Chapter 9: Electrical Design of Overhead Lines

Introduction

An a.c. transmission line has resistance, inductance and capacitance uniformly distributed along its length.

These are known as constants or parameters of the line.

The performance of a transmission line, the efficiency and voltage regulation depends upon these constants.

Therefore, we shall focus our attention on the methods of calculating these constants for a given transmission line.

And special attention will be given to inductance and capacitance.

9.1 Constants of a Transmission Line

A transmission line has resistance, inductance and capacitance uniformly distributed along the whole length of the line.

(i) Resistance. It is the opposition of line conductors to current flow. The resistance is distributed uniformly along the whole length of the line as shown in Fig. 9.1 (*i*), and Fig. 9.1 (*ii*) .

(*ii*) Inductance. When an alternating current flows through a conductor, a changing flux is set up which links the conductor.

Mathematically, inductance is defined as the flux linkages per ampere *i.e.,*

$$\text{Inductance, } L = \psi / I \qquad \text{henry}$$

The inductance is also uniformly distributed along the length of the line as show in Fig. 9.1(*i*), and Fig. 9.1(*ii*).

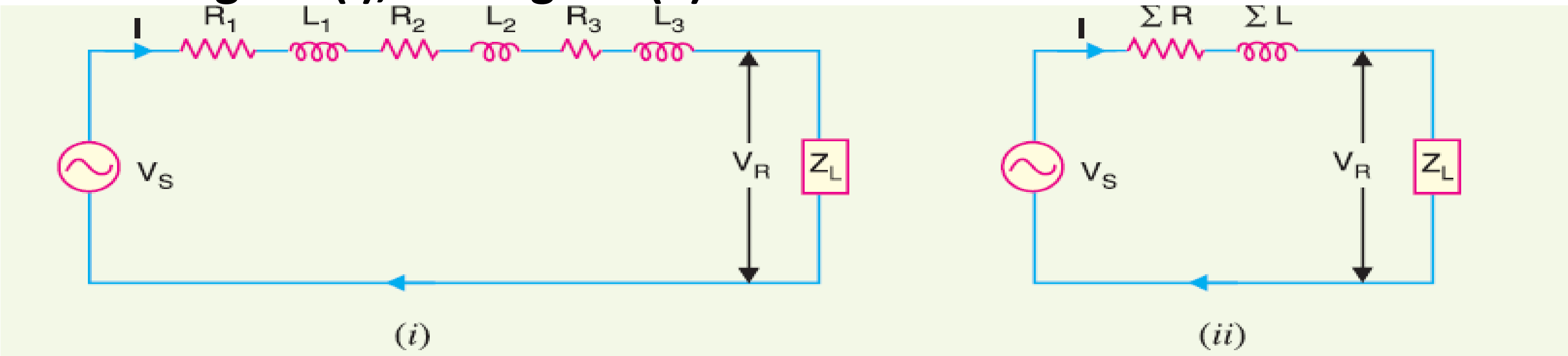

Fig. 9.1

(*iii*) Capacitance. **As any two conductors of an overhead transmission line are separated by air which acts as an insulation, therefore, capacitance exists between any two overhead line conductors.**

The capacitance between the conductors is the charge per unit potential difference *i.e.,*

$$\text{Capacitance, } C = q / v \text{ farad}$$

The capacitance is uniformly distributed along the whole length of the line and may be regarded as a uniform series of capacitors connected between the conductors as shown in Fig. 9.2(*i*) and Fig. 9.2*(ii)*.

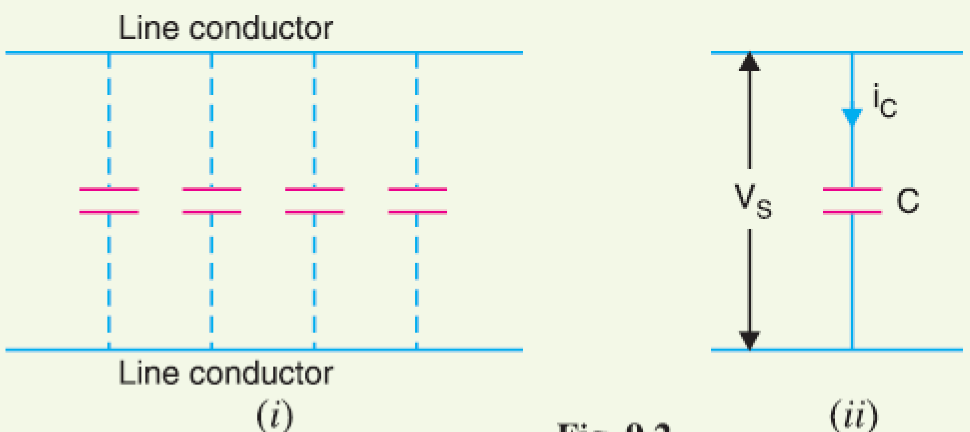

Fig. 9.2

9.2 Resistance of a Transmission Line

The resistance *R* of a line conductor having resistivity ρ, length *l* and area of cross section

***a* is given by ;** $R = \rho\, l / a$

The variation of resistance of metallic conductors with temperature is practically linear over the normal range of operation.

Suppose R_1 and R_2 are the resistances of a conductor at t_1°C and t_2°C($t_2 > t_1$) respectively.

If α_1 is the temperature coefficient at t_1°C, then,

$$R_2 = R_1 [1 + \alpha_1 (t_2 - t_1)]$$

where $$\alpha_1 = \frac{\alpha_0}{1 + \alpha_0 t_1}$$

α_0 **= temperature coefficient at 0° C**

9.3 Skin Effect

When a conductor is carrying steady direct current (dc), this current is uniformly distributed over the whole cross-section of the conductor.

However, an alternating current flowing through the conductor does not distribute uniformly, rather it has the tendency to concentrate near the surface of the conductor as shown in Fig. 9.3. This is known as skin effect.

The tendency of alternating current to concentrate near the surface of a conductor is known as **skin effect.**

Due to skin effect, the effective area of cross-section of the conductor through which current flows is reduced.

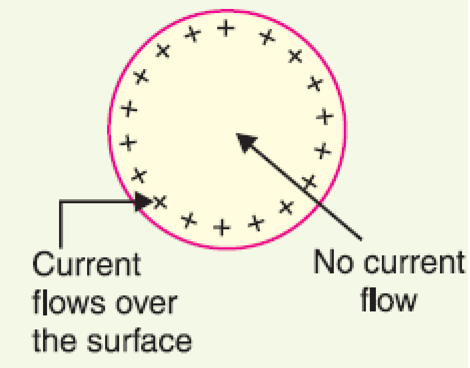

Fig. 9.3

Consequently, the resistance of the conductor is slightly increased when carrying an alternating current.

9.4 Flux Linkages (For more detail see the text book page 205)

As stated earlier, the inductance of a circuit is defined as the flux linkages per unit current.

Therefore, in order to find the inductance of a circuit, the determination of flux linkages is of primary importance.

We shall discuss two important cases of flux linkages.

1. **Flux linkages due to a single current carrying conductor. Consider a long straight cylindrical conductor of radius *r* meters and carrying a current *I* amperes (rms) as shown in Fig. 9.4 (*i*).**

Fig. 9.4

This current will set up magnetic field. The magnetic lines of force will exist inside the conductor as well as outside the conductor.

Both these fluxes will contribute to the inductance of the conductor.

(i) ***Flux linkages due to internal flux.*** **Refer to Fig. 9.4 (*ii*) the total flux linkages from center up to the conductor surface:**

$$\psi_{\text{int}} = \int_0^r \frac{\mu_0 I x^3}{2\pi r^4} dx$$

$$= \frac{\mu_0 I}{8\pi} \text{ weber-turns per metre length}$$

(*i*) *Flux linkages due to internal flux.* Refer to Fig. 9.4 (*ii*)

Total flux linkages of the conductor from surface to infinity,

$$\psi_{ext} = \int_r^\infty \frac{\mu_0 I}{2\pi x} dx \text{ weber-turns}$$

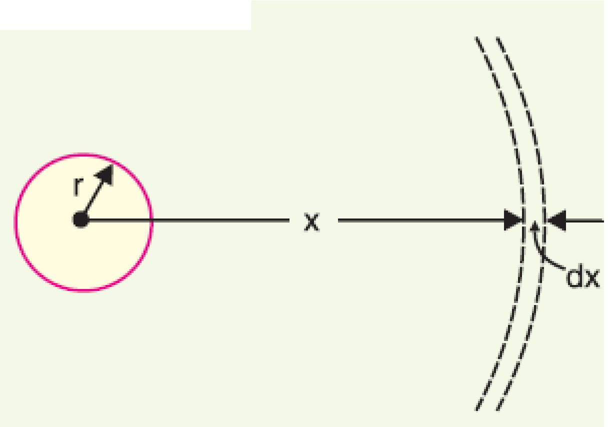

Fig. 9.5

And the

,

$$\therefore \quad \text{Overall flux linkages,} \quad \psi = \psi_{int} + \psi_{ext} = \frac{\mu_0 I}{8\pi} + \int_r^\infty \frac{\mu_0 I}{2\pi x} dx$$

$$\therefore \quad \psi = \frac{\mu_0 I}{2\pi}\left[\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right] \text{ wb-turns/m length}$$

2. Flux linkages in parallel current carrying conductors. **Fig. 9.6 shows the conductors *A,B, C* etc. carrying currents I_A, I_B, I_C *etc* .**

Let us consider the flux linkages with one conductor, say conductor *A*. There will be flux linkages with Conductor *A* due to its own current as discussed previously.

Also there will be flux linkages with this conductor due to the mutual inductance effects of I_B, I_C, I_D etc.

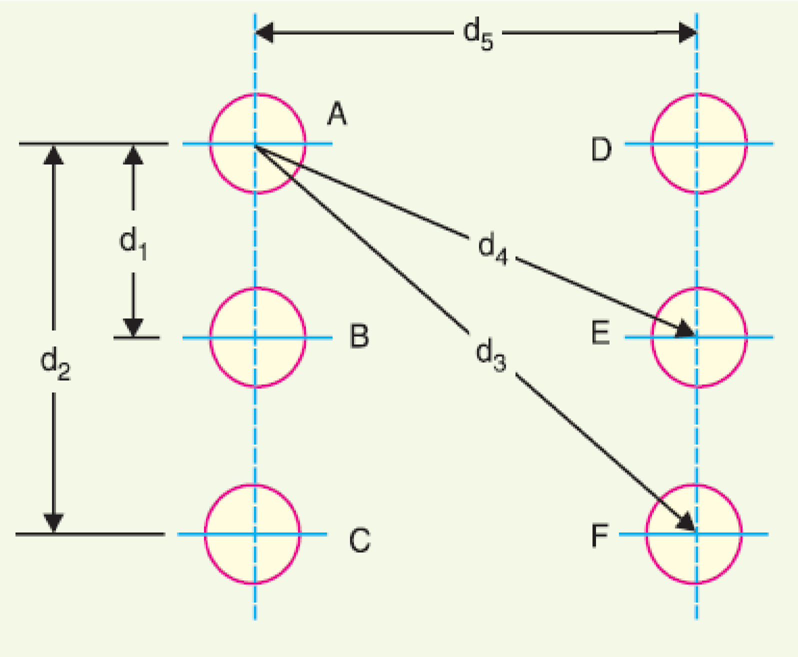

Fig. 9.6

∴ Total flux linkages with conductor A

$$= (i) + (ii) + (iii) + \ldots\ldots$$

$$= \frac{\mu_0\, I_A}{2\pi}\left(\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right) + \frac{\mu_0\, I_B}{2\pi}\int_{d_1}^\infty \frac{dx}{x} + \frac{\mu_0 I_C}{2\pi}\int_{d_2}^\infty \frac{dx}{x} + \ldots$$

Similarly, flux linkages with other conductors can be determined. The above relation provides the basis for evaluating inductance of any circuit.

9.5 Inductance of a Single Phase Two-wire Line

A single phase line consists of two parallel conductors which form a rectangular loop of one turn.

When an alternating current flows through such a loop, a changing magnetic flux is set up.

The changing flux links the loop and hence the loop (or single phase line) possesses inductance.

Consider a single phase overhead line consisting of two parallel conductors *A* and *B* spaced *d* meters apart as shown in Fig. 9.7.

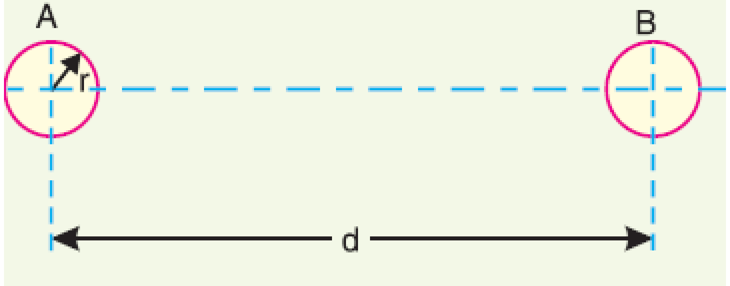

Fig. 9.7

Conductors *A* and *B* carry the same amount of current (*i.e.* $I_A = I_B$), but in the opposite direction because one forms the return circuit of the other.

$$\therefore \qquad I_A + I_B = 0$$

Total flux linkages with conductor A is

$$\psi_A = \text{exp. } (i) + \text{exp } (ii)$$

$$= \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4} + \int_r^{\infty}\frac{dx}{x}\right) + \frac{\mu_0 I_B}{2\pi}\int_d^{\infty}\frac{dx}{x}$$

Inductance of conductor $A, L_A = \dfrac{\psi_A}{I_A}$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e\frac{d}{r}\right]\text{H/m} = \frac{4\pi\times10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e\frac{d}{r}\right]\text{H/m}$$

$$\therefore \quad L_A = 10^{-7}\left[\frac{1}{2} + 2\log_e\frac{d}{r}\right]\text{H/m} \quad ...(i)$$

$$\text{Loop inductance} = 2\,L_A\,\text{H/m} = 10^{-7}\left[1 + 4\log_e\frac{d}{r}\right]\text{H/m}$$

$$\therefore \quad \text{Loop inductance} = 10^{-7}\left[1 + 4\log_e\frac{d}{r}\right]\text{H/m} \quad ...(ii)$$

Note that eq. (*ii*) is the inductance of the two-wire line and is sometimes called loop inductance. However, inductance given by eq. (*i*) is the inductance per conductor and is equal to half the loop inductance.

Expression in alternate form. The expression for the inductance of a conductor can be put in a concise form:

$$L_A = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{d}{r}\right] \text{H/m}$$

$$= 2\times 10^{-7}\left[\frac{1}{4} + \log_e \frac{d}{r}\right]$$

$$= 2\times 10^{-7}\left[\log_e e^{1/4} + \log_e \frac{d}{r}\right]$$

$$\therefore \quad L_A = 2\times 10^{-7} \log_e \frac{d}{r\, e^{-1/4}}$$

If we put $r\, e^{-1/4} = r'$, then,

$$L_A = 2\times 10^{-7} \log_e \frac{d}{r'} \text{ H/m} \qquad \ldots(iii)$$

The term r' $(= r\, e^{-1/4})$ is called *geometric mean radius* (GMR) of the wire.

Therefore, the loop inductance is given by:

Loop inductance $= 2\, L_A = 2\times 2\times 10^{-7} \log_e \dfrac{d}{r'}$ H/m

Note that $r' = 0{\cdot}7788\, r$ is applicable to only solid round conductor.

9.6 Inductance of a 3-Phase Overhead Line

Fig. 9·8 shows the three conductors *A, B* and *C* of a 3-phase line carrying currents I_A, I_B and I_C respectively.

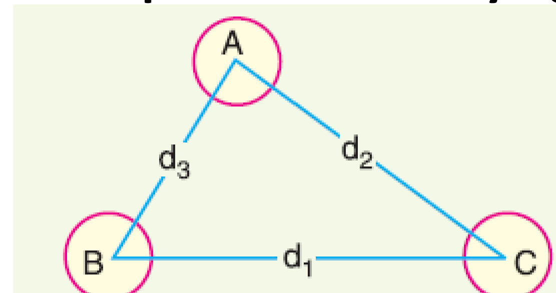

Fig. 9.8

Let d_1, d_2 and d_3 be the spacing between the conductors as shown.

Let us further assume that the loads are balanced *i.e.* $I_A + I_B + I_C = 0$.

Consider the flux linkages with conductor *A.*

There will be flux linkages with conductor *A* due to its own current and also due to the mutual inductance effects of I_B and I_C.

The total flux linkage conductor A due to the three currents is given by:

As $\quad I_A + I_B + I_C = 0,$

$$\therefore \quad \psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d_3 - I_C \log_e d_2\right]$$

(i) Symmetrical spacing.

If the three conductors *A, B* and *C* are placed symmetrically at the corners of an equilateral triangle of side *d,* then, $d_1 = d_2 = d_3 = d$.

Under such conditions, the flux linkages with conductor *A* become :

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d - I_C \log_e d\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - (I_B + I_C)\log_e d\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A + I_A \log_e d\right] \qquad (\because I_B + I_C = -I_A)$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{ werber-turns/m}$$

Inductance of conductor A, $$L_A = \frac{\psi_A}{I_A} \text{ H/m} = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{ H/m}$$

$$= \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{ H/m}$$

$\therefore$ $$L_A = 10^{-7}\left[0 \cdot 5 + 2\log_e \frac{d}{r}\right] \text{ H/m}$$

Derived in a similar way, the expressions for inductance are the same for conductors B and C.

(*ii*) Unsymmetrical spacing. **When 3-phase line conductors are not equidistant from each other, the conductor spacing is said to be unsymmetrical.**

Under such conditions, the flux linkages and inductance of each phase are not the same.

A different inductance in each phase results in unequal voltage drops in the three phases even if the currents in the conductors are balanced.

Therefore, the voltage at the receiving end will not be the same for all phases.

In order that voltage drops are equal in all conductors, we generally interchange the positions of the conductors at regular intervals along the line so that each conductor occupies the original position of every other conductor over an equal distance.

Such an exchange of positions is known as *transposition.*

Fig. 9.9 shows the transposed line. The phase conductors are designated as *A, B* and *C* and the positions occupied are numbered 1, 2 and 3.

The effect of transposition is that each conductor has the same average inductance.

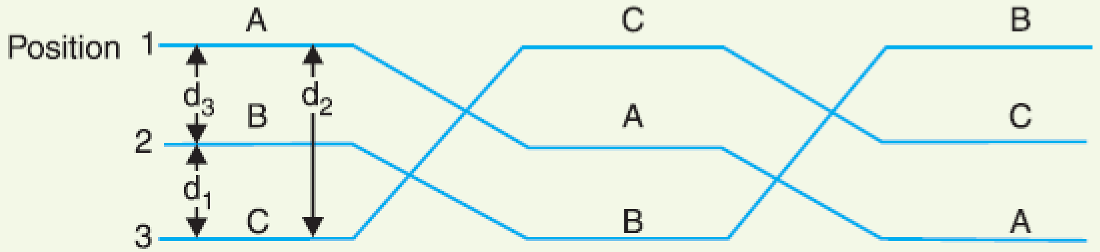

Fig. 9.9

Fig. 9.9 shows a 3-phase transposed line having unsymmetrical spacing. Let us assume that each of the three sections is 1 m in length. Let us further assume balanced conditions *i.e.*, $I_A + I_B + I_C = 0$. Let the line currents be :

$$I_A = I(1 + j\,0)$$
$$I_B = I(-0{\cdot}5 - j\,0{\cdot}866)$$
$$I_C = I(-0{\cdot}5 + j\,0{\cdot}866)$$

As proved above, the total flux linkages per metre length of conductor A is

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right)I_A - I_B \log_e d_3 - I_C \log_e d_2\right]$$

Putting the values of I_A, I_B and I_C , we get,

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4}-\log_e r\right)I - I(-0\cdot 5 - j\,0\cdot 866)\log_e d_3 - I(-0\cdot 5 + j\,0\cdot 866)\log_e d_2\right]$$

$\therefore$ Inductance of conductor A is

$$L_A = \frac{\psi_A}{I_A} = \frac{\psi_A}{I}$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_2\, d_3}}{r} + j\,0\cdot 866\,\log_e \frac{d_3}{d_2}\right]$$

$$= 10^{-7}\left[\frac{1}{2} + 2\,\log_e \frac{\sqrt{d_2\, d_3}}{r} + j\,1\cdot 732\,\log_e \frac{d_3}{d_2}\right] \text{H/m}$$

Similarly inductance of conductors B and C will be :

$$L_B = 10^{-7}\left[\frac{1}{2} + 2\,\log_e \frac{\sqrt{d_3\, d_1}}{r} + j\,1\cdot 732\,\log_e \frac{d_1}{d_3}\right] \text{H/m}$$

$$L_C = 10^{-7}\left[\frac{1}{2} + 2\,\log_e \frac{\sqrt{d_1\, d_2}}{r} + j\,1\cdot 732\,\log_e \frac{d_2}{d_1}\right] \text{H/m}$$

Inducance of each line conductor

$$= \frac{1}{3}(L_A + L_B + L_C)$$

$$= {}^{*}\left[\frac{1}{2} + 2\log_e \frac{\sqrt[3]{d_1\, d_2\, d_3}}{r}\right] \times 10^{-7} \text{ H/m}$$

$$= \left[0 \cdot 5 + 2\log_e \frac{\sqrt[3]{d_1\, d_2\, d_3}}{r}\right] \times 10^{-7} \text{ H/m}$$

If we compare the formula of inductance of an unsymmetrically spaced transposed line with that of symmetrically spaced line, we find that inductance of each line conductor in the two cases will be equal if $d = \sqrt[3]{d_1\, d_2\, d_3}$. The distance d is known as *equivalent equilateral spacing* for unsymmetrically transposed line.

9.7 Concept of Self-GMD and Mutual-GMD

The use of *self geometrical mean distance* (GMD) and *mutual geometrical mean distance* (GMD) simplifies the inductance calculations, of multi-conductor arrangements. The symbols used for these are respectively *Ds* and *Dm.*

(*c*) The principle of geometrical mean distances can be most profitably employed to 3-ϕ double circuit lines. Consider the conductor arrangement of the double circuit shown in Fig. 9·10. Suppose the radius of each conductor is *r*.

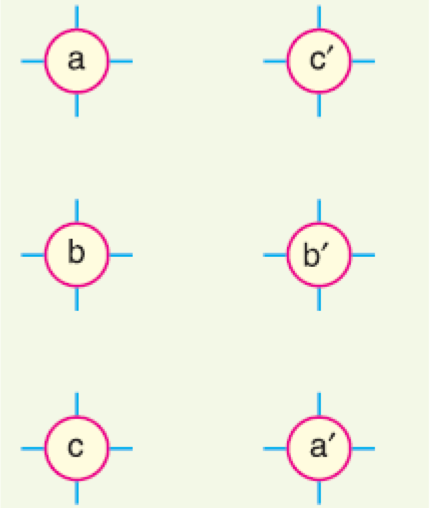

Fig. 9.10

Self-GMD of conductor = 0·7788 *r*

Self-GMD of combination *aa′ is*

$$D_{s1} = (**D_{aa} \times D_{aa'} \times D_{a'a'} \times D_{a'a})^{1/4}$$

Self-GMD of combination *bb′ is*

$$D_{s2} = (D_{bb} \times D_{bb'} \times D_{b'b'} \times D_{b'b})^{1/4}$$

Self-GMD of combination *cc′ is*

$$D_{s3} = (D_{cc} \times D_{cc'} \times D_{c'c'} \times D_{c'c})^{1/4}$$

Equivalent self-GMD of one phase

$$D_s = (D_{s1} \times D_{s2} \times D_{s3})^{1/3}$$

The value of D_s is the same for all the phases as each conductor has the same radius.

Mutual-GMD between phases *A* and *B* is

$$D_{AB} = (D_{ab} \times D_{ab'} \times D_{a'b} \times D_{a'b'})^{1/4}$$

Mutual-GMD between phases *B* and *C* is

$$D_{BC} = (D_{bc} \times D_{bc'} \times D_{b'c} \times D_{b'c'})^{1/4}$$

Mutual-GMD between phases *C* and *A* is

$$D_{CA} = (D_{ca} \times D_{ca'} \times D_{c'a} \times D_{c'a'})^{1/4}$$

Equivalent mutual-GMD, $D_m = (D_{AB} \times D_{BC} \times D_{CA})^{1/3}$

It is worthwhile to note that mutual GMD depends only upon the spacing and is substantially independent of the exact size, shape and orientation of the conductor.

9.8 Inductance Formulas in Terms of GMD

The inductance formulas developed are:

(*i*) *Single phase line*

$$\text{Inductance/conductor/m} = 2 \times 10^{-7} \log_e \frac{D_m}{D_s}$$

where $D_s = 0{\cdot}7788\, r$ and D_m = Spacing between conductors = d

(*ii*) *Single circuit 3-ϕ line*

$$\text{Inductance/phase/m} = 2 \times 10^{-7} \log_e \frac{D_m}{D_s}$$

where $D_s = 0{\cdot}7788\, r$ and $D_m = (d_1\, d_2\, d_3)^{1/3}$

(*iii*) *Double circuit 3-ϕ line*

$$\text{Inductance/phase/m} = 2 \times 10^{-7} \log_e \frac{D_m}{D_s}$$

where $D_s = (D_{s1}\, D_{s2}\, D_{s3})^{1/3}$ and $D_m = (D_{AB} \times D_{BC} \times D_{CA})^{1/3}$

Example 9.1. *A single phase line has two parallel conductors 2 metres apart. The diameter of each conductor is 1·2 cm. Calculate the loop inductance per km of the line.*

Solution.

Spacing of conductors, $d = 2 \text{ m} = 200 \text{ cm}$

Radius of conductor, $r = 1{\cdot}2/2 = 0{\cdot}6 \text{ cm}$

Loop inductance per metre length of the line

$$= 10^{-7}(1 + 4 \log_e d/r) \text{ H}$$
$$= 10^{-7}(1 + 4 \log_e 200/0{\cdot}6) \text{ H}$$
$$= 24{\cdot}23 \times 10^{-7} \text{ H}$$

Inductance Measurement using bridge

Loop inductance per km of the line

$$= 24{\cdot}23 \times 10^{-7} \times 1000 = 24{\cdot}23 \times 10^{-4} \text{ H} = \mathbf{2{\cdot}423 \text{ mH}}$$

Example 9.2. *A single phase transmission line has two parallel conductors 3 m apart, the radius of each conductor being 1 cm. Calculate the loop inductance per km length of the line if the material of the conductor is (i) copper (ii) steel with relative permeability of 100.*

Solution.

Spacing of conductors, $d = 300 \text{ cm}$

Radius of conductor, $r = 1 \text{ cm}$

Loop inductance $= 10^{-7}(\mu_r + 4 \log_e d/r) \text{ H/m}$

(*i*) With copper conductors, $\mu_r = 1$

$\therefore$ Loop inductance/m $= 10^{-7}(1 + 4 \log_e d/r)$ H $= 10^{-7}(1 + 4 \log_e 300/1)$ H

$= 23{\cdot}8 \times 10^{-7}$ H

Loop inductance/km $= 23{\cdot}8 \times 10^{-7} \times 1000 = 2{\cdot}38 \times 10^{-3}$ H $=$ **2·38 mH**

(*ii*) With steel conductors, $\mu_r = 100$

$\therefore$ Loop inductance/m $= 10^{-7}(100 + 4 \log_e 300/1)$ H $= 122{\cdot}8 \times 10^{-7}$ H

Loop inductance/km $= 122{\cdot}8 \times 10^{-7} \times 1000 = 12{\cdot}28 \times 10^{-3}$ H $=$ **12·28 mH**

Example 9.3. *Find the inductance per km of a 3-phase transmission line using 1·24 cm diameter conductors when these are placed at the corners of an equilateral triangle of each side 2 m.*

Solution. Fig. 9·11 shows the three conductors of the three phase line placed at the corners of an equilateral triangle of each side 2 m. Here conductor spacing $d = 2$ m and conductor radius $r = 1{\cdot}24/2 = 0{\cdot}62$ cm.

Inductance/phase/m $= 10^{-7}(0{\cdot}5 + 2 \log_e d/r)$ H

$= 10^{-7}(0{\cdot}5 + 2 \log_e 200/0{\cdot}62)$ H

$= 12 \times 10^{-7}$ H

Inductance/phase/km $= 12 \times 10^{-7} \times 1000$

$= 1{\cdot}2 \times 10^{-3}$ H $=$ **1·2 mH**

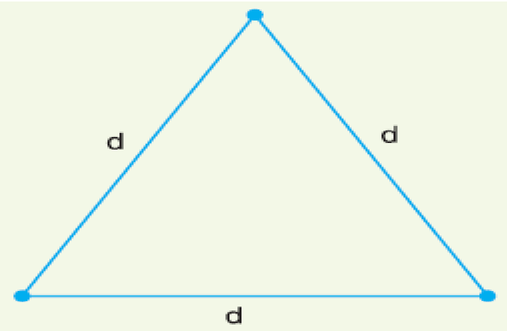

Fig. 9.11

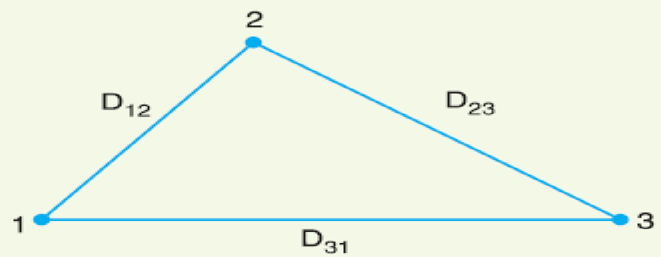

Fig. 9.12

Example 9.4. *The three conductors of a 3-phase line are arranged at the corners of a triangle of sides 2 m, 2·5 m and 4·5 m. Calculate the inductance per km of the line when the conductors are regularly transposed. The diameter of each conductor is 1·24 cm.*

Solution. Fig. 9·12 shows three conductors of a 3-phase line placed at the corners of a triangle of sides $D_{12} = 2$ m, $D_{23} = 2·5$ m and $D_{31} = 4·5$ m. The conductor radius $r = 1·24/2 = 0·62$ cm.

Equivalent equilateral spacing, $D_{eq} = \sqrt[3]{D_{12} \times D_{23} \times D_{31}} = \sqrt[3]{2 \times 2·5 \times 4·5} = 2·82 \text{ m} = 282 \text{ cm}$

$$\text{Inductance/phase/m} = 10^{-7}(0·5 + 2 \log_e D_{eq}/r) \text{ H} = 10^{-7}(0·5 + 2 \log_e 282/0·62) \text{ H}$$
$$= 12·74 \times 10^{-7} \text{ H}$$
$$\text{Inductance/phase/km} = 12·74 \times 10^{-7} \times 1000 = 1·274 \times 10^{-3} \text{ H} = \mathbf{1·274 \text{ mH}}$$

Example 9.5. *Calculate the inductance of each conductor in a 3-phase, 3-wire system when the conductors are arranged in a horizontal plane with spacing such that $D_{31} = 4$ m ; $D_{12} = D_{23} = 2$m. The conductors are transposed and have a diameter of 2·5 cm.*

Solution. Fig. 9.13. shows the arrangement of the conductors of the 3phase line. The conductor radius $r = 2·5/2 = 1·25$ cm.

Equivalent equilateral spacing, $D_{eq} = \sqrt[3]{D_{12} \times D_{23} \times D_{31}} = \sqrt[3]{2 \times 2 \times 4} = 2·52 \text{ m} = 252 \text{ cm}$

$$\text{Inductance/phase/m} = 10^{-7}(0·5 + 2 \log_e D_{eq}/r) \text{ H}$$
$$= 10^{-7}(0·5 + 2 \log_e 252/1·25) \text{ H}$$
$$= 11·1 \times 10^{-7} \text{ H}$$
$$\text{Inductance/phase/km} = 11·1 \times 10^{-7} \times 1000$$
$$= 1·11 \times 10^{-3} \text{ H} = \mathbf{1·11 \text{ mH}}$$

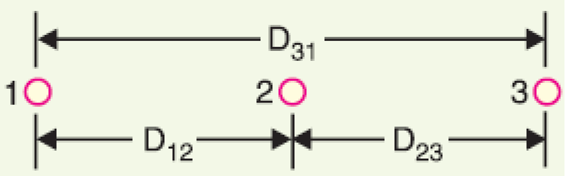

Fig. 9.13

Example 9.7. *Fig. 9.15 shows the spacings of a double circuit 3-phase overhead line. The phase sequence is ABC and the line is completely transposed. The conductor radius in 1·3 cm. Find the inductance per phase per kilometre.*

Solution.

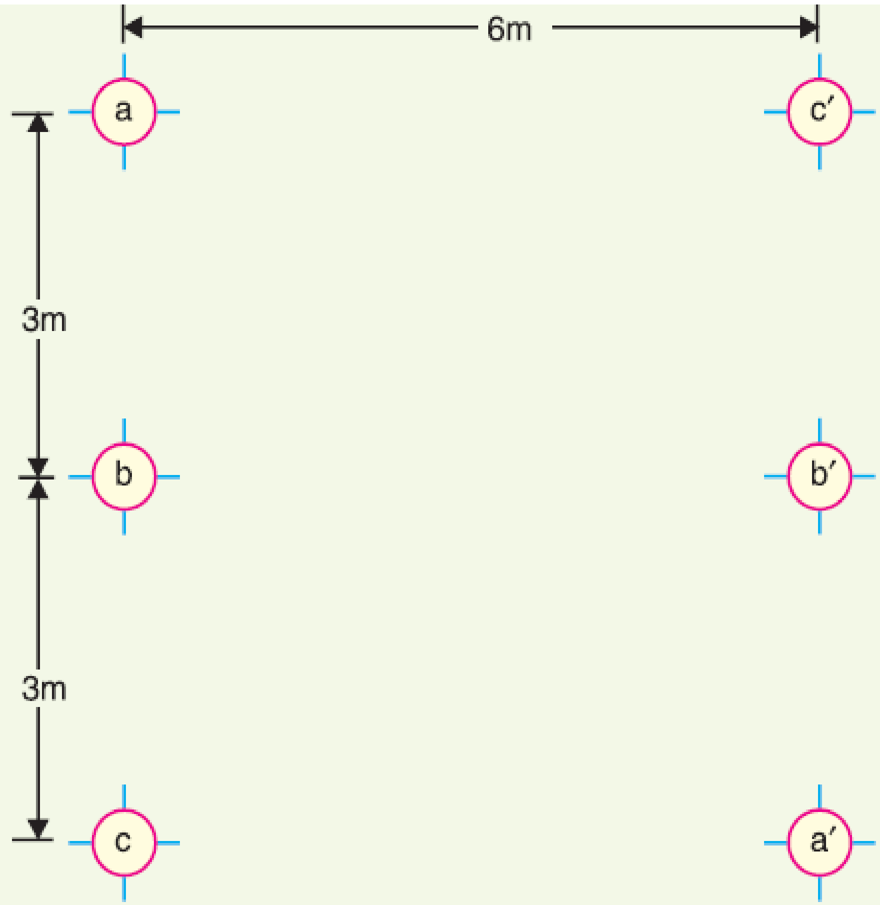

Fig. 9.15

Solution.

$$\text{G.M.R. of conductor} = 1{\cdot}3 \times 0{\cdot}7788 = 1{\cdot}01 \text{ cm}$$

$$\text{Distance } a \text{ to } b' = \sqrt{6^2 + 3^2} = 6{\cdot}7 \text{ m}$$

$$\text{Distance } a \text{ to } d' = \sqrt{6^2 + 6^2} = 8{\cdot}48 \text{ m}$$

Equivalent self G.M.D. of one phase is

$$D_s = \sqrt[3]{D_{s1} \times D_{s2} \times D_{s3}}$$

where D_{s1}, D_{s2} and D_{s3} represent the self-G.M.D. in positions 1, 2 and 3 respectively. Also D_s is the same for all the phases.

$$\text{Now } D_{s1} = \sqrt[4]{D_{aa} \times D_{aa'} \times D_{a'a'} \times D_{a'a}}$$

$$= \sqrt[4]{(1 \cdot 01 \times 10^{-2}) \times (8 \cdot 48) \times (1 \cdot 01 \times 10^{-2}) \times (8 \cdot 48)}$$

$$= 0.292 \text{ m} = D_{s3}$$

$$D_{s2} = \sqrt[4]{D_{bb} \times D_{bb'} \times D_{b'b'} \times D_{b'b}}$$

$$= \sqrt[4]{\left(1 \cdot 01 \times 10^{-2}\right) \times (6) \times \left(1 \cdot 01 \times 10^{-2}\right) \times (6)} = 0{\cdot}246 \text{ m}$$

$$D_s = \sqrt[3]{0 \cdot 292 \times 0 \cdot 246 \times 0 \cdot 292} = 0{\cdot}275 \text{ m}$$

$$\text{Equivalent mutual G.M.D., } D_m = \sqrt[3]{D_{AB} \times D_{BC} \times D_{CA}}$$

where D_{AB}, D_{BC} and D_{CA} represent the mutual G.M.D. between phases A and B, B and C and C and A respectively.

Now $$D_{AB} = \sqrt[4]{D_{ab} \times D_{ab'} \times D_{a'b} \times D_{a'b'}} = \sqrt[4]{3 \times 6.7 \times 6.7 \times 3}$$

$$= 4{\cdot}48 \text{ m} = D_{BC}$$

$$D_{CA} = \sqrt[4]{D_{ca} \times D_{ca'} \times D_{c'a} \times D_{c'a'}} = \sqrt[4]{6 \times 6 \times 6 \times 6} = 6 \text{ m}$$

$\therefore$ $$D_m = \sqrt[3]{4.48 \times 4.48 \times 6} = 4.94 \text{ m}$$

$\therefore$ Inductance per phase per metre length

$$= 10^{-7} \times 2 \log_e D_m/D_s = 10^{-7} \times 2 \log_e 4{\cdot}94/0{\cdot}275$$

$$= 5{\cdot}7 \times 10^{-7} \text{ H}$$

Inductance /phase/km $= 5{\cdot}7 \times 10^{-7} \times 1000 = 0{\cdot}57 \times 10^{-3}$ H $=$ **0·57 mH**

Example 9.10. *In a single phase line (See. Fig. 9.18), conductors a and a′ in parallel form one circuit while conductors b and b′ in parallel form the return path. Calculate the total inductance of the line per km assuming that current is equally shared by the two parallel conductors. Conductor diameter in 2·0 cm.*

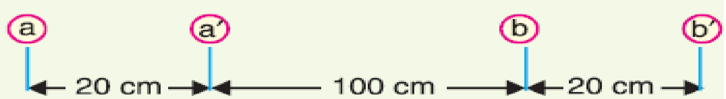

Fig. 9.18

Solution.

Loop inductance/km, $$L = 4 \times 10^{-4} \log_e \frac{D_m}{D_s} \text{ H/km}$$

Mutual G.M.D., $$D_m = \sqrt[4]{D_{ab} \times D_{ab'} \times D_{a'b} \times D_{a'b'}}$$

$$= \sqrt[4]{120 \times 140 \times 100 \times 120} = 119 \text{ cm}$$

Self G.M.D., $$D_s = \sqrt[4]{D_{aa} \times D_{aa'} \times D_{a'a'} \times D_{a'a}}$$

Here $$D_{aa} = D_{a'a'} = 0{\cdot}7788 \text{ cm}; \quad D_{aa'} = D_{a'a} = 20 \text{ cm}$$

$\therefore$ $$D_s = \sqrt[4]{0.7788 \times 0.7788 \times 20 \times 20} = 3{\cdot}94 \text{ cm}$$

$\therefore$ $$L = 4 \times 10^{-4} \log_e \frac{119}{3.94} = 1{\cdot}36 \times 10^{-3} \text{ H/km} = \mathbf{1{\cdot}36 \text{ mH/km}}$$

Assignment # 6

PB1

A 20 km single phase line has two parallel conductors separated by 1·5 metres. The diameter of each conductor is 0·823 cm. If the conductor has a resistance of 0·311 Ω/km, find the loop impedance of this line at 50 Hz. **[19·86 Ω]**

PB2

The three conductors of 3-phase overhead line are arranged in a horizontal plane with a spacing of 4 m between adjacent conductors. The diameter of each conductor is 2 cm. Determine the inductance per km per phase of the line assuming that the lines are transposed. **[1·3 mH]**

PB3

Determine the inductance per km of a 3-phase transmission line using 20 mm diameter conductors when conductors are at the corners of a triangle with spacing of 4,5 and 6 metres. Conductors are regularly transposed. **[1·29 mH/km/phase]**

9.9 Electric Potential

The electric potential at a point due to a charge is the work done in bringing a unit positive charge from infinity to that point.

The concept of electric potential is extremely important for the determination of capacitance in a circuit since the latter is defined as the charge per unit potential.

Thus, the electric potential due to some important conductor arrangements will be discussed.

(i) Potential at a charged single conductor.

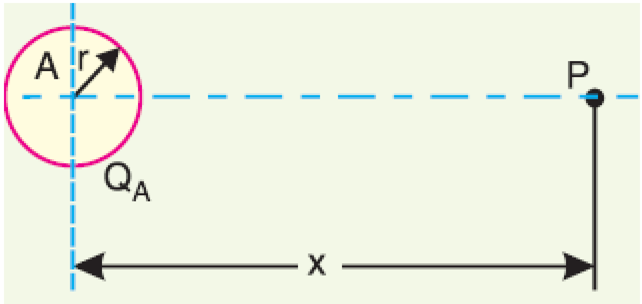

Fig. 9.19

Consider a long straight cylindrical conductor *A* of radius *r* meters as shown in Fig. 9.19.

Let the conductor operates at such a potential (V_A) that charge Q_A coulombs per meter exists on the conductor.

The electric intensity *E* at a distance *x* from the center of the conductor in air is given by:

$$E = \frac{Q_A}{2\pi x \varepsilon_0} \text{ volts/m}$$

where Q_A = charge per meter length

ε_0 = permittivity of free space

As *x* approaches infinity, the value of *E* approaches zero.

Therefore, the potential difference between conductor *A* and infinity distant neutral plane is given by :

$$V_A = \int_r^\infty \frac{Q_A}{2\pi x \varepsilon_0} dx = \frac{Q_A}{2\pi\varepsilon_0} \int_r^\infty \frac{dx}{x}$$

(*ii*) Potential at a conductor in a group of charged conductors.

Overall potential difference between conductor *A, refer to Fig. 9.20,* and infinite neutral plane is, and assuming balanced conditions *i.e.,* $Q_A + Q_B + Q_C = 0$, we have,

$$V_A = \frac{1}{2\pi\,\varepsilon_0}\left[Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_1} + Q_C \log_e \frac{1}{d_2} + \ldots\right]$$

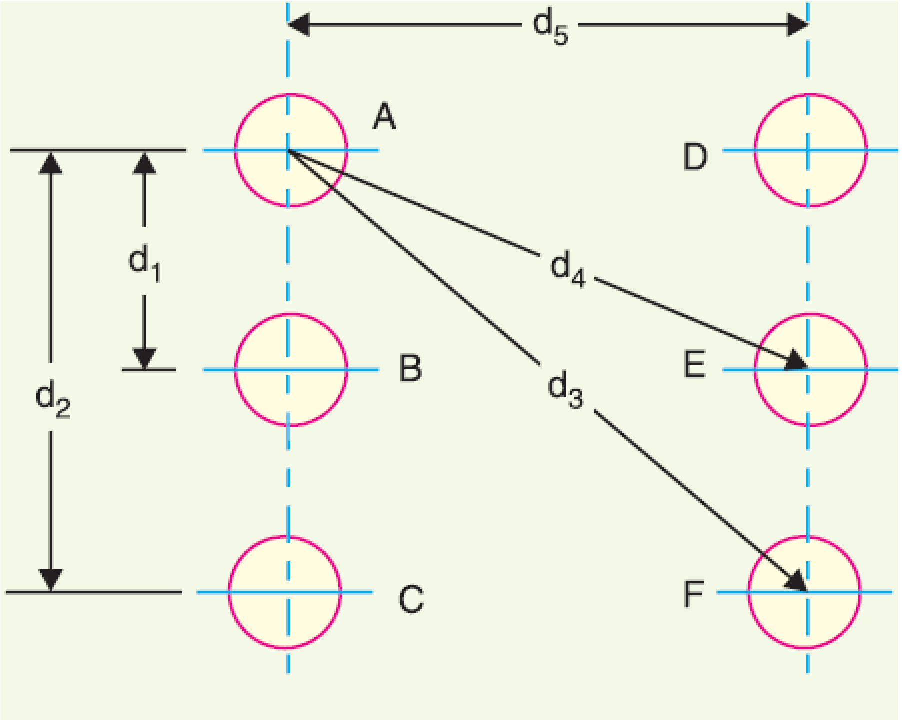

Fig. 9.20

9.10 Capacitance of a Single Phase Two-wire Line

Consider a single phase overhead transmission line consisting of two parallel conductors *A* and *B* spaced *d* meters apart in air, as shown in Fig. 9.21.

Suppose that radius of each conductor is *r* meters.

Let their respective charge be + *Q* and − *Q* coulombs per meter length.

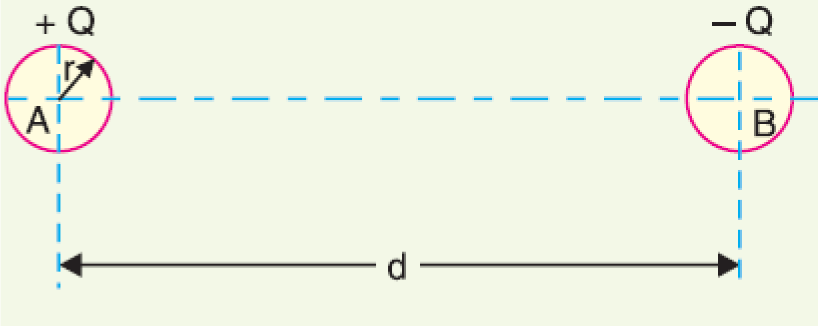

Fig. 9.21

The potential difference between conductors A and B is

$$V_{AB} = 2V_A = \frac{2Q}{2\pi\,\varepsilon_0}\log_e\frac{d}{r}\ \text{volts}$$

∴ Capacitance,

$$C_{AB} = Q/V_{AB} = \frac{Q}{\frac{2Q}{2\pi\,\varepsilon_0}\log_e\frac{d}{r}}\ \text{F/m}$$

∴

$$C_{AB} = \frac{\pi\,\varepsilon_0}{\log_e\frac{d}{r}}\ \text{F / m} \qquad \ldots(i)$$

Capacitance to neutral. Equation (*i*) gives the capacitance between the conductors of a two wire line [See Fig. 9.22].

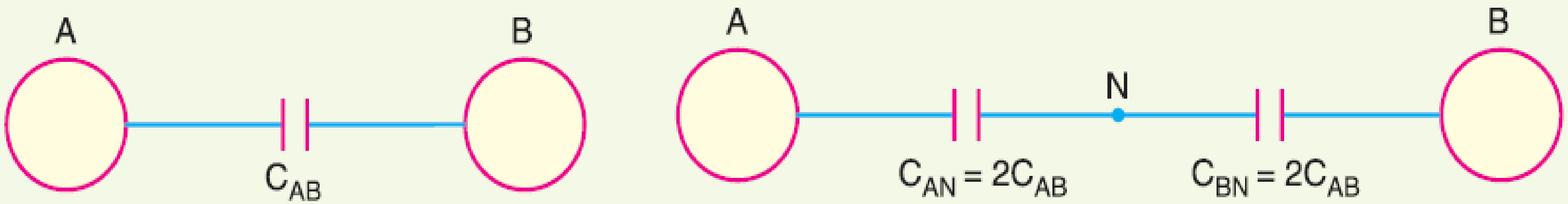

Fig. 9.22

Fig. 9.23

Often it is desired to know the capacitance between one of the conductors and a neutral point between them.

$$\therefore \quad \text{Capacitance to neutral, } C_N = C_{AN} = C_{BN} = 2C_{AB}$$

$$\therefore \quad C_N = \frac{2\pi\varepsilon_0}{\log_e \dfrac{d}{r}} \text{ F / m} \qquad \ldots(ii)$$

Thus the *capacitance to ground* or capacitance to neutral for the two wire line is *twice* the line-to-line capacitance (capacitance between conductors as shown in Fig 9.23).

9.11 Capacitance of a 3-Phase Overhead Line

In a 3-phase transmission line, the capacitance of each conductor is considered instead of capacitance from conductor to conductor.

Here, again two cases arise *viz.,* symmetrical spacing and unsymmetrical spacing.

(i) Symmetrical Spacing. **Fig. 9.24 shows the three conductors *A*, *B* and *C* of the 3-phase overhead transmission line having charges Q_A, Q_B and Q_C per meter length respectively.**

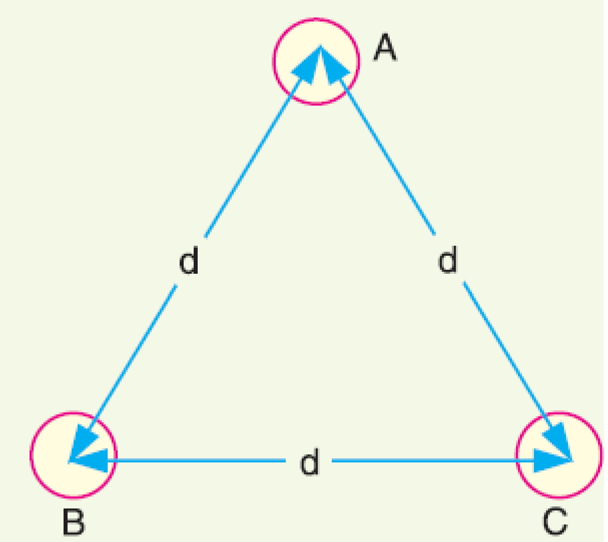

Fig. 9.24

Let the conductors be equidistant (*d* meters) from each other.

Assuming balanced supply, we have, $Q_A + Q_B + Q_C = 0$

$\therefore$ Capacitance of conductor A w.r.t neutral,

$$C_A = \frac{Q_A}{V_A} = \frac{Q_A}{\frac{Q_A}{2\pi\varepsilon_0}\log_e\frac{d}{r}} \text{ F / m} = \frac{2\pi\varepsilon_0}{\log_e\frac{d}{r}} \text{ F / m}$$

$$\therefore \quad C_A = \frac{2\pi\varepsilon_0}{\log_e\frac{d}{r}} \text{ F / m}$$

Note that this equation is identical to capacitance to neutral for two-wire line., the expressions for capacitance of conductors B and C are the identical.

(*ii*) Unsymmetrical spacing.

Fig. 9.25 shows a 3-phase transposed line having unsymmetrical spacing. Let us assume balanced conditions *i.e.* $Q_A + Q_B + Q_C = 0$.

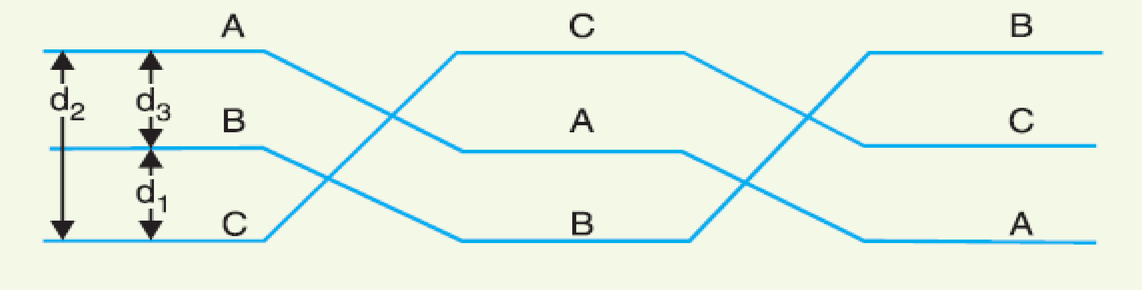

Fig. 9.25

In considering all the three sections of the transposed line for phase *A*, the capacitance to neutral of conductor A can expressed by:

$$C_A = \frac{Q_A}{V_A} = \frac{2\pi\varepsilon_0}{\log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r}} F/m$$

Example 9.11 *A single-phase transmission line has two parallel conductors 3 metres apart, radius of each conductor being 1 cm. Calculate the capacitance of the line per km. Given that* $\varepsilon_0 = 8{\cdot}854 \times 10^{-12}$ *F/m.*

Solution.

Conductor radius, $r = 1$ cm

Spacing of conductors, $d = 3$ m $= 300$ cm

Capacitance of the line $= \dfrac{\pi\varepsilon_0}{\log_e d/r}$ F/m $= \dfrac{\pi \times 8{\cdot}854 \times 10^{-12}}{\log_e 300/1}$ F/m

$= 0{\cdot}4875 \times 10^{-11}$ F/m $= 0{\cdot}4875 \times 10^{-8}$ F/km

$=$ **$0{\cdot}4875 \times 10^{-2}$ μF/km**

Example 9.12. *A 3-phase overhead transmission line has its conductors arranged at the corners of an equilateral triangle of 2 m side. Calculate the capacitance of each line conductor per km. Given that diameter of each conductor is 1·25 cm.*

Solution.

Conductor radius, $r = 1{\cdot}25/2 = 0{\cdot}625$ cm

Spacing of conductors, $d = 2$ m $= 200$ cm

Capacitance of each line conductor

$$= \frac{2\pi\varepsilon_0}{\log_e d/r}\text{ F/m} = \frac{2\pi \times 8{\cdot}854 \times 10^{-12}}{\log_e 200/0{\cdot}625}\text{ F/m}$$

$$= 0{\cdot}0096 \times 10^{-9}\text{ F/m} = 0{\cdot}0096 \times 10^{-6}\text{ F/km} = \mathbf{0{\cdot}0096\ \mu F/km}$$

Example 9.13. *A 3-phase, 50 Hz, 66 kV overhead line conductors are placed in a horizontal plane as shown in Fig. 9.26. The conductor diameter is 1·25 cm. If the line length is 100 km, calculate (i) capacitance per phase, (ii) charging current per phase, assuming complete transposition of the line.*

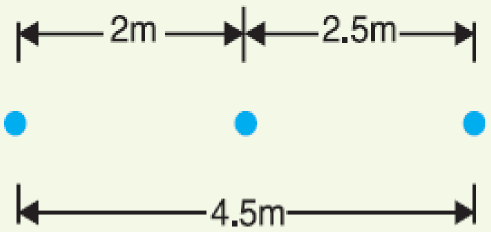

Fig. 9.26

Solution. Fig 9.26 shows the arrangement of conductors of the 3-phase line. The equivalent equilateral spacing is

$$d = \sqrt[3]{d_1 d_2 d_3} = \sqrt[3]{2 \times 2{\cdot}5 \times 4{\cdot}5} = 2{\cdot}82\text{ m}$$

Conductor radius, $r = 1{\cdot}25/2 = 0{\cdot}625$ cm

Conductor spacing , $d = 2{\cdot}82$ m = 282 cm

(*i*) Line to neutral capacitance= $\dfrac{2\,\pi\,\varepsilon_0}{\log_e d/r}$ F/m $= \dfrac{2\,\pi \times 8\cdot 854 \times 10^{-12}}{\log_e 282/0\cdot 625}$ F/m

$$= 0{\cdot}0091 \times 10^{-9}\text{ F/m} = 0{\cdot}0091 \times 10^{-6}\text{ F/km} = 0{\cdot}0091\ \mu\text{F/km}$$

$\therefore$ Line to neutral capacitance for 100 km line is

$$C = 0{\cdot}0091 \times 100 = \mathbf{0{\cdot}91\ \mu F}$$

(*ii*) Charging current per phase is

$$I_C = \frac{V_{ph}}{X_C} = \frac{66{,}000}{\sqrt{3}} \times 2\pi\, f\, C$$

$$= \frac{66{,}000}{\sqrt{3}} \times 2\pi \times 50 \times 0\cdot 91 \times 10^{-6} = \mathbf{10{\cdot}9\ A}$$

TUTORIAL PROBLEMS

1. A single phase transmission line has two parallel conductors 1·5 metres apart, the diameter of each conductor being 0·5 cm. Calculate line to neutral capacitance for a line 80 km long. **[3·48 μF]**

2. A 200 km, 3-phase transmission line has its conductors placed at the corners of an equilateral triangle of 2·5 m side. The radius of each conductor is 1 cm. Calculate :

(*i*) line to neutral capacitance of the line,

(*ii*) charging current per phase if the line is maintained at 66 kV, 50 Hz. **[(*i*) 2·02 μF (*ii*) 24·2 A]**

3. The three conductors *A*, *B* and *C* of a 3-φ line are arranged in a horizontal plane with $D_{AB} = 2$ m and $D_{BC} = 2{\cdot}5$ m. Find line-to-neutral capacitance per km if diameter of each conductor is 1·24 cm. The conductors are transposed at regular intervals. **[0·0091 μF/km]**