

CHAPTER 9

PHASE DIAGRAMS

PROBLEM SOLUTIONS

9.17 A 90 wt% Ag-10 wt% Cu alloy is heated to a temperature within the β + liquid phase region. If the composition of the liquid phase is 85 wt% Ag, determine:

- (a) The temperature of the alloy
- (b) The composition of the β phase
- (c) The mass fractions of both phases

Solution

(a) In order to determine the temperature of a 90 wt% Ag-10 wt% Cu alloy for which β and liquid phases are present with the liquid phase of composition 85 wt% Ag, we need to construct a tie line across the β + L phase region of Figure 9.7 that intersects the liquidus line at 85 wt% Ag; this is possible at about 850°C.

(b) The composition of the β phase at this temperature is determined from the intersection of this same tie line with solidus line, which corresponds to about 95 wt% Ag.

(c) The mass fractions of the two phases are determined using the lever rule, Equations 9.1 and 9.2 with $C_0 = 90$ wt% Ag, $C_L = 85$ wt% Ag, and $C_\beta = 95$ wt% Ag, as

$$W_\beta = \frac{C_0 - C_L}{C_\beta - C_L} = \frac{90 - 85}{95 - 85} = 0.50$$

$$W_L = \frac{C_\beta - C_0}{C_\beta - C_L} = \frac{95 - 90}{95 - 85} = 0.50$$

9.32 For a copper-silver alloy of composition 25 wt% Ag-75 wt% Cu and at 775 °C (1425 °F) do the following:

- (a) Determine the mass fractions of α and β phases.
- (b) Determine the mass fractions of primary α and eutectic microconstituents.
- (c) Determine the mass fraction of eutectic α .

Solution

(a) This portion of the problem asks that we determine the mass fractions of α and β phases for an 25 wt% Ag-75 wt% Cu alloy (at 775°C). In order to do this it is necessary to employ the lever rule using a tie line that extends entirely across the $\alpha + \beta$ phase field. From Figure 9.7 and at 775°C, $C_\alpha = 8.0$ wt% Ag, $C_\beta = 91.2$ wt% Ag, and $C_{\text{eutectic}} = 71.9$ wt% Sn. Therefore, the two lever-rule expressions are as follows:

$$W_\alpha = \frac{C_\beta - C_0}{C_\beta - C_\alpha} = \frac{91.2 - 25}{91.2 - 8.0} = 0.796$$

$$W_\beta = \frac{C_0 - C_\alpha}{C_\beta - C_\alpha} = \frac{25 - 8.0}{91.2 - 8.0} = 0.204$$

(b) Now it is necessary to determine the mass fractions of primary α and eutectic microconstituents for this same alloy. This requires us to utilize the lever rule and a tie line that extends from the maximum solubility of Ag in the α phase at 775°C (i.e., 8.0 wt% Ag) to the eutectic composition (71.9 wt% Ag). Thus

$$W_{\alpha'} = \frac{C_{\text{eutectic}} - C_0}{C_{\text{eutectic}} - C_\alpha} = \frac{71.9 - 25}{71.9 - 8.0} = 0.734$$

$$W_e = \frac{C_0 - C_\alpha}{C_{\text{eutectic}} - C_\alpha} = \frac{25 - 8.0}{71.9 - 8.0} = 0.266$$

(c) And, finally, we are asked to compute the mass fraction of eutectic α , $W_{e\alpha}$. This quantity is simply the difference between the mass fractions of total α and primary α as

$$W_{e\alpha} = W_\alpha - W_{\alpha'} = 0.796 - 0.734 = 0.062$$

9.34 Consider the hypothetical eutectic phase diagram for metals A and B, which is similar to that for the lead-tin system, Figure 9.8. Assume that (1) α and β phases exist at the A and B extremities of the phase diagram, respectively; (2) the eutectic composition is 47 wt% B-53 wt% A; and (3) the composition of the β phase at the eutectic temperature is 92.6 wt% B-7.4 wt% A. Determine the composition of an alloy that will yield primary α and total α mass fractions of 0.356 and 0.693, respectively.

Solution

We are given a hypothetical eutectic phase diagram for which $C_{\text{eutectic}} = 47$ wt% B, $C_{\beta} = 92.6$ wt% B at the eutectic temperature, and also that $W_{\alpha'} = 0.356$ and $W_{\alpha} = 0.693$; from this we are asked to determine the composition of the alloy. Let us write lever rule expressions for $W_{\alpha'}$ and W_{α}

$$W_{\alpha} = \frac{C_{\beta} - C_0}{C_{\beta} - C_{\alpha}} = \frac{92.6 - C_0}{92.6 - C_{\alpha}} = 0.693$$

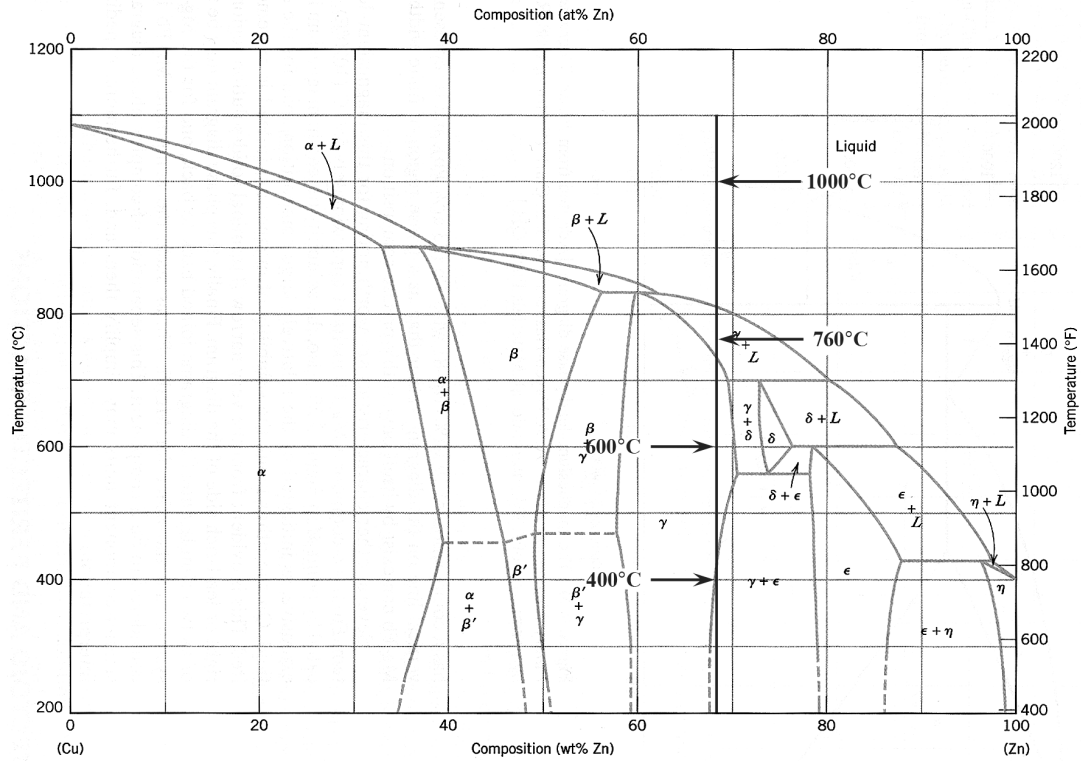
$$W_{\alpha'} = \frac{C_{\text{eutectic}} - C_0}{C_{\text{eutectic}} - C_{\alpha}} = \frac{47 - C_0}{47 - C_{\alpha}} = 0.356$$

Thus, we have two simultaneous equations with C_0 and C_{α} as unknowns. Solving them for C_0 gives $C_0 = 32.6$ wt% B.

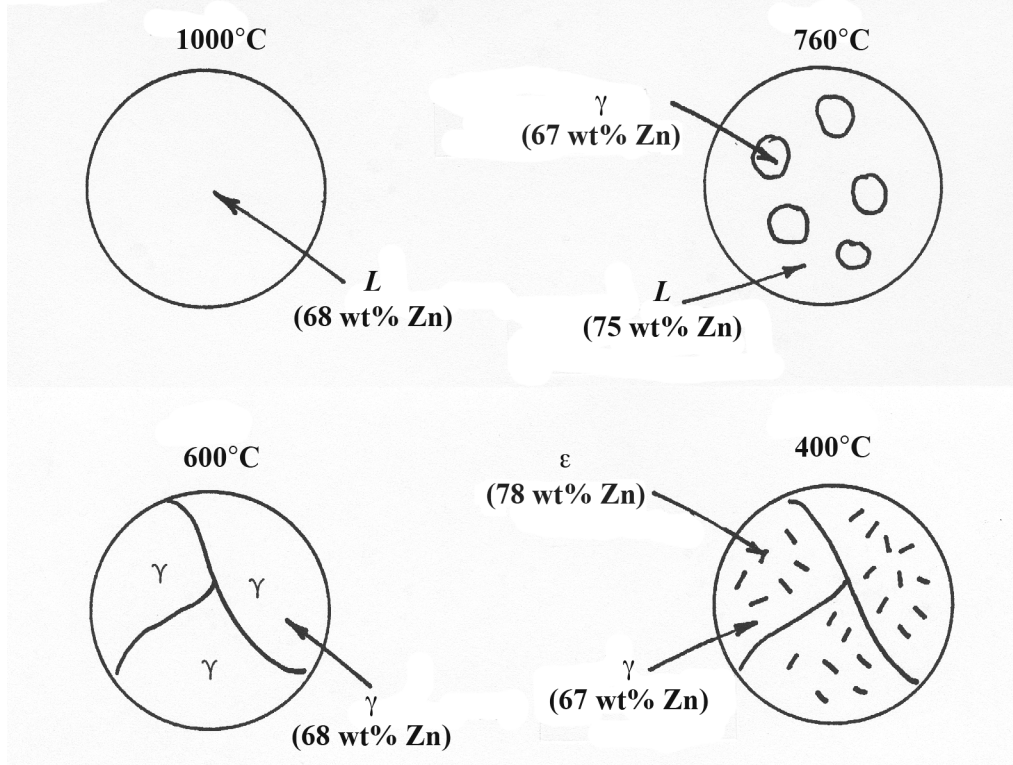
9.36 For a 68 wt% Zn-32 wt% Cu alloy, make schematic sketches of the microstructure that would be observed for conditions of very slow cooling at the following temperatures: 1000 °C (1830 °F), 760 °C (1400 °F), 600 °C (1110 °F), and 400 °C (750 °F). Label all phases and indicate their approximate compositions.

Solution

The illustration below is the Cu-Zn phase diagram (Figure 9.19). A vertical line at a composition of 68 wt% Zn-32 wt% Cu has been drawn, and, in addition, horizontal arrows at the four temperatures called for in the problem statement (i.e., 1000°C, 760°C, 600°C, and 400°C).



On the basis of the locations of the four temperature-composition points, schematic sketches of the four respective microstructures along with phase compositions are represented as follows:



9.50 Consider 1.0 kg of austenite containing 1.15 wt% C, cooled to below 727 °C (1341 °F).

- What is the proeutectoid phase?
- How many kilograms each of total ferrite and cementite form?
- How many kilograms each of pearlite and the proeutectoid phase form?
- Schematically sketch and label the resulting microstructure.

Solution

(a) The proeutectoid phase will be Fe_3C since 1.15 wt% C is greater than the eutectoid composition (0.76 wt% C).

(b) For this portion of the problem, we are asked to determine how much total ferrite and cementite form. Application of the appropriate lever rule expression yields

$$W_{\alpha} = \frac{C_{\text{Fe}_3\text{C}} - C_0}{C_{\text{Fe}_3\text{C}} - C_{\alpha}} = \frac{6.70 - 1.15}{6.70 - 0.022} = 0.83$$

which, when multiplied by the total mass of the alloy (1.0 kg), gives 0.83 kg of total ferrite.

Similarly, for total cementite,

$$W_{\text{Fe}_3\text{C}} = \frac{C_0 - C_\alpha}{C_{\text{Fe}_3\text{C}} - C_\alpha} = \frac{1.15 - 0.022}{6.70 - 0.022} = 0.17$$

And the mass of total cementite that forms is $(0.17)(1.0 \text{ kg}) = 0.17 \text{ kg}$.

(c) Now we are asked to calculate how much pearlite and the proeutectoid phase (cementite) form. Applying Equation 9.22, in which $C_1' = 1.15 \text{ wt\% C}$

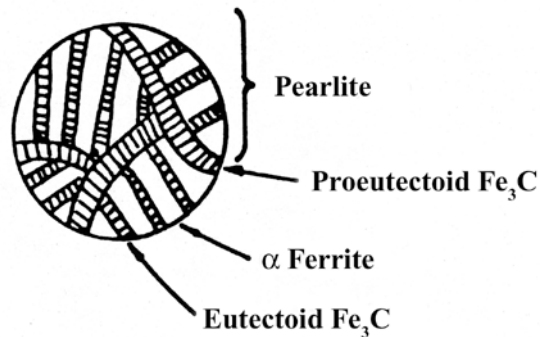
$$W_p = \frac{6.70 - C_1'}{6.70 - 0.76} = \frac{6.70 - 1.15}{6.70 - 0.76} = 0.93$$

which corresponds to a mass of 0.93 kg. Likewise, from Equation 9.23

$$W_{\text{Fe}_3\text{C}'} = \frac{C_1' - 0.76}{5.94} = \frac{1.15 - 0.76}{5.94} = 0.07$$

which is equivalent to 0.07 kg of the total 1.0 kg mass.

(d) Schematically, the microstructure would appear as:



9.53 The microstructure of an iron–carbon alloy consists of proeutectoid ferrite and pearlite; the mass fractions of these two microconstituents are 0.286 and 0.714, respectively. Determine the concentration of carbon in this alloy.

Solution

This problem asks that we determine the carbon concentration in an iron-carbon alloy, given the mass fractions of proeutectoid ferrite and pearlite. From Equation 9.20

$$W_p = 0.714 = \frac{C_0' - 0.022}{0.74}$$

which yields $C_0' = 0.55$ wt% C.

9.54 The mass fractions of total ferrite and total cementite in an iron-carbon alloy are 0.88 and 0.12, respectively. Is this a hypoeutectoid or hypereutectoid alloy? Why?

Solution

In this problem we are given values of W_α and $W_{\text{Fe}_3\text{C}}$ for an iron-carbon alloy (0.88 and 0.12, respectively), and then are asked to specify whether the alloy is hypoeutectoid or hypereutectoid. Employment of the lever rule for total α leads to

$$W_\alpha = 0.88 = \frac{C_{\text{Fe}_3\text{C}} - C_0}{C_{\text{Fe}_3\text{C}} - C_\alpha} = \frac{6.70 - C_0}{6.70 - 0.022}$$

Now, solving for C_0 , the alloy composition, leads to $C_0 = 0.82$ wt% C. Therefore, the alloy is *hypereutectoid* since C_0 is greater than 0.76 wt% C.