# <u>Chapter 8:</u> <u>Systems of Linear Equations & Inequalities</u>

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Systems of equations (and inequalities) are essential to modeling situations with multiple variables and multiple relationships between the variables. At the end of the day, though, the solution set of a system of equations can be easily defined:

#### SOLUTIONS TO A SYSTEM OF EQUATION

1. A point (x, y) is a solution to a system if it makes all equations true.

2. The solution set of a system is the collection of all pairs (x, y) that are solutions to the system (see 1).

*Exercise #1*: Determine if the point (2,5) is a solution to each of the systems provided. Show the work that leads to your answer for each.

(a) y = 4x - 3(2,5) (x,y) y = 4x - 3 5 = 4(2) - 3 5 = 5 2x + y = 9 5 = 4(2) - 3 5 = 5 2(2) + 5 = 9 5 = 5 9 = 9Yes, (2,5) is a solution.

We can solve a system by using a graph. Review this process in the next exercise. *Exercise* #2: Consider the system of equations shown below:

y = 2x + 5 y = 2 - x(a) Graph both equations on the grid shown. Make sure to show all necessary work to graph the lines and don't forget to label each line with its equation.

| y = 2x + 5  | y = -x + 2   |
|---|--|
| y = mx + b  | y = mx + b   |
| $m = \frac{\Delta y}{\Delta x} = \frac{2}{1} = \left(\frac{\uparrow 2}{\rightarrow 1}\right)$ | $m = \frac{\Delta y}{\Delta x} = \frac{-1}{1} = \left(\frac{\downarrow 1}{\rightarrow 1}\right)$ |
| b = 1st point = y - interept  | b = 1st point = y - interept   |
| (0,b) = (0,5)   | (0,b) = (0,2)  |

(b) At what point do the two lines intersect?

(c) Show this point is a solution to the system.

(-1,3) (x,y) y = 2x + 5 3 = 2(-1) + 5 3 = 2 - (-1) 3 = -2 + 5 3 = 3  $Y_{00} (-1,2) \text{ is a solution}$ 

Yes, (-1,3) is a solution.



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*Exercise #3*: Consider the system of equations shown below: 3y = 6x + 15  $y = x^2 + 6x + 5$ 

(a) Graph both equations on the grid shown. Make sure to show all necessary work to graph the linear function, then fill in the provided table for the quadratic function. Don't forget to label each function with its equation.





(b) At what points do the two functions intersect?

(c) Show these points as a solution to the system.

*Exercise #4*: Consider the system of equations shown below:

$$y = x - 1$$
  $y = 2 |x + 1| - 8$ 

(a) Graph both equations on the grid shown. Make sure to show all necessary work to graph the linear function, then fill in the provided table for the absolute value function. Don't forget to label each function with its equation.





(b) At what points do the two functions intersect?

(c) Show these points as a solution to the system.

*Exercise #5*: The functions  $y = -x^2 + 4x$  and y = 4 - x are graphed on the grid shown. Which of the following sets gives all the solutions to the equation  $-x^2 + 4x = 4 - x$ ?



[3] {1,3} [4] {0,4}



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| Name:<br>Algebra I   | Date:<br>Solutions & Solvin                      | Period:<br>ng by Graphing           | 8A HW     |
|--|--|-------------------------------------|-----------|
| 1.) So, now you can put the definition of the graph of an equation to the blanks with one of the words shown: <b>TRUE, INTERSECTIO</b> | ogether with the def<br><b>)N, SOLUTIONS, BC</b> | finition of a system.<br><b>)TH</b> | Fill in   |
| A) To solve a system of equations graphically you find the   |  | of the two gra                      | phs.      |
| B) This works because any intersection point must lie on   |  | graphs.                             |           |
| C) Because intersection points lie on both graphs, they must m   | ake both equations                               |                                     |           |
| D) If intersection points make both equations true, they are   |  | _ to the system of e                | quations. |
| <ul> <li>2.) Determine whether each of the following points is a solution to</li> <li>(a) (3,4)</li> <li>(b) (-10,-1)</li> </ul>       | the given system.<br>)                           |                                     |           |

| ı) (3,4)  |            | (b) (−10,−1)           |             |
|-----------|------------|------------------------|-------------|
| x + y = 7 | y = 2x - 2 | $y = \frac{1}{2}x + 4$ | y = 4x + 30 |

3.) Solve the following system of equations graphically. After graphing, be sure to label each line with its equation and state your final solution as a coordinate pair.

$$y = \frac{1}{3}x + 1$$
  $x + y = 5$ 



4.) Consider the system of equations shown below:

$$y = 2x \qquad \qquad y = -x^2 - 3x$$

(a) Graph both equations on the grid shown. Make sure to show all necessary work to graph the linear function, then fill in the provided table for the quadratic function. Don't forget to label each function with its equation.



(b) At what points do the two functions intersect?

(c) Show these points as a solution to the system.

5.) Solve the following system of equations graphically. After graphing, be sure to label each line with its equation and state your final solution as a coordinate pair.

| у | = | 4 |
|---|---|---|
|---|---|---|

| y =  x | -2 -3 |
|--------|-------|
| x      | у     |
| -1     |       |
| 0      |       |
| 1      |       |
| 2      |       |
| 3      |       |
| 4      |       |
| 5      |       |
| 6      |       |



(b) At what points do the two functions intersect?

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- 6.) The two lines y = 6x + 15 and y = mx 4 intersect at x = -2.
- (a) What is the y –coordinate of their intersection point? (b) What is the value of m?

#### **Review Section:**

7.) Solve the inequality and graph the final solution on the number line provided.

8x - 6(x - 2) > 20 - 2x



- 8.) A nutrition company is marketing a low-calorie snack brownie. A serving size of the snack is 4 brownies and has a total of 50 calories.
  - (a) Determine how many calories 10 brownies would have.
- (b) Determine how many brownies a person has consumed if they have eaten 212.5 calories.

| Name:_ | Homework Answers |
|--------|------------------|
|        |                  |

Algebra I

| Date:              | Period:        |       |
|--------------------|----------------|-------|
| Solutions & Solvin | ng by Graphing | 8A HW |

| 1.) a.) Intersection<br>c.) true | b.) both<br>d.) Solutions     |
|----------------------------------|-------------------------------|
| 2.) a) yes                       | b.) no                        |
| 3.) GRAPH Sc                     | lution: (3,2)                 |
| 4.) GRAPH Sc                     | ution: $(0,0)$ and $(-5,-10)$ |
| 5.) (-5,4) and (9                | 4)                            |
| 6.) $y = 3$ $m =$                | $=-\frac{7}{2}$               |
| 7.) <i>x</i> > 2                 |                               |
| 8.) a) 125 calorie               | s b.) 17 brownies             |

| Name:     | Date: Period:           |    |
|-----------|-------------------------|----|
| Algebra I | Solving by Substitution | 8B |

#### Solving Systems Algebraically by Substitution

There are a variety of ways that we solve a system of equations. In the last lesson we saw how to solve them graphically. In this lesson we will review and understand the basis for solving them by a method known as **substitution**. You have seen this technique in Common Core 8th Grade mathematics, but here we will explore it more deeply.

*Exercise* #1: Consider the system given below and its solution (4,1). 2x + y = 9 y = x - 3

(a) Show that this is a solution to the system using (4,1,).

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The substitution method is used to eliminate one of the variables by replacement when solving a system of equations. Solve using the substitution method.

(b) Substitute x - 3 in for y in the first equation and show that the point (4,1) is still a solution to the new equation.

(c) Solve the system by finishing the substitution from part b. Find y by using the x-value you found.

Substitution is a very important technique and we want to be very good at it. It boils down to one of the most important properties of equality.

|                    | Equ                       | JALS MAY ALWAYS SUBSTITUTE FOR E | QUALS                        |
|--------------------|---------------------------|----------------------------------|------------------------------|
| Exercise #2: Solve | e the following systems o | f equation by using substitut    | ion.                         |
| a) $y = 2x + 5$    | y + 10 = -3x              | b) $4x - 22$                     | $y = 16 \qquad y = -5x + 13$ |

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The algebra of systems allows us to solve all sorts of problems that almost seem like riddles.

*Exercise #3*: Max and his father Kirk are comparing their ages. They know that the sum of their ages is 52 and that Kirk is 7 years older than 4 times Max's age.

a) If Max's age is represented by m, and Kirk's age by k, write a system of equations that describes the two relationships from the problem.

b) Solve the system using substitution to find both of their ages.

*Exercise #4*: Two cell phone plans offer different text packages. the two plans are outlined below: Plan A: \$5.95 per month charge along with a charge of \$0.03 per text message. Plan B: No per month charge, but a charge of \$0.10 per text message.

Is there a certain number of texts, when the two plans cost the same amount? Determine your answer by setting up a system of equations that model the two plans and **solve by substitution**.

*Exercise #5*: A man and a woman start 380 feet away from each other and walk in a straight line towards each other. If she is walking at a rate of 6 feet per second and he is walking at a rate of 2 feet per second, when will they meet? Determine your answer by setting up a system of equations that model the two speeds and **solve by substitution**.

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*Exercise #6*: Solve the following system of equations using substitution method. 4x + 2y = 14 y = 2x - 4

Exercise #7. Solve the following system of equations using substitution method.

y = x + 7y = -2x + 1

- 8. Which choice is the solution to the system of equations: y = 3x and y=2x+7
  - a) (7,10) b)  $\frac{7}{5}, \frac{21}{5}$  c) (-7, -21) (7,21)
- 9. The substitution method is used to solve the following system of equations algebraically:

$$4x - 2y = 6$$
 and  $3x + 27 = 8$ 

Which equivalent equation may be used in this process

a) 
$$3x + 2(2x-3) = 8$$
 b)  $3x+2(3-2x) = 8$  c)  $3(y+3/2)+2y = 8$  d)  $3(3/2-y)+2y=8$ 

| Name:     | Date:                 | Period: |       |
|-----------|-----------------------|---------|-------|
| Algebra I | Solving by Substituti | on      | 8B HW |

#### Solving Systems Algebraically by Substitution Homework

1.) Solve each of the following systems of equations by using substitution. a) y = x + 8 y = 4x - 1 b) y = -3x + 5 2x + y = 6

c) 4x + 3y = 37 y = x - 4 d) x - 5y = -49 y = -2x + 1

e) 3y - 2x = 11 and y + 2x = 9f) x + 2y = 12 y = 2x-4

2.) Given the system of equations shown below  $\frac{1}{2}$ 

$$y = \frac{1}{2}x - 2$$
  $y = -3x + 5$ 

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a) Solve this system of equations graphically uing the grid shown.

b) Solve this system of equations by using substitution.



3.) A rectangle has a perimeter of 42 feet. Its length, *L*, is three feet more than twice its width, *W*.

a) Create an equation in terms of L and W for the perimeter of the rectangle.

b) Create an equation that relates L and W based on the length being three feet more than twice the width.

c) Solve the system of equations that you just created by substitution to find the values of the length and width.

4.) The equations 5x + 2y = 48 and 3x + 2y = 32 represent the money collected from school concert tickets sales during two class periods. If *x* represents the cost for each adult ticket and *y* represents the cost for each student ticket, what is the cost for each adult ticket? (Use Substitution Method to solve.)

#### Review Section:

.5.) The graph below represents a jogger's speed during her 20-minute jog around her neighborhood.

Which statement best describes what the jogger was doing during the 9-12 minute interval of her jog?

- (1) She was standing still.
- (2) She was increasing her speed.
- (3) She was decreasing her speed.
- $(4)\;$  She was jogging at a constant rate.

\_6.) Which table represents a function?





7.) Find the equation of the inequality graphed.





| Name: Homework A                               | nswers                                | Date: Period:           |       |
|--|---------------------------------------|-------------------------|-------|
| Algebra I                                      |                                       | Solving by Substitution | 88 HW |
| 1.) a.) (3,11)<br>c.) (3,7)<br>e.) (2,5)       | b.) (-1,8)<br>d.) (-4,9)<br>f.) (4,4) |                         |       |
| 2.) a.) GRAPH (2, -1)                          | b.) Substitution $(2, -1)$            |                         |       |
| 3.) a.) $42 = 2l + 2w$<br>c.) $w = 6$ $l = 15$ | b.) $l = 2w + 3$                      |                         |       |
| 4.) \$8 for Adult, \$4 for Child               |                                       |                         |       |
| 5.) [4]  |                                       |                         |       |
| 6.) [3]  |                                       |                         |       |
| 7.) $y \ge -3x + 4$                            |                                       |                         |       |

| Name:     | Date:         | Period:   |    |
|-----------|---------------|-----------|----|
| Algebra I | Solving by El | imination | 8C |

#### Solving Systems Algebraically by Elimination

Sometimes solving a system of equations using substitution can be very difficult. For these problems we solve using Linear Combinations (or Elimination). With elimination you solve by eliminating one of the variables. This is accomplished by adding the 2 equations together. Before you can add the equations together, you need one of the two variables to have two things:

1) Same Coefficient

2) Different Signs (one positive and one negative)

#### Steps:

- 1) Make sure that the system of equations is in Standard form [Ax + By = C]
- 2) Criss Cross the leading coefficients (always make the "dropping" number negative)
- 3) Multiply both sides of the equation (left and right!!)
- 4) Add the new equations together to eliminate the first terms, and solve for the remaining variable
- 5) Carry the variable up to the second column and substitute into one of the *original* equations to solve for the second variable
- 6) Check the solution in both original equations.

| <u>First Variable</u>  | Second Variable  | <u>Check</u>  |
|--|--|---|
| 6x + 7y = 11  5x - 6y = -50  | y = 5  | x = -4  y = 5   |
| 5(6x + 7y) = (11)5-6(5x - 6y) = (-50) - 6  | Substitute 5 for y in $5x - 6y = -50$  | 6x + 7y = 11<br>6(-4) + 7(5) = 11<br>11 = 11 (It Checks!)                             |
| $ \begin{array}{r} 80x + 35y &= 55 \\ + - 3x + 36y &= 300 \\ \hline 71y &= 355 \\ \hline 71 &= 71 \\ \hline y &= 5 \end{array} $ | $5x - 6(5) = -50$ $5x - 30 = -50$ $5x - 30 = -50$ $+30 + 30$ $\frac{5x}{5} = -20$ $x = -4$ | 5x - 6y = -50<br>5(-4) - 6(5) = -50<br>-50 = -50 (It Checks!)<br><b>POI = (-4, 5)</b> |

# **Practice:**

1) 7x + 3y = 105x - 6y = 56 Page **17** of **52** 

2) 11x + 5y = 274x + 6y = 60

3) 9x + 7y = 1267x - 9y = -32 Page **18** of **52** 

4) 12x = 5y + 638x + 3y = 23

5) 5x + 9y = 146x = -11y + 18

| Name:     | Date: Period:          |       |
|-----------|------------------------|-------|
| Algebra I | Solving by Elimination | 8C HW |

# Solving Systems Algebraically by Elimination Homework

#### Solve and check!

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1.) Solve the system of equations below using elimination.

 $y - 2x = 3 \qquad \qquad y + 7 = x$ 

2.) Solve the system of equations below using elimination.

| 3x + 2y = 20 | x - y = -5 |
|--------------|------------|
|--------------|------------|

3.) Solve the system of equations below using elimination.

2x + 4y = 2

6x + 3y = -3

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4.) Solve the system of equations below using elimination.

x + 4y = 133x + 2y = 19

5.) A small movie theather sells children's tickets for \$4 each and adult tickets for \$10 each for an animated movie. The theater sells a total of \$388 in ticket sales.

a) If c represents the number of children's tickets sold and *a* represents the number of adult tickets sold, write an equation that models the information shown above.

b) If c represents the number of children's tickets sold and *a* represents the number of adult tickets sold, write an equation that models the fact that a total of 70 tickets were sold for the animated movie.

c) Using the two equations from part a and b, how many children's tickets and how many adult tickets were sold for the animated movie?

- 6.) The point (4, -2) is a solution to the system of equations 2x + y = 6 and x + 5y = -6. Which of the following equations would it **not** be a solution to?
  - [1] 3x + 6y = 0
  - $[2] \ 2x + 10y = -12$
  - [3] 2x + 2y = 12
  - [4] x 4y = 12

\_\_\_\_ 7.) Which of the following points is a solution to the system of equations x - 2y = -11 and 5x + 2y = 29?

- [1] (4,1)
- [2] (5,2)
- [3] (-3,9)
- [4] (3,7)

#### Review Section:

\_\_\_\_\_ 8.) The graph of the function  $f(x) = \sqrt{x+4}$  is shown: The domain of the function is:

- (1)  $\{x | x > 0\}$  (3)  $\{x | x > -4\}$
- (2)  $\{x \mid x \ge 0\}$  (4)  $\{x \mid x \ge -4\}$



9.) Natasha is planning a school celebration and wants to have live music and food for everyone who attends. She has found a band that will charge her \$750 and a caterer who will provide snacks and drinks for \$2.25 per person. If her goal is to keep the average cost per person between \$2.75 and \$3.25, how many people, p, must attend?

| (1) | $225$ | (3) $500$ |
|-----|-------|-----------|
| (2) | $325$ | (4) $750$ |

10.)Alex is selling tickets to a school play. An adult ticket costs \$6.50 and a student ticket costs \$4.00. Alex sells x adult tickets and 12 student tickets. Write a function, f(x), to represent how much money Alex collected from selling tickets.

Homework Answers

Name:\_\_\_\_ Algebra I Date:\_\_\_\_\_ Period:\_\_\_\_\_ Solving by Elimination 8C HW

- 1.) (-10,-17)
- 2.) (2,7)
- 3.) (-1,1)
- 4.) (5,2)
- 5.) a.) 4c + 10a = 388 b.) c + a = 70 c.) 18 adults and 52 children tickets were sold.
- 6.) [3]
- 7.) [4]
- 8.) [4]
- 9.) [4]
- 10.) f(x) = 6.50x + 48

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|------|----|----|----|
|------|----|----|----|

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|-----------|------------------------|-----------------|----|
| Algebra I | The Elimination Method | – More Practice | 8D |

#### The Elimination Method

In previous courses you have seen how to solve systems graphically and how to solve them by substitution. Today's lesson will build on the previous one and formally introduce the technique of solving a system by elimination. Remember from the last lesson that:

SOLUTIONS TO SYSTEMS REMAIN SOLUTIONS IF

1. Properties of equality are used to rewrite either of the equations.

2. The equations are added or subtracted or any rewrite is added or subtracted.

*Exercise* #1: Consider the system of equations shown. Solve the system of equation in two different ways, by eliminating *x* in part a and eliminating *y* in part b.

a) Eliminate *x* to solve

4x + 5y = 12-2x + y = 8

b) Eliminate *y* to solve.

4x + 5y = 12-2x + y = 8

c) Show that the point that you found in part a and part b is a solution to the system of equations.

*Exercise #2*: Solve the following system of equations by elimination and check that your answer is a solution. 5x = 2y + 10 2x + 7y = 43

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There are many applications of solving systems of linear equations by elimination. One of the more interesting ones comes in finding the equation of a line if you know two points that it goes through.

*Exercise #3*: Consider a line that passes through the points (-2, -11) and (3, 14). We want to find its equation in y = mx + b form.

a) Substitute both of the know points into y = mx + b to create a system of two equations with the variables m and b.

b) Solve the system of equations from part a for *m* and *b*. Then write the equation of the line.

*Exercise* #4: Find the equation of the linear function, in y = mx + b form, shown in the table below.

| x | -2 | 2 | 6 | 10 |
|---|----|---|---|----|
| у | 3  | 5 | 7 | 9  |

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|------|----|----|----|
|------|----|----|----|

| Name:     | Date:               | Period: |       |
|-----------|---------------------|---------|-------|
| Algebra I | The Elimination Met | hod     | 8D HW |

### The Elimination Method Homework

1.) Solve each of the following systems of equations by the Method of Elimination.

a) 2x + 5y = 3-2x - y = 5

b) 
$$x = y + 15$$
  
 $4x + 2y = 30$ 

c) 2x + 3y = 176y = -5x + 32 2.) Use the Method of Elimination to find the equation of the line, in y = mx + b form, that passes through the following points. Set up a system first, like we did in Exercise #3 from class. Then, solve the system for the slope, *m* and the y-intercept, *b*. Don't forget to write the equation of the line.

(3,10) and (5,18)

3.) Shana bought sodas and popcorn for the movies. Sodas cost \$3 each and popcorn costs \$4 per bag. Shana bought 7 things from the concession stand, all either sodas or bags of popcorn. Shana spent a total of \$26. Write a system of equations involving the number of sodas, *s*, and the bags of popcorn, *b*. Solve the system to see how many of each Shana bought.

4. )A local theater is showing an animated movie. They charge \$5 per ticket for a child and \$12 per ticket for an adult. They sell a total of 342 tickets and make a total of \$2550. How many of each ticket was sold?

#### Review Section:

4.) What are the zeros of the function  $f(x) = x^2 - 13x - 30$ ?

[1] -10 and 3 [2] 10 and -3

[3] -15 and 2 [4] 15 and -2

5.) If the difference  $(3x^2 - 2x + 5) - (x^2 + 6x - 7)$  is multiplied by  $\frac{1}{2}x^2$ , what is the result in standard form?

6.) Determine the smallest integer that makes -3x + 7 - 5x < 23 true.

Homework Answers Name:\_\_\_\_\_\_ Algebra I

- 1.) a.)  $\left(-\frac{7}{2}, 2\right)$  b.) (10, -5)c.) (-2, 7)
- 2.) y = 4x 2
- 3.) Shana bought 2 sodas and 5 bags of popcorn.
- 4.) [4]
- 5.)  $x^4 4x^3 + 6x^2$
- 6.) The smallest integer is -1.

Date:\_\_\_\_\_ Period:\_\_\_\_\_ The Elimination Method 8D HW

| Name:     | Date:       | Period:    |    |
|-----------|-------------|------------|----|
| Algebra I | Modeling wi | th Systems | 8E |

#### **Modeling with Systems of Equations**

Many real world scenarios can be modeled using systems of equations. In fact, when we have two quantities that are related and two ways in which those quantities are related, then we can often set up and solve a system.

*Exercise #1*: John has nine bills in his wallet that are either five-dollar bills or ten-dollar bills.

a) Fill out the following table to see the dependence of the two variables and how they then determine how much money John has.

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| Number<br>of fives, <i>f</i> | Number<br>of tens, <i>t</i> | Amount of Money, \$ |
|------------------------------|-----------------------------|---------------------|
| 0                            |                             |                     |
| 1                            |                             |                     |
| 2                            |                             |                     |
| 3                            |                             |                     |

b) If *f* represents the number of \$5 bills and *t* represents the number of \$10 bills, then what does the following expression calculate? Explain.

5f + 10t

c) If John has a total of \$55, set up a system of equations involving *f* and *t* that could be used to determine how many of each bill he has. Solve the system. remember that he has 9 total bills.

d) Let's say that we were told that John has 7 bills that were all 5's and 20's and we were also told that he has a total of \$120. Set up and solve a system to help evaluate whether we could have been told true information.

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# There are many different problems that can be modeled with linear systems. Let's try another one where we use information given to determine unit prices.

*Exercise #2*: Samantha went to a concession stand and bought three pretzels and four sodas and paid a total of \$11.25 for them. Amanda went to the same stand and bought five pretzels and two sodas and paid a total of \$8.25.
a) Could pretzels have cost \$1.75 each and sodas \$1.50 each? How can you evaluate based on the information given?
b) Letting x equal the unit cost of a pretzel and letting y equal the unit cost of a soda, write a system of equations that models the information given.

c) Solve the system of equations using the elimination method.

d) If Leah went to the same concession stand and bought two sodas, how many pretzels would she need to buy so that she spent the same amount on both?

We can model information given in a geometric form as well. We should feel relative comfortable working with rectangles and their perimeters. The next questions concerns the relationship between the length and width of a rectangle.

*Exercise #3*: A rectangle has a perimeter of 204 feet. Its length is six feet longer than twice its width. If *L* represents the length of the rectangle and *W* represents the width of the rectangle. Write a system of equations that models the information given in this problem and solve it to find the length and width of this rectangle.

| Name:     | Date:               | Period: |       |
|-----------|---------------------|---------|-------|
| Algebra I | Modeling with Syste | ems     | 8E HW |

#### Modeling with Systems of Equations Homework

1.) A local theater is showing an animated movie. They charge \$5 per ticket for a child and \$12 per ticket for an adult. They sell a total of 342 tickets and make a total of \$2550. We want to try and find out how many of each type of ticket they sold. Let *c* represent the number of children's tickets sold and *a* represent the number of adult tickets sold.

a) Write an equation that represents the fact that 342 total tickets were sold.

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b) Write an equation representing the fact that they made a total of \$2550.

c) Solve the system you created in part a and part b by using the Method of Elimination.

2.) Dana and Mike own a camping supply store and just put in an order for flashlights and sleeping bags. The number of flashlights ordered was five times the number of sleeping bags. the flashlights cost \$12 each and the sleeping bags cost \$45 each. If the total cost for the flashlights and sleeping bags was \$1785, how many flashlights and how many sleeping bags did Dana and Mike order?

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3.) For a concert, there were 206 more tickets sold at the door then were sold in advance. The tickets sold at the door cost \$10 and the tickets sold in advance cost \$6. The total amount of sales for both types of tickets was \$6828. How many door tickets and how many advance tickets were sold?

4.) Christi and Dave went to an office supply store together. Christi bought 15 boxes of paper clips and 7 packages of index cards for a total cost of \$55.40. Dave bought 12 boxes of paper clips and 10 packages of index cards for a total cost of \$61.70. Find the cost of one box of paper clips and the cost of one package of index cards.

## **Review Section**:

\_\_\_\_ 5.) A pattern of blocks is shown. If the patter of blocks continues, which formula(s) could be used to determine the number of blocks in the *n*th term?

| whic<br>bloc   | h formula(s)<br>ks in the <i>n</i> th t | could be used<br>term?        | d to determin  | e the number of |        |        |        |             |
|----------------|---|-------------------------------|----------------|-----------------|--------|--------|--------|-------------|
|                | Ι                                       | II                            | III            |                 |        |        |        |             |
|                | $a_n = n + 4$                           | $a_1 = 2$ $a_n = a_{n-1} + 4$ | $a_n = 4n - 2$ |                 | Term 1 | Term 2 | Term 3 | L<br>Term 4 |
| [1] I<br>[3] I | and II<br>I and III                     | [2] I an<br>[4] III o         | d III<br>nly   |                 |        |        |        |             |

6.) The equation to determine the weekly earnings of an employee at the Hamburger Shack is given by w(x), where *x* represents the number of hours worked.

$$w(x) = f(x) = \begin{cases} 10x, & 0 \le x \le 40\\ 15(x-40) + 400, & x > 40 \end{cases}$$

a) Determine the difference in salary, in dollars, for an employee who works 52 hours versus an employee who works 38 hours.

b) Determine the number of hours an employee must work in order to earn \$445 if you know the employee worked more than 40 hours.

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# Homework Answers

Name:\_\_\_\_ Algebra I Date:Period:Modeling with Systems8E HW

- 1.) a.) c + a = 342
  b.) 5c + 12a = 2550
  c.) 222 children and 120 adult tickets were sold.
- 2.) 85 flashlights and 17 sleeping bags were ordered.
- 3.) 504 tickets were sold at the door and 298 were sold in advance.
- 4.) Paper clips cost \$1.85 and index cards cost \$3.95.
- 5.) [2]
- 6.) a.) \$200 b.) 43 hours

| Name:     | Date:                 | Period:     |    |
|-----------|-----------------------|-------------|----|
| Algebra I | Solving Systems of In | nequalities | 8F |

#### Solving Systems of Inequalities

We can have **systems of inequalities** as well as **systems of equations.** The definition of solving a system still holds: we have to find all points that make all inequalities true.

*Exercise #1*: Consider the system of inequalities shown below. Determine if each of the following points is a solution or not to the system. Show work that justifies your answers.

| x + y > 10                  | $y \ge 3x - 5$         | Γ  |   |   |    |   |
|-----------------------------|------------------------|----|---|---|----|---|
| a) Is $(3,8)$ a solution?   | b) Is (5,9) a solution | 1? |   |   | -  | - |
| (x, y)                      |                        |    | _ |   |    | - |
| $x + y > 10 \qquad y \ge 3$ | 3x - 5                 | -  | _ |   |    | - |
| $3 + 8 > 10 \qquad 8 \ge 3$ | 5(3) - 5               |    |   |   |    |   |
| $11 > 10 \qquad 8 \ge 9$    | 9 — 5                  |    |   |   |    |   |
| TRUE $8 \ge 4$              | 4                      | <  | - |   | ++ | + |
| TRUI                        | E                      | _  |   |   |    |   |
|                             |                        | -  |   |   | ++ | + |
| Yes, (3,8) is a soluti      | ion.                   |    |   |   |    |   |
|                             |                        | -  | _ | _ |    | _ |
|                             |                        |    |   | + |    |   |
|                             |                        |    |   |   |    |   |

c) Graph the solution set to this system of inequalities. State an additional point in the solution set, and an additional point not in the solution set.

*Exercise #2*: On the grid provided, graph the solution to the system of inequalities. State a point that lies in the solution set and one that doesn't lie in the solution set.



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*Exercise #3*: On the grid provided, graph the solution to the system of inequalities. State a point that lies in the solution set and one that doesn't lie in the solution set.



*Exercise #4*: Which of the following points is a solution to the system of inequalities shown below? Show the work that leads to your answer.  $y \le -4x + 2$   $y > \frac{x}{2} + 7$ 

| [1] | (3, -6) | [2] | (0,2) |
|-----|---------|-----|-------|
|     |         |     |       |

[3] (-2,10) [4] (4,10)

*Exercise* #5: Which of the following is a point in the solution set of y > -3x + 5 and  $y \le x - 2$ ?



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*Exercise #6*: For each of the following, provide an explanation or example to support your claim?(M1L22)

a) Is it possible to have a *system of equations* that has no solution?

b. Is it possible to have *a system of equations* that has more than one solution?

c. Is it possible to have *a system of inequalities* that has no solution?

| Name:         | Date: Period:                   |       |
|---------------|---------------------------------|-------|
| Algebra I     | Solving Systems of Inequalities | 8F HW |
| Solving Syste | ems of Inequalities Homework    |       |

1.) Which of the following points is a solution to the system of inequalities shown below?[1] (3,5)[3] (1,-2)y > x + 1 $y \le -2x + 7$ 

[2] (1,3) [4] (2,3)

2.) A system of inequalities is shown graphed below. Which of the following points lies in the solution set of this system?  $\uparrow^{y} f$ 

[1] (-1,2)

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- [2] (1,5)
- [3] (2,-4)
- [4] (4,2)



- 3.) Consider the system of inequalities shown.  $y > \frac{2}{3}x 2$   $y \le \frac{2}{3}x 2$
- a.) Is the origin (0,0) part of the solution set? Determine algebraically.

b.) Graph the solution to the system of inequalities. Then, state one point that lies in the set and one that doesn't.



One Point that Lies in the Solution

One Point that Does Not Lie in the Solution

4.) Graph the solution to the system of inequalities shown below: y + 2x < 6  $x \le 2$ 



State a point that lies in the solution set.

Review Section:

5.) Mo's farm stand sold a total of 165 pounds of apples and peaches. She sold apples for \$1.75 per pound and peaches for \$2.50 per pound. If She made \$337.50, how many pounds of peaches did she sell?

| [1] 11 | [2] 18  |
|--------|---------|
| [3] 65 | [4] 100 |

6.) Given the graph of the line represented by the equation f(x) = -2x + b, if *b* is increased by 4 units, the graph of the new line would be shifted 4 units:

| [1] right | [2] up   |
|-----------|----------|
| [3] left  | [4] down |

7.) To watch a varsity basketball game, spectators must buy a ticket at the door. The cost of an adult ticket is \$3.00 and the cost of a student ticket is \$1.50. If the number of adult tickets sold is represented by *a* and student tickets sold by *s*, which expression represents the amount of money collected at the door from the ticket sales?

| Name:     | Homework Answers |           | Date:              | Period:      | 8F HW |  |
|-----------|------------------|-----------|--------------------|--------------|-------|--|
| Algebra I |                  |           | Solving Systems of | Inequalities |       |  |
| 1.) [2]   |                  |           |                    |              |       |  |
| 2.) [4]   |                  |           |                    |              |       |  |
| 3.) a.) y | es               | b.) GRAPH |                    |              |       |  |
| 4.) GRA   | РН               |           |                    |              |       |  |
| 5.) [3]   |                  |           |                    |              |       |  |
| 6.) [2]   |                  |           |                    |              |       |  |
| 7.) [4]   |                  |           |                    |              |       |  |

| Name:     | Date:               | Period:            |    |
|-----------|---------------------|--------------------|----|
| Algebra I | Modeling with Syste | ms of Inequalities | 8G |

#### Modeling with Systems of Inequalities

There are many situations that arise in business and engineering that necessitate systems of linear inequalities. The region in the xy-plane that solves the systems often represents all of the viable solutions to the system, so being able to visualize this region can be extremely helpful. As always, with modeling, it is important to really read the problems and understand the physical quantities involved.

*Exercise #1*: John mows yards for his father's landscaping business for \$10 per hour and also works at a bakery for \$15 per hour. He can work at most 52 hours per week during the summer. He needs to make at least \$600 per week to cover his living expenses.

a) If John works 14 hours mowing and 30 hours at the bakery, does this satisfy all of the problem's constraints?

b) If x represents the hours John spends mowing and y represents the hours he spends at the bakery write a system of inequalities that describes the scenario.

c) If John must work a minimum of 10 hours for his father, will he be able to make enough money to cover his living expenses? Show the work that leads to your answer. d) Graph the system of inequalities with the help of your calculator on the axes below. Use the space below to think about how to graph these lines.

e) John's father needs him to work a lot at the landscaping business. Show the point on the graph that corresponds the the greatest number of hours that he can work while still covering his expenses.

f)Algebraically, find the greatest number of hours that John can work for his father and still cover his expenses. Explain how you found your answer or show all your algebra below. Hours at Bakery, y

Hours Mowing, x

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*Exercise #2*: Katie works part-time at the Fallbrook Riding Stable. She makes \$5 an hour for exercising horses and \$10 an hour for cleaning stalls. Because Katie is a full-time student, she cannot work more than 12 hours per week. Graph two inequalities that illustrate how many hours Katie needs to work at each job if she plans to earn not less than \$9 per week. (Regentsprep.org)



*Exercise #3*: The drama club at a local high school is trying to raise money by putting on a play. They have only 500 seats in the auditorium that they are using and are selling tickets for these seats at \$5 per child's ticket and \$10 per adult ticket. They must sell at least \$2000 worth of tickets to cover their expenses.

a) If x represents the number of children's tickets sold, and y represents the number of adult tickets sold, write a system of inequalities that models this system. b) Using technology, sketch the region in the coordinate plant that represents solutions to this system of inequalities



Children's Tickets, x

(c) If the students want to sell exactly 500 tickets and make exactly \$2000, how many of each ticket should they sell? Why is this answer not realistic?

| Name:     | Date: Period:                               |
|-----------|---|
| Algebra I | Modeling with Systems of Inequalities 8G HW |

#### Modeling with Systems of Inequalities Homework

1.) Jody is working two jobs, one as a carpenter and one as a website designer. He can work at most 50 hours per week and makes \$35 per hour as a carpenter and \$75 per hour as a website designer. He wants to make at least \$2350 per week but also wants to work at least 10 hours per week as a carpenter. Let c represent the hours he works as a carpenter and let w represent the house he work as a website designer.

a) Write a system of inequalities that models this scenario.

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b) What is the maximum about of money that Jody can make in a week given the system in part a. Explain your reasoning.

c) The graph of the system is shown to the right with its solutions shown shaded. Three lines are graphed. Label each with its equation.

d) Find the coordinates of point A by solving a system of equations by the Method of Elimination.

e) What does the value of x that you found in the solution to part d represent about the number of hours Jody can work as a carpenter. Explain your thinking.



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2.) Kyle works part-time for a local contractor. He makes \$8 an hour if he works with the plumber and \$12 an hour if he works with the mason. Kyle cannot work more than 10 hours per week.Write the system of equalities that represent how many hours Kyle needs to work each job if he plans to earn at least \$100 per week.



Graph the two inequalities and label your graph. Label the solution set with the letter S.

#### **REASONING QUESTION:**

3.) Systems of inequalities can also come in discrete versions where the two variables involved can only take on integer values. Let's look at a simple example of this.

Jennifer is putting together a selection of flowers that has at most 12 flowers in it. She is choosing either roses or carnations. She wants to pick at least three roses and at least two carnations. let r be the number of roses she uses and let c be the number of carnations she uses.

a) Write a system of inequalities that models this scenario.

b) If Jennifer used the minimum number of carnations, what is the maximum number of roses she could use?



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c) What is the fewest flowers Jennifer will use and in what combination?

d) Graph the solution set to the system. Be careful, This should be a collection of points, not a shaded region.

#### **Review Section:**

4. A rocket is launched from the ground and follows a parabolic path represented by the equation  $y = -x^2 + 9x$ . At the same time, a Frisbee is thrown from a window at a height of 16 feet and follows a straight path to the ground represented by -x + 16.

a. Graph the equations that represent the paths of the rocket ang the Frisbee on the same axis.

b. Find the coordinates of the points where the paths intersect.

c. Will the rocket and the Frisbee hit the ground at the same time? Explain.



\_5.) Which graph represents the solution of  $y \le x + 3$  and  $y \ge -2x - 2$ ?



- \_\_\_\_\_ 6.) Last week, a candle store received \$355.60 for selling 20 candles. Small candles sell for \$10.98 and large candles sell for \$27.98. How many large candles did the store sell?
  - [1] 6 [2] 8 [2] 40 [4] 40
  - [3] 10 [4] 12

7.) Which representations are functions? [II] {(0,1), (2,4), (5,7)} [III] [IV] y = 2x + 1[I] У x 2 6 3 -12 4 7 5 --6 5 2 ►X [1] I and II [2] II and IV [3] III only [4] IV only

# **Homework Answers**

Name:\_\_\_\_ Algebra I Date:\_\_\_\_\_ Period:\_\_\_\_\_ Modeling with Systems of Inequalities 8G HW

- 1.) a.)  $c + w \le 50$   $35c + 75 w \ge 2350$   $c \ge 10$ 
  - b.) To make the maximum amount of money, he must work the most as a website designer, because it pays more. He must work 40 hours here because he has to work at least 10 hours as a carpenter.
  - c.) GRAPH
  - d.) (35, 15)
  - e.) A value of c = 35 represents 35 hours working as a carpenter. This represents the maximum number of hours Jody can work as a carpenter and still make at least \$2350 per week.
- 2.) Graph



3.) a.)  $r \ge 3$   $c \ge 2$   $r + c \le 12$ b.) 10 roses c.) 5 flowers d.) GRAPH

4.) a.) See graph, b.) Intersections: (2,14) and (8,8), c.) No. The rocket hits the ground after 9 seconds. At o seconds, the Frisbee is still 7 feet above the ground.





6.) [2] 7.) [2]

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|------|--------------|----|
|------|--------------|----|

| Name:     | <br> |
|-----------|------|
| Algebra I |      |

Date:\_\_\_\_\_ Period:\_\_\_\_ 8H Regents Practice Questions

1. A system of equations is given below.( Aug 2016)

X+2x = 5 2x + y = 4

Which system of equations does not have the same solution:

- 1) 3x + 6y = 15 and 2x + y = 42) 4x + 8y = 20 and 2x + y = 43) x + 2y = 5 and 6x + 3y = 124) x + 2y = 5 and 4x + 2y = 12
- 2. The Celluloid Cinema sold 150 tickets to a movie. Some of these were child tickets and the rest were adult tickets. A child ticket cost \$7.75 and an adult ticket cost \$10.25. If the cinema sold \$1470 worth of tickets, which system of equations could be used to determine how many adult tickets, a, and child tickets, c, were sold? (June 2016)
  - 1) a + c = 150 and 10.25a + 7.75c = 1470 2) a+c = 1470 and 10.25a + 7.75c = 150
  - 3) a+c = 150 and 7.75a + 10.25c = 1470 4) a+c = 1470 and 7.75a +10.25c = 150
- 3. Given the functions  $h(x) = \frac{1}{2}x + 3$  and f(x) = |x|, which value of x makes h(x) = f(x)? (Jan 2016)
  - 1) -2 2) 2 3) 3 4) -6
- 4. Which pair of equation could *not* be used to solve the following equations for x and y?

$$4x + 2y = 22 
-2x + 2y = -8$$
(1)  $4x + 2y = 22 
 $2x - 2y = 8$ 
(3)  $12x + 6y = 66 
 $6x - 6y = 24$ 
(2)  $4x + 2y = 22 
-4x + 4y = -16$ 
(4)  $8x + 4y = 44 
-8x + 8y = -8$$$ 

Part 2.

1. The graph below shows two functions, f(x) and g(x). State all the values of x for which f(x) = g(x). (Aug 2016)



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2. Shawn incorrectly graphed the inequality  $-x - 2y \ge 8$ .

Explain Shawn's mistake.

Graph the inequlity correctly on the set of axis below.



3. For a class picnic, two teachers went to the same store to purchase drinks. One teacher purchased 18 juice boxes and 32 bottles of water, and spent \$19.92. The other teacher purchased 14 juice boxes and 26 bottles of water, and spent \$15.76.

Write a system of equations to represent the costs of a juice box, *j*, and a bottle of water, *w*.

Kara said that the juice boxes might have cost 52 cents each and that the bottles of water might have cost 33 cents each. Use your system of equations to justify that Kara's prices are *not* possible.

Solve your system of equations to determine the actual cost, in dollars, of each juice box and each bottle of water. (Aug. 2016) State the solution and explain how the graph shows the solution.

4. The sum of two numbers, x and y, is more than 8. When you double x and add it to y, the sum is less than 14. Graph the inequalities that represent this scenario on the set of axes below. (June 2016)



Kai says that the point (6,2) is a solution to this system. Determine if he is correct and explain your reasoning.

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5. On the set of axis below, graph:

(June 2016)

$$g(x) = \frac{1}{2}x + 1$$
  
and  
$$f(x) = \begin{cases} 2x + 1, & x \le -1\\ 2 - x^2, & x > -1 \end{cases}$$



How may values of x satisfy the equation f(x) = g(x)? Explain your answer, using evidence from your graphs.

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7. Franco and Caryl went to a bakery to buy desserts. Franco bought 3 packages of cupcakes and 2 packages of brownies for \$19. Caryl bought 2 packages of cupcakes and 4 packages of brownies for \$24. Let x equal the price of one package of cupcakes and y equal the price of one package of brownies. (June 2016)

Write a system of equations that describes the given situation.

On the set of axes below, graph the system of equations



Determine the exact cost of one package of cupcakes and the exact cost of one package of brownies in dollars and cents. Justify your solution.

8. The Reel Good Cinema is conducting a mathematical study. In its theater, there are 200 seats. Adults tickets cost \$12.50 and child tickets cost \$6.25. The cinema's goal is to sell at least \$1500 worth of tickets for the theater.

Write a system of linear inequalities that can be used to find the possible combinations of adult tickets, x, and child tickets, y, that would satisfy the cinema's goal. (June 2015)

#### Page **52** of **52**

Graph the solution to the system of inequalities on the set of axes provided. Label the solution with an **S**.

Marta claims that selling 30 adult tickets and 80 child tickets will result in meeting the cinema's goal. Explain whether she is correct or incorrect, based on the graph drawn. Explain.

