

Chapter 9. Momentum and Collisions

Level : AP Physics

Date :

9.1 Linear Momentum

The linear momentum of a particle of mass m moving with a velocity \mathbf{v} is defined as

$$\mathbf{p} \equiv m\mathbf{v} \text{ [kg}\cdot\text{m/s]}$$

9.3 Nonisolated System: Impulse and Momentum p.252

- If you apply an external force over a certain period of time on a moving point object(=particle), the momentum will change

$$mv_i + Ft = mv_f$$

- A greater force will produce more change in momentum, but *how long* the force acts on the particle also plays an important role in changing the momentum. That is, a '*force over an extended period*' produces greater change in momentum compared to '*brief force*'

$$\text{Force} \times \text{time} = \text{change in momentum}$$

$$\Rightarrow Ft = \Delta m v$$

$$\Rightarrow Ft = \Delta p$$

- $Ft = \text{Impulse}$, and the symbol is ' I ' or ' J '

$$\Rightarrow I = \Delta (m v) \text{ or } I = \Delta p$$

- If the force varies with time, then

$$\Rightarrow d\mathbf{p} = \mathbf{F}dt$$

$$\Rightarrow \mathbf{F} = \frac{d\mathbf{p}}{dt}$$

-We can integrate the above expression to find the change in momentum of a particle when the force acts over some time

$$\Rightarrow \Delta p = p_f - p_i = \int_{t_i}^{t_f} F dt$$

$$\mathbf{I} = \int_{t_i}^{t_f} \mathbf{F} dt = \Delta \mathbf{p}$$

The impulse of the force \mathbf{F} acting on a particle equals the change in momentum of the particle caused by that force

$$\mathbf{p} = m\mathbf{v} \quad \mathbf{F}t = \Delta m \mathbf{v} \quad \mathbf{I} = \int_{t_i}^{t_f} \mathbf{F} dt = \Delta \mathbf{p}$$

Practice questions

1. Jennifer is walking at 1.63m/s. If Jennifer weighs 583N, what is the magnitude of her momentum?

- a) 97kg·m/s b) 137kg·m/s c) 68.6kg·m/s d) 672kg·m/s

2. A 0.065kg tennis ball moving to the right with a speed of 15m/s is struck by a tennis racket, causing it to move to the left with a speed of 15m/s. If the ball remains in contact with the racket for 0.02s, what is the magnitude of the force experienced by the ball?

- a) 0N b) 98N c) 160N d) 1.6×10^5 N

3. A 1kg ball has a velocity of 12m/s downward just before it strikes the ground and bounces up with a velocity of 12m/s upward. What is the change in momentum of the ball?

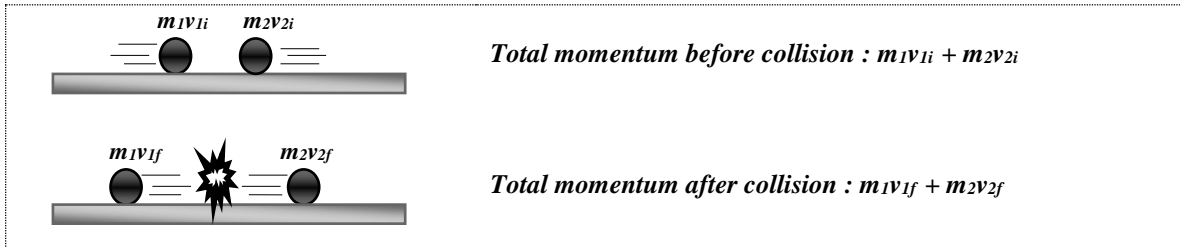
- a) 12kg·m/s, downward b) 12kg·m/s, upward c) 24 kg·m/s, downward d) 24kg·m/s, upward

4. When people jump off from a certain height, we naturally bend our knees when we land so that we increase the stopping time. Let's say Jack, who's mass is $m=60\text{kg}$, is playing on top of the kitchen table which is 1m above the floor. He jumps down and when he lands on the floor he bends his knees. If the stopping time is **0.8s**, then how much force did the floor exert on Jack? If he unwisely lands with his leg stiff, he would stop at **0.01s**. How much force will the floor exert on Jack in this case?

9.4 Isolated System: Collisions in One Dimension p.256

- If any quantity in physics that does not change, that quantity is conserved

**~In an isolated system, when two or more objects collide, the total momentum before the collision and after the collision is conserved if there is no external force acting on the system!!*



total momentum before collision = **total momentum** after collision

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

i) Elastic Collision(perfect bounce with no lost in energy)

For elastic head-on collision, both momentum and kinetic energy are conserved

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

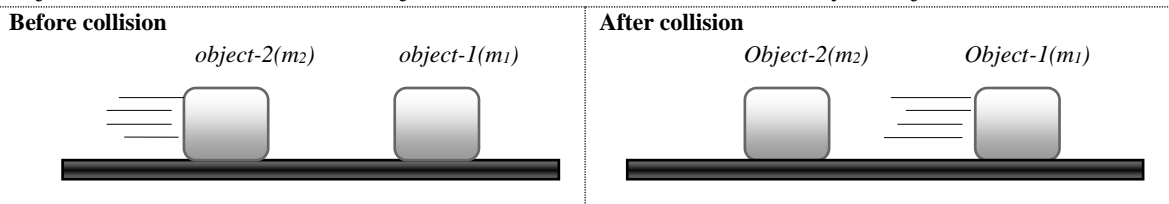
$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

Combining the above two equations can be useful for problem solving

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \text{ and } v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

Case1) Mass of object-2 is equal to object-1 ($m_1 = m_2$)

Object-1($m_1=2\text{kg}$) is standing still and object-2($m_2=2\text{kg}$) with velocity of 4m/s collided with object-1. After the collision, if object-2 stands still, what is the velocity of object-1?

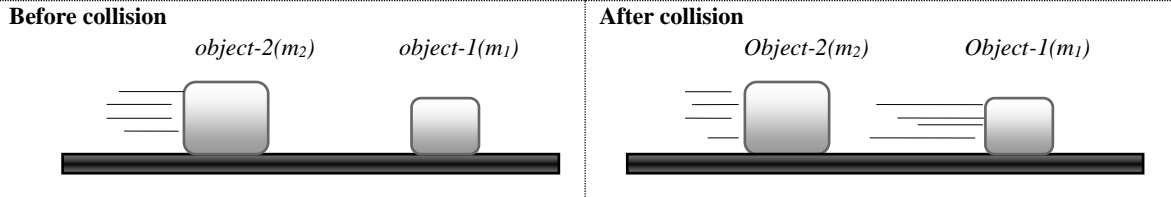


Ans) 4m/s

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

Case2) Mass of object-2 is greater than object-1 ($m_1 < m_2$)

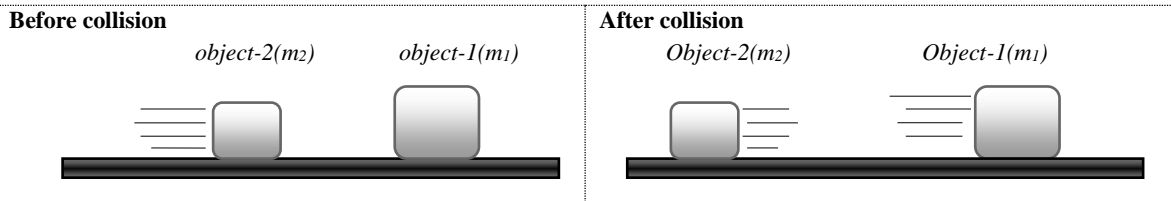
Object-1 ($m_1=2\text{kg}$) is standing still and object-2 ($m_2=4\text{kg}$) collided with object-1 with velocity of 6m/s . After the collision, if the velocity of object-2 is 2m/s , what is the velocity of object-1?



Ans) 8m/s

Case3) Mass of object-2 is less than object-1 ($m_1 > m_2$)

Object-1 ($m_1=4\text{kg}$) is standing still and object-2 ($m_2=2\text{kg}$) collided with object-1 with velocity of 4m/s . After the collision, if the velocity of object-1 is 3m/s , what is the velocity of object-2?



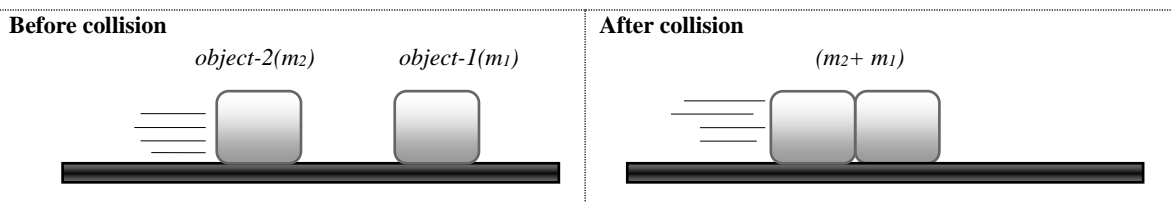
Ans) -2m/s

ii) Inelastic Collision (not a perfect bounce. ex; sticking together after collision)

Kinetic energy is *not* conserved due to energy lost to heat, but momentum is still conserved!

Case4)

Object-1 is standing still and object-2 collided with object-1 with velocity of 4m/s . If the mass of object-2 and object-1 are identical, and the two objects combine after the collision, what will be the velocity of the combined mass?



Ans) 2m/s

$$Ft = \Delta m v$$

$$I = \int_{t_i}^{t_f} F dt = \Delta p$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

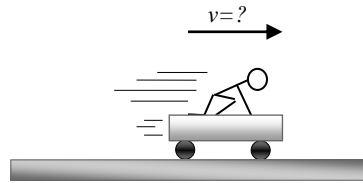
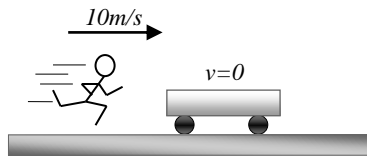
7. A boy of mass 80kg on a roller-skater is holding a ball of mass 2kg. The boy+ball is initially at rest. If the boy pushes the ball, producing a speed 10m/s, what is the *recoil* speed of the boy?

- a) 0.25m/s b) 0.5m/s c) 0.75m/s d) 1m/s

8. An 80kg astronaut carrying a 20kg tool kit initially drifting toward a stationary space shuttle at a speed of 2m/s. If she throws the tool kit toward the shuttle with a speed of 6m/s as seen from the shuttle, her final speed is

- a) 1m/s toward the shuttle b) 1m/s away from the shuttle
c) 2m/s toward the shuttle d) 4m/s toward the shuttle

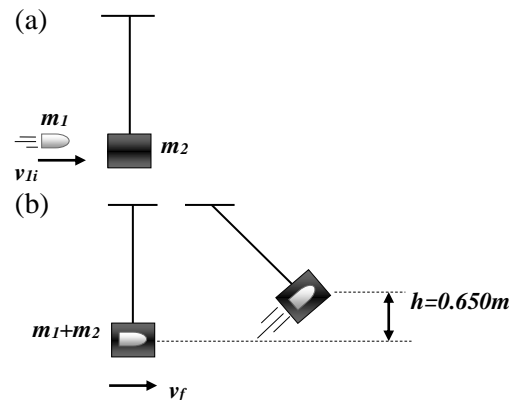
9. A 50kg boy runs at a speed of 10m/s and jumps onto a cart of mass 150kg causing it to move. If the cart is initially at rest, find the speed of the boy-cart.



Ballistic Pendulum *see p.261*

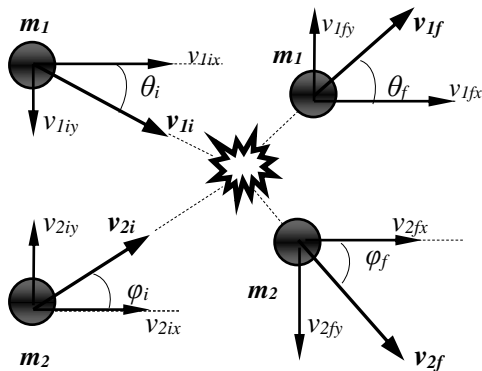
10. A ballistic pendulum is sometimes used in laboratories to measure the speed of a projectile, such as a bullet. The ballistic pendulum shown consists of a block of wood ($m_2=2.5\text{kg}$) suspended by a wire of negligible mass. A bullet of mass $m_1=0.01\text{kg}$ is fired with a speed v_{1i} . Just after the bullet collides with it, the block (with the bullet in it) has a speed of v_f and then swings to a maximum height of 0.65m above the initial position. Find the speed v_{1i} of the bullet, assuming that air resistance is negligible.

- a) 896m/s b) 754m/s c) 545m/s d) 435m/s



Solve Example 9.7 on p.262 'A Two-Body Collision with a Spring'
 Hint : Spring force is a conservative force, so kinetic energy is conserved.

9.5 Isolated System: Collisions in Two Dimensions



$$X\text{-component} : m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

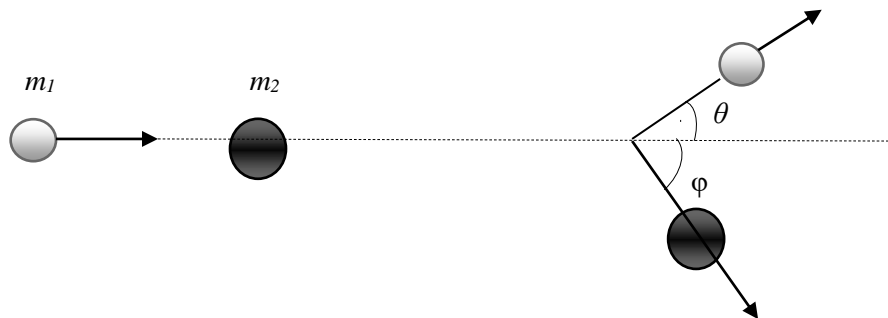
$$Y\text{-component} : m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

Since $v_x = v \cos \theta$, $v_y = v \sin \theta$ and knowing that $\sin(-\theta) = -\sin \theta$ and $\cos(-\theta) = \cos \theta$

$$X: m_1 v_{1i} \cos \theta_i + m_2 v_{2i} \cos \phi_i = m_1 v_{1f} \cos \theta_f + m_2 v_{2f} \cos \phi_f$$

$$Y: -m_1 v_{1i} \sin \theta_i + m_2 v_{2i} \sin \phi_i = m_1 v_{1f} \sin \theta_f - m_2 v_{2f} \sin \phi_f$$

Let us consider a simpler case in which object-1 of mass m_1 collides with object-2 of mass m_2 that is initially at rest. After the collision, object-1 moves at an angle of θ with respect to the horizontal and object-2 moves at an angle of ϕ with respect to the horizontal.



$$X : m_1 v_{1i} = m_1 v_{1f} \cos \theta + m_2 v_{2f} \cos \phi \quad \text{and} \quad Y : 0 = m_1 v_{1f} \sin \theta - m_2 v_{2f} \sin \phi$$

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \text{ (only if collision is elastic)}$$

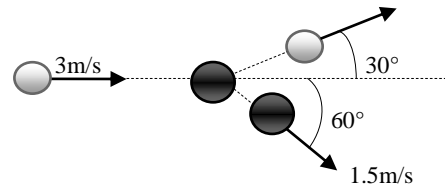
$$X\text{-component} : m_1v_{1ix} + m_2v_{2ix} = m_1v_{1fx} + m_2v_{2fx}$$

$$Y\text{-component} : m_1v_{1iy} + m_2v_{2iy} = m_1v_{1fy} + m_2v_{2fy}$$

$$v_x = v\cos\theta, v_y = v\sin\theta$$

Practice Questions Continue

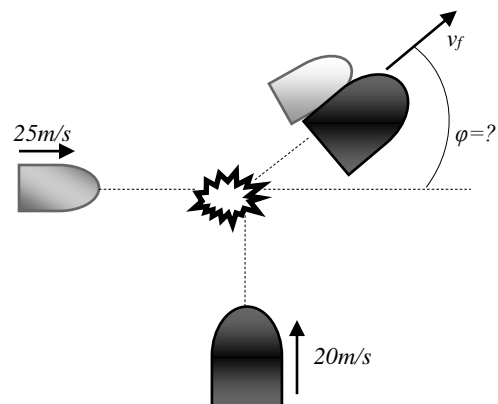
11. In the game of billiards, all the balls have approximately the same mass, about 0.17kg. In the figure, the cue ball strikes another ball at rest such that it follows the path shown. The other ball has a speed of 1.5m/s immediately after the collision. What is the speed of the cue ball after the collision?



- a) 2.1m/s b) 2.6m/s c) 3.3m/s d) 4.9m/s

12. A 1500kg car traveling east with a speed of 25m/s collides at an intersection with a 2500kg van traveling north at a speed of 20m/s. After collision, the two cars *stick* together for brief period of time. Find the direction and the speed of the 'wreckage' after collision.

- a) $\phi=53.1^\circ$, 15.6m/s b) $\phi=53.1^\circ$, 11.9m/s
 c) $\phi=28.4^\circ$, 15.6m/s d) $\phi=28.4^\circ$, 11.9m/s

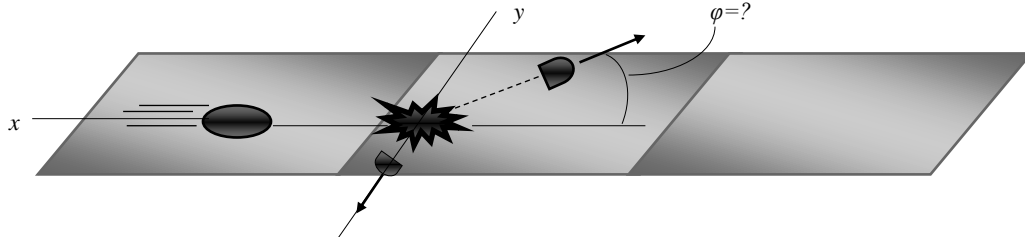


$$X\text{-component} : m_1v_{1ix} + m_2v_{2ix} = m_1v_{1fx} + m_2v_{2fx} \quad v_x = v\cos\theta, v_y = v\sin\theta$$

$$Y\text{-component} : m_1v_{1iy} + m_2v_{2iy} = m_1v_{1fy} + m_2v_{2fy}$$

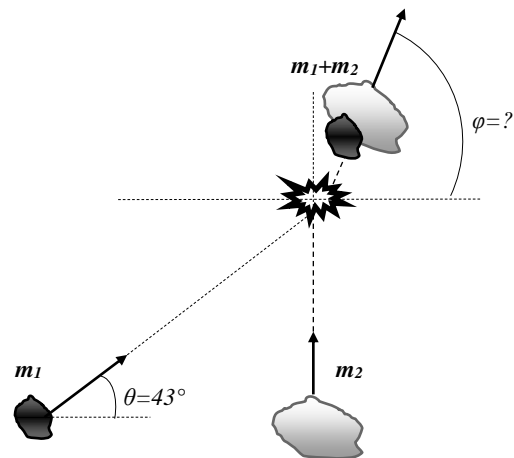
13. A 0.5kg bomb is sliding along an icy pond(frictionless) with a velocity of 2m/s to the right. The bomb explodes into two pieces. After the explosion, a 0.2kg piece moves south at 4m/s. What is the speed of the 0.3kg piece and what angle(ϕ) does it make in respect to the x-axis?

- a) 2.9m/s, $\phi=25^\circ$ b) 2.9m/s, $\phi=39^\circ$ c) 4.3m/s, $\phi=25^\circ$ d) 4.3m/s, $\phi=39^\circ$



14. Two asteroids are drifting in space with trajectories shown. The larger asteroid m_2 is ten times more massive than m_1 . The speed of the smaller asteroid before collision is 45m/s and for m_2 is 15m/s. After the collision they *stick* together briefly. At what angle do the combined asteroids deflect from the x-axis?

- a) 90° b) 80° c) 69° d) 42°

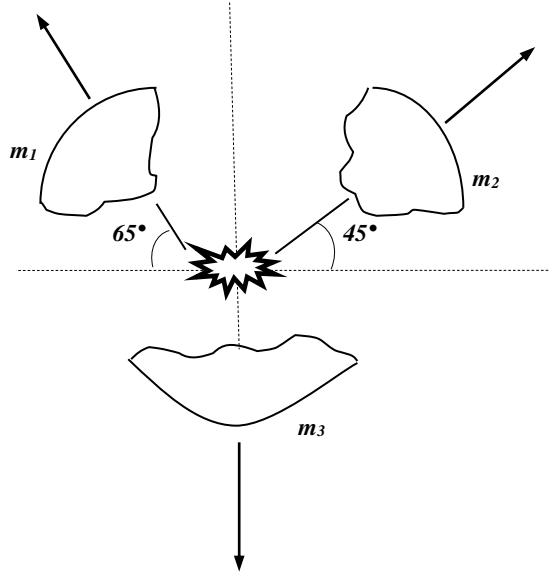


$$X\text{-component} : m_1v_{1ix} + m_2v_{2ix} = m_1v_{1fx} + m_2v_{2fx}$$

$$Y\text{-component} : m_1v_{1iy} + m_2v_{2iy} = m_1v_{1fy} + m_2v_{2fy}$$

$$v_x = v\cos\theta, v_y = v\sin\theta$$

15. A large plate is dropped and breaks into three pieces. The pieces fly apart parallel to the floor. If $m_3 = 1.3\text{kg}$, the speed of m_1 is 3m/s , the speed of m_2 is 1.79m/s and the speed of m_3 is 3.07m/s , then find the mass of m_1 and m_2 .
 a) 1kg b) 2kg c) 3kg d) 4kg

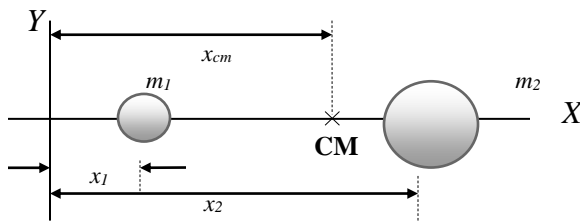


9.6 Center of Mass(p.267)

- Center of Mass(COM) of a system moves as if all the mass of the system were concentrated at that point.

Finding the Center of Mass in 1-dimension

- The COM of two particles is located on the x axis and lies somewhere between the particles.



$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

- The COM for a system of many particles in the x -coordinate

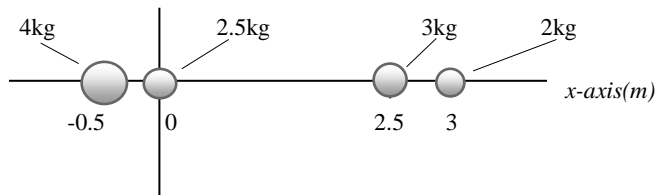
$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \dots + m_nx_n}{m_1 + m_2 + m_3 + \dots + m_n} = \frac{\sum m_i x_i}{\sum m_i}$$

$$x_{cm} = \frac{m_1x_1+m_2x_2+m_3x_3+\dots+m_nx_n}{m_1+m_2+m_3+\dots+m_n} = \frac{\sum m_ix_i}{\sum m_i}$$

16. Find the COM of $m_1=2\text{kg}$, $m_2=4\text{kg}$ where $x_1=2\text{m}$, $x_2=6\text{m}$.

Ans) 4.7m

17. Four objects are situated along the x -axis. A 2kg object is at $+3\text{m}$, a 3kg is at $+2.5\text{m}$, a 2.5kg object is at the origin, and a 4kg object is at -0.5m . Where is the center of mass of these objects?



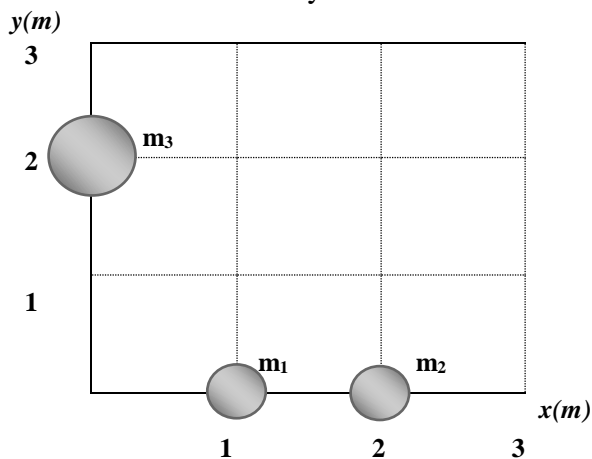
Ans) 1m

Finding the Center of Mass in 2-dimension

- The COM for a system of many particles in the xy -plane

$$x_{cm} = \frac{m_1x_1+m_2x_2+m_3x_3+\dots+m_nx_n}{m_1+m_2+m_3+\dots+m_n}, \quad y_{cm} = \frac{m_1y_1+m_2y_2+m_3y_3+\dots+m_ny_n}{m_1+m_2+m_3+\dots+m_n}$$

Example 9.10 p.269 A system consists of three particles(*point masses where the dimension is irrelevant*) located as shown. The mass of $m_1=m_2=1\text{kg}$ and $m_3=2\text{kg}$. Find the center of mass of the system



Ans) $(0.75, 1)\text{m}$

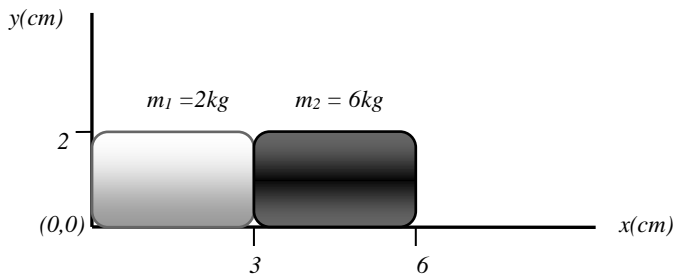
- The COM for a system of many particles in **3-dimension**

$$\mathbf{r}_{cm} = \frac{\sum m_i \mathbf{r}_i}{\sum m_i}$$

where $\mathbf{r}_{cm} = x_{cm}\mathbf{i} + y_{cm}\mathbf{j} + z_{cm}\mathbf{k}$, and $\sum m_i$ is the total mass 'M'

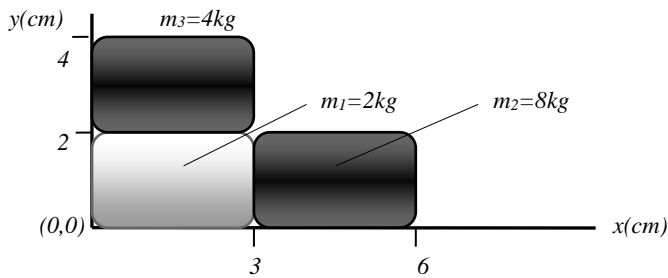
$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \dots + m_nx_n}{m_1 + m_2 + m_3 + \dots + m_n}, \quad y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \dots + m_ny_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

18. Find the center of mass(COM) for the blocks below. All the blocks' mass is evenly distributed, so the COM for individual blocks is at the center. Mark the COM as × below



Answer : COM = (,)cm

19. Find the center of mass(COM) for the blocks below. All the blocks' mass is evenly distributed, so the COM for individual blocks is at the center. Mark the COM as × below



Answer : COM = (,)cm

- If we let the number of elements 'n' to approach infinity, then the COM for **solid object** will be

$$\mathbf{r}_{\text{cm}} = \frac{1}{M} \int \mathbf{r} \, dm$$

- Work on Example 9.11) on p.270 'Center of Mass of a Rod'

9.7 System of Many Particles(p.272)

The velocity of the COM of a system

$$\mathbf{v}_{\text{cm}} = \frac{d\mathbf{r}_{\text{cm}}}{dt} = \frac{1}{M} \sum m_i \frac{d\mathbf{r}_i}{dt} = \frac{\sum m_i \mathbf{v}_i}{M}$$

$$\Rightarrow M\mathbf{v}_{\text{cm}} = \sum m_i \mathbf{v}_i = \sum \mathbf{p}_i = \mathbf{p}_{\text{tot}}$$

The total linear momentum of the system equals the total mass multiplied by the velocity of the COM

=

The total linear momentum of the system is equal to that of a single particle of mass M moving with a velocity v_{cm} .

$$\Sigma \mathbf{F}_{\text{ext}} = M\mathbf{a}_{\text{cm}} = \frac{d\mathbf{p}_{\text{tot}}}{dt}$$

The center of mass of a system of particles of combined mass M moves like an equivalent particle of mass M would move under the influence of the resultant external force on the system

If the resultant external force is zero, then $\Sigma \mathbf{F}_{\text{ext}} = 0$, so $\frac{d\mathbf{p}_{\text{tot}}}{dt} = 0$

$$\Rightarrow \mathbf{p}_{\text{tot}} = M\mathbf{v}_{\text{CM}} \text{ (when } \Sigma \mathbf{F}_{\text{ext}} = 0\text{)}$$

~Continue to the next page~

On p.287 solve questions #51, #52, #55(*Kinetic energy is conserved!*)

Answer for 52. (b) $\mathbf{r}_{CM}=(-2\mathbf{i}-\mathbf{j})m$ (c) $\mathbf{v}_{CM}=(3\mathbf{i}-\mathbf{j})m/s$ (d) $\mathbf{P}=(15\mathbf{i}-5\mathbf{j})kg.m/s$