

CHAPTER

8

Area



*I could fill an entire second life with
working on my prints.*

M. C. ESCHER

Square Limit, M. C. Escher, 1964
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OBJECTIVES

In this chapter you will

- discover area formulas for rectangles, parallelograms, triangles, trapezoids, kites, regular polygons, circles, and other shapes
- use area formulas to solve problems
- learn how to find the surface areas of prisms, pyramids, cylinders, and cones

Areas of Rectangles and Parallelograms

A little learning is a dangerous thing—almost as dangerous as a lot of ignorance.

ANONYMOUS

People work with areas in many occupations. Carpenters calculate the areas of walls, floors, and roofs before they purchase materials for construction. Painters calculate surface areas so that they know how much paint to buy for a job. Decorators calculate the areas of floors and windows to know how much carpeting and drapery they will need. In this chapter you will discover formulas for finding the areas of the regions within triangles, parallelograms, trapezoids, kites, regular polygons, and circles.



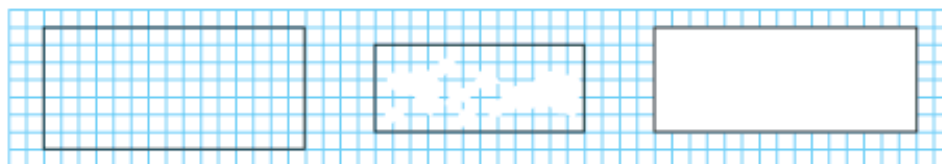
Tile layers need to find floor area to determine how many tiles to buy.

The **area** of a plane figure is the measure of the region enclosed by the figure. You measure the area of a figure by counting the number of square units that you can arrange to fill the figure completely.



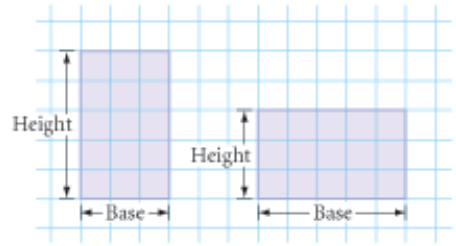
You probably already know many area formulas. Think of the investigations in this chapter as physical demonstrations of the formulas that will help you understand and remember them.

It's easy to find the area of a rectangle.



To find the area of the first rectangle, you can simply count squares. To find the areas of the other rectangles, you could draw in the lines and count the squares, but there's an easier method.

Any side of a rectangle can be called a **base**. A rectangle's **height** is the length of the side that is perpendicular to the base. For each pair of parallel bases, there is a corresponding height.



If we call the bottom side of each rectangle in the figure the base, then the length of the base is the number of squares in each row and the height is the number of rows. So you can use these terms to state a formula for the area. Add this conjecture to your list.

Rectangle Area Conjecture

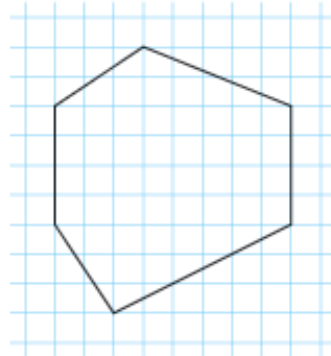
C-74

The area of a rectangle is given by the formula $A = bh$, where A is the area, b is the length of the base, and h is the height of the rectangle.

The area formula for rectangles can help you find the areas of many other shapes.

EXAMPLE A

Find the area of this shape.

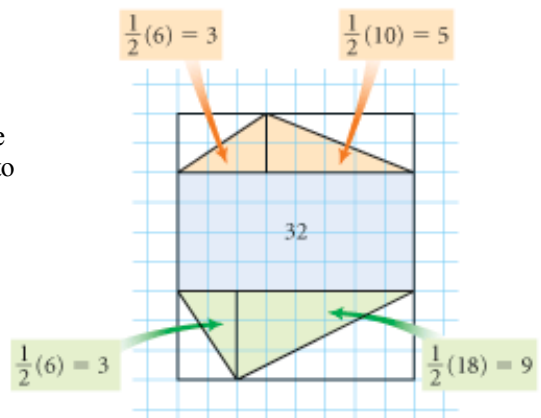


► Solution

The middle section is a rectangle with an area of $4 \cdot 8$, or 32 square units. You can divide the remaining pieces into right triangles, so each piece is actually half a rectangle.

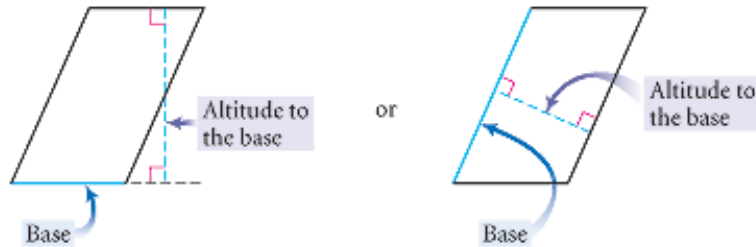
The area of the figure is $32 + 3 + 5 + 9 + 3 = 52$ square units.

There are other ways to find the area of this figure. One way is to find the area of an 8-by-9 rectangle and subtract the areas of four right triangles.

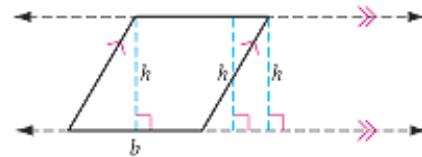


You can also use the area formula for a rectangle to find the area formula for a parallelogram.

Just as with a rectangle, any side of a parallelogram can be called a base. But the height of a parallelogram is not necessarily the length of a side. An **altitude** is any segment from one side of a parallelogram perpendicular to a line through the opposite side. The length of the altitude is the **height**.



The altitude can be inside or outside the parallelogram. No matter where you draw the altitude to a base, its height should be the same, because the opposite sides are parallel.



Investigation

Area Formula for Parallelograms

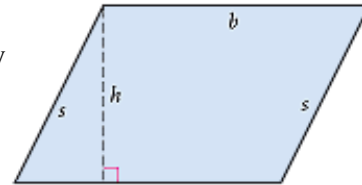
You will need

- heavy paper or cardboard
- a straightedge
- a compass

Using heavy paper, investigate the area of a parallelogram. Can the area be rearranged into a more familiar shape? Different members of your group should investigate different parallelograms so you can be sure your formula works for all parallelograms.

Step 1

Construct a parallelogram on a piece of heavy paper or cardboard. From the vertex of the obtuse angle adjacent to the base, draw an altitude to the side opposite the base. Label the parallelogram as shown.



Step 2

Cut out the parallelogram and then cut along the altitude. You will have two pieces—a triangle and a trapezoid. Try arranging the two pieces into other shapes without overlapping them. Is the area of each of these new shapes the same as the area of the original parallelogram? Why?

Step 3

Is one of your new shapes a rectangle? Calculate the area of this rectangle. What is the area of the original parallelogram? State your next conjecture.

Parallelogram Area Conjecture

C-75

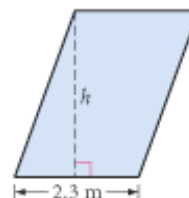
The area of a parallelogram is given by the formula $A = bh$, where A is the area, b is the length of the base, and h is the height of the parallelogram.

▶ To view dynamically how the area of a parallelogram is related to the area of a rectangle, see the **Dynamic Geometry Exploration Area Formula for Parallelograms** at www.keymath.com/DG.

If the dimensions of a figure are measured in inches, feet, or yards, the area is measured in in² (square inches), ft² (square feet), or yd² (square yards). If the dimensions are measured in centimeters or meters, the area is measured in cm² (square centimeters) or m² (square meters). Let's look at an example.

EXAMPLE B

Find the height of a parallelogram that has area 7.13 m² and base length 2.3 m.



▶ **Solution**

$$A = bh$$

Write the formula.

$$7.13 = (2.3)h$$

Substitute the known values.

$$\frac{7.13}{2.3} = h$$

Solve for the height.

$$h = 3.1$$

Divide.

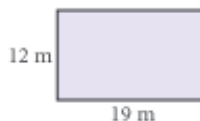
The height measures 3.1 m.



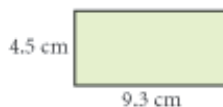
EXERCISES

In Exercises 1–6, each quadrilateral is a rectangle. A represents area and P represents perimeter. Use the appropriate unit in each answer.

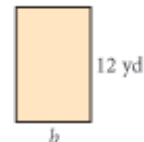
1. $A = ?$



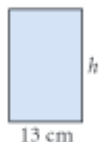
2. $A = ?$



3. $A = 96 \text{ yd}^2$
 $b = ?$



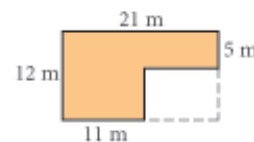
4. $A = 273 \text{ cm}^2$
 $h = ?$



5. $P = 40 \text{ ft}$
 $A = ?$

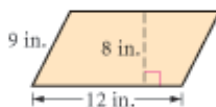


6. Shaded area = ?



In Exercises 7–9, each quadrilateral is a parallelogram.

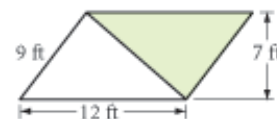
7. $A = ?$



8. $A = 2508 \text{ cm}^2$
 $P = ?$



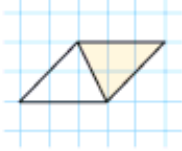
9. Find the area of the shaded region.



10. Sketch and label two different rectangles, each with area 48 cm^2 .

In Exercises 11–13, find the shaded area and explain your method.

11.



12.



13.



14. Sketch and label two different parallelograms, each with area 64 cm^2 .


15. Draw and label a figure with area 64 cm^2 and perimeter 64 cm .

16. The photo shows a Japanese police *koban*. An arch forms part of the roof and one wall. The arch is made from rectangular panels that each measure 1 m by 0.7 m . The arch is 11 panels high and 3 panels wide. What's the total area of the arch?

Cultural CONNECTION

Koban is Japanese for “mini-station,” a small police station. These stations are located in several parts of a city, and officers who work in them know the surrounding neighborhoods and people well. The presence of *kobans* in Japan helps reduce crime and provides communities with a sense of security.

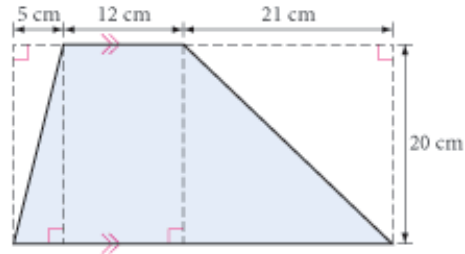


17. What is the total area of the four walls of a rectangular room 4 meters long by 5.5 meters wide by 3 meters high? Ignore all doors and windows.
18. **Application** Ernesto plans to build a pen for his pet iguana. What is the area of the largest rectangular pen that he can make with 100 meters of fencing?
19. The big event at George Washington High School’s May Festival each year is the Cow Drop Contest. A farmer brings his well-fed bovine to wander the football field until—well, you get the picture. Before the contest, the football field, which measures 53 yards wide by 100 yards long, is divided into square yards. School clubs and classes may purchase square yards. If one of their squares is where the first dropping lands, they win a pizza party. If the math club purchases 10 squares, what is the probability that the club will win?
20. **Application** Sarah is tiling a wall in her bathroom. It is rectangular and measures 4 feet by 7 feet. The tiles are square and measure 6 inches on each side. How many tiles does Sarah need? 



For Exercises 21–23, you may choose to use grid pages.

21. Find the area of quadrilateral $ABCD$ with vertices $A(0, 0)$, $B(6, 0)$, $C(14, 16)$, and $D(8, 16)$.
22. Find the area of quadrilateral $EFGH$ with vertices $E(0, 0)$, $F(6, -4)$, $G(8, 0)$, and $H(6, 4)$.
23. Find the area of the trapezoid at right.



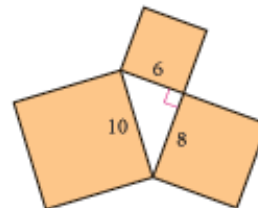
24. **Mini-Investigation** Suppose you measure the sides of a rectangle to calculate its area. Using your ruler, you find that the base of the rectangle is between 15.6 cm and 15.7 cm, and the height is between 12.3 cm and 12.4 cm.
 - a. Find the smallest possible area and the largest possible area.
 - b. What do you think is a reasonable value for the actual area of the rectangle?
 - c. If you use 15.65 cm for the base and 12.35 cm for the height, the calculated area is 193.2775 cm^2 . From your answers to part a, explain why the last four digits in this calculation are not considered significant.

Art CONNECTION

The design at right is a quilt design called the Ohio Star. Traditional quilt block designs range from a simple nine-patch, based on 9 squares, to more complicated designs like Jacob's Ladder or Underground Railroad, which are based on 16, 25, or even 36 squares. Quiltmakers need to calculate the total area of each different type of material before they make a complete quilt.

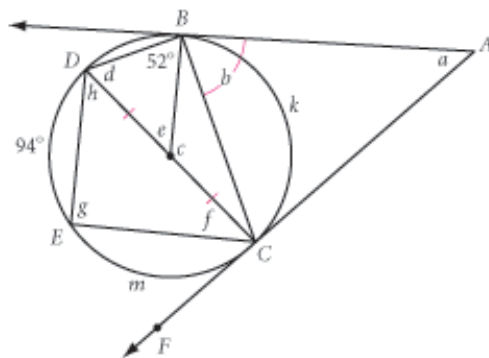


25. **Application** The Ohio Star is a 16-square quilt design. Each block measures 12 inches by 12 inches. One block is shown above. Assume you will need an additional 20% of each fabric to allow for seams and errors.
 - a. Calculate the sum of the areas of all the red patches, the sum of the areas of all the blue patches, and the area of the yellow patch in a single block.
 - b. How many Ohio Star blocks will you need to cover an area that measures 72 inches by 84 inches, the top surface area of a king-size mattress?
 - c. How much fabric of each color will you need? How much fabric will you need for a 15-inch border to extend beyond the edges of the top surface of the mattress?
26. A right triangle with sides measuring 6 cm, 8 cm, and 10 cm has a square constructed on each of its three sides, as shown. Compare the area of the square on the longest side to the sum of the areas of the two squares on the two shorter sides.



Review

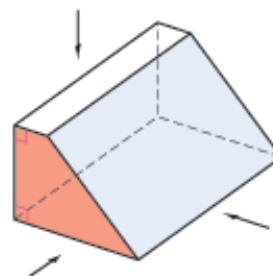
27. **Developing Proof** Copy the figure at right. Find the lettered angle measures and arc measures. \overline{AB} and \overline{AC} are tangents. \overline{CD} is a diameter. Explain how you determined the measures a and b .



28. Given AM as the length of the altitude of an equilateral triangle, construct the triangle.



29. Sketch what the figure at right looks like when viewed from each of these directions.
- Above the figure, looking straight down
 - In front of the figure, that is, looking straight at the red-shaded side
 - The side, looking at the blue-shaded side



project

RANDOM RECTANGLES

What does a typical rectangle look like? A randomly generated rectangle could be long and narrow, or square-like. It could have a large perimeter, but a small area. Or it could have a large area, but a small perimeter. In this project you will randomly generate rectangles and study their characteristics using scatter plots and histograms.

Your project should include

- ▶ A description of how you created your random rectangles, including any constraints you used.
- ▶ A scatter plot of base versus height, a perimeter histogram, an area histogram, and a scatter plot of area versus perimeter.
- ▶ Any other studies or graphs you think might be interesting.
- ▶ Your predictions about the data before you made each graph.
- ▶ An explanation of why each graph looks the way it does.

You can also use a graphing calculator to do this project.



You can use Fathom to generate random base and height values from 0 to 10. Then you can sort them by various characteristics and make a wide range of interesting graphs.

Areas of Triangles, Trapezoids, and Kites

When you add to the truth,
you subtract from it.

THE TALMUD

In Lesson 8.1, you learned the area formula for rectangles, and you used it to discover an area formula for parallelograms. In this lesson you will use those formulas to discover or demonstrate the formulas for the areas of triangles, trapezoids, and kites.



Investigation 1 Area Formula for Triangles

You will need

- heavy paper or cardboard



- Step 1 Cut out a triangle and label its parts as shown. Make and label a copy.
- Step 2 Arrange the triangles to form a figure for which you already have an area formula. Calculate the area of the figure.
- Step 3 What is the area of one of the triangles? Make a conjecture. Write a brief description in your notebook of how you arrived at the formula. Include an illustration.

Triangle Area Conjecture

C-76

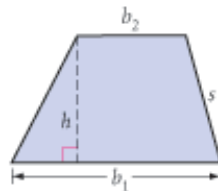
The area of a triangle is given by the formula $\frac{bh}{2}$, where A is the area, b is the length of the base, and h is the height of the triangle.



Investigation 2 Area Formula for Trapezoids

You will need

- heavy paper or cardboard



- Step 1 Construct any trapezoid and an altitude perpendicular to its bases. Label the trapezoid as shown.
- Step 2 Cut out the trapezoid. Make and label a copy.

- Step 3 | Arrange the two trapezoids to form a figure for which you already have an area formula. What type of polygon is this? What is its area? What is the area of one trapezoid? State a conjecture.

Trapezoid Area Conjecture

C-77

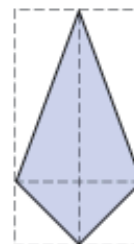
The area of a trapezoid is given by the formula $A = \frac{1}{2}(b_1 + b_2)h$, where A is the area, b_1 and b_2 are the lengths of the two bases, and h is the height of the trapezoid.



Investigation 3 Area Formula for Kites

Can you rearrange a kite into shapes for which you already have the area formula? Do you recall some of the properties of a kite?

Create and carry out your own investigation to discover a formula for the area of a kite. Discuss your results with your group. State a conjecture.



Kite Area Conjecture

C-78

The area of a kite is given by the formula $A = \frac{1}{2}d_1d_2$.



[keymath.com/DG](http://www.keymath.com/DG)

► To view interactive versions of these investigations, see the **Dynamic Geometry Exploration** Areas of Triangles, Trapezoids, and Kites at www.keymath.com/DG.



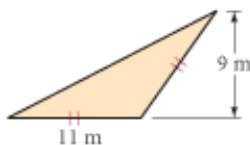
EXERCISES

In Exercises 1–12, use your new area conjectures to solve for the unknown measures.

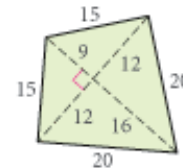
1. $A = ?$



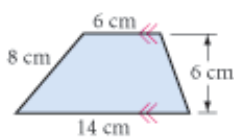
2. $A = ?$



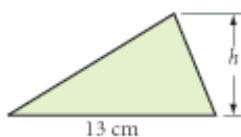
3. $A = ?$



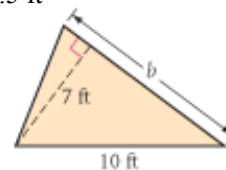
4. $A = ?$



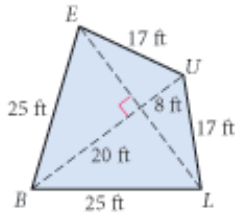
5. $A = 39 \text{ cm}^2$
 $h = ?$



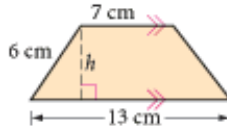
6. $A = 31.5 \text{ ft}^2$
 $b = ?$



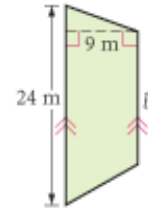
7. $A = 420 \text{ ft}^2$
 $LE = ?$



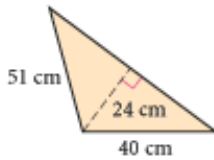
8. $A = 50 \text{ cm}^2$ \textcircled{h}
 $h = ?$



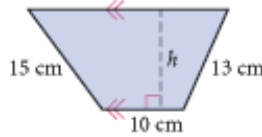
9. $A = 180 \text{ m}^2$
 $b = ?$



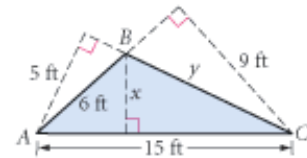
10. $A = 924 \text{ cm}^2$
 $P = ?$



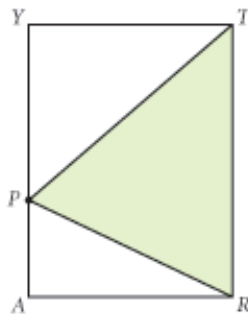
11. $A = 204 \text{ cm}^2$
 $P = 62 \text{ cm}$
 $h = ?$



12. $x = ?$ \textcircled{h}
 $y = ?$



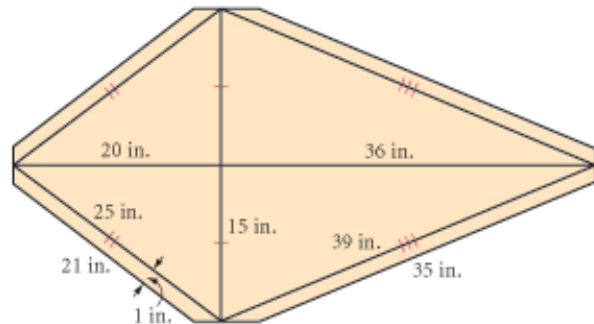
13. Sketch and label two different triangles, each with area 54 cm^2 .
14. Sketch and label two different trapezoids, each with area 56 cm^2 .
15. Sketch and label two different kites, each with area 1092 cm^2 .
16. Sketch and label a triangle and a trapezoid with equal areas and equal heights. How does the base of the triangle compare with the two bases of the trapezoid?
17. P is a random point on side \overline{AY} of rectangle $ARTY$. The shaded area is what fraction of the area of the rectangle? Why?



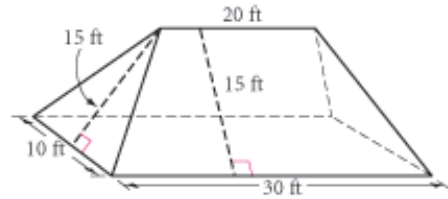
18. One playing card is placed over another, as shown. Is the top card covering half, less than half, or more than half of the bottom card? Explain.



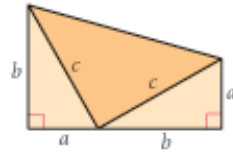
19. **Application** Eduardo has designed this kite for a contest. He plans to cut the kite from a sheet of Mylar plastic and use balsa wood for the diagonals. He will connect all the vertices with string, and fold and glue flaps over the string.
- How much balsa wood and Mylar will he need?
 - Mylar is sold in rolls 36 inches wide. What length of Mylar does Eduardo need for this kite?



20. **Application** The roof on Crystal's house is formed by two congruent trapezoids and two congruent isosceles triangles, as shown. She wants to put new wood shingles on her roof. Each shingle will cover 0.25 square foot of area. (The shingles are 1 foot by 1 foot, but they overlap by 0.75 square foot.) How many shingles should Crystal buy?



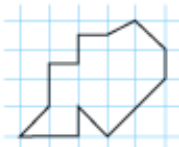
21. A trapezoid has been created by combining two congruent right triangles and an isosceles triangle, as shown. Is the isosceles triangle a right triangle? How do you know? Find the area of the trapezoid two ways: first by using the trapezoid area formula, and then by finding the sum of the areas of the three triangles.



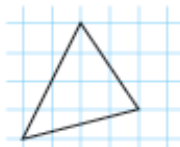
22. Divide a trapezoid into two triangles. Use algebra to derive the formula for the area of the trapezoid by expressing the area of each triangle algebraically and finding their algebraic sum.

Review

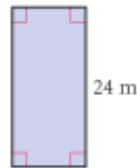
23. $A = ?$



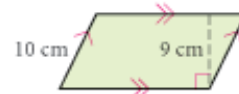
24. $A = ?$



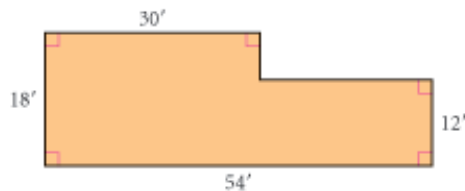
25. $A = 264 \text{ m}^2$
 $P = ?$



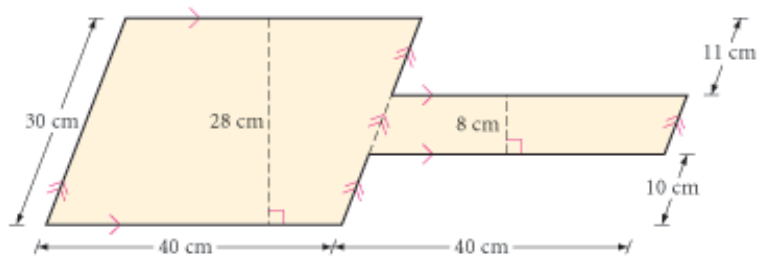
26. $P = 52 \text{ cm}$
 $A = ?$



27. $A = ?$
 $P = ?$

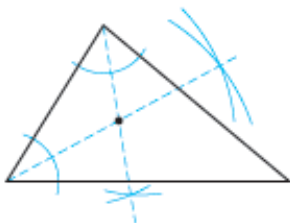


28. $A = ?$
 $P = ?$



29. Identify the point of concurrency from the construction marks.

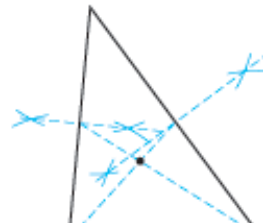
a.



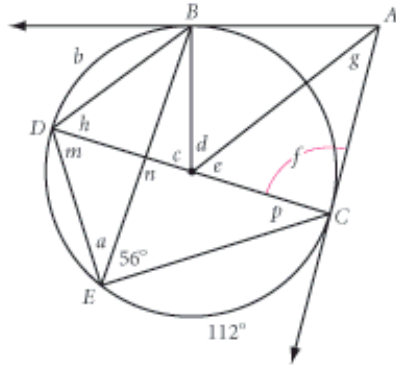
b.



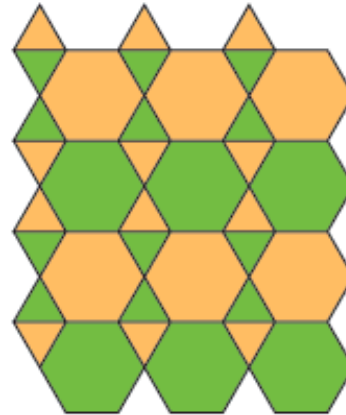
c.



30. **Developing Proof** Trace the figure below. Find the lettered angle measures and arc measures. \overline{AB} and \overline{AC} are tangents. \overline{CD} is a diameter. Explain how you determined the measures d and n .



31. Give the vertex arrangement for this 2-uniform tessellation.

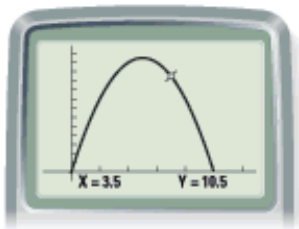


project

MAXIMIZING AREA

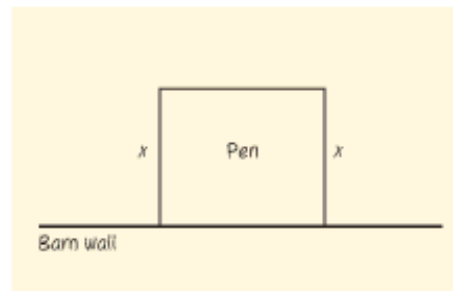
A farmer wants to fence in a rectangular pen using the wall of a barn for one side of the pen and the 10 meters of fencing for the remaining three sides. What dimensions will give her the maximum area for the pen?

You can use the trace feature on your calculator to find the value of x that gives the maximum area. Use your graphing calculator to investigate this problem and find the best arrangement.



Your project should include

- ▶ An expression for the third side length, in terms of the variable x in the diagram.
- ▶ An equation and graph for the area of the pen.
- ▶ The dimensions of the best rectangular shape for the farmer's pen.

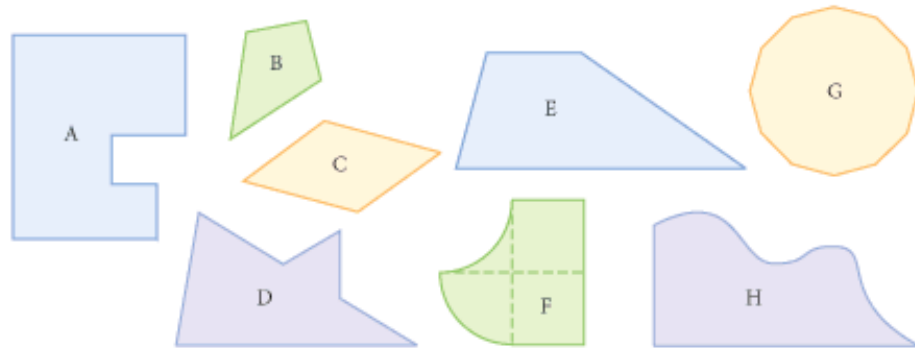


Area Problems

By now, you know formulas for finding the areas of rectangles, parallelograms, triangles, trapezoids, and kites. Now let's see if you can use these area formulas to approximate the areas of irregularly shaped figures.

Optimists look for solutions, pessimists look for excuses.

SUE SWENSON



Investigation

Solving Problems with Area Formulas

You will need

- figures A–H
- centimeter rulers or meterstick

Find the area of each geometric figure your teacher provides. Before you begin to measure, discuss with your group the best strategy for each step. Discuss what units you should use. Different group members might get different results. However, your results should be close. You may average your results to arrive at one group answer. For each figure, write a sentence or two explaining how you measured the area and how accurate you think it is.

Now that you have practiced measuring and calculating area, you're ready to try some application problems. Many everyday projects require you to find the areas of flat surfaces on three-dimensional objects. You'll learn more about surface area in Lesson 8.7.

Career

CONNECTION

Professional housepainters have a unique combination of skills: For large-scale jobs, they begin by measuring the surfaces that they will paint and use measurements to estimate the quantity of materials they will need. They remove old coating, clean the surface, apply sealer, mix color, apply paint, and add finishes. Painters become experienced and specialize their craft through the on-the-job training they receive during their apprenticeships.



In the exercises you will learn how to use area in buying rolls of wallpaper, gallons of paint, bundles of shingles, square yards of carpet, and square feet of tile. Keep in mind that you can't buy $12\frac{11}{16}$ gallons of paint! You must buy 13 gallons. If your calculations tell you that you need 5.25 bundles of shingles, you have to buy 6 bundles. In this type of rounding, you must always round upward.



EXERCISES

- Application** Tammy is estimating how much she should charge for painting 148 rooms in a new motel with one coat of base paint and one coat of finishing paint. The four walls and the ceiling of each room must be painted. Each room measures 14 ft by 16 ft by 10 ft high.
 - Calculate the total area of all the surfaces to be painted with each coat. Ignore doors and windows.
 - One gallon of base paint covers 500 square feet. One gallon of finishing paint covers 250 square feet. How many gallons of each will Tammy need for the job?
- Application** Rashad wants to wallpaper the four walls of his bedroom. The room is rectangular and measures 11 feet by 13 feet. The ceiling is 10 feet high. A roll of wallpaper at the store is 2.5 feet wide and 50 feet long. How many rolls should he buy? (Wallpaper is hung from ceiling to floor. Ignore doors and windows.)
- Application** It takes 65,000 solar cells, each 1.25 in. by 2.75 in., to power the Helios Prototype, shown below. How much surface area, in square feet, must be covered with the cells? The cells on Helios are 18% efficient. Suppose they were only 12% efficient, like solar cells used in homes. How much more surface area would need to be covered to deliver the same amount of power?

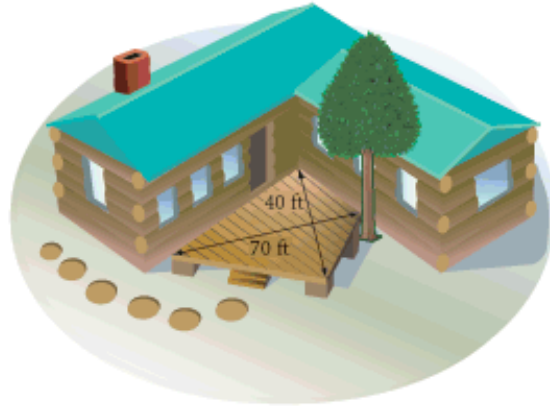


Technology CONNECTION

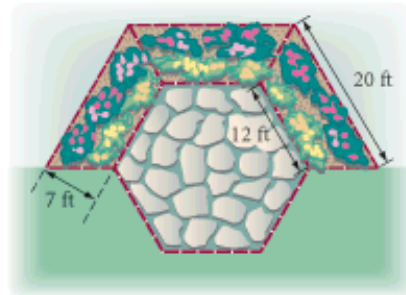
In August 2001, the Helios Prototype, a remotely controlled, nonpolluting solar-powered aircraft, reached 96,500 feet—a record for nonrocket aircraft. Soon, the Helios will likely sustain flight long enough to enable weather monitoring and other satellite functions. For news and updates, go to www.keymath.com/DG.



4. **Application** Harold works at a state park. He needs to seal the redwood deck at the information center to protect the wood. He measures the deck and finds that it is a kite with diagonals 40 feet and 70 feet. Each gallon of sealant covers 400 square feet, and the sealant needs to be applied every six months. How many gallon containers should he buy to protect the deck for the next three years?



5. **Application** A landscape architect is designing three trapezoidal flowerbeds to wrap around three sides of a hexagonal flagstone patio, as shown. What is the area of the entire flowerbed? The landscape architect's fee is \$100 plus \$5 per square foot. What will the flowerbed cost?



Career

CONNECTION

Landscape architects have a keen eye for natural beauty. They study the grade and direction of land slopes, stability of the soil, drainage patterns, and existing structures and vegetation. They look at the various social, economic, and artistic concerns of the client. They also use science and engineering to plan environments that harmonize land features with structures, reducing the impact of urban development upon nature.

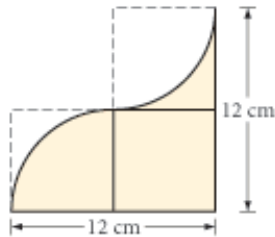


For Exercises 6 and 7, refer to the floor plan at right.

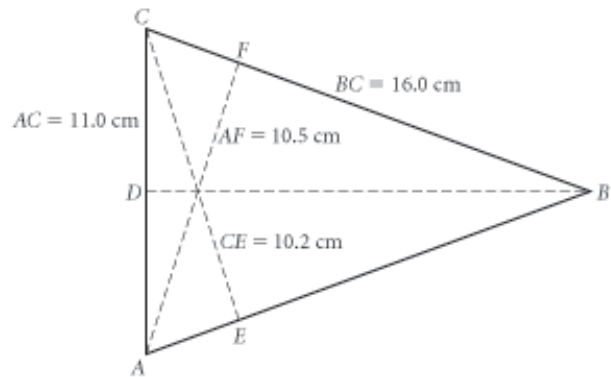
6. **Application** Dareen's family is ready to have wall-to-wall carpeting installed. The carpeting they chose costs \$14 per square yard, the padding \$3 per square yard, and the installation \$3 per square yard. What will it cost them to carpet the three bedrooms and the hallway shown?
7. **Application** Dareen's family now wants to install 1-foot-square terra-cotta tiles in the entryway and kitchen, and 4-inch-square blue tiles on each bathroom floor. The terra-cotta tiles cost \$5 each, and the bathroom tiles cost 45¢ each. How many of each kind will they need? What will all the tiles cost?



8. Find the area of the shaded region.
Assume all the squares are congruent.

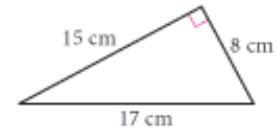


9. \overline{CE} , \overline{BD} , and \overline{AF} are altitudes. Find AB and BD .

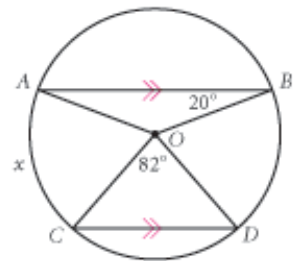


Review

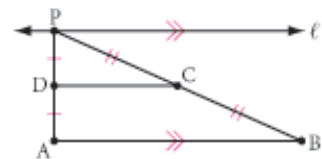
10. A first-century Greek mathematician named Hero is credited with the following formula for the area of a triangle:
 $A = \sqrt{s(s-a)(s-b)(s-c)}$, where A is the area of the triangle, a , b , and c are the lengths of the three sides of the triangle, and s is the semiperimeter (half of the perimeter). Use Hero's formula to find the area of this triangle. Use the formula $A = \frac{1}{2}bh$ to check your answer.



11. **Developing Proof** Explain why x must be 69° in the diagram at right.



12. As P moves from left to right along ℓ , which of the following values changes?
- The area of $\triangle ABP$
 - The area of $\triangle PDC$
 - The area of trapezoid $ABCD$
 - $m\angle A + m\angle PCD + m\angle CPD$
 - None of these



IMPROVING YOUR VISUAL THINKING SKILLS

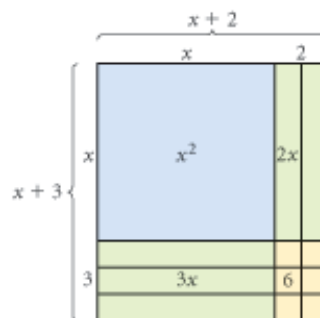
Four-Way Split

How would you divide a triangle into four regions with equal areas? There are at least six different ways it can be done! Make six copies of a triangle and try it.



Products, Factors, and Quadratic Equations

The area of a rectangle is the *product* of two dimensions, the base and height: $A = bh$. So, you can model the product of any two numbers or expressions as the area of a rectangle. For example, the rectangle at right has base $x + 2$ and height $x + 3$. The area of the rectangle is made of four smaller rectangular regions. Multiplying the base and height of each region gives areas of x^2 , $3x$, $2x$, and 6 . Combining the areas gives the product:

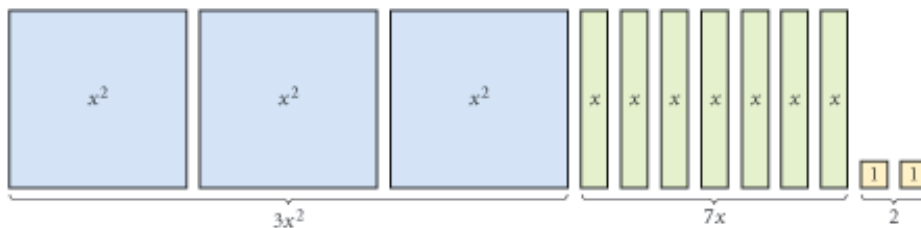


$$(x + 2)(x + 3) = x^2 + 5x + 6$$

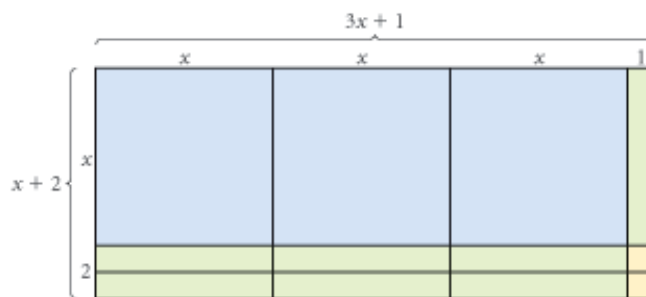
if an expression is the product of two other expressions, you can try to arrange the “pieces” into a rectangle and find the *factors* that were multiplied.

EXAMPLE A | Sketch and label a rectangle to factor $3x^2 + 7x + 2$.

► **Solution** | You may want to first sketch the pieces that make up the area.



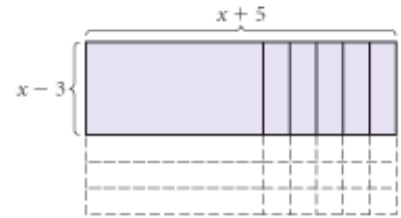
Visualize ways to arrange all of the pieces into a rectangle. if you have algebra tiles or if you cut the pieces out of paper, you can physically experiment with different arrangements. Here is one way to make a rectangle:



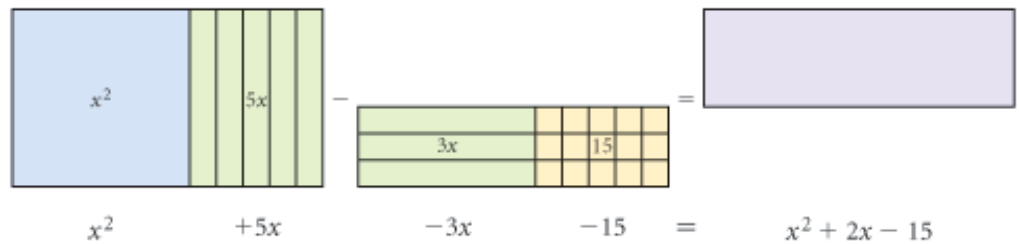
The base and height give the factors:

$$3x^2 + 7x + 2 = (3x + 1)(x + 2)$$

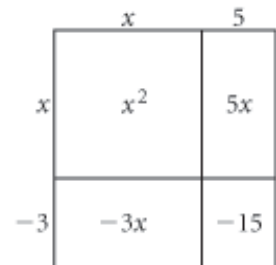
Negative values can be represented by subtraction from a side of a rectangle. For example, the shaded rectangle at right has base $x + 5$ and height $x - 3$, so its area can be expressed as the product $(x + 5)(x - 3)$.



The picture equation below shows why $(x + 5)(x - 3) = x^2 + 2x - 15$. The overall rectangle has combined area $x^2 + 5x$. If you subtract a rectangular region with combined area $3x + 15$, you are left with the shaded rectangle.



When an expression includes large numbers or negative numbers, it is easier to use an abstracted rectangle diagram to represent groups of pieces instead of drawing each individual piece. The rectangle at right shows the product $(x + 5)(x - 3)$. Add the areas of the pieces to write the area as a sum:

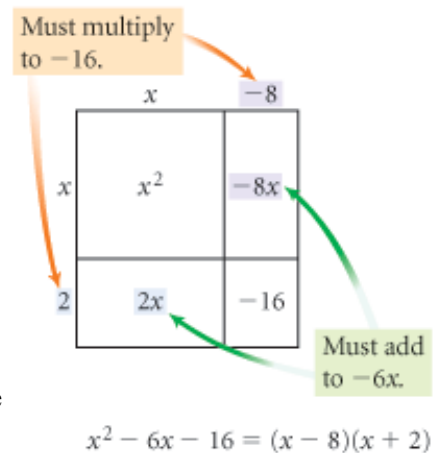


$$(x + 5)(x - 3) = x^2 + 2x - 15.$$

EXAMPLE B | Sketch and label a rectangle diagram to factor $x^2 - 6x - 16$.

► **Solution**

Draw a rectangle diagram with x^2 in the upper-left region and -16 in the lower-right region. The x^2 -region must have dimensions x by x . The remaining two values must multiply to -16 . They must also add to -6 because they are equal to the number of x -pieces, which add to $-6x$. Through some logic and a little trial and error, you can find the combination of numbers that works. In this case, one of the numbers must be negative because the product is negative, and the only factor pairs for 16 are $1 \cdot 16$, $2 \cdot 8$, and $4 \cdot 4$. The rectangle diagram at right shows that the solution is $(x - 8)(x + 2)$.



Problems involving area can result in an equation with an x^2 term, called a **quadratic equation**. A quadratic equation can be written in the form $ax^2 + bx + c = 0$, where a , b , and c represent constants. One method for solving a quadratic equation is to first factor $ax^2 + bx + c$. The **zero-product property** says that if the product of two (or more) factors is zero, then at least one of the factors is zero: if $a \cdot b = 0$, then $a = 0$ or $b = 0$. This gives you a way to solve a factored quadratic equation.

EXAMPLE CSolve $x^2 - 2x = 24$.**► Solution**

$$x^2 - 2x = 24$$

The original equation.

$$x^2 - 2x - 24 = 0$$

Subtract 24 from both sides to set the equation equal to zero.

$$(x + 4)(x - 6) = 0$$

Factor the left side. Use a rectangle diagram

$$x + 4 = 0 \quad \text{or} \quad x - 6 = 0$$

The zero-product property says at least one factor is zero.

$$x = -4 \quad \text{or} \quad x = 6$$

Solve each equation for x .

Check both solutions:

$$(-4)^2 - 2(-4) \stackrel{?}{=} 24$$

$$(6)^2 - 2(6) \stackrel{?}{=} 24$$

Substitute -4 for x , then substitute 6 for x .

$$16 + 8 \stackrel{?}{=} 24$$

$$36 - 12 \stackrel{?}{=} 24$$

Simplify following the order of operations.

$$24 = 24$$

$$24 = 24$$

Both solutions check.

**EXERCISES**

In Exercises 1–6, sketch and label a rectangle to find the product or to factor the expression. As in Example A, show the individual pieces.

1. $(x + 5)(x + 1)$

2. $(x)(2x + 7)$

3. $(3x + 2)(2x + 5)$

4. $6x + 3$

5. $x^2 + 8x + 15$

6. $2x^2 + 11x + 12$

In Exercises 7–18, sketch and label a rectangle diagram to find the product or to factor the expression. As in Example B, do not show the individual pieces.

7. $(x + 15)(x - 11)$

8. $(3x - 7)(4x + 5)$

9. $(x + 4)^2$ or $(x + 4)(x + 4)$

10. $(2x + 5)(2x - 5)$

11. $(a + b)^2$

12. $(a + b)(a - b)$

13. $x^2 + 19x + 60$

14. $x^2 - 10x - 24$

15. $x^2 + x - 20$

16. $x^2 - 6x + 9$

17. $x^2 - 36$

18. $4x^2 - 49$

In Exercises 19–22, solve the quadratic equation.

19. $x^2 + 5x + 4 = 0$

20. $x^2 + 7x = 30$

21. $x(x - 6) = 5x - 24$

22. $(x + 2)(x - 2) = -x^2 - 9x - 8$

23. One base of an isosceles trapezoid is equal to the trapezoid's height and the other base is 4 feet longer than the height. The area of the trapezoid is 48 square feet.

- Let h represent the trapezoid's height in feet. Sketch the trapezoid and label the height.
- Write an expression that includes h for each base of the trapezoid. Add these to your diagram.
- Write an equation that includes h for the area of the trapezoid.
- Solve the equation in part c to find the dimensions of the trapezoid.

IMPROVING YOUR REASONING SKILLS

Tic-Tac-Square



This is one of many variations on tic-tac-toe. Instead of trying to get your X's or O's collinear, the object is to get your X's or O's arranged at the vertices of a square. There are many ways to get four X's or O's arranged at the vertices of a square. Two completed example games are shown below. Numbers are used to show the sequence of moves in each game.

Answer the questions about each of the four unfinished games. You can identify a possible move by the letter of the row and number of the column. For example, the top-right square in the grid is a5.

Example A

		O ₉	O ₃	X ₈
		O ₁	O ₇	O ₅
X ₄	X ₂		X ₆	

In this game, O went first and won.

Example B

	X ₁			
	O ₂	O ₄	X ₃	
X ₇		O ₈		
			X ₅	

In this game, X went first and won.

1.

	1	2	3	4	5
a	X ₆	O ₅	X ₈		
b	O ₁	O ₃	O ₇	O ₁₃	
c		X ₄	X ₂	O ₉	X ₁₄
d	O ₁₇	O ₁₁	X ₁₀	X ₁₂	X ₁₈
e		X ₁₆			O ₁₅

Where should O play to prevent X from winning?

2.

	1	2	3	4	5
a	O ₈	X ₇	X ₉		
b	X ₅	X ₁	O ₂	O ₁₀	
c	O ₆	X ₃	O ₄		
d					
e					

Explain why O's last move, b4, was a mistake

3.

	1	2	3	4	5
a					
b					
c		O ₁	O ₃	X ₂	
d			X ₄		
e					

Where should O play O₅ and O₇ to guarantee a win on O₉?

4.

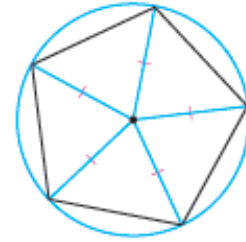
	1	2	3	4	5
a			O ₁₄		
b		O ₁₂	X ₁	X ₃	
c	X ₁₃	X ₅	O ₂	O ₄	X ₁₁
d		O ₈	X ₇	O ₆	O ₁₀
e			X ₉		

Which moves were forced? Who should win? Explain.

If I had to live my life again, I'd make the same mistakes, only sooner.
TALLULAH BANKHEAD

Areas of Regular Polygons

You can divide a regular polygon into congruent isosceles triangles by drawing segments from the center of the polygon to each vertex. The center of the polygon is actually the center of a circumscribed circle, so these congruent segments are sometimes called the radii of a regular polygon.



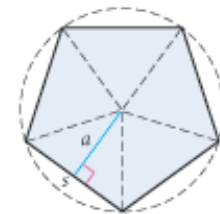
In this investigation you will divide regular polygons into triangles. Then you will write a formula for the area of any regular polygon.



Investigation

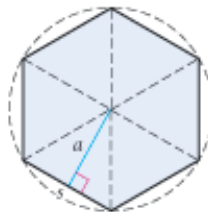
Area Formula for Regular Polygons

Consider a regular pentagon with side length s , divided into congruent isosceles triangles. Each triangle has a base s and a height a .

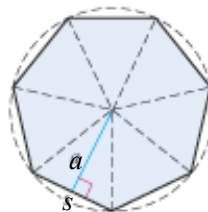


Regular pentagon

- Step 1 What is the area of one isosceles triangle in terms of a and s ?
- Step 2 What is the area of this pentagon in terms of a and s ?
- Step 3 Repeat Steps 1 and 2 with other regular polygons and complete the table below.



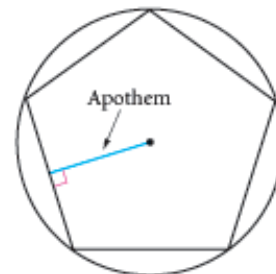
Regular hexagon



Regular heptagon

Number of sides	5	6	7	8	9	10	...	12	...	n
Area of regular polygon							

The distance a appears in the area formula for a regular polygon, and it has a special name—apothem. An **apothem** of a regular polygon is a perpendicular segment from the center of the polygon's circumscribed circle to a side of the polygon. You may also refer to the length of the segment as the apothem.



- Step 4 | What is the perimeter of a regular polygon in terms of n and s ? Use your answer to this question and your last entry in the table to state your next conjecture.

Regular Polygon Area Conjecture

C-79

The area of a regular polygon is given by the formula $\frac{P \cdot a}{2}$ or $\frac{n \cdot s \cdot a}{2}$, where A is the area, P is the perimeter, a is the apothem, s is the length of each side, and n is the number of sides.



EXERCISES

You will need



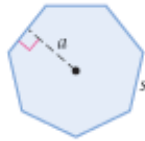
Construction tools
for Exercise 9



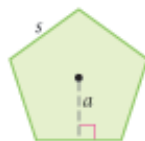
Geometry software
for Exercises 12 and 18

In Exercises 1–8, use the Regular Polygon Area Conjecture to find the unknown length accurate to the nearest unit, or the unknown area accurate to the nearest square unit. Recall that the symbol \approx is used for measurements or calculations that are approximations.

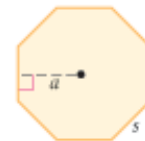
1. $A \approx ?$
 $s = 24$ cm
 $a \approx 24.9$ cm



2. $a \approx ?$
 $s = 107.5$ cm
 $A \approx 19,887.5$ cm²

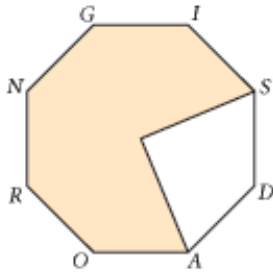


3. $P \approx ?$
 $a = 38.6$ cm
 $A \approx 4940.8$ cm²

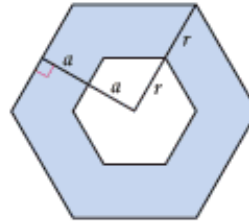


- Regular pentagon: $a = 3$ cm and $s \approx 4.4$ cm, $A \approx ?$
- Regular nonagon: $a = 9.6$ cm and $A \approx 302.4$ cm², $P \approx ?$
- Regular n -gon: $a = 12$ cm and $P \approx 81.6$ cm, $A \approx ?$
- Find the approximate perimeter of a regular polygon if $a = 9$ m and $A \approx 259.2$ m².
- Find the approximate length of each side of a regular n -gon if $a = 80$ feet, $n = 20$, and $A \approx 20,000$ square feet.
- Construction** Draw a segment 4 cm long. Use a compass and straightedge to construct a regular hexagon with sides congruent to this segment. Use the Regular Polygon Area Conjecture and a centimeter ruler to approximate the hexagon's area.
- Draw a regular pentagon with apothem 4 cm. Use the Regular Polygon Area Conjecture and a centimeter ruler to approximate the pentagon's area.
- A square is also a regular polygon. How is the apothem of a square related to the side length? Show that the Regular Polygon Area Conjecture simplifies to s^2 for the area of a square.
- Technology** Use geometry software to construct a circle. Inscribe a pentagon that looks regular and measure its area. Now drag the vertices. How can you drag the vertices to increase the area of the inscribed pentagon? To decrease its area?

13. Find the approximate area of the shaded region of the regular octagon *ROADSIGN*. The apothem measures 20 cm. Segment *GI* measures about 16.6 cm.

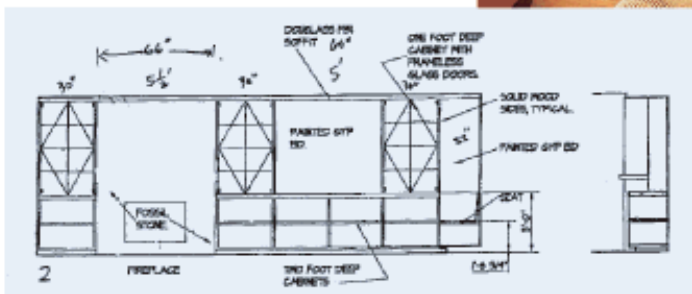


14. Find the approximate area of the shaded regular hexagonal donut. The apothem and sides of the smaller hexagon are half as long as the apothem and sides of the large hexagon. $a \approx 6.9$ cm and $r \approx 8$ cm

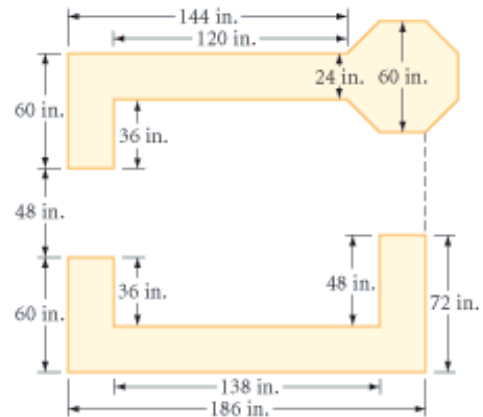


Career CONNECTION

Interior designers, unlike interior decorators, are concerned with the larger planning and technical considerations of interiors, as well as with style and color selection. They have an intuitive sense of spatial relationships. They prepare sketches, schedules, and budgets for client approval and inspect the site until the job is complete.



15. **Application** An interior designer created the kitchen plan shown. The countertop will be constructed of colored concrete. What is its total surface area? If concrete countertops 1.5 inches thick cost \$85 per square foot, what will be the total cost of this countertop?



Review

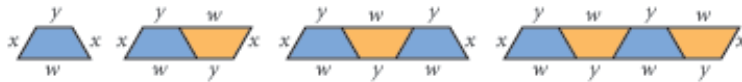
In Exercises 16 and 17, graph the two lines, then find the area bounded by the x -axis, the y -axis, and both lines.

16. $y = \frac{1}{2}x + 5, y = -2x + 10$ (h)

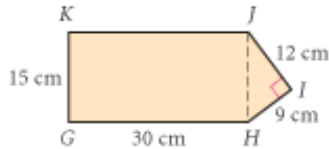
17. $y = -\frac{1}{3}x + 6, y = -\frac{4}{3}x + 12$

18. **Technology** Construct a triangle and its three medians. Compare the areas of the six small triangles that the three medians formed. Make a conjecture and support it with a convincing argument.

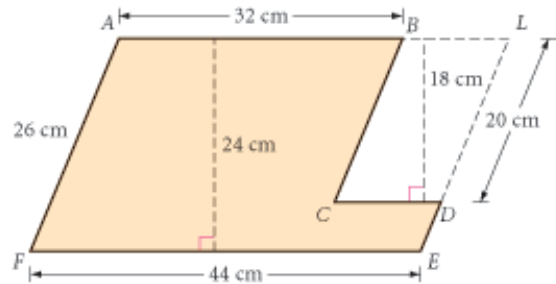
19. If the pattern continues, write an expression for the perimeter of the n th figure in the picture pattern.



20. $GHIK$ is a rectangle. Find the area of pentagon $GHIJK$.



21. $FELA$ and $CDLB$ are parallelograms. Find the area of the shaded region.



IMPROVING YOUR VISUAL THINKING SKILLS

The Squared Square Puzzle

The square shown is called a “squared square.” A square 112 units on a side is divided into 21 squares. The area of square X is 50^2 , or 2500, and the area of square Y is 4^2 , or 16. Find the area of each of the other squares.



Exploration

Pick's Formula for Area

You know how to find the area of polygon regions, but how would you find the area of the dinosaur footprint at right?

You know how to place a polygon on a grid and count squares to find the area. About a hundred years ago, Austrian mathematician Georg Alexander Pick (1859–1943) discovered a relationship, now known as Pick's formula, for finding the area of figures on a square dot grid.

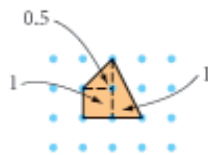
Let's start by looking at polygons on a square dot grid. The dots are called lattice points. Let's count the lattice points in the interior of the polygon and those on its boundary and compare our findings to the areas of the polygon that you get by dividing them into rectangles and triangles.



Polygon A



5 boundary points
1 interior point

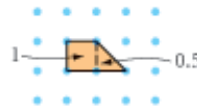


$$\text{Area} = 0.5 + 1 + 1 = 2.5$$

Polygon B



5 boundary points
0 interior points

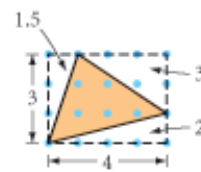


$$\text{Area} = 1 + 0.5 = 1.5$$

Polygon C



3 boundary points
5 interior points



$$\text{Area} = 12 - (1.5 + 3 + 2) = 5.5$$

How can the boundary points and interior points help us find the area? An important technique for finding patterns is to hold one variable constant and see what happens with the other variables. That's what you'll do in the next activity.

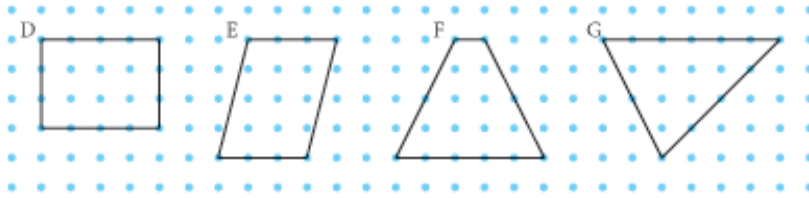
In this activity you will first investigate patterns that emerge when you hold the area constant. Then you will investigate patterns that emerge when you hold the number of interior or boundary points constant. The goal is to find a formula that relates the area of a polygon to the number of interior and boundary points.

Activity

Dinosaur Footprints and Other Shapes

You will need

- square dot paper or graph paper, or
- the worksheets Pick's Formula for Area and Irregular Shapes (optional)
- geoboards (optional)



- Step 1 Confirm that each polygon D through G above has area $A = 12$.
- Step 2 Let b be the number of boundary points and i be the number of interior points. Create and complete a table like this one for polygons with $A = 12$.

	D	E	F	G
Number of boundary points (b)				
Number of interior points (i)				

- Step 3 Study the table for patterns. Do you see a relationship between b and i ? Graph the pairs (b, i) from your table and label each point. What do you notice? Write an equation that fits the points.

Now investigate what happens if you hold the interior points constant.

- Step 4 Polygon B has $b = 5$, $i = 0$, and $A = 1.5$. On square dot paper draw other polygons with *no* interior points. Calculate the area of each polygon.
- Step 5 Copy and complete this table for polygons with $i = 0$.

Number of boundary points (b)	3	4	5	6	7	8
Area			1.5			

- Step 6 Polygon A has $b = 5$, $i = 1$, and $A = 2.5$. On square dot paper draw other polygons with exactly *one* interior point. Calculate the area of each polygon.
- Step 7 Make a table like the one in Step 5, but for polygons with $i = 1$. When you hold the interior points constant, what happens to the area each time one boundary point is added?

Now investigate what happens if you hold the boundary points constant.

- Step 8 Polygon C has $b = 3$, $i = 5$, and $A = 5.5$. On square dot paper draw other polygons with exactly *three* boundary points. Calculate the area of each polygon.
- Step 9 Copy and complete this table for polygons with $b = 3$. When you hold the boundary points constant, what happens to the area each time one interior point is added?

Number of interior points (i)	0	1	2	3	4	5
Area						5.5

To find Pick's formula, it can be helpful to organize all your data in one place.

- Step 10 Copy and complete the table below. Check the patterns by drawing more polygons on square dot paper.

Area (as a function of b and i)

		Number of boundary points (b)					
		3	4	5	6	7	8
Number of interior points (i)	0			1.5			
	1			2.5			
	2						
	3						
	4						
	5	5.5					

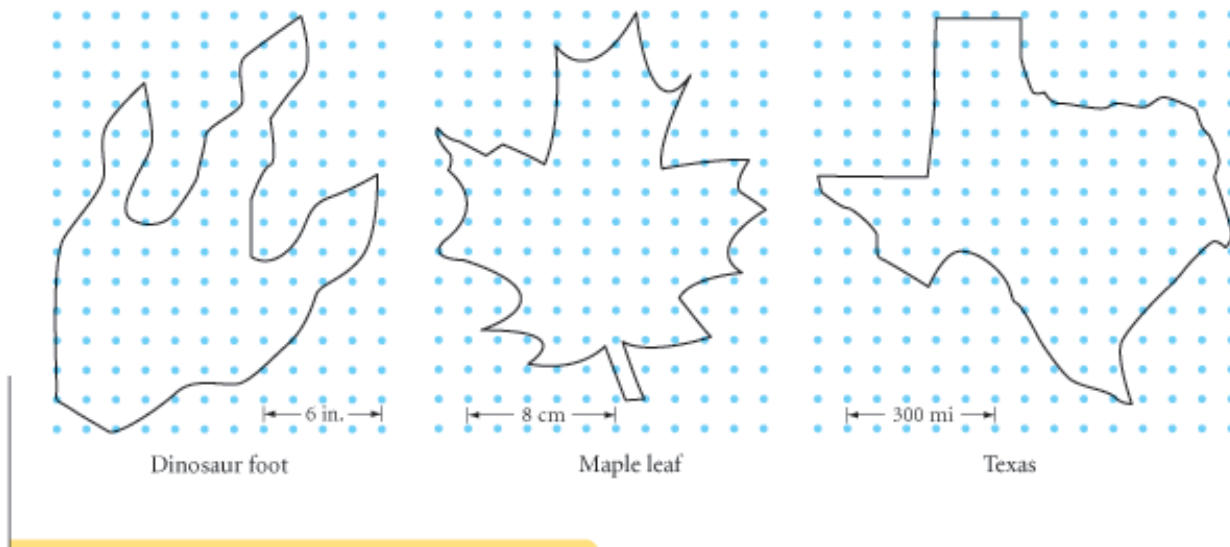
- Step 11 Generalize the formula to find the area if you know the number of boundary points and the number of interior points. Copy and complete the conjecture.

Pick's Formula

If A is the area of a polygon whose vertices are lattice points, b is the number of lattice points on the boundary of the polygon, and i is the number of interior lattice points, then $A = \frac{?}{2} b + \frac{?}{2} i + \frac{?}{2}$.

Pick's formula is useful for approximating the area of an irregularly shaped region.

- Step 12 Use Pick's formula to find the approximate areas of these irregular shapes.



Areas of Circles

So far, you have discovered the formulas for the areas of various polygons. In this lesson you'll discover the formula for the area of a circle. Most of the shapes you have investigated in this chapter could be divided into rectangles or triangles. Can a circle be divided into rectangles or triangles? Not exactly, but in this investigation you will see an interesting way to think about the area of a circle.

*The moon is a dream
of the sun.*

PAUL KLEE



Investigation

Area Formula for Circles

You will need

- a compass
- scissors

Step 1

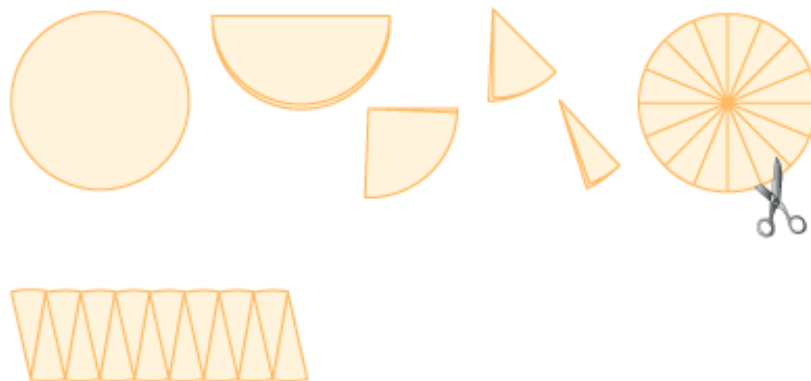
Use your compass to make a large circle. Cut out the circular region.

Step 2

Fold the circular region in half. Fold it in half a second time, then a third time and a fourth time. Unfold your circle and cut it along the folds into 16 wedges.

Step 3

Arrange the wedges in a row, alternating the tips up and down to form a shape that resembles a parallelogram.



If you cut the circle into more wedges, you could rearrange these thinner wedges to look even more like a rectangle, with fewer bumps. You would not lose or gain any area in this change, so the area of this new “rectangle,” skimming off the bumps as you measure its length, would be closer to the area of the original circle.

If you could cut infinitely many wedges, you'd actually have a rectangle with smooth sides. What would its base length be? What would its height be in terms of C , the circumference of the circle?

Step 4

The radius of the original circle is r and the circumference is $2\pi r$. Give the base and the height of a rectangle made of a circle cut into infinitely many wedges. Find its area in terms of r . State your next conjecture.

Circle Area Conjecture

C-80

The area of a circle is given by the formula $A = \pi r^2$, where A is the area and r is the radius of the circle.

How do you use this new conjecture? Let's look at a few examples.

EXAMPLE A

The small apple pie has a diameter of 8 inches, and the large cherry pie has a radius of 5 inches. How much larger is the large pie?

► Solution

First, find each area.

Small pie

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(4)^2 \\ &= \pi(16) \\ &\approx 50.2 \end{aligned}$$



Large pie

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(5)^2 \\ &= \pi(25) \\ &\approx 78.5 \end{aligned}$$



The large pie is 78.5 in^2 , and the small pie is 50.2 in^2 . The difference in area is about 28.3 square inches. So the large pie is more than 50% larger than the small pie, assuming they have the same thickness. Notice that we used 3.14 as an approximate value for π .

EXAMPLE B

If the area of the circle at right is $256\pi \text{ m}^2$, what is the circumference of the circle?

► Solution

Use the area to find the radius, then use the radius to find the circumference.

$$\begin{aligned} A &= \pi r^2 & C &= 2\pi r \\ 256\pi &= \pi r^2 & &= 2\pi(16) \\ 256 &= r^2 & &= 32\pi \\ r &= 16 & &\approx 100.5 \text{ m} \end{aligned}$$



The circumference is 32π meters, or approximately 100.5 meters.

EXERCISES

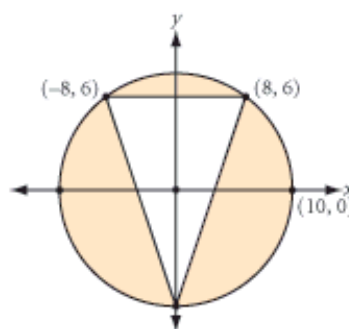
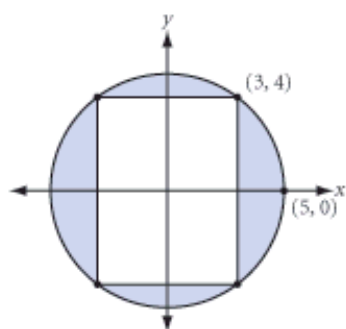
You will need




Geometry software
for Exercise 19

Use the Circle Area Conjecture to solve for the unknown measures in Exercises 1–8. Leave your answers in terms of π , unless the problem asks for an approximation. For approximations, use the π key on your calculator.


- If $r = 3$ in., $A = ?$.
- If $r = 7$ cm, $A = ?$.
- If $r = 0.5$ m, $A \approx ?$.
- If $A = 9\pi$ cm², then $r = ?$.
- If $A = 3\pi$ in², then $r = ?$.
- If $A = 0.785$ m², then $r \approx ?$.
- If $C = 12\pi$ in., then $A = ?$.
- If $C = 314$ m, then $A \approx ?$.
- What is the area of the shaded region between the circle and the rectangle?
- What is the area of the shaded region between the circle and the triangle?



- Sketch and label a circle with an area of 324π cm². Be sure to label the length of the radius.
- Application** The rotating sprinkler arms in the photo at right are all 16 meters long. What is the area of each circular farm? Express your answer to the nearest square meter.
- Application** A small college TV station can broadcast its programming to households within a radius of 60 kilometers. How many square kilometers of viewing area does the station reach? Express your answer to the nearest square kilometer.
- Sampson's dog, Cecil, is tied to a post by a chain 7 meters long. How much play area does Cecil have? Express your answer to the nearest square meter.
- Application** A muscle's strength is proportional to its cross-sectional area. If the cross section of one muscle is a circular region with a radius of 3 cm, and the cross section of a second, identical type of muscle is a circular region with a radius of 6 cm, how many times stronger is the second muscle? 

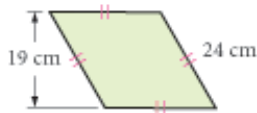


Champion weight lifter Soraya Jimenez extends barbells weighing almost double her own body weight.

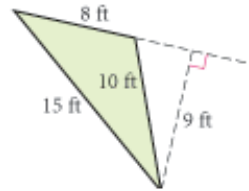
16. What would be a good approximation for the area of a regular 100-gon inscribed in a circle with radius r ? Explain your reasoning. 

Review

17. $A = ?$

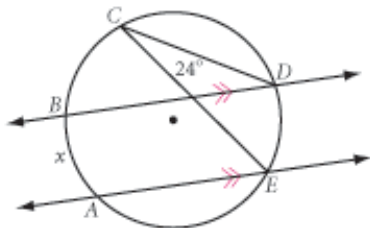


18. $A = ?$

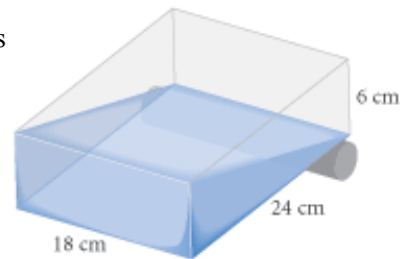


19. **Technology** Construct a parallelogram and a point in its interior. Construct segments from this point to each vertex, forming four triangles. Measure the area of each triangle. Move the point to find a location where all four triangles have equal area. Is there more than one such location? Explain your findings.

20. **Developing Proof** Explain why x must be 48° . 21. **Developing Proof** What's wrong with this picture?



22. The 6-by-18-by-24 cm clear plastic sealed container is resting on a cylinder. It is partially filled with liquid, as shown. Sketch the container resting on its smallest face. Show the liquid level in this position.



IMPROVING YOUR VISUAL THINKING SKILLS

Random Points

What is the probability of randomly selecting from the 3-by-3 grid at right three points that form the vertices of an isosceles triangle?



Any Way You Slice It

In Lesson 8.5, you discovered a formula for calculating the area of a circle. With the help of your visual thinking and problem-solving skills, you can calculate the areas of different sections of a circle.

Cut my pie into four pieces—I don't think I could eat eight.
YOGI BERRA

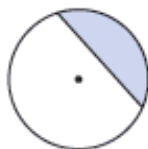


Its makers claimed this was the world's largest slice of pizza.

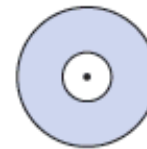
If you cut a slice of pizza, each slice would probably be a sector of a circle. If you could make only one straight cut with your knife, your slice would be a segment of a circle. If you don't like the crust, you'd cut out the center of the pizza; the crust shape that would remain is called an annulus.



Sector of a circle



Segment of a circle



Annulus

A **sector of a circle** is the region between two radii and an arc of the circle.

A **segment of a circle** is the region between a chord and an arc of the circle.

An **annulus** is the region between two concentric circles.

“Picture equations” are helpful when you try to visualize the areas of these regions. The picture equations below show you how to find the area of a sector of a circle, the area of a segment of a circle, and the area of an annulus.

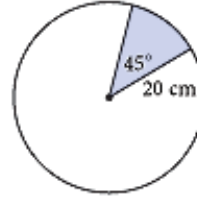
$$\frac{a}{360} \cdot \pi r^2 = A_{\text{sector}}$$

$$\frac{a}{360} \pi r^2 - \frac{1}{2}bh = A_{\text{segment}}$$

$$\pi R^2 - \pi r^2 = A_{\text{annulus}}$$

EXAMPLE A

Find the area of the shaded sector.

**► Solution**The sector is $\frac{45^\circ}{360^\circ}$, or $\frac{1}{8}$, of the circle.

$$\begin{aligned} A_{\text{sector}} &= \frac{\alpha}{360} \cdot \pi r^2 \\ &= \frac{45}{360} \cdot \pi(20)^2 \\ &= \frac{1}{8} \cdot 400\pi \\ &= 50\pi \end{aligned}$$

The area formula for a sector.

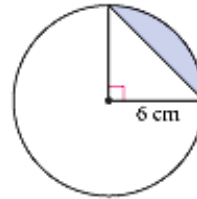
Substitute $r = 20$ and $\alpha = 45$.

Reduce the fraction and square 20.

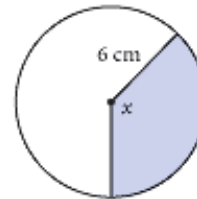
Multiply.

The area is $50\pi \text{ cm}^2$.**EXAMPLE B**

Find the area of the shaded segment.

**► Solution**

According to the picture equation on page 453, the area of a segment is equivalent to the area of the sector minus the area of the triangle. You can use the method in Example A to find that the area of the sector is $\frac{1}{4} \cdot 36\pi \text{ cm}^2$, or $9\pi \text{ cm}^2$. The area of the triangle is $\frac{1}{2} \cdot 6 \cdot 6$, or 18 cm^2 . So the area of the segment is $9\pi - 18 \text{ cm}^2$.

EXAMPLE CThe shaded area is $14\pi \text{ cm}^2$, and the radius is 6 cm. Find x .**► Solution**The sector's area is $\frac{x}{360}$ of the circle's area, which is 36π .

$$\begin{aligned} 14\pi &= \frac{x}{360} \cdot 36\pi \\ \frac{360 \cdot 14\pi}{36\pi} &= x \\ x &= 140 \end{aligned}$$

The central angle measures 140° .

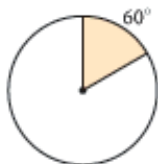
EXERCISES

You will need

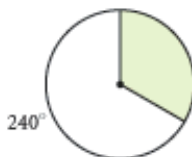


In Exercises 1–8, find the area of the shaded region. The radius of each circle is r . If two circles are shown, r is the radius of the smaller circle and R is the radius of the larger circle.

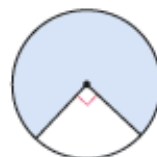
1. $r = 6$ cm



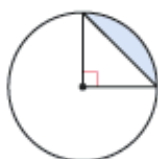
2. $r = 8$ cm



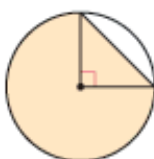
3. $r = 16$ cm



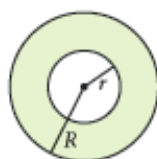
4. $r = 2$ cm



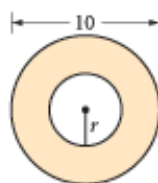
5. $r = 8$ cm



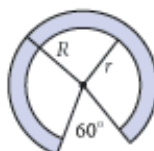
6. $R = 7$ cm
 $r = 4$ cm



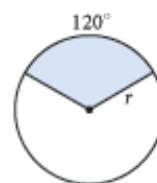
7. $r = 2$ cm



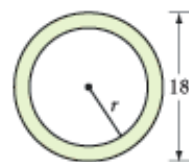
8. $R = 12$ cm
 $r = 9$ cm



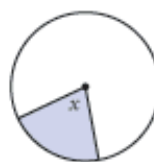
9. The shaded area is 12π cm². Find r .



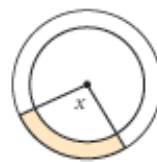
10. The shaded area is 32π cm². Find r .



11. The shaded area is 120π cm², and the radius is 24 cm. Find x .



12. The shaded area is 10π cm². The radius of the large circle is 10 cm, and the radius of the small circle is 8 cm. Find x .



13. **Application** Suppose the pizza slice in the photo at the beginning of this lesson is a sector with a 36° arc, and the pizza has a radius of 20 ft. If one can of tomato sauce will cover 3 ft² of pizza, how many cans would you need to cover this slice?



14. Utopia Park has just installed a circular fountain 8 meters in diameter. The Park Committee wants to pave a 1.5-meter-wide path around the fountain. If paving costs \$10 per square meter, find the cost to the nearest dollar of the paved path around the fountain.



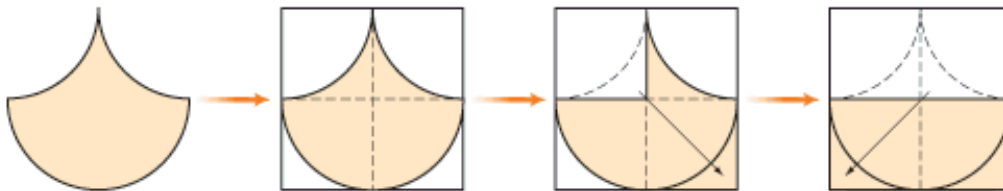
This circular fountain at the Chateau de Villandry in Loire Valley, France, shares a center with the circular path around it. How many concentric arcs and circles do you see in the picture?

Mathematics

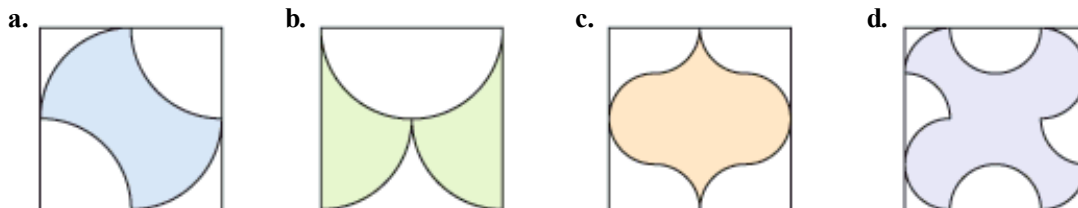
CONNECTION

Attempts to solve the famous problem of squaring a circle—finding a square with the same area as a given circle—led to the creation of some special shapes made up of parts of circles. The diagrams below are based on some that Leonardo da Vinci sketched while attempting to solve this problem.

15. The illustrations below demonstrate how to find a rectangle with the same area as the shaded figure.



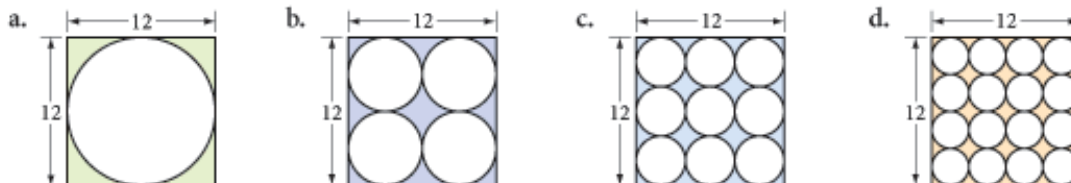
In a series of diagrams, demonstrate how to find a rectangle with the same area as the shaded figure.



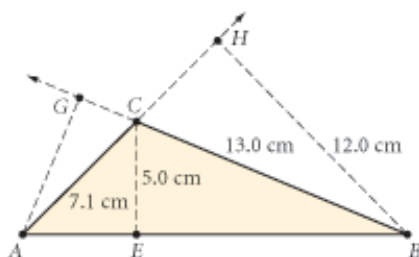
16. **Construction** Reverse the process you used in Exercise 15. On graph paper, draw a 12-by-6 rectangle. Use your compass to divide it into at least four parts, then rearrange the parts into a new curved figure. Draw its outline on graph paper.

Review

17. Each set of circles is externally tangent. What is the area of the shaded region in each figure? What percentage of the area of the square is the area of the circle or circles in each figure? All given measurements are in centimeters.



18. The height of a trapezoid is 15 m and the midsegment is 32 m. What is the area of the trapezoid? h
19. \overline{CE} , \overline{BH} , and \overline{AG} are altitudes. Find AB and AG .



In Exercises 20–23, identify each statement as true or false. If true, explain why. If false, give a counterexample.

20. If the arc of a circle measures 90° and has an arc length of 24π cm, then the radius of the circle is 48 cm.
21. If the measure of each exterior angle of a regular polygon is 24° , then the polygon has 15 sides.
22. If the diagonals of a parallelogram bisect its angles, then the parallelogram is a square.
23. If two sides of a triangle measure 25 cm and 30 cm, then the third side must be greater than 5 cm, but less than 55 cm.

IMPROVING YOUR REASONING SKILLS

Code Equations

Each code below uses the first letters of words that will make the equation true. For example, $12M = a Y$ is an abbreviation of the equation 12 Months = a Year. Find the missing words in each code.

- $45D = \text{an } AA \text{ of an } IRT$
- $7 = S \text{ of a } H$
- $90D = \text{each } A \text{ of a } R$
- $5 = D \text{ in a } P$



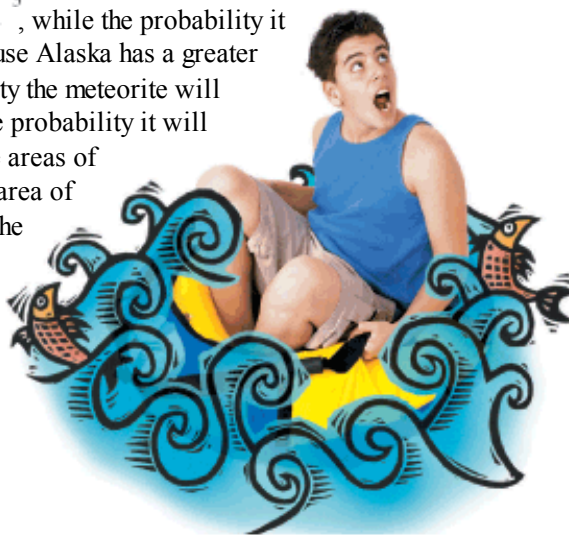
Exploration



Geometric Probability II

You already know that a probability value is a number between 0 and 1 that tells you how likely something is to occur. For example, when you roll a die, three of the six rolls—namely, 2, 3, and 5—are prime numbers. Because each roll has the same chance of occurring, $P(\text{prime number}) = \frac{3}{6}$, or $\frac{1}{2}$.

In some situations, probability depends on area. For example, suppose a meteorite is headed toward Earth. If about $\frac{1}{3}$ of Earth's surface is land, the probability that the meteorite will hit land is about $\frac{1}{3}$, while the probability it will hit water is about $\frac{2}{3}$. Because Alaska has a greater area than Vermont, the probability the meteorite will land in Alaska is greater than the probability it will land in Vermont. If you knew the areas of these two states and the surface area of Earth, how could you calculate the probabilities that the meteorite would land in each state?



Activity

Where the Chips Fall

You will need

- graph paper
- a ruler
- a penny
- a dime

In this activity you will solve several probability problems that involve area.

The Shape of Things

At each level of a computer game, you must choose one of several shapes on a coordinate grid. The computer then randomly selects a point *anywhere* on the grid. If the point is outside your shape, you move to the next level. If the point is on or inside your shape, you lose the game.

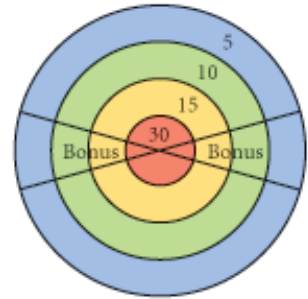
On the first level, a trapezoid, a pentagon, a square, and a triangle are displayed on a grid that goes from 0 to 12 on both axes. The table below gives the vertices of the shapes.

Shape	Vertices
Trapezoid	(1, 12), (8, 12), (7, 9), (4, 9)
Pentagon	(3, 1), (4, 4), (6, 4), (9, 2), (7, 0)
Square	(0, 6), (3, 9), (6, 6), (3, 3)
Triangle	(11, 0), (7, 4), (11, 12)

- Step 1 For each shape, calculate the probability the computer will choose a point on or inside that shape. Express each probability to three decimal places.
- Step 2 What is the probability the computer will choose a point that is on or inside a quadrilateral? What is the probability it will choose a point that is outside all of the shapes?
- Step 3 If you choose a triangle, what is the probability you will move to the next level?
- Step 4 Which shape should you choose to have the best chance of moving to the next level? Why?

Right on Target

You are playing a carnival game in which you must throw one dart at the board shown at right. The score for each region is shown on the board. If your dart lands in a Bonus section, your score is tripled. The more points you get, the better your prize will be. The radii of the circles from the inside to the outside are 4 in., 8 in., 12 in., and 16 in. Assume your aim is not very good, so the dart will hit a random spot. If you miss the board completely, you get to throw again.



- Step 5 What is the probability your dart will land in the red region? Blue region?
- Step 6 Compute the probability your dart will land in a Bonus section. (The central angle measure for each Bonus section is 30° .)
- Step 7 If you score 90 points, you will win the grand prize, a giant stuffed emu. What is the probability you will win the grand prize?
- Step 8 If you score exactly 30 points, you win an “I ♥ Carnivals” baseball cap. What is the probability you will win the cap?
- Step 9 Now imagine you have been practicing your dart game, and your aim has improved. Would your answers to Steps 5–8 change? Explain.

The Coin Toss

You own a small cafe that is popular with the mathematicians in the neighborhood. You devise a game in which the customer flips a coin onto a red-and-white checkered tablecloth with 1-inch squares. If it lands completely within a square, the customer wins and doesn't have to pay the bill. If it lands touching or crossing the boundary of a square, the customer loses.



- Step 10 Assuming the coin stays on the table, what is the probability of the customer winning by flipping a penny? A dime? (*Hint*: Where must the center of the coin land in order to win?)

- Step 11 | If the customer wins only if the coin falls within a red square, what is the probability of winning with a penny? A dime?
- Step 12 | Suppose the game is always played with a penny, and a customer wins if the penny lands completely inside any square (red or white). If your daily proceeds average \$300, about how much will the game cost you per day?

On a Different Note

Two opera stars—Rigoletto and Pollione—are auditioning for a part in an upcoming production. Because the singers have similar qualifications, the director decides to have a contest to see which man can hold a note the longest. Rigoletto has been known to hold a note for any length of time from 6 to 9 minutes. Pollione has been known to hold a note for any length of time from 5 to 7 minutes.

- Step 13 | Draw a rectangular grid in which the bottom side represents the range of times for Rigoletto and the left side represents the range of times for Pollione. Each point in the rectangle represents one possible outcome of the contest.
- Step 14 | On your grid, mark all the points that represent a tie. Use your diagram to find the probability that Rigoletto will win the contest.

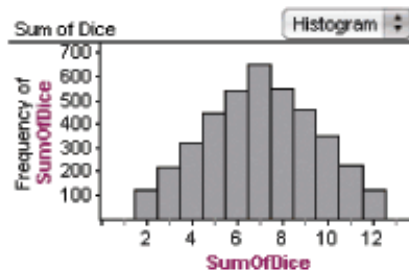
project



DIFFERENT DICE

Understanding probability can improve your chances of winning a game. If you roll a pair of standard 6-sided dice, are you more likely to roll a sum of 6 or 12? It's fairly common to roll a sum of 6, because many combinations of two dice add up to 6. But a 12 is only possible if you roll a 6 on each die.

If you rolled a pair of standard 6-sided dice thousands of times, and recorded the number of times you got each sum, the histogram would resemble the one at right. If you rolled them far fewer times, your histogram would look more irregular.



Would the distribution be different if you used different dice? What if one die had odd numbers and the other had even numbers?

What if you used 8-sided dice? What if you rolled three 6-sided dice instead of two?

Choose one of these scenarios or one that you find interesting to investigate. Make your dice and roll them 20 times. Predict what the graph will look like if you roll the dice 100 times, then check your prediction.

Your project should include

- ▶ Your dice.
- ▶ Histograms of your experimental data, and your predictions and conclusions.



You can use Fathom to simulate real events, such as rolling the dice that you have designed. You can obtain the results of hundreds of events very quickly, and use Fathom's graphing capabilities to make a histogram showing the distribution of different outcomes.

Surface Area

In Lesson 8.3, you calculated the surface areas of walls and roofs. But not all building surfaces are rectangular. How would you calculate the amount of glass necessary to cover a pyramid-shaped building? Or the number of tiles needed to cover a cone-shaped roof?

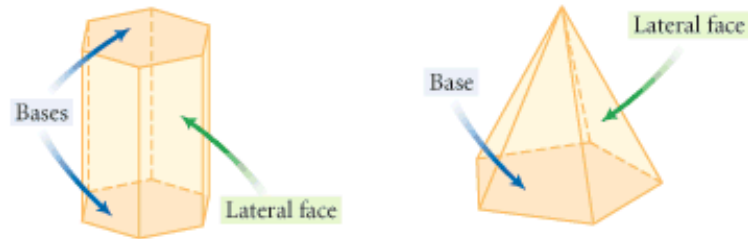
*No pessimist ever
discovered the secrets
of the stars, or sailed
to an uncharted
land, or opened
a new doorway for
the human spirit.*

HELEN KELLER

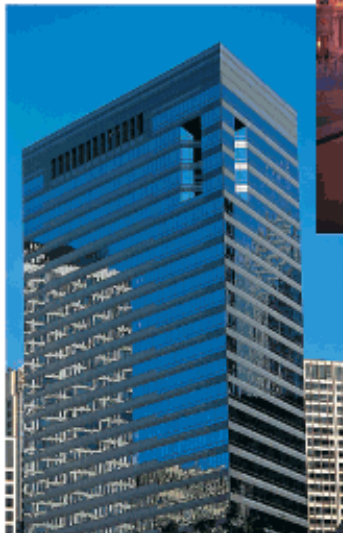
In this lesson you will learn how to find the surface areas of prisms, pyramids, cylinders, and cones. The **surface area** of each of these solids is the sum of the areas of all the faces or surfaces that enclose the solid. For prisms and pyramids, the faces include the solid's **bases** and its **lateral faces**.

In a prism, the bases are two congruent polygons and the lateral faces are rectangles or other parallelograms.

In a pyramid, the base can be any polygon. The lateral faces are triangles.



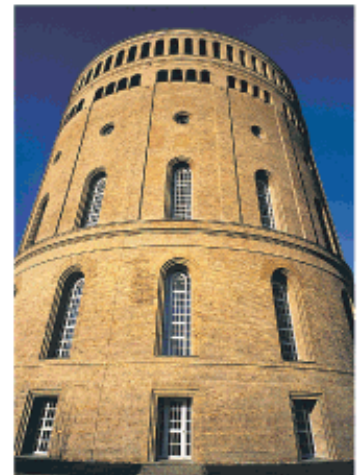
This glass pyramid was designed by I. M. Pei for the entrance of the Louvre museum in Paris, France.



This skyscraper in Chicago, Illinois, is an example of a prism.



A cone is part of the roof design of this Victorian house in Massachusetts.



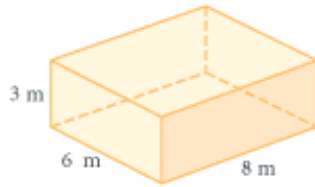
This stone tower is a cylinder on top of a larger cylinder.

To find the surface areas of prisms and pyramids, follow these steps.

- Steps for Finding Surface Area**
1. Draw and label each face of the solid as if you had cut the solid apart along its edges and laid it flat. Label the dimensions.
 2. Calculate the area of each face. If some faces are identical, you only need to find the area of one.
 3. Find the total area of all the faces.

EXAMPLE A

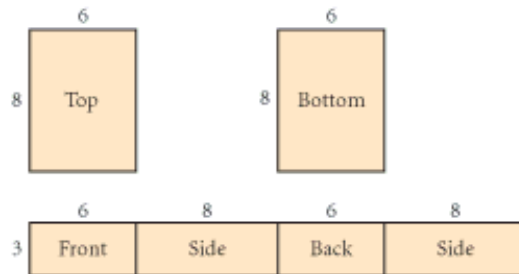
Find the surface area of the rectangular prism.



These shipping containers are rectangular prisms.

► Solution

Draw and label the two congruent bases, and the four lateral faces unfolded into one rectangle. Then find the areas of all the rectangular faces.



$$\begin{aligned}
 \text{surface area} &= 2(\text{base area}) + (\text{lateral surface area}) \\
 &= 2(6 \cdot 8) + 3(6 + 8 + 6 + 8) \\
 &= 2(48) + 3(28) \\
 &= 96 + 84 \\
 &= 180
 \end{aligned}$$

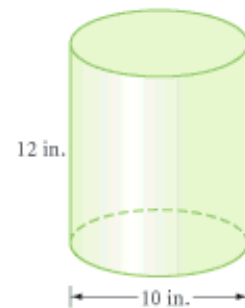
The surface area of the prism is 180 m².

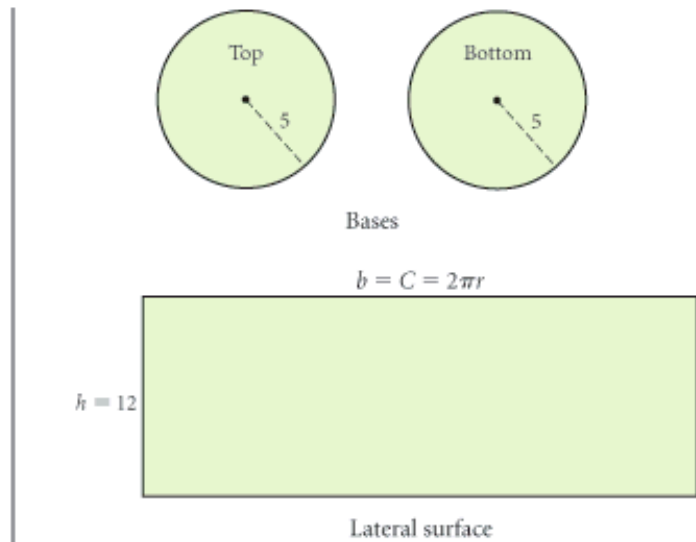
EXAMPLE B

Find the surface area of the cylinder.

► Solution

Imagine cutting apart the cylinder. The two bases are circular regions, so you need to find the areas of two circles. Think of the lateral surface as a wrapper. Slice it and lay it flat to get a rectangular region. You'll need the area of this rectangle. The height of the rectangle is the height of the cylinder. The base of the rectangle is the circumference of the circular base.



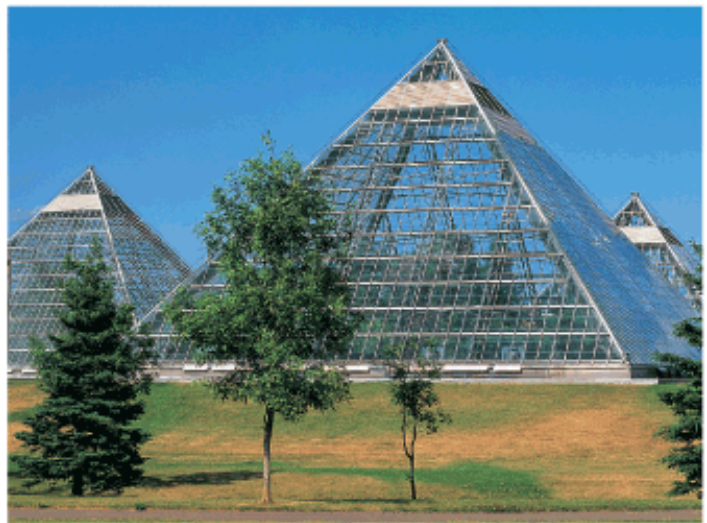


$$\begin{aligned} \text{surface area} &= 2(\pi r^2) + (2\pi r)h \\ &= 2(\pi \cdot 5^2) + (2 \cdot \pi \cdot 5) \cdot 12 \\ &\approx 534 \end{aligned}$$

The surface area of the cylinder is about 534 in².

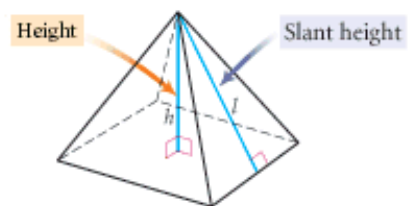


This ice cream plant in Burlington, Vermont, uses cylindrical containers for its milk and cream.



These conservatories in Edmonton, Canada, are glass pyramids.

The surface area of a pyramid is the area of the base plus the areas of the triangular faces. The height of each triangular lateral face is called the **slant height**. The slant height is labeled l and the pyramid height is labeled h .



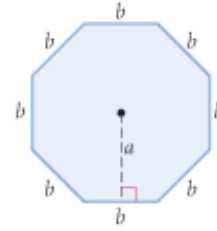
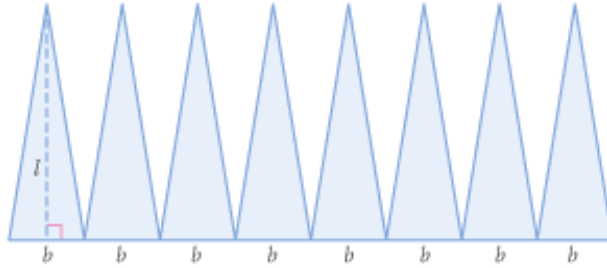
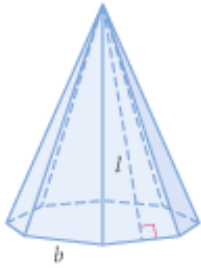
In the investigation you'll find out how to calculate the surface area of a pyramid with a regular polygon base.



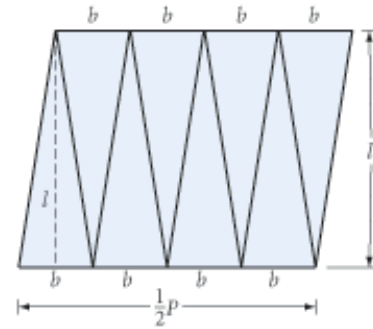
Investigation 1

Surface Area of a Regular Pyramid

You can cut and unfold the surface of a regular pyramid into these shapes.



- Step 1 | What is the area of each lateral face?
- Step 2 | What is the total lateral surface area? What is the total lateral surface area for any pyramid with a regular n -gon base?
- Step 3 | What is the area of the base for any regular n -gon pyramid?
- Step 4 | Use your expressions from Steps 2 and 3 to write a formula for the surface area of a regular n -gon pyramid in terms of n , base length b , slant height l , and apothem a .
- Step 5 | Write another expression for the surface area of a regular n -gon pyramid in terms of height h , apothem a , and perimeter of the base, P .



You can find the surface area of a cone using a method similar to the one you used to find the surface area of a pyramid.

Is the roof of this building in Kashan, Iran, a cone or a pyramid? What makes it hard to tell?

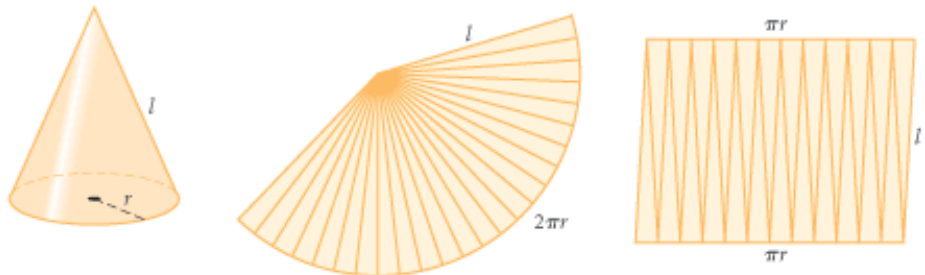




Investigation 2

Surface Area of a Cone

As the number of faces of a pyramid increases, it begins to look like a cone. You can think of the lateral surface as many small triangles or as a sector of a circle.



- Step 1 | What is the area of the base?
- Step 2 | What is the lateral surface area in terms of l and r ? What portion of the circle is the sector? What is the area of the sector?
- Step 3 | Write the formula for the surface area of a cone.

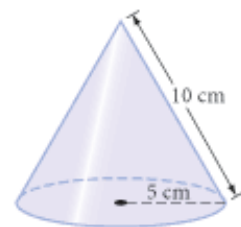
This photograph of Sioux tepees was taken around 1902 in North or South Dakota. Is a tepee shaped more like a cone or a pyramid?



EXAMPLE C | Find the total surface area of the cone.

► **Solution**

$$\begin{aligned} SA &= \pi r l + \pi r^2 \\ &= (\pi)(5)(10) + \pi(5)^2 \\ &= 75\pi \\ &\approx 235.6 \end{aligned}$$



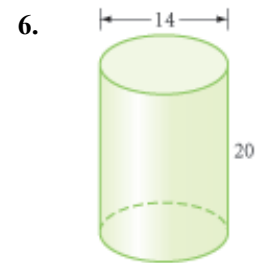
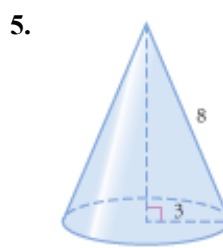
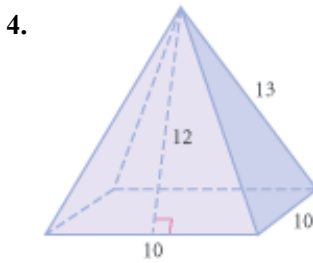
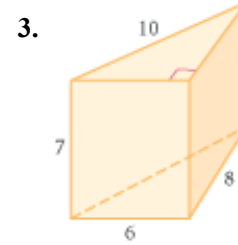
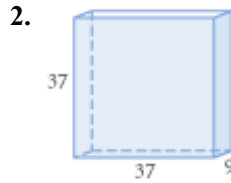
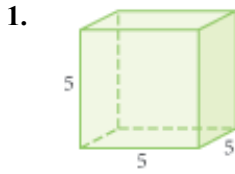
The surface area of the cone is about 236 cm^2 .

EXERCISES

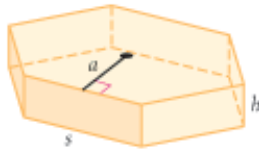
You will need



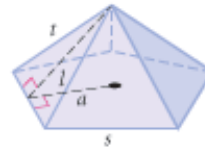
In Exercises 1–10, find the surface area of each solid. All quadrilaterals are rectangles, and all given measurements are in centimeters. Round your answers to the nearest 0.1 cm^2 .



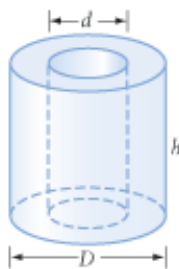
7. The base is a regular hexagon with apothem $a \approx 12.1$, side $s = 14$, and height $h = 7$.



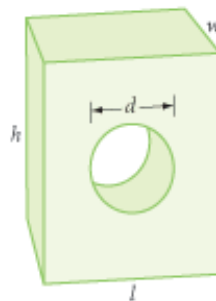
8. The base is a regular pentagon with apothem $a \approx 11$ and side $s = 16$. Each lateral edge $t = 17$, and the height of a face $l = 15$.



9. $D = 8, d = 4, h = 9$



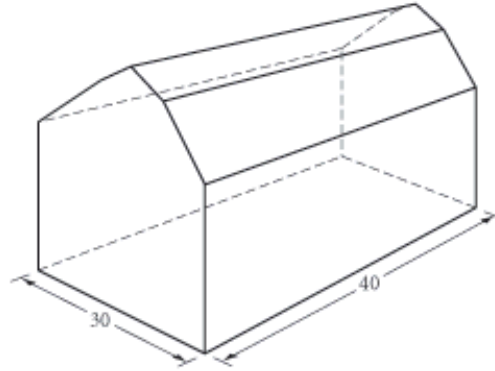
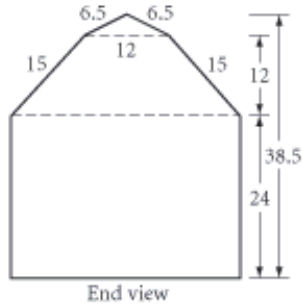
10. $l = 8, w = 4, h = 10, d = 4$



11. Explain how you would find the surface area of this obelisk.



12. **Application** Claudette and Marie are planning to paint the exterior walls of their country farmhouse (all vertical surfaces) and to put new cedar shingles on the roof. The paint they like best costs \$25 per gallon and covers 250 square feet per gallon. The wood shingles cost \$65 per bundle, and each bundle covers 100 square feet. How much will this home improvement cost? All measurements are in feet.



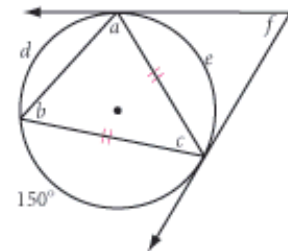
13. **Construction** The shapes of the spinning dishes in the photo are called **frustums** of cones. Think of them as cones with their tops cut off. Use your construction tools to draw pieces that you can cut out and tape together to form a frustum of a cone.



A Sri Lankan dancer balances and spins plates.

Review

14. Use patty paper, templates, or pattern blocks to create a $3^3 \cdot 4^2 / 3^2 \cdot 4 \cdot 3 \cdot 4 / 4^4$ tiling.
15. Trace the figure at right. Find the lettered angle measures and arc measures.



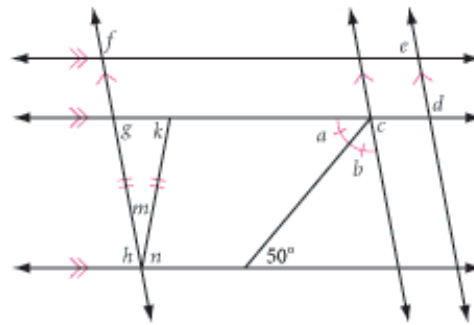
16. **Application** Suppose a circular ranch with a radius of 3 km were divided into 16 congruent sectors. In a one-year cycle, how long would the cattle graze in each sector? What would be the area of each sector?

History CONNECTION

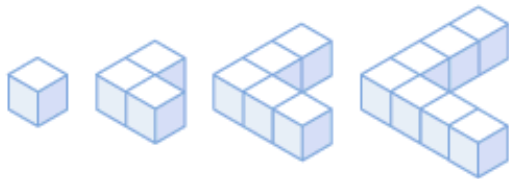
In 1792, visiting Europeans presented horses and cattle to Hawaii's King Kamehameha I. Cattle ranching soon developed when Mexican *vaqueros* came to Hawaii to train Hawaiians in ranching. Today, Hawaiian cattle ranching is big business. "Grazing geometry" is used on Hawaii's Kahua Ranch. Ranchers divide the grazing area into sectors. The cows are rotated through each sector in turn. By the time they return to the first sector, the grass has grown back and the cycle repeats.



17. **Developing Proof** Trace the figure at right. Find the lettered angle measures. Explain how you determined measures f and k .



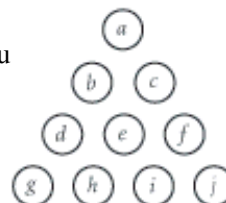
18. If the pattern of blocks continues, what will be the surface area of the 50th solid in the pattern? (Every edge of each block has length 1 unit.)



IMPROVING YOUR VISUAL THINKING SKILLS

Moving Coins

Create a triangle of coins similar to the one shown. How can you move exactly three coins so that the triangle is pointing down rather than up? When you have found a solution, use a diagram to explain it.



Exploration

Alternative Area Formulas

In ancient Egypt, when the yearly floods of the Nile River receded, the river often followed a different course, so the shape of farmers' fields along the banks could change from year to year. Officials then needed to measure property areas in order to keep records and calculate taxes. Partly to keep track of land and finances, ancient Egyptians developed some of the earliest mathematics.

Historians believe that ancient Egyptian tax assessors used this formula to find the area of any quadrilateral:

$$A = \frac{1}{2}(a + c) \cdot \frac{1}{2}(b + d)$$

where a , b , c , and d are the lengths, in consecutive order, of the figure's four sides.

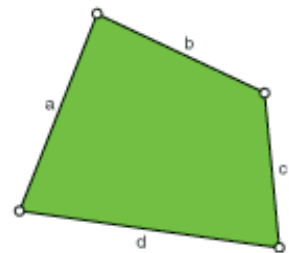
In this activity you will take a closer look at this ancient Egyptian formula, and another formula called Hero's formula, named after Hero of Alexandria.

Activity

Calculating Area in Ancient Egypt

Investigate the ancient Egyptian formula for quadrilaterals.

- | | |
|--------|---|
| Step 1 | Construct a quadrilateral and its interior. |
| Step 2 | Change the labels of the sides to a , b , c , and d , consecutively. |
| Step 3 | Measure the lengths of the sides and use the Sketchpad calculator to find the area according to the ancient Egyptian formula. |
| Step 4 | Select the polygon interior and measure its area. How does the area given by the formula compare to the actual area? Is the ancient Egyptian formula correct? |
| Step 5 | Does the ancient Egyptian formula always favor either the tax collector or the landowner, or does it favor one in some cases and the other in other cases? Explain. |
| Step 6 | Describe the quadrilaterals for which the formula works accurately. For what kinds of quadrilaterals is it slightly inaccurate? Very inaccurate? |
| Step 7 | State the ancient Egyptian formula in words, using the word <i>mean</i> . |



Step 8 | According to Hero's formula, if s is half the perimeter of a triangle with side lengths a , b , and c , the area A is given by the formula

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Use Sketchpad to investigate Hero's formula. Construct a triangle and its interior. Label the sides a , b , and c , and use the Sketchpad calculator to find the triangle's area according to Hero. Compare the result to the measured area of the triangle. Does Hero's formula work for all triangles?

Step 9 | Devise a way of calculating the area of any quadrilateral. Use Sketchpad to test your method.

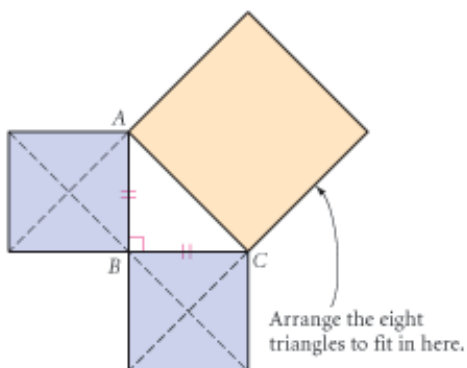
IMPROVING YOUR VISUAL THINKING SKILLS



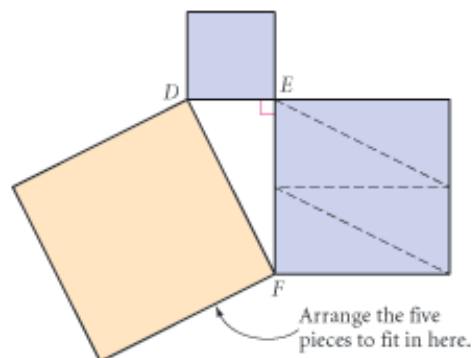
Cover the Square

Trace each diagram below onto another sheet of paper.

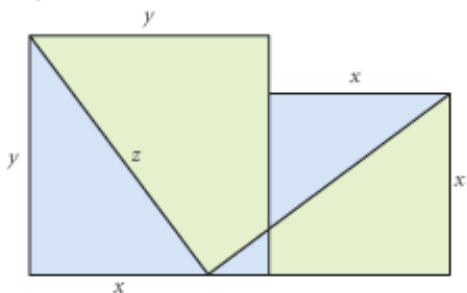
Cut out the four triangles in each of the two small equal squares and arrange them to exactly cover the large square.



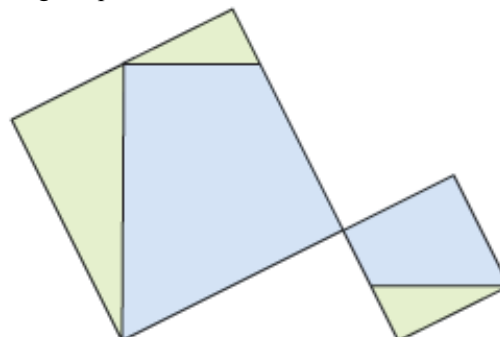
Cut out the small square and the four triangles from the square on leg EF and arrange them to exactly cover the large square.



Two squares with areas x^2 and y^2 are divided into the five regions as shown. Cut out the five regions and arrange them to exactly cover a larger square with an area of z^2 .



Two squares have been divided into three right triangles and two quadrilaterals. Cut out the five regions and arrange them to exactly cover a larger square.



CHAPTER
8
REVIEW

You should know area formulas for rectangles, parallelograms, triangles, trapezoids, regular polygons, and circles. You should also be able to show where these formulas come from and how they're related to one another. Most importantly, you should be able to apply them to solve practical problems involving area, including the surface areas of solid figures. What occupations can you list that use area formulas?



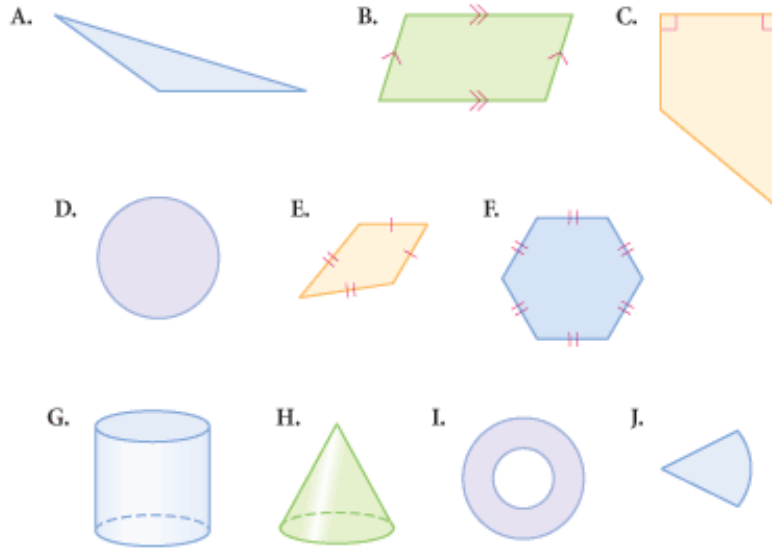
When you use area formulas for real-world applications, you have to consider units of measurement and accuracy. Should you use inches, feet, centimeters, meters, or some other unit? If you work with a circle or a regular polygon, is your answer exact or an approximation?



EXERCISES

▶ For Exercises 1–10, match the area formula with the shaded area.

1. $A = bh$
2. $A = 0.5bh$
3. $A = 0.5h(b_1 + b_2)$
4. $A = 0.5d_1d_2$
5. $A = 0.5aP$
6. $A = \pi r^2$
7. $A = \frac{x}{360} \pi r^2$
8. $A = \pi(R^2 - r^2)$
9. $SA = 2\pi rl + 2\pi r^2$
10. $SA = \pi rl + \pi r^2$



For Exercises 11–13, illustrate each term.

- | | | |
|-------------|-------------|------------------------|
| 11. Apothem | 12. Annulus | 13. Sector of a circle |
|-------------|-------------|------------------------|

For Exercises 14–16, draw a diagram and explain in a paragraph how you derived the area formula for each figure.

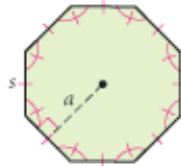
- | | | |
|-------------------|---------------|------------|
| 14. Parallelogram | 15. Trapezoid | 16. Circle |
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Solve for the unknown measures in Exercises 17–25. All measurements are in centimeters.

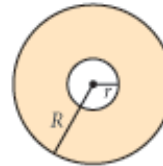
17. $A = ?$



18. $A \approx ?$
 $a = 36$
 $s \approx 41.6$



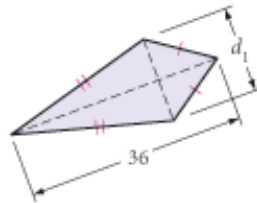
19. $A = ?$
 $R = 8$
 $r = 2$



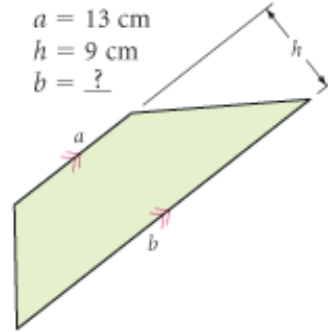
20. $A = 576 \text{ cm}^2$
 $h = ?$



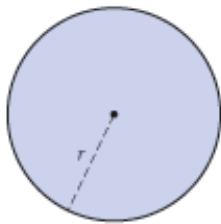
21. $A = 576 \text{ cm}^2$
 $d_1 = ?$



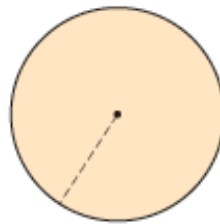
22. $A = 126 \text{ cm}^2$
 $a = 13 \text{ cm}$
 $h = 9 \text{ cm}$
 $b = ?$



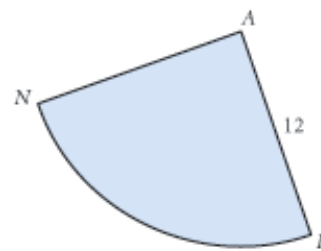
23. $C = 18\pi \text{ cm}$
 $A = ?$



24. $A = 576\pi \text{ cm}^2$
 The circumference is $?.$

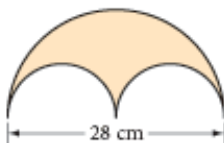


25. $A_{\text{sector}} = 16\pi \text{ cm}^2$
 $m\angle FAN = ?.$

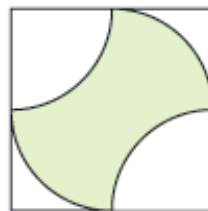


In Exercises 26–28, find the shaded area to the nearest 0.1 cm^2 . In Exercises 27 and 28, the quadrilateral is a square and all arcs are arcs of a circle of radius 6 cm.

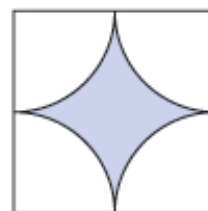
26.



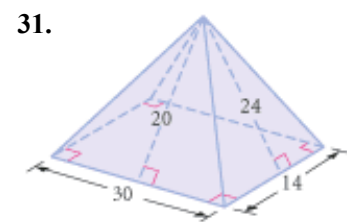
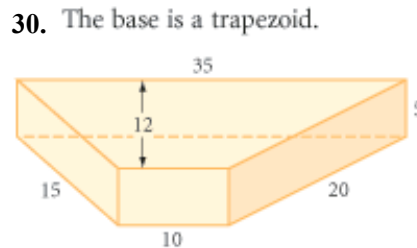
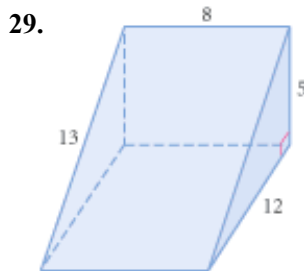
27.



28.



In Exercises 29–31, find the surface area of each prism or pyramid. All given measurements are in centimeters. All quadrilaterals are rectangles, unless otherwise labeled.

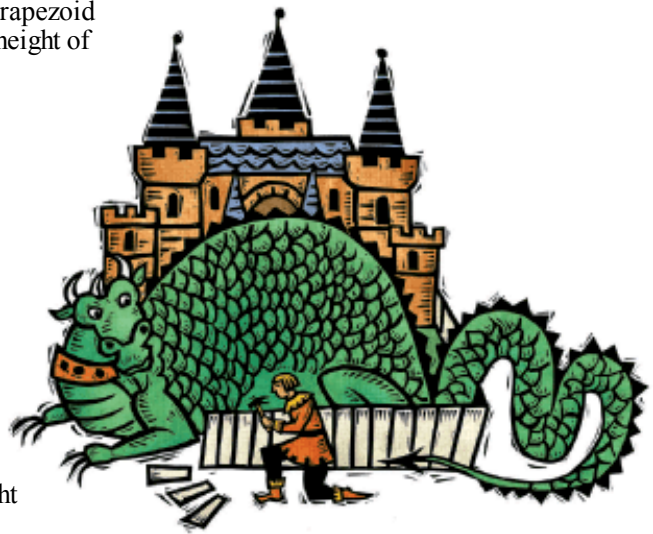


For Exercises 32 and 33, plot the vertices of each figure on graph paper, then find its area.

- 32. Parallelogram $ABCD$ with $A(0, 0)$, $B(14, 0)$, and $D(6, 8)$
- 33. Quadrilateral $FOUR$ with $F(0, 0)$, $O(4, -3)$, $U(9, 5)$, and $R(4, 15)$

- 34. The sum of the lengths of the two bases of a trapezoid is 22 cm, and its area is 66 cm^2 . What is the height of the trapezoid?
- 35. Find the area of a regular pentagon to the nearest tenth of a square centimeter if the apothem measures about 6.9 cm and each side measures 10 cm.

- 36. Find three noncongruent polygons, each with an area of 24 square units, on a 6-by-6 geoboard or a 6-by-6 square dot grid.
- 37. Lancelot wants to make a pen for his pet, Isosceles. What is the area of the largest rectangular pen that Lancelot can make with 100 meters of fencing if he uses a straight wall of the castle for one side of the pen?

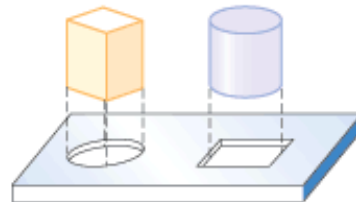


- 38. If you have a hundred feet of rope to arrange into the perimeter of either a square or a circle, which shape will give you the maximum area? Explain.

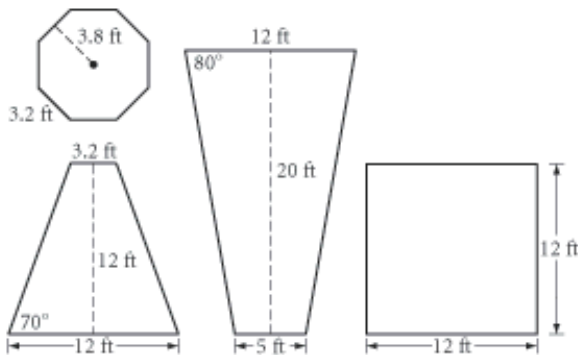


Ropes

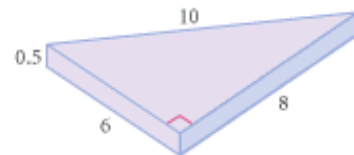
- 39. Which is a better fit (fills more of the hole): a round peg in a square hole or a square peg in a round hole? Explain.



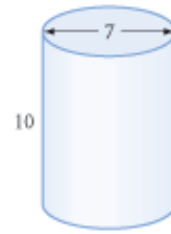
40. Al Dente's Pizzeria sells pizza by the slice, according to the sign. Which slice is the best deal (the most pizza per dollar)?
41. If you need 8 oz of dough to make a 12-inch diameter pizza, how much dough will you need to make a 16-inch pizza on a crust of the same thickness?
42. Which is the biggest slice of pie: one-fourth of a 6-inch diameter pie, one-sixth of an 8-inch diameter pie, or one-eighth of a 12-inch diameter pie? Which slice has the most crust along the curved edge?
43. The Hot-Air Balloon Club at Da Vinci High School has designed a balloon for the annual race. The panels are a regular octagon, eight squares, and sixteen isosceles trapezoids, and club members will sew them together to construct the balloon. They have built a scale model, as shown at right. The approximate dimensions of the four types of panels are below, shown in feet.



- a. What will be the perimeter, to the nearest foot, of the balloon at its widest? What will be the perimeter, to the nearest foot, of the opening at the bottom of the balloon?
- b. What is the total surface area of the balloon to the nearest square foot?
44. You are producing 10,000 of these metal wedges, and you must electroplate them with a thin layer of high-conducting silver. The measurements shown are in centimeters. Find the total cost for silver, if silver plating costs \$1 for each 200 square centimeters. Assume each quadrilateral is a rectangle.

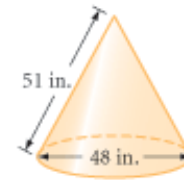


45. The measurements of a chemical storage container are shown in meters. Find the cost of painting the exterior of nine of these large cylindrical containers with sealant. The sealant costs \$32 per gallon. Each gallon covers 18 square meters. Do not paint the bottom faces.

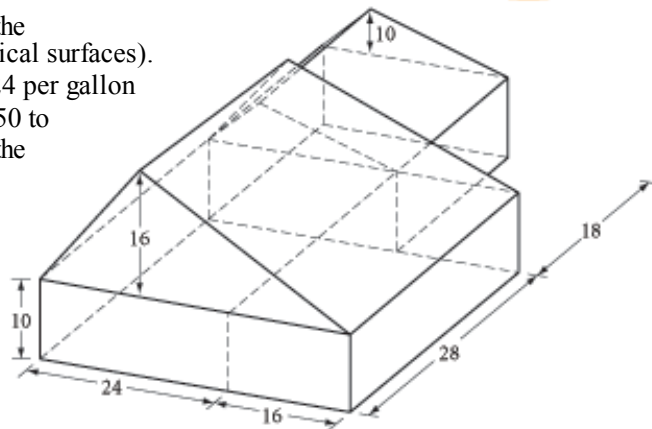


46. Hector is a very cost-conscious produce buyer. He usually buys asparagus in large bundles, each 44 cm in circumference. But today there are only small bundles that are 22 cm in circumference. Two 22 cm bundles are the same price as one 44 cm bundle. Is this a good deal or a bad deal? Why?

47. The measurements of a copper cone are shown in inches. Find the cost of spraying an oxidizer on 100 of these copper cones. The oxidizer costs \$26 per pint. Each pint covers approximately 5000 square inches. Spray only the lateral surface.

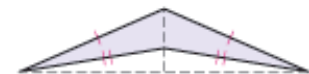


48. Tom and Betty are planning to paint the exterior walls of their cabin (all vertical surfaces). The paint they have selected costs \$24 per gallon and, according to the label, covers 150 to 300 square feet per gallon. Because the wood is very dry, they assume the paint will cover 150 square feet per gallon. How much will the project cost? (All measurements shown are in feet.)



TAKE ANOTHER LOOK

- Technology** Use geometry software to construct these shapes:
 - A triangle whose perimeter can vary, but whose area stays constant
 - A parallelogram whose perimeter can vary, but whose area stays constant
- Developing Proof** True or false? The area of a triangle is equal to half the perimeter of the triangle times the radius of the inscribed circle. Support your conclusion with a convincing argument.
- Developing Proof** Does the area formula for a kite hold for a dart? Support your conclusion with a convincing argument.
- Developing Proof** How can you use the Regular Polygon Area Conjecture to arrive at a formula for the area of a circle? Use a series of diagrams to help explain your reasoning.
- Use algebra to show that the total surface area of a prism with a regular polygon base is given by the formula $SA = P(h + a)$, where h is height of the prism, a is the apothem of the base, and P is the perimeter of the base.



6. Use algebra to show that the total surface area of a cylinder is given by the formula $SA = C(h + r)$, where h is the height of the cylinder, r is the radius of the base, and C is the circumference of the base.
7. **Developing Proof** Here is a different formula for the area of a trapezoid: $A = mh$, where m is the length of the midsegment and h is the height. Does the formula work? Use algebra or a diagram to explain why or why not. Does it work for a triangle?
8. In Lesson 8.3, Exercise 10, you were introduced to Hero's formula for finding the area of a triangle. With the help of Hero's formula and some algebra, derive a formula for the area of any quadrilateral given the length of each of the four sides and one diagonal.
9. **Developing Proof** True or false? If the diagonals of a quadrilateral are perpendicular, then the area of the quadrilateral is half the product of the diagonals. Support your conclusion with either a counterexample or a convincing argument.
10. **Developing Proof** In Lesson 8.6, Exercise 17, you were asked to find the shaded area between the circles inscribed within a square. Did you notice that the ratio of the sum of the areas of all the circles to the area of the square was always the same? Is this always true? Can you prove it?

Assessing What You've Learned



UPDATE YOUR PORTFOLIO Choose one of the more challenging problems you did in this chapter and add it to your portfolio. Write about why you chose it, what made it challenging, what strategies you used to solve it, and what you learned from it.



ORGANIZE YOUR NOTEBOOK Review your notebook to be sure it's complete and well organized. Be sure you have included all of this chapter's area formulas in your conjecture list. Write a one-page chapter summary.



WRITE IN YOUR JOURNAL Imagine yourself five or ten years from now, looking back on the influence this geometry class had on your life. How do you think you'll be using geometry? Will this course have influenced your academic or career goals?



PERFORMANCE ASSESSMENT While a classmate, a friend, a family member, or a teacher observes, demonstrate how to derive one or more of the area formulas. Explain each step, including how you arrive at the formula.



WRITE TEST ITEMS Work with group members to write test items for this chapter. Include simple exercises and complex application problems.



GIVE A PRESENTATION Create a poster, a model, or other visual aid, and give a presentation on how to derive one or more of the area formulas. Or present your findings from one of the Take Another Look activities.