Chapter 8: More on Limits

Lesson 8.1.1

8-1.

a. $\lim\limits_{x\to\infty} a(x) = \lim\limits_{x\to\infty} \frac{1000}{x} = 0$

b. $\lim\limits_{x\to\infty} b(x) = \lim\limits_{x\to\infty}(x^3 - 10000x^2) = \infty$

c. $\lim\limits_{x\to\infty} c(x) = \lim\limits_{x\to\infty} \frac{3x^2+7x}{x^2+1000} = 3$

8-2.

a. $\lim\limits_{x\to\infty} 10x^2 = \infty$

b. $\lim\limits_{x\to\infty}(-0.5^x) = -\lim\limits_{x\to\infty}\left(\tfrac{1}{2}^{x}\right) = 0$

c. $\lim\limits_{x\to\infty} 4\sqrt{x} = \lim\limits_{x\to\infty} 4(x^{1/2}) = \infty$

d. $\lim\limits_{x\to\infty}(5x-3) = \infty$

8-3.

a.
1. $c(x) = 100$
2. $a(x) = 50$
3. $h(x) = 20$
4. $g(x) = 1.1$
5. $b(x) = f(x) = 1$ (tie)
7. $d(x) = 0$
8. $e(x) = -988$

b.
1. $b(x) = 10^{10}$
2. $h(x) = 20000$
3. $c(x) = 10000$
4. $d(x) = 800$
5. $a(x) = 158$
6. $e(x) = 24$
7. $f(x) = 10$
8. $g(x) = 2.59$

c.
1. $e(x) = 1.27 \cdot 10^{30}$
2. $b(x)100^{10} = 1 \cdot 10^{20}$
3. $h(x) = 20 \cdot 100^3 = 20{,}000{,}000$
4. $c(x) = 100 \cdot 100^2 = 1{,}000{,}000$
5. $g(x) = 1.1^{100} = 13{,}780.61$
6. $d(x) = 800 \log 100 = 1600$
7. $a(x) = 50\sqrt{100} = 500$
8. $f(x) = 100$

d.
1. $e(x)$
2. $g(x)$
3. $b(x)$
4. $h(x)$
5. $c(x)$
6. $f(x)$
7. $a(x)$
8. $d(x)$

e. At $x \approx 3127.366$ $a(x)$ passes $d(x)$.

8-4.

a. $\lim\limits_{x\to\infty} a(x) = \lim\limits_{x\to\infty} x^4 = \infty$

b. $\lim\limits_{x\to\infty} b(x) = -\lim\limits_{x\to\infty} 1000000x^3 = -\infty$

8-5.

a. $\lim\limits_{x\to\infty} d(x) = \lim\limits_{x\to\infty}(x^4 - 1000000x^3) = \infty$

b. Big positive wins since x^4 gets larger a lot quicker than x^3.

8-6.

a. The dominant term is $-2y^5$.

b. The dominant term is $20x^9$.

c. The dominant term is x^6.

d. The dominant term is x.

Review and Preview 8.1.1

8-7.

a. $\lim\limits_{x\to\infty} x^6 = \infty$

b. $\lim\limits_{x\to\infty}(x^6 - 12x^5 + 7x^3 - 400) = \infty$

c. $\lim\limits_{x\to\infty}(-3x^7) = -\infty$

d. $\lim\limits_{x\to\infty}(-3x^7 + 1000x^6 + 48,000) = -\infty$

8-8.

All are ∞.

8-9.

a. $\lim\limits_{x\to\infty}(50\log x - x^2) = -\infty$. Dominant term is $-x^2$.

b. $\lim\limits_{x\to\infty}(1.5^x - 300x^4) = \infty$. Dominant term is 1.5^x.

c. $\lim\limits_{x\to\infty}(1000(2)^x - 3^x) = -\infty$. Dominant term is -3^x.

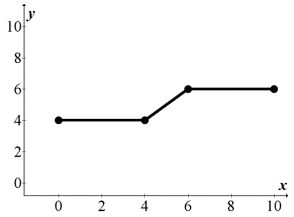
d. $\lim\limits_{x\to\infty}(x - 30\sqrt{x}) = \infty$. Dominant term is x.

8-11.

a. See graph at right.

b. $4 \cdot 4 + 2 \cdot 4 + \frac{1}{2} \cdot 2 \cdot 2 + 4 \cdot 6 = 16 + 10 + 24 = 50$

c. $50 + 6 \cdot 10 = 50 + 60 = 110$



8-12.

a.
$$(2^3)^x = 2^{2(2-x)}$$
$$3x = 4 - 2x$$
$$5x = 4$$
$$x = \tfrac{4}{5}$$

b.
$$200(1.05)^x = 5000$$
$$1.05^x = 25$$
$$\log_{1.05} 1.05^x = \log_{1.05} 25$$
$$x = \frac{\log 25}{\log 1.05}$$
$$x = 65.974$$

Solution continues on next page. →

8-12. *Solution continued from previous page.*

c. $\log_3(x+5) = 4$

$$3^{\log_3(x+5)} = 3^4$$

$$x+5 = 81$$

$$x = 76$$

d. $20m^{1.5} = 1000$

$$m^{1.5} = 50$$

$$\left(m^{3/2}\right)^{2/3} = 50^{2/3}$$

$$m = 13.572$$

8-13.

a. Add 4. b. Divide by 2. c. Add 1.

8-14.

$$\text{slope} = \tfrac{2}{3} \quad \perp \text{ slope} = -\tfrac{3}{2}$$

$$y-5 = -\tfrac{3}{2}(x-7)$$

$$y-5 = -\tfrac{3}{2}x + \tfrac{21}{2}$$

$$2y-10 = -3x+21$$

$$3x+2y = 31$$

8-15.

a. $y = \frac{10}{x}$; vertical asymptote at $x = 0$; horizontal asymptote at $y = 0$

b. $y = 0.5^x$; no vertical asymptote; horizontal asymptote at $y = 0$

c. $y = \frac{1}{x+2}$; vertical asymptote at $x = -2$; horizontal asymptote at $y = 0$

d. $y = 2^x - 3$; no vertical asymptote; horizontal asymptote at $y = -3$

8-16.

a. $3f(x) = 3\left(\frac{1}{x}+20\right) = \frac{3}{x}+60$

b. $-2f(x) = -2\left(\frac{1}{x}+20\right) = -\frac{2}{x}-40$

c. $\frac{1}{5}f(x) = \frac{1}{5}\left(\frac{1}{x}+20\right) = \frac{1}{5x}+4$

d. $f\left(\frac{x}{5}\right) = \frac{1}{x/5}+20 = 1\cdot\frac{5}{x}+20 = \frac{5}{x}+20$

8-17.

$$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = \tfrac{1}{2}$$

$$\sin^2 x - \cos^2 x = \tfrac{1}{2}$$

$$1 - \cos^2 x - \cos^2 x = \tfrac{1}{2}$$

$$-2\cos^2 x = -\tfrac{1}{2}$$

$$\cos^2 x = \tfrac{1}{4}$$

$$\cos x = \pm\tfrac{1}{2}$$

$$x = \pm\tfrac{\pi}{3} + \pi n$$

Lesson 8.1.2

8-18.

The limits for all of the three functions are 2.

a. Answers may vary, but students will probably find iii the easiest to evaluate since they should remember little over big goes to zero leaving $\frac{2}{1}$.

b. $$\frac{2x^3 - x^2}{x^3 + 3x} = \frac{\frac{2x^3}{x^2} - \frac{x^2}{x^2}}{\frac{x^3}{x^2} + \frac{3x}{x^2}} = \frac{2x - 1}{x + \frac{3}{x}} = \frac{\frac{2x}{x} - \frac{1}{x}}{\frac{x}{x} + \frac{\frac{3}{x}}{x}} = \frac{2 - \frac{1}{x}}{1 + \frac{3}{x^2}}$$

8-19.

To change $\frac{x - 2x^2}{5x^3 - 7}$ into $\frac{\frac{1}{x^2} - \frac{2}{x}}{5 - \frac{7}{x^3}}$, divide the numerator and denominator by x^3.

8-20.

a. $$\lim_{x\to\infty} \frac{3x^4 + 2}{6x^4 - 2x} = \lim_{x\to\infty} \frac{\frac{3x^4}{x^4} + \frac{2}{x^4}}{\frac{6x^4}{x^4} - \frac{2x}{x^4}} = \lim_{x\to\infty} \frac{3 + \frac{2}{x^4}}{6 - \frac{2}{x^3}} = \frac{3}{6} = 0.5$$

b. $$\lim_{x\to\infty} \frac{3x^4 + 2}{6x^5 - 2x} = \lim_{x\to\infty} \frac{\frac{3x^4}{x^5} + \frac{2}{x^5}}{\frac{6x^5}{x^5} - \frac{2x}{x^5}} = \lim_{x\to\infty} \frac{\frac{3}{x} + \frac{2}{x^5}}{6 - \frac{2}{x^4}} = \frac{0}{6} = 0$$

c. $$\lim_{x\to\infty} \frac{3x^4 + 2}{6x^3 - 2x} = \lim_{x\to\infty} \frac{\frac{3x^4}{x^3} + \frac{2}{x^3}}{\frac{6x^3}{x^3} - \frac{2x}{x^3}} = \lim_{x\to\infty} \frac{3x + \frac{2}{x^3}}{6 - \frac{2}{x^2}} = \lim_{x\to\infty} \frac{3x}{6} = \infty$$

8-21.

a. The limit is zero (0).

b. The limit will be ∞ or $-\infty$.

c. The limit will be the ratio of the coefficient of the dominant terms.

8-22.

a. $$\lim_{x\to\infty} \frac{8x+170-x^3}{50x^2+17x+100} = \lim_{x\to\infty} \frac{-x^3+8x+170}{50x^2+17x+100} = -\infty$$

b. $$\lim_{x\to\infty} \frac{5x^4-12x+10}{18x+15x^4-3x^3} = \lim_{x\to\infty} \frac{5x^4-12x+10}{15x^4-3x^3+18x} = \frac{1}{3}$$

c. $$\lim_{x\to\infty} \frac{(2x-7)^3}{3x^3-12x+10x^2} = \lim_{x\to\infty} \frac{(4x^2-28x+49)(2x-7)}{3x^3-12x+10x^2} = \lim_{x\to\infty} \frac{8x^3-28x^2-56x^2+196x+98x-343}{3x^3-12x+10x^2} = \frac{8}{3}$$

8-23.

a. $\lim\limits_{x\to\infty} \frac{2^{x+8}}{3^x} = \lim\limits_{x\to\infty} \frac{2^x \cdot 2^8}{3^x}$ Limit = 0 Dominant terms: numerator 2^x, denominator 3^x.

b. $\lim\limits_{x\to\infty} \frac{x+6}{10 \log x}$. Limit = ∞ Dominant terms: numerator x, denominator $\log x$.

c. $\lim\limits_{x\to\infty} \frac{\log 5^x}{2x+3} = \lim\limits_{x\to\infty} \frac{x \log 5}{2x+3} = \frac{\log 5}{2}$ Dominant terms: numerator $x \log 5$, denominator $2x$.

d. $\lim\limits_{x\to\infty} \frac{\sqrt{x^4 - 3x + 1000}}{(2x-3)^2} = \frac{1}{4}$ Dominant terms: numerator $\sqrt{x^4} = x^2$, denominator $(2x)^2 = 4x^2$.

Review and Preview 8.1.2

8-24.

a. $\lim\limits_{x\to-\infty} \frac{2x}{x^3} = \lim\limits_{x\to-\infty} \frac{2x/x}{x^3/x} = \lim\limits_{x\to-\infty} \frac{2}{x^2} = 0$

b. $\lim\limits_{x\to-\infty} \frac{\sqrt{x^2+5}}{2x} = \lim\limits_{x\to-\infty} \frac{\frac{\sqrt{x^2+5}}{x}}{2x/x} = \lim\limits_{x\to-\infty} \frac{\sqrt{\frac{(x^2+5)}{x^2}}}{2} = \lim\limits_{x\to-\infty} \frac{\sqrt{1+\frac{5}{x^2}}}{2} = -\frac{1}{2}$

c. The $\lim\limits_{x\to-\infty} (\cos x)$ does not exist.

d. $\lim\limits_{x\to 0^-} \frac{|x|}{x} = -1$, $\lim\limits_{x\to 0^+} \frac{|x|}{x} = 1$, therefore $\lim\limits_{x\to 0} \frac{|x|}{x}$ does not exist.

e. $\lim\limits_{x\to 0^+} \log_2 x = -\infty$

8-25.

a. It approaches zero since the dominant numerator is x and the dominant denominator is x^2.

b. We have a horizontal asymptote at $y = 0$.

c. $\lim\limits_{x\to -1^-} f(x) = \infty$ and $\lim\limits_{x\to -1^+} f(x) = \infty$.

d. There is an error at $x = -1$ and this is reflected on the graph by a vertical asymptote.

8-26.

a. $\lim\limits_{x\to\infty} f(x) = 1$

b. $\lim\limits_{x\to 3^-} f(x) = \infty$

c. $\lim\limits_{x\to 3^+} f(x) = \infty$

d. $\lim\limits_{x\to -3^-} f(x) = \frac{(-3)^2 - 6(-3) + 10}{(-3-3)^2} = \frac{37}{36}$

e. The function is undefined at $x = 3$.

f. The function's value is $\frac{37}{36}$.

8-27.

a. $x_2 = 4(5) - 10 = 10$

$x_3 = 4(10) - 10 = 40 - 10 = 30$

$x_4 = 4(30) - 10 = 120 - 10 = 110$

$x_5 = 4(110) - 10 = 440 - 10 = 430$

b. $y_2 = 0^2 - 4 = -4$

$y_3 = (-4)^2 - 4 = 16 - 4 = 12$

$y_4 = 12^2 - 4 = 144 - 4 = 140$

$y_5 = 140^2 - 4 = 19,596$

c. $z_2 = \frac{1}{4}$

$z_3 = \frac{1}{1/4} = 1 \cdot \frac{4}{1} = 4$

$z_4 = \frac{1}{4}$

$z_5 = \frac{1}{1/4} = 1 \cdot \frac{4}{1} = 4$

d. $w_2 = 3 \cdot 5 = 15$

$w_3 = 5 \cdot 15 = 75$

$w_4 = 15 \cdot 75 = 1125$

$w_5 = 75 \cdot 1125 = 84,375$

8-28.

$t_2 = 5 + 2(1) = 7,\ t_3 = 7 + 2(2) = 11,\ t_4 = 11 + 2(3) = 17,\ t_5 = 17 + 2(4) = 25$

8-29.

The end behavior of the graph.

8-30.

a. $\lim\limits_{x \to \infty} 3^x = \infty$

b. $\lim\limits_{x \to \infty} (3^x - 6 \cdot 2^x) = \infty$. 3^x is the dominant term in the function.

c. $\lim\limits_{x \to \infty} -5^x = -\infty$

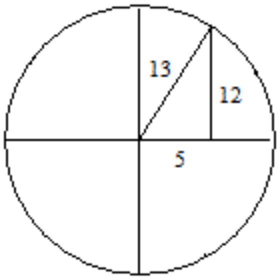
d. $\lim\limits_{x \to \infty} (-5^x + 10 \cdot 4^x) = -\infty$. -5^x is the dominant term in the function.

8-31.

The larger base will determine if the function goes to ∞ or $-\infty$.

8-32.

$\csc A = \frac{1}{\sin A} = \frac{1}{12/13} = \frac{13}{12}$



Lesson 8.1.3

8-33.

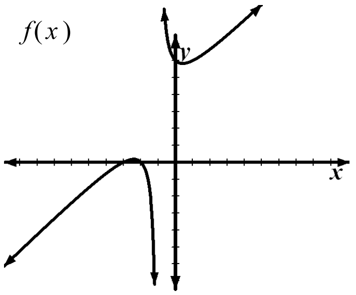
a. $f(x): x = -1$, $g(x): x = -2$, these values would cause division by zero.

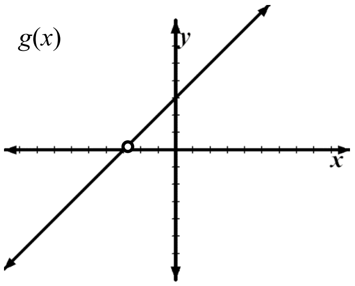
b.

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$f(x)$	–1.5	–0.67	0	0	und	6	6	6.67	7.5	8.4	9.33

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$g(x)$	–2	1	0	und	2	3	4	5	6	7	8

c.





d. In the table y = error. The function is not defined. There appears to be a vertical asymptote for one of the graphs but the other looks linear.

8-34.

They have similar form; domain is all real numbers except one point.

8-35.

$f(x)$ has a vertical asymptote, $g(x)$ has a hole. Common factor in numerator and denominator creates a hole in the graph of a rational function.

8-36.

$y_1 = \frac{x^2-4}{x+2} = \frac{(x+2)(x-2)}{x+2}$ Hole at $x = -2$.

$y_2 = \frac{x-6}{(x+3)(x+4)} = \frac{x-6}{x^2+7x+12}$ Vertical asymptotes at $x = -3$ and $x = -4$.

Horizontal asymptote at $y = 0$.

$y_3 = \frac{4x}{x^2+9}$ Horizontal asymptote at $y = 0$.

8-37.

They simplify to the same function, but have different discontinuities. $f(x) = \frac{x}{x+2}$ has a vertical asymptote at $x = -2$.

$g(x) = \frac{x^2-2x}{x^2-4} = \frac{x(x-2)}{(x+2)(x-2)} = \frac{x}{x+2}$ looks like $f(x)$ but has a hole at $x = 2$.

8-38.

Answers will vary. Examples are given.

a. $\frac{1}{x-3}$ b. $\frac{x^2-9}{x-3}$ c. $\frac{x^3-x}{x^2-1}$

d. $\frac{x^2+2x-24}{x-4}$ e. $\frac{x-2}{x^2+x-6}$ f. $\frac{(x+2)(x^2-9)}{x+2}$

Review and Preview 8.1.3

8-40.

a. $\lim\limits_{x\to-\infty}\frac{x^2+3}{2x^2-5x}=\lim\limits_{x\to-\infty}\frac{x^2/x^2+3/x^2}{2x^2/x^2-5x/x^2}=\lim\limits_{x\to-\infty}\frac{1+3/x^2}{2-5/x}=\frac{1}{2}=0.5$

b. $\lim\limits_{x\to-\infty}\frac{5x^3+7x}{x^4-4x}=\lim\limits_{x\to-\infty}\frac{5x^3/x^3+7x/x^3}{x^4/x^3-4x/x^3}=\lim\limits_{x\to-\infty}\frac{5+7/x^2}{x-4/x^2}=0$

c. $\lim\limits_{x\to-\infty}\frac{-6x^3+8x^2}{15x^2-2}=\lim\limits_{x\to-\infty}\frac{-6x^3/x^2+8x^2/x^2}{15x^2/x^2-2/x^2}=\lim\limits_{x\to-\infty}\frac{-6x+8}{15-2/x^2}=\infty$

8-41.

a. $\lim\limits_{x\to\infty}\frac{5x^2+2x-3}{x^3+1}=0$

b. $\lim\limits_{x\to\infty}\frac{2x^3-7}{3x^3+4x-5}=\frac{2}{3}$

c. $\lim\limits_{x\to 0}\frac{2x^3-7}{3x^3+4x-5}=\frac{2(0^3)-7}{3(0^3)+4(0)-5}=\frac{7}{5}$

8-42.

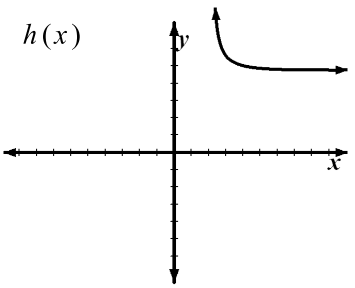
$\lim\limits_{x\to\infty}g(x)=\lim\limits_{x\to\infty}\frac{3x-\sqrt{x}}{\sqrt{x^2-5}}=\lim\limits_{x\to\infty}\frac{\frac{3x}{x}-\sqrt{x/x^2}}{\sqrt{x^2/x^2-5/x^2}}=\lim\limits_{x\to\infty}\frac{3-\sqrt{1/x}}{\sqrt{1-5/x^2}}=3$ The limit equals 3.

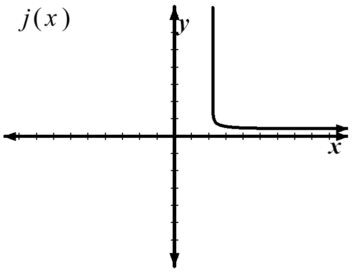
8-43.

a. $\lim\limits_{x\to\infty}h(x)=2+\lim\limits_{x\to\infty}\frac{3x-\sqrt{x}}{\sqrt{x^2-5}}=2+3=5$

b. $\lim\limits_{x\to\infty}j(x)=\lim\limits_{x\to\infty}\log\left(\frac{3x-\sqrt{x}}{\sqrt{x^2-5}}\right)=\log 3$

c. See graphs at right.





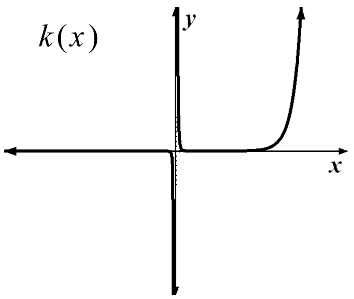
8-44.

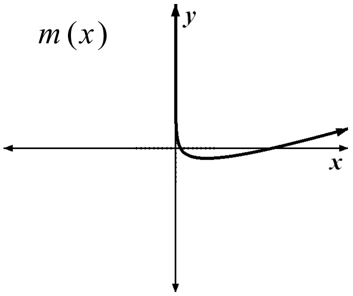
a. $m(x)=\log\left(\frac{2^x}{x^5}\right)=\log(2^x)-\log(x^5)=x\log 2-5\log x\approx 0.301x-5\log x$

b. $\lim\limits_{x\to\infty} k(x)=\lim\limits_{x\to\infty}\left(\frac{2^x}{x^5}\right)=\infty$. The dominant term is 2^x.

$\lim\limits_{x\to\infty} m(x)=\lim\limits_{x\to\infty}\log\left(\frac{2^x}{x^5}\right)=\lim\limits_{x\to\infty}\log(2^x)-\lim\limits_{x\to\infty}\log(x^5)=\infty$

c.





8-45.

a. Area of a side = L^2· Surface area of a cube $=S=6L^2$

b. Length of the diagonal on the face of the cube: $D_1{}^2=L^2+L^2$.

$D_1{}^2=2L^2$

$D_1=L\sqrt{2}$

Length of the longest diagonal: $D^2=L^2+(\sqrt{2}L)^2$

$D^2=L^2+2L^2$

$D^2=3L^2$

$D=L\sqrt{3}$

c. $S=6L^2$

$\sqrt{S/6}=L$

$D=\sqrt{S/6}\cdot\sqrt{3}=\sqrt{3S/6}=\sqrt{S/2}$

8-46.

a. Initial value = \$1.87, multiplier = $1+0.02=1.02$, time is in months.

b. Initial value = \$12,000, multiplier = $1-0.12=0.88$, time is in years.

c. Initial value = \$1,000, multiplier = $1+0.30=1.30$, time is in hours.

d. Initial value = \$5,000, multiplier = $1+0.01=1.01$, time is in months.

8-47.

a. $y=5000\cdot 1.01^{12}$

$y=\$5,634.13$

b. $y=5000\cdot 1.02^{4}$

$y=\$5,412.16$

c. $y=5000\cdot 1.001^{52}$

$y=\$5,266.74$

d. $y=5000\cdot 1.0002^{365}$

$y=\$5,378.61$

Lesson 8.1.4

8-48.

a. $\frac{360^\circ}{12} = 30^\circ$

b. $\cos 15^\circ = \frac{h}{1} \Rightarrow h = \cos 15^\circ$

c. $\sin 15^\circ = \frac{(1/2)b}{1} \Rightarrow 2\sin 15^\circ = b$

d. Area of one triangle: $A = \left(\frac{1}{2}\right) \cdot 2\sin 15^\circ \cdot \cos 15^\circ = \sin 15^\circ \cdot \cos 15^\circ$

Area of polygon: $A = 12\sin 15^\circ \cdot \cos 15^\circ = 3$

8-49.

a. $\tan 15^\circ = \frac{(1/2)b}{1} \Rightarrow \tan 15^\circ = \frac{1}{2}b \Rightarrow 2\tan 15^\circ = b$

b. Area of one triangle: $A = \left(\frac{1}{2}\right) \cdot 2\tan 15^\circ \cdot 1 = \tan 15^\circ$

Area of polygon: $A = 12\tan 15^\circ \approx 3.215$

c. 3 and 3.215

8-50.

a. $\frac{360^\circ}{n}$

b. 1

c. $\cos\left(\frac{360^\circ/2}{n}\right) = \cos\left(\frac{180^\circ}{n}\right)$

d. $2\sin\left(\frac{360^\circ/2}{n}\right) = 2\sin\left(\frac{180^\circ}{n}\right)$

e. n

f. $n\sin\left(\frac{180^\circ}{n}\right)\cos\left(\frac{180^\circ}{n}\right) = \frac{n}{2}\sin\left(\frac{360^\circ}{n}\right)$

8-51.

a. The smallest number of sides of a polygon is 3.

c. The plot shows the areas of the inscribed polygons as n increases.

8-52.

a. $n\tan\left(\frac{360^\circ}{n}\right)$

c. π

d. The value of π gets squeezed between the two functions which act as an upper bound and a lower bound.

8-53.

$$\lim_{n\to\infty} A_c(n) = \pi$$

Review and Preview 8.1.4

8-54.

a. $\lim\limits_{n\to\infty}\left(1+\dfrac{1}{n}\right)=\lim\limits_{n\to\infty}1+\lim\limits_{n\to\infty}\left(\frac{1}{n}\right)=1+0=1$

b. $\lim\limits_{n\to\infty}(1.01)^n=\infty$

8-55.

a. $1050(1.0175)^{20}$

b. $5000\left(1+\frac{.082}{12}\right)^{120}$

c. $P\left(1+\frac{r}{100n}\right)^{nt}$

8-56.

a. First four terms:

$n=1 \quad \frac{2-n}{3+n}=\frac{2-1}{3+1}=\frac{1}{4}$

$n=2 \quad \frac{2-n}{3+n}=\frac{2-2}{3+2}=\frac{0}{5}=0$

$n=3 \quad \frac{2-n}{3+n}=\frac{2-3}{3+3}=-\frac{1}{6}$

$n=4 \quad \frac{2-n}{3+n}=\frac{2-4}{3+4}=-\frac{2}{7}$

$\lim\limits_{n\to\infty}\frac{2-n}{3+n}=-\frac{n}{n}=-1$

b. First four terms:

$n=1 \quad \dfrac{(-2)^n}{3^n}=\dfrac{(-2)^1}{3^1}=-\frac{2}{3}$

$n=2 \quad \dfrac{(-2)^n}{3^n}=\dfrac{(-2)^2}{3^2}=\frac{4}{9}$

$n=3 \quad \dfrac{(-2)^n}{3^n}=\dfrac{(-2)^3}{3^3}=-\frac{8}{27}$

$n=4 \quad \dfrac{(-2)^n}{3^n}=\dfrac{(-2)^4}{3^4}=\frac{16}{81}$

$\lim\limits_{n\to\infty}\dfrac{(-2)^n}{3^n}=0$

c. First four terms:

$n=1 \quad \dfrac{(-2)^n}{2^n}=\dfrac{(-2)^1}{2^1}=-\frac{2}{2}=-1$

$n=2 \quad \dfrac{(-2)^n}{2^n}=\dfrac{(-2)^2}{2^2}=\frac{4}{4}=1$

$n=3 \quad \dfrac{(-2)^n}{2^n}=\dfrac{(-2)^3}{2^3}=-\frac{8}{8}=-1$

$n=4 \quad \dfrac{(-2)^n}{2^n}=\dfrac{(-2)^4}{2^4}=\frac{16}{16}=1$

There is no limit for $\lim\limits_{n\to\infty}\dfrac{(-2)^n}{2^n}$.

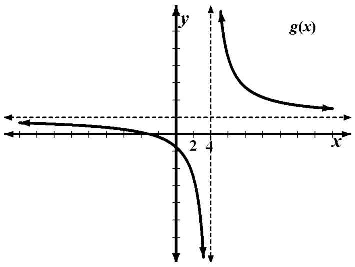
8-57.

See graph at right.

a. $\lim\limits_{x\to\infty}g(x)=2$

$\lim\limits_{x\to-\infty}g(x)=2$

c. $g(x)=\frac{3+2x}{x-4}=\frac{2(x-4)+11}{x-4}=2+\frac{11}{x-4}$



8-58.

a. Increasing

b. Decreasing

c. Decreasing

8-59.

$$\log_b \sqrt{\frac{RQ}{T}} = \frac{1}{2}\log_b\left(\frac{RQ}{T}\right) = \frac{1}{2}(\log_b RQ - \log_b T) =$$

$$\frac{1}{2}(\log_b R + \log_b Q - \log_b T) = \frac{1}{2}(5.4 + 1.2 - (-1.4)) = \frac{1}{2}(8) = 4$$

8-60.

Center of the circle**:** $(0,0)$. Slope from center of circle to point $(3,4)$: $m = \frac{4-0}{3-0} = \frac{4}{3}$

Equation of the line $y - 4 = -\frac{3}{4}(x-3)$ or $y = -\frac{3}{4}(x-3) + 4$

$\perp$ slope $= -\frac{3}{4}$

8-61.

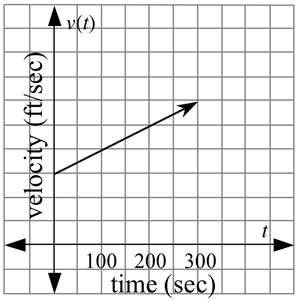
a. See graph at right below.

b. $y = 3 + 0.01t$

c. Area under the curve after 1 minute: $A = \frac{1}{2}\left(3 + (3 + 0.01(60))\right)(60) = 198$ feet

Area under the curve after 5 minutes: $A = \frac{1}{2}\left(3 + (3 + 0.01(300))\right)(300) = 1350$ feet

d. $A = \frac{1}{2}\left(3 + (3 + 0.01T)\right)T = 3T + 0.005T^2$



Lesson 8.2.1

8-62.

a. $\frac{1}{n}$ goes to zero so the expression in the parentheses goes to 1 and the limit might be 1, since $1^n = 1$.

b. Infinity, since the number inside the parentheses is larger than 1.

c. A number around 2.718… It should be surprising that this limit is neither 1 nor infinity.

8-63.

2.718281828

8-64.

a. e^3

b. $e^{0.2}$

8-65.

a. $1000\left(1 + \frac{0.06}{4}\right)^4$

b. $1000\left(1 + \frac{0.06}{12}\right)^{12}$

c. $1000\left(1 + \frac{0.06}{365}\right)^{365}$

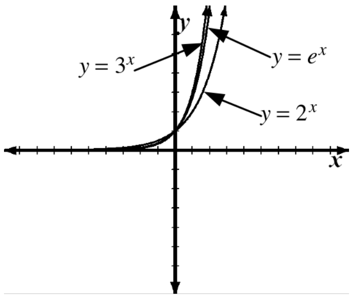
d. $1000\left(1 + \frac{0.06}{24 \cdot 365}\right)^{24 \cdot 365}$

e. $1000e^{0.06}$

f.
- a. \$1061.36
- b. \$10061.68
- c. \$1061.83
- d. \$1061.84
- e. more and more

g. $1200e^{0.075} = \$1293.46$

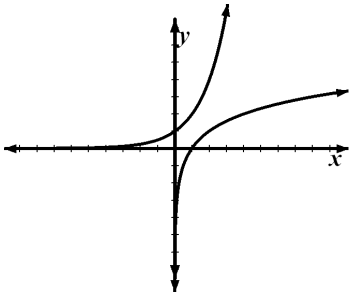
8-66.



8-67.

a. $y = \log_2 x$
See graph at right below.

b. $y = \log_e x$



8-68.

a. $\log_3\left(\frac{1}{3^4}\right) = \log_3 3^{-4} = -4$

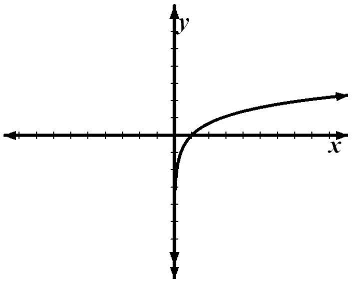
b. $\log_3 54 - \log_3 2 = \log_3\left(\frac{54}{2}\right) = \log_3 27 = \log_3 3^3 = 3$

c. $\ln\left(e^{12}\right) = \ln_e e^{12} = 12 \ln_e e = 12$

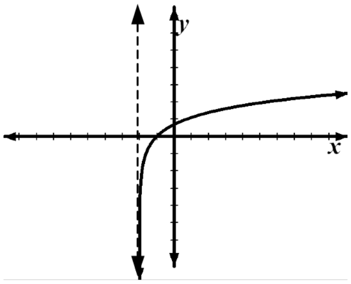
d. $\dfrac{5\ln(e^2)}{\ln(1/e)} = \dfrac{10 \ln e}{\ln e^{-1}} = \dfrac{10}{-1 \ln e} = -10$

8-69.

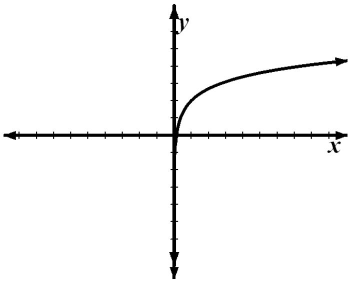
a.



b.



c.



Review and Preview 8.2.1

8-70.

$$\begin{aligned} 17.32 &= 58 \cdot e^{x+3} \\ 0.299 &= e^x e^3 \\ \frac{0.299}{e^3} &= e^x \\ 0.01489 &= e^x \\ \ln(0.01489) &= x \\ x &\approx -4.209 \end{aligned}$$

8-71.

a. $\sum_{k=0}^{4} \frac{1}{k!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} = 2.708333$

b. $\sum_{k=0}^{10} \frac{1}{k!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} + \frac{1}{7!} + \frac{1}{8!} + \frac{1}{9!} + \frac{1}{10!} = 2.71828$

c. $\lim_{n \to \infty} \sum_{k=0}^{n} \frac{1}{k!} = e$

8-72.

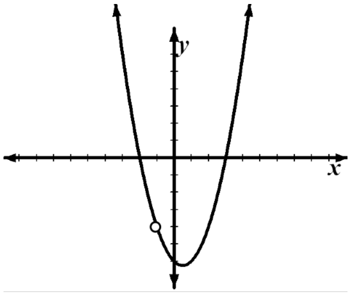
a. $f(-1) = \frac{(-1)^3 - 7(-1) - 6}{(-1+1)} = \frac{2}{0}$ which is undefined.

b. See division at right.

$\lim_{x \to -1} \frac{x^3 - 7x - 6}{x+1} = \lim_{x \to -1} (x^2 - x - 6) = (-1)^2 - (-1) - 6 = -4$

c. Hole at $x = -1$.

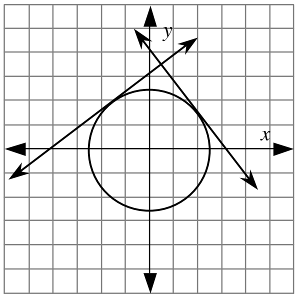
$$\frac{x^3-7x-6}{x+1} \Rightarrow \begin{array}{r} x^2 - x - 6 \\ x+1 \overline{)\, x^3 + 0x^2 - 7x - 6} \\ \underline{-(x^3 + x^2)} \\ -x^2 - 7x \\ \underline{-(-x^2 - 1x)} \\ -6x - 6 \\ \underline{-(-6x - 6)} \\ 0 \end{array}$$



8-73.

See graph at right.

Point of intersection: $(1, 7)$



8-74.

a. 0

b. 0

c. 3

d. 3

8-75.

$t_1 = 1$

$t_2 = 1 + 2(1) + 1 = 4$

$t_3 = 4 + 2(2) + 1 = 9$

$t_4 = 9 + 2(3) + 1 = 16$

$t_5 = 16 + 2(4) + 1 = 25$

8-76.

Let $a = x + 3$

$x^3a^4 + 2x^2a^3 = 0$

$x^2a^3(ax + 2) = 0$

$x^2a^3 = 0$ or $ax + 2 = 0$

If $ax + 2 = 0$

$(x + 3)x + 2 = 0$

$x^2 + 3x + 2 = 0$

$(x + 2)(x + 1) = 0$

$x = -2, -1$

If $x^2a^3 = 0$

$x^2(x + 3)^3 = 0$

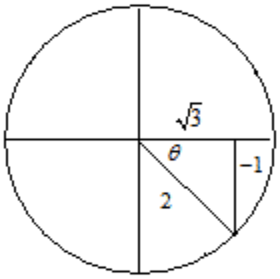
$x = -3, 0$

8-77.

x-intercepts: $0 = 32t - 16t^2 = -16t(2 - t) \Rightarrow t = 0, 2$
Vertex = avg. of x-intercepts or $t = 1$.
At $t = 1$, $s = 32(1) - 16(1^2) = 16$ ft .

8-78.

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$



8-79.

$$f(x) = \frac{2x^2+3x-2}{x^2-4} = \frac{(2x-1)(x+2)}{(x-2)(x+2)} = \frac{(2x-1)}{(x-2)}$$

Hole at $x = -2 \Rightarrow f(-2) = \frac{(2(-2)-1)}{-2-2} = \frac{-5}{-4} = \frac{5}{4} \Rightarrow \left(-2, \frac{5}{4}\right)$

Asymptotes at $x = 2$ and $y = 2$.

Lesson 8.2.2

8-80.

a. $A = 100e^{0.065 \cdot 1}$

$A = \$106.72$

b. $A = 100e^{0.065 \cdot 2}$

$A = \$113.89$

c. $A = 100e^{0.065 \cdot 3}$

$A = \$320.15$

d. $A = 100(e^{0.06})^{17}$

$A = \$301.92$

e. $A = 100(e^{0.06})^t$

8-81.

a. $r = 22$

$G = 1500 \cdot e^{22 \cdot t}$

$G = 1500 \cdot e^{22 \cdot (1/12)}$

$G = 1500 \cdot e^{11/6}$

$G = 9382 \text{ ZWD}$

t-value is $\frac{1}{12}$

b. $A = 500 \cdot \left(1 + \frac{1}{12}\right)^{12}$

$A = \$1,306.52$

c. $A = 500 \cdot e^{1 \cdot 1}$

$A = \$1,359.14$

8-82.

a. $A = 100 \cdot e^{0.05 \cdot 3} = 100e^{0.15}$

$A = \$116.18$

b. $200 = 100 \cdot e^{0.05 \cdot t}$

$2 = e^{0.05 \cdot t}$

$\ln 2 = \ln e^{0.05 \cdot t}$

$\ln 2 = 0.05t$

$t = \frac{\ln 2}{0.05} = 13.863$ years

c. $600 = 300 \cdot e^{0.05 \cdot t}$

$2 = e^{0.05 \cdot t}$

$\ln 2 = \ln e^{0.05 \cdot t}$

$\ln 2 = 0.05t$

$t = \frac{\ln 2}{0.05} = 13.863$ years

8-83.

$$\lim_{n \to \infty} P\left(1 + \frac{r}{n}\right)^{nt} = \lim_{n \to \infty} P\left(1 + \frac{1}{n/r}\right)^{nt}$$

Let $k = \frac{n}{r}$ $\lim_{k \to \infty} P\left(1 + \frac{1}{k}\right)^{krt} = P \lim_{k \to \infty}\left(\left(1 + \frac{1}{k}\right)^{k}\right)^{rt} = Pe^{rt}$

8-84.

a. $C = km^t$

$C = 20(1.07)^{10} = \$39.34$

b. $A = Pe^{rt}$

$A = 20e^{0.07 \cdot 10} = \$40.28$

c. This is a matter of opinion.

8-85.

$y = ae^{kx} = a(e^k)^x$

If $a = c$ and $m = e^k$ then $y = c \cdot m^x$.

8-86.

a. After another 5730 years there would be 25% remaining. After another 5730 years there would be 12.5% remaining.

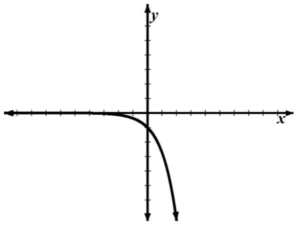
b.
$$\begin{aligned} 0.5 &= e^{k\cdot 5730} \\ -\ln 2 &= \ln e^{5730k} \\ -\ln 2 &= 5730k \\ k &= -\tfrac{\ln 2}{5730} \\ y &= a\cdot e^{-\frac{\ln 2}{5730}x} \end{aligned}$$

c.
$$\begin{aligned} 0.78 &= e^{-\frac{\ln 2}{5730}x} \\ \ln 0.78 &= \ln e^{-\frac{\ln 2}{5730}x} \\ \ln 0.78 &= -\tfrac{\ln 2}{5730}x \\ x &= \ln 0.78 \cdot -\tfrac{5730}{\ln 2} \\ &\sim 2054 \text{ years old} \end{aligned}$$

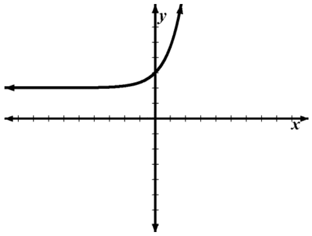
Review and Preview 8.2.2

8-87.

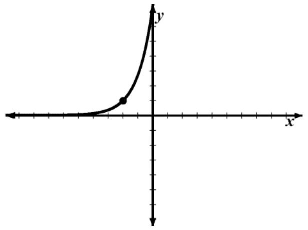
a. $f(x) = -e^x$



b. $g(x) = e^x + 2$



c. $h(x) = e^{x+2}$



8-88.

Compounded quarterly: $A = 10{,}000\left(1+\frac{0.0525}{4}\right)^4 = \$10{,}535.43$

Continuously compounded: $A = 10{,}000e^{0.05\cdot 1} = \$10{,}512.71$

The quarterly-compounding account is better.

8-89.

a.
$$\begin{aligned} a_n &= 6\cdot\left(\tfrac{1}{3}\right)^{n-1} \\ a_1 &= 6\cdot\left(\tfrac{1}{3}\right)^0 = 6 \\ a_2 &= 6\cdot\left(\tfrac{1}{3}\right)^1 = 2 \\ a_3 &= 6\cdot\left(\tfrac{1}{3}\right)^2 = 6\cdot\left(\tfrac{1}{9}\right) = \tfrac{2}{3} \\ &\lim_{n\to\infty} 6\cdot\left(\tfrac{1}{3}\right)^{n-1} = 0 \end{aligned}$$

b.
$$\begin{aligned} a_n &= -2\cdot(1.1)^{n-1} \\ a_1 &= -2\cdot(1.1)^0 = -2 \\ a_2 &= -2\cdot(1.1)^{2-1} = -2.2 \\ a_3 &= -2\cdot(1.1)^{3-1} = -2\cdot 1.21 = -2.42 \\ &\lim_{n\to\infty} -2\cdot(1.1)^{n-1} = -\infty \end{aligned}$$

8-90.

$a_1 = 8$ $\quad a_3 = -\frac{1}{2}\cdot -4 = 2 \quad a_5 = -\frac{1}{2}\cdot -1 = 0.5$

$a_2 = -\frac{1}{2}\cdot 8 = -4 \quad a_4 = -\frac{1}{2}\cdot 2 = -1$

8-91.

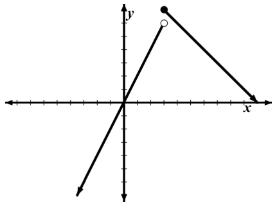
a. 4

b. 4

c. $10^{3\log 4} = 10^{\log_{10} 4^3} = 4^3 = 64$

d. $e^{3\ln 4} = e^{\ln_e 4^3} = 4^3 = 64$

8-92.



8-93.

a. $\lim\limits_{x\to\infty} \frac{5x^2+2x-3}{x^3+1} = \lim\limits_{x\to\infty} \frac{5x^2/x^2+2x/x^2-3/x^2}{x^3/x^2+1/x^2} = \lim\limits_{x\to\infty} \frac{5+2/x-3/x^2}{x+1/x^2} = 0$

b. $\lim\limits_{x\to 0} \frac{2x^3-7}{3x^3+4x-5} = \frac{-7}{-5} = \frac{7}{5}$

8-94.

$$\tan\left(\frac{\pi}{2}-\theta\right) = \frac{\sin(\pi/2-\theta)}{\cos(\pi/2-\theta)} = \frac{\sin(\pi/2)\cos\theta-\cos(\pi/2)\sin\theta}{\cos(\pi/2)\cos\theta+\sin(\pi/2)\sin\theta} = \frac{\cos\theta}{\sin\theta} = \cot\theta$$

8-95.

$$(a+b)^4-(a-b)^4 =$$

$$a^4+4a^3b+6a^2b^2+4ab^3+b^4-(a^4-4a^3b+6a^2b^2-4ab^3+b^4) =$$

$$a^4+4a^3b+6a^2b^2+4ab^3+b^4-a^4+4a^3b-6a^2b^2+4ab^3-b^4 =$$

$$4a^3b+4ab^3 = 8ab(a^2+b^2)$$

8-96.

$$8ab(a^2+b^2)$$

$$8\cos x\sin x(\cos^2 x+\sin^2 x) =$$

$$8\cos x\sin x =$$

$$4(2\cos x\sin x) = 4\sin(2x)$$

Lesson 8.2.3

8-97.

a. The adjacent terms are proportional to each other.

b. $S = \frac{1}{3} + \left(\frac{1}{3} \cdot \frac{1}{3}\right) + \left(\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}\right) + \ldots$

$a_n = \left(\frac{1}{3}\right)^n$

8-98.

a. $S_n = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots. + \frac{1}{3^n}$

$\frac{1}{3} S_n = \quad \frac{1}{9} + \frac{1}{27} + \ldots. + \frac{1}{3^n} + \frac{1}{3^{n+1}}$

b. $\frac{2}{3} S_n = \frac{1}{3} - \frac{1}{3^{n+1}}$

c. $S_n = \frac{1}{2} - \frac{1}{2 \cdot 3^n}$

d. $S_5 = \frac{1}{2} - \frac{1}{2 \cdot 3^5} = \frac{1}{2} - \frac{1}{486} \approx 0.4979$

$S_7 = \frac{1}{2} - \frac{1}{2 \cdot 3^7} = \frac{1}{2} - \frac{1}{4374} \approx 0.4998$

$S_9 = \frac{1}{2} - \frac{1}{2 \cdot 3^9} = \frac{1}{2} - \frac{1}{39366} \approx 0.49997$

The sums seem to be approaching 0.5.

e. $\lim\limits_{n \to \infty} \left(\frac{1}{2} - \frac{1}{2 \cdot 3^n}\right) = \lim\limits_{n \to \infty} \frac{1}{2} - \lim\limits_{x \to \infty} \frac{1}{2 \cdot 3^n} = \frac{1}{2} + 0 = \frac{1}{2}$

8-99.

$$S = 6 + 4 + \frac{8}{3} + \frac{16}{9} + \ldots = 6\left(1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \ldots\right) = 6\left(\left(\frac{2}{3}\right)^0 + \left(\frac{2}{3}\right)^1 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \ldots\right)$$

$$S = 6\sum_{n=0}^{\infty} \frac{2}{3}^n$$

$$S_n = \lim_{x \to \infty} \frac{6(1-(2/3)^n)}{1-2/3} = \frac{6(1-0)}{1/3} = 18$$

8-100.

a. $S_n = \frac{a(1-r^n)}{1-r}$

b. $\lim\limits_{n \to \infty} r^n = 0$

c. $\lim\limits_{n \to \infty} \frac{a(1-r^n)}{1-r} = \frac{a}{1-r}$

8-101.

a. $S_n = 120\left(\frac{1}{2}\right)^n$

$S = \frac{120}{1-1/2} = \frac{120}{1/2} = 240$

b. $S_n = 81\left(-\frac{2}{3}\right)^n$

$S = \frac{81}{1-(-2/3)} = \frac{81}{5/3} = \frac{243}{5} = 48.6$

c. $S_n = 8\left(\frac{3}{2}\right)^n$

Since $r > 1$, the sum goes to infinity. The formula gives a sum of -16.

8-102.

r is greater than 1. The terms get bigger. $\lim\limits_{n \to \infty} a_n = \infty$. Since we keep adding larger quantities the sum will go to infinity.

8-103.

a. $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{5x}=e^{5}$

b. $\lim\limits_{h\to\infty}\left(1+\left(-\frac{1}{h}\right)\right)^{h}=e^{-1}=\frac{1}{e}$

c. $\lim\limits_{z\to\infty}\left(1+\left(\frac{1}{5}\right)\left(\frac{1}{z}\right)\right)^{z}=e^{1/5}$

Review and Preview 8.2.3

8-104.

a. $3.33333\ldots$

b. $$\begin{aligned}10N&=3.33333\ldots\\N&=0.33333\ldots\end{aligned}$$

c. $$\begin{aligned}10N&=3.33333\ldots\\N&=0.33333\ldots\\9N&=3.00000\end{aligned}$$

d. $$\begin{aligned}9N&=3\\N&=\tfrac{1}{3}\end{aligned}$$

8-105.

a. $$\begin{aligned}10N&=5.55555\ldots\\N&=0.55555\ldots\\9N&=5\\N&=\tfrac{5}{9}\end{aligned}$$

b. $$\begin{aligned}10N&=7.77777\ldots\\N&=0.77777\ldots\\9N&=7\\N&=\tfrac{7}{9}\end{aligned}$$

c. $$\begin{aligned}10N&=9.99999\ldots\\N&=0.99999\ldots\\9N&=9\\N&=\tfrac{9}{9}=1\end{aligned}$$

8-106.

$$\begin{aligned}100N&=36.363636\ldots\\N&=\quad 0.363636\ldots\\99N&=36\\N&=\tfrac{36}{99}=\tfrac{4}{11}\end{aligned}$$

8-107.

a. $$\begin{aligned}100N&=60.60\overline{60}\\N&=\quad 0.60\overline{60}\\99N&=60\\N&=\tfrac{60}{99}=\tfrac{20}{33}\end{aligned}$$

b. $$\begin{aligned}100N&=12.12\overline{12}\\N&=\quad 0.12\overline{12}\\99N&=12\\N&=\tfrac{12}{99}=\tfrac{4}{33}\\1.12\overline{12}&=\tfrac{33}{33}+\tfrac{4}{33}=\tfrac{37}{33}\end{aligned}$$

c. $$\begin{aligned}1000N&=123.\overline{123}\\N&=\qquad 0.\overline{123}\\999N&=123\\N&=\tfrac{123}{999}=\tfrac{41}{333}\end{aligned}$$

8-108.

$$\begin{aligned}(1+0.1)^4&=1^4+4\cdot1^3\cdot0.1+6\cdot1^2\cdot0.1^2+4\cdot1\cdot0.1^3+0.1^4\\&=1+4\cdot0.1+6\cdot0.01+4\cdot0.001+0.0001\\&=1+0.4+0.06+0.004+0.0001\\&=1.4641\end{aligned}$$

8-109.

$$y = a \cdot b^x + 10 \;\Rightarrow\; 6 = a \cdot b^4 + 10 \quad 8 = a \cdot b^7 + 10 \;\Rightarrow\; \frac{-2 = a \cdot b^7}{-4 = a \cdot b^4} \Rightarrow \frac{1}{2} = b^3 \quad b = \frac{1}{\sqrt[3]{2}}$$

$$-4 = a \cdot b^4 \qquad -2 = a \cdot b^7$$

$$-4 = a \cdot \left(\frac{1}{\sqrt[3]{2}}\right)^4 \Rightarrow$$

$$y = -8\sqrt[3]{2}\left(\frac{1}{\sqrt[3]{2}}\right)^x + 10$$

$$a = \frac{-4}{\left(\frac{1}{\sqrt[3]{2}}\right)^4} = -4\left(\sqrt[3]{2}\right)^4 = -8\sqrt[3]{2}$$

y-intercept: $y = -8\sqrt[3]{2}\left(\frac{1}{\sqrt[3]{2}}\right)^0 + 10$

$$\Rightarrow\; y = 10 - 8\sqrt[3]{2} \approx -0.079$$

8-110.

Average rate = $\frac{\text{total distance}}{\text{total time}}$ $\quad 60 = \frac{250}{t_1 + t_2} = \frac{250}{\frac{125}{50} + \frac{125}{r_2}} \;\Rightarrow\; \frac{125}{50} + \frac{125}{r_2} = \frac{250}{60}$

$$\frac{125}{r_2} = \frac{250}{60} - \frac{125}{50} = \frac{500}{300} = \frac{5}{3} \;\Rightarrow\; \frac{125}{5} = \frac{r_2}{3} \;\Rightarrow\; r_2 = \frac{125(3)}{5} = 75$$

8-111.

a. $2P = Pe^{rt}$

$2 = e^{rt}$

$\ln 2 = \ln e^{rt}$

$\ln 2 = rt$

$t = \frac{\ln 2}{r}$

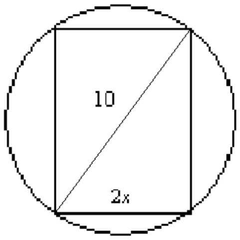
b. $r = \frac{R}{100}$

c. $t = \frac{\ln 2}{r}$

$t = \frac{0.6931}{\frac{R}{100}} = 0.6931 \cdot \frac{100}{R} \approx \frac{70}{R}$

d. We use 70 because it makes the mental arithmetic easier.

8-112.



$(2x)^2 + l^2 = 10^2$

$4x^2 + l^2 = 100$

$l^2 = 100 - 4x^2$

$l = 2\sqrt{25 - x^2}$

a. Perimeter of the rectangle: $P = 2x + 2x + 2\sqrt{25 - x^2} + 2\sqrt{25 - x^2} = 4x + 4\sqrt{25 - x^2}$

b. $A = (2x)\left(2\sqrt{25 - x^2}\right) = 4x\sqrt{25 - x^2}$

8-113.

a. $\ln x = 3$

$e^{\ln x} = e^3$

$x = e^3$

b. $e^x = 200$

$\ln e^x = \ln 200$

$x = \ln 200$

c. $5\ln(x+1) = 20$

$\ln(x+1) = 4$

$e^{\ln(x+1)} = e^4$

$x + 1 = e^4$

$x = e^4 - 1$

d. $e^{(x+2)} + 20 = 100$

$e^{(x+2)} = 80$

$\ln e^{(x+2)} = \ln 80$

$x + 2 = \ln 80$

$x = \ln 80 - 2$

Lesson 8.2.4

8-114.

Infinity

8-115.

a. $\sum_{n=1}^{10} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} = 2.929$

b. $\sum_{n=1}^{20} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{18} + \frac{1}{19} + \frac{1}{20} = 3.598$

c. 5.187

d. 6.793

8-116.

b. $\frac{1}{n^2+n} < \frac{1}{n^2}$ since $n^2 + n > n^2$

8-117.

a. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \frac{1}{15}$

b. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}$

d. It is larger than the series that adds ½ an infinite number of times. Since that series has an infinite sum, the harmonic series (which must be larger) must also be infinite.

e. Infinity.

8-118.

First is the largest, last is the smallest.

8-119.

Infinity. If not, the harmonic series would have a finite sum.

8-120.

a. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \frac{1}{9} - \frac{1}{10}$

b. $\left(1 - \frac{1}{2}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + \ldots > 0$

c. $1 - \left(\frac{1}{2} - \frac{1}{3}\right) - \left(\frac{1}{4} - \frac{1}{5}\right) - \cdots < 1$

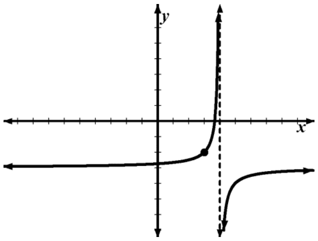
d. The exact sum is $\ln 2 = 0.693$.

Review and Preview 8.2.4

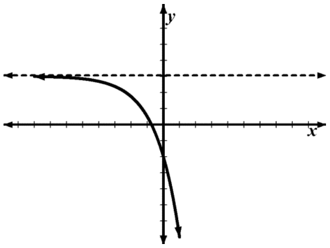
8-121.

Sample graphs:

a.



b.



8-122.

$x_{n+1} = \frac{1}{x_n+1}$ $\quad x_2 = \frac{1}{1+1} = \frac{1}{2}$ $\quad x_4 = \frac{1}{2/3+1} = \frac{1}{5/3} = \frac{3}{5}$

$x_1 = 1$ $\quad x_3 = \frac{1}{1/2+1} = \frac{1}{3/2} = \frac{2}{3}$ $\quad x_5 = \frac{1}{3/5+1} = \frac{1}{8/5} = \frac{5}{8}$

8-123.

a. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

$a_n = \left(\frac{1}{2}\right)^n$

Sum $= \frac{1}{1-1/2} = \frac{1}{1/2} = 2$

b. $16 + 4 + 1 + \frac{1}{4} + \frac{1}{16} + \ldots$

$a_n = 16\left(\frac{1}{4}\right)^n$

Sum $= \frac{16}{1-1/4} = \frac{16}{3/4} = \frac{16\cdot 4}{3} = \frac{64}{3}$

c. $a + 0.9a + 0.81a + 0.729a + \ldots$

$r_n = a\left(\frac{9}{10}\right)^n$

Sum $= \frac{a}{1-9/10} = \frac{a}{1/10} = 10a$

d. The sum would be ∞ because $r > 1$.

8-124.

$$x - \frac{1}{x} = 1$$
$$x^2 - 1 = x$$
$$x^2 - x - 1 = 0$$
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$
$$x = \frac{1 \pm \sqrt{5}}{2}$$

8-125.

$$\frac{1+\sin\theta}{(1-\sin\theta)(1+\sin\theta)} + \frac{1-\sin\theta}{(1+\sin\theta)(1-\sin\theta)} = \frac{1+\sin\theta+1-\sin\theta}{1-\sin^2\theta} == \frac{2}{\cos^2\theta} = 2\sec^2\theta$$

8-126.

a. $f(2) = 10 - 2^2 = 6$

$f(4) = 10 - 4^2 = -6$

$\mathrm{m} = \frac{-6-6}{4-2} = -\frac{12}{2} = -6$

b. $f(2) = 10 - 2^2 = 6$

$f(3) = 10 - 3^2 = 1$

$\mathrm{m} = \frac{6-1}{2-3} = -\frac{5}{1} = -5$

c. $f(2) = 10 - 2^2 = 6$

$f(2.5) = 10 - 2.5^2 = 3.75$

$\mathrm{m} = \frac{6-3.75}{2-2.5} = -\frac{2.25}{0.5} = -4.5$

d. $f(2) = 10 - 2^2 = 6$

$f(2.1) = 10 - 2.1^2 = 5.59$

$\mathrm{m} = \frac{6-5.59}{2-2.1} = -\frac{0.41}{0.1} = -4.1$

e. As the two points get closer and closer, the slope of the line gets closer to -4.

8-127.

a. $\log(x+3) = 2$

$10^{\log(x+3)} = 10^2$

$x + 3 = 100$

$x = 97$

b. $\ln(x+3) = 2$

$e^{\ln(x+3)} = e^2$

$x + 3 = e^2$

$x = e^2 - 3$

c. $\ln 3 + \ln x = 4$

$\ln(3x) = 4$

$e^{\ln(3x)} = e^4$

$3x = e^4$

$x = \frac{e^4}{3}$

d. $\ln(x+4) - \ln x = 2$

$\ln\left(\frac{x+4}{x}\right) = 2$

$e^{\ln\left(\frac{x+4}{x}\right)} = e^2$

$\frac{x+4}{x} = e^2$

$1 + \frac{4}{x} = e^2$

$\frac{4}{x} = e^2 - 1$

$x = \frac{4}{e^2 - 1}$

8-128.

a. $f(1) = 8 - 1^1 = 7$

$f(4) = 8 - 4^2 = -8$

$m = \frac{-8-7}{4-1} = \frac{-15}{3} = -5$

b. $f(1) = 2^1 = 2$

$f(4) = 2^4 = 16$

$m = \frac{16-2}{4-1} = \frac{14}{3} = 4.667$

8-129.

$y = ae^{kx}$

$y = 100e^{kx}$

$85 = 100e^{4k}$

$0.85 = e^{4k}$

$\ln 0.85 = \ln e^{4k}$

$-0.1625 = 4k$

$k = -0.0406$

$50 = 100e^{-0.0406x}$

$0.5 = e^{-0.041x}$

$\ln 0.5 = \ln e^{-0.041x}$

$-0.6925 = -0.0406x$

$x = 17.06 \text{ hours}$

8-130.

$$\begin{aligned} e^{kx} &= m^x \\ \ln e^{kx} &= \ln m^x \\ kx &= x \ln m \\ k &= \ln m \end{aligned}$$

Lesson 8.2.5

8-131.

Month	Baby rabbits	Teen rabbits	Parent rabbits	Total rabbits
1	1	0	0	1
2	0	1	0	1
3	1	0	1	2
4	1	1	1	3
5	2	1	2	5
6	3	2	3	8
7	5	3	5	13
8	8	5	8	21
9	13	8	13	34
10	21	13	21	55
11	34	21	34	89
12	55	34	55	144

8-132.

a. $3+5=8$

$5+8=13$

$8+13=21$

$13+21=34$

$21+34=55$

$34+55=89$

$55+89=144$

b. $a_{n+2}=a_n+a_{n+1}$

8-133.

a. $\frac{a_2}{a_1} = \frac{1}{1} = 1$

$\frac{a_3}{a_2} = \frac{2}{1} = 2$

$\frac{a_4}{a_3} = \frac{3}{2} = 1.3$

$\frac{a_5}{a_4} = \frac{5}{3} = 1.67$

$\frac{a_6}{a_5} = \frac{8}{5} = 1.6$

$\frac{a_7}{a_6} = \frac{13}{8} = 1.625$

$\frac{a_8}{a_7} = \frac{21}{13} = 1.615$

$\frac{a_9}{a_8} = \frac{34}{21} = 1.619$

$\frac{a_{10}}{a_9} = \frac{55}{34} = 1.6176$

$\frac{a_{11}}{a_{10}} = \frac{89}{55} = 1.6182$

$\frac{a_{12}}{a_{11}} = \frac{144}{89} = 1.618$

b. Yes, 1.618.

8-134.

a. $\frac{a_1}{a_2} = \frac{1}{1} = 1$

$\frac{a_2}{a_3} = \frac{1}{2} = 0.5$

$\frac{a_3}{a_4} = \frac{2}{3} = 0.666667$

$\frac{a_4}{a_5} = \frac{3}{5} = 0.6$

$\frac{a_5}{a_6} = \frac{5}{8} = 0.625$

$\frac{a_6}{a_7} = \frac{8}{13} = 0.6154$

$\frac{a_7}{a_8} = \frac{13}{21} = 0.6190$

$\frac{a_8}{a_9} = \frac{21}{34} = 0.6176$

$\frac{a_9}{a_{10}} = \frac{34}{55} = 0.6182$

$\frac{a_{10}}{a_{11}} = \frac{55}{89} = 0.618$

$\frac{a_{11}}{a_{12}} = \frac{89}{144} = 0.6181$

b. Yes, 0.618.

8-135.

$a_{1000} = 1.618033989$. No, it reaches this value by the 25th term. The limit of the calculator's display is reached by this point so it is forced to round off.

8-136.

a. $x_1 = 1$

$x_2 = 1(1-5)+5 = -4+5 = 1$

$x_3 = 1(1-5)+5 = -4+5 = 1$

$x_4 = 1(1-5)+5 = -4+5 = 1$

$x_5 = 1(1-5)+5 = -4+5 = 1$

b. $x_1 = 2$

$x_2 = 2(2-5)+5 = -6+5 = -1$

$x_3 = -1(-1-5)+5 = 6+5 = 11$

$x_4 = 11(11-5)+5 = 66+5 = 71$

$x_5 = 71(71-5)+5 = 4686+5 = 4691$

8-137.

$x = x(x-5)+5$. The only two values are 1 and 5.

$x = x^2 - 5x + 5$

$0 = x^2 - 6x + 5$

$0 = (x-5)(x-1)$

$x = 1, 5$

8-138.

a.
$t_1 = 2$
$t_2 = 2(2) - 1 = 4 - 1 = 3$
$t_3 = 2(3) - 1 = 6 - 1 = 5$
$t_4 = 2(5) - 1 = 10 - 1 = 9$
$t_5 = 2(9) - 1 = 18 - 1 = 17$

b.
$a_1 = 1$
$a_2 = 1 + 2(1) = 1 + 2 = 3$
$a_3 = 3 + 2(2) = 3 + 4 = 7$
$a_4 = 7 + 2(3) = 7 + 6 = 13$
$a_5 = 13 + 2(4) = 13 + 8 = 21$

c.
$f_1 = 1$
$f_2 = 1$
$f_3 = 1 + 1 = 2$
$f_4 = 1 + 2 = 3$
$f_5 = 2 + 3 = 5$

Review and Preview 8.2

8-139.

a.
$a_1 = 1$
$a_2 = 1000$
$a_3 = 1 + 1000 = 1001$
$a_4 = 1000 + 1001 = 2001$
$a_5 = 1001 + 2001 = 3002$
$a_6 = 2001 + 3002 = 5003$
$a_7 = 3002 + 5003 = 8005$
$a_8 = 5003 + 8005 = 13008$
$a_9 = 8005 + 13008 = 21,013$
$a_{10} = 13008 + 21013 = 34,021$
$a_{11} = 21013 + 34021 = 55,034$

b. $\frac{a_{11}}{a_{10}} = \frac{55,034}{34,021} = 1.618$

c. $\frac{a_{10}}{a_{11}} = \frac{34,021}{55,034} = 0.618$

8-140.

$$\frac{a_{n+1}}{a_n} = 1.618$$

8-141.

$a_{n+1} = a_n + 2n + 1$ General term: $a_n = n^2$
$a_1 = 1$
$a_2 = 1 + 2(1) + 1 = 4$
$a_3 = 4 + 2(2) + 1 = 9$
$a_4 = 9 + 2(3) + 1 = 16$
$a_5 = 16 + 2(4) + 1 = 25$

8-142.

a. 1.618

b.
$$L=\sqrt{1+L}$$
$$L^2=1+L$$
$$L^2-L-1=0$$
$$L=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-1)}}{2(1)}$$
$$L=\frac{1\pm\sqrt{5}}{2}$$

8-143.

$$\left(\frac{2x^{-2}y^3}{x^{-5}y^7}\right)^3=\left(\frac{2x^5y^3}{x^2y^7}\right)^3=\left(\frac{2x^3}{y^4}\right)^3=\frac{8x^9}{y^{12}}$$

8-144.

$$f(x+h)=(x+h)^2+(x+h)+1$$
$$=x^2+2xh+h^2+x+h+1$$

$$f(x+h)-f(x)=$$
$$x^2+2xh+h^2+x+h+1-(x^2-x-1)=$$
$$2xh+h+h^2$$

8-145.

a.
$$100N=45.454545\ldots$$
$$N=0.454545\ldots$$
$$99N=45$$
$$N=\tfrac{45}{99}=\tfrac{5}{11}$$

b.
$$100N=81.818181\ldots$$
$$N=0.818181\ldots$$
$$99N=81$$
$$N=\tfrac{81}{99}=\tfrac{9}{11}$$
$$6.818181\ldots=\tfrac{66}{11}+\tfrac{9}{11}=\tfrac{75}{11}$$

c.
$$1000N=675.675675675\ldots$$
$$N=0.675675675\ldots$$
$$999N=675$$
$$N=\tfrac{675}{999}=\tfrac{675}{999}=\tfrac{25}{37}$$

Lesson 8.2.6

8-146.

a. $1 = 2^1 - 1 = 2 - 1 = 1$

b. Assume that $1 + \cdots + 2^{k-1} = 2^k - 1$.

c. If $1 + \cdots + 2^{n-1} = 2^n - 1$, then $1 + \cdots + 2^{n-1} + 2^n =$.

$$2^n - 1 + 2^n =$$

$$2 \cdot 2^n - 1 = 2^{n+1} - 1$$

d. Hence, we have established the truth of the statement for $n = 1$ and proven the induction step, then the statement is true for all positive integers.

8-147.

This is Gauss' method.

Step1: For $n = 1$, $1 = \frac{1(1+1)}{1}\ \frac{1 \cdot 2}{2} = 1$, so the statement is true for $n = 1$.

Step 2: Assume that the statement is true for some integer k.

That is, that $1 + 2 + ... + k = \frac{k(k+1)}{2}$.

Step 3: $1 + 2 + ... + k + (k+1) =$. Thus the induction step is true.

$$\frac{k(k+1)}{2} + (k+1) =$$

$$\frac{k(k+1)}{2} + \frac{2(k+1)}{2} =$$

$$\frac{k(k+1)+2(k+1)}{2} = \frac{(k+2)(k+1)}{2}$$

Step 4: Since we know the statement is true for $n = 1$ and the induction step is true, we know that the statement is true for all positive integers n.

8-148.

a. $3^{2 \cdot 1 - 1} + 1 = 3 + 1 = 4$

b. Assume that $3^{2k-1} + 1$ is divisible by 4 for some integer k.

c. $3^{2(k+1)-1} + 1 = 3^{2k + \underline{\mathbf{2}} - 1} + 1$

$= 3^{2k-1} \cdot 3^{\underline{\mathbf{2}}} + 1$

$= \underline{\ \mathbf{9}\ } \cdot 3^{2k-1} + 1$

$= 9 \cdot 3^{2k-1} + 9 - \underline{\ \mathbf{8}\ }$

$= 9(\underline{\mathbf{3^{2k-1}+1}}) - \underline{\ \mathbf{8}\ }$

$= 9(\underline{\quad \mathbf{4A} \quad}) - \underline{\ \mathbf{8}\ }$

$= \underline{\ \mathbf{36A}\ } - \underline{\ \mathbf{8}\ }$

$= 4(\underline{\ \mathbf{9A - 2}\ })$

d. $3^{2n-1} + 1$

Review and Preview 8.2.6

8-149.

Step 1: $1^2 = \frac{1(1+1)(2\bullet 1+1)}{6} = \frac{1\cdot 2\cdot 3}{6} = \frac{6}{6} = 1$.

Step 2: Assume $1^2 + ... + k^2 = \frac{k(k+1)(2k+1)}{6}$ is true for some integer k.

Step 3: $1^2 + ... + k^2 + (k+1)^2 =$

$$\frac{k(k+1)(2k+1)}{6} + (k+1)^2 =$$

$$\frac{k(k+1)(2k+1)}{6} + \frac{6(k+1)^2}{6} =$$

$$\frac{(k+1)[k(2k+1)+6(k+1)]}{6} =$$

$$\frac{(k+1)(2k^2+k+6k+6)}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$$

so the induction step is true.

Step 4: Since we know the statement is true for $n = 1$ and the induction step is true, we know that the statement is true for all positive integers n.

8-150.

This is an infinite geometric sum. Therefore $S = \frac{a}{1-r}$. Hence $\frac{5}{1-r} = 10$ and $r = \frac{1}{2}$.

8-151.

Step 1: $5^1 - 1 = 4$,

Step 2: Assume that $5^k - 1$ is divisible by 4, therefore $5^k - 1 = 4A$.

Step 3: $5^{k+1} - 1 =$

$$5 \cdot 5^k - 1 =$$

$$5 \cdot 5^k - 5 + 4 =$$

$$5(5^k - 1) + 4 =$$

$$20A + 4 = 4(5A + 1)$$

so $5^{k+1} - 1$ is a multiple of 4.

Step 4: Hence, by mathematical induction we have proven that $5^n - 1$ is divisible by 4 for all natural numbers.

8-152.

$3! = 6 \not> 2^3 = 8$ a. $4! = 24 > 2^4 = 16$ b. Assume that $k! > 2^k$ for $k \geq 4$.

c. $(k+1)! = \underline{\;\mathbf{k!}\;}(k+1)$

$(k+1)! = \underline{\;\mathbf{k!}\;}(k+1) > \underline{\;\mathbf{2^k}\;}(k+1)$

$\underline{\;\mathbf{2^k}\;}(k+1) > 2^k \cdot \underline{\;\mathbf{2}\;} = 2^{\underline{\mathbf{k+1}}}$

$\underline{\;\mathbf{(k+1)!}\;} > \underline{\;\mathbf{2^{k+1}}\;}$

d. Hence, by mathematical induction we have proven that $n! > 2^n$ for $n \geq 4$.

8-153.

Answer to Hint: $\sqrt{n}\sqrt{n+1} > \sqrt{n}\sqrt{n} = \sqrt{n^2} = n$

Step 1: $\frac{1}{\sqrt{1}} \geq \sqrt{1}$

Step 2: Assume $\sum\limits_{j=1}^{k} \frac{1}{\sqrt{j}} \geq \sqrt{k}$ for some integer k.

Step 3: $\sum\limits_{j=1}^{k+1} \frac{1}{\sqrt{j}} =$

$$\sum_{j=1}^{k} \frac{1}{\sqrt{j}} + \frac{1}{\sqrt{k+1}} \geq$$

$$\sqrt{k} + \frac{1}{\sqrt{k+1}} =$$

$$\frac{\sqrt{k}\sqrt{k+1}+1}{\sqrt{k+1}} \geq$$

$$\frac{k+1}{\sqrt{k+1}} = \sqrt{k+1}$$

so the induction step is true.

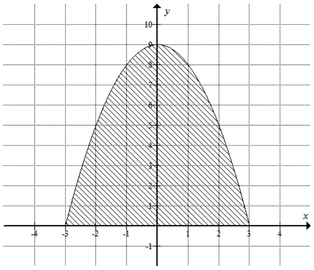
Step 4: Hence, by mathematical induction we have proven that $\sum\limits_{j=1}^{n} \frac{1}{\sqrt{j}} \geq \sqrt{n}$ for any positive integer n.

Closure Chapter 8

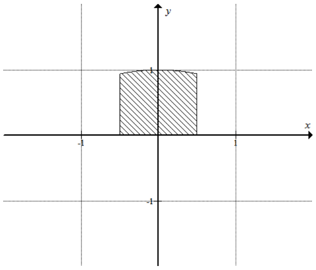
8-154.

a. The area is less than a rectangle.

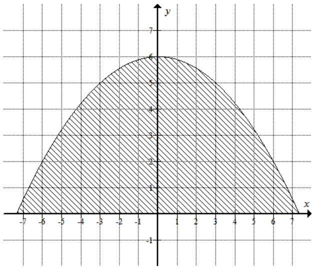
b. 1. $A = \frac{2}{3} \cdot 6 \cdot 9 = 36$



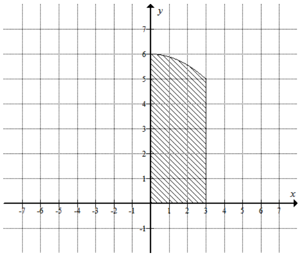
2. $A = \frac{2}{3} \cdot 1 \cdot 1 \approx 0.67$



3. $A = \frac{2}{3} \cdot (2 \cdot 7.348) \cdot 6 \approx 58.788$



8-155.



a. $\int_0^3 f(x)dx = 18$

b. See graph at right.

c. Too large; graph is downward sloping.

d. $R_6 \le Area \le L_6$

e. For 12: Left = 19.094 Right = 16.844.
For 18: Left = 18.574 Right = 17.236

f. The actual area is between the values of the Left Endpoint and Right Endpoint rectangles. As the number of rectangles approaches infinity, the actual area is squeezed between the two values.

CL 8-156.

a. $\lim\limits_{x\to\infty} f(x) = \frac{1}{2}$

b. The $\lim\limits_{x\to 0} f(x)$ does not exist.

c. $\lim\limits_{x\to 1} \frac{x^2-3x-4}{2x^2-8x} = \frac{1-3-4}{2\cdot 1-8} = \frac{-6}{-6} = 1$

d. $\lim\limits_{x\to 4} \frac{(x-4)(x+1)}{2x(x-4)} = \lim\limits_{x\to 4} \frac{(x+1)}{2x} = \frac{5}{8}$

CL 8-157.

a. $\lim\limits_{x\to 5} 2g(x) = 2 \lim\limits_{x\to 5} g(x) = 2 \cdot 13.5 = 27$

b. $\lim\limits_{x\to 5} \frac{g(x)}{2} = \frac{1}{2} \lim\limits_{x\to 5} g(x) = \frac{13.5}{2} = 6.75$

c. $\lim\limits_{x\to 5}(g(x)+10) = \lim\limits_{x\to 5} g(x) + \lim\limits_{x\to 5} 10 = 13.5 + 10 = 23.5$

d. The $\lim\limits_{x\to 6} g(x)$ is unknown as we don't have enough information.

CL 8-158.

a. $\lim\limits_{n\to\infty}\left(1+5\left(\frac{1}{n}\right)\right)^n = e^5$

b. $\lim\limits_{n\to\infty}\left(1+\left(\frac{1}{2}\right)\frac{1}{n}\right)^n = e^{1/2} = \sqrt{e}$

CL 8-159.

a.
$$\begin{aligned} 5e^x &= 30 \\ e^x &= 6 \\ \ln e^x &= \ln 6 \\ x &= \ln 6 \end{aligned}$$

b.
$$\begin{aligned} \ln(x+4) &= 3 \\ e^{\ln(x+4)} &= e^3 \\ x+4 &= e^3 \\ x &= e^3 - 4 \end{aligned}$$

c.
$$\begin{aligned} e^{3x} - 6 &= 22 \\ e^{3x} &= 28 \\ \ln e^{3x} &= \ln 28 \\ 3x &= \ln 28 \\ x &= \frac{\ln 28}{3} \end{aligned}$$

d.
$$\begin{aligned} 5\ln x - 7 &= 18 \\ 5\ln x &= 25 \\ \ln x &= 5 \\ e^{\ln x} &= e^5 \\ x &= e^5 \end{aligned}$$

CL 8-160.

$$a-\tfrac{a}{2}+\tfrac{a}{4}-\tfrac{a}{8}+\ldots$$

$$a_n=a\left(1-\tfrac{1}{2}+\tfrac{1}{4}-\tfrac{1}{8}+\ldots\right)$$

$$a_n=a\left(-\tfrac{1}{2}\right)^n$$

$$S=\tfrac{a}{1-(-1/2)}=\tfrac{a}{3/2}=\tfrac{2}{3}a$$

$$24=\tfrac{2}{3}a$$

$$a=36$$

CL 8-161.

a. $\lim\limits_{x\to\infty}\dfrac{x+\sin x}{2x}=\tfrac{1}{2}$

b. No, because $b(x)$ approaches infinity.

c. $\lim\limits_{x\to\infty}\tfrac{(x-a)^4(x-b)^2(x-c)}{(x-a)^3(x-b)^2(x-c)^2}\approx\tfrac{x^7+\cdots}{x^7+\cdots}=1$

CL 8-162.

$$\lim_{x\to1}\frac{2x^4-2x^3+x^2-9x+8}{3x-3}=\lim_{x\to1}\tfrac{(x-1)(2x^3+x-8)}{3(x-1)}=\tfrac{2(1)^3+1-8}{3}=-\tfrac{5}{3}$$

CL 8-163.

$$T_1=3$$

$$T_2=2T_1-1=2(3)-1=5$$

$$T_3=2T_2-2=2(5)-2=8$$

$$T_4=2T_3-3=2(8)-3=13$$

$$T_5=2T_4-4=2(13)-4=22$$

CL 8-164.

a. $P(t)=1000e^{0.2t}$

$P(7)=1000e^{0.2\cdot7}$

$P(7)=4055$

b. $20,000=1000e^{0.2t}$

$20=e^{0.2t}$

$\ln 20=\ln e^{0.2t}$

$\tfrac{\ln 20}{0.2}=t$

$t\approx14.97$

15 days from his measurement.

CL 8-165.

$$\tfrac{1}{\sec x+\tan x}+\tfrac{1}{\sec x-\tan x}=$$

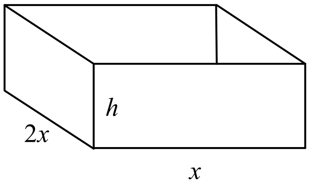
$$\tfrac{(\sec x-\tan x)}{(\sec x+\tan x)(\sec x-\tan x)}+\tfrac{(\sec x+\tan x)}{(\sec x-\tan x)(\sec x+\tan x)}=$$

$$\tfrac{\sec x-\tan x}{\sec^2 x-\tan^2 x}+\tfrac{\sec x+\tan x}{\sec^2 x-\tan^2 x}=$$

$$\tfrac{\sec x-\tan x+\sec x+\tan x}{\sec^2 x-\tan^2 x}=$$

$$\tfrac{2\sec x}{1}=2\sec x$$

CL 8-166.



a. See diagram at right.

b. $V = 30 = x \cdot 2x \cdot h$

$$30 = 2x^2h$$

$$h = \frac{15}{x^2}$$

$$SA = 2 \cdot 2x \cdot x + 2 \cdot 2x \cdot h + 2 \cdot x \cdot h$$

$$SA = 4x^2 + 4xh + 2xh = 4x^2 + 6xh$$

$$SA = 4x^2 + 6xh = 4x^2 + 6x\left(\frac{15}{x^2}\right)$$

$$SA = 4x^2 + 6x\left(\frac{15}{x^2}\right) = 4x^2 + \frac{90}{x}$$

CL 8-167.

$$\text{m} = \frac{7-0}{2-0} = \frac{7}{2}$$

$$d = \sqrt{(2-0)^2 + (7-0)^2} = \sqrt{4+49} = \sqrt{53}$$

$$\tan\theta = 3.5$$

$$\tan^{-1}\tan\theta = \tan^{-1}3.5$$

$$\theta = 74.1^\circ$$

CL 8-168.

$$\frac{40(2)+60(1)+20(.5)}{3.5} = 42.86 \text{ mph}$$