CHAPTER 9: FACTORING EXPRESSIONS AND SOLVING BY FACTORING

Chapter Objectives
By the end of this chapter, students should be able to ✓ Factor a greatest common factor ✓ Factor by grouping including rearranging terms ✓ Factor by applying special-product formulas ✓ Factor trinomials by using a general strategy ✓ Solve equations and applications by factoring

SECTION 9.1: GREATEST COMMON FACTOR AND GROUPING

A. FINDING THE GREATEST COMMON FACTOR

In this lesson, we focus on factoring using the greatest common factor, GCF, of a polynomial. When we multiplied polynomials, we multiplied monomials by polynomials by distributing, such as

$$4x^2(2x^2 - 3x + 8) = 8x^4 - 12x^3 + 32x$$

We work out the same problem, but **backwards**. We will start with $8x^2 - 12x^3 + 32$ and obtain its factored form.

First, we have to identify the GCF of a polynomial. We introduce the GCF of a polynomial by looking at an example in arithmetic. The method in which we obtained the GCF between numbers in arithmetic is the same method we use to obtain the GCF with polynomials.

Definition
The ***factored form*** of a number or expression is the expression written as a product of factors. The ***greatest common factor (GCF)*** of a polynomial is the largest polynomial that is a factor of all terms in the polynomial.

	MEDIA LESSON Determine the GCF of Two Monomials (Duration 2:32)

View the video lesson, take notes and complete the problems below

Find the GCF of $88r^{18}$ and $24r^{13}$.

88 /\ 24 /\

$88r^{18} =$ ________________________________

$24r^{13}$ = ________________________________

GCF = ________________________________

YOU TRY

Find the GCF

a) $24x^3$ and $56x^{15}$

b) $12y^5, 6y^{20}$ and $21y^7$

	MEDIA LESSON Determine the GCF of two monomials (Two variables) (Duration 3:45)

View the video lesson, take notes and complete the problems below

Find the GCF of $108x^5y^3$ and $96x^7y^2$.

$108x^5y^3$= ______________________________

$96x^7y^2$______________________________

GCF=______________________________

108
/\

96
/\

YOU TRY

Find the GCF:

a) $15m^3n^6$ and $45m^4n^2$

b) $24x^4y^2z, 18x^2y^4$, and $12x^3yz^5$

	MEDIA LESSON Find common factor with smaller coefficients (Duration 2:28)

View the video lesson, take notes and complete the problems below

Greatest common factor: ____

On variables we use_______________________________________

Example: Find the greatest common factor.

a) $15a^4 + 10a^2 - 25a^5$

b) $4a^4b^7 - 12a^2b^6 + 20ab^9$

YOU TRY

Find the GCF:

a) $4x^2 - 20x + 10$

b) $6x^4 - 15x^3 + 9x^2$

B. FACTORING THE GREATEST COMMON FACTOR

	MEDIA LESSON Factor using the product method (Duration 6:39)

View the video lesson, take notes and complete the problems below

Identify the greatest common factor. Then factor.

a) $18x^5 + 6x^4 + 24x^3$

18	6	24	
/\	/\	/\	GCF = ____________

____

b) $60a^4b^3 - 15a^3b^4 + 45a^2b^5$

60	15	48	
/\	/\	/\	GCF = ______________

____

____

YOU TRY

Factor:

a) $25x^4 - 15x^6 + 20x^3$

b) $12x^3y^2z - 20x^4y^3z^5 - 16xy^4$

MEDIA LESSON
Factor using the division method (Duration 4:08)

View the video lesson, take notes and complete the problems below

a (b + c) = ______________________________

Put ____________ in the front, and divide. What is left goes in the ____________________.

Example: Factor

a) $9x^4 - 12x^3 + 6x^2$

b) $21a^4b^5 - 14a^3b^7 + 7a^2b^2$

YOU TRY

Factor using the division method.

a) $21x^3 + 14x^2 + 7x$

b) $4x^2 - 20x + 16$

CHECK YOUR SOLUTION:

To check your answer, you can distribute your GCF back into the parenthesis.

MEDIA LESSON
Using distributive property to verify the factored form (Duration 2:46)

View the video lesson, take notes and complete the problems below

Example:

a) $5x^2(6x^2 - 2x + 5)$

b) $-3x^4(6x^3 + 2x - 7)$

YOU TRY

Check your answer from the YOU TRY in the section Factoring using the division method above.

a)

b)

Steps for factoring out the greatest common factor
Step 1. Find the GCF of the expression.
Step 2. Rewrite each term as a product of the GCF and the remaining factors.
Step 3. Rewrite as a product of the GCF and the remaining factors in parenthesis.
Step 4. ✓Verify the factored form by multiplying. The product should be the original expression.

C. A BINOMIAL AS THE GREATEST COMMON FACTOR

As part of a general strategy for factoring, we always look for a GCF. Sometimes the GCF is a monomial, like in the previous examples, or a binomial. Here we discuss factoring a polynomial where the GCF is a binomial.

MEDIA LESSON
Binomial GCF (Duration 2:20)

View the video lesson, take notes and complete the problems below

GCF can be a ____.

Example:

a) $5x(2y-7)+6y(2y-7)$

b) $3x(2x+1)-7(2x+1)$

YOU TRY

Factor:

a) $3a(2a+5b)-4b(2a+5b)$

b) $(9x^2-2)3y-(9x^2-2)5x$

D. FACTOR BY GROUPING

When we have polynomials that have at least 4 terms. Sometimes, we can factor them by using a process known as factor by grouping.

Steps for factoring by grouping
To factor by grouping, we first notice the polynomial expression obtains four terms. **Step 1.** Group two sets of two terms, e.g., ax + ay + bx + by = (ax + ay) + (bx + by). **Step 2.** Factor the GCF from each group, e.g., a(x + y) + b(x + y) **Step 3.** Factor the GCF from the expression, e.g., (x + y)(a + b).

MEDIA LESSON
Factoring by grouping (Duration 4:01)

View the video lesson, take notes and complete the problems below

Grouping: GCF of the ____________ and ____________

Then factor out ____________________________________(if it matches!)

Example:

a) $15xy + 10y - 18x - 12$

b) $6x^2 + 3xy + 2x + y$

YOU TRY

Factor by grouping:

a) $10ab + 15b + 4a + 6$

b) $6x^2 + 9xy - 14x - 21y$

c) $5xy - 8x - 10y + 16$

d) $12ab - 14a - 6b + 7$

E. FACTOR BY GROUPING BY REARRANGING TERMS

Sometimes after completing Step 2, the binomials are not identical (by more than a negative sign). At this point, we must return to the original problem and rearrange the terms so that when we factor by grouping, we obtain identical binomials in Step 2.

	MEDIA LESSON Factor by grouping – rearranging terms (Duration 4:41)

View the video lesson, take notes and complete the problems below

If binomials don't match: ____

Example:

a) $12a^2 - 7b + 3ab - 28a$

b) $6xy - 20 + 8x - 15y$

YOU TRY

Factor:

a) $4a^2 - 21b^3 + 6ab - 14ab^2$

b) $8xy - 12y + 15 - 10x$

EXERCISE

Factor the greatest common factor (GCF) if possible. If not, write "No common factor". Check your answer by multiplying the factors.

1) $9b + 8b^2$

2) $45x^2 - 25$

3) $56 - 35p$

4) $7ab - 35a^2b^2$

5) $-3a^2b + 6a^3b^2$

6) $-5x^2 - 5x^3 - 15x^4$

7) $20x^4 - 30x + 30$

8) $28m^4 + 40m^3 + 8$

9) $30b^9 + 5ab - 15a^2$

10) $-48a^2b^2 - 56a^3b - 50a^5b$

11) $20x^8y^2z^2 + 15x^5y^5z + 35x^3y^3z$

12) $-3mn^8 + 5 - 6m$

13) $30qpr - 5qp + 5q$

14) $50x^2y + 10xy^2 + 70xz^2$

15) $1 + 2n^2$

16) $-18mn^5 + 3mn^3 - 21m^3n^2 + 3mn$

Factor completely. Check your answer using distributive property.

17) $40r^3 - 8r^2 - 25r + 5$

18) $3n^3 - 2n^2 - 9n + 6$

19) $15b^3 + 21b^2 - 35b - 49$

20) $3x^3 + 15x^2 + 2x + 10$

21) $35x^3 - 28x^2 - 20x + 16$

22) $7xy - 49x + 5y - 35$

23) $32xy + 40x^2 + 12y + 15x$

24) $16xy - 56x + 2y - 7$

25) $2xy - 8x^2 + 7y^3 - 28y^2x$

26) $40xy + 35x - 8y^2 - 7y$

27) $35x^3 - 10x^2 - 56x + 10$

28) $16xy - 3x - 6x^2 + 8y$

29) $6x^3 - 48x^2 + 5x - 40$

30) $14v^3 + 10v^2 - 7v - 5$

31) $7n^3 + 21n^2 - 5n - 15$

32) $28p^3 + 21p^2 + 20p + 15$

33) $15ab - 2b^2 - 6a + 5b^3$

34) $42r^3 - 21 - 49r^2 + 18r$

35) $5mn - 10 + 2m - 25n$

36) $-8m + 15n - 40 + 3mn$

37) $4uv + 14u^2 + 12v + 42u$

38) $56x - y + 8xy - 7$

39) $56ab + 14 - 49a - 16b$

40) $24xy + 25y^2 - 20x - 30y^3$

SECTION 9.2: FACTORING TRINOMIALS OF THE FORM $x^2 + bx + c$

A. FACTORING TRINOMIALS OF THE FORM $x^2 + bx + c$

Factoring with **three terms**, or **trinomials**, is the most important technique, especially in further algebra. Since factoring is a product of factors, we first look at multiplying to develop the process of factoring trinomials.

Steps for factoring trinomials of the form $x^2 + bx + c$
Step 1. Find two numbers, p and q, that $p + q = b$ and $p \cdot q = c$
Step 2. Rewrite the expression so that the middle term is split into two terms, p and q.
Step 3. Factor by grouping.
Step 4. ✓ Verify the factored form by finding the product.

MEDIA LESSON
Factoring a trinomial with leading coefficient of 1 (*ac* method) (Duration 10:33)

View the video lesson, take notes and complete the problems below

Factoring trinomials with a leading coefficient of 1.

$$x^2 + bx + c$$

1. Make two sets of parentheses and put the factors of x^2 in the first position of each set of parentheses.

   (x)(x)

2. The second positions are the factors of c that **add** to b.

Example: Factor.

Factors of c

a) $x^2 + 8x + 12$

____

____

b) $x^2 - 4x - 32$

Factors of c

c) $y^2 - 13y + 36$

d) $x^2 + x - 12$

e) $x^2 + 28x + 75$

f) $3x^2 - 27x + 42$

g) $2y^3 - 12y^2 - 80y$

MEDIA LESSON
Factoring trinomials 2 (Duration 5:01)

View the video lesson, take notes and complete the problems below

Factor:

a) $x^2 + 12x + 32$

b) $x^2 + 11x - 60$

c) $x^2 - 9x + 20$

d) $x^2 - 5x - 24$

YOU TRY

a) $x^2 + 9x + 18$

b) $x^2 - 4x + 3$

c) $x^2 - 8x - 20$

d) $a^2 - 9ab + 14b^2$

MEDIA LESSON
Factoring trinomials – "X box" method (Duration 3:21)

View the video lesson, take notes and complete the problems below

If there is a __________ in front of x^2, then the ***ac*** method gives us ____________________________.

a) $x^2 - 2x - 8$

b) $x^2 + 7xy - 8y^2$

YOU TRY

a) $u^2 - 8uv + 15v^2$

b) $m^2 + 2mn - 8n^2$

❖ **PRIME POLYNOMIALS:** If a trinomial (or polynomial) is ***not factorable***, then we say we the trinomial is ***prime***.

For example: factor $x^2 + 2x + 6$.

We identify *b* = 2 and *c* = 6

Factor of *c*	Sum
2, 3	2 + 3 = 5, not *b*
−2, −3	−2 + −3 = −5, not *b*
1, 6	1 + 6 = 7, not *b*
−1, −6	−1+ −6 = −7, not *b*

We can see from the table that there are not any factors of 6 whose sum is 2. In this case, we call this trinomial *not factorable*, or better yet, the trinomial is *prime*.

B. FACTORING TRINOMIALS OF THE FORM x^2 + bx + c WITH A GCF

Factoring the GCF is always the first step in factoring expressions. If all terms have a common factor, we first, factor the GCF and then factor as usual.

	MEDIA LESSON Factoring trinomials with factoring GCF first (Duration 3:39)

View the video lesson, take notes and complete the problems below

a) $7x^2 + 21x - 70$

b) $4x^4y + 36x^3y^2 + 80x^2y^3$

YOU TRY

Factor:

a) $3x^2 - 24x + 45$

b) $4x^2 + 52x + 168$

EXERCISE

Factor completely. If a trinomial is not factorable, write "prime".

1) $p^2 + 17p + 72$

2) $n^2 - 9n + 8$

3) $x^2 - 9x - 10$

4) $b^2 + 12b + 32$

5) $x^2 - 9x - 10$

6) $n^2 - 15n + 56$

7) $p^2 + 15p + 54$

8) $n^2 - 8n + 15$

9) $x^2 - 11xy + 18y^2$

10) $x^2 + xy - 12y^2$

11) $x^2 + 4xy - 12y^2$

12) $5a^2 + 60a + 100$

13) $6a^2 + 24a - 192$

14) $6x^2 + 18xy + 12y^2$

15) $6x^2 + 96xy + 378y^2$

16) $x^2 - x - 72$

17) $x^2 + x - 30$

18) $x^2 + 13x + 40$

19) $m^2 + 13x - 36$

20) $6m^2 - 36mn - 162n^2$

21) $b^2 - 17b + 70$

22) $x^2 - 3x - 18$

23) $a^2 - 6a - 27$

24) $p^2 + 7p - 30$

25) $m^2 - 15mn + 50n^2$

26) $x^2 + x + 1$

27) $x^2 + 10xy + 16y^2$

28) $u^2 - 9uv + 14v^2$

29) $x^2 + 14xy + 45y^2$

30) $5n^2 - 45n + 40$

31) $5v^2 + 20v - 25$

32) $m^2 - 2mn + n^2$

SECTION 9.3: FACTORING TRINOMIALS OF THE FORM $ax^2 + bx + c$

A. FACTORING TRINOMIALS OF THE FORM $ax^2 + bx + c$ BY GROUPING

When the leading coefficient a is not 1, it takes a few more steps to factor the trinomial. There are many ways to factor this type of trinomials. You are going to learn 2 methods in this section. The first one is factor by grouping and the second one is the "bottoms- up" method.
First, let's take a look at the grouping method.

Steps for factoring trinomials of the form $ax^2 + bx + c$ using the grouping method
Step 1. Find two numbers, $\boldsymbol{p}$ and $\boldsymbol{q}$, that $\boldsymbol{p} + \boldsymbol{q} = \boldsymbol{b}$ and $\boldsymbol{p} \cdot \boldsymbol{q} = \boldsymbol{a} \cdot \boldsymbol{c}$
Step 2. Rewrite the expression so that the middle term is split into two terms, p and q.
Step 3. Factor by grouping.
Step 4. ✓ Verify the factored form by finding the product.

MEDIA LESSON
Factor trinomials when leading coefficient is not 1 - Grouping method (Duration 4:21)

View the video lesson, take notes and complete the problems below

a) $4x^2 - 4x - 15$

b) $20x^2 + 19x + 3$

YOU TRY

Factor using the grouping method and verify your answer by multiplying the two binomials. Show your work.

a) $3x^2 + 11x + 6$

b) $8x^2 - 2x - 15$

B. FACTORING TRINOMIALS OF THE FORM $ax^2 + bx + c$ BY THE "BOTTOMS UP" METHOD

Steps for factoring trinomials of the form $ax^2 + bx + c$ using the "bottoms-up" method
Step 1. Multiply $a \cdot c$, then write a new trinomial in the form of $x^2 + bx + a{\cdot}c$
Step 2. Factor as you normally would with trinomials with the leading coefficient of 1.
Step 3. Divide the constants in each binomial factor by the original value of a.
Step 4. Simplify the fractions formed.
Step 5. If the simplified fractions does not have the denominator of 1, move the denominator to the coefficient of the variable.
Step 6. ✓ Verify the factored form by finding the product

MEDIA LESSON
Factor trinomials when the leading coefficient is NOT 1 - Bottoms up method (Duration 4:20)

View the video lesson, take notes and complete the problems below

a) $4x^2 - 4x - 15$

b) $20x^2 + 19x + 3$

YOU TRY

Factor the trinomials using the "bottoms up" method. Show your work.

c) $3x^2 + 11x + 6$

d) $8x^2 - 2x - 15$

C. FACTORING TRINOMIALS OF THE FORM $ax^2 + bx + c$ BY THE TRIAL AND ERROR METHOD

Factoring by trial-and-error is just a guess and check process when you try to add up different products to get the middle term ***bx***. This sometimes works out faster than other methods above and sometimes not. If you want a step-by-step process that always works, this method may not be the best method for you.

	MEDIA LESSON Factor trinomials when the leading coefficient is NOT 1 – Trial and error method (Duration 5:22)

View the video lesson, take notes and complete the problems below

a) $4x^2 - 4x - 15$

$(2x$ ________________ $)(2x$ ________________ $)$

____

____

$(4x$ ____________ $)(4x$ ____________ $)$

____

b) $20x^2 + 19x + 3$

$(5x$ ________________ $)(4x$ ________________ $)$

____

____

$(10x$ ________________ $)(2x$ ________________ $)$

____

____

$(20x$ ________________ $)(x$ ________________ $)$

____

____

$(5x$ ________________ $)(4x$ ________________ $)$

____

____

$(10x$ ________________ $)(2x$ ________________ $)$

____

$(20x$ ________________ $)(x$ ________________ $)$

YOU TRY

Factor the trinomials below by using the trial-error method:

a) $10x^2 - 27x + 5$

D. FACTORING TRINOMIALS OF THE FORM ax^2 + bx + c WITH A GCF IN THE COEFFICIENTS

As always, when factoring, we will first look for a GCF in the coefficients, factor the GCG, then factor the trinomial as usual.

Stop at 7:00	MEDIA LESSON Factoring trinomials of the form ax^2 + bx + c with GCF in the coeffients (Duration 1:45)

View the video lesson, take notes and complete the problems below

Example: $18x^2 - 15x - 12$

YOU TRY

a) $24x^2 - 22x + 4$

b) $-18x^3 - 33x^2 + 30x$

EXERCISE

Factor completely by using grouping method. Show your work.

1) $7x^2 - 48x + 36$

2) $7b^2 + 15b + 2$

3) $5a^2 - 13a - 28$

4) $2x^2 - 5x + 2$

5) $2b^2 - b - 3$

6) $5k^2 + 13k + 6$

7) $3x^2 - 17x + 20$

8) $6x^2 - 39x - 21$

9) $6x^2 - 29x + 20$

10) $4x^2 + 9x + 2$

11) $4x^2 + 13xy + 3y^2$

12) $3u^2 + 13uv - 10v^2$

Factor completely by using the "bottoms up" method. Show your work.

13) $4m^2 - 9m - 9$

14) $4x^2 + 13x + 3$

15) $6p^2 - 11p - 7$

16) $4r^2 + r - 3$

17) $4r^2 + 3r - 7$

18) $3r^2 - 4r - 4$

19) $3x^2 + 10x - 8$

20) $2x^2 - 5x - 3$

21) $2y^2 + 15y + 7$

22) $7a^2 - 11a + 4$

23) $4x^2 + 16x + 16$

24) $24a^2 - 30a + 9$

25) $10x^3 + 15x^2 - 10x$

26) $2x^3y + 12x^2y + 18xy$

27) $5t^2 + 15t + 10$

28) $2x^2 + 8x + 6$

29) $7x^2 - 2xy - 5y^2$

30) $24x\hat{}2 - 52xy + 8y\hat{}2$

31) $4x^2 + 13xy + 3y^2$

32) $3u^2 + 13uv - 10v^2$

SECTION 9.4: SPECIAL PRODUCTS

In the previous chapter, we recognized two special products: difference of two squares and perfect square trinomials. In this section, we discuss these special products to factor expressions.

A. DIFFERENCE OF TWO SQUARES

When we see a binomial where both the 1st and 2nd terms are perfect square and one subtracts another, you have the difference of two squares. You can apply the following formula to factor quickly.

Difference of two squares
$\boldsymbol{a^2 - b^2 = (a+b)(a-b)}$

	MEDIA LESSON Factor a Difference of Squares (Duration 4:19)

View the video lesson, take notes and complete the problems below

Example: Factoring binomials

a) $x^2 - 36$

b) $x^2 - 16$

c) $100 - 9x^2$

d) $2x^2 - 18$

❖ **Warning:** The sum of squares $\boldsymbol{a^2 + b^2}$ does **NOT** factor. It is always prime.

	MEDIA LESSON Factoring a difference of squares with two variables (Duration 1:48)

View the video lesson, take notes and complete the problems below

Factor: $x^2 - 49y^2$

YOU TRY

Factor completely:

a) $x^2 - 9$

b) $25 - m^2$

c) $9a^2 - 25b^2$

d) $5x^2 - 45y^2$

B. PERFECT SQUARE TRINOMIALS

In this section, we discuss two special types of trinomials that are called the perfect square trinomials. In order to have a perfect square trinomial, we need to have the 1st and the 2nd terms squared and the middle term is twice the 1st and the 2nd terms. This pattern allows us to be more efficient when we factor trinomials.

Perfect square trinomials

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

	MEDIA LESSON Factor perfect square trinomials (Duration 5:53)

View the video lesson, take notes and complete the problems below

Given a perfect square trinomial, factor the trinomial into the square of a binomial:

1. ____
2. ____
3. ____
____

Example:

1. $36x^2 + 60x + 25$

2. $49x^2 - 28x + 4$

YOU TRY

Factor by using the perfect square formula:

a) $x^2 - 6x + 9$

b) $4x^2 + 20xy + 25y^2$

C. FACTORING SPECIAL PRODUCTS WITH A GCF IN THE COEFFICIENTS

Sometimes, we have to factor a GCF out of a polynomial before we can apply the difference of two squares binomial or the perfect square trinomial formulas.

MEDIA LESSON Factor a difference of squares with a common factor (Duration 3:05)

View the video lesson, take notes and complete the problems below

a) Example: $4x^2 - 36$

YOU TRY

a) Factor completely: $72x^2 - 8$

b) Factor the GCF and apply the perfect square formula: $3x^2 - 18x + 27$

D. A SUM OR DIFFERENCE OF TWO CUBES

Sum or difference of two cubes
There are special formulas for a sum or difference of two cubes.
Difference of two cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
Sum of two cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

We can also use the acronym SOAP for the formulas for factoring a sum or difference of two cubes.

Same: binomial has the same sign as the expression

Opposite: middle term of the trinomial has the opposite sign than the expression

Always

Positive: last term of the trinomial is always positive

SOAP is an easier way of remembering the signs in the formula because the formulas for the sum and difference of two cubes are the same except for the signs. Let's take a look:

$$a^3 \underbrace{-}_{sign} b^3 = (a \underbrace{-}_{same} b)(a^2 \underbrace{+}_{opposite} ab \underbrace{+}_{positive} b^2)$$

$$a^3 \underbrace{+}_{sign} b^3 = (a \underbrace{+}_{same} b)(a^2 \underbrace{-}_{opposite} ab \underbrace{+}_{positive} b^2)$$

	MEDIA LESSON Factor a Sum or Difference of Cubes (Duration 4:58)

View the video lesson, take notes and complete the problems below

Special product – Cubes

Sum of cubes: ____

____

Difference of cubes: ____

____

Example: Factor.

a) $m^3 + 125$

b) $8a^3 - 27y^3$

YOU TRY

a) $m^3 - 27$

b) $125p^3 + 8r^3$

Sometimes, you have to factor out the common factor in the coefficients before you can apply the Sum or Difference of Cubes formulas.

	MEDIA LESSON Factor a Sum or Difference of Cubes when coefficients have common factor (Duration 3:34)

View the video lesson, take notes and complete the problems below

Factor completely:

c) $4x^3 - 32$

d) $2x^3 + 250$

	MEDIA LESSON Factor a Sum or Difference of Cubes – Caution (Duration 5:22)

View the video lesson, take notes and complete the problems below

Example: Factor completely $64x^6 + 216y^3$.

64 /\ 216 /\

YOU TRY

a) $5x^3 - 40$

b) $128a^4b^2 + 54ab^5$

EXERCISE

Name the special product and factor completely. Show your work.

	Name		Name
1) $r^2 - 16$	__________________	2) $v^2 - 25$	__________________
3) $p^2 - 4$	__________________	4) $9k^2 - 4$	__________________
5) $3x^2 - 27$	__________________	6) $16x^2 - 36$	__________________
7) $10a^2 - 250b^2$	__________________	8) $a^2 - 2a + 1$	__________________
9) $x^2 + 6x + 9$	__________________	10) $x^2 - 6x + 9$	__________________
11) $k^2 + 4k + 4$	__________________	12) $k^2 - 4k + 4$	__________________
13) $25p^2 - 10p + 1$	__________________	14) $8x^2 - 24xy + 18y^2$	__________________
15) $8 - m^3$	__________________	16) $x^3 - 64$	__________________
17) $216 - u^3$	__________________	18) $125a^3 - 64$	__________________
19) $x^3 + 27y^3$	__________________	20) $x^3 + 64$	__________________
21) $32m^3 - 4n^3$	__________________	22) $x^2 - 1$	__________________
23) $4a^2 - 20ab + 25b^2$	__________________	24) $9a^2 - 1$	__________________
25) $x^2 - 9$	__________________	26) $125x^3 + 27y^3$	__________________
27) $4v^2 - 1$	__________________	28) $25a^2 + 30ab + 9b^2$	__________________
29) $5n^2 - 20$	__________________	30) $x^2 + 8xy + 16y^2$	__________________
31) $4m^2 - 64n^2$	__________________	32) $20x^2 + 20xy + 5y^2$	__________________
33) $n^2 - 8n + 16$	__________________	34) $x^3 + 8$	__________________
35) $x^2 + 2x + 1$	__________________	36) $64x^3 - 27$	__________________
37) $125x^3 + 216$	__________________	38) $18m^2 - 24mn + 8n^2$	__________________
39) $x^4 - y^4$	__________________	40) $z^4 - 16$	__________________

SECTION 9.5: FACTORING, A GENERAL STRATEGY

A General Strategy To Factoring

Step 1. Factor out the greatest common factor, if possible.
Step 2. Determine the number of terms in the polynomial.
Step 3. a) Two Terms

- Difference of two squares: $a^2 - b^2 = (a + b)(a - b)$
- Difference of two cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- Sum of two cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

b) Three Terms

- Perfect square trinomial: $a^2 + 2ab + b^2 = (a + b)^2$ or $a^2 - 2ab + b^2 = (a - b)^2$
- Old fashion way:
  - $x^2 + bx + c = (x + p)(x + q)$: using the *ac* method
  - $ax^2 + bx + c$: Factor by grouping or by the "bottoms up" method.

c) Four Terms

- Factor by grouping, rearranging terms, if needed.

Step 4. Check your work by multiplying out the product of factors.

MEDIA LESSON
General factoring strategy (Duration 5:00)

View the video lesson, take notes and complete the problems below

- Always do ___________ first!
-

2 terms:	3 terms:	4 terms:

Example:

a) $25x^2 - 16$ ________________________

b) $x^2 - x - 20$ ________________________

c) $xy + 2y + 5x + 10$ ________________________

EXERCISE

Apply the general factoring strategy to factor the following polynomials completely. Show your work.

1) $24az - 18ah + 60yz - 45yh$

2) $2x^3 - 128y^3$

3) $54u^3 - 16$

4) $x^2 - 4xy + 3y^2$

5) $m^2 - 4n^2$

6) $128 + 54x^3$

7) $n^3 + 7n^2 + 10n$

8) $5x^2 + 2x$

9) $mn - 12x + 3m - 4xn$

10) $27m^2 - 48n^2$

11) $16x^2 + 48xy + 36y^2$

12) $2x^3 + 5x^2y + 3y^2x$

13) $5x^2 - 22x - 15$

14) $x^3 - 27y^3$

15) $3m^3 - 6m^2n - 24n^2m$

16) $3ac + 15ad^2 + x^2c + 5x^2d^2$

17) $16a^2 - 9b^2$

18) $32x^2 - 18y^2$

19) $v^2 - v$

20) $9n^3 - 3n^2$

SECTION 9.6: SOLVE BY FACTORING

When solving linear equations, such as $2x - 5 = 21$, we can solve by isolating the variable on one side and a number on the other side. However, in this chapter, we have an x^2 term, so if it looks different, then it is different. Hence, we need a new method for solving trinomial equations. One method is using the zero product rule. There are other methods for solving trinomial equations, but that is for a future chapter.

Definition
A polynomial equation is any equation that contains a polynomial expression. A trinomial equation is written in the form $$ax^2 + bx + c = 0$$ where a, b, c are coefficients, and $a \neq 0$. If the trinomial equations have the highest power is 2, they are also called as q**uadratic equations.**

A. ZERO PRODUCT RULE

Zero product rule
If a; b are non-zero factors, then $a \cdot b = 0$ implies $a = 0$ or $b = 0$ or both $a = b = 0$.

	MEDIA LESSON Solve equations by using the Zero product rule (Duration 4:04))

View the video lesson, take notes and complete the problems below

Zero product rule: ____

To solve we set each ______________________ equal to zero

Example:

a) $(5x - 1)(2x + 5) = 0$

b) $2x(x - 6)(2x + 3) = 0$

YOU TRY

Solve for x:

a) $x(x + 7) = 0$

b) $(2x - 3)(5x + 1) = 0$

B. SOLVE EQUATIONS BY FACTORING

Steps for solving trinomial equations
Step 1. Write the given equation in the form $ax^2 + bx + c = 0$.
Step 2. Factor the left side of the equation into a product of factors.
Step 3. Use the zero product rule to set each factor equal to zero and then solve for the unknown.
Step 4. Verify the solution(s).

MEDIA LESSON
Solve quadratic equations by factoring the GCF (Duration 2:38)

View the video lesson, take notes and complete the problems below

Example: Solve

a) $x^2 + 4x = 0$

b) $14x^2 - 35x = 0$

YOU TRY

Solve the equations by factoring:

a) $n^2 - 9n = 0$

b) $7n^2 - 28n = 0$

MEDIA LESSON
Factor and solve quadratic equations when ***a*** = 1 (Duration 5:24)

View the video lesson, take notes and complete the problems below

Example: Solve

a) $x^2 + 6x + 8 = 0$

b) $x^2 + 23x - 50 = 0$

c) $x^2 - 8x + 15 = 0$

d) $x^2 - 5x - 24 = 0$

YOU TRY

Solve the equations by factoring

a) $c^2 + 5c + 6 = 0$

b) $y^2 - 9y + 14 = 0$

	MEDIA LESSON Factor and solve quadratic equation with a negative leading coefficient (Duration 5:10)

View the video lesson, take notes and complete the problems below

Example: Solve by factoring.

a) $-x^2 + 7x + 18 = 0$

b) $-x^2 - 12x - 36 = 0$

YOU TRY

Solve by factoring:

a) $-x^2 + x + 6 = 0$

b) $-y^2 - 3y + 18 = 0$

MEDIA LESSON
Factor & solve quadratic equations with common factor in the coefficients (Duration 4:27)

View the video lesson, take notes and complete the problems below

Example: Solve by factoring.

a) $3x^2 + 15x + 18 = 0$

b) $8x^2 - 72x + 162 = 0$

YOU TRY

Solve by factoring:

c) $3x^2 - 24x + 45 = 0$

d) $4x^2 + 52x + 168 = 0$

MEDIA LESSON
Factor and solve a quadratic equation when the leading coefficient is NOT 1 (Duration 6:17)

View the video lesson, take notes and complete the problems below

Example: Solve by factoring.

a) $4x^2 + 25x - 21 = 0$

b) $3x^2 - 23x + 30 = 0$

YOU TRY

Solve by factoring

a) $3x^2 + 11x + 6 = 0$

b) $8x^2 - 2x - 15 = 0$

MEDIA LESSON
Factor and solve a quadratic equation using the difference of 2 squares (Duration 3:58)

View the video lesson, take notes and complete the problems below

Example: Solve by factoring

a) $x^2 - 49 = 0$

b) $4x^2 = 81$

MEDIA LESSON
Factor and solve a quadratic equation using the difference of 2 squares with GCF (Duration 5:03)

View the video lesson, take notes and complete the problems below

Example: Solve by factoring

c) $48x^2 - 75 = 0$

d) $2x^2 = 32$

YOU TRY

Factor binomials and solve the equation

a) $x^2 - 9 = 0$

b) $8x^2 = 50$

MEDIA LESSON Factor and solve a quadratic equation using the perfect square trinomials (Duration 4:52)	

View the video lesson, take notes and complete the problems below

Example: Solve by factoring.

a) $x^2 + 4x + 4 = 0$

b) $x^2 - 10x + 25 = 0$

c) $4x^2 - 12x + 9 = 0$

YOU TRY

Solve by factoring:

a) $x^2 - 2x + 1 = 0$

b) $9x^2 + 6x + 1 = 0$

C. SIMPLIFY THE EQUATION

Sometimes the equation isn't so straightforward. We may have to do some preliminary work so that the equation takes the form of a trinomial equation and then we can use the zero product rule.

MEDIA LESSON
Solve by factoring – Simplify first (Duration 4:57)

View the video lesson, take notes and complete the problems below

Example:

a) $2x(x+4) = 3x - 3$

b) $(2x-3)(3x+1) = -8x - 1$

YOU TRY

Simplify the following equations and solve by factoring.

a) $(x-7)(x+3) = -9$

$3x^2 + 4x - 5 = 7x^2 + 4x - 14$

EXERCISE

Solve each equation by factoring. Show your work.

1) $(k-7)(k+2)=0$

2) $(x-1)(x+4)=0$

3) $6x^2-150=0$

4) $2n^2+10n-28=0$

5) $7x^2+26x+15=0$

6) $5n^2-9n-2=0$

7) $x^2-4x-8=-8$

8) $x^2-5x-1=-5$

9) $49p^2+371p-163=5$

10) $7x^2+17x-20=-8$

11) $7r^2+84=-49r$

12) $x^2-6x=16$

13) $3v^2+7v=40$

14) $35x^2+120x=-45$

15) $4k^2+18k-23=6k-7$

16) $9x^2-46+7x=7x+8x^2+3$

17) $2m^2+19m+40=-2m$

18) $40p^2+183p-168=p+5p^2$

19) $(a+4)(a-3)=0$

20) $(2x+5)(x-7)=0$

21) $p^2+4p-32=0$

22) $m^2-m-30=0$

23) $40r^2-285r-280=0$

24) $2b^2-3b-2=0$

25) $v^2-8v-3=-3$

26) $a^2-6a+6=-2$

27) $7k^2+57k+13=5$

28) $7n^2-28n=0$

29) $6b^2=5+7b$

30) $9n^2+39n=-36$

31) $a^2+7a-9=-3+6a$

32) $x^2+10x+30=6$

33) $5n^2+41n+40=-2$

34) $24x^2+11x-80=3x$

CHAPTER REVIEW

KEY TERMS AND CONCEPTS	
Look for the following terms and concepts as you work through the workbook. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the text or in the media lesson.	
Factored form	
Greatest common factor (GCF)	