CHAPTER 8

Radar Measurement and Tracking

This chapter enables the reader to:

- Understand the radar measurement error sources and factors affecting the accuracy of radar measurements;
- Quantify the accuracy of radar measurements of target range, angle, and velocity;
- Know how target features are measured by radar, and quantify the accuracy of measuring common features;
- Understand the enhancement in measurement accuracy and other features provided by multiradar measurements and single-radar multilateration;
- Understand measurement-smoothing and target-tracking concepts, and common tracking implementation techniques.

While radar search and target detection are essential to radar use, most applications also require measurement of target characteristics. These may include target position and velocity, as described in Sections 8.1 through 8.3, and indicators of target size, shape and rotation, as described in Section 8.4. The use of multiple radar to improve measurement accuracy is discussed in Section 8.5. Combining radar measurements into target tracks, issues associated with radar tracking, and tracking methods are discussed in Sections 8.6 and 8.7.

Radar can provide measurement of target position in range and in two angular coordinates, relative to the radar location. The angular coordinates are usually defined as elevation angle relative to the local horizontal, and azimuth, which may be measured relative to true north, or for phased arrays, relative to array broadside azimuth. This measurement geometry is illustrated in Figure 8.1. Radars employing coherent processing may also directly measure the target radial velocity.

To measure target characteristics, the target must be resolved by the radar. This requires that the target be separated from other targets by the radar resolution in at least one of the measurement coordinates [1, pp. 3–6]. Radar resolution in angle is usually defined by the beamwidth in two orthogonal angular coordinates, usually azimuth and elevation (see Section 3.2). The required target separation, D, in an angular coordinate, often called cross-range separation, is given by:

$$D = R\theta \tag{8.1}$$

where θ is the radar beamwidth in the angular coordinate, and R is the target range.

Radar range resolution, R, is given by:

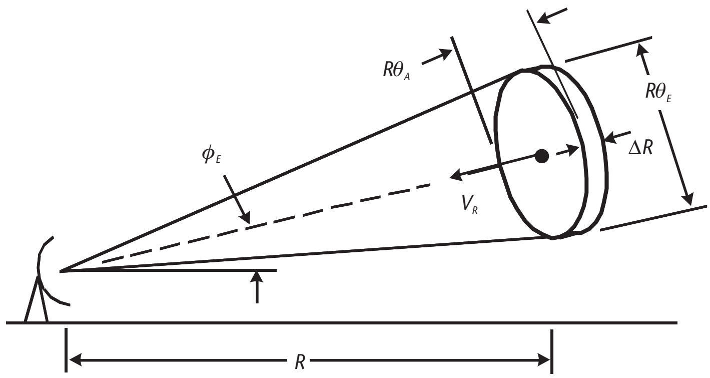

Figure 8.1 Geometry for radar measurement and target resolution.

$$\Delta R = \frac{c\,\tau_R}{2} = \frac{c}{2B} \tag{8.2}$$

where τ_R is the radar resolution in time, which is approximately equal to the reciprocal of the signal bandwidth, B (see Chapter 4). The geometry of the radar resolution cell is shown in Figure 8.1.

Targets that occupy the same range and angle resolution cell may be resolved in radial velocity by radars that employ coherent processing. The velocity resolution, ΔV, is given by:

$$\Delta V = \frac{\lambda f_R}{2} = \frac{\lambda}{2\tau} \tag{8.3}$$

where f_R is the Doppler-frequency resolution of the radar waveform, which is approximately equal to the reciprocal of the total waveform duration, τ (see Chapter 4).

Note that resolution is required in only one dimension to allow the measurement of target characteristics.

The accuracy with which a radar may measure a target characteristic is determined by several error sources:

- A S/N-dependent random measurement error. These errors vary with $(S/N)^{-1/2}$.
- A random measurement error having fixed standard deviation, due to noise sources in the latter stages of the radar receiver. These errors are usually small, and correspond to S/N-dependent errors produced at large S/N values. Thus, they set a limit on how far random errors may be reduced by increasing S/N.
- A bias error associated with the radar calibration and measurement process. These errors may vary randomly with drift in radar calibration, only with correlation times that are long compared with the usual radar observation period.
- Errors due to radar propagation conditions, or uncertainties in correcting for the propagation conditions, as discussed in Chapter 9. Components of these errors may vary randomly from observation to observation like the radar random measurement errors, or remain fixed for extended periods like the radar bias errors.

- Errors from interference sources such as radar clutter (see Sections 9.1 and 9.2), and radar jamming signals (see Sections 10.2 and 10.3).
- Errors due to target scintillation and glint (see Section 9.4).

The first three error sources above, S/N-dependent, fixed random, and bias, are discussed in this chapter. The errors from propagation, clutter, jamming, scintillation, and glint may be evaluated using techniques described elsewhere in this book and in the references. These errors may be combined with the S/N-dependent, fixed random, and bias errors to produce three major error components. This is illustrated by an example in Section 12.5.

The radar accuracy is characterized in the analyses in this section by the standard deviation of a Gaussian distribution, designated by σ, which reasonably models measurement-error distributions for many cases of interest [1, p. 201]. For many applications, it is appropriate to use multiples of σ to characterize the outer bound of the error distribution. The probability that a one-dimensional measurement will occur between several specified error bounds for this distribution is given in Table 8.1. For example, half the measurements will occur between $\pm 0.675\ \sigma$. Only 1 in 370 measurements will occur outside a ± 3-σ bound.

8.1 Range Measurement Accuracy

A pulsed radar determines radar range, R, by measuring the time interval, t, between the transmitted and received signal:

$$R = ct/2 \tag{8.4}$$

In early radars, the receive time was measured by observing the pulse return on a display, such as an A scope. Some later radars use automatic range measurement, using two contiguous range gates, called early and late gates. When the energy in the two gates is equal, the crossover time between the gates is at the center of the received pulse. Many modern radars sample the received signal and determine the target range by fitting the sample data to a replica of the pulse, such as shown in Figure 4.1(a).

The range-measurement accuracy is characterized by the rms measurement error, σ_R, given by the root-sum-square (rss) of the three error components.

$$\sigma_R = \left(\sigma_{RN}{}^2 + \sigma_{RF}{}^2 + \sigma_{RB}{}^2\right)^{1/2} \tag{8.5}$$

Table 8.1 Error Bounds and Their Occurrence Probabilities for a Gaussian Distribution.

Error Bound	*Probability of Measurement Occurring Within the Error Bound*
0.675 σ	0.5
1 σ	0.683
2 σ	0.955
3 σ	0.997
4 σ	0.9994

where:

σ_{RN} = S/N -dependent random range measurement error;

σ_{RF} = range fixed random error, the rss of the radar fixed random range error and the random range error from propagation;

σ_{RB} = range bias error, the rss of the radar range bias error and the range bias error from propagation.

The S/N-dependent error usually dominates the radar range error. It is random, with a standard deviation given by:

$$\sigma_{RN} = \frac{\Delta R}{\sqrt{2(S/N)}} = \frac{c}{2B\sqrt{2(S/N)}} \tag{8.6}$$

For single-pulse measurements, the value of S/N in (8.6) is the single-pulse S/N (assuming S/N is large enough that the detection loss may be neglected; see Section 5.4).

When multiple pulses are used, the integrated S/N should be used in (8.6) [1, pp. 35–44 and 82–86]. This is given by (5.14) for coherent integration, and by (5.17) for noncoherent integration. With phased-array radar and dish-tracking radar, the target is usually assumed to be near the center of the beam, as discussed above. When it is not, an appropriate beamshape loss, L_{BS}, should be used in calculating S/N (see Chapter 7). With rotating search radar, measurements are made as the beam sweeps past the target, and a beamshape loss, L_{BS} (usually 1.6 dB), should be included in calculating S/N (see Section 7.1). If pulse integration produces a signal-processing loss, L_{SPI}, the integrated S/N should be reduced by this loss (see Section 5.4).

The fixed random range error may limit the range measurement accuracy for large values of S/N. Internal radar noise typically produces an equivalent S/N of 25 to 35 dB, which results in random fixed errors of 1/25th to 1/80th of the range resolution. Random range errors due to propagation are usually small, except when multipath conditions exist (see Chapter 9).

The magnitude of radar range bias errors usually depends on the care taken to reduce them. Since bias errors are constant for a series of measurements, or for multiple targets in the same general area, they do not affect radar tracking or the relative locations of targets. Thus, little effort is made to reduce range bias errors in many radars, and they may have values of tens of meters. When absolute target position is important, careful calibration may reduce radar range bias errors to the level of fixed range random errors. Range bias errors from propagation conditions are usually small, and are discussed in Chapter 9.

For example, a radar having a waveform bandwidth, B = 1 MHz has a range resolution, ΔR = 150m. If the S/N (either single-pulse or integrated) is 15 dB, σ_{RN} = 18.9m. If the fixed error, σ_{RF}, is 0.02 times the resolution (3m), and the bias error, σ_{RB} = 10m, the overall range-measurement accuracy, σ_R = 21.6m. The relative error between observations or targets is calculated without the bias error: σ_R = 19.1m.

Additional examples of range measurement error calculations are shown in Table 8.2. The second column shows the range error from using a single pulse at the S/N shown. The total error is the rss of the three components shown in the column. The corresponding relative error is shown in column 3. In this case, the bias error is

Table 8.2 Range Measurement Examples

Parameter	*Single Pulse*	*Single Pulse (Relative)*	*Coherent Integration*	*Noncoherent Integration*
ΔR	15m	15m	15m	15m
Pulse S/N	8 dB	8 dB	8 dB	8 dB
n	1	1	10	10
Integrated S/N	N/A	N/A	18 dB	17.4 dB
σ_{RN}	4.22m	4.22m	1.34m	1.44m
σ_{RF}	0.5m	0.5m	0.5m	0.5m
σ_{RB}	1.0m	N/A	1.0m	1.0m
σ_R	4.37m	4.25m	1.75m	1.82m

not included in the rss combination, leading to a lower error value. Columns 4 and 5 show the impact of pulse integration. Ten pulses are coherently integrated in column 4, and noncoherently integrated in column 5, giving the integrated S/N values shown. The S/N-dependent errors are calculated using these integrated S/N values, and the total error values calculated using these smaller S/N-dependent error values.

8.2 Angular Measurement Accuracy

Radar angular measurements are commonly made using monopulse receive antennas that produce simultaneous receive beams slightly offset in angle to either side of the transmit beam. The difference pattern formed by these beams may be used to measure target angular position with a single signal transmission, as described in Section 3.2.

Other angle-measurement techniques involve transmission and reception of multiple signals at different angles around the target. With rotating search radar, target angular position may be measured by finding the center of a series of pulse returns as the antenna sweeps past the target, as shown in Figure 7.1. Early tracking radar either used a similar technique, or transmitted in several beam positions around the target and found the angular position where the signal return was maximum. For nonfluctuating targets, these techniques may produce an angular measurement accuracy comparable to that of monopulse radar [1, pp. 33–35].

The measurement accuracy in each angular coordinate is characterized by the rms measurement error, σ_A, given by the rss of the three error components.

$$\sigma_A = \left(\sigma_{AN}^2 + \sigma_{AF}^2 + \sigma_{AB}^2\right)^{1/2} \tag{8.7}$$

where:

σ_{AN} = S/N -dependent random angular measurement error;

σ_{AF} = angular fixed random error, the rss of the radar fixed random angle error and the random angle error from propagation;

σ_{AB} = angle bias error, the rss of the radar angle bias error and the angle bias error from propagation.

The S/N-dependent error usually dominates the radar angle errors. It is random, with a standard deviation given for monopulse radar by:

$$\sigma_{AN} = \frac{\theta}{k_M \sqrt{2(S/N)}} \tag{8.8}$$

where θ is the radar beamwidth in the angular coordinate of the measurement, and k_M is the monopulse pattern difference slope (see Section 3.2). The value of k_M is typically 1.6 [1, pp. 24–32]. For single-pulse measurements, the single-pulse S/N is used in (8.8), while for multipulse measurements, the integrated S/N is used, as discussed in Section 8.1 for range-measurement accuracy. As in the previous discussion, signal-processing losses from integration and beamshape losses should be used, when appropriate, in calculating S/N.

Radar clutter and jamming may affect the radar sum and difference channels differently than Gaussian noise sources. In such cases, treating their impacts as contributions to S/N using (8.8) is not valid, and more complex analysis is needed for accurate results [1, pp. 135–142, 158–158, 215–229].

Equation (8.8) with a value of $k_M = 1.6$ also gives the approximate angular error for nonmonopulse radar that employ multipulse measurements of nonfluctuating targets [1, pp. 33–44]. Corrections for fluctuating targets employing multipulse measurements of nonmonopulse radar are discussed in [1, pp. 171–182].

As with range measurement, the fixed angular random errors may limit the angular measurement accuracy for large values of S/N. Typical internal radar noise levels discussed above would produce angular errors 1/40th to 1/125th of the beamwidth. This is sometimes referred to as a maximum beam-splitting ratio of 40 to 125. Random angular errors due to propagation are usually small (see Chapter 9).

The magnitude of radar angular bias errors depends on the care taken to reduce them. Since bias errors are constant for a series of measurements, or for multiple targets in the same general area, they do not affect radar tracking or the relative locations of targets. With careful calibration, radar angular bias errors from radar calibration may be reduced to the level of fixed angular errors. Angular bias errors from propagation may be significant if not corrected, especially elevation-angle errors at low elevation angles, as discussed in Chapter 9. The radar alignment errors may be reduced by tracking targets with known locations, or by tracking satellites over a portion of their trajectories. Tropuspheric propagation errors may be reduced by correcting for known or estimated conditions, as discussed in Section 9.3.

For example, for an azimuth radar beamwidth of 1 degree and S/N = 12 dB, σ_{AN} = 0.11 degrees, or 1.9 mrad. If σ_{AF} = 0.2 mrad, and σ_{AB} = 0.5 mrad, then the overall azimuth measurement error is σ_A = 2.0 mrad.

Another angular error source, not analyzed here, is target glint. Glint is the effect of target scatterers separated in the cross-range direction producing fluctuations of the apparent angle-of-arrival of the signal return. These may exceed the angular extent of the target, and may be a major source of angular error at short ranges (e.g., with target-seeking radar) [1, pp. 164–171].

With phased-array radar, the parameters that determine angular measurement errors may vary with the beam scan angle off-broadside. The array beamwidth, θ_d, in an angular coordinate at a scan angle off-broadside of φ in that coordinate is given by:

$$\theta_{\varphi} = \theta_{B} / \cos\varphi \tag{8.9}$$

where θ_B is the broadside beamwidth in the angular coordinate (see Section 3.3).

The fixed and bias angular errors, σ_{AF} and σ_{AB}, may have components that are both independent of scan angle, and scan-dependent components that are defined in sine space [2, pp. 2–19]. The latter produce angular measurement errors that are approximately proportional to 1/cos φ. Errors that are independent of scan angle may include atmospheric errors and bias errors from unknown array orientation. Scan-dependent errors may result from errors in element spacing and relative phasing. The value of the error at a particular scan angle is the rss of the nonvarying and the varying error components.

In the previous example, if the azimuth scan angle off-broadside is 30 degrees, the azimuth beamwidth increases to 1.15 degrees and σ_{AN} = 2.2 mrad. If an additional scan-dependent fixed random error component is 0.0001 sin φ (often called 0.1 msine), $\sigma_{AF} = (0.20^2 + 0.11^2)^{1/2} = 0.23$ mrad. Similarly, with an additional scan-dependent azimuth bias error of 0.3 msine, σ_{AB} = 0.61 mrad. The overall azimuth measurement error, σ_A = 2.3 mrad.

These angle-measurement error examples are summarized in Table 8.3. The measurement and error parameters are given in the first column. The errors for a dish radar are calculated in column 2, where the beam is not broadened by off-broadside scan, and the scan-dependent errors do not apply. The S/N-dependent error is 1.9 mrad, and the overall error is found by rss in combination with the fixed random and bias error values. Column 3 shows the results for a phased array. The beamwidth at 30° scan angle is 1.15°, and the S/N dependent error increases to 2.2 mrad (assuming the S/N remains at 12 dB). The fixed random and bias errors are found by rss combining the nonscan-dependent and scan-dependent components, and the resulting overall error is calculated as the rss of these terms.

Depending on the accuracy required, the angular measurement error components may be calculated at each scan angle of interest, or average values may be used.

The measurement error normal to the radar LOS, called the cross-range error, is found by multiplying the angular measurement error (in radians), by the target

Table 8.3 Examples of Angle-Measurement Errors

Parameter	*Dish Radar*	*Array Radar*
θ	1°	1° (broadside)
φ	N/A	30°
θ_{φ}	N/A	1.15°
S/N	12 dB	12 dB
σ_{AN}	1.9 mrad	2.2 mrad
Fixed	0.2 mrad	0.2 mrad
Scan-dependent fixed	N/A	0.1 msine
σ_{AF}	0.2 mrad	0.23 mrad
Bias	0.5 mrad	0.5 mrad
Scan-dependent bias	N/A	0.3 msine
σ_{AB}	0.5 mrad	0.61 mrad
σ_{A}	2.0 mrad	2.3 mrad

range. The error in measuring the cross-range target position in an angular coordinate direction, σ_D, is given by:

$$\sigma_D = R\sigma_A \tag{8.10}$$

The resulting target uncertainty volume has standard deviations in the cross-range dimensions of $R_{\sigma A1}$ and $R_{\sigma A2}$, where the numbers refer to the orthogonal angular coordinates, and a standard deviation in the range dimension σ_R. For most radars, the cross-range errors that result from angular-measurement errors at useful ranges far exceed the range-measurement errors. The resulting target uncertainty volume is a relatively flat, circular, or elliptical disk that is normal to the radar LOS, as illustrated in Figure 8.1.

8.3 Velocity Measurement Accuracy

Target radial velocity may be measured in two ways:

- From the Doppler-frequency shift of the received signal;
- From multiple range measurements.

Measurements using Doppler-frequency shift almost always give significantly better accuracy than noncoherent processing of range measurements.

A coherent radar may measure the target radial velocity, V_R, from the Doppler-frequency shift of the received signal:

$$V_R = \lambda f_D / 2 \tag{8.11}$$

where f_D is the Doppler-frequency shift and λ is the radar signal wavelength.

The radial-velocity measurement accuracy from measuring Doppler-frequency shift is characterized by the rms measurement error, σ_V, given by the rss of the three error components.

$$\sigma_V = \left(\sigma_{VN}{}^2 + \sigma_{VF}{}^2 + \sigma_{VB}{}^2\right)^{1/2} \tag{8.12}$$

where:

σ_{VN} = S/N -dependent random radial-velocity measurement error;

σ_{VF} = radial-velocity fixed random error, the rss of the radar fixed random radial-velocity error, and the random radial-velocity error from propagation;

σ_{VB} = radial-velocity bias error, the rss of the radar radial-velocity bias error, and the radial-velocity bias error from propagation.

The S/N-dependent error usually dominates the radar radial-velocity error. It is random, with a standard deviation given by [1, pp. 101–103]:

$$\sigma_{VN} = \frac{\lambda}{2\tau\sqrt{2(S/N)}} = \frac{\Delta V}{\sqrt{2(S/N)}} \tag{8.13}$$

where τ is the duration of the radar waveform that is coherently processed, and ΔV is the radial-velocity resolution given by (8.3). As with the measurements in the previous two sections, the single-pulse or integrated S/N may be used, and appropriate integration processing and beamshape losses should be applied.

The fixed random radial-velocity error may limit the measurement accuracy for very large values of S/N. Random radial-velocity errors due to propagation are usually small. The magnitude of radar radial-velocity bias errors depends on the care taken to reduce them. With careful calibration, they may be reduced to the level of random radial-velocity errors. Bias errors from propagation conditions are usually small, as discussed in Chapter 9.

The target radial velocity also may be found from the difference of two range measurements, divided by the time between the measurements:

$$V_R = \frac{R_1 - R_2}{t_1 - t_2} \tag{8.14}$$

where R_1 and R_2 are the ranges and t_1 and t_2 are the respective measurement times. The resulting radial-velocity accuracy is given by [1, p. 356]:

$$\sigma_V = \frac{\sqrt{2}\,\sigma_R}{(t_1 - t_2)} \quad \text{(two pulse)} \tag{8.15}$$

The bias error, σ_{RB}, should not be included in σ_R for this calculation, since it will be constant for the two measurements.

For n periodic measurements over a time period, t_N, this result may be extended to [1. p. 357]:

$$\sigma_V = \frac{\sqrt{12}\,\sigma_R}{\sqrt{n}\,t_N} \quad \text{(pulse train, } n \geq 6\text{)} \tag{8.16}$$

The PRF for these parameters is equal to n/t_N. This result is for a pulse train having many pulses. For $n < 6$, (8.15) calculates a smaller error than does (8.16), and should be used.

For comparable processing times, the measurement of Doppler frequency provides much greater accuracy in V_R than the noncoherent modes described above. For example, at S band ($\lambda = 0.09$m), with a pulse duration $\tau = 1$ ms and S/N = 15 dB, $\sigma_{VN} =$ 5.7 m/s using Doppler processing. Assuming a range resolution of 15m ($B = 10$ MHz), and S/N = 15 dB, $\sigma_{RN} = 1.9$m. Two pulses separated by 1 ms will give $\sigma_{VN} = 2{,}680$ m/s, far greater than the error with Doppler processing. The two pulses would have to be separated by 471 ms to provide accuracy comparable to that using Doppler processing. Using these pulses at a PRF of 1,000 Hz, 110 pulses over a period of 110 ms would be needed to provide the same radial-velocity accuracy as Doppler processing.

These examples are summarized in Table 8.4. The coherent Doppler measurement parameters are given in the second column of the table. The velocity resolution is 45 m/s, and the S/N dependent random Doppler-velocity measurement error in the column is shown to be 5.7 m/s. When multiple range measurements are used, the S/N-dependent random range error in column 3 is shown to be 1.9m. For two pulses

Table 8.4 Radial-Velocity Measurement Examples

Parameter	*Doppler Velocity*	*2-Pulse Range*	*Multipulse Range*
λ	0.09m	N/A	N/A
τ	1 ms	N/A	N/A
ΔV	45 m/s	N/A	N/A
S/N	15 dB	15 dB	15 dB
ΔR	N/A	15m	15m
σ_{RN}	N/A	1.9m	1.9m
Pulse Separation	N/A	1 ms	N/A
Number of Pulses	N/A	2	110
PRF	N/A	N/A	1,000 Hz
$t_1 - t_2$ or t_N	N/A	1 ms	110 ms
σ_{VN}	5.7 m/s	2,680 m/s	5.7 m/s

separated by 1 ms, the resulting radial velocity error is 2,680 m/s, as shown in column 3. Column 4 shows the result when 110 pulses are used over a period of 110 ms. The measurement error is then 5.7 m/s, which is equal to the error with a single pulse when using Doppler measurement.

The preceding has addressed radial-velocity measurement. Angular or cross-range velocity measurements may be made using the noncoherent measurement techniques described above and in (8.14) to 8.16). The resulting cross-range velocity accuracy, σ_C, is given by:

$$\sigma_c = \frac{\sqrt{2}R\sigma_A}{(t_2 - t_1)} \quad \text{(two pulse)} \tag{8.17}$$

$$\sigma_c = \frac{\sqrt{12}\,R\sigma_A}{\sqrt{n}\,t_N} \quad \text{(pulse train, } n \geq 6\text{)} \tag{8.18}$$

The values of φ_A should be calculated without the angular bias, φ_{AB}

As with position measurement, the angular or cross-range velocity errors usually far exceed the radial-velocity error. This is especially true when coherent processing is used to measure radial velocity from target Doppler shift.

8.4 Measurement of Target Features

Radar can measure a number of target features. Such measurements may allow the class of target or perhaps even the type of target to be identified. The following paragraphs describe several common radar measurements:

- Target RCS, measured from the amplitude of the return signal;
- Target radial length, measured from the time duration of the returned signal;
- Target rotational velocity, measured by the Doppler-frequency spread of the returned signal.

Additional information on the target may be inferred from the variation of these measurements with time. The interpretation of radar measurements of target characteristics is a complex process that will not be addressed here. This is not intended as a comprehensive list of potential measurements of target features, which would be beyond the scope of this book.

The target RCS may be determined by measuring the returned S/N and calculating the RCS from the radar equation (see Section 5.1). The RCS measurement accuracy is characterized by the rms measurement error, σ_S, given by the rss of the three error components.

$$\sigma_S = \left(\sigma_{SN}^2 + \sigma_{SF}^2 + \sigma_{SB}^2\right)^{1/2} \tag{8.19}$$

where:

σ_{SN} = S/N-dependent random RCS measurement error;

σ_{SF} = RCS fixed random error, the rss of the radar fixed random RCS error, and the random RCS error from propagation;

σ_{SB} = RCS bias error, the rss of the radar RCS bias error, and the RCS bias error from propagation.

The S/N-dependent RCS error is proportional to the error in measuring the received power. It is random, with a standard deviation approximately given by [3, pp. 4-1–4-8]:

$$\sigma_{SN} \approx \frac{\sqrt{2}\,\sigma}{\sqrt{(S/N)}} \tag{8.20}$$

where σ in the numerator refers to the target RCS. As with the measurement errors discussed in the previous sections, either the single-pulse or integrated S/N may be used in (8.20). Any losses for beamshape and integration processing will affect both the S/N value and the calculation of RCS.

For example, with S/N = 20 dB, the standard deviation of the S/N dependent error is ±14%. If the RCS value is 10 m^2, the standard deviation is ±1.4 m^2.

The random fixed errors and random propagation errors will usually be small, as discussed earlier. Bias errors in RCS measurement result when the radar parameters used to calculate the RCS are not accurately known. These may be significant unless care is taken to calibrate the radar. Such bias errors also may be reduced by calibrating the radar using a target of known RCS value. If the radar parameters (e.g., transmitter peak power) drift with time, frequent calibration may be necessary, and some RCS bias may be produced. RCS bias errors may also be produced by atmospheric and rain attenuation (discussed in Chapter 9). These errors may be reduced by estimating the attenuation, but the accuracy of doing this may be questionable, especially with rain. Note that RCS-measurement bias errors caused by uncorrected signal-path attenuation or changes in radar parameters are factors multiplying RCS.

Target RCS may depend on the transmitted and received signal polarizations. Most radars that transmit linear polarization receive the same-sense linear polarization, and those that transmit circular polarization receive the opposite-sense circular polarization. These are often referred to as the principal receive polarizations. Some radar also have the capability to receive the orthogonal polarization, either using two receivers, or on successive pulses with a single receiver. A few radars also have the capability for transmitting two orthogonal polarizations on successive pulses. The RCS measurements made by these radars may provide additional information on target characteristics [4, pp. 13–21].

Target length in the radial dimension may be measured by radar having range resolution, ΔR, that is less than the target radial dimension, a. This is illustrated in Figure 8.2 for a target having two scatterers that are resolved in range [Figure 8.2(a)], and for a target having several scatterers that are not resolved [Figure 8.2(b)]. In the first case, the range of each scatterer may be measured and the target radial length is determined by their range separation:

$$a = R_2 - R_1 \tag{8.21}$$

where the subscripts correspond to the two measurements.

The accuracy of each range measurement is given by (8.5), and the accuracy of the radial length measurement, σ_L, is given by:

$$\sigma_L = \left(\sigma_{R1}^{\ 2} + \sigma_{R2}^{\ 2}\right)^{1/2} \tag{8.22}$$

where the accuracies of the two measurements may be different due to differences in the S/N for the two scatterers. Since the bias errors, σ_{RB}, will be the same for both measurements, they should not be used in calculating the range measurement errors.

When the individual target scatterers cannot be resolved, as shown in Figure 8.2(b), the accuracy of the radial length measurement cannot be evaluated with precision. However, it may be approximated, using an average value of the S/N for cal-

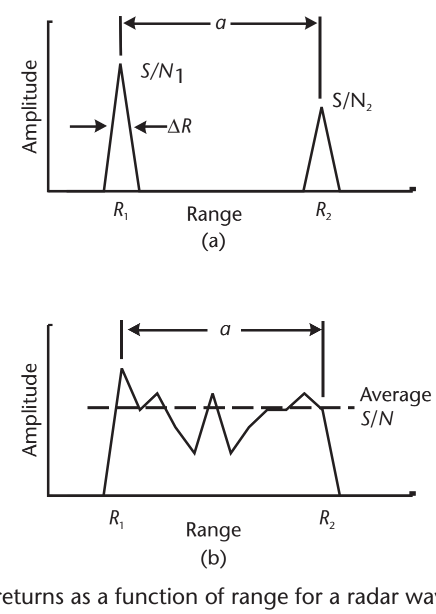

Figure 8.2 Target signal returns as a function of range for a radar waveform having range resolution less than the target radial dimension. (a) Resolved scatterers. (b) Unresolved scatterers.

culating the range measurement error, σ_R. The resulting length-measurement accuracy may be estimated by:

$$\sigma_L \approx \sqrt{2}\sigma_R \tag{8.23}$$

The spread in Doppler-frequency shift in the signal return from a target is determined by the relative radial velocities of the target scatterers. This is illustrated in Figure 8.3(a) for a disk or cylinder with diameter a, having scatterers at its periphery, and rotating with an angular velocity ω_T. If the angle between the radar LOS and the target rotational axis is γ, ΔV_R is given by:

$$\Delta V_R = a\omega_T \sin\gamma \tag{8.24}$$

The resulting spread in the target Doppler-frequency shift, Δf_D, is illustrated in Figure 8.3(b), and is given by:

$$\Delta f_D = \frac{2a\omega_T f}{c}\sin\gamma \tag{8.25}$$

For example, for a cylindrical target having a diameter of 2m, rotating at one rpm ($\omega = 2\pi/60$), and with the rotation axis 30 degrees from the radar LOS, ΔV_R = 0.105 m/s. At C band (5.5 GHz), the resulting Doppler-frequency spread, Δf_D = 3.8 Hz.

To measure the spread in Doppler-frequency shift, Δf_D, the waveform frequency resolution, f_R, must be smaller than Δf_D, or equivalently, the waveform velocity resolution, ΔV, must be smaller than the target radial velocity spread, ΔV_R. In addition, if the radar waveform has velocity ambiguities, they must be separated by an amount greater than ΔV_R (see Section 4.5 and Figure 4.9). The accuracy of measurement of the Doppler-frequency spread, σ_F, is determined similarly to that of the target length.When the scatterers are not resolved, it is given by:

$$\sigma_F \approx \sqrt{2}\,\sigma_V \tag{8.26}$$

A more complex measurement, called range-Doppler imaging, involves generating a two-dimensional plot of signal return versus radial range and radial Doppler

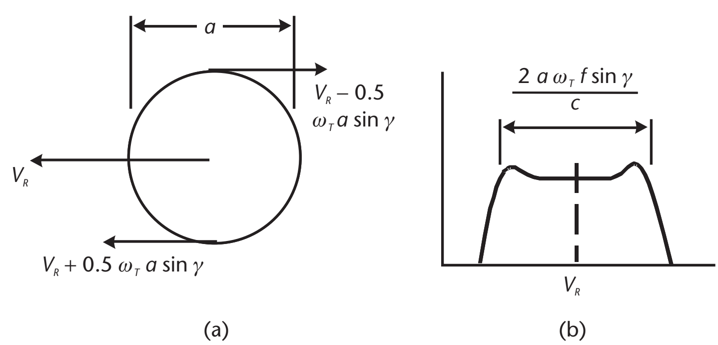

Figure 8.3 Target rotational geometry and the resulting Doppler-frequency spectrum. (a) Rotational geometry. (b) Spectrum.

velocity for a rotating target. In each resolvable range cell, a spectrum similar to that shown in Figure 8.3(b) is generated. The position of scatterers in this spectrum corresponds to their locations on the target. The target scatterers on this plot may be interpreted as a target image in the range and cross-range dimensions. Range-Doppler imaging may be used to determine the configuration of rotating objects. It may also be used on stationary objects when the LOS rotation is produced by a moving radar.

8.5 Multiradar Measurements

The use of two or more radars to measure target characteristics may offer a number of advantages. These include:

- Improved position and velocity accuracy by using multiple-range and radial-velocity measurements to reduce cross-range measurement errors;
- Target feature measurements from multiple viewing angles and at multiple frequencies;
- Increased opportunity for providing favorable location, measurement geometry, and radar frequency for making radar measurements.

The accuracy improvements provided by the first item above is addressed in this section. The last two factors depend on details of the system application.

In most pulsed radars, the range accuracy is significantly better than the cross-range accuracy (the angular accuracy multiplied by the target range). Similarly, the radial-velocity accuracy is significantly better than the cross-range velocity accuracy. When two or more radars observe the target, their range measurements may be combined to greatly improve the overall position and velocity measurement accuracy. This technique is sometimes called multilateration.

This is illustrated in Figure 8.4, which shows two radars observing a target with LOS that are separated by an angle α. The two radars have range measurement errors of σ_{R1} and σ_{R2}. The cross-range errors are large, so the target position uncertainty for each radar is shown by parallel lines. The position measurement error in

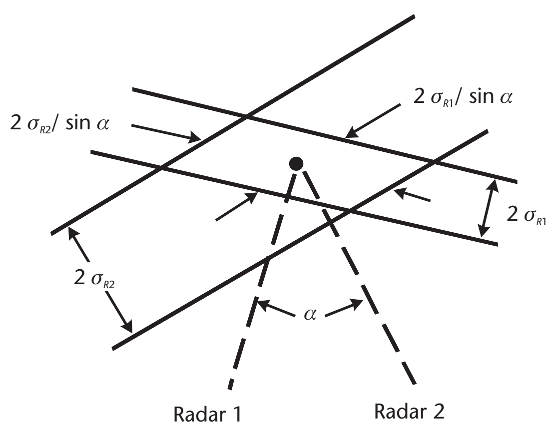

Figure 8.4 Geometry for target position determination using range measurements by two radar.

the plane defined by the radars and the target is shown in the figure. The maximum position measurement error in this plane, σ_{PD}, is approximately given by:

$$\sigma_{PD} \approx \frac{\sigma_R}{\sin\alpha} \tag{8.27}$$

where σ_R is the larger of the two range measurement errors.

For the most favorable geometry, $\alpha = 90°$, and $\sigma_{PD} = \sigma_R$. When α is less than 90°, the measurement error increases. This is sometimes referred to as geometric dilution of precision (GDOP). When α approaches zero, there is no longer any benefit to using two radars. Then the error from (8.27) becomes very large, and the cross-range accuracy is determined by the radar angular measurement error.

A similar result is obtained for measuring target velocity in the plane defined by the radars and the target. The maximum target velocity error in the plane, σ_{VD}, is approximately given by:

$$\sigma_{VD} \approx \frac{\sigma_V}{\sin\alpha} \tag{8.28}$$

where σ_V is larger of the two radial velocity measurement errors.

For example, if two radars observe a target with aspect angles that differ by 45 degrees and have range-measurement accuracies of 1.5m and 3.0m, the resulting maximum position error in the plane of the radars and target is approximately 4.2m. If the two radial velocity-measurement accuracies are 5 m/s and 2 m/s, the resulting radial velocity measurement error is approximately 7.1 m/s.

The measurement-error reduction discussed above occurs in the plane defined by the target and the radars. Adding radars in that plane may further reduce the measurement errors in that plane, but will not affect the out-of-plane cross-range errors. This is often approximately the situation for several ground-based radars observing surface or low-altitude airborne targets. Reducing the out-of-plane errors using this technique requires adding one or more radar well out of the plane defined by the other radars and the target. The accuracy provided by these may be analyzed by defining other planes containing two or more radars and the target.

The preceding analyses assume that the radar measurements are made simultaneously, or that the target is stationery. This requirement may be relaxed when radar measurements are smoothed or processed in a tracking filter, as described in the following section.

Under some circumstances, a single radar may be used to provide the advantages of multilateration. For example, a radar flying past a fixed ground target may provide multilateration measurements. Similarly, a fixed radar may provide multilateration measurements on a target flying past the radar. In this latter case, the characteristics of the target path must be known (or estimated) to correctly combine the radar measurements. This technique may be used on targets assumed to fly straight paths, and on missiles and satellites on ballistic trajectories.

Use of multiple radars for target measurement assumes that the measurements on a target by the multiple radars may be correctly associated with that target. This is usually possible when a single target is in the beam of each radar. When multiple targets are in the radar beam, incorrect measurement associations may occur, and ghost targets may be generated. This is analogous to the ghosting problem for passive tracking, discussed in Chapter 10.

8.6 Measurement Smoothing and Tracking

Radar measurement data may be smoothed to increase the measurement accuracy, estimate target trajectory parameters, and predict future target position. The combining of two or more measurements to determine radial and cross-range velocity is discussed in Section 8.3, and the resulting measurement accuracies are given in (8.15) through (8.18).

The random components of radar position measurements may be reduced by averaging measurements. For example, when n range measurements are combined, the resulting smoothed range error, σ_{RS}, is given by:

$$\sigma_{RS} = \left(\frac{\sigma_{RN}^2 + \sigma_{RF}^2}{n} + \sigma_{RB}^2 \right)^{1/2} \tag{8.29}$$

The S/N-dependent random error and the random fixed range error are reduced by $n^{1/2}$. The bias error is not reduced by smoothing. It should not be included in the calculation when relative range measurements are considered, as discussed in Section 8.1. Note that measurement errors from radar clutter may not be independent from measurement-to-measurement, and thus may not be reduced by $n^{1/2}$ [1, pp. 136–142].

For the example given in Section 8.1, if 20 range measurements are averaged, the S/N-dependent random range error is reduced from 18.9m to 4.2m, and the fixed random range error is reduced from 3m to 0.7m. The resulting range-measurement accuracy from (8.29) is 10.9m, compared to 21.6m with no smoothing. The smoothed measurement error is seen to be dominated by the 10m bias error, which is not reduced by smoothing. If this bias term is not included, the measurement error with smoothing is 4.3m.

The smoothed angular measurement error, σ_{AS}, is similarly given by:

$$\sigma_{AS} = \left(\frac{\sigma_{AN}^2 + \sigma_{AF}^2}{n} + \sigma_{AB}^2 \right)^{1/2} \tag{8.30}$$

Future target position may be predicted, based on measurements of position and velocity. For a prediction time, t_P, the accuracy of the predicted range position, σ_{PR}, and cross-range position, σ_{PC}, are given by:

$$\sigma_{PR} = \left[\sigma_R^2 + \left(\sigma_V t_P \right)^2 \right]^{1/2} \quad \text{(non-maneuvering)} \tag{8.31}$$

$$\sigma_{PC} = \left[\left(R\sigma_A \right)^2 + \left(\sigma_C t_P \right)^2 \right]^{1/2} \quad \text{(non-maneuvering)} \tag{8.32}$$

Equations (8.31) and (8.32) assume that the target velocity remains constant during the prediction time, and that the observation geometry does not change significantly during this time. They are therefore valid only for relatively short prediction periods. For longer time periods, the LOS to the target may rotate, so that a cross-range prediction error component appears in the range direction.

For long prediction times, the predicted error due to velocity measurement [the second term in (8.31) and (8.32)] usually dominates the prediction accuracy, and the errors due to angular measurement errors are usually much greater than the range measurement errors. The largest semi-axis of the predicted target position error ellipsoid, σ_P, is then given approximately by:

$$\sigma_P \approx \sigma_C t_P = \frac{\sqrt{12} R \sigma_A t_P}{\sqrt{n}\, t_N} \quad \text{(non-maneuvering)} \tag{8.33}$$

where σ_C and σ_A are the larger of the two cross-range and angular measurement errors, respectively. When the target velocity remains constant, (8.33) may be used for long prediction times. It may also be used for predicting the position errors of orbital targets, whose flight paths follow Keplerian laws, but with less accuracy, due to the effect of orbital mechanics on the shape and orientation of the error ellipsoid.

For the angle-measurement example given in Section 8.2, the azimuth error (excluding bias error) is 2 mrad. Assuming that this error is larger than the elevation error, if the angular velocity is measured by 30 pulses at a 10-Hz rate over a period of 3 sec at a target range of 100 km, the predicted target position error for a prediction time of 50 sec is 2.11 km.

The preceding discussion has dealt with nonmaneuvering targets. These include aircraft with straight, level, and constant-velocity flight paths, and exoatmospheric objects in Keplerian orbits. The measurements on such targets may be smoothed over long periods of time to reduce the random error components, as indicated by (8.29) and (8.30), and their positions may be predicted well into the future, as indicated by (8.33). Many other targets may have random maneuvers or other accelerations. The measurement-smoothing times and prediction times for these targets are limited by their acceleration characteristics [5, p. 459].

Tracking filters are usually used for smoothing the measurements and predicting the future positions of maneuvering targets. Simple tracking filters use fixed smoothing coefficients. The α-β filter uses an α parameter for smoothing target position, and a β parameter for smoothing target velocity. The values of these parameters are a compromise between providing good smoothing to reduce random measurement errors, and providing rapid response to target maneuvers. Target acceleration may produce a dynamic-lag error in such filters that may dominate the total error in some cases. Some such filters add a third parameter, γ, for smoothing target acceleration [6, pp. 184–186].

In Kalman filters, the radar measurements are matched to a model of the measurement errors and target dynamics. If these are accurately modeled, the Kalman filter will minimize the mean-square measurement error. Kalman filters are somewhat more complex to implement than fixed-parameter filters, but they are widely used in modern radar because of their capabilities for dealing with missing data, variable measurement noise, and variable target dynamics [7, pp. 19–44]. Radars such as dish-tracking radars continuously observe a target and provide measurements at a high data rate. Other radars may be limited in the measurement rate they provide on a target. For example, rotating search radars operate in a track-while-scan (TWS) mode discussed below, and generate a measurement in each rotational period. Such low measurement rates may limit the accuracy provided by the tracking filter on maneuvering targets [5, pp. 445–446].

Multifunction phased-array radars may limit the measurement rate in order to track multiple targets or perform other radar functions. However, the tracking rate must be high enough to provide the required tracking accuracy for maneuvering targets, and to assure that the radar does not lose track of the target between observations. Phased-array radars usually illuminate the predicted target position with a beam. For successful tracking, the target should appear within that beam most of the time.

The error in predicting the target position is approximately given by the rss of the cross-range target prediction error, given in (8.32), and by the error due to target maneuver. For a maximum target acceleration of a_T, the error in predicting target position due to maneuver, σ_M, is given by:

$$\sigma_M = \frac{a_T t_P^{\,2}}{2} \tag{8.34}$$

To assure that the predicted target position is within the radar beam for most observations the 3-σ value of the total prediction error should be less than half the beamwidth times the radar range:

$$\left[\left(\sigma_C t_P\right)^2 + \left(\frac{a_T t_P^{\,2}}{2} \right)^2 \right]^{1/2} \le \frac{R\theta}{6} \tag{8.35}$$

When multiple targets are in track by a radar, it is important that new measurements be associated with the correct target track. This is often done using a nearest-neighbor assignment of new measurements. However, for closely spaced targets, a technique that incorporates all target observations in the neighborhood of the predicted target position is sometimes used [7, pp. 9–10]. Association of target measurements may be improved by increasing the measurement rate above that which is needed to maintain track on a single target.

8.7 Radar Tracking Techniques

Many techniques are used to implement tracking in radar systems. In early rotating surveillance radars (see Section 7.2), the target positions on successive scans were marked on a PPI display, and the positions connected. Later, digital processing provided automatic measurement of target position and calculation of target course and speed. Such techniques are called track-while-scan (TWS).

Dish-tracking radars usually employ a measurement technique to determine the target offset from the beam center. These include:

- Sequential lobing, where successive beams are transmitted and received slightly offset in angle from the main beam;
- Conical scan, where the antenna scans a small cone around the target position;
- Monopulse measurement, where target angular position is measured with a single pulse by using difference antenna patterns (see Section 3.2).

A servo is used to direct and maintain the antenna pointing in the direction of the target. The angular tracking filter parameters are incorporated into the servo control loop.

Phased-array radars often track many targets simultaneously, and have great flexibility in determining when tracking measurements are made. These radars usually use monopulse angular measurements. The tracking filter is implemented in a digital computer, which also controls the radar measurement process. Many such radars use recursive Kalman filters for tracking, but others collect the radar data and employ batch processing.

When multiple radars in a system are observing a target, the tracking filter may use measurement data from more than one radar. This may provide the advantages of multilateration, described in Section 8.5, as well as more frequent track updates and the possibility of more favorable measurement geometry. Such multiradar tracking may employ fusion of individual radar tracks, or it may combine all collected measurement data into a single track. Critical issues in implementing such techniques include accurate knowledge of relative position and orientation of all radars, radar calibration (especially reduction of bias errors), and correct association of targets between radar.

8.8 Problems

The following problems are provided to assist in reviewing this chapter and to ensure a basic understanding of the material. For maximum benefit, the problems should first be solved without using the VBA custom radar functions. Solutions to these problems are given in Appendix E, Section E.8.

1. In how many dimensions must a target be resolved from other targets in order to make measurements on it?
2. A radar has a range resolution of 15m (bandwidth = 10 MHz), a fixed random range error of 1m, and a range bias error of 5m. If the S/N = 18 dB, what are the standard deviations of the relative and of the absolute range errors? What are the 3-σ values of these errors? What is the 3-σ absolute range-measurement error if 50 pulses are smoothed to make the measurement?

3. A phased-array radar has a beamwidth on broadside of 2.5 degrees, a fixed random angle error of 2 mrad, a fixed scan-dependent random angle error of 1 msine, an angle bias error of 3 mrad, and a scan-dependent angle bias error of 1.5 msine. If the S/N = 12 dB, what is the 1-σ angular measurement error on broadside? What is the error at a 30-degree scan angle? What is the cross-range measurement error at a range of 150 km for these cases?
4. An X-band radar (9.5 GHz), has a pulse duration of 1 ms and a pulse-compression ratio of 1,000. If the S/N = 15 dB, what is the error in measuring radial velocity using Doppler-frequency shift? What is the accuracy of measuring radial velocity noncoherently using two such pulses separated by 20 ms? Neglect fixed random and bias errors.
5. A radar observes a target having RCS = 10 m^2 with S/N = 18 dB. If the fixed random measurement error is 5% of the RCS, what is the RCS measurement error, neglecting bias? If rain attenuation of 2 dB is not accounted for in the radar calibration, what is the resulting RCS measurement bias error? What is the overall measurement error?
6. A radar has range resolution of 15m and beamwidth of 1.5 degrees. It makes 20 periodic measurements on the target at a range of 200 km over a total period of 10 sec, with S/N = 12 dB for each measurement. What are the measurement errors for range, angle, radial velocity (noncoherent measurements), and cross-range velocity? What are the predicted position errors in range and cross-range, if the prediction time is 50 sec? Neglect fixed random and bias errors.
7. For a one-dimensional Gaussian error distribution, what error bound will include all but 4.5% of the observations?
8. A dish radar observes a target with S/N = 12 dB. The signal bandwidth is 10 MHz, the fixed range error is 2m, and the range bias error is 10m. The beamwidth is 2 degrees, the fixed angle error is 1 mrad, and the angle bias error is 1.5 mrad. The waveform duration is 10 ms, the frequency is 9.5 GHz (X band), the fixed radial-velocity error is 0.2 m/s, and the radial velocity bias error is 0.1 m/s.

   a. Find the 3-σ errors in absolute and relative range, angle and radial-velocity (from Doppler-frequency measurement).

   b. Find the 3-σ absolute and relative cross-range measurement errors at 500 km range.

   c. Find the 3-σ absolute and relative range and angle measurement errors when 10 measurements are averaged.

   d. Find the 3-σ radial and cross-range velocity errors when 10 pulses are used at a PRF of 5 Hz, at a target range of 500 km. Assume radial velocity is obtained from range measurements.
9. Find the 1-σ angular-measurement error for a phased array when the beam is scanned at an angle of 45 degrees in the angular measurement coordinate. The broadside beamwidth is 1 degree, S/N = 15 dB, the fixed errors are 0.2 mrad and 0.3 msine (scan-dependent), and the bias errors are 0.1 mrad and 0.2 msine (scan-dependent).

10. What is the 1-σ radial length measurement error for a radar having 0.5m range resolution, S/N = 20 dB, and fixed range error of 0.1m? What is the impact of decreasing S/N to 12 dB?
11. Consider two identical radars viewing a target with LOS separated by 45 degrees. The S/N for the two radars are 20 dB and 12 dB. Both radars have range resolution 20m, fixed range error 1m, range bias error 1.5m, velocity resolution 10 m/s, fixed velocity error 0.5 m/s, and velocity bias error 0.7 m/s. Find the approximate maximum 3-σ position and velocity errors.
12. A radar has azimuth and elevation beamwidths of 1 and 3 degrees, respectively. It observes a nonmaneuvering target at a range of 500 km for 10 sec at a PRF of 5 Hz, and S/N = 12 dB. What is the 3-σ value of the largest error semi-axis, predicted 150 sec after the observation period? Neglect fixed and bias errors.

8.9 VBA Software Functions for Radar Measurement and Tracking

8.9.1 Function RangeError_m

Purpose Calculates the standard deviation of the radar range-measurement error.

Reference Equations (8.5), (8.6), and (8.29).

Features Combines calculated S/N-dependent random range error with fixed random range error and range bias error. Range bias error may be omitted to calculate the relative range error. Smoothing of random error components over multiple measurements may be modeled.

Input Parameters (with units specified)

RangeRes_m = radar range resolution (m). A value of 1/B may be used, where B is the radar signal bandwidth.

SNR_dB = measurement S/N (dB). This may be the single-pulse S/N or integrated S/N as appropriate. The value input should take into account any beamshape loss or additional signal-processing loss for pulse integration. For single-pulse S/N values less than about 12 dB, the detection loss (5.16) should also be included.

RangeFixEr_m = composite fixed random range error (m). This parameter is the rss of the radar fixed random range error and any random range errors due to propagation and other sources.

RangeBiasEr_m (optional) = composite range bias error (m). This parameter is the rss of the radar range bias error and any range bias errors due to propagation and other sources. If left blank, a zero value will be assumed for this parameter.

N_Smooth_Integer (optional) = number of range measurements that are smoothed in the calculated range accuracy (integer). If left blank, a single-pulse, or single integrated pulse-group measurement will be assumed. No output is generated for values less than 1.

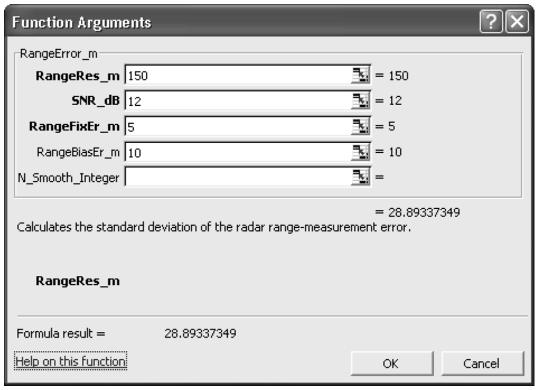

Figure 8.5 Excel parameter box for Function RangeError_m.

Function Output The standard deviation of the range-measurement error for the parameters specified (m).

The Excel Function Arguments parameter box for Function RangeError_m is shown in Figure 8.5, with sample parameters and a solution.

8.9.2 Function AngleError_mR

Purpose Calculates the standard deviation of the radar angular-measurement error.

Reference Equations (8.7), (8.8), (8.9), and (8.30).

Features Combines calculated S/N-dependent random angle error with fixed random angle error and angle bias error. A monopulse measurement with a difference slope $k_M = 1.6$ is assumed. Angle bias error may be omitted to calculate the relative angle error. Allows modeling of phased-array beam broadening and scan angle dependent fixed random and bias angle errors. Smoothing of random error components over multiple measurements may be modeled.

Input Parameters (with units specified)

Beamwidth_mR = radar antenna beamwidth on array broadside in the angular measurement coordinate (mrad).

SNR_dB = measurement S/N (dB). This may be the single-pulse S/N or integrated S/N as appropriate. The value input should take into account any beamshape loss or additional signal-processing loss for pulse integration. For single-pulse S/N values less than about 12 dB, the detection loss (5.16) should also be included.

AngleFixEr_mR = composite fixed random angle error in the measurement coordinate (mrad). This parameter is the rss of the radar fixed random angle error and any random angle errors due to propagation and other sources.

ScanAngle_deg (optional) = scan angle for phased arrays in the measurement coordinate (degrees). If this parameter is omitted, a value of zero will be assumed.

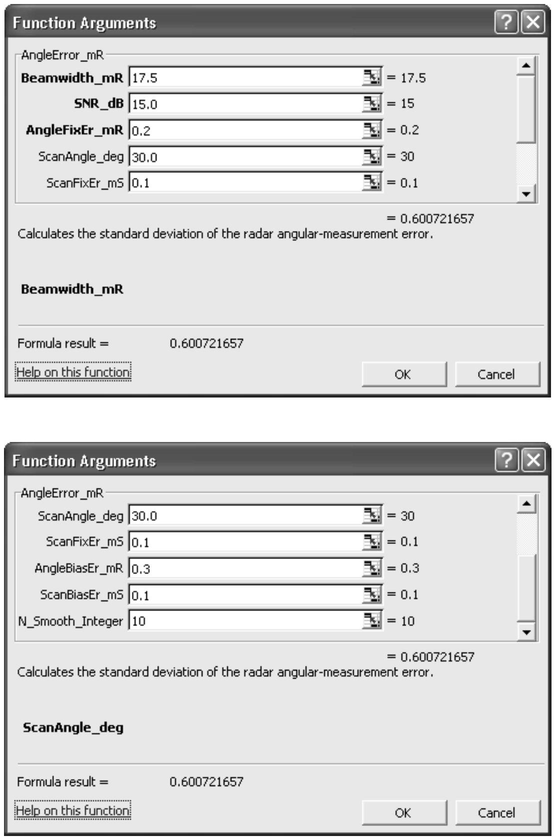

Figure 8.6 Excel parameter box for Function AngleError_mR.

ScanFixEr_mS (optional) = scan-angle dependent radar random angle error in the measurement coordinate for phased-arrays (msine). If this parameter is omitted, a value of zero will be assumed.

AngleBiasEr_mR (optional) = composite angle bias error in the measurement coordinate (mrad). This parameter is the rss of the radar angle bias error in the measurement coordinate and any angle bias errors due to propagation and other sources. If left blank, a zero value will be assumed for this parameter.

ScanBiasEr_mS (optional) = scan-angle dependent radar angle bias error in the measurement coordinate for phased arrays (msine) If left blank, a zero value will be assumed for this parameter.

N_Smooth_Integer (optional) = number of angle measurements that are smoothed in the calculated angle accuracy (integer). If left blank, a single-pulse, or integrated pulse-group measurement will be assumed. No output is generated for values less than 1.

Function Output The standard deviation of the angular-measurement error for the parameters specified (mrad). The Excel Function Arguments parameter box for Function AngleError_mR is shown in Figure 8.6, with sample parameters and a solution.

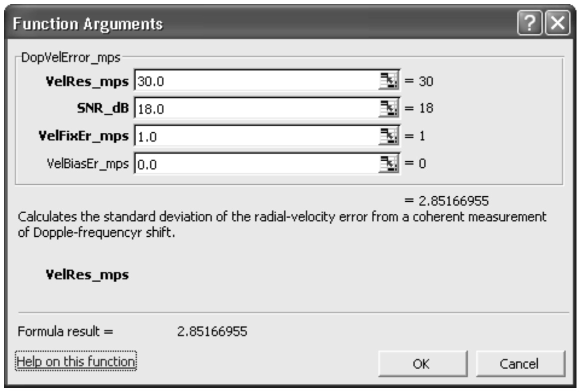

Figure 8.7 Excel parameter box for Function AngleError_mR.

8.9.3 Function DopVelError_mps

Purpose Calculates the standard deviation of the radar radial-velocity measurement error from a coherent measurement of Doppler-frequency shift.

Reference Equations (8.12) and (8.13).

Features Combines calculated S/N-dependent random radial-velocity error with fixed random radial-velocity error and radial-velocity bias error. Radial-velocity bias error may be omitted to calculate the relative radial-velocity error.

Input Parameters (with units specified)

VelRes_mps = radar radial-velocity resolution (m/s). A value of $\lambda/2\tau$ may be used, where λ is the radar signal wavelength, and τ is the duration of the coherently processed waveform.

SNR_dB = measurement S/N (dB). This may be the single-pulse S/N or integrated S/N as appropriate. The value input should take into account any beamshape loss or additional signal-processing loss for pulse integration. For single-pulse S/N values less than about 12 dB, the detection loss (5.16) should also be included.

VelFixEr_mps = composite fixed random radial-velocity error (m/s). This parameter is the rss of the radar fixed random radial-velocity error and any random radial-velocity errors due to propagation and other sources.

VelBiasEr_mps (optional) = composite radial-velocity bias error (m/s). This parameter is the rss of the radar radial-velocity bias error and any radial-velocity bias errors due to propagation and other sources. If left blank, a zero value will be assumed for this parameter.

Function Output The standard deviation of the radial-velocity measurement error from coherent measurement of Doppler-frequency shift for the parameters specified (m/s).

The Excel Function Arguments parameter box for Function DopVelError_mps is shown in Figure 8.7, with sample parameters and a solution.

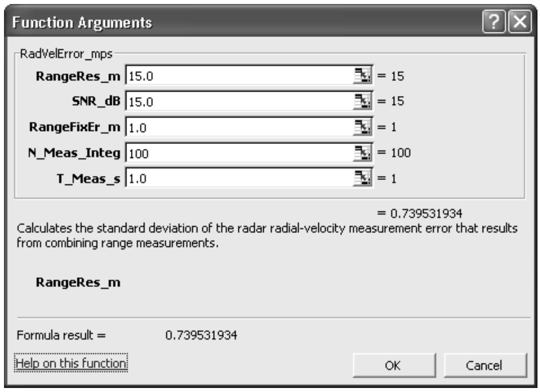

Figure 8.8 Excel parameter box for Function RadVelError_mps.

8.9.4 Function RadVelError_mps

Purpose Calculates the standard deviation of the radar radial-velocity measurement error that results from combining range measurements.

Reference Equations (8.15) and (8.16).

Features Calculates the range measurement error from radar range error-parameter inputs. Calculates the radial-velocity error from either the difference of two range measurements, or from processing a train of six or more range measurements.

Input Parameters (with units specified)

RangeRes_m = radar range resolution (m). A value of 1/*B* may be used, where *B* is the radar signal bandwidth.

SNR_dB = range measurement S/N (dB). This may be the single-pulse S/N or integrated S/N as appropriate. The value input should take into account any beamshape loss or additional signal-processing loss for pulse integration. For single-pulse S/N values less than about 12 dB, the detection loss (5.16) should also be included.

RangeFixEr_m = composite fixed random range-measurement error (m). This parameter is the rss of the radar fixed random range error and any random range errors due to propagation and other sources.

N_Meas_Integer = number of range measurements used in the radial-velocity measurement (integer). If 2 to 5 is input, (8.15) is used. If 6 or greater is input (8.16) is used. No result is produced for input values less than 2, indicated by an output of –1.

T_Meas_s = duration of measurements (sec). For two measurements, this is the time separation of the measurements. For a pulse train, this is the duration of the pulse train used.

Function Output The standard deviation of radial-velocity measurement error from combining range measurements for the parameters specified (m/s).

The Excel Function Arguments parameter box for Function RadVelError_mps is shown in Figure 8.8, with sample parameters and a solution.

8.9.5 Function CrossVelError_mps

Purpose Calculates the standard deviation of the radar cross-range velocity measurement error that results from combining angle measurements.

Reference Equations (8.17) and (8.18).

Features Calculates the angular measurement error from radar angle error-parameter inputs. Calculates the cross-range velocity error from either the difference of two angle measurements, or from processing a train of angle measurements, using the specified target range. A value of $k_M = 1.6$ is assumed. Allows modeling of phased-array beam broadening and scan-dependent errors.

Input Parameters (with units specified)

Beamwidth_mR = radar antenna beamwidth on the array broadside in the cross-range measurement coordinate (mrad).

SNR_dB = angle measurement S/N (dB). This may be the single-pulse S/N or integrated S/N as appropriate. The value input should take into account any beamshape loss or additional signal-processing loss for pulse integration. For single-pulse S/N values less than about 12 dB, the detection loss (5.16) should also be included.

AngleFixEr_mR = composite fixed random angle error in the measurement coordinate (mrad). This parameter is the rss of the radar fixed random angle error and any random angle errors due to propagation and other sources.

TgtRange_km = range of target (km).

N_Meas_Integer = number of angle measurements used in the cross-range velocity measurement (integer). If 2 to 5 is input, (8.17) is used. If 6 or greater is input, (8.18) is used. No result is produced for input values less than 2, indicated by an output of −1.

T_Meas_s = duration of measurements (sec). For 2 measurements, this is the time separation of the measurements. For a pulse train, this is the duration of the pulse train used.

ScanAngle_deg (optional) = scan angle in the measurement coordinate for phased arrays (degrees). If this parameter is omitted, a value of zero will be assumed.

ScanFixEr_mS (optional) = scan-angle dependent radar random angle error in the measurement coordinate for phased-arrays (msine). If this parameter is omitted, a value of zero will be assumed.

Function Output The standard deviation of cross-range velocity measurement error from combining angle measurements for the parameters specified (m/s).

The Excel Function Arguments parameter box for Function CrossVelError_mps is shown in Figure 8.9, with sample parameters and a solution.

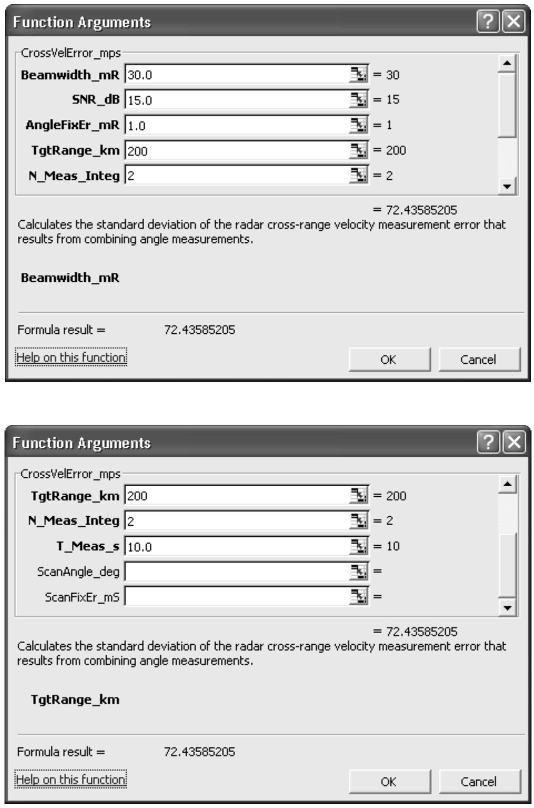

Figure 8.9 Excel parameter box for Function CrossVelError_mps.

8.9.6 Function PredictError_km

Purpose Calculates the approximate standard deviation of the error in predicted position from radar measurements for nonmaneuvering targets.

Reference Equation (8.7) to (8.9), (8.30), and (8.33).

Features Calculates the angular measurement error from radar angle-error parameter inputs. Calculates the cross-range velocity error from processing a train of angle measurements and the specified target range. Calculates predicted position error from the cross-range velocity error and the given prediction time. The cross-range coordinate having the larger measurement error (beamwidth) should be used to produce the largest predicted-position error semi-axis. A value of k_M = 1.6 is assumed. Allows modeling of phased-array beam broadening and scan-dependent errors.

Input Parameters (with units specified)

Beamwidth_mR = radar antenna beamwidth on array broadside in the cross-range measurement coordinate (mrad).

SNR_dB = angle measurement S/N (dB). This may be the single-pulse S/N or integrated S/N as appropriate. The value input should take into account any beamshape loss or additional signal-processing loss for pulse integration. For

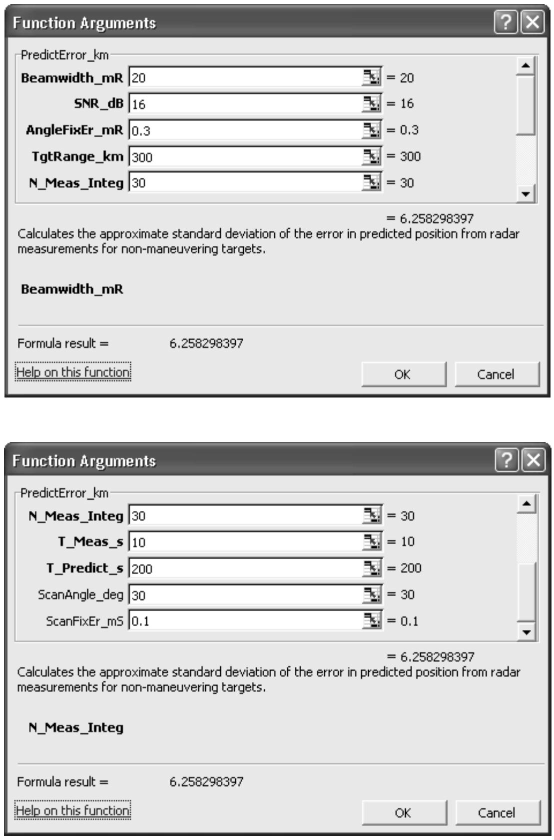

Figure 8.10 Excel parameter box for Function PredictError_km.

single-pulse S/N values less than about 12 dB, the detection loss (5.16) should also be included.

AngleFixEr_mR = composite fixed random angle error in the measurement coordinate (mrad). This parameter is the rss of the radar fixed random angle error and any random angle errors due to propagation and other sources.

TgtRange_km = range of target (km).

N_Meas_Integer = number of range measurements in the pulse train used for the cross-range velocity measurement (integer). No result is produced for an input of less than 2, indicated by an output of −1.

T_Meas_s = duration of the pulse train used for cross-range velocity measurement (sec).

T_Predict_s = time after cross-range velocity measurement that prediction error is calculated for (sec).

ScanAngle_deg (optional) = Scan angle in the measurement coordinate for phased arrays (degrees). If this parameter is omitted, a value of zero will be assumed.

ScanFixEr_mS (optional) = Scan-angle dependent radar random angle error in the measurement coordinate for phased-arrays (msine). If this parameter is omitted, a value of zero will be assumed.

Function Output The standard deviation predicted position error in the coordinate of the cross-range velocity measurement (km).

The Excel Function Arguments parameter box for Function PredictError_km is shown in Figure 8.10, with sample parameters and a solution.

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