GLENCOE MATHEMATICS

Algebra 2

Chapter 9 Resource Masters

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Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks.

Study Guide and Intervention Workbook	0-07-828029-X
Skills Practice Workbook	0-07-828023-0
Practice Workbook	0-07-828024-9

ANSWERS FOR WORKBOOKS The answers for Chapter 9 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

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Algebra 2
Chapter 9 Resource Masters

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Contents

Teacher's Guide to Using the Chapter 9 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 9 Resource Masters* includes the core materials needed for Chapter 9. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Algebra 2 TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 9-1. Encourage them to add these pages to their Algebra 2 Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Study Guide and Intervention
Each lesson in *Algebra 2* addresses two objectives. There is one Study Guide and Intervention master for each objective.

WHEN TO USE Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

WHEN TO USE These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics
One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 9 Resource Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

  All of the above tests include a free-response Bonus question.
- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Algebra 2. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of algebra concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and quantitative-comparison questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 518–519. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

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9 Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 9. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
asymptote A·suhm(p)·TOHT		
complex fraction		
constant of variation		
continuity KAHN·tuhn·OO·uh·tee		
direct variation		
inverse variation IHN·VUHRS		
joint variation		

(continued on the next page)

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Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
point discontinuity		
rational equation		
rational expression		
rational function		
rational inequality		

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9-1 Study Guide and Intervention

Multiplying and Dividing Rational Expressions

Simplify Rational Expressions A ratio of two polynomial expressions is a **rational expression**. To simplify a rational expression, divide both the numerator and the denominator by their greatest common factor (GCF).

Multiplying Rational Expressions	For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$, if $b \neq 0$ and $d \neq 0$.
Dividing Rational Expressions	For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$, if $b \neq 0$, $c \neq 0$, and $d \neq 0$.

Lesson 9-1

Example **Simplify each expression.**

a. $\frac{24a^5b^2}{(2ab)^4}$

$$\frac{24a^5b^2}{(2ab)^4} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 3 \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{a}} \cdot a \cdot \overset{1}{\cancel{b}} \cdot \overset{1}{\cancel{b}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 2 \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{b}} \cdot \underset{1}{\cancel{b}} \cdot b \cdot b} = \frac{3a}{2b^2}$$

b. $\frac{3r^2s^3}{5t^4} \cdot \frac{20t^2}{9r^3s}$

$$\frac{3r^2s^3}{5t^4} \cdot \frac{20t^2}{9r^3s} = \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{r}} \cdot \overset{1}{\cancel{r}} \cdot \overset{1}{\cancel{s}} \cdot s \cdot s \cdot 2 \cdot 2 \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{t}} \cdot \overset{1}{\cancel{t}}}{\underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{t}} \cdot \underset{1}{\cancel{t}} \cdot t \cdot t \cdot \underset{1}{\cancel{3}} \cdot 3 \cdot \underset{1}{\cancel{r}} \cdot \underset{1}{\cancel{r}} \cdot r \cdot \underset{1}{\cancel{s}}} = \frac{2 \cdot 2 \cdot s \cdot s}{3 \cdot r \cdot t \cdot t} = \frac{4s^2}{3rt^2}$$

c. $\frac{x^2 + 8x + 16}{2x - 2} \div \frac{x^2 + 2x - 8}{x - 1}$

$$\frac{x^2 + 8x + 16}{2x - 2} \div \frac{x^2 + 2x - 8}{x - 1} = \frac{x^2 + 8x + 16}{2x - 2} \cdot \frac{x - 1}{x^2 + 2x - 8}$$

$$= \frac{\overset{1}{\cancel{(x + 4)}}(x + 4)\overset{1}{\cancel{(x - 1)}}}{2\underset{1}{\cancel{(x - 1)}}(x - 2)\underset{1}{\cancel{(x + 4)}}} = \frac{x + 4}{2(x - 2)}$$

Exercises

Simplify each expression.

1. $\frac{(-2ab^2)^3}{20ab^4}$

2. $\frac{4x^2 - 12x + 9}{9 - 6x}$

3. $\frac{x^2 + x - 6}{x^2 - 6x - 27}$

4. $\frac{3m^3 - 3m}{6m^4} \cdot \frac{4m^5}{m + 1}$

5. $\frac{c^2 - 3c}{c^2 - 25} \cdot \frac{c^2 + 4c - 5}{c^2 - 4c + 3}$

6. $\frac{(m - 3)^2}{m^2 - 6m + 9} \cdot \frac{m^3 - 9m}{m^2 - 9}$

7. $\frac{6xy^4}{25z^3} \div \frac{18xz^2}{5y}$

8. $\frac{16p^2 - 8p + 1}{14p^4} \div \frac{4p^2 + 7p - 2}{7p^5}$

9. $\frac{2m - 1}{m^2 - 3m - 10} \div \frac{4m^2 - 1}{4m + 8}$

9-1 Study Guide and Intervention *(continued)*

Multiplying and Dividing Rational Expressions

Simplify Complex Fractions A **complex fraction** is a rational expression whose numerator and/or denominator contains a rational expression. To simplify a complex fraction, first rewrite it as a division problem.

Example **Simplify** $\dfrac{\frac{3s-1}{s}}{\frac{3s^2+8s-3}{s^4}}$.

$\dfrac{\frac{3s-1}{s}}{\frac{3s^2+8s-3}{s^4}} = \dfrac{3s-1}{s} \div \dfrac{3s^2+8s-3}{s^4}$ Express as a division problem.

$= \dfrac{3s-1}{s} \cdot \dfrac{s^4}{3s^2+8s-3}$ Multiply by the reciprocal of the divisor.

$= \dfrac{\overset{1}{\cancel{(3s-1)}}\overset{s^3}{\cancel{s^4}}}{\underset{1}{\cancel{s}}\underset{1}{\cancel{(3s-1)}}(s+3)}$ Factor.

$= \dfrac{s^3}{s+3}$ Simplify.

Exercises

Simplify.

1. $\dfrac{\frac{x^3y^2z}{a^2b^2}}{\frac{a^3x^2y}{b^2}}$

2. $\dfrac{\frac{a^2bc^3}{x^2y^2}}{\frac{ab^2}{c^4x^2y}}$

3. $\dfrac{\frac{b^2-1}{3b+2}}{\frac{b+1}{3b^2-b-2}}$

4. $\dfrac{\frac{b^2-100}{b^2}}{\frac{3b^2-31b+10}{2b}}$

5. $\dfrac{\frac{x-4}{x^2+6x+9}}{\frac{x^2-2x-8}{3+x}}$

6. $\dfrac{\frac{a^2-16}{a+2}}{\frac{a^2+3a-4}{a^2+a-2}}$

7. $\dfrac{\frac{2x^2+9x+9}{x+1}}{\frac{10x^2+19x+6}{5x^2+7x+2}}$

8. $\dfrac{\frac{b+2}{b^2-6b+8}}{\frac{b^2+b-2}{b^2-16}}$

9. $\dfrac{\frac{x^2-x-2}{x^3+6x^2-x-30}}{\frac{x+1}{x+3}}$

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9-1 Skills Practice

Multiplying and Dividing Rational Expressions

Lesson 9-1

Simplify each expression.

1. $\frac{21x^3y}{14x^2y^2}$

2. $\frac{5ab^3}{25a^2b^2}$

3. $\frac{(x^6)^3}{(x^3)^4}$

4. $\frac{8y^2(y^6)^3}{4y^{24}}$

5. $\frac{18}{2x - 6}$

6. $\frac{x^2 - 4}{(x - 2)(x + 1)}$

7. $\frac{3a^2 - 24a}{3a^2 + 12a}$

8. $\frac{3m}{2n} \cdot \frac{n^3}{6}$

9. $\frac{24e^3}{5f^2} \cdot \frac{10(ef)^3}{8e^5f}$

10. $\frac{5s^2}{s^2 - 4} \cdot \frac{s + 2}{10s^5}$

11. $\frac{7g}{y^2} \div 21g^3$

12. $\frac{80y^4}{49z^5v^7} \div \frac{25y^5}{14z^{12}v^5}$

13. $\frac{3x^2}{x + 2} \div \frac{3x}{x^2 - 4}$

14. $\frac{q^2 + 2q}{6q} \div \frac{q^2 - 4}{3q^2}$

15. $\frac{w^2 - 5w - 24}{w + 1} \cdot \frac{w^2 - 6w - 7}{w + 3}$

16. $\frac{t^2 + 19t + 84}{4t - 4} \cdot \frac{2t - 2}{t^2 + 9t + 14}$

17. $\frac{x^2 - 5x + 4}{2x - 8} \div (3x^2 - 3x)$

18. $\frac{16a^2 + 40a + 25}{3a^2 - 10a - 8} \div \frac{4a + 5}{a^2 - 8a + 16}$

19. $\frac{\frac{c^2}{2d^2}}{-\frac{c^6}{5d}}$

20. $\frac{\frac{a^2 - b^2}{4a}}{\frac{a + b}{2a}}$

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9-1 Practice

Multiplying and Dividing Rational Expressions

Simplify each expression.

1. $\frac{9a^2b^3}{27a^4b^4c}$

2. $\frac{(2m^3n^2)^3}{-18m^5n^4}$

3. $\frac{10y^2 + 15y}{35y^2 - 5y}$

4. $\frac{2k^2 - k - 15}{k^2 - 9}$

5. $\frac{25 - v^2}{3v^2 - 13v - 10}$

6. $\frac{x^4 + x^3 - 2x^2}{x^4 - x^3}$

7. $\frac{-2u^3y}{15xz^5} \cdot \frac{25x^3}{14u^2y^2}$

8. $\frac{a + y}{6} \cdot \frac{4}{y + a}$

9. $\frac{n^5}{n - 6} \cdot \frac{n^2 - 6n}{n^8}$

10. $\frac{a - y}{w + n} \cdot \frac{w^2 - n^2}{y - a}$

11. $\frac{x^2 - 5x - 24}{6x + 2x^2} \cdot \frac{5x^2}{8 - x}$

12. $\frac{x - 5}{10x - 2} \cdot \frac{25x^2 - 1}{x^2 - 10x + 25}$

13. $\frac{a^5y^3}{wy^7} \div \frac{a^3w^2}{w^5y^2}$

14. $\left(\frac{2xy}{w^2}\right)^3 \div \frac{24x^2}{w^5}$

15. $\frac{x + y}{6} \div \frac{x^2 - y^2}{3}$

16. $\frac{3x + 6}{x^2 - 9} \div \frac{6x^2 + 12x}{4x + 12}$

17. $\frac{2s^2 - 7s - 15}{(s + 4)^2} \div \frac{s^2 - 10s + 25}{s + 4}$

18. $\frac{9 - a^2}{a^2 + 5a + 6} \div \frac{2a - 6}{5a + 10}$

19. $\frac{\frac{2x + 1}{x}}{\frac{4 - x}{x}}$

20. $\frac{\frac{x^2 - 9}{4}}{\frac{3 - x}{8}}$

21. $\frac{\frac{x^3 + 2^3}{x^2 - 2x}}{\frac{(x + 2)^3}{x^2 + 4x + 4}}$

22. GEOMETRY A right triangle with an area of $x^2 - 4$ square units has a leg that measures $2x + 4$ units. Determine the length of the other leg of the triangle.

23. GEOMETRY A rectangular pyramid has a base area of $\frac{x^2 + 3x - 10}{2x}$ square centimeters and a height of $\frac{x^2 - 3x}{x^2 - 5x + 6}$ centimeters. Write a rational expression to describe the volume of the rectangular pyramid.

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9-1 Reading to Learn Mathematics

Multiplying and Dividing Rational Expressions

Pre-Activity **How are rational expressions used in mixtures?**

Read the introduction to Lesson 9-1 at the top of page 472 in your textbook.

- Suppose that the Goodie Shoppe also sells a candy mixture of chocolate mints and caramels. If this mixture is made with 4 pounds of chocolate mints and 3 pounds of caramels, then ____________ of the mixture is mints and ____________ of the mixture is caramels.
- If the store manager adds another y pounds of mints to the mixture, what fraction of the mixture will be mints?

Lesson 9-1

Reading the Lesson

1. a. In order to simplify a rational number or rational expression, ________________ the numerator and ________________ and divide both of them by their ______________________________.

b. A rational expression is undefined when its ________________ is equal to __________. To find the values that make the expression undefined, completely ________________ the original ________________ and set each factor equal to __________.

2. a. To multiply two rational expressions, ________________ the ________________ and multiply the denominators.

b. To divide two rational expressions, ________________ by the ________________ of the ________________.

3. a. Which of the following expressions are complex fractions?

i. $\frac{7}{12}$ **ii.** $\frac{\frac{3}{8}}{\frac{5}{16}}$ **iii.** $\frac{r+5}{r-5}$ **iv.** $\frac{\frac{z+1}{z}}{z}$ **v.** $\frac{\frac{r^2-25}{9}}{\frac{r+5}{3}}$

b. Does a complex fraction express a multiplication or division problem? How is multiplication used in simplifying a complex fraction?

Helping You Remember

4. One way to remember something new is to see how it is similar to something you already know. How can your knowledge of division of fractions in arithmetic help you to understand how to divide rational expressions?

9-1 Enrichment

Reading Algebra

In mathematics, the term *group* has a special meaning. The following numbered sentences discuss the idea of group and one interesting example of a group.

01 To be a group, a set of elements and a binary operation must satisfy four conditions: the set must be closed under the operation, the operation must be associative, there must be an identity element, and every element must have an inverse.

02 The following six functions form a group under the operation of composition of functions: $f_1(x) = x$, $f_2(x) = \frac{1}{x}$, $f_3(x) = 1 - x$, $f_4(x) = \frac{(x-1)}{x}$, $f_5(x) = \frac{x}{(x-1)}$, and $f_6(x) = \frac{1}{(1-x)}$.

03 This group is an example of a noncommutative group. For example, $f_3 \circ f_2 = f_4$, but $f_2 \circ f_3 = f_6$.

04 Some experimentation with this group will show that the identity element is f_1.

05 Every element is its own inverse except for f_4 and f_6, each of which is the inverse of the other.

Use the paragraph to answer these questions.

1. Explain what it means to say that a set is *closed* under an operation. Is the set of positive integers closed under subtraction?

2. Subtraction is a noncommutative operation for the set of integers. Write an informal definition of noncommutative.

3. For the set of integers, what is the identity element for the operation of multiplication? Justify your answer.

4. Explain how the following statement relates to sentence 05:

$$(f_6 \cdot f_4)(x) = f_6[f_4(x)] = f_6\left(\frac{1}{(1-x)}\right) = \frac{1}{\frac{1-(x-1)}{x}} = x = f_1(x).$$

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9-2 Study Guide and Intervention

Adding and Subtracting Rational Expressions

LCM of Polynomials To find the least common multiple of two or more polynomials, factor each expression. The LCM contains each factor the greatest number of times it appears as a factor.

Example 1 **Find the LCM of $16p^2q^3r$, $40pq^4r^2$, and $15p^3r^4$.**

$16p^2q^3r = 2^4 \cdot p^2 \cdot q^3 \cdot r$

$40pq^4r^2 = 2^3 \cdot 5 \cdot p \cdot q^4 \cdot r^2$

$15p^3r^4 = 3 \cdot 5 \cdot p^3 \cdot r^4$

$\text{LCM} = 2^4 \cdot 3 \cdot 5 \cdot p^3 \cdot q^4 \cdot r^4$

$= 240p^3q^4r^4$

Example 2 **Find the LCM of $3m^2 - 3m - 6$ and $4m^2 + 12m - 40$.**

$3m^2 - 3m - 6 = 3(m + 1)(m - 2)$

$4m^2 + 12m - 40 = 4(m - 2)(m + 5)$

$\text{LCM} = 12(m + 1)(m - 2)(m + 5)$

Exercises

Find the LCM of each set of polynomials.

1. $14ab^2, 42bc^3, 18a^2c$

2. $8cdf^3, 28c^2f, 35d^4f^2$

3. $65x^4y, 10x^2y^2, 26y^4$

4. $11mn^5, 18m^2n^3, 20mn^4$

5. $15a^4b, 50a^2b^2, 40b^8$

6. $24p^7q, 30p^2q^2, 45pq^3$

7. $39b^2c^2, 52b^4c, 12c^3$

8. $12xy^4, 42x^2y, 30x^2y^3$

9. $56stv^2, 24s^2v^2, 70t^3v^3$

10. $x^2 + 3x, 10x^2 + 25x - 15$

11. $9x^2 - 12x + 4, 3x^2 + 10x - 8$

12. $22x^2 + 66x - 220, 4x^2 - 16$

13. $8x^2 - 36x - 20, 2x^2 + 2x - 60$

14. $5x^2 - 125, 5x^2 + 24x - 5$

15. $3x^2 - 18x + 27, 2x^3 - 4x^2 - 6x$

16. $45x^2 - 6x - 3, 45x^2 - 5$

17. $x^3 + 4x^2 - x - 4, x^2 + 2x - 3$

18. $54x^3 - 24x, 12x^2 - 26x + 12$

9-2 Study Guide and Intervention *(continued)*

Adding and Subtracting Rational Expressions

Add and Subtract Rational Expressions To add or subtract rational expressions, follow these steps.

Step 1	If necessary, find equivalent fractions that have the same denominator.
Step 2	Add or subtract the numerators.
Step 3	Combine any like terms in the numerator.
Step 4	Factor if possible.
Step 5	Simplify if possible.

Example **Simplify** $\dfrac{6}{2x^2 + 2x - 12} - \dfrac{2}{x^2 - 4}$**.**

$$\frac{6}{2x^2 + 2x - 12} - \frac{2}{x^2 - 4}$$

$= \dfrac{6}{2(x + 3)(x - 2)} - \dfrac{2}{(x - 2)(x + 2)}$ Factor the denominators.

$= \dfrac{6(x + 2)}{2(x + 3)(x - 2)(x + 2)} - \dfrac{2 \cdot 2(x + 3)}{2(x + 3)(x - 2)(x + 2)}$ The LCD is $2(x + 3)(x - 2)(x + 2)$.

$= \dfrac{6(x + 2) - 4(x + 3)}{2(x + 3)(x - 2)(x + 2)}$ Subtract the numerators.

$= \dfrac{6x + 12 - 4x - 12}{2(x + 3)(x - 2)(x + 2)}$ Distributive Property

$= \dfrac{2x}{2(x + 3)(x - 2)(x + 2)}$ Combine like terms.

$= \dfrac{x}{(x + 3)(x - 2)(x + 2)}$ Simplify.

Exercises

Simplify each expression.

1. $\dfrac{-7xy}{3x} + \dfrac{4y^2}{2y}$

2. $\dfrac{2}{x - 3} - \dfrac{1}{x - 1}$

3. $\dfrac{4a}{3bc} - \dfrac{15b}{5ac}$

4. $\dfrac{3}{x + 2} + \dfrac{4x + 5}{3x + 6}$

5. $\dfrac{3x + 3}{x^2 + 2x + 1} + \dfrac{x - 1}{x^2 - 1}$

6. $\dfrac{4}{4x^2 - 4x + 1} - \dfrac{5x}{20x^2 - 5}$

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9-2

Skills Practice

Adding and Subtracting Rational Expressions

Find the LCM of each set of polynomials.

1. $12c, 6c^2d$

2. $18a^3bc^2, 24b^2c^2$

3. $2x - 6, x - 3$

4. $5a, a - 1$

5. $t^2 - 25, t + 5$

6. $x^2 - 3x - 4, x + 1$

Simplify each expression.

7. $\frac{3}{x} + \frac{5}{y}$

8. $\frac{3}{8p^2q} + \frac{5}{4p^2q}$

9. $\frac{2c - 7}{3} + 4$

10. $\frac{2}{m^2n} + \frac{5}{n}$

11. $\frac{12}{5y^2} - \frac{2}{5yz}$

12. $\frac{7}{4gh} + \frac{3}{4h^2}$

13. $\frac{2}{a + 2} - \frac{3}{2a}$

14. $\frac{5}{3b + d} - \frac{2}{3bd}$

15. $\frac{3}{w - 3} - \frac{2}{w^2 - 9}$

16. $\frac{3t}{2 - x} + \frac{5}{x - 2}$

17. $\frac{m}{m - n} - \frac{m}{n - m}$

18. $\frac{4z}{z - 4} + \frac{z + 4}{z + 1}$

19. $\frac{1}{x^2 + 2x + 1} + \frac{x}{x + 1}$

20. $\frac{2x + 1}{x - 5} - \frac{4}{x^2 - 3x - 10}$

21. $\frac{n}{n - 3} + \frac{2n + 2}{n^2 - 2n - 3}$

22. $\frac{3}{y^2 + y - 12} - \frac{2}{y^2 + 6y + 8}$

Lesson 9-2

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9-2 Practice

Adding and Subtracting Rational Expressions

Find the LCM of each set of polynomials.

1. x^2y, xy^3

2. a^2b^3c, abc^4

3. $x + 1, x + 3$

4. $g - 1, g^2 + 3g - 4$

5. $2r + 2, r^2 + r, r + 1$

6. $3, 4w + 2, 4w^2 - 1$

7. $x^2 + 2x - 8, x + 4$

8. $x^2 - x - 6, x^2 + 6x + 8$

9. $d^2 + 6d + 9, 2(d^2 - 9)$

Simplify each expression.

10. $\frac{5}{6ab} - \frac{7}{8a}$

11. $\frac{5}{12x^4y} - \frac{1}{5x^2y^3}$

12. $\frac{1}{6c^2d} + \frac{3}{4cd^3}$

13. $\frac{4m}{3mn} + 2$

14. $2x - 5 - \frac{x - 8}{x + 4}$

15. $\frac{4}{a - 3} + \frac{9}{a - 5}$

16. $\frac{16}{x^2 - 16} + \frac{2}{x + 4}$

17. $\frac{2 - 5m}{m - 9} + \frac{4m - 5}{9 - m}$

18. $\frac{y - 5}{y^2 - 3y - 10} + \frac{y}{y^2 + y - 2}$

19. $\frac{5}{2x - 12} - \frac{20}{x^2 - 4x - 12}$

20. $\frac{2p - 3}{p^2 - 5p + 6} - \frac{5}{p^2 - 9}$

21. $\frac{1}{5n} - \frac{3}{4} + \frac{7}{10n}$

22. $\frac{2a}{a - 3} - \frac{2a}{a + 3} + \frac{36}{a^2 - 9}$

23. $\dfrac{\frac{2}{x - y} + \frac{1}{x + y}}{\frac{1}{x - y}}$

24. $\dfrac{\frac{r + 6}{r} - \frac{1}{r + 2}}{\frac{r^2 + 4r + 3}{r^2 + 2r}}$

25. GEOMETRY The expressions $\frac{5x}{2}$, $\frac{20}{x + 4}$, and $\frac{10}{x - 4}$ represent the lengths of the sides of a triangle. Write a simplified expression for the perimeter of the triangle.

26. KAYAKING Mai is kayaking on a river that has a current of 2 miles per hour. If r represents her rate in calm water, then $r + 2$ represents her rate with the current, and $r - 2$ represents her rate against the current. Mai kayaks 2 miles downstream and then back to her starting point. Use the formula for time, $t = \frac{d}{r}$, where d is the distance, to write a simplified expression for the total time it takes Mai to complete the trip.

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9-2 Reading to Learn Mathematics

Adding and Subtracting Rational Expressions

Pre-Activity **How is subtraction of rational expressions used in photography?**

Read the introduction to Lesson 9-2 at the top of page 479 in your textbook.

A person is standing 5 feet from a camera that has a lens with a focal length of 3 feet. Write an equation that you could solve to find how far the film should be from the lens to get a perfectly focused photograph.

Reading the Lesson

1. a. In work with rational expressions, LCD stands for ____________________________________ and LCM stands for ____________________________________. The LCD is the ___________ of the denominators.

b. To find the LCM of two or more numbers or polynomials, _________________ each number or _________________. The LCM contains each _________________ the _________________ number of times it appears as a _________________.

2. To add $\frac{x^2 - 3}{x^2 - 5x + 6}$ and $\frac{x - 4}{x^3 - 4x^2 + 4x}$, you should first factor the _________________ of each fraction. Then use the factorizations to find the ___________ of $x^2 - 5x + 6$ and $x^3 - 4x^2 + 4x$. This is the ___________ for the two fractions.

3. When you add or subtract fractions, you often need to rewrite the fractions as equivalent fractions. You do this so that the resulting equivalent fractions will each have a denominator equal to the ___________ of the original fractions.

4. To add or subtract two fractions that have the same denominator, you add or subtract their _________________ and keep the same _________________.

5. The sum or difference of two rational expressions should be written as a polynomial or as a fraction in _________________.

Helping You Remember

6. Some students have trouble remembering whether a common denominator is needed to add and subtract rational expressions or to multiply and divide them. How can your knowledge of working with fractions in arithmetic help you remember this?

Lesson 9-2

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9-2 Enrichment

Superellipses

The circle and the ellipse are members of an interesting family of curves that were first studied by the French physicist and mathematician Gabriel Lamé (1795–1870). The general equation for the family is

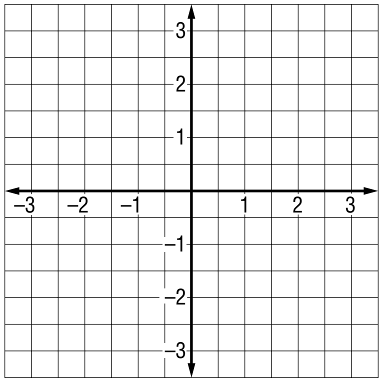
$\left|\frac{x}{a}\right|^n + \left|\frac{y}{b}\right|^n = 1$, with $a \neq 0$, $b \neq 0$, and $n > 0$.

For even values of n greater than 2, the curves are called **superellipses.**

1. Consider two curves that are *not* superellipses. Graph each equation on the grid at the right. State the type of curve produced each time.

a. $\left|\frac{x}{2}\right|^2 + \left|\frac{y}{2}\right|^2 = 1$

b. $\left|\frac{x}{3}\right|^2 + \left|\frac{y}{2}\right|^2 = 1$

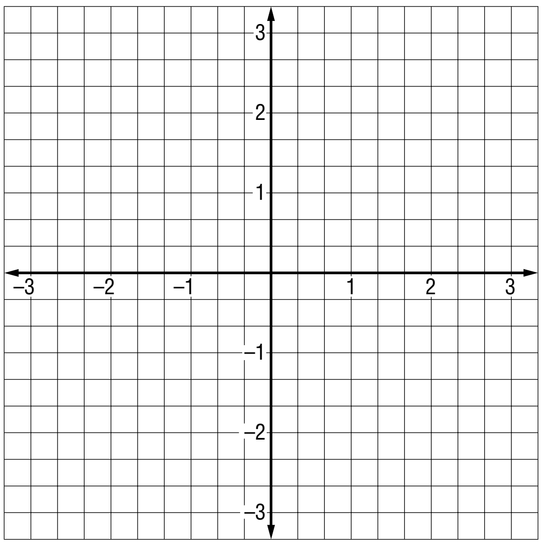


2. In each of the following cases you are given values of *a, b,* and *n* to use in the general equation. Write the resulting equation. Then graph. Sketch each graph on the grid at the right.

a. $a = 2, b = 3, n = 4$

b. $a = 2, b = 3, n = 6$

c. $a = 2, b = 3, n = 8$



3. What shape will the graph of $\left|\frac{x}{2}\right|^n + \left|\frac{y}{2}\right|^n$ approximate for greater and greater even, whole-number values of n?

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9-3 Study Guide and Intervention

Graphing Rational Functions

Vertical Asymptotes and Point Discontinuity

Rational Function	an equation of the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial expressions and $q(x) \neq 0$
Vertical Asymptote of the Graph of a Rational Function	An asymptote is a line that the graph of a function approaches, but never crosses. If the simplified form of the related rational expression is undefined for $x = a$, then $x = a$ is a vertical asymptote.
Point Discontinuity of the Graph of a Rational Function	Point discontinuity is like a hole in a graph. If the original related expression is undefined for $x = a$ but the simplified expression is defined for $x = a$, then there is a hole in the graph at $x = a$.

Example **Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \frac{4x^2 + x - 3}{x^2 - 1}$.**

First factor the numerator and the denominator of the rational expression.

$f(x) = \frac{4x^2 + x - 3}{x^2 - 1} = \frac{(4x - 3)(x + 1)}{(x + 1)(x - 1)}$

The function is undefined for $x = 1$ and $x = -1$.

Since $\frac{(4x - 3)(x + 1)}{(x + 1)(x - 1)} = \frac{4x - 3}{x - 1}$, $x = 1$ is a vertical asymptote. The simplified expression is defined for $x = -1$, so this value represents a hole in the graph.

Exercises

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \frac{4}{x^2 + 3x - 10}$

2. $f(x) = \frac{2x^2 - x - 10}{2x - 5}$

3. $f(x) = \frac{x^2 - x - 12}{x^2 - 4x}$

4. $f(x) = \frac{3x - 1}{3x^2 + 5x - 2}$

5. $f(x) = \frac{x^2 - 6x - 7}{x^2 + 6x - 7}$

6. $f(x) = \frac{3x^2 - 5x - 2}{x + 3}$

7. $f(x) = \frac{x + 1}{x^2 - 6x + 5}$

8. $f(x) = \frac{2x^2 - x - 3}{2x^2 + 3x - 9}$

9. $f(x) = \frac{x^3 - 2x^2 - 5x + 6}{x^2 - 4x + 3}$

Lesson 9-3

NAME _____________________________ DATE __________ PERIOD _____

9-3 Study Guide and Intervention *(continued)*

Graphing Rational Functions

Graph Rational Functions Use the following steps to graph a rational function.

Step 1	First see if the function has any vertical asymptotes or point discontinuities.
Step 2	Draw any vertical asymptotes.
Step 3	Make a table of values.
Step 4	Plot the points and draw the graph.

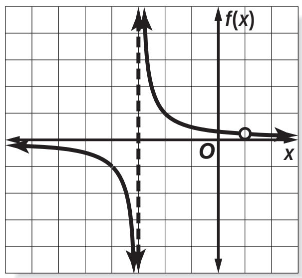
Example **Graph $f(x) = \dfrac{x - 1}{x^2 + 2x - 3}$.**

$\dfrac{x - 1}{x^2 + 2x - 3} = \dfrac{x - 1}{(x - 1)(x + 3)}$ or $\dfrac{1}{x + 3}$

Therefore the graph of $f(x)$ has an asymptote at $x = -3$ and a point discontinuity at $x = 1$.

Make a table of values. Plot the points and draw the graph.

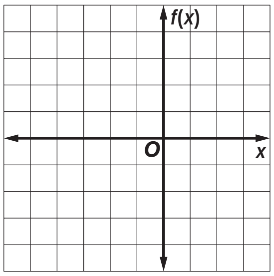
x	−2.5	−2	−1	−3.5	−4	−5
***f*(*x*)**	2	1	0.5	−2	−1	−0.5



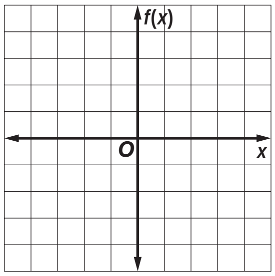
Exercises

Graph each rational function.

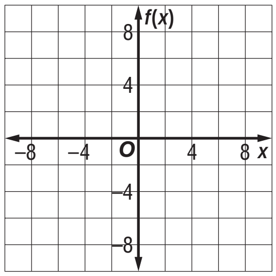
1. $f(x) = \dfrac{3}{x + 1}$



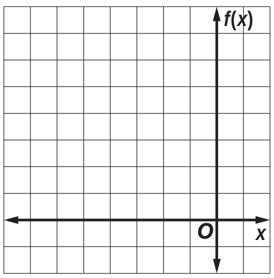
2. $f(x) = \dfrac{2}{x}$



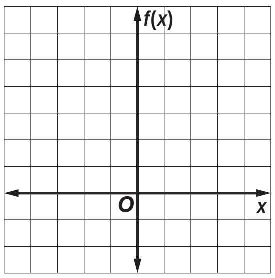
3. $f(x) = \dfrac{2x + 1}{x - 3}$



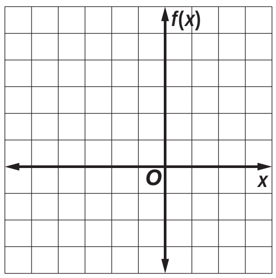
4. $f(x) = \dfrac{2}{(x + 3)^2}$



5. $f(x) = \dfrac{x^2 - x - 6}{x - 3}$



6. $f(x) = \dfrac{x^2 - 6x + 8}{x^2 - x - 2}$



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9-3 Skills Practice

Graphing Rational Functions

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \dfrac{3}{x^2 - 2x - 8}$

2. $f(x) = \dfrac{10}{x^2 - 13x + 36}$

3. $f(x) = \dfrac{x + 12}{x^2 + 10x - 24}$

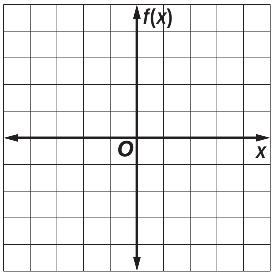
4. $f(x) = \dfrac{x - 1}{x^2 - 4x + 3}$

5. $f(x) = \dfrac{x^2 + 8x + 12}{x + 2}$

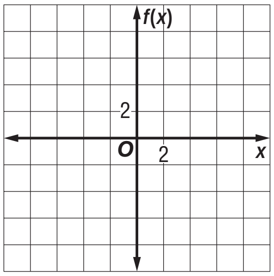
6. $f(x) = \dfrac{x^2 + x - 12}{x - 3}$

Graph each rational function.

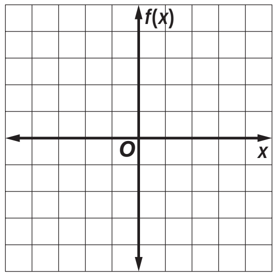
7. $f(x) = \dfrac{-3}{x}$



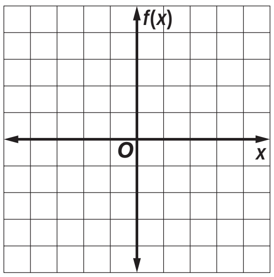
8. $f(x) = \dfrac{10}{x}$



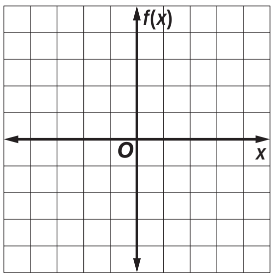
9. $f(x) = \dfrac{-4}{x}$



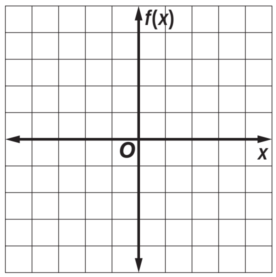
10. $f(x) = \dfrac{2}{x - 1}$



11. $f(x) = \dfrac{x}{x + 2}$



12. $f(x) = \dfrac{x^2 - 4}{x - 2}$



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9-3 Practice

Graphing Rational Functions

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \dfrac{6}{x^2 + 3x - 10}$

2. $f(x) = \dfrac{x - 7}{x^2 - 10x + 21}$

3. $f(x) = \dfrac{x - 2}{x^2 + 4x + 4}$

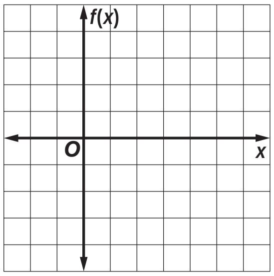
4. $f(x) = \dfrac{x^2 - 100}{x + 10}$

5. $f(x) = \dfrac{x^2 - 2x - 24}{x - 6}$

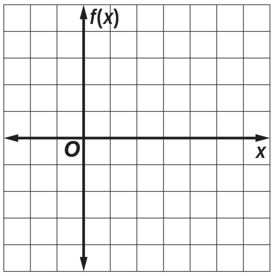
6. $f(x) = \dfrac{x^2 + 9x + 20}{x + 5}$

Graph each rational function.

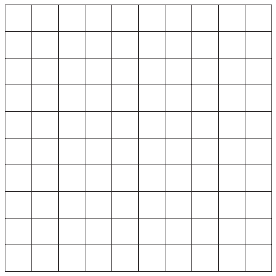
7. $f(x) = \dfrac{-4}{x - 2}$



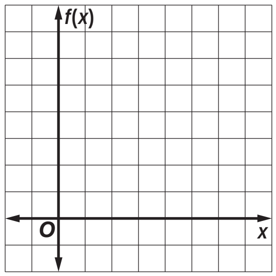
8. $f(x) = \dfrac{x - 3}{x - 2}$



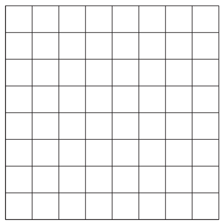
9. $f(x) = \dfrac{3x}{(x + 3)^2}$



10. PAINTING Working alone, Tawa can give the shed a coat of paint in 6 hours. It takes her father x hours working alone to give the shed a coat of paint. The equation $f(x) = \dfrac{6 + x}{6x}$ describes the portion of the job Tawa and her father working together can complete in 1 hour. Graph $f(x) = \dfrac{6 + x}{6x}$ for $x \geq 0, y \geq 0$. If Tawa's father can complete the job in 4 hours alone, what portion of the job can they complete together in 1 hour?



11. LIGHT The relationship between the illumination an object receives from a light source of I foot-candles and the square of the distance d in feet of the object from the source can be modeled by $I(d) = \dfrac{4500}{d^2}$. Graph the function $I(d) = \dfrac{4500}{d^2}$ for $0 \leq I \leq 80$ and $0 \leq d \leq 80$. What is the illumination in foot-candles that the object receives at a distance of 20 feet from the light source?



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9-3 Reading to Learn Mathematics

Graphing Rational Functions

Pre-Activity **How can rational functions be used when buying a group gift?**

Read the introduction to Lesson 9-3 at the top of page 485 in your textbook.

- If 15 students contribute to the gift, how much would each of them pay?
- If each student pays \$5, how many students contributed?

Reading the Lesson

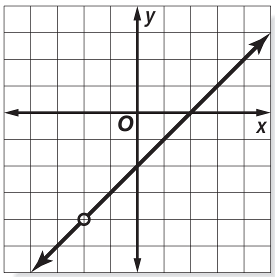
1. Which of the following are rational functions?

A. $f(x) = \frac{1}{x - 5}$ **B.** $g(x) = \sqrt{x}$ **C.** $h(x) = \frac{x^2 - 25}{x^2 + 6x + 9}$

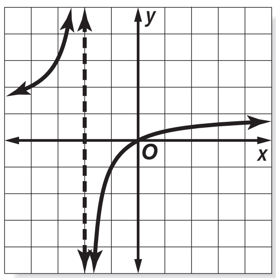
2. a. Graphs of rational functions may have breaks in ______________. These may occur as vertical ______________ or as point ______________.

b. The graphs of two rational functions are shown below.

I.



II.



Graph I has a ____________________ at $x =$ ________.

Graph II has a ____________________ at $x =$ ________.

Match each function with its graph above.

$f(x) = \frac{x}{x + 2}$ $g(x) = \frac{x^2 - 4}{x + 2}$

Helping You Remember

3. One way to remember something new is to see how it is related to something you already know. How can knowing that division by zero is undefined help you to remember how to find the places where a rational function has a point discontinuity or an asymptote?

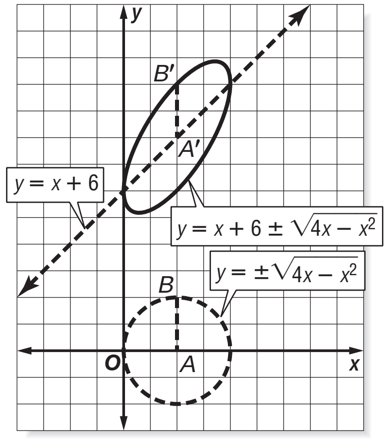
Lesson 9-3

9-3 Enrichment

Graphing with Addition of y-Coordinates

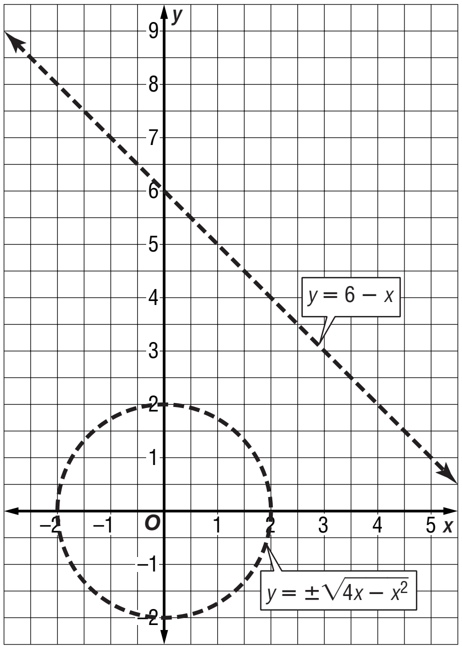
Equations of parabolas, ellipses, and hyperbolas that are "tipped" with respect to the x- and y-axes are more difficult to graph than the equations you have been studying.

Often, however, you can use the graphs of two simpler equations to graph a more complicated equation. For example, the graph of the ellipse in the diagram at the right is obtained by adding the y-coordinate of each point on the circle and the y-coordinate of the corresponding point of the line.

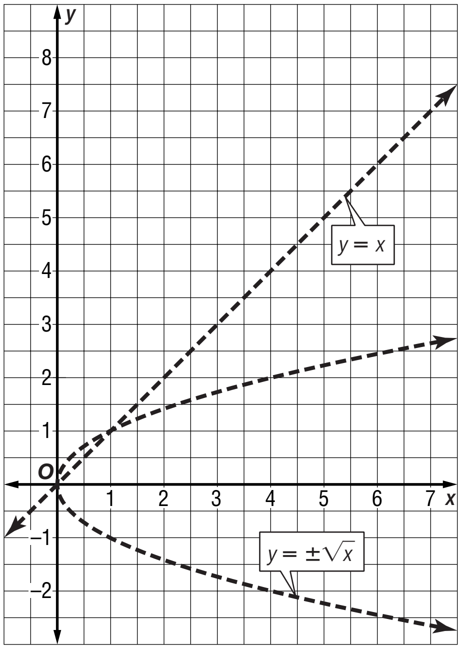


Graph each equation. State the type of curve for each graph.

1. $y = 6 - x \pm \sqrt{4 - x^2}$



2. $y = x \pm \sqrt{x}$



Use a separate sheet of graph paper to graph these equations. State the type of curve for each graph.

3. $y = 2x \pm \sqrt{7 + 6x - x^2}$

4. $y = -2x \pm \sqrt{-2x}$

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9-4 Study Guide and Intervention

Direct, Joint, and Inverse Variation

Direct Variation and Joint Variation

Direct Variation	y varies directly as x if there is some nonzero constant k such that $y = kx$. k is called the constant of variation.
Joint Variation	y varies jointly as x and z if there is some number k such that $y = kxz$, where $x \neq 0$ and $z \neq 0$.

Example **Find each value.**

a. If y varies directly as x and $y = 16$ when $x = 4$, find x when $y = 20$.

$\frac{y_1}{x_1} = \frac{y_2}{x_2}$ Direct proportion

$\frac{16}{4} = \frac{20}{x_2}$ $y_1 = 16$, $x_1 = 4$, and $y_2 = 20$

$16x_2 = (20)(4)$ Cross multiply.

$x_2 = 5$ Simplify.

The value of x is 5 when y is 20.

b. If y varies jointly as x and z and $y = 10$ when $x = 2$ and $z = 4$, find y when $x = 4$ and $z = 3$.

$\frac{y_1}{x_1z_1} = \frac{y_2}{x_2z_2}$ Joint variation

$\frac{10}{2 \cdot 4} = \frac{y_2}{4 \cdot 3}$ $y_1 = 10$, $x_1 = 2$, $z_1 = 4$, $x_2 = 4$, and $z_2 = 3$

$120 = 8y_2$ Simplify.

$y_2 = 15$ Divide each side by 8.

The value of y is 15 when $x = 4$ and $z = 3$.

Exercises

Find each value.

1. If y varies directly as x and $y = 9$ when $x = 6$, find y when $x = 8$.

2. If y varies directly as x and $y = 16$ when $x = 36$, find y when $x = 54$.

3. If y varies directly as x and $x = 15$ when $y = 5$, find x when $y = 9$.

4. If y varies directly as x and $x = 33$ when $y = 22$, find x when $y = 32$.

5. Suppose y varies jointly as x and z. Find y when $x = 5$ and $z = 3$, if $y = 18$ when $x = 3$ and $z = 2$.

6. Suppose y varies jointly as x and z. Find y when $x = 6$ and $z = 8$, if $y = 6$ when $x = 4$ and $z = 2$.

7. Suppose y varies jointly as x and z. Find y when $x = 4$ and $z = 11$, if $y = 60$ when $x = 3$ and $z = 5$.

8. Suppose y varies jointly as x and z. Find y when $x = 5$ and $z = 2$, if $y = 84$ when $x = 4$ and $z = 7$.

9. If y varies directly as x and $y = 14$ when $x = 35$, find y when $x = 12$.

10. If y varies directly as x and $x = 200$ when $y = 50$, find x when $y = 1000$.

11. If y varies directly as x and $y = 39$ when $x = 52$, find y when $x = 22$.

12. If y varies directly as x and $x = 60$ when $y = 75$, find x when $y = 42$.

13. Suppose y varies jointly as x and z. Find y when $x = 6$ and $z = 11$, if $y = 120$ when $x = 5$ and $z = 12$.

14. Suppose y varies jointly as x and z. Find y when $x = 5$ and $z = 10$, if $y = 12$ when $x = 8$ and $z = 6$.

15. Suppose y varies jointly as x and z. Find y when $x = 7$ and $z = 18$, if $y = 351$ when $x = 6$ and $z = 13$.

16. Suppose y varies jointly as x and z. Find y when $x = 5$ and $z = 27$, if $y = 480$ when $x = 9$ and $z = 20$.

9-4 Study Guide and Intervention *(continued)*

Direct, Joint, and Inverse Variation

Inverse Variation

Inverse Variation	y varies inversely as x if there is some nonzero constant k such that $xy = k$ or $y = \frac{k}{x}$.

Example **If a varies inversely as b and $a = 8$ when $b = 12$, find a when $b = 4$.**

$\frac{a_1}{b_2} = \frac{a_2}{b_1}$ Inverse variation

$\frac{8}{4} = \frac{a_2}{12}$ $a_1 = 8$, $b_1 = 12$, $b_2 = 4$

$8(12) = 4a_2$ Cross multiply.

$96 = 4a_2$ Simplify.

$24 = a_2$ Divide each side by 4.

When $b = 4$, the value of a is 24.

Exercises

Find each value.

1. If y varies inversely as x and $y = 12$ when $x = 10$, find y when $x = 15$.

2. If y varies inversely as x and $y = 9$ when $x = 45$, find y when $x = 27$.

3. If y varies inversely as x and $y = 100$ when $x = 38$, find y when $x = 76$.

4. If y varies inversely as x and $y = 32$ when $x = 42$, find y when $x = 24$.

5. If y varies inversely as x and $y = 36$ when $x = 10$, find y when $x = 30$.

6. If y varies inversely as x and $y = 75$ when $x = 12$, find y when $x = 10$.

7. If y varies inversely as x and $y = 18$ when $x = 124$, find y when $x = 93$.

8. If y varies inversely as x and $y = 90$ when $x = 35$, find y when $x = 50$.

9. If y varies inversely as x and $y = 42$ when $x = 48$, find y when $x = 36$.

10. If y varies inversely as x and $y = 44$ when $x = 20$, find y when $x = 55$.

11. If y varies inversely as x and $y = 80$ when $x = 14$, find y when $x = 35$.

12. If y varies inversely as x and $y = 3$ when $x = 8$, find y when $x = 40$.

13. If y varies inversely as x and $y = 16$ when $x = 42$, find y when $x = 14$.

14. If y varies inversely as x and $y = 9$ when $x = 2$, find y when $x = 5$.

15. If y varies inversely as x and $y = 23$ when $x = 12$, find y when $x = 15$.

NAME ______________________________ DATE ____________ PERIOD _____

9-4 Skills Practice

Direct, Joint, and Inverse Variation

State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation.

1. $c = 12m$

2. $p = \frac{4}{q}$

3. $A = \frac{1}{2}bh$

4. $rw = 15$

5. $y = 2rst$

6. $f = 5280m$

7. $y = 0.2s$

8. $vz = -25$

9. $t = 16rh$

10. $R = \frac{8}{w}$

11. $\frac{a}{b} = \frac{1}{3}$

12. $C = 2\pi r$

Find each value.

13. If y varies directly as x and $y = 35$ when $x = 7$, find y when $x = 11$.

14. If y varies directly as x and $y = 360$ when $x = 180$, find y when $x = 270$.

15. If y varies directly as x and $y = 540$ when $x = 10$, find x when $y = 1080$.

16. If y varies directly as x and $y = 12$ when $x = 72$, find x when $y = 9$.

17. If y varies jointly as x and z and $y = 18$ when $x = 2$ and $z = 3$, find y when $x = 5$ and $z = 6$.

18. If y varies jointly as x and z and $y = -16$ when $x = 4$ and $z = 2$, find y when $x = -1$ and $z = 7$.

19. If y varies jointly as x and z and $y = 120$ when $x = 4$ and $z = 6$, find y when $x = 3$ and $z = 2$.

20. If y varies inversely as x and $y = 2$ when $x = 2$, find y when $x = 1$.

21. If y varies inversely as x and $y = 6$ when $x = 5$, find y when $x = 10$.

22. If y varies inversely as x and $y = 3$ when $x = 14$, find x when $y = 6$.

23. If y varies inversely as x and $y = 27$ when $x = 2$, find x when $y = 9$.

24. If y varies directly as x and $y = -15$ when $x = 5$, find x when $y = -36$.

9-4 Practice

Direct, Joint, and Inverse Variation

State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation.

1. $u = 8wz$	**2.** $p = 4s$	**3.** $L = \frac{5}{k}$	**4.** $xy = 4.5$
5. $\frac{C}{d} = \pi$	**6.** $2d = mn$	**7.** $\frac{1.25}{g} = h$	**8.** $y = \frac{3}{4x}$

Find each value.

9. If y varies directly as x and $y = 8$ when $x = 2$, find y when $x = 6$.

10. If y varies directly as x and $y = -16$ when $x = 6$, find x when $y = -4$.

11. If y varies directly as x and $y = 132$ when $x = 11$, find y when $x = 33$.

12. If y varies directly as x and $y = 7$ when $x = 1.5$, find y when $x = 4$.

13. If y varies jointly as x and z and $y = 24$ when $x = 2$ and $z = 1$, find y when $x = 12$ and $z = 2$.

14. If y varies jointly as x and z and $y = 60$ when $x = 3$ and $z = 4$, find y when $x = 6$ and $z = 8$.

15. If y varies jointly as x and z and $y = 12$ when $x = -2$ and $z = 3$, find y when $x = 4$ and $z = -1$.

16. If y varies inversely as x and $y = 16$ when $x = 4$, find y when $x = 3$.

17. If y varies inversely as x and $y = 3$ when $x = 5$, find x when $y = 2.5$.

18. If y varies inversely as x and $y = -18$ when $x = 6$, find y when $x = 5$.

19. If y varies directly as x and $y = 5$ when $x = 0.4$, find x when $y = 37.5$.

20. GASES The volume V of a gas varies inversely as its pressure P. If $V = 80$ cubic centimeters when $P = 2000$ millimeters of mercury, find V when $P = 320$ millimeters of mercury.

21. SPRINGS The length S that a spring will stretch varies directly with the weight F that is attached to the spring. If a spring stretches 20 inches with 25 pounds attached, how far will it stretch with 15 pounds attached?

22. GEOMETRY The area A of a trapezoid varies jointly as its height and the sum of its bases. If the area is 480 square meters when the height is 20 meters and the bases are 28 meters and 20 meters, what is the area of a trapezoid when its height is 8 meters and its bases are 10 meters and 15 meters?

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9-4 Reading to Learn Mathematics

Direct, Joint, and Inverse Variation

Pre-Activity **How is variation used to find the total cost given the unit cost?**

Read the introduction to Lesson 9-4 at the top of page 492 in your textbook.

- For each additional student who enrolls in a public college, the total high-tech spending will _________________ (increase/decrease) by _________________.
- For each decrease in enrollment of 100 students in a public college, the total high-tech spending will _________________ (increase/decrease) by _________________.

Reading the Lesson

1. Write an equation to represent each of the following variation statements. Use k as the constant of variation.

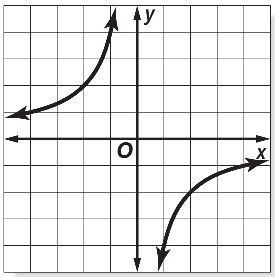
a. m varies inversely as n.

b. s varies directly as r.

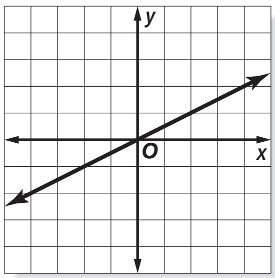
c. t varies jointly as p and q.

2. Which type of variation, direct or inverse, is represented by each graph?

a.



b.



Helping You Remember

3. How can your knowledge of the equation of the slope-intercept form of the equation of a line help you remember the equation for direct variation?

Lesson 9-4

9-4 Enrichment

Expansions of Rational Expressions

Many rational expressions can be transformed into **power series.** A power series is an infinite series of the form $A + Bx + Cx^2 + Dx^3 + \ldots$. The rational expression and the power series normally can be said to have the same values only for certain values of x. For example, the following equation holds only for values of x such that $-1 < x < 1$.

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots \text{ for } -1 < x < 1$$

Example **Expand $\frac{2+3x}{1+x+x^2}$ in ascending powers of x.**

Assume that the expression equals a series of the form $A + Bx + Cx^2 + Dx^3 + \ldots$.
Then multiply both sides of the equation by the denominator $1 + x + x^2$.

$$\frac{2+3x}{1+x+x^2} = A + Bx + Cx^2 + Dx^3 + \ldots$$

$$2 + 3x = (1 + x + x^2)(A + Bx + Cx^2 + Dx^3 + \ldots)$$

$$\begin{aligned} 2 + 3x = A &+ Bx + Cx^2 + Dx^3 + \ldots \\ &+ Ax + Bx^2 + Cx^3 + \ldots \\ & + Ax^2 + Bx^3 + \ldots \end{aligned}$$

$$2 + 3x = A + (B + A)x + (C + B + A)x^2 + (D + C + B)x^3 + \ldots$$

Now, match the coefficients of the polynomials.

$2 = A$

$3 = B + A$

$0 = C + B + A$

$0 = D + C + B + A$

Finally, solve for A, B, C, and D and write the expansion.

$A = 2$, $B = 1$, $C = -3$, and $D = 0$

Therefore, $\frac{2+3x}{1+x+x^2} = 2 + x - 3x^2 + \ldots$

Expand each rational expression to four terms.

1. $\frac{1-x}{1+x+x^2}$

2. $\frac{2}{1-x}$

3. $\frac{1}{1+x}$

NAME ________________________________ DATE ____________ PERIOD _____

9-5 Study Guide and Intervention

Classes of Functions

Identify Graphs You should be familiar with the graphs of the following functions.

Function	Description of Graph
Constant	a horizontal line that crosses the y-axis at a
Direct Variation	a line that passes through the origin and is neither horizontal nor vertical
Identity	a line that passes through the point (a, a), where a is any real number
Greatest Integer	a step function
Absolute Value	V-shaped graph
Quadratic	a parabola
Square Root	a curve that starts at a point and curves in only one direction
Rational	a graph with one or more asymptotes and/or holes
Inverse Variation	a graph with 2 curved branches and 2 asymptotes, $x = 0$ and $y = 0$ (special case of rational function)

Exercises

Identify the function represented by each graph.

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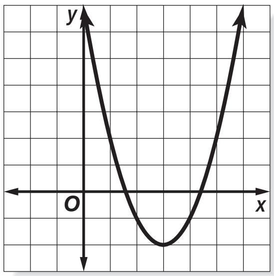

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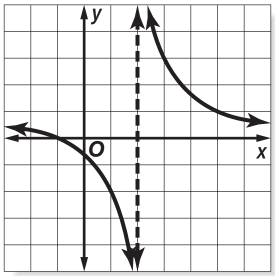

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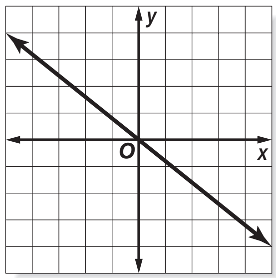

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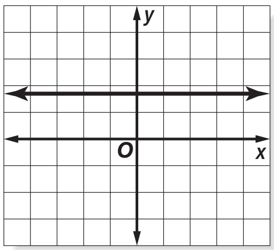

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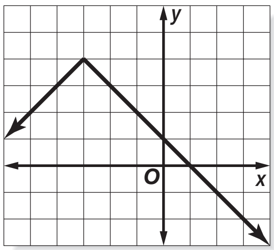

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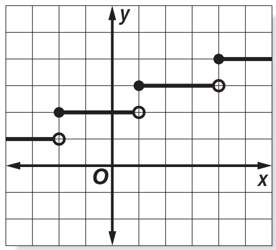

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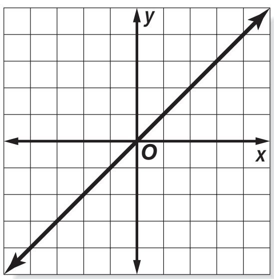

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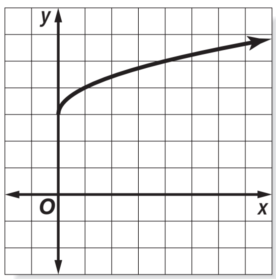

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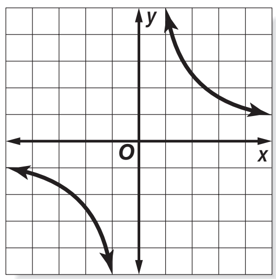


Lesson 9-5

NAME ______________________________ DATE ____________ PERIOD _____

9-5 Study Guide and Intervention *(continued)*

Classes of Functions

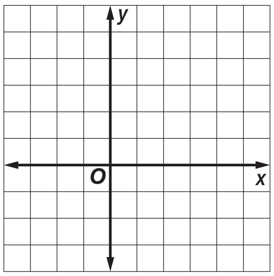
Identify Equations You should be able to graph the equations of the following functions.

Function	General Equation
Constant	$y = a$
Direct Variation	$y = ax$
Identity	$y = x$
Greatest Integer	equation includes a variable within the greatest integer symbol, $[\![\ ]\!]$
Absolute Value	equation includes a variable within the absolute value symbol, $\vert\ \vert$
Quadratic	$y = ax^2 + bx + c$, where $a \neq 0$
Square Root	equation includes a variable beneath the radical sign, $\sqrt{\ }$
Rational	$y = \frac{p(x)}{q(x)}$
Inverse Variation	$y = \frac{a}{x}$

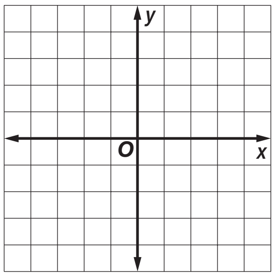
Exercises

Identify the function represented by each equation. Then graph the equation.

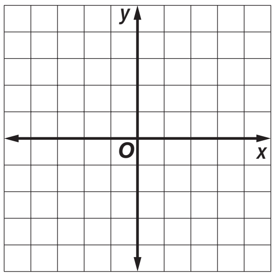
1. $y = \frac{6}{x}$



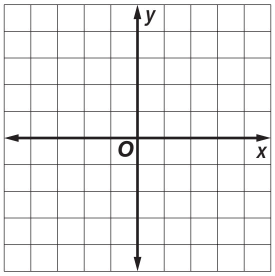
2. $y = \frac{4}{3}x$



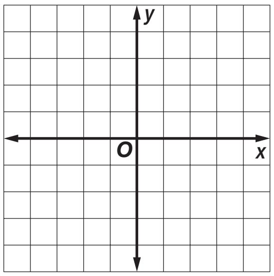
3. $y = -\frac{x^2}{2}$



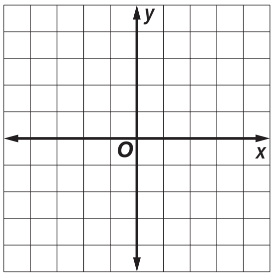
4. $y = |3x| - 1$



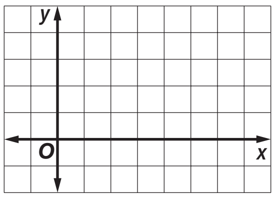
5. $y = -\frac{2}{x}$



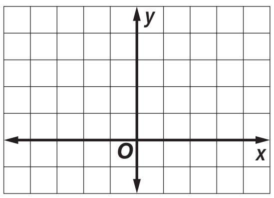
6. $y = \left[\!\left[\frac{x}{2}\right]\!\right]$



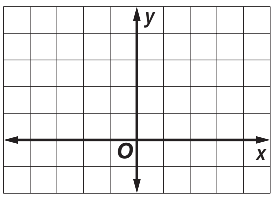
7. $y = \sqrt{x - 2}$



8. $y = 3.2$



9. $y = \frac{x^2 + 5x + 6}{x + 2}$



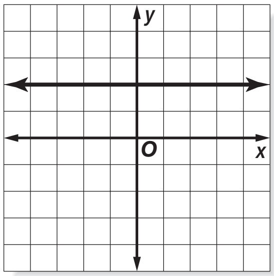
NAME ______________________________ DATE ____________ PERIOD _____

9-5 Skills Practice

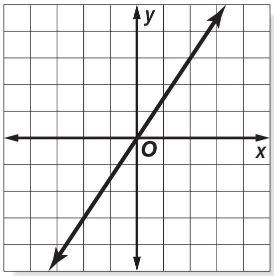
Classes of Functions

Identify the type of function represented by each graph.

1.



2.



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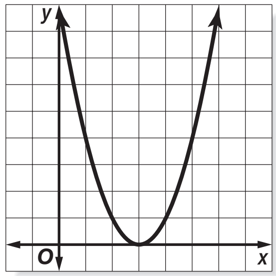
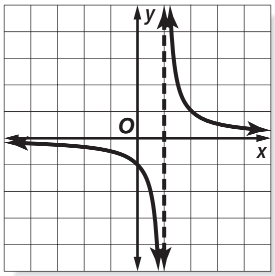


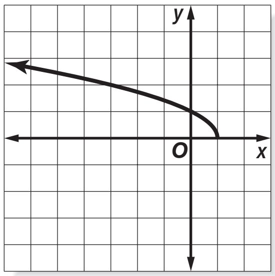
Match each graph with an equation below.

A. $y = |x - 1|$ **B.** $y = \dfrac{1}{x - 1}$ **C.** $y = \sqrt{1 - x}$ **D.** $y = [\![x]\!] - 1$

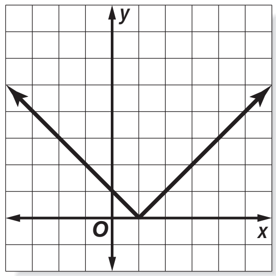
4.



5.

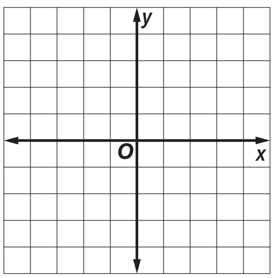


6.

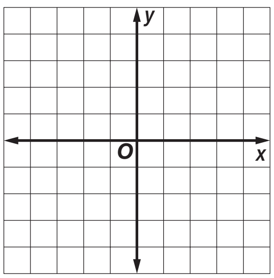


Identify the type of function represented by each equation. Then graph the equation.

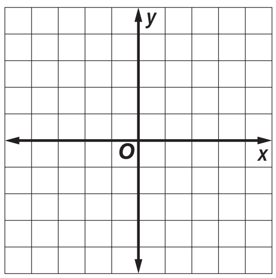
7. $y = \dfrac{2}{x}$



8. $y = 2[\![x]\!]$



9. $y = -3x$



Lesson 9-5

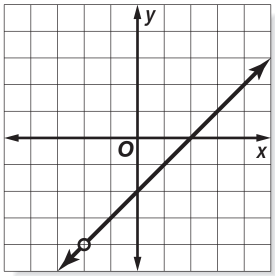
NAME ______________________________ DATE __________ PERIOD _____

9-5 Practice

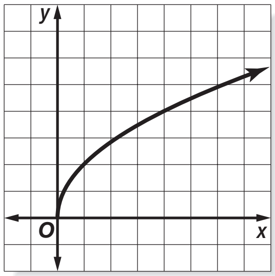
Classes of Functions

Identify the type of function represented by each graph.

1.



2.



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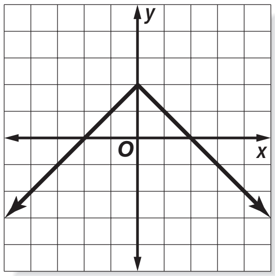
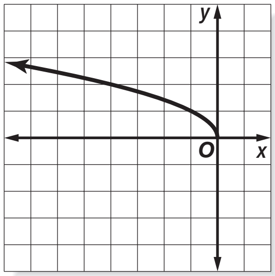


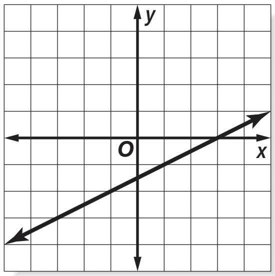
Match each graph with an equation below.

A. $y = |2x + 1|$ **B.** $y = [\![2x + 1]\!]$ **C.** $y = \frac{x - 3}{2}$ **D.** $y = \sqrt{-x}$

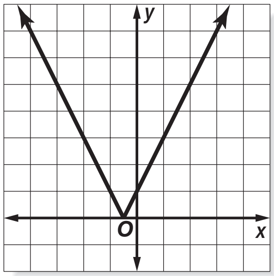
4.



5.

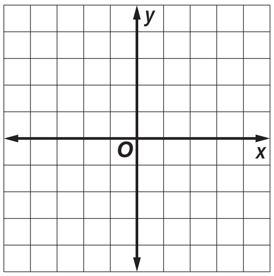


6.

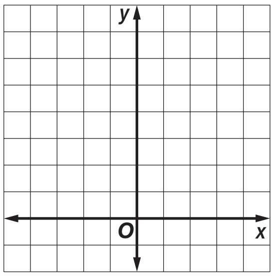


Identify the type of function represented by each equation. Then graph the equation.

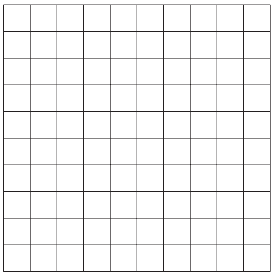
7. $y = -3$



8. $y = 2x^2 + 1$



9. $y = \frac{x^2 + 5x + 6}{x + 2}$



10. BUSINESS A startup company uses the function $P = 1.3x^2 + 3x - 7$ to predict its profit or loss during its first 7 years of operation. Describe the shape of the graph of the function.

11. PARKING A parking lot charges \$10 to park for the first day or part of a day. After that, it charges an additional \$8 per day or part of a day. Describe the graph and find the cost of parking for $6\frac{1}{2}$ days.

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9-5 Reading to Learn Mathematics

Classes of Functions

Pre-Activity **How can graphs of functions be used to determine a person's weight on a different planet?**

Read the introduction to Lesson 9-5 at the top of page 499 in your textbook.

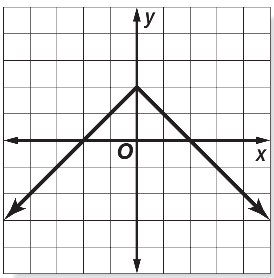
- Based on the graph, estimate the weight on Mars of a child who weighs 40 pounds on Earth.
- Although the graph does not extend far enough to the right to read it directly from the graph, use the weight you found above and your knowledge that this graph represents direct variation to estimate the weight on Mars of a woman who weighs 120 pounds on Earth.

Reading the Lesson

1. Match each graph below with the type of function it represents. Some types may be used more than once and others not at all.

I. square root **II.** quadratic **III.** absolute value **IV.** rational

V. greatest integer **VI.** constant **VII.** identity

a.


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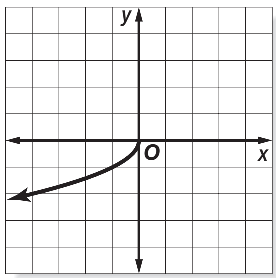

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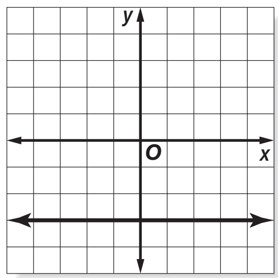
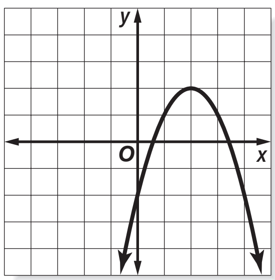


d.


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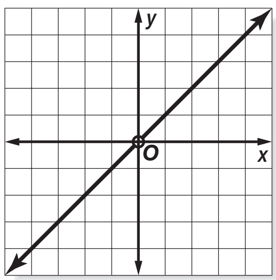

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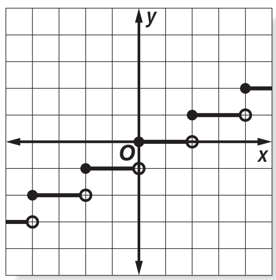

Helping You Remember

2. How can the symbolic definition of absolute value that you learned in Lesson 1-4 help you to remember the graph of the function $f(x) = |x|$?

9-5 Enrichment

Partial Fractions

It is sometimes an advantage to rewrite a rational expression as the sum of two or more fractions. For example, you might do this in a calculus course while carrying out a procedure called integration.

You can resolve a rational expression into partial fractions if two conditions are met:

(1) The degree of the numerator must be less than the degree of the denominator; and

(2) The factors of the denominator must be known.

Example **Resolve $\frac{3}{x^3 + 1}$ into partial fractions.**

The denominator has two factors, a linear factor, $x + 1$, and a quadratic factor, $x^2 - x + 1$. Start by writing the following equation. Notice that the degree of the numerators of each partial fraction is less than its denominator.

$$\frac{3}{x^3 + 1} = \frac{A}{x + 1} + \frac{Bx + C}{x^2 - x + 1}$$

Now, multiply both sides of the equation by $x^3 + 1$ to clear the fractions and finish the problem by solving for the coefficients A, B, and C.

$$\frac{3}{x^3 + 1} = \frac{A}{x + 1} + \frac{Bx + C}{x^2 - x + 1}$$

$$3 = A(x^2 - x + 1) + (x + 1)(Bx + C)$$

$$3 = Ax^2 - Ax + A + Bx^2 + Cx + Bx + C$$

$$3 = (A + B)x^2 + (B + C - A)x + (A + C)$$

Equating each term, $0x^2 = (A + B)x^2$

$$0x = (B + C - A)x$$

$$3 = (A + C)$$

Therefore, $A = 1$, $B = -1$, $C = 2$, and $\frac{3}{x^3 + 1} = \frac{1}{x + 1} + \frac{-x + 2}{x^2 - x + 1}$.

Resolve each rational expression into partial fractions.

1. $\frac{5x - 3}{x^2 - 2x - 3} = \frac{A}{x + 1} + \frac{B}{x - 3}$

2. $\frac{6x + 7}{(x + 2)^2} = \frac{A}{x + 2} + \frac{B}{(x + 2)^2}$

3. $\frac{4x^3 - x^2 - 3x - 2}{x^2(x + 1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x + 1} + \frac{D}{(x + 1)^2}$

9-6 Study Guide and Intervention

Solving Rational Equations and Inequalities

Solve Rational Equations A **rational equation** contains one or more rational expressions. To solve a rational equation, first multiply each side by the least common denominator of all of the denominators. Be sure to exclude any solution that would produce a denominator of zero.

Example **Solve $\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$.**

$\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$	Original equation
$10(x+1)\left(\frac{9}{10} + \frac{2}{x+1}\right) = 10(x+1)\left(\frac{2}{5}\right)$	Multiply each side by $10(x + 1)$.
$9(x+1) + 2(10) = 4(x+1)$	Multiply.
$9x + 9 + 20 = 4x + 4$	Distributive Property
$5x = -25$	Subtract 4x and 29 from each side.
$x = -5$	Divide each side by 5.

Check

$\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$	Original equation
$\frac{9}{10} + \frac{2}{-5+1} \stackrel{?}{=} \frac{2}{5}$	$x = -5$
$\frac{9}{10} + \frac{2}{-4} \stackrel{?}{=} \frac{2}{5}$	Simplify.
$\frac{18}{20} - \frac{10}{20} \stackrel{?}{=} \frac{2}{5}$	Simplify.
$\frac{8}{20} \stackrel{?}{=} \frac{2}{5}$	Simplify.
$\frac{2}{5} = \frac{2}{5}$	

Exercises

Solve each equation.

1. $\frac{2y}{3} - \frac{y+3}{6} = 2$

2. $\frac{4t-3}{5} - \frac{4-2t}{3} = 1$

3. $\frac{2x+1}{3} - \frac{x-5}{4} = \frac{1}{2}$

4. $\frac{3m+2}{5m} + \frac{2m-1}{2m} = 4$

5. $\frac{4}{x-1} = \frac{x+1}{12}$

6. $\frac{x}{x-2} + \frac{4}{x-2} = 10$

7. NAVIGATION The current in a river is 6 miles per hour. In her motorboat Marissa can travel 12 miles upstream or 16 miles downstream in the same amount of time. What is the speed of her motorboat in still water?

8. WORK Adam, Bethany, and Carlos own a painting company. To paint a particular house alone, Adam estimates that it would take him 4 days, Bethany estimates $5\frac{1}{2}$ days, and Carlos 6 days. If these estimates are accurate, how long should it take the three of them to paint the house if they work together?

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9-6 Study Guide and Intervention *(continued)*

Solving Rational Equations and Inequalities

Solve Rational Inequalities To solve a rational inequality, complete the following steps.

Step 1 State the excluded values.
Step 2 Solve the related equation.
Step 3 Use the values from steps 1 and 2 to divide the number line into regions. Test a value in each region to see which regions satisfy the original inequality.

Example **Solve $\frac{2}{3n} + \frac{4}{5n} \leq \frac{2}{3}$.**

Step 1 The value of 0 is excluded since this value would result in a denominator of 0.

Step 2 Solve the related equation.

$\frac{2}{3n} + \frac{4}{5n} = \frac{2}{3}$ Related equation

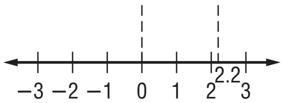
$15n\left(\frac{2}{3n} + \frac{4}{5n}\right) = 15n\left(\frac{2}{3}\right)$ Multiply each side by 15*n*.

$10 + 12 = 10n$ Simplify.

$22 = 10n$ Simplify.

$2.2 = n$ Simplify.

Step 3 Draw a number with vertical lines at the excluded value and the solution to the equation.



Test $n = -1$.
$-\frac{2}{3} + \left(-\frac{4}{5}\right) \leq \frac{2}{3}$ is true.

Test $n = 1$.
$\frac{2}{3} + \frac{4}{5} \leq \frac{2}{3}$ is *not* true.

Test $n = 3$.
$\frac{2}{9} + \frac{4}{15} \leq \frac{2}{3}$ is true.

The solution is $n < 0$ or $n \geq 2.2$.

Exercises

Solve each inequality.

1. $\frac{3}{a + 1} \geq 3$

2. $\frac{1}{x} \geq 4x$

3. $\frac{1}{2p} + \frac{4}{5p} > \frac{2}{3}$

4. $\frac{3}{2x} - \frac{2}{x} > \frac{1}{4}$

5. $\frac{4}{x - 1} + \frac{5}{x} < 2$

6. $\frac{3}{x^2 - 1} + 1 > \frac{2}{x - 1}$

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9-6 Skills Practice

Solving Rational Equations and Inequalities

Lesson 9-6

Solve each equation or inequality. Check your solutions.

1. $\frac{x}{x-1} = \frac{1}{2}$

2. $2 = \frac{4}{n} + \frac{1}{3}$

3. $\frac{9}{3x} = \frac{-6}{2}$

4. $3 - z = \frac{2}{z}$

5. $\frac{2}{d+1} = \frac{1}{d-2}$

6. $\frac{s-3}{5} = \frac{8}{s}$

7. $\frac{2x+3}{x+1} = \frac{3}{2}$

8. $-\frac{12}{y} = y - 7$

9. $\frac{x-2}{x+4} = \frac{x+1}{x+10}$

10. $\frac{3}{k} - \frac{4}{3k} > 0$

11. $2 - \frac{3}{v} < \frac{5}{v}$

12. $n + \frac{3}{n} < \frac{12}{n}$

13. $\frac{1}{2m} - \frac{3}{m} < -\frac{5}{2}$

14. $\frac{1}{2x} < \frac{2}{x} - 1$

15. $\frac{15}{x} + \frac{9x-7}{x+2} = 9$

16. $\frac{3b-2}{b+1} = 4 - \frac{b+2}{b-1}$

17. $2 = \frac{5}{2q} + \frac{2q}{q+1}$

18. $8 - \frac{4}{z} = \frac{8z-8}{z+2}$

19. $\frac{1}{n+3} + \frac{5}{n^2-9} = \frac{2}{n-3}$

20. $\frac{1}{w+2} + \frac{1}{w-2} = \frac{4}{w^2-4}$

21. $\frac{x-8}{2x+2} + \frac{x}{2x+2} = \frac{2x-3}{x+1}$

22. $\frac{12s+19}{s^2+7s+12} - \frac{3}{s+3} = \frac{5}{s+4}$

23. $\frac{2e}{e^2-4} + \frac{1}{e-2} = \frac{2}{e+2}$

24. $\frac{8}{t^2-9} + \frac{4}{t+3} = \frac{2}{t-3}$

NAME ____ DATE ____________ PERIOD _____

9-6 Practice

Solving Rational Equations and Inequalities

Solve each equation or inequality. Check your solutions.

1. $\frac{12}{x} + \frac{3}{4} = \frac{3}{2}$

2. $\frac{x}{x - 1} - 1 = \frac{x}{2}$

3. $\frac{p + 10}{p^2 - 2} = \frac{4}{p}$

4. $\frac{s}{s + 2} + s = \frac{5s + 8}{s + 2}$

5. $\frac{5}{y - 5} = \frac{y}{y - 5} - 1$

6. $\frac{1}{3x - 2} + \frac{5}{x} = 0$

7. $\frac{5}{t} < \frac{9}{2t + 1}$

8. $\frac{1}{2h} + \frac{5}{h} = \frac{3}{h - 1}$

9. $\frac{4}{w - 2} = \frac{-1}{w + 3}$

10. $5 - \frac{3}{a} < \frac{7}{a}$

11. $\frac{4}{5x} + \frac{1}{10} < \frac{3}{2x}$

12. $8 + \frac{3}{y} > \frac{19}{y}$

13. $\frac{4}{p} + \frac{1}{3p} < \frac{1}{5}$

14. $\frac{6}{x - 1} = \frac{4}{x - 2} + \frac{2}{x + 1}$

15. $g + \frac{g}{g - 2} = \frac{2}{g - 2}$

16. $b + \frac{2b}{b - 1} = 1 - \frac{b - 3}{b - 1}$

17. $2 = \frac{x + 2}{x - 3} + \frac{x - 2}{x - 6}$

18. $5 - \frac{3d + 2}{d - 1} = \frac{2d - 4}{d + 2}$

19. $\frac{1}{n + 2} + \frac{1}{n - 2} = \frac{3}{n^2 - 4}$

20. $\frac{c + 1}{c - 3} = 4 - \frac{12}{c^2 - 2c - 3}$

21. $\frac{3}{k - 3} + \frac{4}{k - 4} = \frac{25}{k^2 - 7k + 12}$

22. $\frac{4v}{v - 1} - \frac{5v}{v - 2} = \frac{2}{v^2 - 3v + 2}$

23. $\frac{y}{y + 2} + \frac{7}{y - 5} = \frac{14}{y^2 - 3y - 10}$

24. $\frac{x^2 + 4}{x^2 - 4} + \frac{x}{2 - x} = \frac{2}{x + 2}$

25. $\frac{r}{r + 4} + \frac{4}{r - 4} = \frac{r^2 + 16}{r^2 - 16}$

26. $3 = \frac{6a - 1}{2a + 7} + \frac{22}{a + 5}$

27. BASKETBALL Kiana has made 9 of 19 free throws so far this season. Her goal is to make 60% of her free throws. If Kiana makes her next x free throws in a row, the function $f(x) = \frac{9 + x}{19 + x}$ represents Kiana's new ratio of free throws made. How many successful free throws in a row will raise Kiana's percent made to 60%?

28. OPTICS The lens equation $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ relates the distance p of an object from a lens, the distance q of the image of the object from the lens, and the focal length f of the lens. What is the distance of an object from a lens if the image of the object is 5 centimeters from the lens and the focal length of the lens is 4 centimeters?

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9-6 Reading to Learn Mathematics

Solving Rational Equations and Inequalities

Pre-Activity **How are rational equations used to solve problems involving unit price?**

Read the introduction to Lesson 9-6 at the top of page 505 in your textbook.

- If you increase total number of minutes of long-distance calls from March to April, will your long-distance phone bill increase or decrease?
- Will your actual cost per minute increase or decrease?

Reading the Lesson

1. When solving a rational equation, any possible solution that results in 0 in the denominator must be excluded from the list of solutions.

2. Suppose that on a quiz you are asked to solve the rational inequality $\frac{3}{z+2} - \frac{6}{z} > 0$. Complete the steps of the solution.

Step 1 The excluded values are __________ and __________.

Step 2 The related equation is ______________.

To solve this equation, multiply both sides by the LCD, which is ______________. Solving this equation will show that the only solution is -4.

Step 3 Divide a number line into __________ regions using the excluded values and the solution of the related equation. Draw dashed vertical lines on the number line below to show these regions.

Consider the following values of $\frac{3}{z+2} - \frac{6}{z}$ for various test values of z.

If $z = -5$, $\frac{3}{z+2} - \frac{6}{z} = 0.2$.

If $z = -3$, $\frac{3}{z+2} - \frac{6}{z} = -1$.

If $z = -1$, $\frac{3}{z+2} - \frac{6}{z} = 9$.

If $z = 1$, $\frac{3}{z+2} - \frac{6}{z} = -5$.

Using this information and your number line, write the solution of the inequality.

Helping You Remember

3. How are the processes of adding rational expressions with different denominators and of solving rational expressions alike, and how are they different?

9-6 Enrichment

Limits

Sequences of numbers with a rational expression for the general term often approach some number as a finite limit. For example, the reciprocals of the positive integers approach 0 as n gets larger and larger. This is written using the notation shown below. The symbol ∞ stands for infinity and $n \to \infty$ means that n is getting larger and larger, or "n goes to infinity."

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n}, \ldots \qquad \lim_{n \to \infty} \frac{1}{n} = 0$

Example **Find $\lim_{n \to \infty} \frac{n^2}{(n+1)^2}$**

It is not immediately apparent whether the sequence approaches a limit or not. But notice what happens if we divide the numerator and denominator of the general term by n^2.

$$\frac{n^2}{(n+1)^2} = \frac{n^2}{n^2 + 2n + 1}$$

$$= \frac{\frac{n^2}{n^2}}{\frac{n^2}{n^2} + \frac{2n}{n^2} + \frac{1}{n^2}}$$

$$= \frac{1}{1 + \frac{2}{n} + \frac{1}{n^2}}$$

The two fractions in the denominator will approach a limit of 0 as n gets very large, so the entire expression approaches a limit of 1.

Find the following limits.

1. $\lim_{n \to \infty} \frac{n^3 + 5n}{n^4 - 6}$

2. $\lim_{n \to \infty} \frac{1 - n}{n^2}$

3. $\lim_{n \to \infty} \frac{2(n + 1) + 1}{2n + 1}$

4. $\lim_{n \to \infty} \frac{2n + 1}{1 - 3n}$

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9 Chapter 9 Test, Form 1

SCORE _____

Assessment

Write the letter for the correct answer in the blank at the right of each question.

Simplify each expression.

1. $\frac{24mn}{18m^2}$

A. $\frac{3m}{4n}$ **B.** $\frac{4mn}{3}$ **C.** $\frac{4n}{3m}$ **D.** $\frac{4}{3}$ **1.** _____

2. $\frac{6a + 12}{5} \cdot \frac{10}{a + 2}$

A. 12 **B.** 24 **C.** $12a + 12$ **D.** $24a$ **2.** _____

3. $\frac{y}{x^2 - y^2} \div \frac{y^2}{x - y}$

A. $\frac{1}{y(x + y)}$ **B.** $\frac{y^3}{x^3 - x^2y - xy^2 + y^3}$ **C.** $\frac{x + y}{y}$ **D.** $\frac{1}{y(x - y)}$ **3.** _____

4. $\frac{\frac{m^2}{5n^3}}{\frac{m}{n^2}}$

A. $5mn$ **B.** $\frac{m}{5n}$ **C.** $\frac{1}{5}mn$ **D.** $\frac{m^2}{n}$ **4.** _____

5. $\frac{10}{pq} + \frac{4}{q}$

A. $\frac{10 + 4p}{pq^2}$ **B.** $\frac{14}{q(p + 1)}$ **C.** $\frac{10p + 4}{pq}$ **D.** $\frac{10 + 4p}{pq}$ **5.** _____

6. $\frac{4}{k + 1} + \frac{9}{2(k + 1)}$

A. $\frac{13}{2(k + 1)}$ **B.** $\frac{17}{2(k + 1)}$ **C.** $\frac{11}{k + 1}$ **D.** $\frac{8}{9}$ **6.** _____

For Questions 7 and 8, find the LCM of each set of polynomials.

7. $10x^2$, $30xy^2$

A. $30x^2y^2$ **B.** $300x^3y^2$ **C.** $10x$ **D.** $40x^2y^2$ **7.** _____

8. $3z + 12$, $6z + 24$

A. $18(z + 4)$ **B.** $3(z + 4)$ **C.** $6(z + 4)$ **D.** $z + 4$ **8.** _____

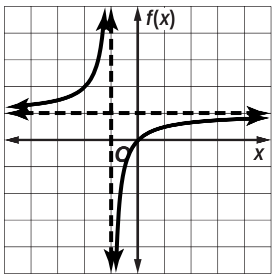
9. Which is an equation of the vertical asymptote of the graph of $f(x) = \frac{x - 1}{x - 2}$?

A. $y = 1$ **B.** $y = 2$ **C.** $x = 2$ **D.** $x = 1$ **9.** _____

10. Which rational function is graphed?

A. $f(x) = \frac{2}{x + 1}$ **B.** $f(x) = \frac{2}{x - 1}$

C. $f(x) = \frac{x}{x - 1}$ **D.** $f(x) = \frac{x}{x + 1}$ **10.** _____



9 Chapter 9 Test, Form 1 *(continued)*

11. The equation $z = 30x$ represents a(n) __?__ variation.

A. direct **B.** joint **C.** inverse **D.** combined **11.** ______

12. Suppose y varies jointly as x and z. If $y = 24$ when $x = 2$ and $z = 3$, find y when $x = 1$ and $z = 5$.

A. 5 **B.** 20 **C.** 10 **D.** 4 **12.** ______

13. The equation $m = \frac{4}{n}$ represents a(n) __?__ variation.

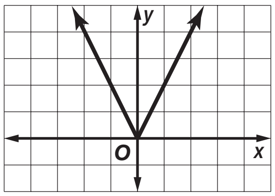
A. direct **B.** joint **C.** inverse **D.** reverse **13.** ______

14. If y varies inversely as x and $y = 2$ when $x = 10$, find y when $x = 5$.

A. 1 **B.** 4 **C.** 25 **D.** 100 **14.** ______

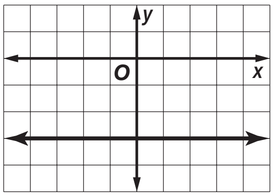
For Questions 15 and 16, identify the function represented by each graph.

15.



A. absolute value
B. greatest integer
C. direct variation
D. quadratic **15.** ______

16.



A. identity
B. constant
C. inverse variation
D. rational **16.** ______

17. Identify the type of function represented by $y = \sqrt{16x}$.

A. direction variation **B.** quadratic
C. inverse variation **D.** square root **17.** ______

18. Solve $\frac{x}{x-2} = \frac{7}{5}$.

A. -7 **B.** 5 **C.** 7 **D.** $-\frac{5}{7}$ **18.** ______

19. Solve $y + 4 = \frac{5}{y}$.

A. $-5, 1$ **B.** $-1, 5$ **C.** ± 1 **D.** $\varnothing$ **19.** ______

20. Solve $\frac{9}{m-5} < 3$.

A. $m < 5$ or $m > 8$ **B.** $m < -2$ or $m > 5$
C. $-2 < m < 5$ **D.** $5 < m < 8$ **20.** ______

Bonus Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \frac{x^2 - 9}{x^2 - 3x}$. **B:** ______________

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9 Chapter 9 Test, Form 2A

SCORE ______

Assessment

Write the letter for the correct answer in the blank at the right of each question.

1. For what value(s) of m is the expression $\frac{m^2 - 2m + 1}{2m^2 + m - 3}$ undefined?

A. $-\frac{3}{2}, 0, 1$ **B.** $-1, \frac{3}{2}$ **C.** $-\frac{3}{2}, 1$ **D.** $\frac{3}{2}$ **1.** ______

Simplify each expression.

2. $\frac{x^2 + 5x + 4}{x^2 + 2x + 1} \cdot \frac{2x + 2}{x + 4}$

A. $\frac{1}{2}$ **B.** 2 **C.** $\frac{(x + 4)^2}{2(x + 1)^2}$ **D.** $\frac{x + 4}{2(x + 1)}$ **2.** ______

3. $\frac{a + b}{3} \div \frac{a^2 + b^2}{12}$

A. $\frac{a + b}{4(a^2 + b^2)}$ **B.** $\frac{4}{a + b}$ **C.** $\frac{4}{a - b}$ **D.** $\frac{4(a + b)}{a^2 + b^2}$ **3.** ______

4. $\dfrac{\frac{4s^2 - 36}{8s^2 - 24s}}{\frac{12s + 36}{2s^2 - 6s}}$

A. $\frac{s - 3}{12}$ **B.** $12s - 36$ **C.** $\frac{s + 3}{s - 3}$ **D.** 3 **4.** ______

5. $\frac{6n}{n^2 - 9} - \frac{3}{n + 3}$

A. $\frac{3}{n + 3}$ **B.** $\frac{3}{n - 3}$ **C.** $\frac{6n - 3}{n^2 - n + 12}$ **D.** $\frac{6n - 3}{n^2 - 9}$ **5.** ______

6. $\frac{m}{m - 5} - \frac{2}{5 - m}$

A. $\frac{2m}{m - 5}$ **B.** $\frac{m - 2}{m - 5}$ **C.** $\frac{m + 2}{m - 5}$ **D.** $\frac{2m}{(m - 5)^2}$ **6.** ______

For Questions 7 and 8, find the LCM of each set of polynomials.

7. $5p - 20, 15p - 60$

A. $75(p - 4)$ **B.** $15(p - 4)$ **C.** $p - 4$ **D.** $5(p - 4)$ **7.** ______

8. $t^2 - 8t + 15, t^2 - t - 20$

A. $(t + 3)(t - 5)(t + 4)$ **B.** $(t + 3)(t - 5)(t - 4)$

C. $(t - 3)(t + 5)(t - 4)$ **D.** $(t - 3)(t - 5)(t + 4)$ **8.** ______

9. Determine the equations of any vertical asymptotes of the graph of $f(x) = \frac{x^2 + 5x + 6}{x - 1}$.

A. $x = 1$ **B.** $x = -2$

C. $x = -2, x = -3$ **D.** $y = 1$ **9.** ______

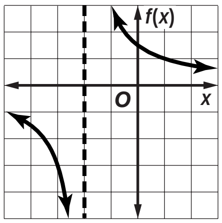
10. Determine the values of x for any holes in the graph of $f(x) = \frac{x + 5}{x^2 + 6x + 5}$.

A. $x = 5$ **B.** $x = -5$

C. $x = 1$ **D.** $x = -1, x = -5$ **10.** ______

9 Chapter 9 Test, Form 2A *(continued)*

11. Which rational function is graphed?



A. $f(x) = \frac{3}{x + 2}$ **B.** $f(x) = \frac{3}{x - 2}$

C. $f(x) = \frac{x}{x + 2}$ **D.** $f(x) = \frac{x}{x - 2}$ **11.** ______

12. If y varies directly as x and $y = 4$ when $x = -2$, find y when $x = 30$.

A. $-\frac{4}{15}$ **B.** 60 **C.** -60 **D.** $\frac{4}{15}$ **12.** ______

13. The area A of a triangle varies jointly as the lengths of its base b and height h. If $A = 75$ when $b = 15$ and $h = 10$, find A when $b = 8$ and $h = 6$.

A. 12 **B.** 48 **C.** 24 **D.** 96 **13.** ______

14. If y varies inversely as x and $y = 2$ when $x = 6$, find y when $x = 36$.

A. $\frac{1}{6}$ **B.** 6 **C.** 3 **D.** $\frac{1}{3}$ **14.** ______

15. The distance a car can travel on a certain amount of fuel varies inversely with its speed. If a car traveling 50 miles per hour can travel 300 miles on 10 gallons of fuel, how far could the car travel on 10 gallons of fuel at 60 miles per hour?

A. 250 mi **B.** 360 mi **C.** 275 mi **D.** 300 mi **15.** ______

16. Identify the type of function represented by $y = (x + 1)^2 - 4$.

A. square root **B.** rational

C. inverse variation **D.** quadratic **16.** ______

17. Identify the type of function represented by $y = \frac{x^2 - 9}{x - 3}$.

A. quadratic **B.** rational

C. inverse variation **D.** direct variation **17.** ______

18. Solve $\frac{n}{n - 4} + n = \frac{12 - 4n}{n - 4}$.

A. $-4, 3$ **B.** $-3, 4$ **C.** -4 **D.** 3 **18.** ______

19. Solve $4 - \frac{1}{b} < \frac{3}{b}$.

A. $b > 0$ **B.** $b < 0$ or $b > 1$ **C.** $0 < b < 1$ **D.** $b < 1$ **19.** ______

20. Tomas can do a job in 4 hours. Julia can do the same job in 6 hours. How many hours will it take the two of them to do the job if they work together?

A. 3.5 **B.** 2.4 **C.** 5 **D.** 2 **20.** ______

Bonus Simplify $\frac{1 + \frac{3}{x}}{1 + \frac{4}{x} + \frac{3}{x^2}}$. **B:** ______________

NAME ______________________ DATE ____________ PERIOD ______

9 Chapter 9 Test, Form 2B

SCORE ______

Assessment

Write the letter for the correct answer in the blank at the right of each question.

1. For what value(s) of x is the expression $\frac{x^2 - 4x + 4}{2x^2 - 3x - 2}$ undefined?

A. $-\frac{1}{2}, 0, 2$ **B.** $-\frac{1}{2}, 2$ **C.** $-2, \frac{1}{2}$ **D.** $-\frac{1}{2}$ **1.** ______

Simplify each expression.

2. $\frac{t^2 - 2t - 3}{t^2 - 1} \cdot \frac{3t - 3}{t^2 - 4t + 3}$

A. $\frac{t^2 - 6t + 9}{3t - 3}$ **B.** $\frac{3(t - 3)}{t^2 - 1}$ **C.** 3 **D.** $\frac{3}{t - 1}$ **2.** ______

3. $\frac{m + 2n}{6} \div \frac{m^2 - 4n^2}{10}$

A. $\frac{5}{3(m - 2n)}$ **B.** $\frac{5}{3(m + 2n)}$

C. $\frac{4}{m - 2n}$ **D.** $\frac{m^3 - 4mn^2 + 2m^2n - 8n^3}{60}$ **3.** ______

4. $\dfrac{\frac{3b^2 - 12}{6b^2 + 12b}}{\frac{5b - 10}{10b^2 + 20b}}$

A. $\frac{b + 2}{b - 2}$ **B.** $b - 2$ **C.** $2b + 4$ **D.** $b + 2$ **4.** ______

5. $\frac{30}{m^2 - 25} + \frac{3}{m + 5}$

A. $\frac{3m + 25}{m^2 - 25}$ **B.** $\frac{33}{m^2 - 25}$ **C.** $\frac{3}{m - 5}$ **D.** $\frac{3(m + 15)}{(m + 5)(m - 5)}$ **5.** ______

6. $\frac{7}{m - 6} - \frac{m}{6 - m}$

A. $\frac{7 - m}{m - 6}$ **B.** $\frac{m + 7}{m - 6}$ **C.** $\frac{m - 7}{m - 6}$ **D.** $\frac{7}{6 - m}$ **6.** ______

For Questions 7 and 8, find the LCM of each set of polynomials.

7. $7m - 21, 14m - 42$

A. $m - 3$ **B.** $98(m - 3)$ **C.** $7(m - 3)$ **D.** $14(m - 3)$ **7.** ______

8. $t^2 - t - 12, t^2 + 2t - 24$

A. $(t + 3)(t - 4)(t + 6)$ **B.** $(t - 3)(t + 4)(t + 6)$

C. $(t - 3)(t - 4)(t - 6)$ **D.** $(t + 3)(t + 4)(t - 6)$ **8.** ______

9. Determine the equations of any vertical asymptotes of the graph of $f(x) = \frac{2x + 3}{x^2 + 2x - 3}$.

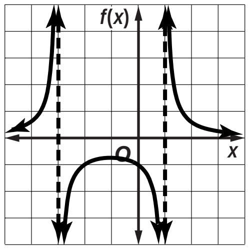
A. $x = -1$ **B.** $x = 3$ **C.** $x = -3, x = 1$ **D.** $y = 2$ **9.** ______

10. Determine the values of x for any holes in the graph of $f(x) = \frac{x + 3}{x^2 + 5x + 6}$.

A. $x = -3$ **B.** $x = 3$ **C.** $x = -2, x = -3$ **D.** $x = -2$ **10.** ______

Chapter 9 Test, Form 2B *(continued)*

11. Which rational function is graphed?



A. $f(x) = \frac{x-3}{x-1}$ **B.** $f(x) = \frac{3}{(x+3)(x-1)}$

C. $f(x) = \frac{x-3}{x+1}$ **D.** $f(x) = \frac{3}{(x-3)(x+1)}$

11. ______

12. If y varies jointly as x and z and $y = 60$ when $x = 10$ and $z = -3$, find y when $x = 8$ and $z = 15$.

A. -240 **B.** 15 **C.** 240 **D.** -15

12. ______

13. SALES An appliance store manager noted that weekly sales varied directly with the amount of money spent on advertising. If last week's sales were \$10,000 and \$2000 was spent on advertising, what should sales be during a week that \$1200 was spent on advertising?

A. \$4800 **B.** \$6000 **C.** \$16,667 **D.** \$50,000

13. ______

14. If y varies inversely as x and $y = 5$ when $x = 5$, find y when $x = 45$.

A. $\frac{3}{2}$ **B.** $\frac{2}{3}$ **C.** $\frac{5}{9}$ **D.** $\frac{9}{5}$

14. ______

15. The distance a car can travel on a certain amount of fuel varies inversely with its speed. If a car traveling 50 miles per hour can travel 336 miles on 10 gallons of fuel, how far could the car travel on 10 gallons of fuel at 60 miles per hour?

A. 315 mi **B.** 320 mi **C.** 403.2 mi **D.** 280 mi

15. ______

16. Identify the type of function represented by $y = |x - 5|$.

A. direct variation **B.** absolute value

C. inverse variation **D.** constant

16. ______

17. Identify the type of function represented by $y = 4$.

A. greatest integer **B.** direct variation

C. constant **D.** identity

17. ______

18. Solve $\frac{n}{n-3} + n = \frac{7n-18}{n-3}$.

A. 3 **B.** 6 **C.** 3, 6 **D.** -3, 6

18. ______

19. Solve $7 - \frac{3}{m} > \frac{18}{m}$.

A. $m < 0$ or $m > 3$ **B.** $0 < m < 3$

C. $m > 3$ **D.** $m < 0$

19. ______

20. The sum of a number and 16 times its reciprocal is 10. Find the number(s).

A. -8 or -2 **B.** 2 or 8 **C.** 4 **D.** ± 4

20. ______

Bonus Simplify $\dfrac{1 - \frac{2}{x}}{1 - \frac{1}{x} - \frac{2}{x^2}}$.

B: ______

NAME ______ DATE ______ PERIOD ______

9 Chapter 9 Test, Form 2C

SCORE ______

1. For what value(s) of x is the expression $\frac{x^2 - 9}{2x^2 - 3x - 9}$ undefined? **1.** ______

Simplify each expression.

2. $\frac{x^3}{x^2 - 64} \div \frac{x^2}{x + 8}$ **2.** ______

3. $\frac{3b^2 + 3b - 6}{b^2 - 6b + 5} \cdot \frac{b^2 - 25}{6b + 12}$ **3.** ______

4. $\dfrac{\frac{3m^2 - 75}{6m^2 + 30m}}{\frac{4m - 20}{9m^2 + 45m}}$ **4.** ______

5. $\frac{2}{x - 2} - \frac{8}{x^2 - 4}$ **5.** ______

6. $\frac{5}{3m - 1} - \frac{2}{1 - 3m}$ **6.** ______

Find the LCM of each set of polynomials.

7. $4m^3n$, $9mn^4$, $18m^4n^2$ **7.** ______

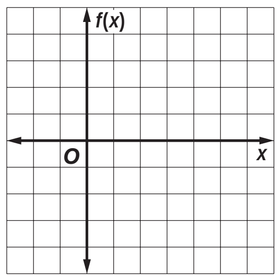
8. $n^2 - 2n - 8$, $n^2 + 2n - 24$ **8.** ______

For Questions 9 and 10, determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

9. $f(x) = \frac{x + 1}{x - 3}$ **9.** ______

10. $f(x) = \frac{x^2 - 2x - 8}{x + 2}$ **10.** ______

11. Graph the rational function $f(x) = \frac{x + 3}{x - 2}$. **11.**



12. If y varies jointly as x and z and $y = 6$ when $x = 4$ and $z = 12$, find y when $x = 24$ and $z = 5$. **12.** ______

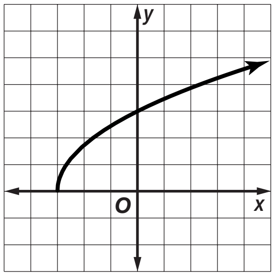
9 Chapter 9 Test, Form 2C *(continued)*

13. **PHOTOGRAPHS** A film-developing company noted that, in a particular town, the number of customers requesting online delivery of their vacation pictures varied directly with the number of households having high-speed Internet access. Currently, 5000 households in the town have high-speed Internet access and 80 customers request online delivery of their photographs. If this trend continues, how many customers should the film-developing company expect to request online delivery when 12,000 households have high-speed Internet access? **13.** ________

14. If y varies inversely as x and $y = 25$ when $x = 6$, find y when $x = 150$. **14.** ________

15. **WILDFIRES** Firefighters battling wildfires in western states noted that the percentage P of the fire remaining uncontained varied inversely with the amount of precipitation A that fell the previous day. If k is the constant of variation, write an equation that expresses P as a function of A. **15.** ________

16. Identify the type of function represented by the graph. **16.** ________



17. Identify the type of function represented by $y = -\frac{2}{3}x$. **17.** ________

For Questions 18 and 19, solve each equation or inequality.

18. $x + \frac{2x}{x-2} = \frac{3x-2}{x-2}$ **18.** ________

19. $9 + \frac{2}{m} > \frac{47}{m}$ **19.** ________

20. **PAINTING** Alice can paint a room in 8 hours. Her assistant can paint the same room in 12 hours. How long will it take if the two of them work together? **20.** ________

Bonus Solve $\dfrac{\frac{1}{x+2} + \frac{1}{x-3}}{\frac{1}{x+2} - \frac{1}{x-3}} = 1$. **B:** ________

NAME ______________________ DATE ____________ PERIOD ______

9 Chapter 9 Test, Form 2D

SCORE ______

1. For what value(s) of x is the expression $\frac{x^2 - x - 6}{2x^2 - x - 10}$ undefined?   1. ________

Simplify each expression.

2. $\frac{x^4}{x^2 - 25} \div \frac{x^2}{x + 5}$ 2. ________

3. $\frac{m^2 + 2m - 8}{3m^2 + 15m + 12} \cdot \frac{4m^2 - 4}{8m^2 - 16m}$ 3. ________

4. $\dfrac{\frac{12y^2 - 48}{8y^2 + 16y}}{\frac{9y - 18}{4y^2 + 8y}}$ 4. ________

5. $\frac{3}{x - 3} - \frac{18}{x^2 - 9}$ 5. ________

6. $\frac{3}{2n - 1} - \frac{2}{1 - 2n}$ 6. ________

Find the LCM of each set of polynomials.

7. $7s^2t$, $6st^4$, $14s^3t^2$ 7. ________

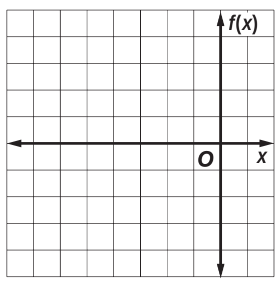
8. $n^2 + 6n + 5$, $n^2 + 3n - 10$ 8. ________

For Questions 9 and 10, determine the equations of any vertical asymptotes and the values of *x* for any holes in the graph of each rational function.

9. $f(x) = \frac{x - 6}{x^2 - 2x - 24}$ 9. ________

10. $f(x) = \frac{3}{x^2 + 7x + 10}$ 10. ________

11. Graph the rational function $f(x) = \frac{x}{x + 2}$.   11.



12. If y varies jointly as x and z and $y = 12$ when $x = 18$ and $z = 6$, find y when $x = 81$ and $z = 7$.   12. ________

Chapter 9 Test, Form 2D *(continued)*

13. RESTAURANTS In a certain county, the planning commission noted that the number of restaurant permits renewed each year varied directly with the number of tourists visiting the county during the previous year. Last year, 400,000 tourists visited the county and 1200 restaurants renewed their permits. This year, 350,000 tourists are projected to visit the county. How many restaurant permits will be renewed if the trend continues?

13. ______________

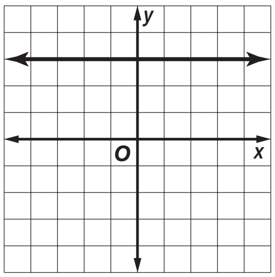
14. If y varies inversely as x and $y = 12$ when $x = 6$, find y when $x = 8$.

14. ______________

15. GOVERNMENT Part of a model used by a state government indicates that revenue R varies inversely with the percentage of eligible workers who are unemployed U. If the constant of variation is k, write an equation that expresses R as a function of U.

15. ______________

16. Identify the type of function represented by the graph.



16. ______________

17. Identify the type of function represented by $y = \frac{11}{x}$.

17. ______________

For Questions 18 and 19, solve each equation or inequality.

18. $\frac{2x}{x-3} - \frac{1}{2} = \frac{2}{2x-6}$

18. ______________

19. $\frac{8r-3}{r} < \frac{45}{r}$

19. ______________

20. GARDENING Joyce can plant a garden in 120 minutes, and Jim can do the same job in 80 minutes. How long will it take to plant the garden if both of them work together?

20. ______________

Bonus Solve $\dfrac{\frac{1}{x-5} + \frac{1}{x+1}}{\frac{1}{x-5} - \frac{1}{x+1}} = 1$.

B: ______________

NAME ______________________ DATE ____________ PERIOD ________

9

Chapter 9 Test, Form 3

SCORE ________

Assessment

1. For what value(s) of x is the expression $\dfrac{2x^2 - x - 10}{6x^3 - 13x^2 - 5x}$ undefined? 1. ________

For Questions 2–6, simplify each expression.

2. $\dfrac{3x^2 + 12x + 12}{2x^2 + x - 6} \cdot \dfrac{4x^2 - 9}{3x^3 + x^2 - 10x}$ 2. ________

3. $\dfrac{g^2 + 5g + 4}{5g + 5} \div \dfrac{g^2 + 8g + 16}{g^2 + g - 12}$ 3. ________

4. $\dfrac{\dfrac{3m + 4n}{4m - 3n}}{\dfrac{3m + 4n}{4m + 3n}}$ 4. ________

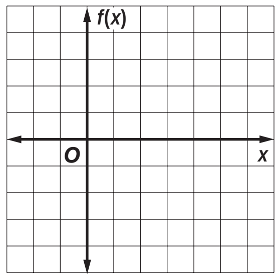
5. $\dfrac{9a^2 + 4b^2}{9a^2 - 4b^2} + \dfrac{3a}{2b - 3a} + \dfrac{2b}{3a + 2b}$ 5. ________

6. $\dfrac{(2 - n)\left(\dfrac{1}{2} + \dfrac{1}{n}\right)}{}$ 6. ________

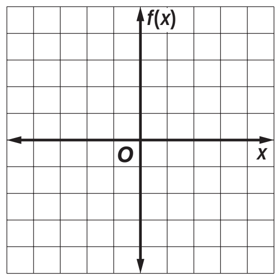
7. Find the LCM of $c^2 + 2cd + d^2$, $c^2 - d^2$, and $c - d$. 7. ________

For Questions 8 and 9, determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function. Then graph each function.

8. $f(x) = \dfrac{-2}{(x - 3)^2}$ 8. ________



9. $f(x) = \dfrac{x^2 - 4}{2x + 4}$ 9. ________



10. If y varies jointly as x and z and $y = \frac{1}{5}$ when $x = \frac{1}{3}$ and $z = 15$, find y when $x = 10$ and $z = \frac{1}{4}$. 10. ________

TELECOMMUNICATIONS For Questions 11 and 12, use the information below and in the table.

City	Population in 2000
Atlanta	416,000
Charleston	97,000
Raleigh	276,000
Tallahassee	151,000

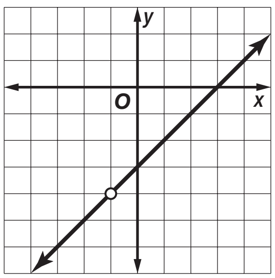
The average number of daily phone calls C between two cities is directly proportional to the product of the populations P_1 and P_2 of the cities and inversely proportional to the square of the distance d between the cities. That is, $C = \frac{kP_1P_2}{d^2}$.

11. Atlanta and Charleston are located approximately 324 miles apart and the average number of daily phone calls between the cities is 7700. Find the constant of variation k to the nearest hundredth. **11.** ____________

12. About 17,100 calls are made each day between Atlanta and Tallahassee. Find the distance between the cities to the nearest mile. **12.** ____________

13. The current I in an electrical circuit varies inversely with the resistance R in the circuit. If the current is 1.2 when the resistance is 6, write an equation relating the current and the resistance. Then find the current when the resistance is 0.18. **13.** ____________

14. Identify the type of function represented by the graph. **14.** ____________



15. Identify the type of function represented by $xy = 0.3$. **15.** ____________

For Questions 16–19, solve each equation or inequality.

16. $\frac{5}{y-3} + \frac{10}{y^2 - y - 6} = \frac{y}{y+2}$ **16.** ____________

17. $\frac{2}{n-5} = \frac{3n+1}{n^2 - 3n - 10} - \frac{1}{n+2}$ **17.** ____________

18. $\frac{1}{6x} + \frac{2}{3x} < \frac{5}{9}$ **18.** ____________

19. $\frac{4}{1-z} > z + 3$ **19.** ____________

20. NUMBER THEORY A fraction has a value of $\frac{3}{5}$. If the numerator is decreased by 8 and the denominator is increased by 3, its value is $\frac{1}{4}$. Find the original fraction. **20.** ____________

Bonus Simplify $\frac{2 + \frac{3}{x}}{\frac{2}{x} - \frac{3}{x^2}} \cdot \frac{\frac{3}{x^2} - \frac{2}{x}}{\frac{3}{x^2} + \frac{2}{x^3}}$ and state any value(s) of x for which the expression is undefined. **B:** ____________

NAME ______________________ DATE ____________ PERIOD ______

9 Chapter 9 Open-Ended Assessment

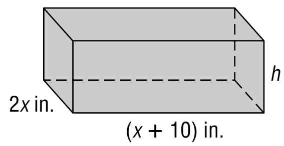
SCORE ______

Assessment

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solutions in more than one way or investigate beyond the requirements of the problem.

1. Write three different rational expressions that are equivalent to the expression $\frac{a}{a - 5}$.

2. The volume of the rectangular box shown is given by $V = (2x^3 + 26x^2 + 60x)$ cubic inches.



a. Explain how to find an expression in terms of x for the height h of the box.

b. In terms of x, h = ____?____ in simplest form.

c. Explain how you could check the expression you found in part **b**. Then check your expression.

3. Write two polynomials for which the LCM is $3y^2 - 12$.

4. Compare and contrast the graphs of the rational functions $f(x) = \frac{(x + 2)(x - 3)}{x + 2}$ and $g(x) = \frac{(x + 2)(x - 3)}{x(x + 2)}$.

5. You decide to invest 10% of your before-tax income in a retirement fund, so you have your employer deduct this money from your weekly paycheck.

a. Write an equation to represent the amount deducted from your paycheck d for investment in your retirement fund for a week during which you worked h hours at r dollars per hour.

b. Is your equation a direct, joint, or inverse variation? Explain your choice.

c. If you earn $9.50 per hour and worked 36 hours last week, explain how to determine the amount deducted last week for your retirement fund.

6. The Franklin Electronics Company has determined that, after its first 50 CD players are produced, the average cost of producing one CD player can be approximated by the function $C(x) = \frac{60x + 17{,}000}{x - 50}$, where x represents the number of CD players produced. Consumer research has indicated that the company should charge the consumer $80 per CD player in order to maximize its profit. Thus, the revenue from the sale of each CD player can be represented by the function $R(x) = 80$.

a. Identify the function represented by $C(x)$. Explain your choice.

b. Identify the function represented by $R(x)$. Explain your choice.

c. The company wants to determine how many CD players must be produced and sold in order to ensure that the revenue from each one is greater than the average cost of producing each one. Write an inequality whose solution represents the information for which the company is looking.

d. Solve your inequality and interpret your solution in the context of the problem.

NAME ______________________ DATE ____________ PERIOD ______

9 Chapter 9 Vocabulary Test/Review

SCORE ______

asymptote	continuity	joint variation	rational expression
complex fraction	direct variation	point discontinuity	rational function
constant of variation	inverse variation	rational equation	rational inequality

Underline or circle the correct word or phrase to complete each sentence.

1. The equation $y = \frac{3}{x}$ is an example of (*direct variation, inverse variation, joint variation*).

2. $r(x) = \frac{x^2 + 6x + 9}{x^2 + 5x + 6}$ is an example of a (*complex fraction, rational function, rational expression*).

3. The graph of $y = \frac{3}{x + 5}$ has a(n) (*asymptote, point discontinuity, constant of variation*).

4. Adding or subtracting rational expressions requires you to find a(n) (*least common denominator, asymptote, complex fraction*).

5. The formula for simple interest, $I = Prt$, is an example of (*direct variation, inverse variation, joint variation*).

6. The graph of $y = \frac{x + 5}{x - 3}$ has a break in (*asymptote, discontinuity, continuity*) at $x = 3$.

7. $\frac{2}{t} + \frac{3}{t^2} < 1$ is an example of a (*rational inequality, rational equation, rational function*).

8. If you walk at a steady speed, your speed and the time it takes to walk 1 mile are (*asymptotes, inversely proportional, direct variations*) to each other.

9. The equation $C = \pi d$ gives the circumference of a circle in terms of its diameter. Here, π is called the (*constant of variation, point discontinuity, asymptote*).

10. If the rational expression in a rational function is not written in lowest terms, the graph of the function may have a (*continuity, constant of variation, point discontinuity*).

In your own words—
Define each term.

11. rational expression

12. complex fraction

NAME ____________________ DATE __________ PERIOD ______

SCORE ______

9 Chapter 9 Quiz

(Lessons 9–1 and 9–2)

For Questions 1–4, simplify each expression.

1. $\frac{12a^3n}{x^2n^4} \cdot \frac{6x^7n^5}{9a^5n^2}$

2. $\frac{x^2 - 6x + 8}{3x - 12} \div \frac{x^2 - 4}{x^2 + 5x + 6}$

3. $\frac{2x^2 - x - 3}{x + 4} \cdot \frac{x^2 - 2x - 24}{x + 1}$

4. $\dfrac{\frac{p^2 - 3p}{p^2 - 6p + 9}}{\frac{20}{4p - 12}}$

5. **Standardized Test Practice** For what value(s) of x is the expression $\frac{x^2 - 5x - 14}{x^2 + 7x + 10}$ undefined?

A. $-5, 2$ **B.** $0, 2, 5$ **C.** -2 **D.** $0, 2$ **E.** $-5, -2$

Find the LCM of each set of polynomials.

6. $12a^2, 15b^3, 20ab^2$

7. $5x^2 - 20, 3x + 6$

8. $2t^2 - 3t + 1, 2t^2 + 7t - 4$

Simplify each expression.

9. $\frac{7}{m^2n} - \frac{2}{5mn}$

10. $\frac{5y}{y^2 - 3y} - \frac{7}{3 - y}$

1. ______________
2. ______________
3. ______________
4. ______________
5. ______________
6. ______________
7. ______________
8. ______________
9. ______________
10. ______________

NAME ____________________ DATE __________ PERIOD ______

SCORE ______

9 Chapter 9 Quiz

(Lesson 9–3)

For Questions 1–3, determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

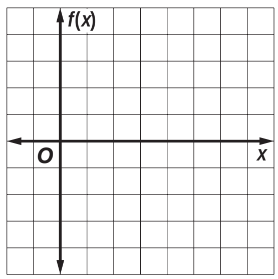
1. $f(x) = \frac{3}{x^2 + x - 2}$

2. $f(x) = \frac{x + 3}{x^2 + 2x - 3}$

3. $f(x) = \frac{x^2 + 2x - 8}{x + 4}$

4. Graph $f(x) = \frac{4}{x - 3}$.

1. ______________
2. ______________
3. ______________
4.



NAME ________________ DATE ________ PERIOD ______

9 Chapter 9 Quiz

SCORE ______

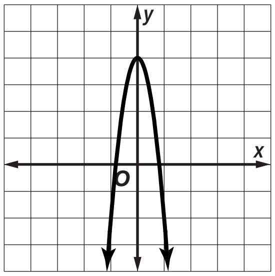
(Lessons 9–4 and 9–5)

1. State whether $rt = 30$ represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation. **1.** ____________

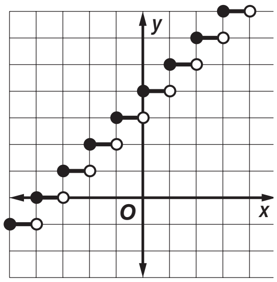
2. Suppose y varies jointly as x and z. Find y when $x = 1$ and $z = 4$, if $y = 96$ when $x = 4$ and $z = 8$. **2.** ____________

Identify the type of function represented by each graph.

3.



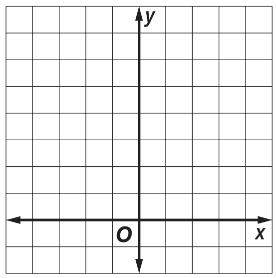
4.



3. ____________

4. ____________

5. Identify the type of function represented by $y = 3|x| + 2$. Then graph the equation. **5.** ____________



NAME ________________ DATE ________ PERIOD ______

9 Chapter 9 Quiz

SCORE ______

(Lesson 9–6)

For Questions 1–4, solve each equation or inequality.

1. $\frac{6}{x + 2} = \frac{x - 7}{x + 2} + \frac{1}{4}$ **1.** ____________

2. $\frac{t - 5}{t - 3} = \frac{t - 3}{t + 3} + \frac{1}{t - 3}$ **2.** ____________

3. $3 + \frac{2}{t} > \frac{8}{t}$ **3.** ____________

4. $\frac{6}{m + 5} > 2$ **4.** ____________

5. **NUMBER THEORY** The ratio of two less than a number to six more than that number is 2 to 3. Find the number. **5.** ____________

NAME ______________________ DATE ____________ PERIOD ______

9

Chapter 9 Mid-Chapter Test

(Lessons 9–1 through 9–3)

SCORE ______

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

1. For what value(s) of x is the expression $\frac{2x(x-3)}{(x+4)(x^2-9)}$ undefined?

A. $-4, 9$ **B.** $-4, -3, 0, 3$ **C.** $-4, 0, 3, 9$ **D.** $-4, -3, 3$ **1.** ______

For Questions 2–5, simplify each expression.

2. $\frac{9y^2-1}{2y-1} \cdot \frac{1-2y}{3y-1}$

A. $-3y-1$ **B.** $3y+1$ **C.** $-3y+1$ **D.** $3y-1$ **2.** ______

3. $\frac{c^2-c-20}{c^2-6c+5} \div \frac{c^2-16}{3c-3}$

A. $\frac{3}{c-4}$ **B.** $\frac{3}{c+4}$ **C.** $\frac{c+4}{3}$ **D.** $\frac{c-4}{3}$ **3.** ______

4. $\dfrac{\frac{3m^2-12}{4m^2+8m}}{\frac{6m-12}{8m^2+16m}}$

A. $\frac{9(m-2)}{16m^2(m+2)}$ **B.** $\frac{m(m^2-4)}{m-2}$ **C.** $m+2$ **D.** $\frac{4(m+2)}{3}$ **4.** ______

5. $\frac{1}{5} - \frac{3}{4w} + \frac{3}{10w}$

A. $\frac{4w-21}{20w}$ **B.** $\frac{4w-9}{20w}$ **C.** $\frac{1}{20w}$ **D.** $-\frac{1}{4w}$ **5.** ______

Part II

6. Simplify $\frac{x}{x^2+x-6} - \frac{1}{x^2-6x+8}$. **6.** ______

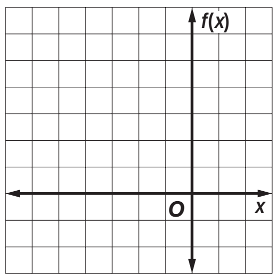
For Questions 7 and 8, find the LCM for each set of polynomials.

7. $12s^3, 18s^2t, 24t^4$ **7.** ______

8. $9c-15, 21c-35$ **8.** ______

9. Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \frac{x+3}{x^2-x-12}$. **9.** ______

10. Graph $f(x) = \frac{4}{(x+2)^2}$. **10.**



Assessment

NAME ______________________ DATE ____________ PERIOD ______

9 Chapter 9 Cumulative Review

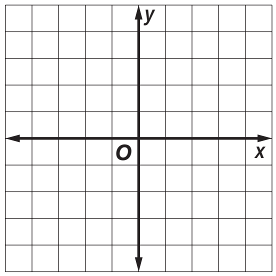
(Chapters 1–9)

1. Determine whether $C = \begin{bmatrix} 1 & 5 \\ -3 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} & \frac{1}{16} \\ & -\frac{5}{16} \end{bmatrix}$ are inverses. (Lesson 4-7) **1.** ____________

2. Simplify the expression $\left(w^{\frac{1}{3}}\right)^{\frac{2}{5}}$. (Lesson 5-7) **2.** ____________

3. Solve $x^2 + 2x + 2 = 0$ by completing the square. (Lesson 6-4) **3.** ____________

4. Graph $y \geq x^2 - 4x$. (Lesson 6-7) **4.**



5. Use synthetic substitution to find $f(3)$ for $f(x) = 3x^3 - 7x^2 + 5x - 10$. (Lesson 7-4) **5.** ____________

6. List all of the possible rational zeros of $2x^4 - 5x^3 + 3x^2 - 12x - 6$. (Lesson 7-6) **6.** ____________

7. Write an equation for a circle with center at $(0, -3)$ that passes through $(5, 7)$. (Lessons 8-1 and 8-3) **7.** ____________

8. Write an equation for the ellipse whose major axis is 10 units long and parallel to the x-axis, whose minor axis is 6 units long, and whose center is at $(1, -2)$. (Lesson 8-4) **8.** ____________

9. State whether the graph of $5x^2 + 5y^2 - 10x + 15y = 10$ is an *ellipse*, *circle*, *parabola*, or *hyperbola*. (Lesson 8-6) **9.** ____________

10. Simplify $\dfrac{\frac{9y^2 - 36}{5y^2 + 10y}}{\frac{6y - 12}{10y^2 + 20y}}$. (Lesson 9-1) **10.** ____________

11. Suppose y varies jointly as x and z. Find y when $x = 16$ and $z = 5$, if $y = 9$ when $x = 3$ and $z = 12$. (Lesson 9-4) **11.** ____________

12. Evita adds a 75% acid solution to 8 milliliters of solution that is 15% acid. The function that represents the percent of acid in the resulting solution is $f(x) = \frac{8(0.15) + x(0.75)}{8 + x}$, where x is the amount of 75% acid solution added. How much 75% acid solution should be added to create a solution that is 50% acid? (Lesson 9-6) **12.** ____________

9 Standardized Test Practice

(Chapters 1–9)

Part 1: Multiple Choice

Instructions: Fill in the appropriate oval for the best answer.

1. If 6 more than the product of a number and -2 is greater than 10, which of the following could be that number?

A. -3 **B.** -2 **C.** 0 **D.** 3 **1.** Ⓐ Ⓑ Ⓒ Ⓓ

2. If the diameter of a circle is doubled, then the area is multiplied by _______.

E. 2 **F.** 4 **G.** 8 **H.** 16 **2.** Ⓔ Ⓕ Ⓖ Ⓗ

3. Which represents an irrational number?

A. $-\frac{1}{3}$ **B.** 1 **C.** $\sqrt{2}$ **D.** $\sqrt{9}$ **3.** Ⓐ Ⓑ Ⓒ Ⓓ

4. If $a < 0$, which of the following must be true?

E. $a - 2 < 2 - a$ **F.** $-2a < a^2$

G. $a - 2 < 2a$ **H.** $a^2 > a + 2$ **4.** Ⓔ Ⓕ Ⓖ Ⓗ

5. A cube is equal in volume to a rectangular solid with edges that measure 4, 6, and 9. What is the measure of an edge of the cube?

A. 216 **B.** 36 **C.** 108 **D.** 6 **5.** Ⓐ Ⓑ Ⓒ Ⓓ

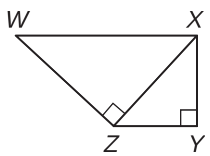
6. If $abc = 30$ and $b = c$, then a equals which of the following?

E. $\frac{30}{c^2}$ **F.** $\frac{15}{c}$ **G.** $30c^2$ **H.** $15c$ **6.** Ⓔ Ⓕ Ⓖ Ⓗ

7. What is the value of $(a - b)^3$ if $b = a + 2$?

A. -8 **B.** -6 **C.** 6 **D.** 8 **7.** Ⓐ Ⓑ Ⓒ Ⓓ

8. In the figure, *WXZ* and *XYZ* are isosceles right triangles. If $XY = 8$, find the perimeter of quadrilateral *WXYZ*.



E. $16 + 16\sqrt{2}$ **F.** $24 + 8\sqrt{2}$

G. $32 + 8\sqrt{2}$ **H.** $32 + 16\sqrt{2}$ **8.** Ⓔ Ⓕ Ⓖ Ⓗ

9. In a 30-day month, how many weekend days fall on dates that are prime numbers if the first day of the month is Thursday?

A. 2 **B.** 3 **C.** 4 **D.** 5 **9.** Ⓐ Ⓑ Ⓒ Ⓓ

10. Sonia purchased 5 pencils and 2 pens for \$5.10. Wai purchased 8 of the same type of pencil and 6 of the same type of pen, and spent \$13.20. What is the cost of 2 pencils and one pen?

E. \$2.10 **F.** \$3.90 **G.** \$1.80 **H.** \$2.40 **10.** Ⓔ Ⓕ Ⓖ Ⓗ

Assessment

9 Standardized Test Practice *(continued)*

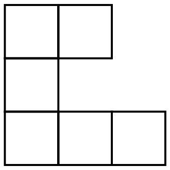
Part 2: Grid In

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate oval that corresponds to that entry.

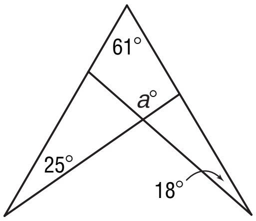
11. 3 is 12% of what number?

12. If $w = 4x$, $y = 10z$, $x = 3$, and $z = \frac{1}{2}$, what is the value of $\frac{2}{y} - \frac{3}{w}$?

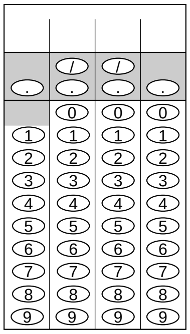
13. How many rectangles can be found in the figure shown?



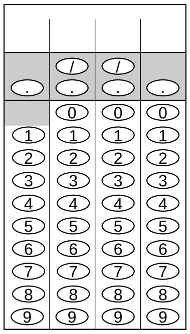
14. What is the value of a in the figure shown?



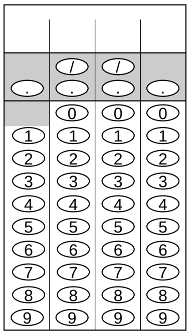
11.



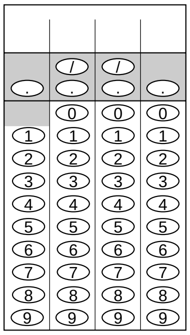
12.



13.

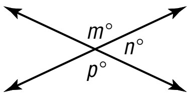


14.



Part 3: Quantitative Comparison

Instructions: Compare the quantities in columns A and B. Shade in
- (A) if the quantity in column A is greater;
- (B) if the quantity in column B is greater;
- (C) if the quantities are equal; or
- (D) if the relationship cannot be determined from the information given.

	Column A		Column B	
15.		$R < h + 1$		**15.** (A) (B) (C) (D)
	R		h	
16.		 $m + n + p = 3m$		**16.** (A) (B) (C) (D)
	n		p	
17.		$\frac{3}{t} = 2$; $\frac{2}{s} = 3$		**17.** (A) (B) (C) (D)
	t		s	
18.		$y \neq 0$		**18.** (A) (B) (C) (D)
	$3y + 1$		$\frac{3y^3 + y^2}{y^2}$	

NAME ______________________ DATE ____________ PERIOD ______

9

Standardized Test Practice

Student Record Sheet *(Use with pages 518–519 of the Student Edition.)*

Part 1 *Multiple Choice*

Select the best answer from the choices given and fill in the corresponding oval.

1 (A) (B) (C) (D)

2 (A) (B) (C) (D)

3 (A) (B) (C) (D)

4 (A) (B) (C) (D)

5 (A) (B) (C) (D)

6 (A) (B) (C) (D)

7 (A) (B) (C) (D)

8 (A) (B) (C) (D)

9 (A) (B) (C) (D)

Part 2 *Short Response/Grid In*

Solve the problem and write your answer in the blank.

For Questions 14–20, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

10 ____________

11 ____________

12 ____________

13 ____________

14 ____________

15 ____________

16 ____________

17 ____________

18 ____________

19 ____________

20 ____________

Part 3 *Quantitative Comparison*

Select the best answer from the choices given and fill in the corresponding oval.

21 (A) (B) (C) (D)

22 (A) (B) (C) (D)

23 (A) (B) (C) (D)

24 (A) (B) (C) (D)

25 (A) (B) (C) (D)

Answers

NAME ______________________ DATE ________ PERIOD _____

9-1 Study Guide and Intervention

Multiplying and Dividing Rational Expressions

Simplify Rational Expressions A ratio of two polynomial expressions is a **rational expression**. To simplify a rational expression, divide both the numerator and the denominator by their greatest common factor (GCF).

Multiplying Rational Expressions	For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$, if $b \neq 0$ and $d \neq 0$.
Dividing Rational Expressions	For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$, if $b \neq 0$, $c \neq 0$, and $d \neq 0$.

Example **Simplify each expression.**

a. $\dfrac{24a^5b^2}{(2ab)^4}$

$$\frac{24a^5b^2}{(2ab)^4} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 3 \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot a \cdot \cancel{b} \cdot \cancel{b}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{b} \cdot \cancel{b} \cdot b \cdot b} = \frac{3a}{2b^2}$$

b. $\dfrac{3r^2s^3}{5t^4} \cdot \dfrac{20t^2}{9r^3s}$

$$\frac{3r^2s^3}{5t^4} \cdot \frac{20t^2}{9r^3s} = \frac{\cancel{3} \cdot \cancel{r} \cdot \cancel{r} \cdot \cancel{s} \cdot s \cdot s \cdot 2 \cdot 2 \cdot \cancel{5} \cdot \cancel{t} \cdot \cancel{t}}{\cancel{5} \cdot \cancel{t} \cdot \cancel{t} \cdot t \cdot t \cdot \cancel{3} \cdot 3 \cdot \cancel{r} \cdot \cancel{r} \cdot r \cdot \cancel{s}} = \frac{2 \cdot 2 \cdot s \cdot s}{3 \cdot r \cdot t \cdot t} = \frac{4s^2}{3rt^2}$$

c. $\dfrac{x^2 + 8x + 16}{2x - 2} \div \dfrac{x^2 + 2x - 8}{x - 1}$

$$\frac{x^2 + 8x + 16}{2x - 2} \div \frac{x^2 + 2x - 8}{x - 1} = \frac{x^2 + 8x + 16}{2x - 2} \cdot \frac{x - 1}{x^2 + 2x - 8}$$

$$= \frac{\cancel{(x + 4)}(x + 4)\cancel{(x - 1)}}{2\cancel{(x - 1)}(x - 2)\cancel{(x + 4)}} = \frac{x + 4}{2(x - 2)}$$

Exercises

Simplify each expression.

1. $\dfrac{(-2ab^2)^3}{20ab^4}$ $-\dfrac{2a^2b^2}{5}$
2. $\dfrac{4x^2 - 12x + 9}{9 - 6x}$ $\dfrac{3 - 2x}{3}$
3. $\dfrac{x^2 + x - 6}{x^2 - 6x - 27}$ $\dfrac{x - 2}{x - 9}$
4. $\dfrac{3m^3 - 3m}{6m^4} \cdot \dfrac{4m^5}{m + 1}$ $2m^2(m - 1)$
5. $\dfrac{c^2 - 3c}{c^2 - 25} \cdot \dfrac{c^2 + 4c - 5}{c^2 - 4c + 3}$ $\dfrac{c}{c - 5}$
6. $\dfrac{(m - 3)^2}{m^2 - 6m + 9} \cdot \dfrac{m^3 - 9m}{m^2 - 9}$ m
7. $\dfrac{6xy^4}{25z^3} \div \dfrac{18xz^2}{5y}$ $\dfrac{y^5}{15z^5}$
8. $\dfrac{16p^2 - 8p + 1}{14p^4} \div \dfrac{4p^2 + 7p - 2}{7p^5}$ $\dfrac{p(4p - 1)}{2(p + 2)}$
9. $\dfrac{2m - 1}{m^2 - 3m - 10} \div \dfrac{4m^2 - 1}{4m + 8}$ $\dfrac{4}{(2m + 1)(m - 5)}$

Lesson 9-1

NAME ______________________ DATE ________ PERIOD _____

9-1 Study Guide and Intervention *(continued)*

Multiplying and Dividing Rational Expressions

Simplify Complex Fractions A **complex fraction** is a rational expression whose numerator and/or denominator contains a rational expression. To simplify a complex fraction, first rewrite it as a division problem.

Example **Simplify** $\dfrac{\frac{3s - 1}{s}}{\frac{3s^2 + 8s - 3}{s^4}}$.

$$\frac{\frac{3s - 1}{s}}{\frac{3s^2 + 8s - 3}{s^4}} = \frac{3s - 1}{s} \div \frac{3s^2 + 8s - 3}{s^4}$$ Express as a division problem.

$$= \frac{3s - 1}{s} \cdot \frac{s^4}{3s^2 + 8s - 3}$$ Multiply by the reciprocal of the divisor.

$$= \frac{\cancel{(3s - 1)}\cancel{s^4}^{\,s^3}}{\cancel{s}\cancel{(3s - 1)}(s + 3)}$$ Factor.

$$= \frac{s^3}{s + 3}$$ Simplify.

Exercises

Simplify.

1. $\dfrac{\frac{x^3y^2z}{a^2b^2}}{\frac{a^3x^2y}{b^2}}$ $\dfrac{xyz}{a^5}$
2. $\dfrac{\frac{a^2bc^3}{x^2y^2}}{\frac{ab^2}{c^4x^2y}}$ $\dfrac{ac^7}{by}$
3. $\dfrac{\frac{b^2 - 1}{3b + 2}}{\frac{b + 1}{3b^2 - b - 2}}$ $(b - 1)^2$
4. $\dfrac{\frac{b^2 - 100}{b^2}}{\frac{3b^2 - 31b + 10}{2b}}$ $\dfrac{2(b + 10)}{b(3b - 1)}$
5. $\dfrac{\frac{x - 4}{x^2 + 6x + 9}}{\frac{x^2 - 2x - 8}{3 + x}}$ $\dfrac{1}{(x + 3)(x + 2)}$
6. $\dfrac{\frac{a^2 - 16}{a + 2}}{\frac{a^2 + 3a - 4}{a^2 + a - 2}}$ $a - 4$
7. $\dfrac{\frac{2x^2 + 9x + 9}{x + 1}}{\frac{10x^2 + 19x + 6}{5x^2 + 7x + 2}}$ $x + 3$
8. $\dfrac{\frac{b + 2}{b^2 - 6b + 8}}{\frac{b^2 + b - 2}{b^2 - 16}}$ $\dfrac{b + 4}{(b - 1)(b - 2)}$
9. $\dfrac{\frac{x^2 - x - 2}{x^3 + 6x^2 - x - 30}}{\frac{x + 1}{x + 3}}$ $\dfrac{1}{x + 5}$

NAME ______________________ DATE __________ PERIOD _____

9-1 Skills Practice

Multiplying and Dividing Rational Expressions

Simplify each expression.

1. $\frac{21x^3y}{14x^2y^2}$ $\frac{3x}{2y}$

2. $\frac{5ab^3}{25a^2b^2}$ $\frac{b}{5a}$

3. $\frac{(x^6)^3}{(x^3)^4}$ x^6

4. $\frac{8y^2(y^6)^3}{4y^{24}}$ $\frac{2}{y^4}$

5. $\frac{18}{2x-6}$ $\frac{9}{x-3}$

6. $\frac{x^2-4}{(x-2)(x+1)}$ $\frac{x+2}{x+1}$

7. $\frac{3a^2-24a}{3a^2+12a}$ $\frac{a-8}{a+4}$

8. $\frac{3m}{2n}\cdot\frac{n^3}{6}$ $\frac{mn^2}{4}$

9. $\frac{24e^3}{5f^2}\cdot\frac{10(ef)^3}{8e^5f}$ $6e$

10. $\frac{5s^2}{s^2-4}\cdot\frac{s+2}{10s^5}$ $\frac{1}{2s^3(s-2)}$

11. $\frac{7g}{y^2}\div 21g^3$ $\frac{1}{3g^2y^2}$

12. $\frac{80y^4}{49z^5v^7}\div\frac{25y^5}{14z^{12}v^5}$ $\frac{32z^7}{35v^2y}$

13. $\frac{3x^2}{x+2}\div\frac{3x}{x^2-4}$ $x(x-2)$

14. $\frac{q^2+2q}{6q}\div\frac{q^2-4}{3q^2}$ $\frac{q^2}{2(q-2)}$

15. $\frac{w^2-5w-24}{w+1}\cdot\frac{w^2-6w-7}{w+3}$ $(w-8)(w-7)$

16. $\frac{t^2+19t+84}{4t-4}\cdot\frac{2t-2}{t^2+9t+14}$ $\frac{t+12}{2(t+2)}$

17. $\frac{x^2-5x+4}{2x-8}\div(3x^2-3x)$ $\frac{1}{6x}$

18. $\frac{16a^2+40a+25}{3a^2-10a-8}\div\frac{4a+5}{a^2-8a+16}$ $\frac{(4a+5)(a-4)}{3a+2}$

19. $\frac{\frac{c^2}{2d^2}}{-\frac{c^6}{5d}}$ $-\frac{5}{2c^4d}$

20. $\frac{\frac{a^2-b^2}{4a}}{\frac{a+b}{2a}}$ $\frac{a-b}{2}$

Lesson 9-1

NAME ______________________ DATE __________ PERIOD _____

9-1 Practice (Average)

Multiplying and Dividing Rational Expressions

Simplify each expression.

1. $\frac{9a^2b^3}{27a^4b^4c}$ $\frac{1}{3a^2bc}$

2. $\frac{(2m^3n^2)^3}{-18m^5n^4}$ $-\frac{4m^4n^2}{9}$

3. $\frac{10y^2+15y}{35y^2-5y}$ $\frac{2y+3}{7y-1}$

4. $\frac{2k^2-k-15}{k^2-9}$ $\frac{2k+5}{k+3}$

5. $\frac{25-v^2}{3v^2-13v-10}$ $-\frac{v+5}{3v+2}$

6. $\frac{x^4+x^3-2x^2}{x^4-x^3}$ $\frac{x+2}{x}$

7. $\frac{-2u^3y}{15xz^5}\cdot\frac{25x^3}{14u^2y^2}$ $-\frac{5ux^2}{21yz^5}$

8. $\frac{a+y}{6}\cdot\frac{4}{y+a}$ $\frac{2}{3}$

9. $\frac{n^5}{n-6}\cdot\frac{n^2-6n}{n^8}$ $\frac{1}{n^2}$

10. $\frac{a-y}{w+n}\cdot\frac{w^2-n^2}{y-a}$ $n-w$

11. $\frac{x^2-5x-24}{6x+2x^2}\cdot\frac{5x^2}{8-x}$ $-\frac{5x}{2}$

12. $\frac{x-5}{10x-2}\cdot\frac{25x^2-1}{x^2-10x+25}$ $\frac{5x+1}{2(x-5)}$

13. $\frac{a^5y^3}{wy^7}\div\frac{a^3w^2}{w^5y^2}$ $\frac{a^2w^2}{y^2}$

14. $\left(\frac{2xy}{w^2}\right)^3\div\frac{24x^2}{w^5}$ $\frac{xy^3}{3w}$

15. $\frac{x+y}{6}\div\frac{x^2-y^2}{3}$ $\frac{1}{2(x-y)}$

16. $\frac{3x+6}{x^2-9}\div\frac{6x^2+12x}{4x+12}$ $\frac{2}{x(x-3)}$

17. $\frac{2s^2-7s-15}{(s+4)^2}\div\frac{s^2-10s+25}{s+4}$ $\frac{2s+3}{(s+4)(s-5)}$

18. $\frac{9-a^2}{a^2+5a+6}\div\frac{2a-6}{5a+10}$ $-\frac{5}{2}$

19. $\frac{\frac{2x+1}{x}}{\frac{4-x}{x}}$ $\frac{2x+1}{4-x}$

20. $\frac{\frac{x^2-9}{4}}{\frac{3-x}{8}}$ $-2(x+3)$

21. $\frac{\frac{x^3+2^3}{x^2-2x}}{\frac{(x+2)^3}{x^2+4x+4}}$ $\frac{x^2-2x+4}{x(x-2)}$

22. **GEOMETRY** A right triangle with an area of x^2-4 square units has a leg that measures $2x+4$ units. Determine the length of the other leg of the triangle.
$x-2$ units

23. **GEOMETRY** A rectangular pyramid has a base area of $\frac{x^2+3x-10}{2x}$ square centimeters and a height of $\frac{x^2-3x}{x^2-5x+6}$ centimeters. Write a rational expression to describe the volume of the rectangular pyramid. $\frac{x+5}{6}$ cm^3

Answers

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9-1 Reading to Learn Mathematics

Multiplying and Dividing Rational Expressions

Pre-Activity **How are rational expressions used in mixtures?**

Read the introduction to Lesson 9-1 at the top of page 472 in your textbook.

- Suppose that the Goodie Shoppe also sells a candy mixture of chocolate mints and caramels. If this mixture is made with 4 pounds of chocolate mints and 3 pounds of caramels, then **$\frac{4}{7}$** of the mixture is mints and **$\frac{3}{7}$** of the mixture is caramels.
- If the store manager adds another y pounds of mints to the mixture, what fraction of the mixture will be mints? **$\frac{4+y}{7+y}$**

Reading the Lesson

1. a. In order to simplify a rational number or rational expression, **factor** the numerator and **denominator** and divide both of them by their **greatest common factor**.

b. A rational expression is undefined when its **denominator** is equal to **0**. To find the values that make the expression undefined, completely **factor** the original **denominator** and set each factor equal to **0**.

2. a. To multiply two rational expressions, **multiply** the **numerators** and multiply the denominators.

b. To divide two rational expressions, **multiply** by the **reciprocal** of the **divisor**.

3. a. Which of the following expressions are complex fractions? **ii, iv, v**

i. $\frac{7}{12}$ **ii.** $\frac{\frac{3}{8}}{\frac{5}{16}}$ **iii.** $\frac{r+5}{r-5}$ **iv.** $\frac{\frac{z+1}{z}}{z}$ **v.** $\frac{\frac{r^2-25}{9}}{\frac{r+5}{3}}$

b. Does a complex fraction express a multiplication or division problem? **division** How is multiplication used in simplifying a complex fraction? **Sample answer: To divide the numerator of the complex fraction by the denominator, multiply the numerator by the reciprocal of the denominator.**

Helping You Remember

4. One way to remember something new is to see how it is similar to something you already know. How can your knowledge of division of fractions in arithmetic help you to understand how to divide rational expressions? **Sample answer: To divide rational expressions, multiply the first expression by the reciprocal of the second. This is the same "invert and multiply" process that is used when dividing arithmetic fractions.**

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9-1 Enrichment

Reading Algebra

In mathematics, the term *group* has a special meaning. The following numbered sentences discuss the idea of group and one interesting example of a group.

01 To be a group, a set of elements and a binary operation must satisfy four conditions: the set must be closed under the operation, the operation must be associative, there must be an identity element, and every element must have an inverse.

02 The following six functions form a group under the operation of composition of functions: $f_1(x) = x$, $f_2(x) = \frac{1}{x}$, $f_3(x) = 1 - x$, $f_4(x) = \frac{(x-1)}{x}$, $f_5(x) = \frac{x}{(x-1)}$, and $f_6(x) = \frac{1}{(1-x)}$.

03 This group is an example of a noncommutative group. For example, $f_3 \circ f_2 = f_4$, but $f_2 \circ f_3 = f_6$.

04 Some experimentation with this group will show that the identity element is f_1.

05 Every element is its own inverse except for f_4 and f_6, each of which is the inverse of the other.

Use the paragraph to answer these questions.

1. Explain what it means to say that a set is *closed* under an operation. Is the set of positive integers closed under subtraction? **Performing the operation on any two elements of the set results in an element of the same set. No, 3 and 4 are positive integers but 3 − 4 is not.**

2. Subtraction is a noncommutative operation for the set of integers. Write an informal definition of noncommutative. **The order in which the elements are used with the operation can affect the result.**

3. For the set of integers, what is the identity element for the operation of multiplication? Justify your answer. **1, because, for every integer a, $a \cdot 1 = a$ and $1 \cdot a = a$.**

4. Explain how the following statement relates to sentence 05:

$$(f_6 \cdot f_4)(x) = f_6[f_4(x)] = f_6\left(\frac{1}{(1-x)}\right) = \frac{1}{1 - \frac{(x-1)}{x}} = x = f_1(x).$$

It shows that f_4 is the inverse of f_6.

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9-2 Study Guide and Intervention

Adding and Subtracting Rational Expressions

LCM of Polynomials To find the least common multiple of two or more polynomials, factor each expression. The LCM contains each factor the greatest number of times it appears as a factor.

Example 1 **Find the LCM of $16p^2q^3r$, $40pq^4r^2$, and $15p^3r^4$.**

$16p^2q^3r = 2^4 \cdot p^2 \cdot q^3 \cdot r$
$40pq^4r^2 = 2^3 \cdot 5 \cdot p \cdot q^4 \cdot r^2$
$15p^3r^4 = 3 \cdot 5 \cdot p^3 \cdot r^4$
$\text{LCM} = 2^4 \cdot 3 \cdot 5 \cdot p^3 \cdot q^4 \cdot r^4$
$= 240p^3q^4r^4$

Example 2 **Find the LCM of $3m^2 - 3m - 6$ and $4m^2 + 12m - 40$.**

$3m^2 - 3m - 6 = 3(m + 1)(m - 2)$
$4m^2 + 12m - 40 = 4(m - 2)(m + 5)$
$\text{LCM} = 12(m + 1)(m - 2)(m + 5)$

Exercises

Find the LCM of each set of polynomials.

1. $14ab^2, 42bc^3, 18a^2c$
$126a^2b^2c^3$

2. $8cdf^3, 28c^2f, 35d^4f^2$
$280c^2d^4f^3$

3. $65x^4y, 10x^2y^2, 26y^4$
$130x^4y^4$

4. $11mn^5, 18m^2n^3, 20mn^4$
$1980m^2n^5$

5. $15a^4b, 50a^2b^2, 40b^8$
$600a^4b^8$

6. $24p^7q, 30p^2q^2, 45pq^3$
$360p^7q^3$

7. $39b^2c^2, 52b^4c, 12c^3$
$156b^4c^3$

8. $12xy^4, 42x^2y, 30x^2y^3$
$420x^2y^4$

9. $56stv^2, 24s^2v^2, 70t^3v^3$
$840s^2t^3v^3$

10. $x^2 + 3x, 10x^2 + 25x - 15$
$5x(x + 3)(2x - 1)$

11. $9x^2 - 12x + 4, 3x^2 + 10x - 8$
$(3x - 2)^2(x + 4)$

12. $22x^2 + 66x - 220, 4x^2 - 16$
$44(x - 2)(x + 2)(x + 5)$

13. $8x^2 - 36x - 20, 2x^2 + 2x - 60$
$4(x - 5)(x + 6)(2x + 1)$

14. $5x^2 - 125, 5x^2 + 24x - 5$
$5(x - 5)(x + 5)(5x - 1)$

15. $3x^2 - 18x + 27, 2x^3 - 4x^2 - 6x$
$6x(x - 3)^2(x + 1)$

16. $45x^2 - 6x - 3, 45x^2 - 5$
$15(5x + 1)(3x - 1)(3x + 1)$

17. $x^3 + 4x^2 - x - 4, x^2 + 2x - 3$
$(x - 1)(x + 1)(x + 3)(x + 4)$

18. $54x^3 - 24x, 12x^2 - 26x + 12$
$6x(3x + 2)(3x - 2)(2x - 3)$

Lesson 9-2

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9-2 Study Guide and Intervention *(continued)*

Adding and Subtracting Rational Expressions

Add and Subtract Rational Expressions To add or subtract rational expressions, follow these steps.

Step 1 If necessary, find equivalent fractions that have the same denominator.
Step 2 Add or subtract the numerators.
Step 3 Combine any like terms in the numerator.
Step 4 Factor if possible.
Step 5 Simplify if possible.

Example **Simplify $\frac{6}{2x^2 + 2x - 12} - \frac{2}{x^2 - 4}$.**

$\frac{6}{2x^2 + 2x - 12} - \frac{2}{x^2 - 4}$

$= \frac{6}{2(x + 3)(x - 2)} - \frac{2}{(x - 2)(x + 2)}$ Factor the denominators.

$= \frac{6(x + 2)}{2(x + 3)(x - 2)(x + 2)} - \frac{2 \cdot 2(x + 3)}{2(x + 3)(x - 2)(x + 2)}$ The LCD is $2(x + 3)(x - 2)(x + 2)$.

$= \frac{6(x + 2) - 4(x + 3)}{2(x + 3)(x - 2)(x + 2)}$ Subtract the numerators.

$= \frac{6x + 12 - 4x - 12}{2(x + 3)(x - 2)(x + 2)}$ Distributive Property

$= \frac{2x}{2(x + 3)(x - 2)(x + 2)}$ Combine like terms.

$= \frac{x}{(x + 3)(x - 2)(x + 2)}$ Simplify.

Exercises

Simplify each expression.

1. $\frac{-7xy}{3x} + \frac{4y^2}{2y}$ **$-\frac{y}{3}$**

2. $\frac{2}{x - 3} - \frac{1}{x - 1}$ **$\frac{x + 1}{(x - 1)(x - 3)}$**

3. $\frac{4a}{3bc} - \frac{15b}{5ac}$ **$\frac{4a^2 - 9b^2}{3abc}$**

4. $\frac{3}{x + 2} + \frac{4x + 5}{3x + 6}$ **$\frac{4x + 14}{3x + 6}$**

5. $\frac{3x + 3}{x^2 + 2x + 1} + \frac{x - 1}{x^2 - 1}$ **$\frac{4}{x + 1}$**

6. $\frac{4}{4x^2 - 4x + 1} - \frac{5x}{20x^2 - 5}$ **$\frac{-2x^2 + 9x + 4}{(2x + 1)(2x - 1)^2}$**

Answers

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9-2 Skills Practice

Adding and Subtracting Rational Expressions

Find the LCM of each set of polynomials.

1. $12c, 6c^2d$ $12c^2d$
2. $18a^3bc^2, 24b^2c^2$ $72a^3b^2c^2$
3. $2x - 6, x - 3$ $2(x - 3)$
4. $5a, a - 1$ $5a(a - 1)$
5. $t^2 - 25, t + 5$ $(t + 5)(t - 5)$
6. $x^2 - 3x - 4, x + 1$ $(x - 4)(x + 1)$

Simplify each expression.

7. $\frac{3}{x} + \frac{5}{y}$ $\frac{5x + 3y}{xy}$
8. $\frac{3}{8p^2q} + \frac{5}{4p^2q}$ $\frac{13}{8p^2q}$
9. $\frac{2c - 7}{3} + 4$ $\frac{2c + 5}{3}$
10. $\frac{2}{m^2n} + \frac{5}{n}$ $\frac{2 + 5m^2}{m^2n}$
11. $\frac{12}{5y^2} - \frac{2}{5yz}$ $\frac{12z - 2y}{5y^2z}$
12. $\frac{7}{4gh} + \frac{3}{4h^2}$ $\frac{7h + 3g}{4gh^2}$
13. $\frac{2}{a + 2} - \frac{3}{2a}$ $\frac{a - 6}{2a(a + 2)}$
14. $\frac{5}{3b + d} - \frac{2}{3bd}$ $\frac{15bd - 6b - 2d}{3bd(3b + d)}$
15. $\frac{3}{w - 3} - \frac{2}{w^2 - 9}$ $\frac{3w + 7}{(w - 3)(w + 3)}$
16. $\frac{3t}{2 - x} + \frac{5}{x - 2}$ $\frac{5 - 3t}{x - 2}$
17. $\frac{m}{m - n} - \frac{m}{n - m}$ $\frac{2m}{m - n}$
18. $\frac{4z}{z - 4} + \frac{z + 4}{z + 1}$ $\frac{5z^2 + 4z - 16}{(z - 4)(z + 1)}$
19. $\frac{1}{x^2 + 2x + 1} + \frac{x}{x + 1}$ $\frac{x^2 + x + 1}{(x + 1)^2}$
20. $\frac{2x + 1}{x - 5} - \frac{4}{x^2 - 3x - 10}$ $\frac{2x^2 + 5x - 2}{(x - 5)(x + 2)}$
21. $\frac{n}{n - 3} + \frac{2n + 2}{n^2 - 2n - 3}$ $\frac{n + 2}{n - 3}$
22. $\frac{3}{y^2 + y - 12} - \frac{2}{y^2 + 6y + 8}$ $\frac{y + 12}{(y + 4)(y - 3)(y + 2)}$

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9-2 Practice (Average)

Adding and Subtracting Rational Expressions

Find the LCM of each set of polynomials.

1. x^2y, xy^3 x^2y^3
2. a^2b^3c, abc^4 $a^2b^3c^4$
3. $x + 1, x + 3$ $(x + 1)(x + 3)$
4. $g - 1, g^2 + 3g - 4$ $(g - 1)(g + 4)$
5. $2r + 2, r^2 + r, r + 1$ $2r(r + 1)$
6. $3, 4w + 2, 4w^2 - 1$ $6(2w + 1)(2w - 1)$
7. $x^2 + 2x - 8, x + 4$ $(x + 4)(x - 2)$
8. $x^2 - x - 6, x^2 + 6x + 8$ $(x + 2)(x + 4)(x - 3)$
9. $d^2 + 6d + 9, 2(d^2 - 9)$ $2(d - 3)(d + 3)^2$

Simplify each expression.

10. $\frac{5}{6ab} - \frac{7}{8a}$ $\frac{20 - 21b}{24ab}$
11. $\frac{5}{12x^4y} - \frac{1}{5x^2y^3}$ $\frac{25y^2 - 12x^2}{60x^4y^3}$
12. $\frac{1}{6c^2d} + \frac{3}{4cd^3}$ $\frac{2d^2 + 9c}{12c^2d^3}$
13. $\frac{4m}{3mn} + 2$ $\frac{2(2 + 3n)}{3n}$
14. $2x - 5 - \frac{x - 8}{x + 4}$ $\frac{2(x + 3)(x - 2)}{x + 4}$
15. $\frac{4}{a - 3} + \frac{9}{a - 5}$ $\frac{13a - 47}{(a - 3)(a - 5)}$
16. $\frac{16}{x^2 - 16} + \frac{2}{x + 4}$ $\frac{2}{x - 4}$
17. $\frac{2 - 5m}{m - 9} + \frac{4m - 5}{9 - m}$ $\frac{7 - 9m}{m - 9}$
18. $\frac{y - 5}{y^2 - 3y - 10} + \frac{y}{y^2 + y - 2}$ $\frac{2y - 1}{(y + 2)(y - 1)}$
19. $\frac{5}{2x - 12} - \frac{20}{x^2 - 4x - 12}$ $\frac{5}{2(x + 2)}$
20. $\frac{2p - 3}{p^2 - 5p + 6} - \frac{5}{p^2 - 9}$ $\frac{2p^2 - 2p + 1}{(p - 2)(p + 3)(p - 3)}$
21. $\frac{1}{5n} - \frac{3}{4} + \frac{7}{10n}$ $\frac{3(6 - 5n)}{20n}$
22. $\frac{2a}{a - 3} - \frac{2a}{a + 3} + \frac{36}{a^2 - 9}$ $\frac{12}{a - 3}$
23. $\frac{\frac{2}{x - y} + \frac{1}{x + y}}{\frac{1}{x - y}}$ $\frac{3x + y}{x + y}$
24. $\frac{\frac{r + 6}{r} - \frac{1}{r + 2}}{\frac{r^2 + 4r + 3}{r^2 + 2r}}$ $\frac{r + 4}{r + 1}$

25. **GEOMETRY** The expressions $\frac{5x}{2}$, $\frac{20}{x + 4}$, and $\frac{10}{x - 4}$ represent the lengths of the sides of a triangle. Write a simplified expression for the perimeter of the triangle. $\frac{5(x^3 - 4x - 16)}{2(x - 4)(x + 4)}$

26. **KAYAKING** Mai is kayaking on a river that has a current of 2 miles per hour. If r represents her rate in calm water, then $r + 2$ represents her rate with the current, and $r - 2$ represents her rate against the current. Mai kayaks 2 miles downstream and then back to her starting point. Use the formula for time, $t = \frac{d}{r}$, where d is the distance, to write a simplified expression for the total time it takes Mai to complete the trip. $\frac{4r}{(r + 2)(r - 2)}$ h

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9-2 Reading to Learn Mathematics

Adding and Subtracting Rational Expressions

Pre-Activity **How is subtraction of rational expressions used in photography?**

Read the introduction to Lesson 9-2 at the top of page 479 in your textbook.

A person is standing 5 feet from a camera that has a lens with a focal length of 3 feet. Write an equation that you could solve to find how far the film should be from the lens to get a perfectly focused photograph.

$\frac{1}{q} = \frac{1}{3} - \frac{1}{5}$

Reading the Lesson

1. a. In work with rational expressions, LCD stands for **least common denominator** and LCM stands for **least common multiple**. The LCD is the **LCM** of the denominators.

b. To find the LCM of two or more numbers or polynomials, **factor** each number or **polynomial**. The LCM contains each **factor** the **greatest** number of times it appears as a **factor**.

2. To add $\frac{x^2 - 3}{x^2 - 5x + 6}$ and $\frac{x - 4}{x^3 - 4x^2 + 4x}$, you should first factor the **denominator** of each fraction. Then use the factorizations to find the **LCM** of $x^2 - 5x + 6$ and $x^3 - 4x^2 + 4x$. This is the **LCD** for the two fractions.

3. When you add or subtract fractions, you often need to rewrite the fractions as equivalent fractions. You do this so that the resulting equivalent fractions will each have a denominator equal to the **LCD** of the original fractions.

4. To add or subtract two fractions that have the same denominator, you add or subtract their **numerators** and keep the same **denominator**.

5. The sum or difference of two rational expressions should be written as a polynomial or as a fraction in **simplest form**.

Helping You Remember

6. Some students have trouble remembering whether a common denominator is needed to add and subtract rational expressions or to multiply and divide them. How can your knowledge of working with fractions in arithmetic help you remember this?

Sample answer: In arithmetic, a common denominator is needed to add and subtract fractions, but not to multiply and divide them. The situation is the same for rational expressions.

Lesson 9-2

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9-2 Enrichment

Superellipses

The circle and the ellipse are members of an interesting family of curves that were first studied by the French physicist and mathematician Gabriel Lamé (1795–1870). The general equation for the family is

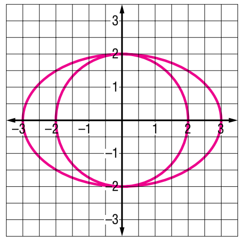
$\left|\frac{x}{a}\right|^n + \left|\frac{y}{b}\right|^n = 1$, with $a \neq 0$, $b \neq 0$, and $n > 0$.

For even values of n greater than 2, the curves are called **superellipses.**

1. Consider two curves that are *not* superellipses. Graph each equation on the grid at the right. State the type of curve produced each time.

a. $\left|\frac{x}{2}\right|^2 + \left|\frac{y}{2}\right|^2 = 1$ **circle**

b. $\left|\frac{x}{3}\right|^2 + \left|\frac{y}{2}\right|^2 = 1$ **ellipse**



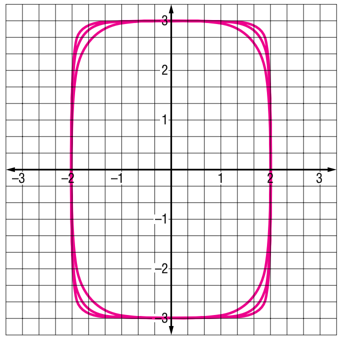
2. In each of the following cases you are given values of a, b, and n to use in the general equation. Write the resulting equation. Then graph. Sketch each graph on the grid at the right.

a. $a = 2, b = 3, n = 4$

b. $a = 2, b = 3, n = 6$

c. $a = 2, b = 3, n = 8$

See students' graphs.



3. What shape will the graph of $\left|\frac{x}{2}\right|^n + \left|\frac{y}{2}\right|^n$ approximate for greater and greater even, whole-number values of n?

a rectangle that is 6 units long and 4 units wide, centered at the origin

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9-3 Study Guide and Intervention

Graphing Rational Functions

Vertical Asymptotes and Point Discontinuity

Rational Function	an equation of the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial expressions and $q(x) \neq 0$
Vertical Asymptote of the Graph of a Rational Function	An asymptote is a line that the graph of a function approaches, but never crosses. If the simplified form of the related rational expression is undefined for $x = a$, then $x = a$ is a vertical asymptote.
Point Discontinuity of the Graph of a Rational Function	Point discontinuity is like a hole in a graph. If the original related expression is undefined for $x = a$ but the simplified expression is defined for $x = a$, then there is a hole in the graph at $x = a$.

Example **Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \frac{4x^2 + x - 3}{x^2 - 1}$.**

First factor the numerator and the denominator of the rational expression.

$f(x) = \frac{4x^2 + x - 3}{x^2 - 1} = \frac{(4x - 3)(x + 1)}{(x + 1)(x - 1)}$

The function is undefined for $x = 1$ and $x = -1$.

Since $\frac{(4x - 3)(x + 1)}{(x + 1)(x - 1)} = \frac{4x - 3}{x - 1}$, $x = 1$ is a vertical asymptote. The simplified expression is defined for $x = -1$, so this value represents a hole in the graph.

Exercises

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \frac{4}{x^2 + 3x - 10}$
asymptotes: $x = 2$, $x = -5$

2. $f(x) = \frac{2x^2 - x - 10}{2x - 5}$
hole: $x = \frac{5}{2}$

3. $f(x) = \frac{x^2 - x - 12}{x^2 - 4x}$
asymptote: $x = 0$; hole $x = 4$

4. $f(x) = \frac{3x - 1}{3x^2 + 5x - 2}$
asymptote: $x = -2$; hole: $x = \frac{1}{3}$

5. $f(x) = \frac{x^2 - 6x - 7}{x^2 + 6x - 7}$
asymptotes: $x = 1$, $x = -7$

6. $f(x) = \frac{3x^2 - 5x - 2}{x + 3}$
asymptote: $x = -3$

7. $f(x) = \frac{x + 1}{x^2 - 6x + 5}$
asymptotes: $x = 1$, $x = 5$

8. $f(x) = \frac{2x^2 - x - 3}{2x^2 + 3x - 9}$
asymptote: $x = -3$; hole: $x = \frac{3}{2}$

9. $f(x) = \frac{x^3 - 2x^2 - 5x + 6}{x^2 - 4x + 3}$
holes: $x = 1$, $x = 3$

Lesson 9-3

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9-3 Study Guide and Intervention *(continued)*

Graphing Rational Functions

Graph Rational Functions Use the following steps to graph a rational function.

Step 1	First see if the function has any vertical asymptotes or point discontinuities.
Step 2	Draw any vertical asymptotes.
Step 3	Make a table of values.
Step 4	Plot the points and draw the graph.

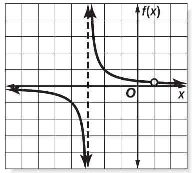
Example **Graph $f(x) = \frac{x - 1}{x^2 + 2x - 3}$.**

$\frac{x - 1}{x^2 + 2x - 3} = \frac{x - 1}{(x - 1)(x + 3)}$ or $\frac{1}{x + 3}$

Therefore the graph of $f(x)$ has an asymptote at $x = -3$ and a point discontinuity at $x = 1$.

Make a table of values. Plot the points and draw the graph.

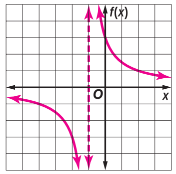
x	−2.5	−2	−1	−3.5	−4	−5
$f(x)$	2	1	0.5	−2	−1	−0.5



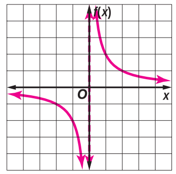
Exercises

Graph each rational function.

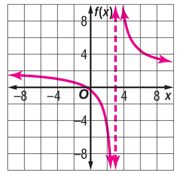
1. $f(x) = \frac{3}{x + 1}$



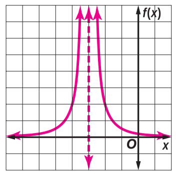
2. $f(x) = \frac{2}{x}$



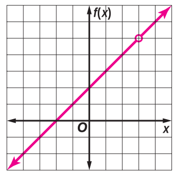
3. $f(x) = \frac{2x + 1}{x - 3}$



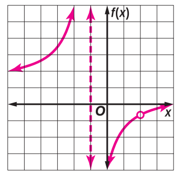
4. $f(x) = \frac{2}{(x + 3)^2}$



5. $f(x) = \frac{x^2 - x - 6}{x - 3}$



6. $f(x) = \frac{x^2 - 6x + 8}{x^2 - x - 2}$



NAME ______________________ DATE __________ PERIOD _____

9-3 Skills Practice

Graphing Rational Functions

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \frac{3}{x^2 - 2x - 8}$ asymptotes: $x = 4$, $x = -2$

2. $f(x) = \frac{10}{x^2 - 13x + 36}$ asymptotes: $x = 4$, $x = 9$

3. $f(x) = \frac{x + 12}{x^2 + 10x - 24}$ asymptote: $x = 2$; hole: $x = -12$

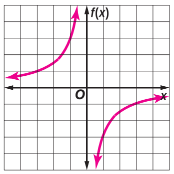
4. $f(x) = \frac{x - 1}{x^2 - 4x + 3}$ asymptote: $x = 3$; hole: $x = 1$

5. $f(x) = \frac{x^2 + 8x + 12}{x + 2}$ hole: $x = -2$

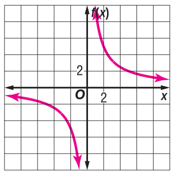
6. $f(x) = \frac{x^2 + x - 12}{x - 3}$ hole: $x = 3$

Graph each rational function.

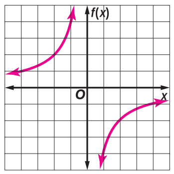
7. $f(x) = \frac{-3}{x}$



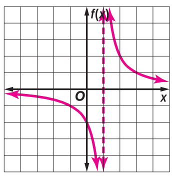
8. $f(x) = \frac{10}{x}$



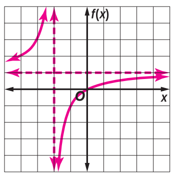
9. $f(x) = \frac{-4}{x}$



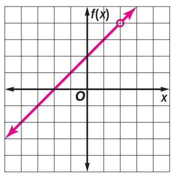
10. $f(x) = \frac{2}{x - 1}$



11. $f(x) = \frac{x}{x + 2}$



12. $f(x) = \frac{x^2 - 4}{x - 2}$



Lesson 9-3

NAME ______________________ DATE __________ PERIOD _____

9-3 Practice (Average)

Graphing Rational Functions

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \frac{6}{x^2 + 3x - 10}$ asymptotes: $x = 2$, $x = -5$

2. $f(x) = \frac{x - 7}{x^2 - 10x + 21}$ asymptote: $x = 3$; hole: $x = 7$

3. $f(x) = \frac{x - 2}{x^2 + 4x + 4}$ asymptote: $x = -2$

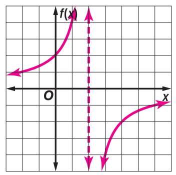
4. $f(x) = \frac{x^2 - 100}{x + 10}$ hole: $x = -10$

5. $f(x) = \frac{x^2 - 2x - 24}{x - 6}$ hole: $x = 6$

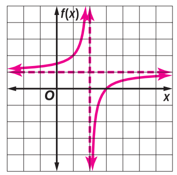
6. $f(x) = \frac{x^2 + 9x + 20}{x + 5}$ hole: $x = -5$

Graph each rational function.

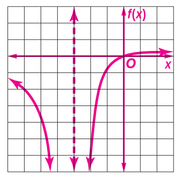
7. $f(x) = \frac{-4}{x - 2}$



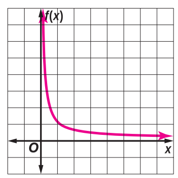
8. $f(x) = \frac{x - 3}{x - 2}$



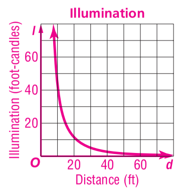
9. $f(x) = \frac{3x}{(x + 3)^2}$



10. PAINTING Working alone, Tawa can give the shed a coat of paint in 6 hours. It takes her father x hours working alone to give the shed a coat of paint. The equation $f(x) = \frac{6 + x}{6x}$ describes the portion of the job Tawa and her father working together can complete in 1 hour. Graph $f(x) = \frac{6 + x}{6x}$ for $x \geq 0, y \geq 0$. If Tawa's father can complete the job in 4 hours alone, what portion of the job can they complete together in 1 hour? $\frac{5}{12}$



11. LIGHT The relationship between the illumination an object receives from a light source of I foot-candles and the square of the distance d in feet of the object from the source can be modeled by $I(d) = \frac{4500}{d^2}$. Graph the function $I(d) = \frac{4500}{d^2}$ for $0 \leq I \leq 80$ and $0 \leq d \leq 80$. What is the illumination in foot-candles that the object receives at a distance of 20 feet from the light source? 11.25 foot-candles



Answers

NAME ______________ DATE ________ PERIOD _____

9-3 Reading to Learn Mathematics

Graphing Rational Functions

Pre-Activity **How can rational functions be used when buying a group gift?**

Read the introduction to Lesson 9-3 at the top of page 485 in your textbook.

- If 15 students contribute to the gift, how much would each of them pay? **$10**
- If each student pays $5, how many students contributed? **30 students**

Reading the Lesson

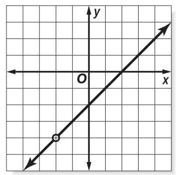
1. Which of the following are rational functions? **A and C**

A. $f(x) = \frac{1}{x - 5}$ **B.** $g(x) = \sqrt{x}$ **C.** $h(x) = \frac{x^2 - 25}{x^2 + 6x + 9}$

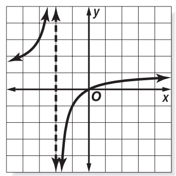
2. a. Graphs of rational functions may have breaks in **continuity**. These may occur as vertical **asymptotes** or as point **discontinuities**.

b. The graphs of two rational functions are shown below.

I.



II.



Graph I has a **point discontinuity** at $x =$ **−2**.

Graph II has a **vertical asymptote** at $x =$ **−2**.

Match each function with its graph above.

$f(x) = \frac{x}{x + 2}$ **II** $g(x) = \frac{x^2 - 4}{x + 2}$ **I**

Helping You Remember

3. One way to remember something new is to see how it is related to something you already know. How can knowing that division by zero is undefined help you to remember how to find the places where a rational function has a point discontinuity or an asymptote?

Sample answer: A point discontinuity or vertical asymptote occurs where the function is undefined, that is, where the denominator of the related rational expression is equal to 0. Therefore, set the denominator equal to zero and solve for the variable.

Lesson 9-3

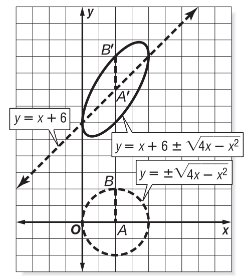
NAME ______________ DATE ________ PERIOD _____

9-3 Enrichment

Graphing with Addition of y-Coordinates

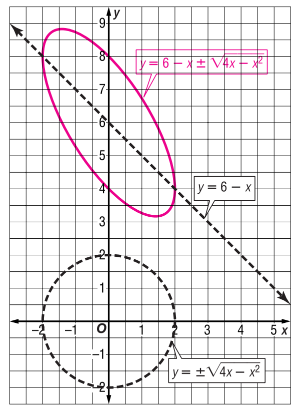
Equations of parabolas, ellipses, and hyperbolas that are "tipped" with respect to the x- and y-axes are more difficult to graph than the equations you have been studying.

Often, however, you can use the graphs of two simpler equations to graph a more complicated equation. For example, the graph of the ellipse in the diagram at the right is obtained by adding the y-coordinate of each point on the circle and the y-coordinate of the corresponding point of the line.

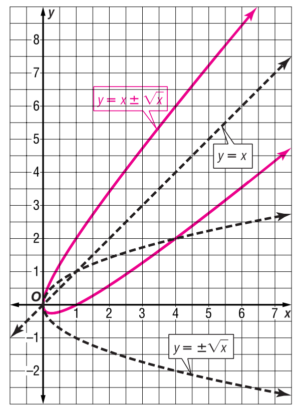


Graph each equation. State the type of curve for each graph.

1. $y = 6 - x \pm \sqrt{4 - x^2}$ **ellipse**



2. $y = x \pm \sqrt{x}$ **parabola**



Use a separate sheet of graph paper to graph these equations. State the type of curve for each graph.

3. $y = 2x \pm \sqrt{7 + 6x - x^2}$ **ellipse**
See students' graphs.

4. $y = -2x \pm \sqrt{-2x}$ **parabola**
See students' graphs.

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9-4 Study Guide and Intervention

Direct, Joint, and Inverse Variation

Direct Variation and Joint Variation

Direct Variation	*y* varies directly as *x* if there is some nonzero constant *k* such that $y = kx$. *k* is called the constant of variation.
Joint Variation	*y* varies jointly as *x* and *z* if there is some number *k* such that $y = kxz$, where $x \neq 0$ and $z \neq 0$.

Example **Find each value.**

a. If *y* varies directly as *x* and $y = 16$ when $x = 4$, find *x* when $y = 20$.

$\frac{y_1}{x_1} = \frac{y_2}{x_2}$ Direct proportion

$\frac{16}{4} = \frac{20}{x_2}$ $y_1 = 16, x_1 = 4$, and $y_2 = 20$

$16x_2 = (20)(4)$ Cross multiply.

$x_2 = 5$ Simplify.

The value of *x* is 5 when *y* is 20.

b. If *y* varies jointly as *x* and *z* and $y = 10$ when $x = 2$ and $z = 4$, find *y* when $x = 4$ and $z = 3$.

$\frac{y_1}{x_1 z_1} = \frac{y_2}{x_2 z_2}$ Joint variation

$\frac{10}{2 \cdot 4} = \frac{y_2}{4 \cdot 3}$ $y_1 = 10, x_1 = 2, z_1 = 4, x_2 = 4$, and $z_2 = 3$

$120 = 8y_2$ Simplify.

$y_2 = 15$ Divide each side by 8.

The value of *y* is 15 when $x = 4$ and $z = 3$.

Exercises

Find each value.

1. If *y* varies directly as *x* and $y = 9$ when $x = 6$, find *y* when $x = 8$. **12**
2. If *y* varies directly as *x* and $y = 16$ when $x = 36$, find *y* when $x = 54$. **24**
3. If *y* varies directly as *x* and $x = 15$ when $y = 5$, find *x* when $y = 9$. **27**
4. If *y* varies directly as *x* and $x = 33$ when $y = 22$, find *x* when $y = 32$. **48**
5. Suppose *y* varies jointly as *x* and *z*. Find *y* when $x = 5$ and $z = 3$, if $y = 18$ when $x = 3$ and $z = 2$. **45**
6. Suppose *y* varies jointly as *x* and *z*. Find *y* when $x = 6$ and $z = 8$, if $y = 6$ when $x = 4$ and $z = 2$. **36**
7. Suppose *y* varies jointly as *x* and *z*. Find *y* when $x = 4$ and $z = 11$, if $y = 60$ when $x = 3$ and $z = 5$. **176**
8. Suppose *y* varies jointly as *x* and *z*. Find *y* when $x = 5$ and $z = 2$, if $y = 84$ when $x = 4$ and $z = 7$. **30**
9. If *y* varies directly as *x* and $y = 14$ when $x = 35$, find *y* when $x = 12$. **4.8**
10. If *y* varies directly as *x* and $x = 200$ when $y = 50$, find *x* when $y = 1000$. **4000**
11. If *y* varies directly as *x* and $y = 39$ when $x = 52$, find *y* when $x = 22$. **16.5**
12. If *y* varies directly as *x* and $x = 60$ when $y = 75$, find *x* when $y = 42$. **33.6**
13. Suppose *y* varies jointly as *x* and *z*. Find *y* when $x = 6$ and $z = 11$, if $y = 120$ when $x = 5$ and $z = 12$. **132**
14. Suppose *y* varies jointly as *x* and *z*. Find *y* when $x = 5$ and $z = 10$, if $y = 12$ when $x = 8$ and $z = 6$. **12.5**
15. Suppose *y* varies jointly as *x* and *z*. Find *y* when $x = 7$ and $z = 18$, if $y = 351$ when $x = 6$ and $z = 13$. **567**
16. Suppose *y* varies jointly as *x* and *z*. Find *y* when $x = 5$ and $z = 27$, if $y = 480$ when $x = 9$ and $z = 20$. **360**

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9-4 Study Guide and Intervention *(continued)*

Direct, Joint, and Inverse Variation

Inverse Variation

Inverse Variation	*y* varies inversely as *x* if there is some nonzero constant *k* such that $xy = k$ or $y = \frac{k}{x}$.

Example **If *a* varies inversely as *b* and $a = 8$ when $b = 12$, find *a* when $b = 4$.**

$\frac{a_1}{b_2} = \frac{a_2}{b_1}$ Inverse variation

$\frac{8}{4} = \frac{a_2}{12}$ $a_1 = 8, b_1 = 12, b_2 = 4$

$8(12) = 4a_2$ Cross multiply.

$96 = 4a_2$ Simplify.

$24 = a_2$ Divide each side by 4.

When $b = 4$, the value of *a* is 24.

Exercises

Find each value.

1. If *y* varies inversely as *x* and $y = 12$ when $x = 10$, find *y* when $x = 15$. **8**
2. If *y* varies inversely as *x* and $y = 9$ when $x = 45$, find *y* when $x = 27$. **15**
3. If *y* varies inversely as *x* and $y = 100$ when $x = 38$, find *y* when $x = 76$. **50**
4. If *y* varies inversely as *x* and $y = 32$ when $x = 42$, find *y* when $x = 24$. **56**
5. If *y* varies inversely as *x* and $y = 36$ when $x = 10$, find *y* when $x = 30$. **12**
6. If *y* varies inversely as *x* and $y = 75$ when $x = 12$, find *y* when $x = 10$. **90**
7. If *y* varies inversely as *x* and $y = 18$ when $x = 124$, find *y* when $x = 93$. **24**
8. If *y* varies inversely as *x* and $y = 90$ when $x = 35$, find *y* when $x = 50$. **63**
9. If *y* varies inversely as *x* and $y = 42$ when $x = 48$, find *y* when $x = 36$. **56**
10. If *y* varies inversely as *x* and $y = 44$ when $x = 20$, find *y* when $x = 55$. **16**
11. If *y* varies inversely as *x* and $y = 80$ when $x = 14$, find *y* when $x = 35$. **32**
12. If *y* varies inversely as *x* and $y = 3$ when $x = 8$, find *y* when $x = 40$. **0.6**
13. If *y* varies inversely as *x* and $y = 16$ when $x = 42$, find *y* when $x = 14$. **48**
14. If *y* varies inversely as *x* and $y = 9$ when $x = 2$, find *y* when $x = 5$. **3.6**
15. If *y* varies inversely as *x* and $y = 23$ when $x = 12$, find *y* when $x = 15$. **18.4**

NAME ______________ DATE __________ PERIOD _____

9-4 Skills Practice

Direct, Joint, and Inverse Variation

State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation.

1. $c = 12m$ **direct; 12**
2. $p = \frac{4}{q}$ **inverse; 4**
3. $A = \frac{1}{2}bh$ **joint; $\frac{1}{2}$**
4. $rw = 15$ **inverse; 15**
5. $y = 2rst$ **joint; 2**
6. $f = 5280m$ **direct; 5280**
7. $y = 0.2s$ **direct; 0.2**
8. $vz = -25$ **inverse; −25**
9. $t = 16rh$ **joint; 16**
10. $R = \frac{8}{w}$ **inverse; 8**
11. $\frac{a}{b} = \frac{1}{3}$ **direct; $\frac{1}{3}$**
12. $C = 2\pi r$ **direct; 2π**

Find each value.

13. If y varies directly as x and $y = 35$ when $x = 7$, find y when $x = 11$. **55**

14. If y varies directly as x and $y = 360$ when $x = 180$, find y when $x = 270$. **540**

15. If y varies directly as x and $y = 540$ when $x = 10$, find x when $y = 1080$. **20**

16. If y varies directly as x and $y = 12$ when $x = 72$, find x when $y = 9$. **54**

17. If y varies jointly as x and z and $y = 18$ when $x = 2$ and $z = 3$, find y when $x = 5$ and $z = 6$. **90**

18. If y varies jointly as x and z and $y = -16$ when $x = 4$ and $z = 2$, find y when $x = -1$ and $z = 7$. **14**

19. If y varies jointly as x and z and $y = 120$ when $x = 4$ and $z = 6$, find y when $x = 3$ and $z = 2$. **30**

20. If y varies inversely as x and $y = 2$ when $x = 2$, find y when $x = 1$. **4**

21. If y varies inversely as x and $y = 6$ when $x = 5$, find y when $x = 10$. **3**

22. If y varies inversely as x and $y = 3$ when $x = 14$, find x when $y = 6$. **7**

23. If y varies inversely as x and $y = 27$ when $x = 2$, find x when $y = 9$. **6**

24. If y varies directly as x and $y = -15$ when $x = 5$, find x when $y = -36$. **12**

NAME ______________ DATE __________ PERIOD _____

9-4 Practice (Average)

Direct, Joint, and Inverse Variation

State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation.

1. $u = 8wz$ **joint; 8**
2. $p = 4s$ **direct; 4**
3. $L = \frac{5}{k}$ **inverse; 5**
4. $xy = 4.5$ **inverse; 4.5**
5. $\frac{C}{d} = \pi$ **direct; π**
6. $2d = mn$ **joint; $\frac{1}{2}$**
7. $\frac{1.25}{g} = h$ **inverse; 1.25**
8. $y = \frac{3}{4x}$ **inverse; $\frac{3}{4}$**

Find each value.

9. If y varies directly as x and $y = 8$ when $x = 2$, find y when $x = 6$. **24**

10. If y varies directly as x and $y = -16$ when $x = 6$, find x when $y = -4$. **1.5**

11. If y varies directly as x and $y = 132$ when $x = 11$, find y when $x = 33$. **396**

12. If y varies directly as x and $y = 7$ when $x = 1.5$, find y when $x = 4$. **$\frac{56}{3}$**

13. If y varies jointly as x and z and $y = 24$ when $x = 2$ and $z = 1$, find y when $x = 12$ and $z = 2$. **288**

14. If y varies jointly as x and z and $y = 60$ when $x = 3$ and $z = 4$, find y when $x = 6$ and $z = 8$. **240**

15. If y varies jointly as x and z and $y = 12$ when $x = -2$ and $z = 3$, find y when $x = 4$ and $z = -1$. **8**

16. If y varies inversely as x and $y = 16$ when $x = 4$, find y when $x = 3$. **$\frac{64}{3}$**

17. If y varies inversely as x and $y = 3$ when $x = 5$, find x when $y = 2.5$. **6**

18. If y varies inversely as x and $y = -18$ when $x = 6$, find y when $x = 5$. **−21.6**

19. If y varies directly as x and $y = 5$ when $x = 0.4$, find x when $y = 37.5$. **3**

20. GASES The volume V of a gas varies inversely as its pressure P. If $V = 80$ cubic centimeters when $P = 2000$ millimeters of mercury, find V when $P = 320$ millimeters of mercury. **500 cm^3**

21. SPRINGS The length S that a spring will stretch varies directly with the weight F that is attached to the spring. If a spring stretches 20 inches with 25 pounds attached, how far will it stretch with 15 pounds attached? **12 in.**

22. GEOMETRY The area A of a trapezoid varies jointly as its height and the sum of its bases. If the area is 480 square meters when the height is 20 meters and the bases are 28 meters and 20 meters, what is the area of a trapezoid when its height is 8 meters and its bases are 10 meters and 15 meters? **100 m^2**

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9-4 Reading to Learn Mathematics

Direct, Joint, and Inverse Variation

Pre-Activity **How is variation used to find the total cost given the unit cost?**

Read the introduction to Lesson 9-4 at the top of page 492 in your textbook.

- For each additional student who enrolls in a public college, the total high-tech spending will **increase** (increase/decrease) by **$149**.
- For each decrease in enrollment of 100 students in a public college, the total high-tech spending will **decrease** (increase/decrease) by **$14,900**.

Reading the Lesson

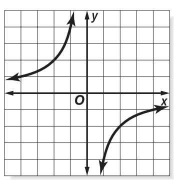
1. Write an equation to represent each of the following variation statements. Use k as the constant of variation.

a. m varies inversely as n. $m = \frac{k}{n}$

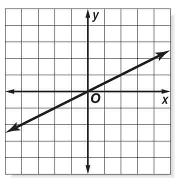
b. s varies directly as r. $s = kr$

c. t varies jointly as p and q. $t = kpq$

2. Which type of variation, direct or inverse, is represented by each graph?

a. 

inverse

b. 

direct

Helping You Remember

3. How can your knowledge of the equation of the slope-intercept form of the equation of a line help you remember the equation for direct variation?

Sample answer: The graph of an equation expressing direct variation is a line. The slope-intercept form of the equation of a line is $y = mx + b$. In direct variation, if one of the quantities is 0, the other quantity is also 0, so $b = 0$ and the line goes through the origin. The equation of a line through the origin is $y = mx$, where m is the slope. This is the same as the equation for direct variation with $k = m$.

NAME ______________________ DATE __________ PERIOD _____

9-4 Enrichment

Expansions of Rational Expressions

Many rational expressions can be transformed into **power series.** A power series is an infinite series of the form $A + Bx + Cx^2 + Dx^3 + \ldots$. The rational expression and the power series normally can be said to have the same values only for certain values of x. For example, the following equation holds only for values of x such that $-1 < x < 1$.

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots \text{ for } -1 < x < 1$$

Example **Expand $\frac{2 + 3x}{1 + x + x^2}$ in ascending powers of x.**

Assume that the expression equals a series of the form $A + Bx + Cx^2 + Dx^3 + \ldots$. Then multiply both sides of the equation by the denominator $1 + x + x^2$.

$$\frac{2 + 3x}{1 + x + x^2} = A + Bx + Cx^2 + Dx^3 + \ldots$$

$$2 + 3x = (1 + x + x^2)(A + Bx + Cx^2 + Dx^3 + \ldots)$$

$$\begin{aligned} 2 + 3x = A + Bx + Cx^2 + Dx^3 + \ldots \\ + Ax + Bx^2 + Cx^3 + \ldots \\ + Ax^2 + Bx^3 + \ldots \end{aligned}$$

$$2 + 3x = A + (B + A)x + (C + B + A)x^2 + (D + C + B)x^3 + \ldots$$

Now, match the coefficients of the polynomials.

$2 = A$
$3 = B + A$
$0 = C + B + A$
$0 = D + C + B + A$

Finally, solve for A, B, C, and D and write the expansion.

$A = 2, B = 1, C = -3$, and $D = 0$

Therefore, $\frac{2 + 3x}{1 + x + x^2} = 2 + x - 3x^2 + \ldots$

Expand each rational expression to four terms.

1. $\frac{1 - x}{1 + x + x^2}$ $1 - 2x + x^2 + x^3 + \ldots$

2. $\frac{2}{1 - x}$ $2 - 2x + 2x^2 - 2x^3 + \ldots$

3. $\frac{1}{1 + x}$ $1 - x + x^2 - x^3 + \ldots$

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9-5 Study Guide and Intervention

Classes of Functions

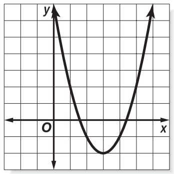
Identify Graphs You should be familiar with the graphs of the following functions.

Function	Description of Graph
Constant	a horizontal line that crosses the y-axis at a
Direct Variation	a line that passes through the origin and is neither horizontal nor vertical
Identity	a line that passes through the point (a, a), where a is any real number
Greatest Integer	a step function
Absolute Value	V-shaped graph
Quadratic	a parabola
Square Root	a curve that starts at a point and curves in only one direction
Rational	a graph with one or more asymptotes and/or holes
Inverse Variation	a graph with 2 curved branches and 2 asymptotes, $x = 0$ and $y = 0$ (special case of rational function)

Exercises

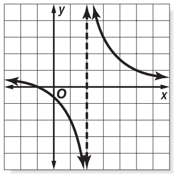
Identify the function represented by each graph.

1.



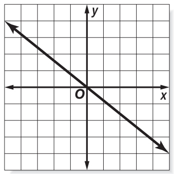
quadratic

2.



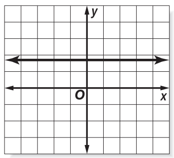
rational

3.



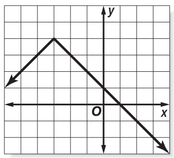
direct variation

4.



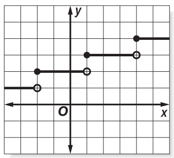
constant

5.



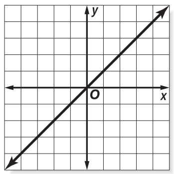
absolute value

6.



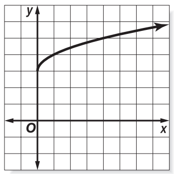
greatest integer

7.



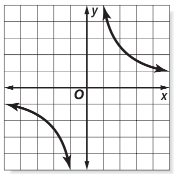
identity

8.



square root

9.



inverse variation

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9-5 Study Guide and Intervention *(continued)*

Classes of Functions

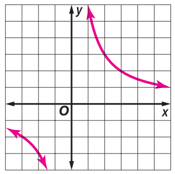
Identify Equations You should be able to graph the equations of the following functions.

Function	General Equation
Constant	$y = a$
Direct Variation	$y = ax$
Identity	$y = x$
Greatest Integer	equation includes a variable within the greatest integer symbol, $[\![\ ]\!]$
Absolute Value	equation includes a variable within the absolute value symbol, $\vert\ \vert$
Quadratic	$y = ax^2 + bx + c$, where $a \neq 0$
Square Root	equation includes a variable beneath the radical sign, $\sqrt{\ }$
Rational	$y = \frac{p(x)}{q(x)}$
Inverse Variation	$y = \frac{a}{x}$

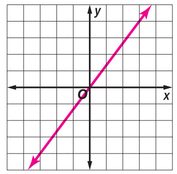
Exercises

Identify the function represented by each equation. Then graph the equation.

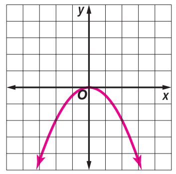
1. $y = \frac{6}{x}$ inverse variation



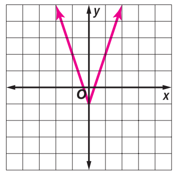
2. $y = \frac{4}{3}x$ direct variation



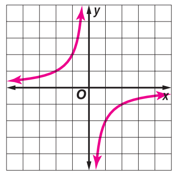
3. $y = -\frac{x^2}{2}$ quadratic



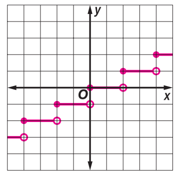
4. $y = |3x| - 1$ absolute value



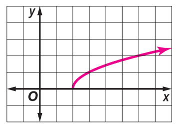
5. $y = -\frac{2}{x}$ inverse variation



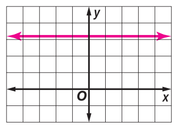
6. $y = [\![\frac{x}{2}]\!]$ greatest integer



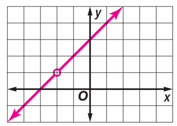
7. $y = \sqrt{x - 2}$ square root



8. $y = 3.2$ constant



9. $y = \frac{x^2 + 5x + 6}{x + 2}$ rational

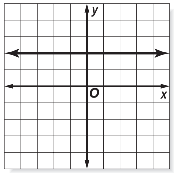


NAME ______________________ DATE ________ PERIOD _____

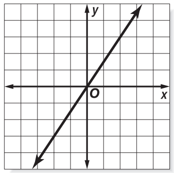
9-5 Skills Practice

Classes of Functions

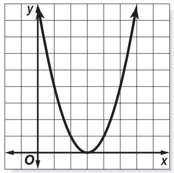
Identify the type of function represented by each graph.

1. 

constant

2. 

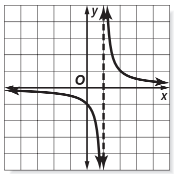
direct variation

3. 

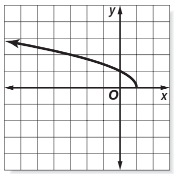
quadratic

Match each graph with an equation below.

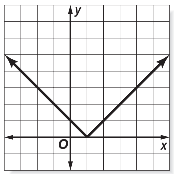
A. $y = |x - 1|$ **B.** $y = \frac{1}{x - 1}$ **C.** $y = \sqrt{1 - x}$ **D.** $y = [\![x]\!] - 1$

4. 

B

5. 

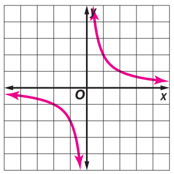
C

6. 

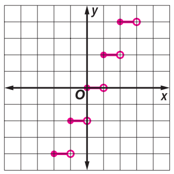
A

Identify the type of function represented by each equation. Then graph the equation.

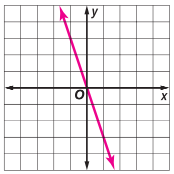
7. $y = \frac{2}{x}$ inverse variation or rational



8. $y = 2[\![x]\!]$ greatest integer



9. $y = -3x$ direct variation



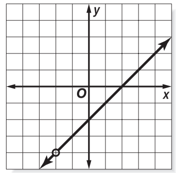
Lesson 9-5

NAME ______________________ DATE ________ PERIOD _____

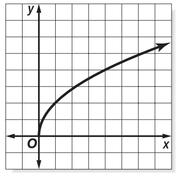
9-5 Practice (Average)

Classes of Functions

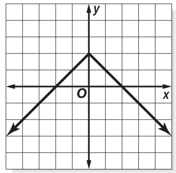
Identify the type of function represented by each graph.

1. 

rational

2. 

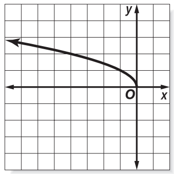
square root

3. 

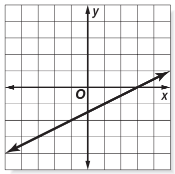
absolute value

Match each graph with an equation below.

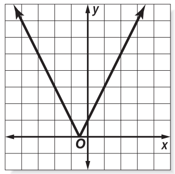
A. $y = |2x + 1|$ **B.** $y = [\![2x + 1]\!]$ **C.** $y = \frac{x - 3}{2}$ **D.** $y = \sqrt{-x}$

4. 

D

5. 

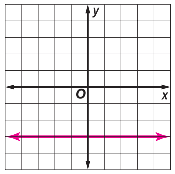
C

6. 

A

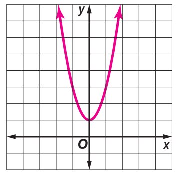
Identify the type of function represented by each equation. Then graph the equation.

7. $y = -3$



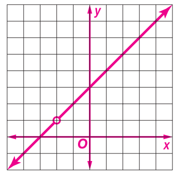
constant

8. $y = 2x^2 + 1$



quadratic

9. $y = \frac{x^2 + 5x + 6}{x + 2}$



rational

10. **BUSINESS** A startup company uses the function $P = 1.3x^2 + 3x - 7$ to predict its profit or loss during its first 7 years of operation. Describe the shape of the graph of the function. The graph is U-shaped; it is a parabola.

11. **PARKING** A parking lot charges $10 to park for the first day or part of a day. After that, it charges an additional $8 per day or part of a day. Describe the graph and find the cost of parking for $6\frac{1}{2}$ days. The graph looks like a series of steps, similar to a greatest integer function, but with open circles on the left and closed circles on the right; $58.

Answers

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9-5 Reading to Learn Mathematics

Classes of Functions

Pre-Activity **How can graphs of functions be used to determine a person's weight on a different planet?**

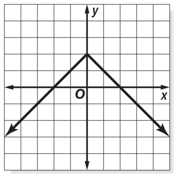
Read the introduction to Lesson 9-5 at the top of page 499 in your textbook.

- Based on the graph, estimate the weight on Mars of a child who weighs 40 pounds on Earth.
  about 15 pounds
- Although the graph does not extend far enough to the right to read it directly from the graph, use the weight you found above and your knowledge that this graph represents direct variation to estimate the weight on Mars of a woman who weighs 120 pounds on Earth.
  about 45 pounds

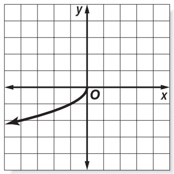
Reading the Lesson

1. Match each graph below with the type of function it represents. Some types may be used more than once and others not at all.

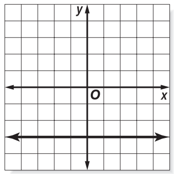
I. square root **II.** quadratic **III.** absolute value **IV.** rational
V. greatest integer **VI.** constant **VII.** identity

a. 

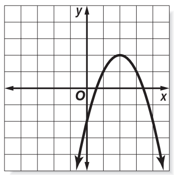
III

b. 

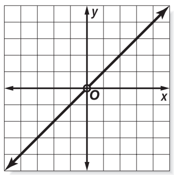
I

c. 

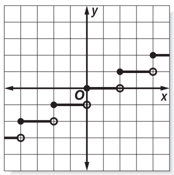
VI

d. 

II

e. 

IV

f. 

V

Helping You Remember

2. How can the symbolic definition of absolute value that you learned in Lesson 1-4 help you to remember the graph of the function $f(x) = |x|$? **Sample answer: Using the definition of absolute value, $f(x) = x$ if $x \geq 0$ and $f(x) = -x$ if $x < 0$. Therefore, the graph is made up of pieces of two lines, one with slope 1 and one with slope −1, meeting at the origin. This forms a V-shaped graph with "vertex" at the origin.**

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9-5 Enrichment

Partial Fractions

It is sometimes an advantage to rewrite a rational expression as the sum of two or more fractions. For example, you might do this in a calculus course while carrying out a procedure called integration.

You can resolve a rational expression into partial fractions if two conditions are met:

(1) The degree of the numerator must be less than the degree of the denominator; and

(2) The factors of the denominator must be known.

Example **Resolve $\frac{3}{x^3 + 1}$ into partial fractions.**

The denominator has two factors, a linear factor, $x + 1$, and a quadratic factor, $x^2 - x + 1$. Start by writing the following equation. Notice that the degree of the numerators of each partial fraction is less than its denominator.

$$\frac{3}{x^3 + 1} = \frac{A}{x + 1} + \frac{Bx + C}{x^2 - x + 1}$$

Now, multiply both sides of the equation by $x^3 + 1$ to clear the fractions and finish the problem by solving for the coefficients A, B, and C.

$$\frac{3}{x^3 + 1} = \frac{A}{x + 1} + \frac{Bx + C}{x^2 - x + 1}$$

$$3 = A(x^2 - x + 1) + (x + 1)(Bx + C)$$

$$3 = Ax^2 - Ax + A + Bx^2 + Cx + Bx + C$$

$$3 = (A + B)x^2 + (B + C - A)x + (A + C)$$

Equating each term, $0x^2 = (A + B)x^2$

$0x = (B + C - A)x$

$3 = (A + C)$

Therefore, $A = 1$, $B = -1$, $C = 2$, and $\frac{3}{x^3 + 1} = \frac{1}{x + 1} + \frac{-x + 2}{x^2 - x + 1}$.

Resolve each rational expression into partial fractions.

1. $\frac{5x - 3}{x^2 - 2x - 3} = \frac{A}{x + 1} + \frac{B}{x - 3}$ **$A = 2$, $B = 3$**

2. $\frac{6x + 7}{(x + 2)^2} = \frac{A}{x + 2} + \frac{B}{(x + 2)^2}$ **$A = 6$, $B = -5$**

3. $\frac{4x^3 - x^2 - 3x - 2}{x^2(x + 1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x + 1} + \frac{D}{(x + 1)^2}$ **$A = 1$, $B = -2$, $C = 3$, $D = -4$**

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9-6 Study Guide and Intervention

Solving Rational Equations and Inequalities

Solve Rational Equations A **rational equation** contains one or more rational expressions. To solve a rational equation, first multiply each side by the least common denominator of all of the denominators. Be sure to exclude any solution that would produce a denominator of zero.

Example **Solve $\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$.**

$\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$ Original equation

$10(x+1)\left(\frac{9}{10} + \frac{2}{x+1}\right) = 10(x+1)\left(\frac{2}{5}\right)$ Multiply each side by $10(x + 1)$.

$9(x+1) + 2(10) = 4(x+1)$ Multiply.

$9x + 9 + 20 = 4x + 4$ Distributive Property

$5x = -25$ Subtract 4x and 29 from each side.

$x = -5$ Divide each side by 5.

Check

$\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$ Original equation

$\frac{9}{10} + \frac{2}{-5+1} \stackrel{?}{=} \frac{2}{5}$ $x = -5$

$\frac{9}{10} + \frac{2}{-4} \stackrel{?}{=} \frac{2}{5}$ Simplify.

$\frac{18}{20} - \frac{10}{20} \stackrel{?}{=} \frac{2}{5}$ Simplify.

$\frac{8}{20} \stackrel{?}{=} \frac{2}{5}$ Simplify.

$\frac{2}{5} = \frac{2}{5}$

Exercises

Solve each equation.

1. $\frac{2y}{3} - \frac{y+3}{6} = 2$ **5**

2. $\frac{4t-3}{5} - \frac{4-2t}{3} = 1$ **2**

3. $\frac{2x+1}{3} - \frac{x-5}{4} = \frac{1}{2}$ **$-\frac{13}{5}$**

4. $\frac{3m+2}{5m} + \frac{2m-1}{2m} = 4$ **$-\frac{1}{24}$**

5. $\frac{4}{x-1} = \frac{x+1}{12}$ **± 7**

6. $\frac{x}{x-2} + \frac{4}{x-2} = 10$ **$\frac{8}{3}$**

7. **NAVIGATION** The current in a river is 6 miles per hour. In her motorboat Marissa can travel 12 miles upstream or 16 miles downstream in the same amount of time. What is the speed of her motorboat in still water? **42 mph**

8. **WORK** Adam, Bethany, and Carlos own a painting company. To paint a particular house alone, Adam estimates that it would take him 4 days, Bethany estimates $5\frac{1}{2}$ days, and Carlos 6 days. If these estimates are accurate, how long should it take the three of them to paint the house if they work together? **about $1\frac{2}{3}$ days**

Lesson 9-6

NAME ______________________ DATE __________ PERIOD _____

9-6 Study Guide and Intervention *(continued)*

Solving Rational Equations and Inequalities

Solve Rational Inequalities To solve a rational inequality, complete the following steps.

Step 1 State the excluded values.
Step 2 Solve the related equation.
Step 3 Use the values from steps 1 and 2 to divide the number line into regions. Test a value in each region to see which regions satisfy the original inequality.

Example **Solve $\frac{2}{3n} + \frac{4}{5n} \le \frac{2}{3}$.**

Step 1 The value of 0 is excluded since this value would result in a denominator of 0.

Step 2 Solve the related equation.

$\frac{2}{3n} + \frac{4}{5n} = \frac{2}{3}$ Related equation

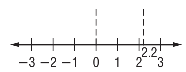
$15n\left(\frac{2}{3n} + \frac{4}{5n}\right) = 15n\left(\frac{2}{3}\right)$ Multiply each side by $15n$.

$10 + 12 = 10n$ Simplify.

$22 = 10n$ Simplify.

$2.2 = n$ Simplify.

Step 3 Draw a number with vertical lines at the excluded value and the solution to the equation.



Test $n = -1$.
$-\frac{2}{3} + \left(-\frac{4}{5}\right) \le \frac{2}{3}$ is true.

Test $n = 1$.
$\frac{2}{3} + \frac{4}{5} \le \frac{2}{3}$ is *not* true.

Test $n = 3$.
$\frac{2}{9} + \frac{4}{15} \le \frac{2}{3}$ is true.

The solution is $n < 0$ or $n \ge 2.2$.

Exercises

Solve each inequality.

1. $\frac{3}{a+1} \ge 3$ **$-1 < a \le 0$**

2. $\frac{1}{x} \ge 4x$ **$x \le -\frac{1}{2}$ or $0 < x \le \frac{1}{2}$**

3. $\frac{1}{2p} + \frac{4}{5p} > \frac{2}{3}$ **$0 < p < \frac{39}{20}$**

4. $\frac{3}{2x} - \frac{2}{x} > \frac{1}{4}$ **$-2 < x < 0$**

5. $\frac{4}{x-1} + \frac{5}{x} < 2$ **$x < 0$ or $\frac{1}{2} < x < 1$ or $x > 5$**

6. $\frac{3}{x^2-1} + 1 > \frac{2}{x-1}$ **$x < -1$ or $0 < x < 1$ or $x > 2$**

Answers

NAME ______________________ DATE ________ PERIOD _____

9-6 Skills Practice

Solving Rational Equations and Inequalities

Solve each equation or inequality. Check your solutions.

1. $\frac{x}{x-1} = \frac{1}{2}$ **−1**
2. $2 = \frac{4}{n} + \frac{1}{3}$ **$\frac{12}{5}$**
3. $\frac{9}{3x} = \frac{-6}{2}$ **−1**
4. $3 - z = \frac{2}{z}$ **1, 2**
5. $\frac{2}{d+1} = \frac{1}{d-2}$ **5**
6. $\frac{s-3}{5} = \frac{8}{s}$ **−5, 8**
7. $\frac{2x+3}{x+1} = \frac{3}{2}$ **−3**
8. $-\frac{12}{y} = y - 7$ **3, 4**
9. $\frac{x-2}{x+4} = \frac{x+1}{x+10}$ **8**
10. $\frac{3}{k} - \frac{4}{3k} > 0$ **$k > 0$**
11. $2 - \frac{3}{v} < \frac{5}{v}$ **$0 < v < 4$**
12. $n + \frac{3}{n} < \frac{12}{n}$ **$n < -3$ or $0 < n < 3$**
13. $\frac{1}{2m} - \frac{3}{m} < -\frac{5}{2}$ **$0 < m < 1$**
14. $\frac{1}{2x} < \frac{2}{x} - 1$ **$0 < x < \frac{3}{2}$**
15. $\frac{15}{x} + \frac{9x-7}{x+2} = 9$ **3**
16. $\frac{3b-2}{b+1} = 4 - \frac{b+2}{b-1}$ **4**
17. $2 = \frac{5}{2q} + \frac{2q}{q+1}$ **−5**
18. $8 - \frac{4}{z} = \frac{8z-8}{z+2}$ **$\frac{2}{5}$**
19. $\frac{1}{n+3} + \frac{5}{n^2-9} = \frac{2}{n-3}$ **−4**
20. $\frac{1}{w+2} + \frac{1}{w-2} = \frac{4}{w^2-4}$ **$\varnothing$**
21. $\frac{x-8}{2x+2} + \frac{x}{2x+2} = \frac{2x-3}{x+1}$ **$\varnothing$**
22. $\frac{12s+19}{s^2+7s+12} - \frac{3}{s+3} = \frac{5}{s+4}$ **2**
23. $\frac{2e}{e^2-4} + \frac{1}{e-2} = \frac{2}{e+2}$ **−6**
24. $\frac{8}{t^2-9} + \frac{4}{t+3} = \frac{2}{t-3}$ **5**

NAME ______________________ DATE ________ PERIOD _____

9-6 Practice (Average)

Solving Rational Equations and Inequalities

Solve each equation or inequality. Check your solutions.

1. $\frac{12}{x} + \frac{3}{4} = \frac{3}{2}$ **16**
2. $\frac{x}{x-1} - 1 = \frac{x}{2}$ **−1, 2**
3. $\frac{p+10}{p^2-2} = \frac{4}{p}$ **$-\frac{2}{3}$, 4**
4. $\frac{s}{s+2} + s = \frac{5s+8}{s+2}$ **4**
5. $\frac{5}{y-5} = \frac{y}{y-5} - 1$ **all reals except 5**
6. $\frac{1}{3x-2} + \frac{5}{x} = 0$ **$\frac{5}{8}$**
7. $\frac{5}{t} < \frac{9}{2t+1}$ **$t < -5$ or $-\frac{1}{2} < t < 0$**
8. $\frac{1}{2h} + \frac{5}{h} = \frac{3}{h-1}$ **$\frac{11}{5}$**
9. $\frac{4}{w-2} = \frac{-1}{w+3}$ **−2**
10. $5 - \frac{3}{a} < \frac{7}{a}$ **$0 < a < 2$**
11. $\frac{4}{5x} + \frac{1}{10} < \frac{3}{2x}$ **$0 < x < 7$**
12. $8 + \frac{3}{y} > \frac{19}{y}$ **$y < 0$ or $y > 2$**
13. $\frac{4}{p} + \frac{1}{3p} < \frac{1}{5}$ **$p < 0$ or $p > \frac{65}{3}$**
14. $\frac{6}{x-1} = \frac{4}{x-2} + \frac{2}{x+1}$ **$\varnothing$**
15. $g + \frac{g}{g-2} = \frac{2}{g-2}$ **−1**
16. $b + \frac{2b}{b-1} = 1 - \frac{b-3}{b-1}$ **−2**
17. $2 = \frac{x+2}{x-3} + \frac{x-2}{x-6}$ **$\frac{14}{3}$**
18. $5 - \frac{3d+2}{d-1} = \frac{2d-4}{d+2}$ **6**
19. $\frac{1}{n+2} + \frac{1}{n-2} = \frac{3}{n^2-4}$ **$\frac{3}{2}$**
20. $\frac{c+1}{c-3} = 4 - \frac{12}{c^2-2c-3}$ **$-\frac{5}{3}$, 5**
21. $\frac{3}{k-3} + \frac{4}{k-4} = \frac{25}{k^2-7k+12}$ **7**
22. $\frac{4v}{v-1} - \frac{5v}{v-2} = \frac{2}{v^2-3v+2}$ **−1, −2**
23. $\frac{y}{y+2} + \frac{7}{y-5} = \frac{14}{y^2-3y-10}$ **0**
24. $\frac{x^2+4}{x^2-4} + \frac{x}{2-x} = \frac{2}{x+2}$ **$\varnothing$**
25. $\frac{r}{r+4} + \frac{4}{r-4} = \frac{r^2+16}{r^2-16}$ **all reals except −4 and 4**
26. $3 = \frac{6a-1}{2a+7} + \frac{22}{a+5}$ **−2**

27. **BASKETBALL** Kiana has made 9 of 19 free throws so far this season. Her goal is to make 60% of her free throws. If Kiana makes her next x free throws in a row, the function $f(x) = \frac{9+x}{19+x}$ represents Kiana's new ratio of free throws made. How many successful free throws in a row will raise Kiana's percent made to 60%? **6**

28. **OPTICS** The lens equation $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ relates the distance p of an object from a lens, the distance q of the image of the object from the lens, and the focal length f of the lens. What is the distance of an object from a lens if the image of the object is 5 centimeters from the lens and the focal length of the lens is 4 centimeters? **20 cm**

NAME ______________________ DATE __________ PERIOD _____

9-6 Reading to Learn Mathematics

Solving Rational Equations and Inequalities

Pre-Activity **How are rational equations used to solve problems involving unit price?**

Read the introduction to Lesson 9-6 at the top of page 505 in your textbook.

- If you increase total number of minutes of long-distance calls from March to April, will your long-distance phone bill increase or decrease? **increase**
- Will your actual cost per minute increase or decrease? **decrease**

Reading the Lesson

1. When solving a rational equation, any possible solution that results in 0 in the denominator must be excluded from the list of solutions.

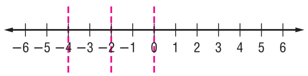
2. Suppose that on a quiz you are asked to solve the rational inequality $\frac{3}{z+2} - \frac{6}{z} > 0$. Complete the steps of the solution.

Step 1 The excluded values are **−2** and **0**.

Step 2 The related equation is $\frac{3}{z+2} - \frac{6}{z} = 0$.

To solve this equation, multiply both sides by the LCD, which is **$z(z + 2)$**. Solving this equation will show that the only solution is −4.

Step 3 Divide a number line into **4** regions using the excluded values and the solution of the related equation. Draw dashed vertical lines on the number line below to show these regions.



Consider the following values of $\frac{3}{z+2} - \frac{6}{z}$ for various test values of z.

If $z = -5$, $\frac{3}{z+2} - \frac{6}{z} = 0.2$.

If $z = -3$, $\frac{3}{z+2} - \frac{6}{z} = -1$.

If $z = -1$, $\frac{3}{z+2} - \frac{6}{z} = 9$.

If $z = 1$, $\frac{3}{z+2} - \frac{6}{z} = -5$.

Using this information and your number line, write the solution of the inequality.

$z < -4$ or $-2 < z < 0$

Helping You Remember

3. How are the processes of adding rational expressions with different denominators and of solving rational expressions alike, and how are they different? **Sample answer: They are alike because both use the LCD of all the rational expressions in the problem. They are different because in an addition problem, the LCD remains after the fractions are added, while in solving a rational equation, the LCD is eliminated.**

NAME ______________________ DATE __________ PERIOD _____

9-6 Enrichment

Limits

Sequences of numbers with a rational expression for the general term often approach some number as a finite limit. For example, the reciprocals of the positive integers approach 0 as n gets larger and larger. This is written using the notation shown below. The symbol ∞ stands for infinity and $n \to \infty$ means that n is getting larger and larger, or "n goes to infinity."

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n}, \ldots \qquad \lim_{n\to\infty} \frac{1}{n} = 0$$

Example **Find $\lim_{n\to\infty} \frac{n^2}{(n+1)^2}$**

It is not immediately apparent whether the sequence approaches a limit or not. But notice what happens if we divide the numerator and denominator of the general term by n^2.

$$\frac{n^2}{(n+1)^2} = \frac{n^2}{n^2 + 2n + 1} = \frac{\frac{n^2}{n^2}}{\frac{n^2}{n^2} + \frac{2n}{n^2} + \frac{1}{n^2}} = \frac{1}{1 + \frac{2}{n} + \frac{1}{n^2}}$$

The two fractions in the denominator will approach a limit of 0 as n gets very large, so the entire expression approaches a limit of 1.

Find the following limits.

1. $\lim_{n\to\infty} \frac{n^3 + 5n}{n^4 - 6}$ **0**

2. $\lim_{n\to\infty} \frac{1 - n}{n^2}$ **0**

3. $\lim_{n\to\infty} \frac{2(n+1) + 1}{2n + 1}$ **1**

4. $\lim_{n\to\infty} \frac{2n + 1}{1 - 3n}$ **$-\frac{2}{3}$**

Chapter 9 Assessment Answer Key

Form 1
Page 553

1. C
2. A
3. A
4. B
5. D
6. B
7. A
8. C
9. C
10. D

Page 554

11. A
12. B
13. C
14. B
15. A
16. B
17. D
18. C
19. A
20. A

B: asymptote: $x = 0$; hole: $x = 3$

Form 2A
Page 555

1. C
2. B
3. D
4. A
5. B
6. C
7. B
8. D
9. A
10. B

(continued on the next page)

Chapter 9 Assessment Answer Key

Form 2A *(continued)*
Page 556

11. A
12. C
13. C
14. D
15. A
16. D
17. B
18. A
19. C
20. B

B: $\frac{x}{x+1}$

Form 2B
Page 557

1. B
2. D
3. A
4. D
5. C
6. B
7. D
8. A
9. C
10. A

Page 558

11. B
12. A
13. B
14. C
15. D
16. B
17. C
18. B
19. A
20. B

B: $\frac{x}{x+1}$

Answers

Chapter 9 Assessment Answer Key

Form 2C

Page 559

1. $-\frac{3}{2}, 3$

2. $\frac{x}{x-8}$

3. $\frac{b+5}{2}$

4. $\frac{9(m+5)}{8}$

5. $\frac{2}{x+2}$

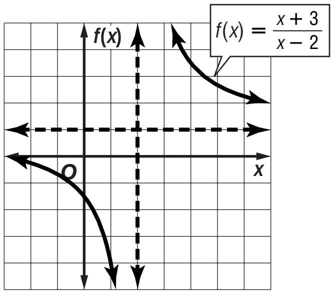
6. $\frac{7}{3m-1}$

7. $36m^4n^4$

8. $(n+2)(n-4)(n+6)$

9. asymptote: $x = 3$

10. hole: $x = -2$

11. 

12. 15

Page 560

13. 192 customers

14. 1

15. $P = \frac{k}{A}$

16. square root

17. direct variation

18. 1

19. $m < 0$ or $m > 5$

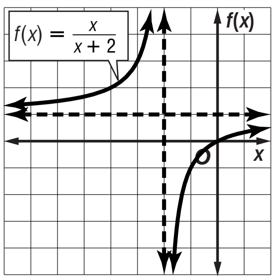
20. 4.8 h

B: $\varnothing$

Chapter 9 Assessment Answer Key

Form 2D

Page 561

1. $-2, \frac{5}{2}$
2. $\frac{x^2}{x - 5}$
3. $\frac{m - 1}{6m}$
4. $\frac{2(y + 2)}{3}$
5. $\frac{3}{x + 3}$
6. $\frac{5}{2n - 1}$
7. $42s^3t^4$
8. $(n + 1)(n + 5)(n - 2)$
9. asymptote: $x = -4$; hole: $x = 6$
10. asymptotes: $x = -5$, $x = -2$
11. 

12. 63

Page 562

13. 1050 permits
14. 9
15. $R = \frac{k}{U}$
16. constant
17. inverse variation
18. $-\frac{1}{3}$
19.
20. 48 min

B: $\varnothing$

Answers

Chapter 9 Assessment Answer Key

Form 3

Page 563

1. $-\frac{1}{3}, 0, \frac{5}{2}$

2. $\frac{3(2x + 3)}{x(3x - 5)}$

3. $\frac{g - 3}{5}$

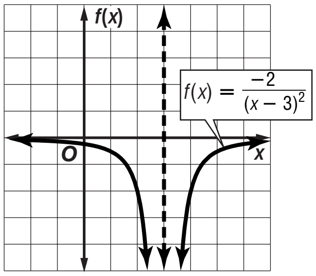
4. $\frac{4m + 3n}{4m - 3n}$

5. 0

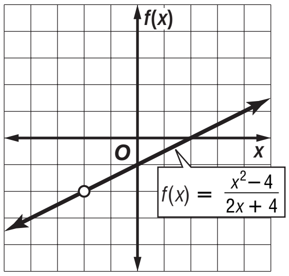
6. −1

7. $(c - d)(c + d)^2$

8. asymptote: $x = 3$



9. hole: $x = -2$



10. $\frac{1}{10}$

Page 564

11. 0.02

12. 271 mi

13. $I = \frac{7.2}{R}$; 40

14. rational

15. inverse variation

16. 10

17. ∅

18. $x < 0$ or $x > \frac{3}{2}$

19. $z < -1$ or $-1 < z < 1$

20. $\frac{15}{25}$

B: $-\frac{x^2(2x + 3)}{3x + 2}$; $x \neq -\frac{2}{3}, 0, \frac{3}{2}$

Chapter 9 Assessment Answer Key

Page 565, Open-Ended Assessment
Scoring Rubric

Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts of *simplifying rational expressions, determining vertical asymptotes and point discontinuity of rational functions, solving joint variation problems, identifying equations as different types of functions,* and *solving rational equations and inequalities.* • Uses appropriate strategies to solve problems. • Computations are correct. • Written explanations are exemplary. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows an understanding of the concepts of *simplifying rational expressions, determining vertical asymptotes and point discontinuity of rational functions, solving joint variation problems, identifying equations as different types of functions,* and *solving rational equations and inequalities.* • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Satisfies all requirements of problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows an understanding of most of the concepts of *simplifying rational expressions, determining vertical asymptotes and point discontinuity of rational functions, solving joint variation problems, identifying equations as different types of functions,* and *solving rational equations and inequalities.* • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work is shown to substantiate the final computation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts of *simplifying rational expressions, determining vertical asymptotes and point discontinuity of rational functions, solving joint variation problems, identifying equations as different types of functions,* and *solving rational equations and inequalities.* • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Does not satisfy requirements of problems. • No answer may be given.

Chapter 9 Assessment Answer Key

Page 565, Open-Ended Assessment
Sample Answers

In addition to the scoring rubric found on page A25, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1. Each student response must include three expressions which, when simplified, "reduce to" $\frac{a}{a-5}$.
Sample answer: $\frac{3a}{3a-15}$, $\frac{a^2}{a^2-5a}$, $\frac{a(a+1)}{(a-5)(a+1)}$.

2a. Students should explain that the height can be found by dividing the volume by the product of the length and width of the box.

2b. $(x + 3)$ in.

2c. Sample answer: Substitute a value for x in each of the given expressions for the length, width, and volume, and the same value for x in the expression found for h, and then check that $V = \ell wh$.
CHECK For $x = 5$,
length $= (5) + 10 = 15$ in.
width $= 2(5) = 10$ in.
volume $= 2(5)^3 + 26(5)^2 + 60(5)$
$= 1200$ in^3
height $= (5) + 3 = 8$ in.
Verify $V = \ell wh$: $1200 = (15)(10)(8)$ ✓

3. Each student response must include two polynomials in which 3, $y + 2$, and $y - 2$ each appears as a factor of at least one of those polynomials, but which have no other factor. Sample answer: $y^2 - 4$, $3(y + 2)$.

4. Student responses should indicate that the graph of $f(x)$ has a hole at $x = -2$, but no vertical asymptote. Its graph is a straight line with a hole in it at $(-2, -5)$. The graph of $g(x)$ also has a hole at $x = -2$, but has a vertical asymptote at $x = 0$. Its graph is not a straight line, but two curves having a hole in the graph at $\left(-2, \frac{5}{2}\right)$.

5a. $d = 0.10hr$

5b. joint variation; the amount deducted varies directly as the product of two quantities, the hourly wage and the number of hours worked.

5c. Students should indicate that they should substitute $r = 9.50$ and $h = 36$ in the formula they wrote in part **a**.
The amount deducted was $34.20.

6a. Students should conclude that $C(x)$ is a rational function since it is of the form $y = \frac{p(x)}{q(x)}$, where $p(x) = 60x + 17{,}000$ and $q(x) = x - 50$ are polynomial functions.

6b. Students should indicate that $R(x)$ is a constant function since it is of the form $y = a$, where a is any number.

6c. $80 > \frac{60x + 17{,}000}{x - 50}$

6d. $x > 1050$; The company must produce and sell at least 1050 CD players in order to ensure that the revenue from each one is greater than the average cost of producing each one.

Chapter 9 Assessment Answer Key

Vocabulary Test/Review
Page 566

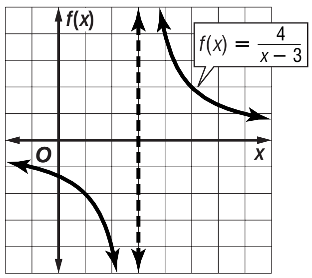
1. inverse variation
2. rational function
3. asymptote
4. least common denominator
5. joint variation
6. continuity
7. rational inequality
8. inversely proportional
9. constant of variation
10. point discontinuity
11. Sample answer: A rational expression is the ratio of two polynomials. The denominator cannot be 0.
12. Sample answer: A complex fraction is a fraction in which the numerator, denominator, or both, contain fractions.

Quiz (Lessons 9–1 and 9–2)
Page 567

1. $\frac{8x^5}{a^2}$
2. $\frac{x+3}{3}$
3. $(2x - 3)(x - 6)$
4. $\frac{p}{5}$
5. E
6. $60a^2b^3$
7. $15(x - 2)(x + 2)$
8. $(t - 1)(2t - 1)(t + 4)$
9. $\frac{35 - 2m}{5m^2n}$
10. $\frac{12}{y - 3}$

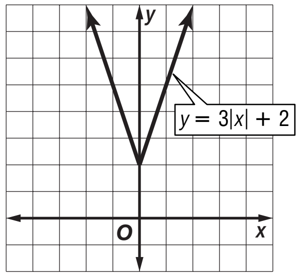
Quiz (Lesson 9–3)
Page 567

1. asymptotes: $x = -2$, $x = 1$
2. asymptote: $x = 1$; hole: $x = -3$
3. hole: $x = -4$
4.



Quiz (Lessons 9–4 and 9–5)
Page 568

1. inverse; 30
2. 12
3. quadratic
4. greatest integer
5. absolute value



Quiz (Lesson 9–6)
Page 568

1. 10
2. 9
3. $t < 0$ or $t > 2$
4. $-5 < m < -2$
5. 18

Chapter 9 Assessment Answer Key

Mid-Chapter Test
Page 569

1. D

2. A

3. A

4. C

5. B

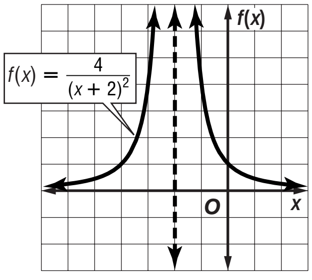
6. $\dfrac{x^2 - 5x - 3}{(x + 3)(x - 2)(x - 4)}$

7. $72s^3t^4$

8. $21(3c - 5)$

9. asymptote: $x = 4$; hole: $x = -3$

10.



Cumulative Review
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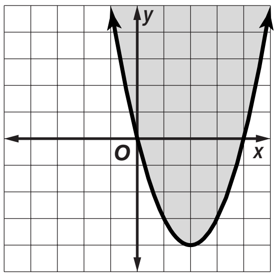
1. yes

2. $w^{\frac{2}{15}}$

3. $-1 \pm i$

4.



5. 23

6. $\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}$

7. $x^2 + (y + 3)^2 = 125$

8. $\dfrac{(x - 1)^2}{25} + \dfrac{(y + 2)^2}{9} = 1$

9. circle

10. $3(y + 2)$

11. 20

12. 11.2 mL

Chapter 9 Assessment Answer Key

Standardized Test Practice

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1. **A** B C D

2. E **F** G H

3. A B **C** D

4. **E** F G H

5. A B C **D**

6. **E** F G H

7. **A** B C D

8. E F **G** H

9. A **B** C D

10. E F G **H**

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11. 25

12. 3/20

13. 13

14. 104

15. A B C **D**

16. A B **C** D

17. **A** B C D

18. A B **C** D

Answers