0.88). Calculate the horsepower that the pump supplies to the oil if there is a differential reading of 120 cm of mercury in the U-tube manometer. Assume α= 1.0 at all locations.



Solution

$$V_1 = \frac{Q}{A_1} = \frac{0.16}{\left(\frac{\pi}{4}\right)0.3^2} = 2.26$$

Similarly

$$V_2 = \frac{Q}{A_2} = \frac{0.16}{\left(\frac{\pi}{4}\right)0.15^2} = 9.04$$

Manometer equation

$$\left(\frac{p_2}{\gamma}+z_2\right)-\left(\frac{p_1}{\gamma}+z_1\right) = \frac{h_m(S_m-S_o)}{S_o} = \frac{1.2(13.76-0.88)}{0.88} = 17.5636\,m$$

Energy equation reduces to

$$\left(\frac{p_1}{\gamma}+z_1+\alpha_1\frac{\bar{V}_1^2}{2g}\right)+h_p = \left(\frac{p_2}{\gamma}+z_2+\alpha_2\frac{\bar{V}_2^2}{2g}\right)$$

$$h_p = \left(\frac{p_2}{\gamma}+z_2\right)-\left(\frac{p_1}{\gamma}+z_1\right)+\left(\frac{\bar{V}_2^2}{2g}-\frac{\bar{V}_1^2}{2g}\right) = \left(17.5636+\frac{(9.04)^2-(2.26)^2}{2*9.81}\right) = 21.5m$$

$$Power = h_pQ\gamma = 21.5*0.16*9.81*880 = \mathbf{29.7kW}$$

Power = 39.81 HP

7.4 Power Equation

$$Power = h \quad Q\gamma$$
$$Turbine\ Power = \dot{W}_T = h_T Q\gamma$$
$$Pump\ Power = \dot{W}_p = h_p Q\gamma$$

Efficiency the ratio of power output to power input

$$\eta = \frac{power\ out\ from\ a\ machine\ \text{or system}}{power\ input\ to\ a\ machine\ \text{or system}} = \frac{P_{out}}{P_{in}}$$

Mechanical efficiency of the pump is η_p, the power output delivered by the pump to the flow is

$$\dot{W}_p = \eta_p \dot{W}_s$$

Where $\dot{W}_s$ power supplied to pump, usually by a rotating shaft that is connected to a motor.
For a turbine, the output power $\dot{W}_s$ is usually delivered by a rotating shaft to a generator
Mechanical efficiency of the turbine is η_T, the output power supplied by the turbine is

$$\dot{W}_s = \eta_T \dot{W}_T$$

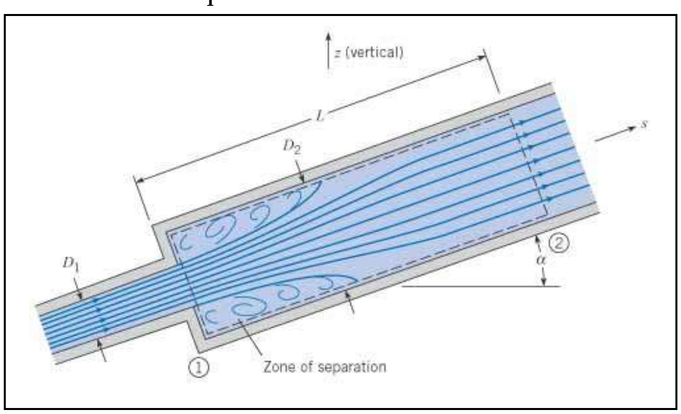
where $\dot{W}_T$ is the power input to the turbine from the flow.

7.5 Contrasting the Bernoulli Equation and the Energy Equation

The Bernoulli equation and the energy equation are derived in different ways.
The Bernoulli equation was derived by applying Newton's second law to a particle and then integrating the resulting equation along a streamline. The energy equation was derived by starting with the first law of thermodynamics and then using the Reynolds transport theorem. Consequently, the Bernoulli equation involves only mechanical energy, whereas the energy equation includes both mechanical and thermal energy.
The Bernoulli equation is applied by selecting two points on a streamline and then equating terms at these points:
In addition, these two points can be anywhere in the flow field for the special case of irrotational flow. The energy equation is applied by selecting an inlet section and an outlet section in a pipe and then equating terms as they apply to the pipe.
The two equations have different assumptions. The Bernoulli equation applies to steady, incompressible, and inviscid flow. The energy equation applies to steady, viscous, incompressible flow in a pipe with additional energy being added through a pump or extracted through a turbine.
Under special circumstances the energy equation can be reduced to the Bernoulli equation. If the flow is inviscid, there is no head loss; that is, $h_L = 0$. If the "pipe" is regarded as a small stream tube enclosing a streamline, then $\alpha = 1$. There is no pump or turbine along a streamline, so $h_p = h_t = 0$. In this case the energy equation is identical to the Bernoulli equation. Note that the energy equation cannot be developed starting with the Bernoulli equation.

7.6 Transitions
Abrupt Expansion

An *abrupt or sudden expansion* in a pipe or duct is a change from a smaller section area to a larger section area.



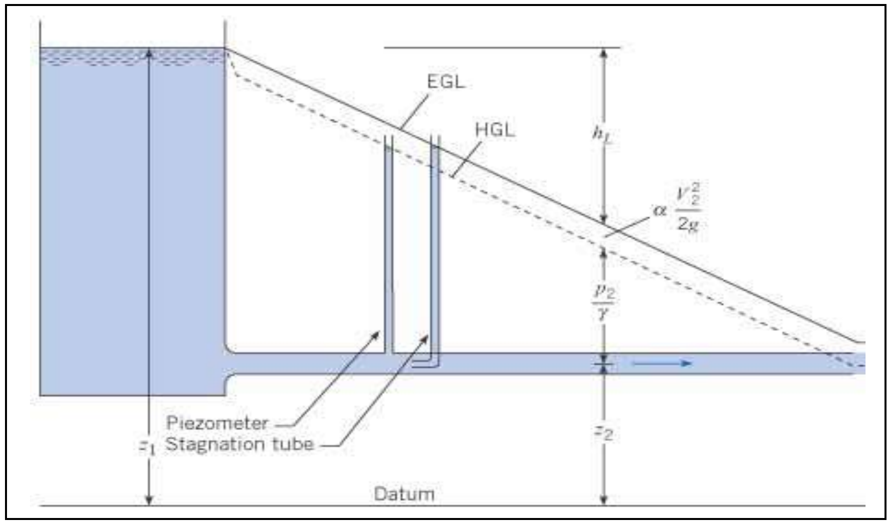
7.7 Hydraulic and Energy Grade Lines

$$\text{EGL} = \begin{pmatrix}\text{velocity}\\ \text{head}\end{pmatrix} + \begin{pmatrix}\text{pressure}\\ \text{head}\end{pmatrix} + \begin{pmatrix}\text{elevation}\\ \text{head}\end{pmatrix}$$

$$\text{EGL} = \alpha\frac{V^2}{2g} + \frac{P}{\gamma} + z = \begin{pmatrix}\text{total}\\ \text{head}\end{pmatrix}$$

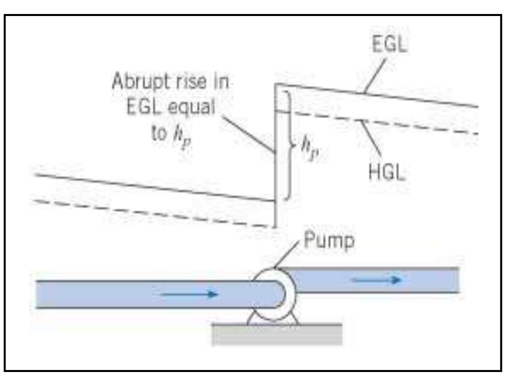
$$\text{HGL} = \begin{pmatrix}\text{pressure}\\ \text{head}\end{pmatrix} + \begin{pmatrix}\text{elevation}\\ \text{head}\end{pmatrix}$$

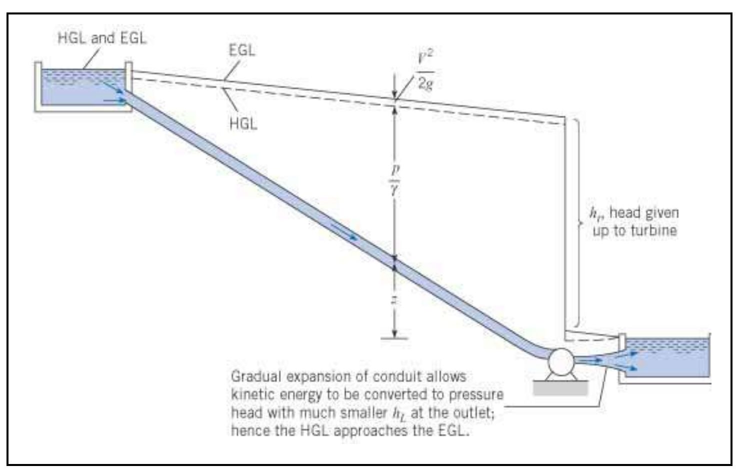
$$\text{HGL} = \frac{P}{\gamma} + z = \begin{pmatrix}\text{piezometric}\\ \text{head}\end{pmatrix}$$

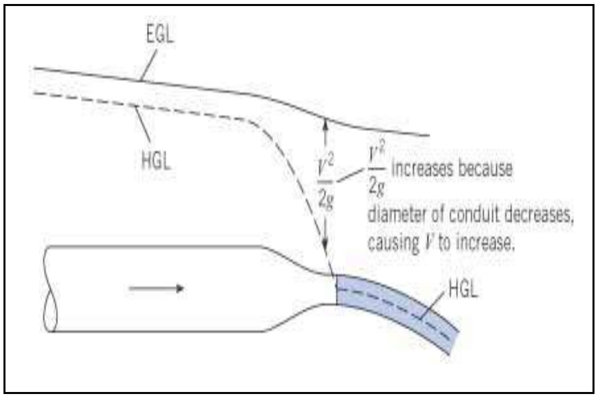


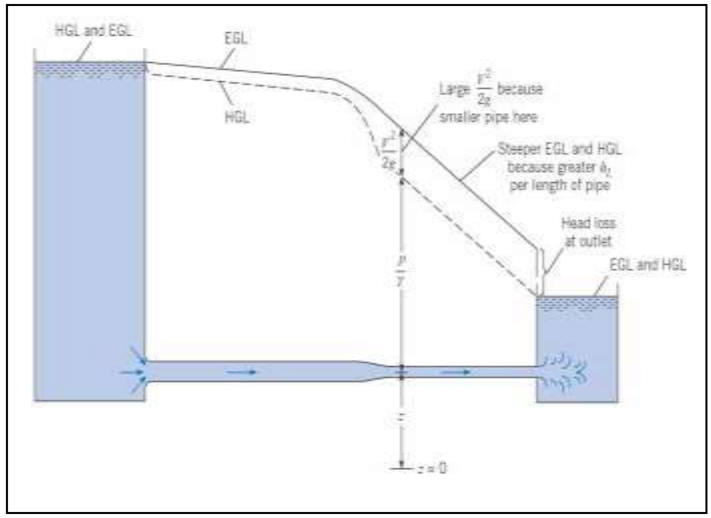
Tips for Drawing HGLs and EGLs

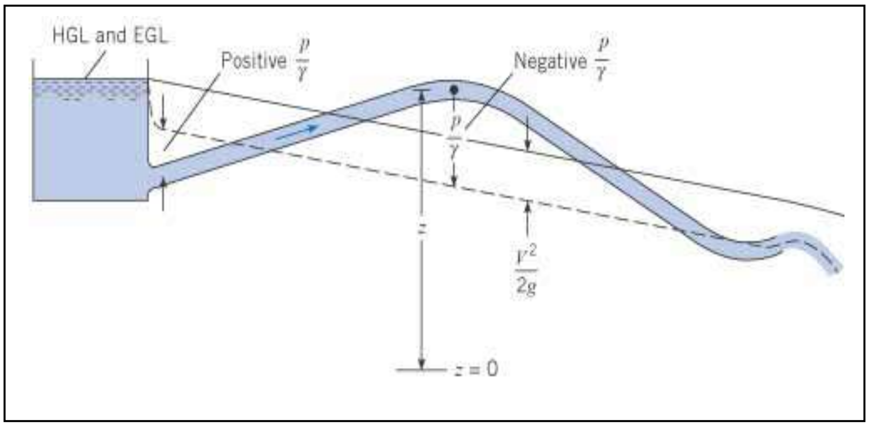
1. In a lake or reservoir, the HGL and EGL will coincide with the liquid surface. Also, both the HGL and EGL will indicate piezometric head. For example, see Fig. 7.7.
2. A pump causes an abrupt rise in the EGL and HGL by adding energy to the flow. For example, see Fig. 7.8.
3. For steady flow in a Pipe of constant diameter and wall roughness, the slope (7*hL*/7*L*) of the EGL and the HGL will be constant. For example, see Fig. 7.7
4. Locate the HGL below the EGL by a distance of the velocity head ($\alpha V2/2g$).
5. Height of the EGL decreases in the flow direction unless a pump is present.
6. A turbine causes an abrupt drop in the EGL and HGL by removing energy from the flow. For example, see Fig. 7.9.
7. Power generated by a turbine can be increased by using a gradual expansion at the turbine outlet. As shown in Fig. 7.9, the expansion converts kinetic energy to pressure. If the outlet to a reservoir is an abrupt expansion, as in Fig. 7.11, this kinetic energy is lost.
8. When a pipe discharges into the atmosphere the HGL is coincident with the system because $p/\gamma = 0$ at these points. For example, in Figures 7.10 and 7.12, the HGL in the liquid jet is drawn through the jet itself.
9. When a flow passage changes diameter, the distance between the EGL and the HGL will change (see Fig.7.10 and Fig. 7.11) because velocity changes. In addition, the slope on the EGL will change because the head loss per length will be larger in the conduit with the larger velocity
10. If the HGL falls below the pipe, then p/γ is negative, indicating subatmospheric pressure (see Fig. 7.12) and a potential location of cavitation.






EGL
HGL
$\frac{V^2}{2g}$
$\frac{V^2}{2g}$ increases because diameter of conduit decreases, causing V to increase.
HGL


HGL and EGL
EGL
HGL
Large $\frac{V^2}{2g}$ because smaller pipe here
$\frac{V^2}{2g}$
Steeper EGL and HGL because greater h_L per length of pipe
Head loss at outlet
EGL and HGL
$\frac{p}{\gamma}$
z
$z = 0$


HGL and EGL
Positive $\frac{p}{\gamma}$
Negative $\frac{p}{\gamma}$
$\frac{p}{\gamma}$
z
$\frac{V^2}{2g}$
$z = 0$