CHAPTER 8: EXPONENTS AND POLYNOMIALS

Chapter Objectives
By the end of this chapter, the student should be able to ✓ Simplify exponential expressions with positive and/or negative exponents ✓ Multiply or divide expressions in scientific notation ✓ Evaluate polynomials for specific values ✓ Apply arithmetic operations to polynomials ✓ Apply special-product formulas to multiply polynomials ✓ Divide a polynomial by a monomial or by applying long division

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SECTION 8.1: EXPONENTS RULES AND PROPERTIES

A. PRODUCT RULE OF EXPONENTS

MEDIA LESSON
Product rule of exponents (Duration 2:57)

View the video lesson, take notes and complete the problems below.

$a^3 \cdot a^2 = (a\ a\ a)(a\ a) = a^5$

Product rule: $\boldsymbol{a^m \cdot a^n = a^{m+n}}$

______________________________!

Example 1: $(2x^3)(4x^2)(-3x)$
= ______________________________

Example 2: $(5a^3b^7)(2a^9b^2c^4)$
= ______________________________

❖ **Warning!** The rule can only apply when you have the same base.

YOU TRY

Simplify:

a) $5^3 5^{10}$

b) $x^1 x^3 x^2$

c) $(2x^3y^5z)(5xy^2z^3)$

B. QUOTIENT RULE OF EXPONENTS

MEDIA LESSON
Quotient rule of exponents (Duration 3:12)

View the video lesson, take notes and complete the problems below.

$$\frac{a^5}{a^3} = \frac{a \cdot a \cdot a \cdot a \cdot a}{a \cdot a \cdot a} = a^2$$

Quotient Rule: $\boldsymbol{\frac{a^m}{a^n} = a^{m-n}}$

____

Example 1: $\frac{a^7b^2}{a^3b}$
= ____________________________

Example 2: $\frac{8m^7n^4}{6m^5n}$
= ____________________________

YOU TRY

Simplify.

a) $\frac{7^{13}}{7^5}$

b) $\frac{5a^3b^5c^2}{2ab^3c}$

c) $\frac{3x^5}{x^3y}$

C. POWER RULE OF EXPONENTS

	MEDIA LESSON Power rule of exponents (Duration 5:00)

View the video lesson, take notes and complete the problems below.

$(ab)^3$=______________________________ = ________ Power of a product: $\boldsymbol{(ab)^m = a^m b^m}$
$\left(\frac{a}{b}\right)^3$ =____________________ =_____________ Power of a Quotient: $\boldsymbol{\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}}$, if b is not 0.
$(a^2)^3$ = _____________________ = ______ Power of a Power: $\boldsymbol{(a^m)^n = a^{m \cdot n}}$

Example 1: $(5a^4b)^3$

Example 2: $\left(\frac{5m^3}{9n^4}\right)^2$

❖ **Warning!** It is important to be careful to only use the power of a product rule with multiplication inside parenthesis. This property is not allowed for addition or subtraction, i.e.

$$(a+b)^m \neq a^m + b^m$$
$$(a-b)^m \neq a^m - b^m$$

YOU TRY

Simplify:

a) $\left(\frac{x^3}{y^2}\right)^5$

b) $\left(\frac{2^3}{5^2}\right)^7$

c) $(x^3yz^2)^4$

d) $(4x^2y^5)^3$

e) $\left(\frac{a^3b}{c^8d^5}\right)^2$

f) $\left(\frac{4xy}{8z}\right)^2$

D. ZERO AS AN EXPONENT

	MEDIA LESSON Zero as Exponent (Duration 3:51)

View the video lesson, take notes and complete the problems below.

$\frac{a^3}{a^3}$= ____

Zero Power Rule: $\boldsymbol{a^0 = 1}$

Example 1: $(5x^3yz^5)^0$

Example 2: $(3x^2y^0)(5x^0y^4)$

YOU TRY

Simplify the expressions completely

a) $(3x^2)^0$

b) $\frac{2m^0n^6}{3n^5}$

E. NEGATIVE EXPONENTS

	MEDIA LESSON Negative Exponents (Duration 4:44)

View the video lesson, take notes and complete the problems below.

$\frac{a^3}{a^5}$ = ____

= ____

Negative Exponent Rule: $\boldsymbol{a^{-m} = \frac{1}{a^m}}$ $\qquad \frac{1}{a^{-m}} = a^m \qquad \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m = \frac{b^m}{a^m}$

where *a* and *b* are not 0.

Example 1: $\frac{7x^{-5}}{3^{-1}yz^{-4}}$

Example 2: $\frac{2}{5a^{-4}}$

❖ **Warning!** It is important to note a negative exponent does not imply the expression is negative, only the reciprocal of the base. Hence, negative exponents imply reciprocals**.**

YOU TRY

a) $\frac{3}{5^{-1}x}$

b) $\frac{a^3b^2c}{2d^{-1}e^{-4}}$

F. PROPERTIES OF EXPONENTS

Putting all the rules together, we can simplify more complex expression containing exponents. Here we apply all the rules of exponents to simplify expressions.

Exponent Rules		
Product $a^m \cdot a^n = a^{m+n}$	Quotient $\frac{a^m}{a^n} = a^{m-n}$	Power of Power $(a^m)^n = a^{m \cdot n}$
Power of a Product $(ab)^m = a^m b^m$	Power of a Quotient $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	Zero Power $a^0 = 1$
Negative Power $a^{-m} = \frac{1}{a^m}$	Reciprocal of Negative Power $\frac{1}{a^{-m}} = a^m$	Negative Power of a Quotient $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m = \frac{b^m}{a^m}$

	MEDIA LESSON Properties of Exponents (Duration 5:00)

View the video lesson, take notes and complete the problems below.

Example 1: $(4x^5y^2z)^2(2x^4y^{-2}z^3)^4$	Example 2: $\frac{(2x^2y^3)^4(x^4y^{-6})^{-2}}{(x^{-6}y^4)^2}$

YOU TRY

Simplify and write your final answers in positive exponents.

a) $\frac{4x^{-5}y^{-3} \cdot 3x^3y^{-2}}{6x^{-5}y^3}$

b) $\frac{(3ab^3)^{-2} \cdot ab^{-3}}{2a^{-4}b^0}$

EXERCISE

Simplify. Be sure to follow the simplifying rules and write answers with positive exponents.

1) $4 \cdot 4^4 \cdot 4^4$

2) $4 \cdot 2^2$

3) $3m \cdot 4mn$

4) $2m^4n^2 \cdot 4nm^2$

5) $(3^3)^4$

6) $(4^4)^2$

7) $(2u^3v^2)^2$

8) $(2a^4)^4$

9) $\frac{4^5}{4^3}$

10) $x^2y^4 \cdot xy^2$

11) $(xy)^3$

12) $\frac{3^7}{3^3}$

13) $\frac{3^2}{3}$

14) $\frac{3nm^2}{3n}$

15) $\frac{4x^3y^4}{3xy^3}$

16) $\frac{x^2y^4}{4xy}$

17) $3x \cdot 4x^2$

18) $(u^2v^2 \cdot 2u^4)^3$

19) $(x^3y^4 \cdot 2x^2y^3)^2$

20) $2x(x^4y^4)^4$

21) $\frac{2x^7y^5}{3x^3y \cdot 4x^2y^3}$

22) $\left(\frac{(2x)^3}{x^3}\right)^2$

23) $\left(\frac{2y^{17}}{(2x^2y^4)^4}\right)^3$

24) $\left(\frac{2mn^4 \cdot 2m^4n^4}{mn^4}\right)^3$

25) $\frac{2xy^5 \cdot 2x^2y^3}{2xy^4 \cdot y^3}$

26) $\frac{2x^2y^2z^6 \cdot 2zx^2y^2}{(x^2z^3)^2}$

27) $\frac{2y}{(x^0y^2)^4}$

28) $\frac{2ba^7 \cdot 2b^4}{ba^2 \cdot 3a^3b^4}$

29) $\frac{2a^2b^2a^7}{(ba^4)^2}$

30) $\frac{yx^2 \cdot (y^4)^2}{2y^4}$

31) $\frac{2a^2b^2a^7}{(ba^4)^2}$

32) $\frac{n^3(n^4)^2}{2mn}$

33) $\frac{(2y^3x^2)^2}{2x^2y^4x^2}$

34) $\frac{2q^3p^3r^4 \cdot 2p^3}{(qrp^3)^2}$

35) $2x^4y^{-2} \cdot (2xy^3)^4$

36) $\frac{2x^{-3}y^2}{3x^{-3}y^3 \cdot 3x^0}$

37) $\frac{uv^{-1}}{2u^0v^4 \cdot 2uv}$

38) $\left(\frac{2a^2b^3}{a^{-1}}\right)^4$

39) $\frac{2xy^2 \cdot 4x^3y^{-4}}{4x^{-4}y^{-4} \cdot 4x}$

40) $\frac{2b^4c^{-2} \cdot (2b^3c^2)^{-4}}{a^{-2}b^4}$

SECTION 8.2 SCIENTIFIC NOTATION

A. INTRODUCTION TO SCIENTIFIC NOTATION

One application of exponent properties is scientific notation. Scientific notation is used to represent really large or really small numbers, like the numbers that are too large or small to display on the calculator.

For example, the distance light travels per year in miles is a very large number (5,879,000,000,000) and the mass of a single hydrogen atom in grams is a very small number (0.00000000000000000000000167). Basic operations, such as multiplication and division, with these numbers, would be quite cumbersome. However, the exponent properties allow for simpler calculation.

	MEDIA LESSON Introduction of scientific notation (Watch from 0:00 – 9:00)

View the video lesson, take notes and complete the problems below.

10^0 =___________

10^1 =____________

10^2 =_____________

10^3 = _____________

10^{100} = __________________________

Avogadro number: 602,200,000,000,000,000,000,000 = _______________________________

	MEDIA LESSON Definition of scientific notation (Duration 4:59)

View the video lesson, take notes and complete the problems below.

Standard Form (Standard Notation): ____

Scientific Notation: ____

b: ____

b positive: ____________________________________

b negative: ___________________________________

Example: Convert to Scientific Notation

a) 48,100,000,000 = __________________ b) 0.0000235 = _________________

Definition
Scientific notation is a notation for representing extremely large or small numbers in form of

$$a\ x\ 10^b$$

where 1 < a < 10 and *b* is number of decimal places from the right or left we moved to obtain ***a***.

A few notes regarding scientific notation:

- ***b*** is the way we convert between scientific and standard notation.
- ***b*** represents the number of times we multiply by 10. (Recall, multiplying by 10 moves the decimal point of a number one place value.)
- We decide which direction to move the decimal (left or right) by remembering that in standard notation, positive exponents are numbers greater than ten and negative exponents are numbers less than one (but larger than zero).

Case 1. If we move the decimal to the left with a number in standard notation, then ***b*** will be positive.
Case 2. If we move the decimal to the right with a number in standard notation, then ***b*** will be negative.

B. CONVERT NUMBERS TO SCIENTIFIC NOTATION

MEDIA LESSON
Convert standard notation to scientific notation (Duration 1:40)

View the video lesson, take notes and complete the problems below.

Example: Convert to scientific notation

8,150,000 =

0.00000245 =

YOU TRY

Convert the following number to scientific notation

a) 14,200

b) 0.0042

c) How long is a light-year?
The light-year is a measure of distance, not time. It is the total distance of a beam of light that travels in one year is almost 6 trillion (6,000,000,000,000) miles in a straight line. Express a light year in scientific notation. (Source: NASA Glenn Educational Programs Office https://www.grc.nasa.gov/www/k-12/aerores.htm)

C. CONVERT NUMBERS FROM SCIENTIFIC NOTATION TO STANDARD NOTATION

To convert a number from scientific notation of the form

$$\boldsymbol{a\ x\ 10^b}$$

to standard notation, we can follow these rules of thumb.

- If $\boldsymbol{b}$ is positive, this means the original number was greater than 10, we move the decimal to the right $\boldsymbol{b}$ times.
- If $\boldsymbol{b}$ is negative, this means the original number was less than 1 (but greater than zero), we move the decimal to the left $\boldsymbol{b}$ times.

MEDIA LESSON
Convert scientific notation to standard notation (Duration 2:22)

View the video lesson, take notes and complete the problems below.

Example: Rewrite in standard notation (decimal notation)

a) 7.85×10^6 b) 1.6×10^{-4}

YOU TRY

Covert the following scientific notation to standard notation

a) 3.21×10^5 b) 7.4×10^{-3}

D. MULTIPLY AND DIVIDE NUMBERS IN SCIENTIFIC NOTATION

Converting numbers between standard notation and scientific notation is important in understanding scientific notation and its purpose. We multiply and divide numbers in scientific notation using the exponent properties. If the immediate result is not written in scientific notation, we will complete an additional step in writing the answer in scientific notation.

Steps for multiplying and dividing numbers in scientific notation

Step 1. Rewrite the factors as multiplying or dividing a-values and then multiplying or dividing $\mathbf{10^b}$ values.

Step 2. Multiply or divide the $\boldsymbol{a}$ values and apply the product or quotient rule of exponents to add or subtract the exponents, $\boldsymbol{b}$, on the base 10s, respectively.

Step 3. Be sure the result is in scientific notation. If not, then rewrite in scientific notation.

MEDIA LESSON
Multiply and divide scientific notation (Duration 2:47)

View the video lesson, take notes and complete the problems below

- Multiply/ Divide the ____
- Use ____on the 10s

Example:

a) $(3.4 \times 10^5)(2.7 \times 10^{-2})$ b) $\dfrac{5.32\times10^4}{1.9\times10^{-3}}$

MEDIA LESSON
Multiply scientific notations with simplifying final answer step (Duration 3:47)

View the video lesson, take notes and complete the problems below.

Example:

a) $(1.2 \times 10^4)(5.3 \times 10^3)$ b) $(9 \times 10^1)(7 \times 10^9)$

	MEDIA LESSON Divide scientific notations with simplifying final answer step (Duration 3:44)

View the video lesson, take notes and complete the problems below.

a) $\frac{7\times10^{12}}{2\times10^{7}}$

b) $\frac{2.4\times10^{7}}{4.8\times10^{2}}$

YOU TRY

Multiply or divide.

a) $(2.1\ x\ 10^{-7})(3.7\ x\ 10^{5})$

b) $\frac{4.96\ x\ 10^{4}}{3.1\ x\ 10^{-3}}$

c) $(4.7\ x\ 10^{-3})(6.1\ x\ 10^{9})$

d) $(2\times10^{6})(8.8\times10^{5})$

e) $\frac{8.4\times10^{5}}{7\times10^{2}}$

f) $\frac{2.014\ x\ 10^{-3}}{3.8\ x\ 10^{-7}}$

E. SCIENTIFIC NOTATION APPLICATIONS

	MEDIA LESSON Scientific notation application example 1 (Duration 2:36)

View the video lesson, take notes and complete the problems below.

Example 1: There were approximately 50,000 finishers of the 2015 New York City Marathon. Each finisher ran a distance of 26.2 miles. If you add together the total number miles ran by all the runners, how many times around the earth would the marathon runners have run? Assume the circumference of the earth to be approximately $2.5\ \times\ 10^{4}$ miles.

Total distance = ____

____

	MEDIA LESSON Scientific notation application example 2 (Duration 3:24)

View the video lesson, take notes and complete the problems below.

Example 2: If a computer can conduct 400 trillion operations per second, how long would it take the computer to perform 500 million operations?

400 trillion = ____

500 million = ____

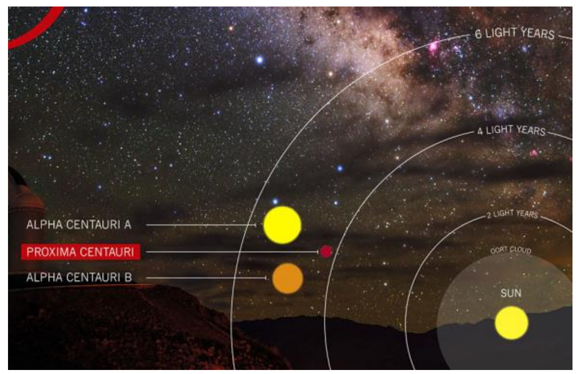
Number of Operations: ____

Rate of Operations: ____

____

YOU TRY

a) It takes approximately 3.7×10^4 hour for the light on Proxima Centauri, the next closet star to our sun, to reach us from there. The speed of light is 6.71×10^8 miles per hour. What is the distance from there to earth? Given $distance = rate \times time$. Express your answer in scientific notation



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a) If the North Pole and the South Pole ice were to melt, the north polar ice would make essentially no contribution since it is float ice. However, the south polar ice would make a considerable contribution since it overlays the Antarctic land mass and is not float ice. If Antarctic ice melted, it would become approximately 1.5×10^9 gallons of water. If it takes roughly, $6\ x\ 10^6$ gallons of water to fill 1 foot of the earth, estimate how many feet the earth's oceans would rise? Express your answer in the standard form. (Source: NASA Glenn Educational Programs Office https://www.grc.nasa.gov/www/k-12/aerores.htm)

EXERCISES

Write each number in scientific notation.

1) 885 **2)** 0.081 **3)** 0.000039

4) 0.000744 **5)** 1.09 **6)** 15,000

Write each number in standard notation.

7) 8.7×10^5 **8)** 9×10^{-4} **9)** 2×10^0

10) 2.56×10^2 **11)** 5×10^4 **12)** 6×10^{-5}

Simplify. Write each answer in scientific notation.

13) $(7 \times 10^1)(2 \times 10^3)$ **14)** $(5.26 \times 10^5)(3.16 \times 10^2)$ **15)** $(2.6 \times 10^{-2})(6 \times 10^{-2})$

16) $(3.6 \times 10^0)(6.1 \times 10^{-3})$ **17)** $(6.66 \times 10^{-4})(4.23 \times 10^1)$ **18)** $(3.15 \times 10^3)(8.8 \times 10^{-5})$

19) $\frac{4.81 \times 10^6}{9.62 \times 10^2}$ **20)** $\frac{5.33 \times 10^6}{2 \times 10^3}$ **21)** $\frac{4.08 \times 10^{-6}}{5.1 \times 10^{-4}}$

22) $\frac{9 \times 10^4}{3 \times 10^{-2}}$ **23)** $\frac{3.22 \times 10^{-3}}{7 \times 10^{-6}}$ **24)** $\frac{1.3 \times 10^{-6}}{6.5 \times 10^0}$

25) $\frac{5.8 \times 10^3}{5.8 \times 10^{-3}}$ **26)** $\frac{5 \times 10^6}{2.5 \times 10^2}$ **27)** $\frac{8.4 \times 10^5}{7 \times 10^{-2}}$

Scientific Notation Applications

(Source: NASA Glenn Educational Programs Office https://www.grc.nasa.gov/www/k-12/aerores.htm)

28) The mass of the sun is 1.98×10^{33} grams. If a single proton has a mass of 1.6×10^{-24} grams, how many protons are in the sun?

29) Pluto is located at a distance of 5.9×10^{14} centimeters from Earth. At the speed of light, $2.99 \times 10^{10}\ cm/sec$, approximately how many hours does it take a light signal (or radio message) to travel to Pluto and return? Write your answer standard form.

30) The planet Osiris was discovered by astronomers in 1999 and is at a distance of 150 light-years (1 light-year = 9.2×10^{12} kilometers).

a) How many kilometers is Osiris from earth? Express your answer in scientific notation.

b) If an interstellar probe were sent to investigate this world up close, traveling at a maximum speed of 700 km/sec or 7×10^2 km/sec, how many seconds would it take to reach Osiris?

c) There is about 3.15×10^6 seconds in a year. How many years would it take to reach Osiris?

SECTION 8.3: POLYNOMIALS

A. INTRODUCTION TO POLYNOMIALS

	MEDIA LESSON Algebraic Expression Vocabulary (Duration 5:52)

View the video lesson, take notes and complete the problems below.

Definitions
Terms: Parts of an algebraic expression separated by addition or subtraction (+ or −) symbols. **Constant Term:** A number with no variable factors. A term whose value never changes. **Factors:** Numbers or variable that are multiplied together **Coefficient:** The number that multiplies the variable.

Example 1: Consider the algebraic expression $4x^5 + 3x^4 - 22x^2 - x + 17$

a. List the terms: ____

b. Identify the constant term. ____

Example 2: Complete the table below

	$-4m$	$-x$	$\frac{1}{2}bh$	$\frac{2r}{5}$
List of Factors				
Identify the Coefficient				

Example 3: Consider the algebraic expression $5y^4 - 8y^3 + y^2 - \frac{y}{4} - 7$

a. How many terms are there? ________________________

b. Identify the constant term. ________________________

c. What is the coefficient of the first term? ________________________

d. What is the coefficient of the second term ________________________

e. What is the coefficient of the third term? ________________________

f. List the factors of the fourth term. ________________________

YOU TRY

Consider the algebraic expression $3x^5 + 4x^4 - 2x + 8$

a) How many terms are there? ________________________

b) Identify the constant term. ________________________

c) What is the coefficient of the first term? ________________________

d) What is the coefficient of the second term ________________________

e) What is the coefficient of the third term? ________________________

f) List the factors of the third term. ________________________

	MEDIA LESSON Introduction to polynomials (Duration 7:12)

View the video lesson, take notes and complete the problems below.

Minute 2:31

Definitions

$$12x^5 - 2x^2 + x - 7$$

Polynomial: An algebraic expression composed of the sum of terms containing a single variable raised to a non-negative integer exponent.

Monomial: A polynomial consisting of one term, example: __________________

Binomial: A polynomial consisting of two terms, example: __________________

Trinomial: A polynomial consisting of three terms, example: __________________

Leading Term: The term that contains the highest power of the variable in a polynomial,

example: __________________

Leading Coefficient: The coefficient of the leading term, example: __________________

Constant Term: A number with no variable factors. A term whose value never changes.

Example: __________________

Degree: The highest exponent in a polynomial, example: __________________

Example 1: Complete the table below.

Polynomial	Name	Leading Coefficient	Constant Term	Degree
$24a^6 + a^2 + 5$				
$2m^3 + m^2 - 2m - 8$				
$5x^2 + x^3 - 7$				
$-2x + 4$				
$4x^3$				

YOU TRY

Complete the table below.

Polynomial	Name	Leading Coefficient	Constant Term	Degree
$n^2 - 2n + 8$				
$7y^2$				
$6x - 7$				

MEDIA LESSON
Introduction to polynomials 2 (Duration 2:58)

View the video lesson, take notes and complete the problems below.

Given: $9y + 7y^3 - 5 - 4y^2$

1st term: ________________	Degree:__________________	Coefficient:_______________
2nd term: ________________	Degree:__________________	Coefficient:_______________
3rd term: ________________	Degree:__________________	Coefficient:_______________
4th term: ________________	Degree:__________________	Coefficient:_______________

Leading coefficient: _________________
Degree of leading term: ______________
Degree of polynomial: ________________
Write the polynomial in descending order: ____
(Or write the polynomial in the standard form)

Standard form of a polynomial

The standard form of a polynomial is where the polynomial is written with **descending exponents**.
For example: Rewrite the polynomial in standard form and identify the coefficients, variable terms, and degree of the polynomial

$$-12x^2 + x^3 - x + 2$$

The standard form of the above polynomial is $x^3 - 12x^2 - x + 2$.

The coefficients are 1; −12; −1, and 2; the variable terms are $x^3, -12x^2, -x$. The degree of the polynomial is 3 because that is the highest degree of all terms.

YOU TRY

Write the following polynomials in the descending order or in standard form:

a) $3x - 9x^3 + 2x^6 + 7x^2 - 3 + x^4$

b) $5m^2 - 5m^4 + 3 - 4m^3 - 2m^7$

B. EVALUATING POLYNOMIAL EXPRESSIONS

MEDIA LESSON
Evaluating algebraic expressions (Duration 7:48)

View the video lesson, take notes and complete the problems below.

To evaluate an algebraic or variable expression, _________________ the value of the variables into the expression. Then evaluate using the order of operations.

Example 1: If we are given $5x - 12$ and $x = 17$ we can evaluate.

$5x - 12$

= 5 (____) – 12

= _____________________

Example 2: Let $x = -3, y = 7, z = -2$

Evaluate $x - 3y + 7$

Evaluate $2x^2 + 5y - z^3$

Example 3: Let $= 2$. Evaluate $\frac{9}{y} - 8y + 2$.

Example 4: Let $x = 3, y = -5$. Evaluate $4x - 3y^2$

Example 5: Let $= -2$.
Evaluate $3x^2 - x^2 + 2x + 9$.

Example 6: Let $x = 2, y = -3$. Evaluate $\frac{x^2y^2}{x^2 - 2y^3}$

YOU TRY

a) Evaluate $2x^2 - 4x + 6$ when $x = -4$.

b) Evaluate $-x^2 + 2x + 6$ when $= 3$.

C. ADD AND SUBTRACT POLYNOMIALS

Combining like terms review

	MEDIA LESSON Combine like terms 1 (Duration 4:36)

View the video lesson, take notes and complete the problems below.

Definition
Like terms: Two or more terms are like terms if they have the same variable or variables with the same exponents.

Which of these terms are like terms? $-2x^3,\ 2x,\ 2y,\ 7x^3,\ 4y,\ 6x^2,\ y^2$

Like terms: ____

Like terms: ____

To combine like terms, we ____. The variable factors ___________________.

Example: Simplify each polynomials, if possible.

a) $4x^3 - 7x^3$

b) $2y^2 + 4y - y^2 + 2 - 9y - 5 + 2y$

MEDIA LESSON
Combine like terms 2 (Duration 2:15)

View the video lesson, take notes and complete the problems below

Combine like terms.

a) $x^2y + 3xy^2 + 4x^2y$

b) $-7m - 4 + 2m + 9$

YOU TRY

Combine like terms.

a) $5x^2 + 2x - 5x^2 - 3x + 1$

b) $3xy^2 - 2x^2 + 6 + 3y - 5xy^2 - 3$

c) $3x^2yz + 9x^2 - 5xy^2z - 3y^2 + 5x^2$

d) $3x^2 - 3x + 5y^2 - ax^2 + 7 - x - 10y^2$

Add and subtract polynomials

MEDIA LESSON
Add and subtract polynomials (Duration 3:53)

View the video lesson, take notes and complete the problems below.

To add polynomials: ____

To subtract polynomials: ____

a) $(5x^2 - 7x + 9) + (2x^2 + 5x - 14)$

b) $(3x^3 - 4x + 7) - (8x^3 + 9x - 2)$

MEDIA LESSON
Add and subtract polynomials (Duration 5:04)

View the video lesson, take notes and complete the problems below

c) $(2x^5 - 6x^3 - 12x^2 - 4) + (-11x^5 + 8x + 2x^2 + 6)$

d) $(-9y^3 - 6y^2 - 11x + 2) - (-9y^4 - 8y^3 + 4x^2 + 2x)$

YOU TRY

Perform the operation below.

a) $(4x^3 - 2x + 8) + (3x^3 - 9x^2 - 11)$

b) $(5x^2 - 2x + 7) - (3x^2 + 6x - 4)$

c) $(2x^2 - 4x + 3) + (5x^2 - 6x + 1) - (x^2 - 9x + 8)$

D. MULTIPLY POLYNOMIAL EXPRESSIONS

1. Distributive property review

	MEDIA LESSON Distribute property review (Duration 6:08)

View the video lesson, take notes and complete the problems below.

Distributive Property $a(b + c) = ab + ac$

$a = 2$
$b = 3$
$c = 4$

Example: Use the distributive property to expand each of the following expressions

a) $5(2x + 4)$

b) $-3(x^2 - 2x + 7)$

c) $-(5x^4 - 8)$

d) $\frac{2}{5}\left(\frac{x}{4} - \frac{1}{3}\right)$

YOU TRY

Use the distributive property to expand each of the following expressions.

a) $4(-5x^2 + 9x - 3)$

b) $-7(-2m^2 + m - 2)$

2. Multiply a polynomial by a monomial

	MEDIA LESSON Multiply a polynomial by a monomial (Duration 2:46)

View the video lesson, take notes and complete the problems below.

To multiply a monomial by a polynomial: ____

Example 1: $5x^2(6x^2 - 2x + 5)$

Example 2: $-3x^4(6x^3 + 2x - 7)$

YOU TRY

Multiply.

a) $4x^3(5x^2 - 2x + 5)$

b) $2a^3b(3ab^2 - 4a)$

3. *Multiplying with binomials*

	MEDIA LESSON Multiply binomials (Duration 4:27)

View the video lesson, take notes and complete the problems below.

To multiply a binomial by a binomial: ____

____

This process is often called ____________, which stands for ____________________________________

Example:

a) $(4x - 2)(5x + 1)$

b) $(3x - 7)(2x - 8)$

YOU TRY

Multiply.

a) $(3x + 5)(x + 13)$

b) $(4x + 7y)(3x - 2y)$

4. *Multiply with trinomials*

	MEDIA LESSON Multiply with trinomials (Duration 5:00)

View the video lesson, take notes and complete the problems below.

Multiplying trinomials is just like _______________, we just have _____________________________.

Example:

a) $(2x - 4)(3x^2 - 5x + 1)$

b) $(2x^2 - 6x + 1)(4x^2 - 2x - 6)$

YOU TRY

Multiply.

a) $(2x - 5)(4x^2 - 7x + 3)$

b) $(5x^2 + x - 10)(3x^2 - 10x - 6)$

E. SPECIAL PRODUCTS

There are a few shortcuts that we can take when multiplying polynomials. If we can recognize when to use them, we should so that we can obtain the results even quicker. In future chapters, we will need to be efficient in these techniques since multiplying polynomials will only be one of the steps in the problem. These two formulas are important to commit to memory. The more familiar we are with them, the next two chapters will be so much easier.

1. Difference of two squares

	MEDIA LESSON Difference of two squares (Duration 2:33)

View the video lesson, take notes and complete the problems below.

Sum and difference

$(a + b)(a - b)$ = ________________________________

= ________________________________

Sum and difference shortcut:

$(a + b)(a - b)$ = ______________________

Example:

a) $(x + 5)(x - 5)$

b) $(6x - 2)(6x + 2)$

YOU TRY

Simplify:

a) $(3x + 7)(3x - 7)$

b) $(8 - x^2)(8 + x^2)$

2. Perfect square trinomials

Another shortcut used to multiply binomials is called perfect square trinomials. These are easy to recognize because this product is the square of a binomial. Let's take a look at an example.

	MEDIA LESSON Perfect Square (Duration 3:40)

View the video lesson, take notes and complete the problems below.

Perfect square

$(a+b)^2$ = ____

Perfect square shortcut:

$(a+b)^2$ = ______________________________

Example:

a) $(x-4)^2$

b) $(2x+7)^2$

YOU TRY

Simplify:

a) $(x-5)^2$

b) $(2x+9)^2$

c) $(3x-7y)^2$

d) $(6-2m)^2$

F. POLYNOMIAL DIVISION

Dividing polynomials is a process very similar to long division of whole numbers. Before we look at long division with polynomials, we will first master dividing a polynomial by a monomial.

1. Polynomial division with monomials

	MEDIA LESSON Dividing polynomials by monomials - Separated fractions method (Duration 8:14)

View the video lesson, take notes and complete the problems below.

We divide a polynomial by a monomial by rewriting the expression as separated fractions rather than one fraction. We use the fact: $\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$

Example:

a) $\frac{-6w^8}{30w^3}$

b) $\frac{3x-6}{2}$

c) $\frac{6x^3+2x^2-4}{4x}$

d) $\frac{20a^2+35a-4}{-5a^2}$

YOU TRY

Simplify.

a) $\frac{9x^5+6x^4-18x^3-24x^2}{3x^2}$

b) $\frac{8x^3+4x^2-2x+6}{4x^2}$

	MEDIA LESSON Long division review (Duration 3:55)

View the video lesson, take notes and complete the problems below.

Long division review

$5\overline{)2632}$

Long division steps:

1. ____
2. ____
3. ____
4. ____
5. ____

This method may seem elementary, but it isn't the arithmetic we want to review, it is the method. We use the same method as we did in arithmetic, but now with polynomials.

	MEDIA LESSON Dividing polynomials by monomials – Long division method (Duration 5:00)

View the video lesson, take notes and complete the problems below.

Example:

a) $\frac{3x^5+18x-9x^3}{3x^2}$

b) $\frac{15a^6-25a^5+5a^4}{5a^4}$

YOU TRY

Divide using the long division method.

a) $\frac{8x^6+20x^4+4x^3}{4x^3}$

b) $\frac{n^4-n^3+n^2}{n}$

c) $\frac{12x^4-24x^3+3x^2}{6x}$

2. Polynomial division with polynomials

	MEDIA LESSON Divide a polynomial by a polynomial (Duration 5:00)

View the video lesson, take notes and complete the problems below.

Polynomial division with polynomials

On division step, only focus on the ________________________

Example 1: Divide $\frac{x^3-2x^2-15x+30}{x+4}$.

Example 2: Divide $\frac{4x^3-6x+12x+8}{2x-1}$.

YOU TRY

a) $\frac{x^2+8x+12}{x+1} =$

b) $\frac{3x^3-5x^2-32x+7}{x-4} =$

c) $\frac{6x^3-8x^2+10x+103}{2x+4}=$

	MEDIA LESSON Divide a polynomial by a polynomial - rewriting the remainder as an expression (Duration 5:10)

View the video lesson, take notes and complete the problems below.

Example: Divide $\frac{x^3+8x^2-17x-15}{x+3}$.

YOU TRY

Divide the polynomials and write the remainder as an expression.

a) $\frac{x^2-5x+7}{x-2}=$

b) $\frac{x^3-4x^2-6x+4}{x-1}=$

3. Polynomial division with missing terms

Sometimes when dividing with polynomials, there may be a missing term in the dividend. We do not ignore the term, we just write in 0 as the coefficient.

	MEDIA LESSON Polynomial division with missing terms (Duration 5:00)

View the video lesson, take notes and complete the problems below.

Divide polynomials – Missing terms
The exponents must ____________________________________.

If one is missing, we will add ____.

Example 1: $\frac{3x^3-50x+4}{x-4}$

Example 2: $\frac{2x^3+4x^2+9}{x+3}$

YOU TRY

a) $\frac{2x^3-4x+42}{x+3} =$

b) $\frac{3x^3-3x^2+4}{x-3} =$

EXERCISES

Evaluate the expression for the given value. Show your work.

1) $-a^3 - a^2 + 6a - 21$ when $a = -4$

2) $n^2 - 3n - 11$ when $n = -6$

3) $n^3 - 7n^2 + 15n - 20$ when $n = 2$

4) $n^3 - 9n^2 + 23n - 21$ when $n = 5$

5) $-5n^4 - 11n^3 - 9n^2 - n - 5$ when $n = 2$

6) $x^4 - 5x^3 - x + 13$ when $x = 1$

7) $x^2 + 9x + 23$ when $x = -3$

8) $-x^3 + x^2 - x + 11$ when $x = 6$

9) $-x^4 - 6x^3 + x^2 - 24$ when $x = -1$

10) $m^4 + m^3 + 2m^2 + 13m + 5$ when $m = 3$

Simplify. Write the answer in standard form. Show your work.

11) $(5p - 5p^4) - (8p - 8p^4)$

12) $(3n^2 - n^3) - (2n^3 - 7n^2)$

13) $(8n + n^4) - (3n - 4n^4)$

14) $(1 + 5p^3) - (1 - 8p^3)$

15) $(5n^4 + 6n^3) + (8 - 3n^3 - 5n^4)$

16) $(3 + b^4) + (7 + 2b + b^4)$

17) $(8x^3 + 1) - (5x^4 - 6x^3 + 2)$

18) $(2a + 2a^4) - (3a^2 - 6a + 3)$

19) $(4p^2 - 3 - 2p) - (3p^2 - 6p + 3)$

20) $(4b^3 + 7b^2 - 3) + (8 + 5b^2 + b^3)$

21) $(3 + 2n^2 + 4n^4) + (n^3 - 7n^2 - 4n^4)$

22) $(n - 5n^4 + 7) + (n^2 - 7n^4 - n)$

23) $(8r^4 - 5r^3 + 5r^2) + (2r^2 + 2r^3 - 7r^4 + 1)$

24) $(6x - 5x^4 - 4x^2) - (2x - 7x^2 - 4x^4 - 8) - (8 - 6x^2 - 4x^4)$

Multiply and simplify. Show your work.

25) $6(p - 7)$

26) $5m^4(4m + 4)$

27) $(8b + 3)(7b - 5)$

28) $(3v - 4)(5v - 2)$

29) $(5x + y)(6x - 4y)$

30) $(7x + 5y)(8x + 3y)$

31) $(6n - 4)(2n^2 - 2n + 5)$

32) $(8n^2 + 4n + 6)(6n^2 - 5n + 6)$

33) $3(3x - 4)(2x + 1)$

34) $7(x - 5)(x - 2)$

35) $(6x + 3)(6x^2 - 7x + 4)$

36) $(5k^2 + 3k + 3)(3k^2 + 3k + 6)$

37) $(2a^2 + 6a + 3)(7a^2 - 6a + 1)$

38) $3n^2(6n + 7)$

39) $(7u^2 + 2u - 3)(u^2 + 4)$

40) $3x^2(2x + 3)(6x + 9)$

Find each product by applying the special products formulas. Show your work.

41) $(x + 8)(x - 8)$

42) $(1 + 3p)(1 - 3p)$

43) $(1 - 7n)(1 + 7n)$

44) $(5n - 8)(5n + 8)$

45) $(4x + 8)(4x - 8)$

46) $(4y - x)(4y + x)$

47) $(4m - 2n)(4m + 2n)$

48) $(6x - 2y)(6x + 2y)$

49) $(a + 5)^2$

50) $(x - 8)^2$

51) $(p + 7)^2$

52) $(7 - 5n)^2$

53) $(5m-3)^2$

54) $(5x+7y)^2$

55) $(2x+2y)^2$

56) $(5+2r)^2$

57) $(2+5x)^2$

58) $(4v-7)(4v+7)$

59) $(n-5)(n+5)$

60) $(4k+2)^2$

61) $(a-4)(a+4)$

62) $(x-3)(x+3)$

63) $(8m+5)(8m-5)$

64) $(2r+3)(2r-3)$

65) $(b-7)(b+7)$

66) $(7a+7b)(7a-7b)$

67) $(3y-3x)(3y+3x)$

68) $(1+5n)^2$

69) $(v+4)^2$

70) $(1-6n)^2$

71) $(7k-7)^2$

72) $(4x-5)^2$

73) $(3a+3b)^2$

74) $(4m-n)^2$

75) $(8x+5y)^2$

76) $(m-7)^2$

77) $(8n+7)(8n-7)$

78) $(b+4)(b-4)$

79) $(7x+7)^2$

Divide: Show your work.

80) $\frac{20x^4+x^3+2x^2}{4x^3}$

81) $\frac{5n^4+n^3+40n^2}{5n}$

82) $\frac{12x^4+24x^3+3x^2}{6x}$

83) $\frac{5x^5+18x^3+4x+9}{9x}$

84) $\frac{3k^4+4k^2+2}{8k^2}$

85) $\frac{10n^4+5n^3+2n^2}{n^2}$

Divide and write your remainder as an expression. Show your work.

86) $\frac{v^2-2v-89}{v-10}$

87) $\frac{x^2-2x-71}{x+8}$

88) $\frac{n^2+13n+32}{n+5}$

89) $\frac{10x^2-19x+9}{10x-9}$

90) $\frac{a^2-4a-38}{a-8}$

91) $\frac{45p^2-56p+19}{9p-4}$

92) $\frac{27b^2+87b+35}{3b+8}$

93) $\frac{4r^2-r-1}{4r+3}$

94) $\frac{n^2-4}{n-2}$

95) $\frac{x^3-26x-41}{x+4}$

96) $\frac{4x^2-4x+2}{2x-5}$

97) $\frac{a^3+5a^2-4a-5}{a+7}$

98) $\frac{p^3+5p^2+3p-5}{p+1}$

99) $\frac{x^3-46x+22}{x+7}$

100) $\frac{2x^3+12x^2-20}{2x+6}$

101) $\frac{4v^3+4v+19}{4v+12}$

102) $\frac{r^3-r^2-16r+8}{r-4}$

103) $\frac{12n^3+12n^2-15n-4}{2n+3}$

CHAPTER REVIEW

KEY TERMS AND CONCEPTS	
Look for the following terms and concepts as you work through the workbook. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the text or in the media lesson.	
Product rule of exponents	
Quotient rule of exponents	
Power rule of a product	
Power rule of a quotient	
Power rule of a Power	
Zero power rule	
Negative exponent rule	
Reciprocal of negative rule	
Negative power of a quotient rule	
Scientific notation	
Standard notation (Decimal notation)	
Polynomial	
Monomial	

Binomial	
Trinomial	
Leading Term	
Leading Coefficient	
Degree of a Polynomial	
Constant Term	

Simplify. Be sure to follow the simplifying rules and write answers with positive exponents.

1) $4 \cdot 4^4 \cdot 4^2$

2) $3 \cdot 3^2 3^2$

3) $(4^3)^4$

4) $(3^2)^2$

5) $(xy)^3$

6) $\frac{2x^4y^5 \cdot 2z^{10}x^2y^7}{(xy^2z^2)^4}$

7) $(2x^2y^2)^4x^{-4}$

8) $\frac{2y^2}{(2x^0y^4)^{-4}}$

9) $\frac{(a^4)^4}{2b}$

10) $\left(\frac{2y^{-4}}{x^2}\right)^{-2}$

11) $\frac{2x^{-2}y^0 \cdot 2xy^4}{(xy^0)^{-1}}$

12) $\left(\frac{(2x^{-3}y^0z^{-1})^3 \cdot x^{-3}y^2}{2x^3}\right)^{-2}$

Write each number in scientific notation.

13) 3458

14) 00.00067

Write each number in standard notation.

15) 9.123×10^{-3}

16) 9×10^4

Simplify. Write each answer in scientific notation.

17) $\frac{3.22 \times 10^{-3}}{7 \times 10^{-6}}$

18) $(3.15 \times 10^3)(8 \times 10^{-1})$

19) $\frac{3.2 \times 10^{-3}}{5.02 \times 10^0}$

20) $(2 \times 10^4)(6 \times 10^1)$

Evaluate the expression for the given value. Show your work.

21) $x^2 - 6x^4 + 2x^2 - 2$ when $x = -2$

22) $-y^2 - y - 1$ when $y = -2$

Multiply and simplify. Show your work.

23) $(5x + 4)(5x - 4)$

24) $(4x - 2)(6x^2 - 2x + 1)$

25) $(x + 4x^2 + 3)(x - x^2 + 3)$

26) $(2x + 4)^2$

Divide and write your remainder as an expression. Show your work.

27) $\frac{2x^6+4x^4+6x^2}{2x}$

28) $\frac{9x^3+45x^2+27x-5}{9x+9}$

29) $\frac{4x^2-33x+28}{4x-5}$

30) $\frac{24b^3-38b^2+29b-60}{4b-7}$