

# CHAPTER 8: Hypothesis Testing

In this chapter we will learn ....

- To use an inferential method called a hypothesis test
- To analyze evidence that data provide
- To make decisions based on data

## Major Methods for Making Statistical Inferences about a Population

- **The traditional Method**
- **The p-value Method**
- **Confidence Interval**

## Section 8-1: Steps in Hypothesis Testing – Traditional Method

The main goal in many research studies is to check whether the data collected support certain statements or predictions.

**Statistical Hypothesis – a conjecture about a population parameter. This conjecture may or may not be true.**

**Example:** The mean income for a resident of Denver is equal to the mean income for a resident of Seattle.

- Population parameter is mean income
- One population consists of residents of Denver while the other consists of residents of Seattle.

There are two types of statistical hypotheses:

**Null Hypothesis ( $H_0$ )** – a statistical hypothesis that states that there is **no** difference between a parameter and a specific value, or that there is **no** difference between two parameters.

**Alternative Hypothesis ( $H_1$ )** – a statistical hypothesis that states the existence of a **difference** between a parameter and a specific value, or states that there is a **difference** between two parameters.

*Can you formulate a null and alternative hypothesis for the income example?*

We tend to want to reject the null hypothesis so we assume it is true and look for enough evidence to conclude it is incorrect.

We tend to want to accept the alternative hypothesis. If the null hypothesis is rejected then we must accept that the alternative hypothesis is true.

Note:  **$H_0$  will ALWAYS have an equal sign** (and possibly a less than or greater than symbol, depending on the alternative hypothesis). The alternative hypothesis has a range of values that are alternatives to the one in  $H_0$ .

The null and alternative hypotheses are stated together. The following are typical hypothesis for means, where  $k$  is a specified number.

**Two-tailed test**

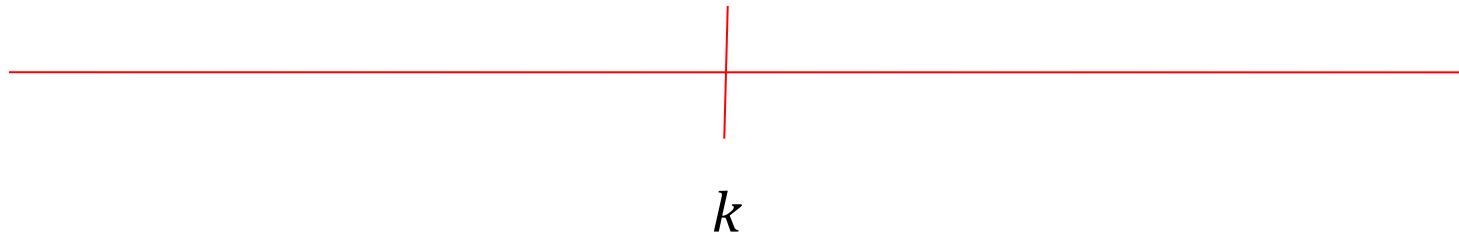
$$H_0: \mu = k$$
$$H_1: \mu \neq k$$

**Right-tailed test**

$$H_0: \mu = k$$
$$H_1: \mu > k$$

**Left-tailed test**

$$H_0: \mu = k$$
$$H_1: \mu < k$$



*Note:* Right-tailed and left-tailed tests are distinguished by the way the greater than or less than points. It is the direction where are alternative places the true mean.

**Examples:** State the  $H_0$  and  $H_1$  for each case.

A researcher thinks that if expectant mothers use vitamins, the birth weight of the babies will increase. The average birth weight of the population is 8.6 pounds.

$$H_0: \mu = 8.6$$

$$H_1: \mu > 8.6$$

An engineer hypothesizes that the mean number of defects can be decreased in a manufacturing process of compact disks by using robots instead of humans for certain tasks. The mean number of defective disks per 1000 is 18.

$$H_0 :$$

$$H_1 :$$

A psychologist feels that playing soft music during a test will change the results of the test. The psychologist is not sure whether the grades will be higher or lower. In the past, the mean of the scores was 73.

$H_0 :$

$H_1 :$

When a researcher conducts a study, he or she is generally looking for evidence to support a claim of some type of difference. In this case, the claim should be stated as the alternative hypothesis. **Because of this, the alternative hypothesis is sometimes called the research hypothesis.**

Keywords help to indicate what the null and/or alternative hypotheses should be.

**Table 8–1** Hypothesis-Testing Common Phrases

$>$	$<$
Is greater than	Is less than
Is above	Is below
Is higher than	Is lower than
Is longer than	Is shorter than
Is bigger than	Is smaller than
Is increased	Is decreased or reduced from
$=$	$\neq$
Is equal to	Is not equal to
Is the same as	Is different from
Has not changed from	Has changed from
Is the same as	Is not the same as



After stating the hypotheses, the researcher designs the study.

- Select the correct statistical test
- Choose an appropriate level of significance
- Formulate a plan for conducting the study

**Statistical Test** – uses the data obtained from a sample to make a decision about whether the null hypothesis should be rejected.





**Test Value** (test statistic) – the numerical value obtained from a statistical test.

When we make a conclusion from a statistical test there are two types of errors that we could make. They are called: Type I and Type II Errors

**Type I error** – reject  $H_0$  when  $H_0$  is true.

**Type II error** – do not reject  $H_0$  when  $H_0$  is false.

Results of a statistical test:

	$H_0$ is True	$H_0$ is False
Reject $H_0$	<b>Type I Error</b> 	Correct Decision 
Do not Reject $H_0$	Correct Decision 	<b>Type II Error</b> 

**Example:** Decision Errors in a Legal Trial . What are  $H_0$  and  $H_1$ ?

	$H_0$ true	$H_0$ false
Reject $H_0$	<b>Error</b> Type I	Correct decision
Do not reject $H_0$	Correct decision	<b>Error</b> Type II

$H_0$ : Defendant is innocent.

$H_1$ : Defendant is not innocent, i.e., guilty

If you are the defendant, which is the worse error? Why?

- The decision of the jury does not prove that the defendant did or did not commit the crime.
- The decision is based on the evidence presented.
- If the evidence is strong enough the defendant will be convicted in most cases, if it is weak the defendant will be acquitted.
- So the decision to reject the null hypothesis does not prove anything.
- The question is how large of a difference is enough to say we have enough evidence to reject the null hypothesis?

**Significance level** - is the maximum probability of committing a Type I error. This probability is symbolized by  $\alpha$ .  
 $P(\text{Type I error} | H_0 \text{ is true}) = \alpha$

**Critical or Rejection Region** – the range of values for the test value that indicate a significant difference and that the null hypothesis should be rejected.

**Non-critical or Non-rejection Region** – the range of values for the test value that indicates that the difference was probably due to chance and that the null hypothesis should not be rejected.

**Critical Value (CV)** – separates the critical region from the non-critical region, i.e., when we should reject  $H_0$  from when we should not reject  $H_0$ .

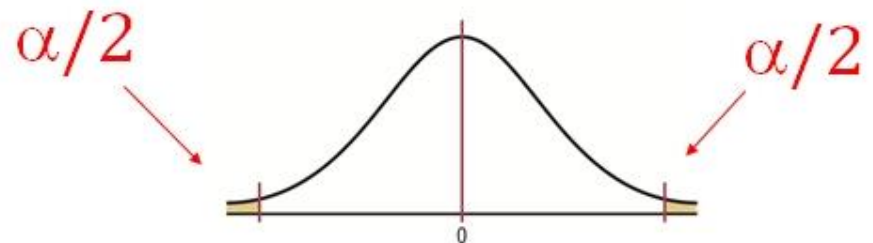
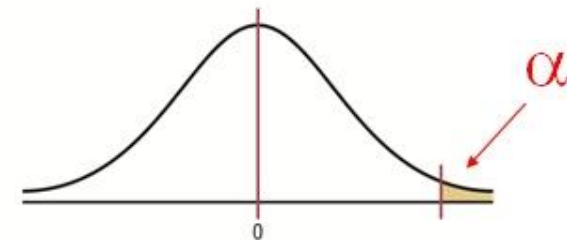
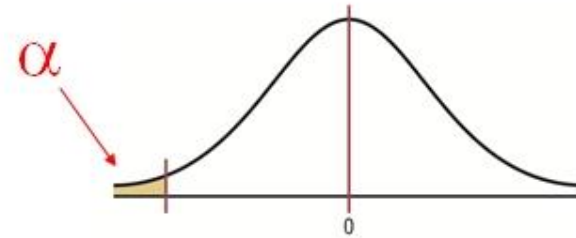
- The location of the critical value depends on the inequality sign of the alternative hypothesis.
- Depending on the distribution of the test value, you will use different tables to find the critical value.

**One-tailed test** – indicates that the null hypothesis should be rejected when the test value is in the critical region on one side.

⇒ **Left-tailed test** – when the critical region is on the left side of the distribution of the test value.

⇒ **Right-tailed test** – when the critical region is on the right side of the distribution of the test value.

**Two-tailed test** – the null hypothesis should be rejected when the test value is in either of two critical regions on either side of the distribution of the test value.



To obtain the critical value, the researcher must choose the significance level,  $\alpha$ , and know the distribution of the test value.

- The distribution of the test value indicates the shape of the distribution curve for the test value. This will have a shape that we know (like the standard normal or  $t$  distribution).
- Let's assume that the test value has a standard normal distribution.
- We should use Table E (the standard normal table) or Table F (using the bottom row of the  $t$  distribution, which is equivalent to a standard normal distribution) to find the critical value.



## Finding the Critical Values for Specific $\alpha$ Values, Using Table E

**Step 1:** Draw a figure for the distribution of the test values and indicate the appropriate area for the rejection region.

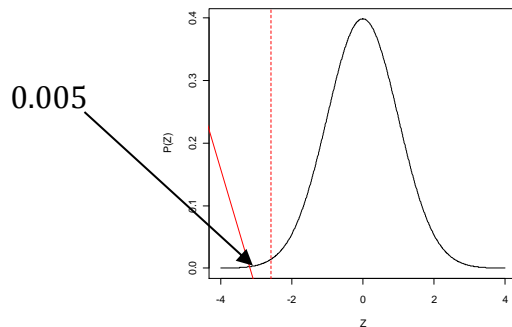
- If the test is left-tailed, the critical region, with area equal to  $\alpha$ , will be on the left side of the distribution curve.
- If the test is right-tailed, the critical region, with area equal to  $\alpha$ , will be on the right side of the distribution curve.
- If the test is two-tailed,  $\alpha$  must be divided by 2; the critical regions will be in each end of the distribution curve - half the area in the left part of the distribution and half of the area in the right part of the distribution.

## Step 2:

- For a left-tailed test, use the  $z$  value that corresponds to the area equivalent to  $\alpha$  in Table E, i.e.,  $z_{1-\alpha}$ , the  $\alpha$  percentile of the distribution.
- For a right-tailed test, use the  $z$  value that corresponds to the area equivalent to  $1 - \alpha$  in Table E, i.e.,  $z_{\alpha}$ , the  $1 - \alpha$  percentile of the distribution.
- For a two-tailed test, use the  $z$  value that corresponds to  $\alpha/2$  for the left lower CV. It will be negative. Change the sign to positive and you will get the critical value for the right side.

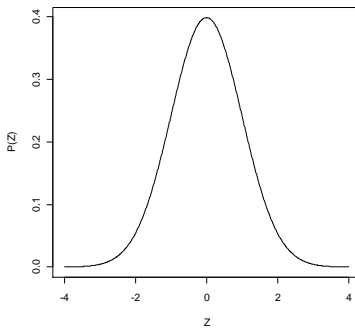
**Example:** Find the critical value(s) for each situation and draw the appropriate figure, showing the critical region.

Left-tailed test with  $\alpha = 0.005$

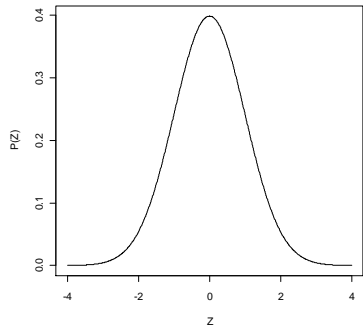


Looking up 0.005 in the Z table  
We have  $Z = -2.575$ .

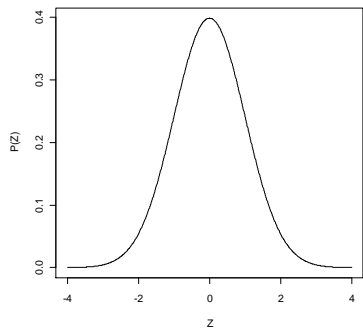
Right-tailed test with  $\alpha = 0.01$



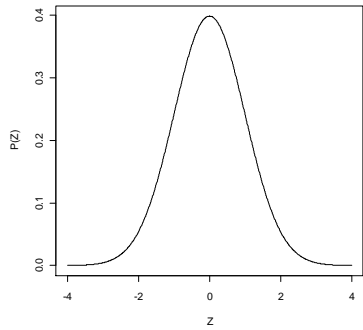
## Two-tailed test with $\alpha = 0.1$



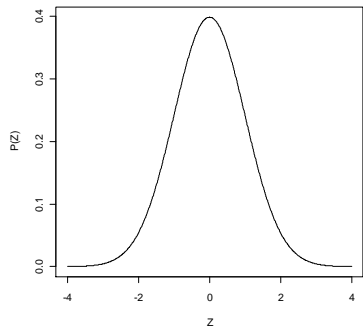
## Left-tailed test with $\alpha = 0.2$



## Right-tailed test with $\alpha = 0.15$



## Two-tailed test with $\alpha = 0.09$



## Hypothesis Test Procedure (Traditional Method)

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s) from the appropriate table.

Step 3 Compute the test value.

Step 4 Make the decision to reject or not reject the null hypothesis.

Step 5 Summarize the results.

## Section 8-2: z Test for a Mean

**MOTIVATING SCENARIO:** It has been reported that the average credit card debt for college seniors is \$3262.

The student senate at a large university feels that their seniors have a debt much less than this, so it conducts a study of 50 randomly selected seniors and finds that the average debt is \$2995, and **the population standard deviation** is \$1100.

**Can we support the student senate's claim using the data collected?**

## How...the z Test for a Mean

A **statistical test** uses the data obtained from a sample to make a decision about whether the null hypothesis should be rejected.

The numerical value obtained from a statistical test is called the **test value**.

You will notice that our statistical tests will resemble the general formula for a z-score:

$$\text{Test Value} = \frac{\text{observed value} - \text{expected value}}{\text{standard error}}$$



## The z test for Means

The z test is a statistical test for the mean of a population. It can be used when  $n \geq 30$ , or when the population is normally distributed and  $\sigma$  is known.

The formula for the z-test is:  $z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$ , where

$\bar{X} =$   $\sigma =$

$\mu =$   $n =$

We use our standard normal distribution...our z table!

## Hypothesis Test Procedure (Traditional Method)

**Step 1** State the hypotheses and identify the claim.

**Step 2** Find the critical value(s) from the appropriate table.

**Step 3** Compute the test value.

**Step 4** Make the decision to reject or not reject the null hypothesis.

**Step 5** Summarize the results.

**Example:** It has been reported that the average credit card debt for college seniors is \$3262. The student senate at a large university feels that their seniors have a debt **much less** than this, so it conducts a study of 50 randomly selected seniors and finds that the average debt is \$2995, and the population standard deviation is \$1100. Let's conduct the test based on a Type I error of  $\alpha=0.05$ .

**Step 1** State the hypotheses and identify the claim.

$$H_0: \mu = \$3262$$

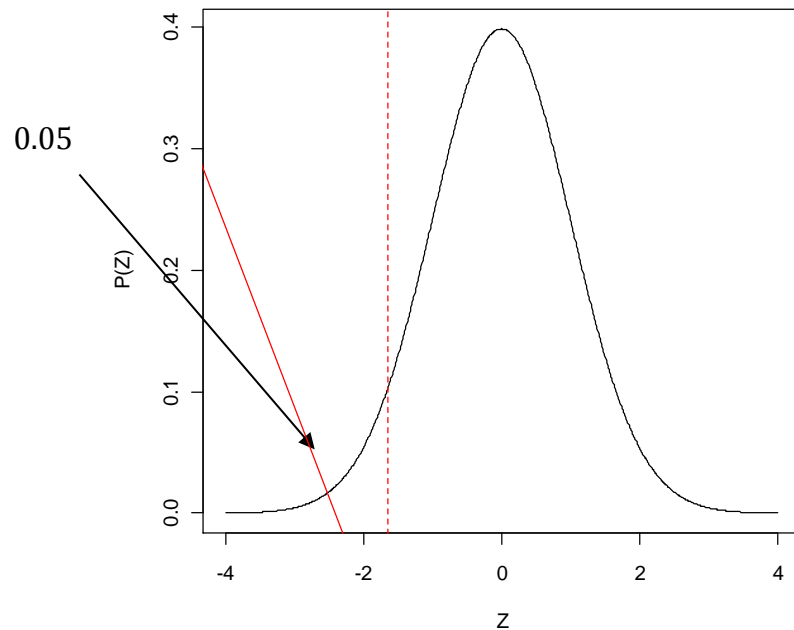
$$H_1: \mu < \$3262$$



CLAIM

**Step 2** Find the critical value(s) from the appropriate table.

Left-tailed test,  $\alpha=0.05 \Rightarrow Z$  will be negative and have probability 0.05 underneath it



**Z = -1.645 or**  
**Z = -1.65**

Table E The Standard Normal Distribution						
Cumulative Standard Normal Distribution						
z	.00	.01	.02	.03	.04	.05
-3.4	.0003	.0003	.0003	.0003	.0003	.0003
-3.3	.0005	.0005	.0005	.0004	.0004	.0004
-3.2	.0007	.0007	.0006	.0006	.0006	.0006
-3.1	.0010	.0009	.0009	.0009	.0008	.0008
-3.0	.0013	.0013	.0013	.0012	.0012	.0011
-2.9	.0019	.0018	.0018	.0017	.0016	.0016
-2.8	.0026	.0025	.0024	.0023	.0023	.0022
-2.7	.0035	.0034	.0033	.0032	.0031	.0030
-2.6	.0047	.0045	.0044	.0043	.0041	.0040
-2.5	.0062	.0060	.0059	.0057	.0055	.0054
-2.4	.0082	.0080	.0078	.0075	.0073	.0071
-2.3	.0107	.0104	.0102	.0099	.0096	.0094
-2.2	.0139	.0136	.0132	.0129	.0125	.0122
-2.1	.0179	.0174	.0170	.0166	.0162	.0158
-2.0	.0228	.0222	.0217	.0212	.0207	.0202
-1.9	.0287	.0281	.0274	.0268	.0262	.0256
-1.8	.0359	.0351	.0344	.0336	.0329	.0322
-1.7	.0446	.0436	.0427	.0418	.0409	.0401
-1.6	.0548	.0537	.0526	.0516	.0505	.0495

**Step 3** Compute the test value.

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{2995 - 3262}{\frac{1100}{\sqrt{50}}} = -1.716341$$

**Step 4** Make the decision to reject or not reject the null hypothesis.

Since this is a left-tailed test, our rejection region consists of values of Z that are smaller than our critical value of Z = -1.645.

Since our test value (-1.716341) is less than our critical value (-1.645), we reject the null hypothesis.

## **Step 5** Summarize the results.

We have evidence to support the student senate claim that the university's seniors have credit card debt that is less than the reported average debt.

This is based on a Type I error rate of 0.05. This means we falsely make the claim above 5% of the time.

**Example:** The medical Rehabilitation Education Foundation reports that the average cost of rehabilitation for stroke victims is \$24,672.

To see if the average cost of rehab is different at a particular hospital, a researcher selects a random sample of 35 stroke victims at the hospital and finds the average cost of their rehab is \$25,250. The standard deviation of the population is \$3251.

At  $\alpha = 0.01$ , can it be concluded that the average cost of stroke rehabilitation at a particular hospital is different from \$24,672?

**Step 1** State the hypotheses and identify the claim.

**Step 2** Find the critical value(s) from the appropriate table.

**Step 3** Compute the test value.



**Step 4** Make the decision to reject or not reject the null hypothesis.

**Step 5** Summarize the results.

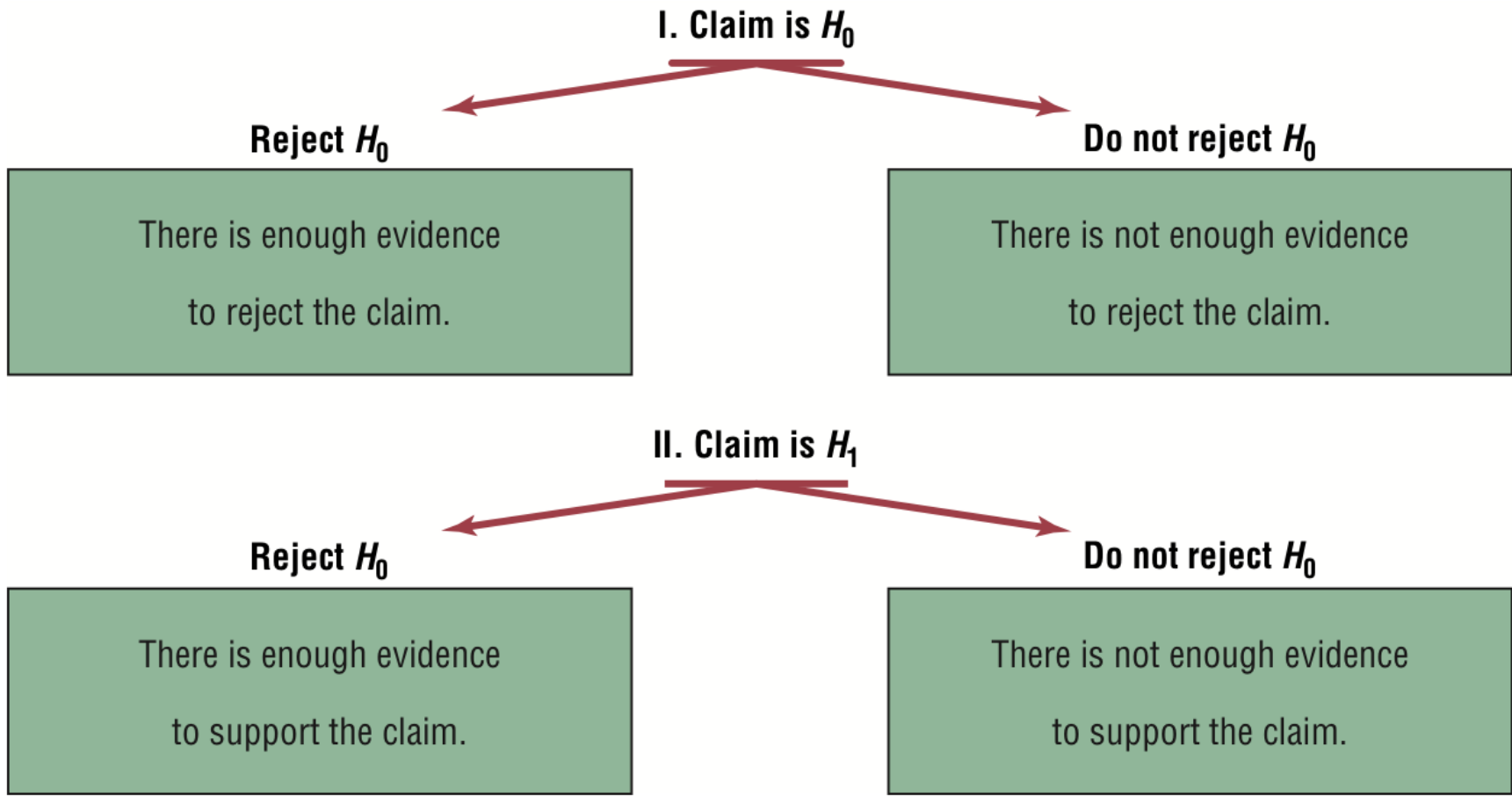
## **IMPORTANT NOTE:**

**When the null hypothesis is not rejected, we do not accept it as true. There is merely not enough evidence to say that it is false.**

Consider the jury trial analogy. We don't find people innocent, only guilty or not guilty. If someone is found not guilty, it does not mean that they were proved innocent; it only means that there is not enough evidence to reach a guilty verdict.

**We conclude the alternative hypothesis (when we reject the null) because the data clearly support that conclusion.**

See the following slide for ways of describing your results.



See page 415 to help you remember what you can say in summarizing the results based on where the claim is ( $H_0$  or  $H_1$ ) and whether you rejected  $H_0$  or not.

## P-Value Method for Hypothesis Testing

We often test hypotheses at common levels of significance ( $\alpha = 0.05$ , or  $0.01$ ). Recall that the choice of alpha depends on the seriousness of the Type I error. There is another approach that utilizes a P-value.

The P-Value (or probability value) is the probability of getting a sample statistic (such as the mean) or a more extreme sample statistic in the direction of the alternative hypothesis when the null hypothesis is true.

The P-value is the actual area under the standard normal distribution curve of the test value or a more extreme value (further in the tail).

## A General Rule for Finding P-values using the $Z$ distribution or the $t$ distribution:

Suppose that  $z^*$  is the test statistic of a  $z$  test and  $t^*$  is the test statistic of a  $t$  test

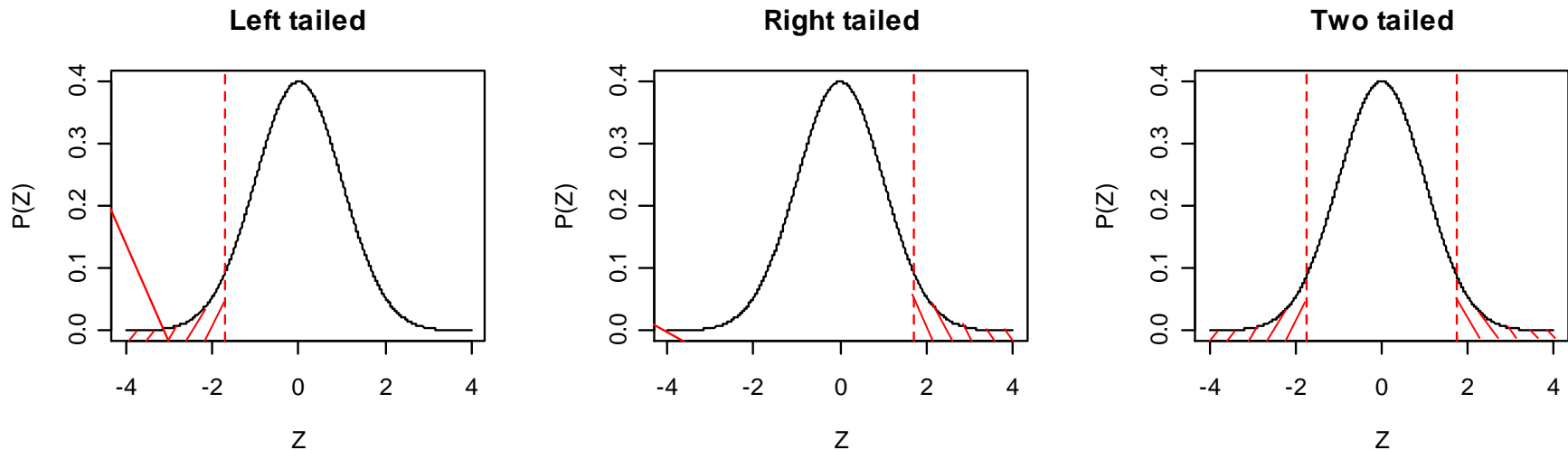
**Left-tailed test:** p-value =  $P(Z \leq z^*)$  or p-value =  $P(T \leq t^*)$   
(depending on whether we are doing a  $z$  test or a  $t$  test).

**Right-tailed test:** p-value =  $P(Z \geq z^*)$  or p-value =  $P(T \geq t^*)$   
(depending on whether we are doing a  $z$  test or a  $t$  test).

**Two-tailed test:** p-value =  $2P(Z \geq |z^*|)$  or p-value =  $2P(T \geq |t^*|)$   
(depending on whether we are doing a  $z$  test or a  $t$  test).

The smaller the P-value, the stronger the evidence is against  $H_0$ .

We use Table E to find P-Values that use a z Test.



## *Examples:*

Suppose you have a left-tailed test and find the area in the tail to be 0.0489. What is the P-value? Would you reject this at  $\alpha = 0.05$ ?  $\alpha = 0.01$ ?

Suppose you have a two-tailed test and find the area in one tail to be 0.0084. What is the P-value? Would you reject this at  $\alpha = 0.05$ ?  $\alpha = 0.01$ ?

## P-Value

If p-value is  $\leq \alpha$  then we \_\_\_\_\_.

If p-value is  $> \alpha$  then we \_\_\_\_\_.

Let's reconsider the hypothesis tests we did earlier and find the corresponding P-values.



**Example:** It has been reported that the average credit card debt for college seniors is \$3262. The student senate at a large university feels that their seniors have a debt much less than this, so it conducts a study of 50 randomly selected seniors and finds that the average debt is \$2995, and the population standard deviation is \$1100. Let's conduct the test based on a Type I error of  $\alpha=0.05$ .

**Step 3** Compute the test value and find the P-value.

$$Z = -1.72$$

**Step 4 and Step 5**

**Example:** The medical Rehabilitation Education Foundation reports that the average cost of rehabilitation for stroke victims is \$24,672. To see if the average cost of rehab is different at a particular hospital, a researcher selects a random sample of 35 stroke victims at the hospital and find the average cost of their rehab is \$25,250. The standard deviation of the population is \$3,251. At  $\alpha = 0.01$ , can it be concluded that the average cost of stroke rehabilitation at a particular hospital is different from \$24,672?

**Step 3** Compute the test value and find the P-value.

$$Z = 1.05$$

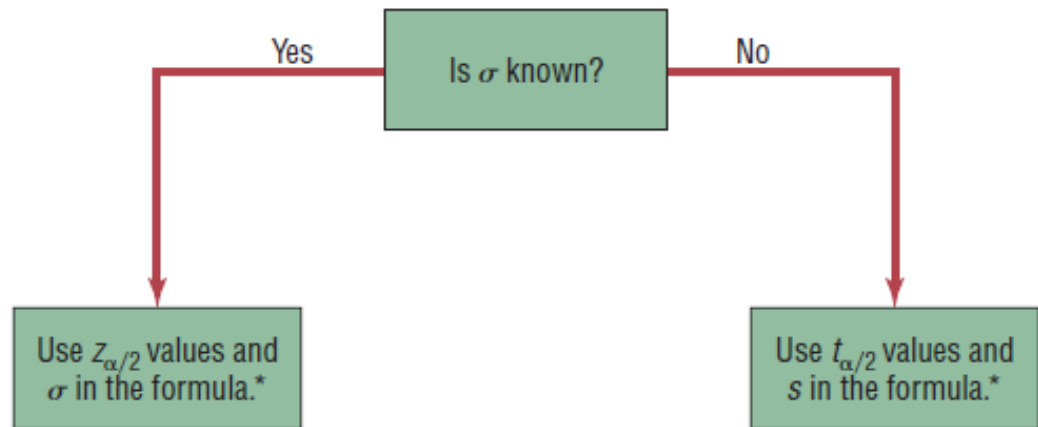
**Step 4 and Step 5**

## Section 8-3: $t$ Test for a Mean

When a population is normally or approximately normally distributed, but the **population standard deviation is unknown**, the  $z$  test is inappropriate for testing hypotheses involving means. Instead **we will use the  $t$  test when  $\sigma$  is unknown and the distribution of the variable is approximately normal.**

**Figure 8–25**

Using the  $z$  or  $t$  Test

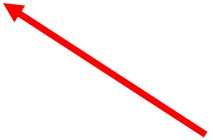


\*If  $n < 30$ , the variable must be normally distributed.

The **one-sample  $t$  test** is a statistical test for the mean of a population and is used when the population is normally or approximately **normally distributed** and  **$\sigma$  is unknown**.

The formula for the test value of the one-sample  $t$  test is:

$$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} \quad d.f. = n-1$$



*Recall from Chapter 7 that the  $t$  distribution is a family of curves and requires us to know the degrees of freedom in order to find the appropriate  $t$  value.*

## Finding Critical Values Using Table F

- Determine if the test is one-tailed or two tailed.
- Determine the degrees of freedom
- Find the significance level  $\alpha$  by looking in the appropriate row for the number of tails and down the column with the degrees of freedom.
- If the degrees of freedom is not in the table, always round DOWN to the nearest table value (this is a conservative approach).
- If the test is a left-tailed test, then the critical value is the **NEGATIVE** of the value given in the table.

**Example:** Find the critical  $t$  value for  $\alpha = 0.01$  with sample size of 13 for a left-tailed test.

- ◆ Left tailed means the critical  $t$  value will be negative
- ◆  $n=13$  means the degrees of freedom are  $n-1 = 12$
- ◆ The critical value is  $-2.681$

Table F		The $t$ Distribution				
Confidence intervals		80%	90%	95%	98%	99%
One tail, $\alpha$		0.10	0.05	0.025	0.01	0.005
d.f.	Two tails, $\alpha$	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947

***Example:*** Find the critical  $t$  value for  $\alpha = 0.05$  with sample size of 21 for a right-tailed test.

***Example:*** Find the critical  $t$  value for  $\alpha = 0.10$  with sample size of 59 for a two-tailed test.

## Finding P-Values Using Table F

To find a p-value:

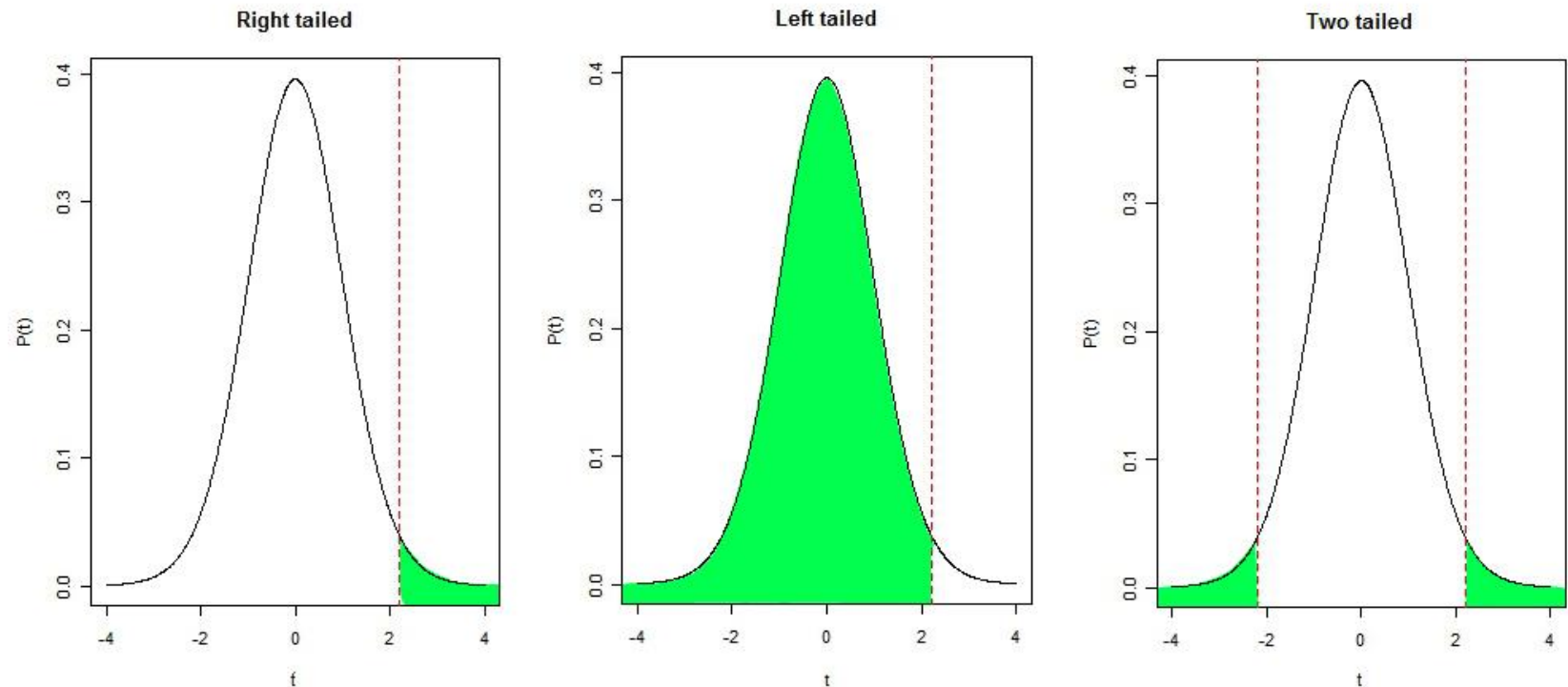
- Draw a picture of the area we're trying to find.
- Find row corresponding to  $df = n - 1$ .
- Find the 2 values in the row that the value of the test statistic falls between.
- Find the  $\alpha$ 's that corresponds to these values
- The  $\alpha$ 's are the values that the p-value falls between



You can calculate the exact p-value using your calculator:

- 2<sup>nd</sup> -> VARS (DISTR)
- tcdf(lowerbound, upperbound, df)
  - lowerbound corresponds to the value on the left that you are computing probability in between
  - upperbound corresponds to the value on the right that you are computing probability in between
  - use EE99 for  $\infty$
  - use -EE99 for  $-\infty$
- You will need to multiply by 2 if you are performing a two-tailed test.

**Example:** Let  $n = 29$  and your test value be  $t = 2.21$ . Find the p-value for a right-tailed hypothesis test. Find the p-value for a left-tailed test. Find the p-value for a two-tailed hypothesis test.



Degrees of freedom =  $29 - 1 = 28$

<b>Table F</b>		<b>The <math>t</math> Distribution</b>				
	<b>Confidence intervals</b>	<b>80%</b>	<b>90%</b>	<b>95%</b>	<b>98%</b>	<b>99%</b>
	<b>One tail, <math>\alpha</math></b>	<b>0.10</b>	<b>0.05</b>	<b>0.025</b>	<b>0.01</b>	<b>0.005</b>
<b>d.f.</b>	<b>Two tails, <math>\alpha</math></b>	<b>0.20</b>	<b>0.10</b>	<b>0.05</b>	<b>0.02</b>	<b>0.01</b>
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947
16		1.337	1.746	2.120	2.583	2.921
17		1.333	1.740	2.110	2.567	2.898
18		1.330	1.734	2.101	2.552	2.878
19		1.328	1.729	2.093	2.539	2.861
20		1.325	1.725	2.086	2.528	2.845
21		1.323	1.721	2.080	2.518	2.831
22		1.321	1.717	2.074	2.508	2.819
23		1.319	1.714	2.069	2.500	2.807
24		1.318	1.711	2.064	2.492	2.797
25		1.316	1.708	2.060	2.485	2.787
26		1.315	1.706	2.056	2.479	2.779
27		1.314	1.703	2.052	2.473	2.771
28		1.313	1.701	2.048	2.467	2.763
29		1.311	1.699	2.044	2.462	2.756
30		1.310	1.697	2.041	2.458	2.750
40		1.303	1.691	2.029	2.447	2.733
50		1.299	1.688	2.023	2.442	2.728
60		1.297	1.687	2.021	2.440	2.726
70		1.296	1.686	2.020	2.439	2.725
80		1.295	1.686	2.019	2.438	2.724
90		1.295	1.685	2.018	2.438	2.724
100		1.295	1.685	2.018	2.438	2.724

## Right-tailed p-value

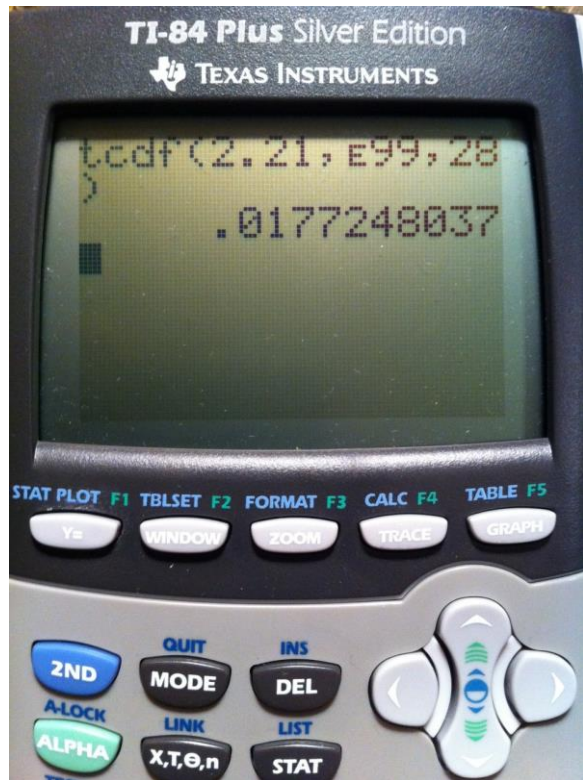
- is between 0.025 and 0.01 based on the table
- is 0.018 based on the calculator

## Left-tailed p-value

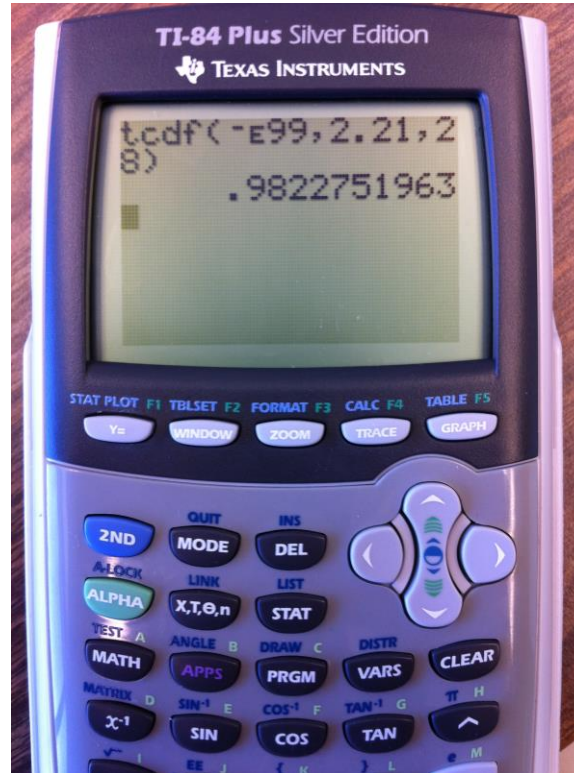
- is between 0.975 and 0.99 based on the table
- is 0.982 based on the calculator

## Two-tailed p-value

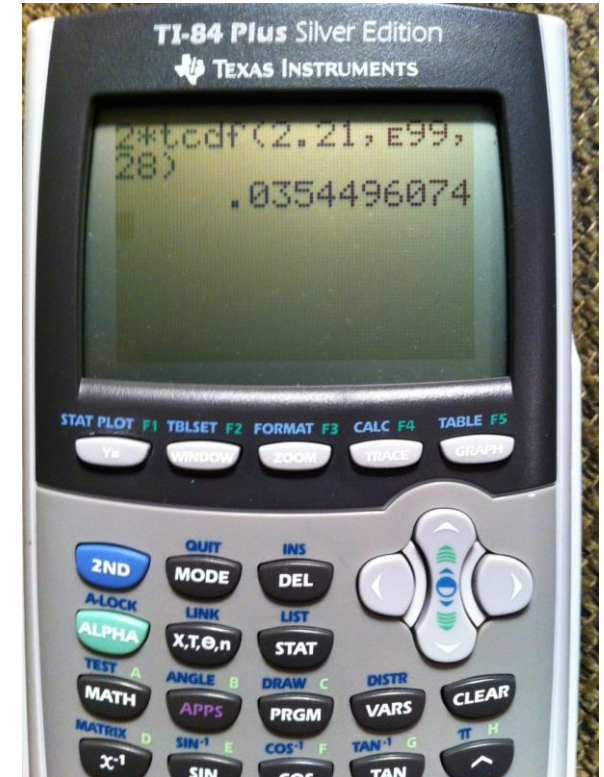
- is between 0.05 and 0.02 based on the table
- is 0.035 based on the calculator



**Right tailed**



**Left tailed**



**Two-tailed**

**Example:** Let  $n = 11$  and your test value be  $t = -1.35$ . Find the p-value for a left-tailed hypothesis test. Find the p-value for a right-tailed hypothesis test. Find the p-value for a two-tailed hypothesis test.

**Example:** We wish to check that normal body temperature may be less than 98.6 degrees. In a random sample of  $n = 18$  individuals, the sample mean was found to be 98.217 and the standard deviation was .684. Assume the population is normally distributed. Use  $\alpha = 0.05$ .

**Step 1 State the hypotheses and identify the claim.**

$$H_0: \mu = 98.6$$

$$H_1: \mu < 98.6 \quad \Leftarrow \quad \text{CLAIM}$$

**Step 2 Find the critical value(s) from the appropriate table.**

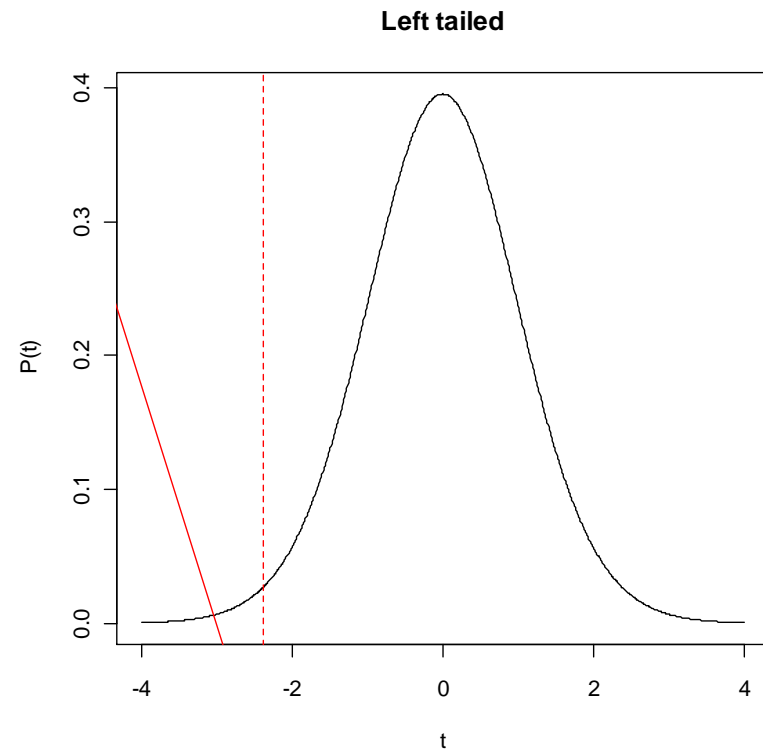
Left tailed,  $\alpha = 0.05$ ,  $df = 18 - 1 = 17 \Rightarrow t$  critical value = 1.740

**Step 3 Compute the test value and determine the P-value.**

$$t = \frac{98.217 - 98.6}{\frac{0.684}{\sqrt{18}}} = -2.375631 \approx -2.38$$

p-value is between 0.01 and 0.025

p-value = 0.0146





## **Step 4** Make the decision to reject or not reject the null hypothesis.

Since our p-value is less than our  $\alpha = 0.05$ , we reject the null hypothesis.

The same conclusion is reached by looking at the critical value. Our test value is smaller than the critical value of -1.74.

You only need to do it one way. The decision will always match.

## **Step 5** Summarize the results.

We have enough evidence to support the claim that average body temperature is less than 98.6 degrees.

**Example:** A certain company would like to determine the amount of time employees waste at work each day. A random sample of 10 of its employees shows a mean time of 121.80 minutes wasted per day with a standard deviation of 9.45 minutes per day. Does the data provide evidence that the mean amount of time wasted by employees each day is more than 120 minutes? Test at  $\alpha=.05$ . Assume the population is at least approximately normally distributed.

Let  $\mu$  = mean daily wasted time for employees of this company.

**Step 1 State the hypotheses and identify the claim.**

**Step 2 Find the critical value(s) from the appropriate table.**

**Step 3 Compute the test value and determine the P-value.**

**Step 4 Make the decision to reject or not reject the null hypothesis.**

**Step 5 Summarize the results.**

Suppose the previous test were two-tailed instead of one-tailed.

What would the critical values/regions be?

What would the p-value for the test be?

What would our decision be?

## Section 8-4: z-Test for a Proportion

A hypothesis test involving proportions can be considered as a binomial experiment when there are only two outcomes and the probability of success does not change from trial to trial.

However, when both  $n\hat{p}$  and  $n\hat{q}$  are each greater than or equal to 5, the Central Limit Theorem kicks in and a normal distribution can be used to describe the distribution of the sample proportions.

**Recall:** When the central limit theorem applies to data from a binomial distribution, then  $\hat{p}$  can be well approximated by a normal distribution with mean  $p$  and standard deviation  $\sqrt{pq/n}$ , i.e.,

$\hat{p}$  is approximately Normal with mean  $=p$  and standard error  $= \sqrt{\frac{pq}{n}}$ .

When performing our hypothesis test, we will make an assumption about the value of  $p$  in our null hypothesis, e.g.,

$$H_0 : p = k$$

where  $k$  is some fixed value. We can use this to determine to perform a hypothesis test for  $p$ .

## Formula for the z-Test for Proportions

Test value: 
$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

$\hat{p} =$

$p =$

$q =$

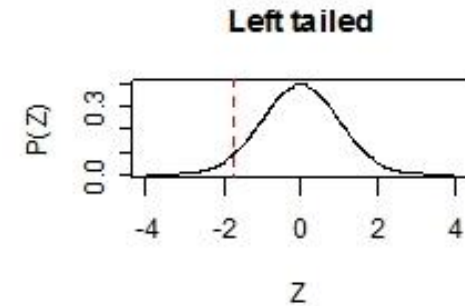
$n =$



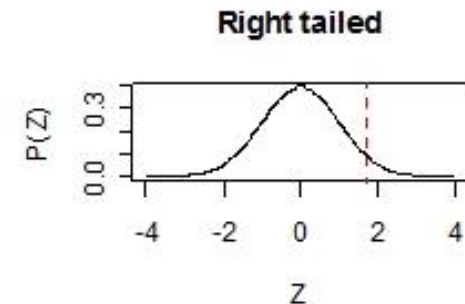


The p-values can be found according to the following procedure.  
If  $z^*$  is the test value of our test then:

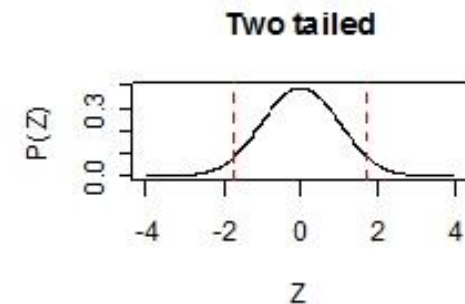
**Left-tailed test:**  $p\text{-value} = P(Z \leq z^*)$



**Right-tailed test:**  $p\text{-value} = P(Z \geq z^*)$



**Two-tailed test:**  $p\text{-value} = 2P(Z \geq |z^*|)$



**Example:** An educator estimates that the dropout rate for seniors at high schools in Colorado is 12%. Last year in a random sample of 300 Colorado seniors, 27 withdrew from school. At  $\alpha = 0.05$ , is there enough evidence to reject the educator's claim?

**Step 1** State the hypotheses and identify the claim.

$$H_0: p = 0.12 \quad H_1: p \neq 0.12$$



CLAIM

**Step 2** Find the critical value(s) from the appropriate table.

Two-tailed,  $\alpha=0.05$  ..... critical value is  $Z = \pm 1.96$   
(*peak back at slide 333*)

**Step 3** Compute the test value and determine the P-value.

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{27}{300} - 0.12}{\sqrt{\frac{0.12(1-0.12)}{300}}}$$

$$= \frac{-0.03}{0.01876166} = -1.60$$

$$\text{p-value} = 2*(0.0548)$$

$$= 0.1096$$

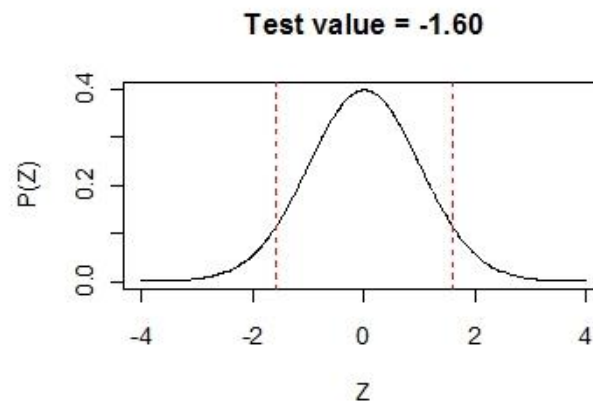


Table E The Standard Normal Distribution			
Cumulative Standard Normal Distribution			
z	.00	.01	.02
-3.4	.0003	.0003	.0003
-3.3	.0005	.0005	.0005
-3.2	.0007	.0007	.0006
-3.1	.0010	.0009	.0009
-3.0	.0013	.0013	.0013
-2.9	.0019	.0018	.0018
-2.8	.0026	.0025	.0024
-2.7	.0035	.0034	.0033
-2.6	.0047	.0045	.0044
-2.5	.0062	.0060	.0059
-2.4	.0082	.0080	.0078
-2.3	.0107	.0104	.0102
-2.2	.0139	.0136	.0132
-2.1	.0179	.0174	.0170
-2.0	.0228	.0222	.0217
-1.9	.0287	.0281	.0274
-1.8	.0359	.0351	.0344
-1.7	.0446	.0436	.0427
-1.6	.0548	.0537	.0526

**Step 4** Make the decision to reject or not reject the null hypothesis.

Noting that our p-value is larger than our Type I error rate of 0.05, we fail to reject the null hypothesis.

You come to the same decision by noting that our test value of -1.60 falls within the non-critical region of -1.96 to 1.96.

**Step 5** Summarize the results.

We do not have sufficient evidence to reject the claim that the dropout rate for seniors at high schools in Colorado is 12%.

*Check check! Were  $n\hat{p}$  and  $n\hat{q}$  both greater than or equal to 5?*

Let's generate some questions about Math 2830 students that would allow us to test a hypothesis concerning proportions using our class as the sample.

1.

2.

3.

4.

Question # \_\_\_\_\_

What level of significance would we like to use? \_\_\_\_\_

Our class data:

**Step 1** State the hypotheses and identify the claim.

**Step 2** Find the critical value(s) from the appropriate table.

**Step 3** Compute the test value and determine the P-value.

**Step 4** Make the decision to reject or not reject the null hypothesis.

**Step 5** Summarize the results.



Question # \_\_\_\_\_

What level of significance would we like to use? \_\_\_\_\_

Our class data:

**Step 1** State the hypotheses and identify the claim.

**Step 2** Find the critical value(s) from the appropriate table.

**Step 3** Compute the test value and determine the P-value.

**Step 4** Make the decision to reject or not reject the null hypothesis.

**Step 5** Summarize the results.