CHAPTER 9 Series–Parallel Analysis of AC Circuits

Chapter Outline

9.1 AC SERIES CIRCUITS

In DC analysis of series-parallel circuits, what type of numbers was utilized? ___________________________

If the signal source is AC instead of DC and the circuit contains inductors and/or capacitors, what type of numbers would you expect to be utilized? Why?

Good news: <u>the circuit analysis ***techniques*** are the same</u>.

What is the phase shift of the voltage with respect to the current for an inductor? _______________ Why?

What is the phase shift of the voltage with respect to the current for a capacitor? _______________ Why?

What is the phase shift of the voltage with respect to the current for a resistor? _______________ Why?

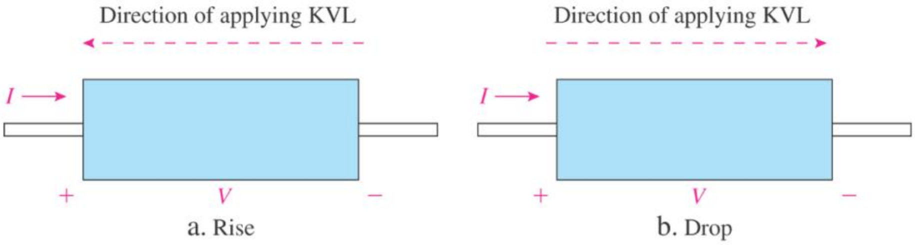
What would you expect the phase shift to be for AC signals in series, parallel, and series–parallel circuits with resistances, inductors, *and* capacitors? Why?

How is series–parallel circuit analysis with AC sources approached? Review the circuit laws, concepts, and results for DC circuit analysis by stating in words each relation that follows:

1. $I_1 = I_2 = I_3 = \ldots = I_N$ ______________________________ **(9.1)**

2. $\sum_{n=1}^{N} V_n = +V_{\text{rise1}} + V_{\text{rise2}} + V_{\text{rise3}} + \ldots - V_{\text{drop1}} - V_{\text{drop2}} - V_{\text{drop3}} - \ldots = 0$ ______________________________ **(9.2)**

How are the signs for voltage rises and voltage drops assigned (**Figure 9.1** below)?



Is the key to the sign of the voltage rise (or drop) the current direction or the direction that KVL is applied?

3. $R_T = R_1 + R_2 + R_3 + \ldots + R_N = \sum_{n=1}^{N} R_n$ ______________________________ **(9.3)**

4. $V_x = V_T \dfrac{R_x}{R_T}$ ______________________________ **(9.4)**

Which of these laws and concepts are applicable to AC series circuits? Answer the following questions:

Does constant current throughout a series circuit change for any signal? ______________________________

Does KVL change for any signal? Why or why not? ______________________________

Are 3 and 4 above valid? Why or why not? ______________________________

If the same DC circuit analysis techniques are valid for AC, and resistances are used in DC circuit analysis, then what must be used in AC circuit analysis? Why?

State in words the circuit laws, concepts, and results for **AC series circuits**:

1. $\tilde{I}_1 = \tilde{I}_2 = \tilde{I}_3 = \ldots = \tilde{I}_N$ ______________________________ **(9.5)**

2. $\sum_{n=1}^{N} \tilde{V}_n = +\tilde{V}_{\text{rise1}} + \tilde{V}_{\text{rise2}} + \tilde{V}_{\text{rise3}} + \ldots - \tilde{V}_{\text{drop1}} - \tilde{V}_{\text{drop2}} - \tilde{V}_{\text{drop3}} - \ldots = 0$ ______________________________ **(9.6)**

How is a voltage polarity assigned with AC signals considering the polarity is alternating?

3. $\tilde{Z}_T = \tilde{Z}_1 + \tilde{Z}_2 + \tilde{Z}_3 + \ldots + \tilde{Z}_N = \sum_{n=1}^{N} \tilde{Z}_n$ ____ **(9.7)**

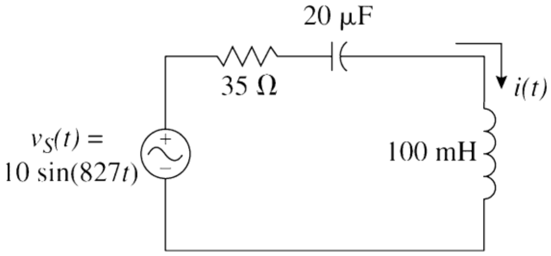
4. $\tilde{V}_x = \tilde{V}_T \frac{\tilde{Z}_x}{\tilde{Z}_T}$ ____ **(9.8)**

Example 9.1.1 (Explain each step.)

For the circuit shown in **Figure 9.2** below, determine (a) $\tilde{Z}_T$, (b) $\tilde{I}$, (c) $\tilde{V}_R$ and $\tilde{V}_L$ by the VDR, and (d) $\tilde{V}_C$ by Ohm's law. (e) Verify KVL.

______________: $v_S(t) = 10 \sin(827t)$ V
$R = 35\ \Omega$
$C = 20\ \mu F$
$L = 100$ mH

______________: a. $\tilde{Z}_T$
b. $\tilde{I}$
c. $\tilde{V}_R$ and $\tilde{V}_L$ by VDR
d. $\tilde{V}_C$ by Ohm's law
e. Verify KVL



Strategy:

Determine $\tilde{Z}_R$, $\tilde{Z}_L$, and $\tilde{Z}_C$

$$\tilde{Z}_T = \tilde{Z}_R + \tilde{Z}_C + \tilde{Z}_L$$

Label a circuit diagram with voltage polarities and current directions.

$$\tilde{I} = \frac{\tilde{V}_S}{\tilde{Z}_T}$$

$$\tilde{V}_R = \tilde{V}_S \frac{\tilde{Z}_R}{\tilde{Z}_T}, \quad \tilde{V}_L = \tilde{V}_S \frac{\tilde{Z}_L}{\tilde{Z}_T}$$

$$\tilde{V}_C = \tilde{I}\,\tilde{Z}_C$$

$$+\tilde{V}_S - \tilde{V}_R - \tilde{V}_C - \tilde{V}_L = 0$$

Solution:

$$\tilde{Z}_R = R = 35\,\Omega$$

$$\tilde{Z}_C = -j\frac{1}{\omega C} = -j\frac{1}{(827)(20\mu)} = -j60.460\,\Omega$$

$$\tilde{Z}_L = +j\omega L = +j(827)(0.1) = +j82.7\,\Omega$$

a. $\tilde{Z}_T = \tilde{Z}_R + \tilde{Z}_C + \tilde{Z}_L = 35 - j60.46 + j82.7 = 35.000 + j22.240\,\Omega = 35.0 + j22.2\,\Omega$

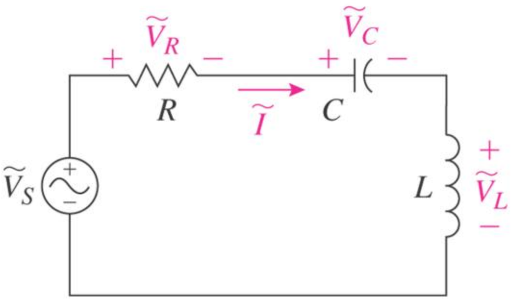
Note: All voltages and currents are expressed as peak values in this example. See **Figure 9.3** below.

b. $\tilde{I} = \frac{\tilde{V}_S}{\tilde{Z}_T} = \frac{10\angle 0°}{35 + j22.24} = 0.24115\angle -32.434° = 0.241\angle -32.4°\,\text{A}$

c. $\tilde{V}_R = \tilde{V}_S\frac{\tilde{Z}_R}{\tilde{Z}_T} = 10\angle 0°\frac{35}{35 + j22.24} = 8.4402\angle -32.433° = 8.44\angle -32.4°\ \text{V}$

Determine $\tilde{V}_L$ in a similar manner. Answer: $\tilde{V}_L = 19.943\angle +57.567° = 19.9\angle +57.6°\ \text{V}$

d. $\tilde{V}_C = \tilde{I}\,\tilde{Z}_C = (0.24115\angle -32.434°)(-j60.46)$
$= 14.579\angle -122.434° = 14.6\angle -122.4°\ \text{V}$



e. (Refer to Figure 9.3) $+\tilde{V}_S - \tilde{V}_R - \tilde{V}_C - \tilde{V}_L = 0$

Perform KVL.

Is KVL satisfied?

9.2 AC PARALLEL CIRCUITS

State the important laws, concepts, and results that were found for DC parallel circuits in words:

1. $V_S = V_1 = V_2 = V_3 = \ldots = V_N$ ____ **(9.9)**

2. $\sum_{n=1}^{N} I_n = +I_1 + I_2 + I_3 + \ldots + I_N = 0$ ____ **(9.10)**

How is the sign of the current assigned to if it is entering the node? Leaving the node?

3. $R_T = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \ldots + \dfrac{1}{R_N}}$ ________________________________ in general **(9.11)**

$R_T = R_1 \| R_2 = \dfrac{R_1 R_2}{R_1 + R_2}$ ____________________________ for two parallel resistors **(9.12)**

4. $I_x = \dfrac{I_T R_T}{R_x}$ ____ in general **(9.13)**

$I_1 = \dfrac{I_T R_2}{R_1 + R_2}$, $I_2 = \dfrac{I_T R_1}{R_1 + R_2}$ ________________________ for two parallel resistors **(9.14)**

5. $G = \dfrac{I}{V} = \dfrac{1}{R}$ ____ **(9.15)**

6. $G_T = G_1 + G_2 + \ldots + G_N$ ____ **(9.16)**

Which of these laws and concepts are applicable to AC series circuits? Answer the following questions:

Does constant voltage across a parallel circuit change for any signal?__________________________________

Does KCL change for any signal? Why or why not?____

Are 3 - 6 above valid? Why or why not?____

If the same DC circuit analysis techniques are valid for AC, and resistances are used in DC circuit analysis, then what must be used in AC circuit analysis? Why?

State in words the circuit laws, concepts, and results for **AC parallel circuits**:

1. $\tilde{V}_S = \tilde{V}_1 = \tilde{V}_2 = \tilde{V}_3 = \ldots = \tilde{V}_N$ ______________________________ **(9.17)**

2. $\sum_{n=1}^{N} \tilde{I}_n = +\tilde{I}_1 + \tilde{I}_2 + \tilde{I}_3 + \ldots + \tilde{I}_N = 0$ ______________________________ **(9.18)**

3. $\tilde{Z}_T = \dfrac{1}{\dfrac{1}{\tilde{Z}_1} + \dfrac{1}{\tilde{Z}_2} + \ldots + \dfrac{1}{\tilde{Z}_N}}$ (general or special case?) ______________________________ **(9.19)**

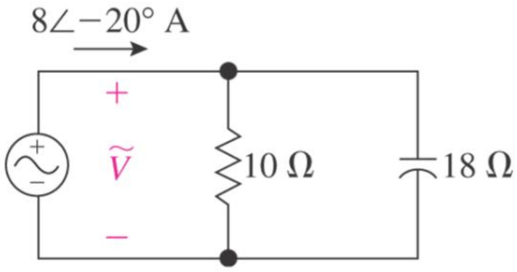
$\tilde{Z}_T = \tilde{Z}_1 \| \tilde{Z}_2 = \dfrac{\tilde{Z}_1 \tilde{Z}_2}{\tilde{Z}_1 + \tilde{Z}_2}$ (general or special case?) ______________________________ **(9.20)**

4. $\tilde{I}_x = \dfrac{\tilde{I}_T \tilde{Z}_T}{\tilde{Z}_x}$ (general or special case?) ______________________________ **(9.21)**

$\tilde{I}_1 = \dfrac{\tilde{I}_T \tilde{Z}_2}{\tilde{Z}_1 + \tilde{Z}_2}$, $\tilde{I}_2 = \dfrac{\tilde{I}_T \tilde{Z}_1}{\tilde{Z}_1 + \tilde{Z}_2}$ (general or special case?) ______________________________ **(9.22)**

Example 9.2.1 (Explain each step.)

For the circuit shown in **Figure 9.4** below, determine (a) $\tilde{Z}_T$, (b) $\tilde{V}$, (c) $\tilde{I}_R$ by the CDR, and (d) $\tilde{I}_C$ by Ohm's law. (e) Verify KCL.



________________: $R = 10\ \Omega$
$X_C = 18\ \Omega$
$\tilde{I}_T = 8\angle -20°$ A

________________: a. $\tilde{Z}_T$ T
b. $\tilde{V}$
c. $\tilde{I}_R$ by the CDR
d. $\tilde{I}_C$ by Ohm's law
e. Verify KCL

________________: Determine $\tilde{Z}_R$ and $\tilde{Z}_C$

$\tilde{Z}_T = \dfrac{\tilde{Z}_R \tilde{Z}_C}{\tilde{Z}_R + \tilde{Z}_C}$

Label a circuit diagram with voltage polarities and current directions.

$\tilde{V} = \tilde{I}_T \tilde{Z}_T$

$\tilde{I}_R = \dfrac{\tilde{I}_T \tilde{Z}_T}{\tilde{Z}_R}$ or $\tilde{I}_R = \dfrac{\tilde{I}_T \tilde{Z}_C}{\tilde{Z}_R + \tilde{Z}_C}$ (which equation is easier? note the previous step)

$$\tilde{I}_C = \frac{\tilde{V}}{\tilde{Z}_C}$$

Apply KCL to the "top" node.

Solution:

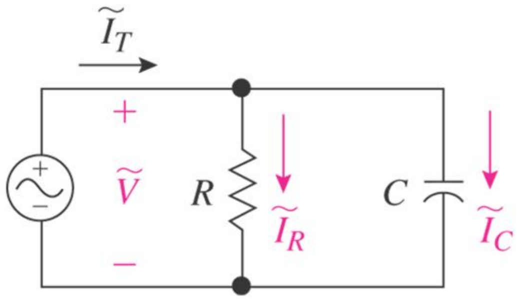
$\tilde{Z}_R = R = 10\,\Omega$

$\tilde{Z}_C = -jX_C = -j18\,\Omega$

a. $\tilde{Z}_T = \dfrac{\tilde{Z}_R \tilde{Z}_C}{\tilde{Z}_R + \tilde{Z}_C} = \dfrac{(10)(-j18)}{10 - j18} = 8.7416\angle -29.055° = 8.74\angle -29.1°\ \Omega$

b. See **Figure 9.5** below for voltage polarity and current direction assignments.

$$\tilde{V} = \tilde{I}_T \tilde{Z}_T = (8\angle -20°)(8.7416\angle -29.055°)$$
$$= 69.933\angle -49.055° = 69.9\angle -49.1°\ \text{V}$$



c. $\tilde{I}_R = \dfrac{\tilde{I}_T \tilde{Z}_T}{\tilde{Z}_R} = \dfrac{(8\angle -20°)(8.7416\angle -29.055°)}{10}$

$$= 6.9933\angle -49.055° = 6.99\angle -49.1°\ \text{A}$$

d. $\tilde{I}_C = \dfrac{\tilde{V}}{\tilde{Z}_C} = \dfrac{69.933\angle -49.055°}{-j18} = 3.8851\angle +40.945° = 3.89\angle +40.9°\ \text{A}$

e. KCL at _______________ node: $+\tilde{I}_T - \tilde{I}_R - \tilde{I}_C = 0$

Perform KCL:

Is KCL satisfied?

What is the inverse of resistance (G = 1/R)? ____

What is the inverse of reactance? $B = \frac{1}{X}$ ____ **(9.23)**

Identify: $B_L = \frac{1}{X_L}$ _________________________ $B_C = \frac{1}{X_C}$ _________________________ **(9.24)**

If susceptance is the inverse of reactance, what is the inverse of impedance? __________________________

$$\tilde{Y} = \frac{1}{\tilde{Z}}$$ **(9.25)**

Is $\tilde{Y}$ a complex number? Why or why not? ____

How does admittance $(\tilde{Y})$ relate to conductance (G) and susceptance (B)? Explain each step:

$\tilde{Z} = R \pm jX$ ____ **(9.26)**

Why is the plus sign used with inductive reactance?____

Why is the minus sign used with capacitive reactance? ____

$\tilde{Y} = G \pm jB$ ____ **(9.27)**

Conductance is the __________ part of admittance and susceptance is the ____________ part of admittance.

$\tilde{Y} = \frac{1}{\tilde{Z}} = \frac{\tilde{I}}{\tilde{V}} = \frac{I}{V}\angle(\theta_I - \theta_V)$ ____ **(9.28)**

Why is the sign minus used with inductive susceptance?____

Why is the plus sign used with capacitive susceptance? ____

What is the unit of admittance? _____________________________________ abbreviation? __________

5. $\tilde{Y} = \frac{\tilde{I}}{\tilde{V}} = \frac{1}{\tilde{Z}}$ (S) ____ **(9.29)**

6. $\tilde{Y}_T = \tilde{Y}_1 + \tilde{Y}_2 + \ldots + \tilde{Y}_N = \sum_{n=1}^{N} \tilde{Y}_n$ ____ **(9.30)**

Example 9.2.2

(Explain each step.)

For the circuit shown previously in **Figure 9.4** (repeated below), determine (a) $\tilde{Y}_T$ from G and B, and (b) $\tilde{Y}_T$ from $\tilde{Z}_T$.

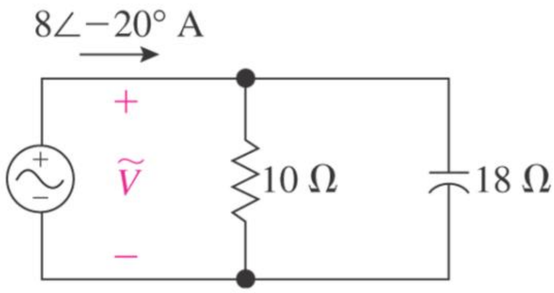
Given: R = 10 Ω
X_C = 18 Ω
$\tilde{Z}_T = 8.7416\angle -29.055°\ \Omega = 7.6415 - j4.24535\ \Omega$ (from Example 9.2.1)

Desired: a. $\tilde{Y}_T$ from G and B

b. $\tilde{Y}_T$ from $\tilde{Z}_T$

Strategy: a. $G = 1/R$, $B = 1/X$, $\tilde{Y}_T = G + jB$

b. $\tilde{Y}_T = 1/\tilde{Z}_T$



Solution:

a. $G = \frac{1}{R} = \frac{1}{10} = 0.1\,\text{S}$

$$B_C = \frac{1}{X_C} = \frac{1}{18} = 0.055556\,\text{S}$$

$$\tilde{Y}_T = G + jB = 0.1 + j0.05556 = 0.100 + j0.0556\,\text{S}$$

b. $\tilde{Y}_T = \frac{1}{\tilde{Z}} = \frac{1}{8.7416\angle -29.055^\circ} = 0.114396\angle +29.055^\circ$

$$= 0.0999993 + j0.055556 = 0.100 + j0.0556\,\text{S}$$

Why do the results from parts (a) and (b) match?

Observation about the impedance–admittance relationship: start with $\tilde{Z}_T$ of the circuit in Figure 9.4 (repeated above). Explain each step:

$$\tilde{Z}_T = 7.6415 - j4.24535\,\Omega = R - jX$$

Consider:

$$G = \frac{1}{R} = \frac{1}{7.6415} = 0.13086 = 0.131\,\text{S} \quad \textbf{(9.31)}$$

$$B_C = \frac{1}{X_C} = \frac{1}{4.24535} = 0.23555 = 0.236\,\text{S} \quad \textbf{(9.32)}$$

Compare these results with those of Example 9.2.2:

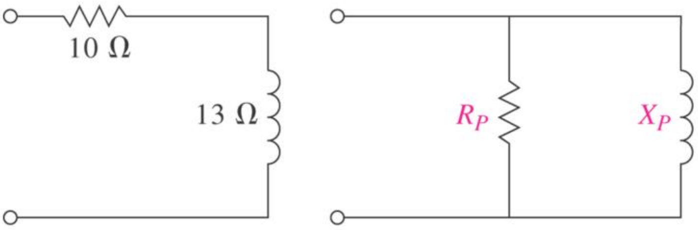
$$G = \frac{1}{R} = \frac{1}{10} = 0.1 = 0.100\,\text{S} \quad \textbf{(9.33)}$$

$$B_C = \frac{1}{X_C} = \frac{1}{18} = 0.055556 = 0.0556\,\text{S} \quad \textbf{(9.34)}$$

These answers do not match! Why not? Which approach is correct?

Example 9.2.3 (fill in)

Determine the (a) total equivalent admittance and (b) parallel equivalent circuit for the series circuit shown in **Figure 9.6a** (below).



a. Series circuit b. Equivalent parallel circuit

________________: $R_S = 10\ \Omega$
$X_S = 13\ \Omega$ (inductive)

________________: a. $\tilde{Y}_T$
b. R_P and X_P

________________: a. $\tilde{Y}_T = \dfrac{1}{\tilde{Z}_T}$, express in rectangular form
b. $R_P = 1/G_p$, and $X_P = 1/B_P$

Solution: (fill in)

Answers: a. $\tilde{Y}_T = 0.037175 - j0.048327\ \text{S} = 0.0372 - j0.0483\ \text{S}$ b. $R_P = 26.9\ \Omega$, $X_P = 20.7\ \Omega$

Example 9.2.4 (fill in)

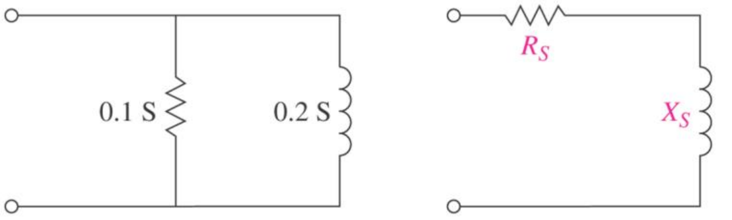
Determine the resistance and reactance values of the series equivalent circuit for the parallel circuit shown in **Figure 9.7a** (below).

Given:

Desired:

Strategy:

Solution:



a. Parallel circuit b. Equivalent series circuit

Answer: $R_S = 2.000\ \Omega$; $X_S = 4.000\ \Omega$

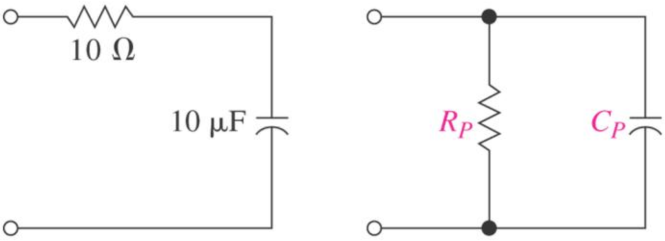
Example 9.2.5 (fill in)

Determine the component values for the parallel equivalent circuit of the series circuit shown in **Figure 9.8a** (below) if ω = 20 krads/s.

Given:

Desired:

Strategy:



a. Series circuit b. Equivalent parallel circuit

Solution:

Answer: $R_P = 12.5\,\Omega$, $C_P = 2.00\,\mu\text{F}$

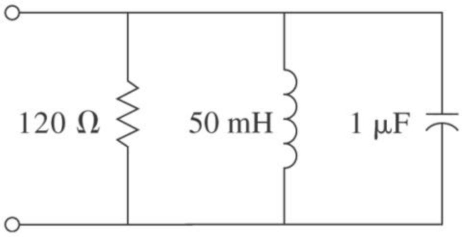
Example 9.2.6 (fill in)

For the circuit in **Figure 9.9** (below), determine (a) the total equivalent admittance, and (b) the values for the components of the series equivalent circuit. The frequency is 1 kHz.

Given:

Desired:

Strategy: a.



$$\tilde{Y}_T = \tilde{Y}_R + \tilde{Y}_C + \tilde{Y}_L = G_P + jB_C - jB_L = \frac{1}{R_P} + j\omega C - j\frac{1}{\omega L}$$

b. $\tilde{Z}_T = \frac{1}{\tilde{Y}_T} = R_S + jX_S,\ X_S \to C$

Solution:

Answers: a. $\tilde{Y}_T = (8.3333 + j3.1001)\text{ mS} = (8.33 + j3.10)\text{ mS}$

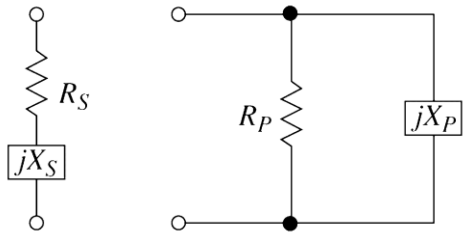
b. $\tilde{Z}_T = 105.41 - j39.215\,\Omega$, R_S = 105 Ω; X_S = 39.215 Ω (capacitive), $C_S = 4.06\,\mu\text{F}$

Why are there only two reactive components in the series equivalent circuit?

Generalization of Series–Parallel Conversions to "Handbook" Equations

(Explain each step.)

Case 1: Series-to-Parallel Conversion—see Fig. 9.10:



$$Z_T = R_S \pm jX_S,\ Y_T = G_P \mp jB_P \quad \textbf{(9.35)}$$

$$\tilde{Y}_T = \frac{1}{(R_S \pm jX_S)} \quad \textbf{(9.36)}$$

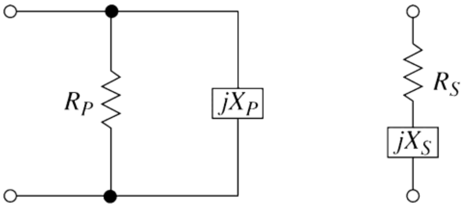
$$\tilde{Y}_T = \frac{1}{(R_S \pm jX_S)} \cdot \frac{(R_S \mp jX_S)}{(R_S \mp jX_S)} = \frac{R_S \mp jX_S}{R_S^2 + X_S^2} \quad \textbf{(9.37)}$$

$$\tilde{Y}_T = \frac{R_S}{R_S^2 + X_S^2} \mp j\frac{X_S}{R_S^2 + X_S^2} \quad \textbf{(9.38)}$$

$$G_P = \frac{R_S}{R_S^2 + X_S^2},\quad \boxed{R_P = \frac{R_S^2 + X_S^2}{R_S}} \quad \textbf{(9.39)}$$

$$\mp jB_P = \mp\frac{jX_S}{R_S^2 + X_S^2},\quad \boxed{X_P = \frac{R_S^2 + X_S^2}{X_S}} \quad \textbf{(9.40)}$$

Case 2: Parallel-to-Series Conversion—see Fig. 9.11:



$$\tilde{Z}_T = R_P \,\|\, (\pm jX_P),\ \tilde{Z}_T = R_S \pm jX_S \quad \textbf{(9.41)}$$

$$\tilde{Z}_T = \frac{R_P\,(\pm jX_P)}{(R_P \pm jX_P)} \quad \textbf{(9.42)}$$

$$\tilde{Z}_T = \frac{R_P\,(\pm jX_P)}{(R_P \pm jX_P)} \cdot \frac{(R_P \mp jX_P)}{(R_P \mp jX_P)} = \frac{\pm jX_P R_P^2 \pm jX_P\,(\pm jX_P) R_P}{R_P^2 + X_P^2} \quad \textbf{(9.43)}$$

$$\tilde{Z}_T = \frac{R_P X_P^2 \pm jR_P^2 X_P}{R_P^2 + X_P^2} = \frac{R_P X_P^2}{R_P^2 + X_P^2} \pm j\frac{R_P^2 X_P}{R_P^2 + X_P^2} \quad \textbf{(9.44)}$$

$$\boxed{R_S = \frac{R_P\, X_P^2}{R_P^2 + X_P^2}} \quad \textbf{(9.45)}$$

$$\pm jX_S = \pm j\frac{R_P^2 X_P}{R_P^2 + X_P^2},\quad \boxed{X_S = \frac{R_P^2 X_P}{R_P^2 + X_P^2}} \quad \textbf{(9.46)}$$

9.3 AC Series–Parallel Circuits

KVL, total equivalent impedance, the VDR, and the fact that the current is the same are generally applied to what type of circuit?

KCL, total equivalent impedance and admittance, the CDR, and the fact that the voltage is the same are generally applied to what type of circuit?

The following general guidelines for DC ***series–parallel*** circuit analysis are extended to AC:

1. The *general approach* to analyzing an AC series–parallel circuit is to identify the groups of ____________ that are in series and those that are in parallel and to apply the appropriate series and parallel circuit laws to the impedances in that group.

2. There are usually multiple approaches to analyzing an AC series–parallel circuit.
   - *think* about your strategy → formulate an *efficient* approach

3. A single-source series–parallel circuit is fundamentally a series circuit or fundamentally a parallel circuit.
   - The source is the key.
   - If a single impedance is in series with the source on either side (or both sides) of the source, then the circuit is fundamentally a ______________________ circuit.
   - If the current path from the source divides on both sides of the source, then the circuit is fundamentally a ________________________ circuit.

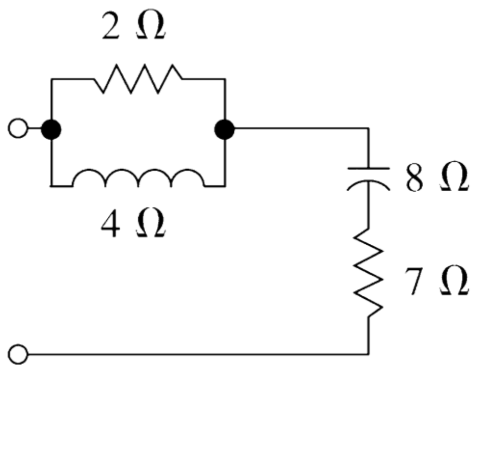
4. A group of *series* impedances, usually a "leg" in a parallel circuit, is called a _____________________.

5. In the VDR and the CDR, the total voltage or current becomes the total voltage or the current for that group of impedances, not necessarily the entire circuit.

Example 9.3.1 (Explain each step.)
Determine the total equivalent admittance of the circuit in **Figure 9.12** (below).
Given: circuit in Figure 9.12

Desired:

Strategy: $\tilde{Y}_1 = \frac{1}{2} + \frac{1}{j4}, \ \tilde{Z}_T = \frac{1}{\tilde{Y}_1} - j8 + 7, \ \tilde{Y}_T = \frac{1}{\tilde{Z}_T}$



Solution:

$\tilde{Y}_1 = \frac{1}{2} + \frac{1}{+j4} = 0.5 - j0.25 \text{ S}$ (identify group 1)

$$\tilde{Z}_T = \frac{1}{0.5 - j0.25} - j8 + 7 = 8.6000 - j7.2000\,\Omega$$

$$\tilde{Y}_T = \frac{1}{\tilde{Z}_T} = \frac{1}{8.6 - j7.2} = 0.08916\angle + 39.936°$$
$$= 0.0892\angle + 39.9° \text{ S} = 0.0684 + j0.0572 \text{ S}$$

Example 9.3.2 (Explain each step.)

Determine (a) $\tilde{I}_x$ and (b) $\tilde{V}_o$ for the circuit shown in **Figure 9.13** (below).

Given: circuit in Figure 9.13

Desired: a. $\tilde{I}_x$

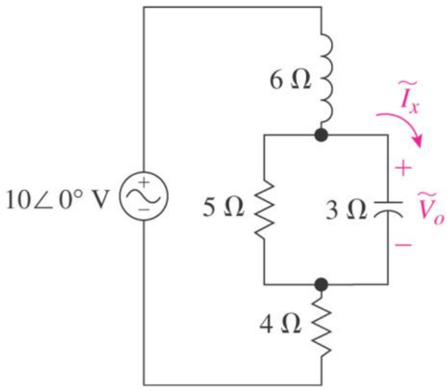
b. $\tilde{V}_o$

Strategy: $\tilde{Z}_x = [5 \| (-j3)],\ \tilde{Z}_T = +j6 + \tilde{Z}_x + 4$ (identify $\tilde{Z}_x$)

$\tilde{I}_T = \dfrac{\tilde{V}_S}{\tilde{Z}_T}$

$\tilde{V}_o = \tilde{I}_T \tilde{Z}_x$

$\tilde{I}_x = \dfrac{\tilde{V}_o}{\tilde{Z}_C}$



Solution:

$$\tilde{Z}_x = \frac{(5)(-j3)}{5-j3} = 2.5725\angle -59.036°\ \Omega$$

$$\tilde{Z}_T = +j6 + 2.5725\angle -59.036° + 4 = 6.5372\angle +35.478°\ \Omega$$

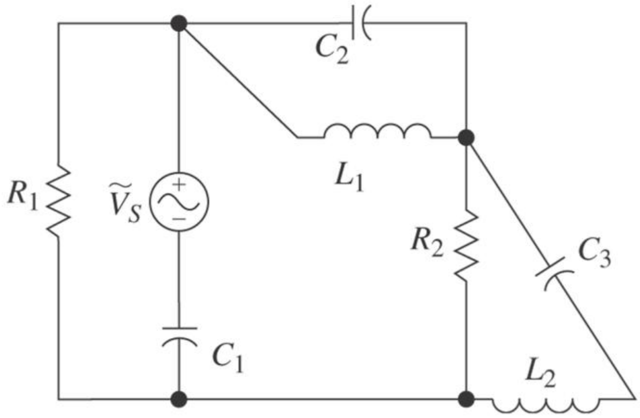
$$\tilde{I}_T = \frac{\tilde{V}_S}{\tilde{Z}_T} = \frac{10\angle 0°}{6.5372\angle +35.478°} = 1.5297\angle -35.478°\ \text{A}$$

$$\tilde{V}_o = \tilde{I}_T \tilde{Z}_x = (1.5297\angle -35.478°)(2.5725\angle -59.036°)$$
$$= 3.9352\angle -94.514° = 3.94\angle -94.5°\ \text{V}$$

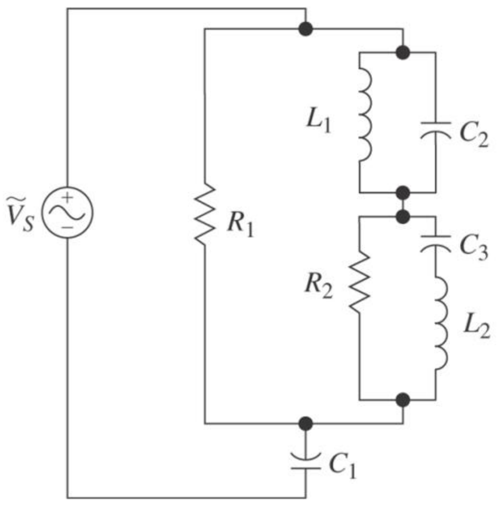
$$\tilde{I}_x = \frac{\tilde{V}_o}{\tilde{Z}_C} = \frac{3.9352\angle -94.514°}{3\angle -90°} = 1.3117\angle -4.514°\ \text{A} = 1.31\angle -4.5°\ \text{A}$$

Example 9.3.3

Redraw the circuit in **Figure 9.14** (below), clearly showing series and parallel groups of components.

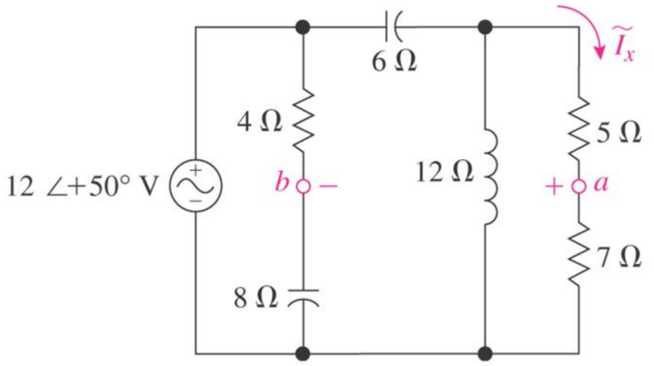


Solution: (Try to redraw the circuit first, then check it with the answer in **Figure 9.15** on the next page.)



Example 9.3.4 (fill in steps)

Determine (a) $\tilde{I}_x$ and (b) $\tilde{V}_{ab}$ for the circuit shown in **Figure 9.16** (below).



Given: circuit in Figure 9.16

Desired: a. $\tilde{I}_x$

b. $\tilde{V}_{ab}$

Strategy: Redraw and label the circuit (next to Fig. 9.16)

-
-
-

Solution: (use separate paper)

Answers: a. $\tilde{I}_x = 1.41\angle +95.0°$ A b. $\tilde{V}_{ab} = 12.1\angle +152.4°$ V

9.4 ANALYSIS OF MULTIPLE-SOURCE AC CIRCUITS USING SUPERPOSITION

Consider AC series–parallel circuits that contain *several sources* as well as several impedances.

What condition on the sources must hold true if superposition with phasors is to be used? Why? (Hint: reactances)

In superposition the circuit is analyzed one source at a time. The other sources are ________________________.

How does one deactivate an AC voltage source? Why does it guarantee 0 V?

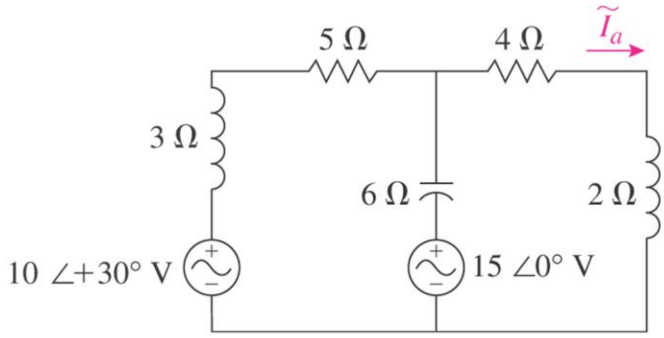
How is an AC current source deactivated? Why does it guarantee 0 A?

Write the general superposition procedure

This last step is actually the superposition step: the phasor voltages and currents are *superposed.*

Example 9.4.1

Determine the current $\tilde{I}_a$ and $i_a(t)$ in **Figure 9.18** (below).
Peak values for sources are shown.



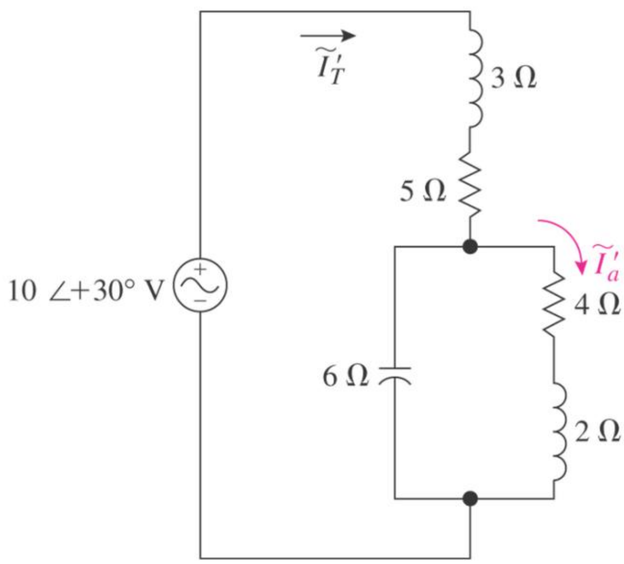
Given: circuit in Figure 9.18

Desired: $\tilde{I}_a$ and $i_a(t)$
$\tilde{I}_a$ uses the peak value of $i_a(t)$.

Strategy: superposition

Solution:

Deactivate the 15 V source; redraw the circuit (see **Figure 9.19** below).



Explain each step:

substrategy: $\tilde{Z}'_T = j3 + 5 + [-j6 \,||\, (4 + j2)]$

$$\tilde{I}'_T = \frac{10\angle +30^\circ}{\tilde{Z}'_T}$$

CDR $\rightarrow \tilde{I}'_a$

$$\tilde{Z}'_T = 5 + j3 + \cfrac{1}{\cfrac{1}{-j6} + \cfrac{1}{4 + j2}}$$

$$= 5 + j3 + 4.7434\angle -18.435^\circ$$

$$= 9.6177\angle +8.973^\circ\ \Omega$$

$$\tilde{I}'_T = \frac{10\angle +30^\circ}{9.6177\angle +8.973^\circ} = 1.0398\angle +21.027^\circ\text{ A}$$

$$\tilde{I}'_a = \frac{(1.0398\angle +21.027^\circ)(6\angle -90^\circ)}{(-j6) + (4 + j2)} = 1.1028\angle -23.973^\circ\text{ A}$$

Fill in the rest of the solution steps on separate paper:

Deactivate the 10 V source; redraw the circuit; determine the substrategy; determine $\tilde{I}''_a$ and $\tilde{I}_a$.

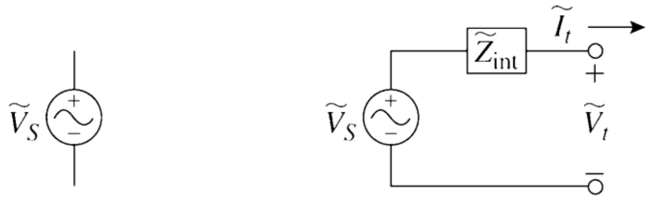
Answers: $\tilde{I}''_a = 1.6076\angle +66.991^\circ$ A , $\tilde{I}_a = 1.9342\angle +32.234^\circ = 1.93\angle +32.2^\circ$ A , $i_a(t) = 1.93 \sin(\omega t + 32.2^\circ)$ A

AC sources

The properties of ideal AC sources are

- *An ideal AC voltage source has a constant*_________________ *and can deliver any* ______________
- *An ideal AC current source delivers a constant* __________________ *at any* ___________________

Explain a practical AC voltage source *model* shown in **Figure 9.21b** (below).



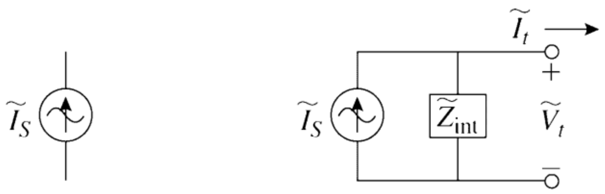
a. Ideal voltage source b. Model of a practical voltage source

How was the following equation obtained?_______________________

$$\tilde{V}_t = \tilde{V}_S - \tilde{V}_{Z_{\text{int}}} = \tilde{V}_S - \tilde{I}_t \tilde{Z}_{\text{int}} \qquad \textbf{(9.47)}$$

What does the previous equation predict about the terminal voltage of a practical AC source?

Explain a practical AC current source *model* shown in **Figure 9.22b** (below).



a. Ideal current source b. Model of a practical current source

How was the following equation obtained?_______________________

$$\tilde{I}_t = \tilde{I}_S - \tilde{I}_{Z_{\text{int}}} = \tilde{I}_S - \frac{\tilde{V}_t}{\tilde{Z}_{\text{int}}} \qquad \textbf{(9.48)}$$

What does the previous equation predict about the terminal current of a practical AC source?

Which quantity was not calculated in this chapter that was determined in previous chapters (conspicuous by its absence)?

Learning Objectives

Discussion: Can you perform each learning objective for this chapter? (Examine each one.)

As a result of successfully completing this chapter, you should be able to:

1. Utilize phasors in AC series–parallel circuit voltage and current calculations.
2. Describe the fundamental properties of series and parallel circuits and subcircuits.
3. Calculate all voltages and currents in single-source AC series, parallel, and series–parallel circuits.
4. Perform series–parallel circuit conversions.
5. Describe why and how superposition is used in multiple-source AC series, parallel, and series–parallel circuits.
6. Calculate all voltages and currents in multiple-source AC series, parallel, and series–parallel circuits.
7. Explain what an AC current source is and how to analyze AC series–parallel circuits that contain a current source.