Chapter 8: Simple Stellar Populations

Simple Stellar Population consists of stars born at the same time and having the same *initial* element composition. Stars of different masses follow different evolutionary tracks.

Theoretical isochrones

Old SSPs

- Age estimates
- Abundances of light elements
- Reddening and metallicity estimates
- Distance estimates
- Luminosity functions and estimates of the IMF

Young SSPs

- Age estimates
- Distance estimates

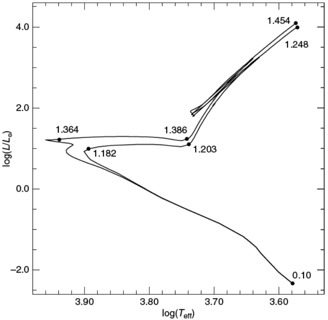
Review

Outline

Theoretical isochrones

In a theoretical HRD, stars of an SSP are located along an **isochrone** (which originates from the Greek word meaning "same age"). This line connects the points belong to the various theoretical evolutionary tracks at the same age.

- By applying to each point a set of appropriate bolometric corrections, the isochrone can then be converted to an observational CMD.
- The turnover (**TO**) point is the the bluest point along the isochrone MS, where the central hydrogen is exhausted.
- For a particular isochrone, the stellar mass range is very large along the MS.
- The mass evolving along RGB and successive phases is approximately constant; i.e., the stars there essentially all evolved from the same ZAMS mass.



HRD of two isochrones with ages equal to 2 and 3 Gyr ($Z = 0.001$). Stellar masses (in solar mass units) at representative evolving points are marked.

The evolutionary phase of a star along an isochrone may be characterized with the curvilinear coordinate s, starting from at the bottom of the ZAMS and increasing when moving towards more advanced phases.
The s value at a point is uniquely determined by the age of this isochrone t and the ZAMS mass M of the star, $s = s(t, M)$.
This function can then be inverted to write $t = t(M, s)$. Then we have

$$dt(M, s) = \left(\frac{dt}{dM}\right)_s dM + \left(\frac{dt}{ds}\right)_M ds = 0,$$

which leads to

$$\left(\frac{dM}{ds}\right)_t = -\left(\frac{dM}{dt}\right)_s \left(\frac{dt}{ds}\right)_M,$$

where $\left(\frac{dM}{dt}\right)_s$ is the ZAMS mass change rate for stars entering the phase, and $\left(\frac{dt}{ds}\right)_M$ is the time that a star with the mass M stays in the phase.

Theoretical isochrones also predict the relative number of stars at different evolutionary phases.
The number of stars in a particular phase interval is

$$\frac{dN}{ds} = -\frac{dN}{dM}\left(\frac{dM}{dt}\right)_s \left(\frac{dt}{ds}\right)_M$$

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Note that the mass used here needs to be ZAMS mass, whereas the value of the evolving mass (which determines the surface gravity) is always used to calculate the bolometric corrections to a generic photometric system.

Outline

Old SSPs

'Old' here denotes ages greater than ~ 4 Gyr (corresponding to stellar masses lower than $\sim 1.3 M_{\odot}$. This encompasses stars over the look back time range from redshift $z = 0$ up to $z \sim 2$ (when the age of the universe is ~ 4 Gyr).

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Theoretical models suggest that two special events significantly mark the spectral evolution of an SSP during its lifetime:

1. After $\sim 10^8$ yrs, due to the sudden appearance of red and bright Asymptotic Giant Branch (AGB) stars (AGB transition).
2. After $\sim 6 \times 10^8$ yrs (with the corresponding stellar mass $M \lesssim 2.5 M_{\odot}$ and with He-burning starting from degenerate cores), due to the development of the full Red Giant Branch (RGB transition).

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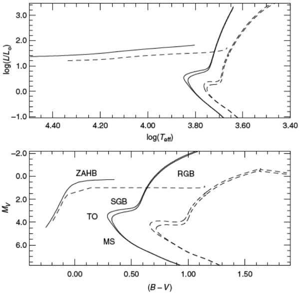
So, the age considered in this section is far beyond from the RGB phase transition.

Important properties of old SSP

- The brightness and color of the TO are affected by both age and Z.
- The lower MS ($\gtrsim$ 2 mag below the TO) and the RGB are unaffected by age, but are sensitive to Z.
  A star of certain Z and mass behaves somewhat like a star of lower Z and lower mass, in terms of the MS lifetime, brightness, and color.
- The brightness of the ZAHB is also unaffected by age, but sensitive to Z.
  The brightness is determined mostly by the He core mass at the He flash, which *decreases* with increasing Z.



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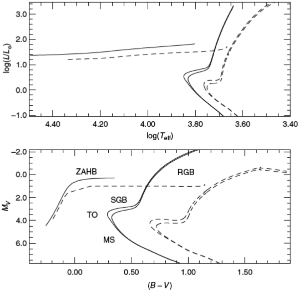

HRD and CMD of two pairs of isochrones from the ZAMS to the ZAHB, with ages t=10 and 12.5 Gyr, Z=0.0001 (solid line) and 0.02 (dashed line).

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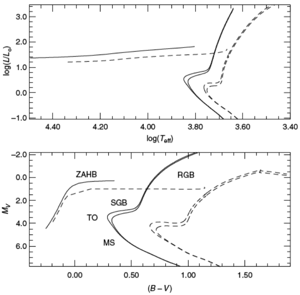

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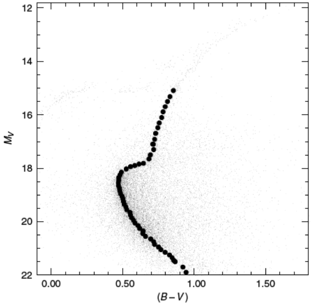
These properties can be used as tools to determine such important parameters as ages, Z, and distances to SSPs, providing the foundation for understanding the cosmic evolution.

Age estimates

- Ideally, one should do the isochrone fit to an observed CMD, using all the information in the HR diagram.
- But, even for a generic SSP, this approach may be difficult because of the systematic uncertainties in both the model and in measurements.
  An empirical CMD typically shows non-negligible scatters, caused by photometric errors, blending effects, and the presence of unresolved binaries, hence causing significant errors in the parameter determination of a fit.
- Sometimes, however, it is still desirable to use the approach to gain information about the CMD, even in more complicated situations than the SSP (e.g., to estimate the star formation history of a galaxy), when such errors may be tolerated.

A comparison is the use of a ’fiducial line’ (connecting peaks in color or magnitude ranges in the empirical CMD) as the observed CMD, in a way that is resistant to the errors.

The figure shows the fiducial line for the MS, SGB and RGB of the globular cluster M5 (filled circles) superimposed on the observed CMD.



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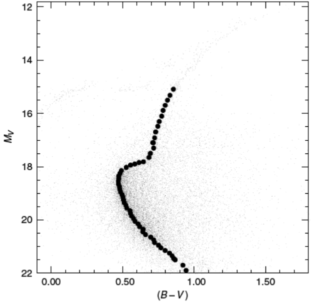

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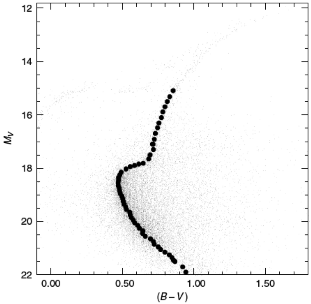

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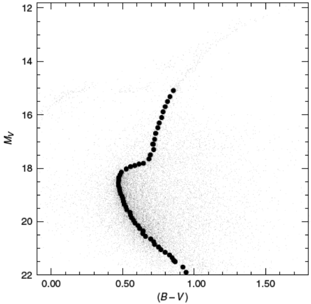

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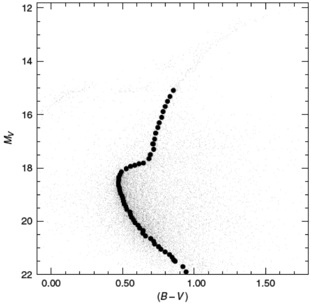

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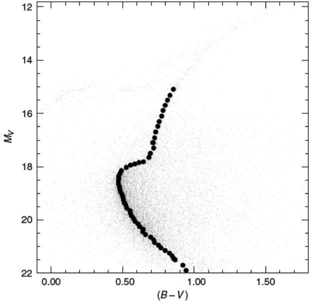

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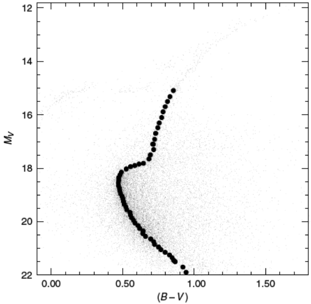

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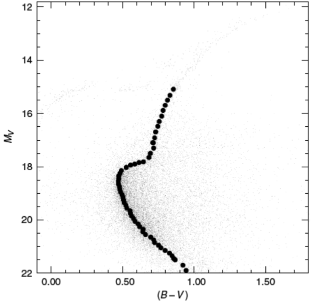

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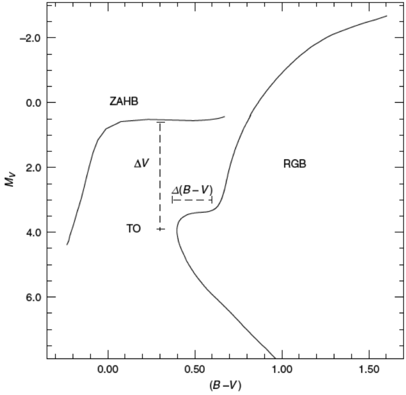
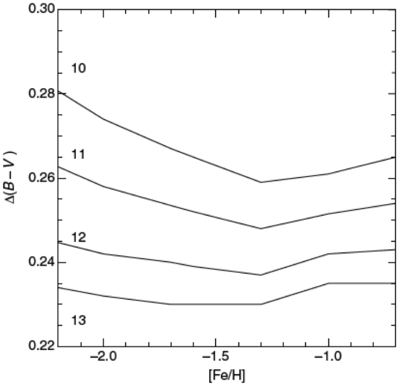
We'd like to use indicators that are less affected by the uncertainties or errors. For example, the direct comparison of the model and observed TO positions, either in color or in brightness, needs accurate correction for the effects, as well as the extinction and distance.

It is preferred to use differential quantities like the magnitude (usually V) difference between ZAHB and TO or the color [usually either $(B-V)$ or $(V-I)$] difference between the TO and the base of the RGB. The difference changes with the age of the SSP.



Graphical representation of the ΔV (vertical) and $\Delta(B-V)$(horizontal) age indicators for old SSPs.

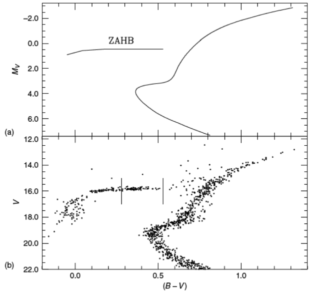


Theoretical values of $\Delta(B-V)$ as a function of [Fe/H] and age (in units of Gyr).

Use the vertical method as an example:

$$V_{\rm ZAHB} = < V_{\rm HB} > +0.05[{\rm Fe/H}]+0.20$$

where the ZAHB in a cluster CMD means the lower envelope of the HB star distribution, while $< V_{\rm HB} >$ is the mean level of objects in the RR Lyrae instability strip or at its red side. The blue side is not horizontal in a typical CMD (because of the choice of the color) and thus cannot be used.



Comparison of the CMDs of a 12 Gyr old metal-poor isochrone (upper panel) and the globular cluster M15 (lower panel). The position of the RR Lyrae instability strip is marked on the CMD.

Abundances of light elements

Light elements, such as He and Li, and their isotopes originate primarily in the primordial nucleosynthesis during the Big Bang. The abundance measurements of their abundances can therefore provide the observational constraints on the theory and on the cosmological baryon density in particular.

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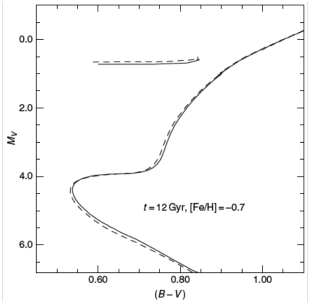

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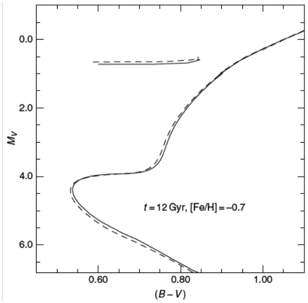


Two isochrones with the same age and metallicity, Y equals to 0.25 (solid line) or 0.27 (dashed line).

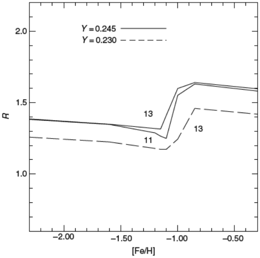
The initial He abundance Y affects the properties of SSPs (chiefly due to the dependence of the molecular weight on Y):

- A star with a higher Y appears brighter and bluer (if other parameters remain the same).
- An increase of Y decreases the mass of a star evolving at the TO and in post-MS phases and shifts the MS to left and the ZAHB to upward in a CMD.

These shifts can thus be used as measurements of Y.

Helium abundance measurement

- *The parameter* **R**, defined as the number ratio of HB stars to RGB stars brighter than the ZAHB level, increases with Y. This parameter has a very weak dependence on [Fe/H], except for the effect due to the RGB bump.

  The figure shows R-parameter values as a function of [Fe/H], Y, and age (in Gyr).

- *The parameter* **A** is defined to be the luminosity-to-mass ratio of a RR Lyrae star, $A = \log(L/L_\odot) - 0.81\log(M/M_\odot)$.
  - Increasing Y increases the luminosity of the HB (hence the luminosity of RR Lyrae stars), which is only partly offset by the dependence of M on Y.
  - The stellar evolution model predicts $A \propto 1.20Y$ and $\log P = 11.497 + 0.84A - 3.481\log T_{eff}$, from which we can determine A and then Y if both P and T_{eff} can measured observationally.
  - The error in Y arises chiefly from the difficulty in getting an accurate T_{eff}.

Li abundance measurement

Spectroscopically, old stars are too cold to show He lines. But Li abundance can be measured.

- Lithium is a very fragile element, easily destroyed in stellar interiors when $T \gtrsim 2.5 \times 10^6$ K. Such temperature can be achieved in stellar cores during their pre-MS contraction and even in outer regions during the MS phase.
- If the surface convection zone extends to such a region at some points of the stellar evolution, depending on the opacity and hence the temperature of a star, the Li abundance at its stellar surface can then be substantially altered.
- Indeed, the measurements of the Li abundance as a function of T_{eff} shows a relative flat part and then a sharp down-turn at $T_{eff} < 5700$ K.

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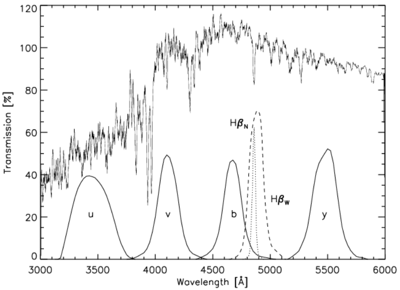
There exist discrepancies between the Big Bang nucleosynthesis predictions and stellar measurements, presumably due to uncertainties in the stellar nuclear process. Reliable test of the nucleosynthesis should be made with the abundance measurements for the IGM.

Reddening estimates

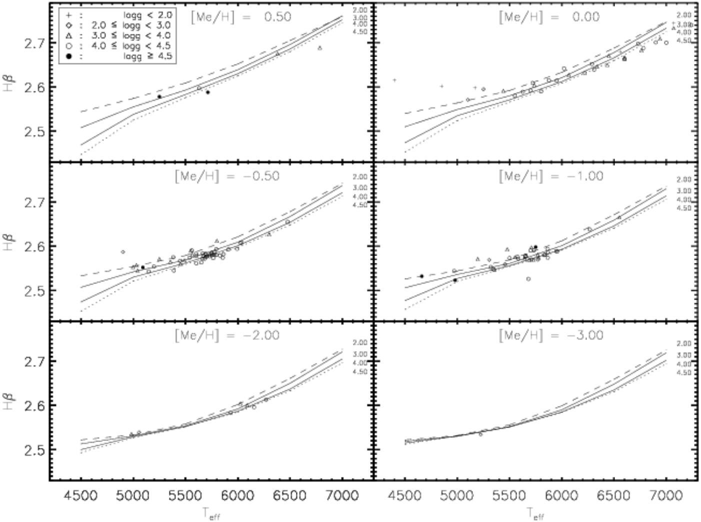
Strömgrem photometry:

- This photometry system is designed to isolate parts of the stellar spectra from which one can build color indices sensitive to specific properties of stars.
- The system consists of four filers *uvby* ($\approx$ 200 Å wide each), plus a pair of narrower filters β_n (30 Å) and β_w (100 Å), centered at 4860Å, that measure the strength of the H_β line and its adjacent continuum.
- The measurements of the broad-band color indices, such as $E(b-y)$, can be used to determine the extinction $E(B-V) = 1.4E(b-y)$.



The uvby-Hβ transmission functions. As a comparison, the flux of a model with $T_{eff} = 6000$ K, $\log(g) = 4.0$ and [Me/H] = 0.0 is plotted.

- In contrast, the age-dependent index, $\beta (= \beta_w - \beta_n)$, is sensitive to T_{eff} (hence defining the intrinsic spectral shape), but not to reddening.



Theoretical Hβ values versus effective temperature for models, together with values for the standard stars.

Metallicity Estimate

The de-reddened CMD can then be used to measure Z.

For example, the slope of the RGB of an old SSP is affected by Z and is insensitive to age. It is customary to use a reference that is empirically (and mostly locally) constructed, based on clusters with good photometric and spectroscopic determinations of [Fe/H].

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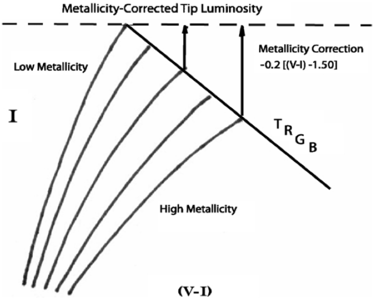
The metallicity dependence of the luminosity of the Tip of the RGB (TRGB) may also be used. With increasing metallicity, the H-burning rate in the shell becomes more efficient. As results, the thermal condition for igniting He is reached at a lower-mass and hotter He core.



Schematic representation of a variety of red giant branches in the I vs. V - I CMD (see Madore, B. F., et al. 2008, ApJ.)

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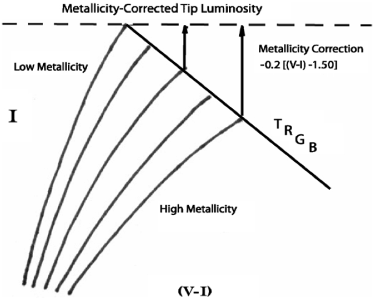

Schematic representation of a variety of red giant branches in the I vs. V - I CMD (see Madore, B. F., et al. 2008, ApJ.)

Of course, spectroscopy may also be used to estimate the metallicity of individual stars. But this is not always practical, especially for a distant stellar cluster.

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Tip of the RGB method:
The bolometric luminosity of the TRGB is determined by the mass of the He core at the He flash, with little dependence on the age if greater than ~ 4 Gyr (i.e., initial mass lower than $\sim 1.8M_{\odot}$). Such stars all develop very similar levels of electron degeneracy within the He core, and the mass of the He core has to reach almost the same value before He-burning ignites.

- In practice, the TRGB method is found to be best done in the I-band, with the weakest dependence on the metallicity,

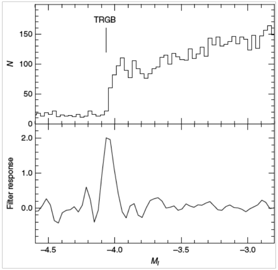
$$M_I^{TRGB} = 0.14[\mathrm{Fe/H}]^2 + 0.48[\mathrm{Fe/H}] - 3.629$$

$$V - I = 0.58[\mathrm{Fe/H}]^2 + 2.472[\mathrm{Fe/H}] + 0.4013,$$

  from which a solution of M_I^{TRGB} vs. $V - I$ can be found.

- Once the I-band apparent magnitude of the TRGB (I_{TRGB}) is determined with a proper extinction correction, the distance modulus can be estimated from

$$(m - M)_0 = I_{0,TRGB} - M_I^{TRGB}.$$



An example of LF of a globular cluster-like stellar population around the TRGB (upper panel), plus the response of the edge-detection algorithm applied to the same LF (lower panel).

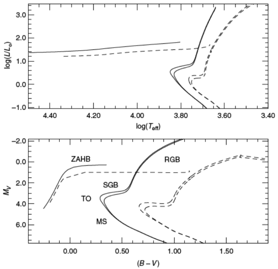
The method has been applied to the field halo population of external galaxies and is used for galaxies up to the Virgo distance.

Horizontal branch fitting method:

As in case of TRGB stars, the ZAHB brightness of low mass stars is determined by the value of the He core mass at the He flash, depending only on the initial metallicity:

$$M_V(ZAHB) = 0.17[\mathrm{Fe/H}] + 0.78.$$

The estimated distances for a sample of Galactic GCs are consistent with present MS-fitting distances (see below).



HRD and CMD with ages t=10 and 12.5 Gyr, Z=0.0001 (solid line) and 0.02 (dashed line).

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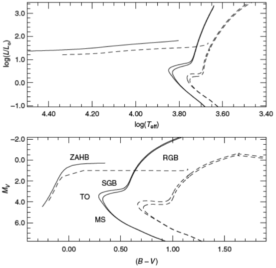


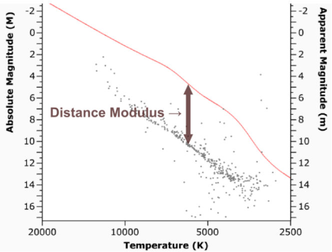
HRD and CMD with ages t=10 and 12.5 Gyr, Z=0.0001 (solid line) and 0.02 (dashed line).

In addition, both red clump and RGB bump have also been used for distance measurements. Examples are the determination of the orientation of the stellar bar in the central region of our Galaxy, using red clump stars. Furthermore, they have been used to map out extinction toward the Galactic nuclear region.

MS-fitting method:

The *lower* part of the MS can be used as a template and compared to the observed MS in an SSP.

- The template can be theoretical and empirical. To accurately correct for the composition effect, an empirical MS is built by considering local field stars of known [Fe/H] (e.g., determined from spectroscopy) with distances determined geometrically.
- Because of the steep slope of the lower MS, a small uncertainty in the color can affect the measurement a lot. An uncertainty in colors (not due to the uncertainty in the reddening) translates into an uncertainty of five times larger in the derived distance modulus.
- Typical errors on the best MS-fitting distance are of the order of $\sim 0.07 - 0.08$ mag.



The difference between the apparent magnitude of the cluster and the absolute magnitude of the main sequence line is the distance modulus used in the main sequence fitting technique.

Summary: Specific diagnostics in the CMD of old SSP

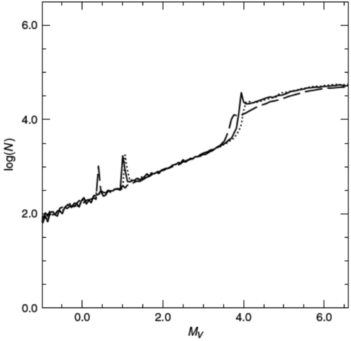
- Age: TO (vertical and horizontal methods); WD number turn-off.
- Stellar temperature and redenning: Strömgrem photometry (H_β and uvby filters); SED fits.
- Abundances: helium (R- or A-parameter), lithium (spectroscopy), and metallicity (slope of RGB, insensitive to the age and A_V)
- distance: TRGB-fitting, HB-fitting, and MS-fitting.

Luminosity function analysis

The luminosity function (LF) of an SSP is a traditional tool to assess the level of agreement between theoretical stellar evolution models and real stars. The shape of the LF of post-MS evolutionary phases of an SSP is fully determined by the evolutionary times of stars with an essentially identical mass evolving along those phases, independent of the choice of the IMF.

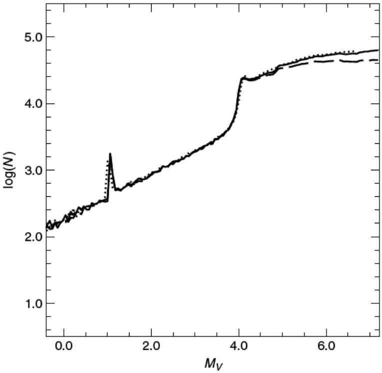
LF diagnostics and IMF estimate

- The shape of the TO and its brightness are affected by the age; the SGB population along a young isochrone (hence a relatively high initial mass) causes a more peaked shape before dropping into the RGB than along an old one.
- The local maximum along the RGB is due to the RGB bump. Comparison of the predicted bump brightness with SSP observations provides a powerful test for the extension of the convection in stellar envelopes; the higher metallicity corresponds to higher opacity, hence deeper the convection reaches.



LFs covering MS, SGB and RGB for three isochrones (computed using a Salpeter IMF, $dn/dm \propto m^{-2.35}$) with, respectively, $t = 14$ Gyr, $Z = 0.008$ (dotted line); $t = 12$Gyr, $Z = 0.008$ (solid line); $t = 14$ Gyr, $Z = 0.002$ (dashed line). All LFs assume Y=0.254 and are normalized to the same number of RGB stars at $M_V = 20$

- The slope of the post-MS LFs is essentially independent of age and of the initial Z, because the luminosity evolution is pre-dominantly due to the growth of the electron degenerate He core mass.
- The star counts along the MS is sensitive to the IMF due to the large mass range covered.
- After the observed and theoretical LFs are normalized to the same number of stars along the RGB, the IMF of an SSP (old or young) can be determined by comparing the shape of the observed LF along the MS with the theoretical counterparts.



As in the lat figure, but for isochrones computed for, respectively, $Y = 0.254$ and Salpeter IMF $dn/dm \propto m^{-2.35}$ (solid line); $Y = 0.254$ and an IMF $dn/dm \propto m^{-1.35}$ (dashed line), and $Y = 0.273$ (dotted line); all have $t = 14$ Gyr and $Z = 0.008$.

In practice, one needs to consider the dynamic (mass-segregation) effect on the LF of a particular region of a globular cluster or even in the central regions of galactic bulges (such as in M31).

More massive objects tend to sink toward the cluster center, whereas lower mass ones move outward.

Therefore, one actually determine the so-called present-day mass function (PDMF). The relationship between the PDMF and IMF has to be carefully deduced from the dynamical modeling of the observed population.

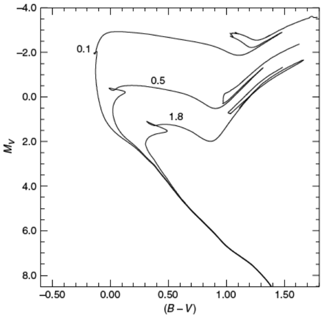
The dynamic effect is also important for central regions of young clusters, even on 10^6 year time scales.

Outline

Age estimates

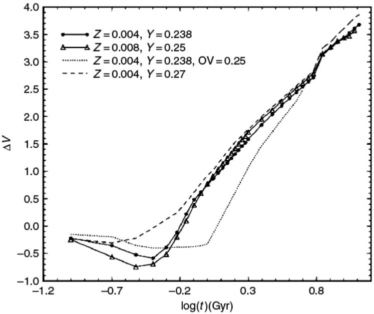
The isochrones of 'young' SSPs (younger than $\sim$ 4 Gyr; e.g., typically open clusters) are distinctly different from those for old SSPs. Precise age determination for young SSPs are difficult.

- The young isochrones show the hook-like feature at the TO due to the overall contraction of young stars when the hydrogen abundance in the core becomes $X < 0.05$ but before the complete exhaustion.
- For very young ages the vertical TO region covers a large luminosity range. The SGB and RGB phase is almost completely depopulated because of the much faster evolutionary timescale (the He core mass at the end of the central H-burning reaches the Schönberg-Chandrasekhar limit; He-burning starts before the core could become degenerate).
- The horizontal method is of no use here.



Isochrones for a solar chemical composition and three marked ages (in units of Gyr).

- The vertical method may still be used if a sizable sample of red clump stars is present, although the dependence on the age and metallicity is different from that of old SSPs.
- For ages between $\sim$ 0.5 and 4 Gyr the He-burning phase is usually a red clump of stars (because of the presence of relatively massive envelopes). For younger stars, the phase moves progressively to the blue side of the CMD, describing increasingly larger loops to the blue.



ΔV as a function of age for young isochrones with various initial chemical compositions.

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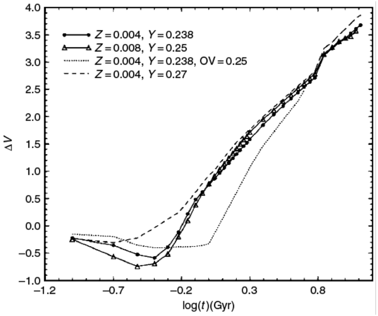

ΔV as a function of age for young isochrones with various initial chemical compositions.

If the distance and extinction to the observed SSP are known (e.g., clusters in the Galactic bulge and in external galaxies), a direct fit of theoretical isochrones to the TO can be used to estimate the age. Similar methods as used for the old SSPs may be used for faint MS stars of young SSPs.

Distance estimates

- The TRGB and He-burning phases are of less use because they are both affected by the SSP age; the TRGB does not exist for ages below $\sim 0.1 - 1$ Gyr.
- The most important technique is to make use of the Cepheid period-luminosity relationships, whenever a number of such variables is detected in the SSP.
- Classical Cepheids are Pop I variable stars of 4-20$M_{\odot}$ in the yellow supergiant phase.
- Only a few Cepheids in the Galaxy have parallax error below 30%. They also have large uncertainties in the extinction correction.
- The template P-L relationship traditionally used has been determined on the Cepheids in the LMC.

Outline

Review

Key concepts: simple stellar population, isochrone, TO of an isochrone.

1. What determines the stellar number density along the post-MS isochrone?
2. How do various features in a CMD depend on the parameters of an isochrone?
3. What are some basic ways to estimate the age, metallicity, and distance?
4. Using the high-resolution HST imaging capability, what may be the way to determine the distance to a dwarf spheroid galaxy that is probably in our Local Group and consists of primarily stars more than 4 Gyr old?
5. How may a present day mass function of a stellar cluster different from its IMF?
6. What are the so-called red-clump stars?